Initial data of black hole localized on Karch-Randall brane

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Abstract

Based on the conjectured duality in the Karch-Randall braneworld models, it is suggested that a static brane-localized black hole larger than bulk curvature length does exist in this model. It is also suggested that the Hawking-Page transition of the four-dimensional black hole may be reproduced as a change in the shape of five-dimensional black hole in the bulk. In order to test these suggestions, we try to construct time-symmetric initial data of brane-localized black hole in the Karch-Randall braneworld model and study its properties. I will explain the method of the analysis and illustrate expected results.

1 Introduction

The Randall-Sundrum II (RS-II) model [1] is a brane world model, which provides a way to realize our four-dimensional world in a higher-dimensional spacetime. This model is composed of five-dimensional bulk spacetime with negative cosmological constant and a four-dimensional brane with positive tension. Weak gravitational field on the brane obeys the usual four-dimensional Newton law with a correction suppressed at a large distance from the gravitational source [1, 2], though the extra-dimension extends infinitely in this model. If we weaken the brane tension a little, a small negative cosmological constant appears in the effective Einstein gravity on the four-dimensional brane, and the spacetime on the brane becomes AdS_4. It is known that there is an “almost” massless graviton even in this model, and four-dimensional gravity is realized if length scale of interest is shorter than the curvature scale on the brane. This model with an AdS brane is called the Karch-Randall (KR) model [3]. Since four-dimensional gravity is realized effectively in these models, it is difficult to distinguish them from ordinary four-dimensional models as long as we investigate only in the weak gravity regime. Essential features of these models will appear in strong gravitation phenomena on them. In order to assess viability of these models observationally, we should study how these models behave when strongly gravitation exists in them.

Now, consider a gravitational collapse on the brane in each of these models for an example of such strong gravitation phenomena. Naively thinking, a static black hole (BH) whose horizon is localized near the brane will form as a final state of this collapse, since ordinary four-dimensional gravity is realized on the brane. There are exact static solutions of black objects, which are black strings (BS) in those models [4,5], and they also are candidates of the black object which is formed after the collapse. However, it seems unlikely that a BS is formed as a result of gravitational collapse, since it is singular at the bulk AdS horizon and also unstable due to so-called Gregory-Laflamme instability [6]. Thus, a brane-localized BH seems to be most likely to form after the collapse on the brane.

Contrary to this expectation of brane-localized BH formation, such an exact static solution has not been discovered yet though much effort has been devoted into this issue (e.g. Ref. [7]). Numerical solution of a static brane-localized BH was constructed when the horizon size is not much larger than the bulk curvature scale, but the construction becomes harder as the horizon size becomes larger [8, 9]. A work on brane-localized BH issue [10] reports that there are some problems in numerical solution construction even when the brane-localized BH is smaller than the bulk curvature scale. These facts do not exclude the possibility that a static solution of brane-localized BH larger than the bulk curvature scale does exist, but we do not have any strong evidence of its existence. As an explanation of the lack of static solution, there is a conjecture that brane-localized static BHs larger than bulk curvature scale do not exist in RS-II model based on the AdS/CFT correspondence in this model [11, 12].

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There are several works related to this conjecture (e.g. [13,14]), but no definite conclusion is obtained yet. It is desirable to investigate the properties of static BH solution directly in order to test the validity of this conjecture, but it is technically difficult to construct a static large BH solution numerically. Thus, we consider time-symmetric initial data which have a brane-localized apparent horizon (AH), expecting that their properties may give some insight into the brane-localized BH.

We have done this initial data construction and analysis of their properties in [15]. It was confirmed in this work that there exists an initial data with an arbitrariry large blane-localized BH. We analyzed their themodynamic properties by comparing this initial data with a black string solution which have the same four-dimensional ADM mass. This analysis revealed that such a large brane-localized BH have smaller entopy than than the black string of the same mass has, and thus it seems to be unstable and tend to evolve into some other black object. This tendency changes as we decrease the size of the brane-localized BH; when it is smaller than the bulk curvature scale, it have more entropy than the black string has. This result is consistent that a static stable brane-localized BH does exists when it is smaller than the bulk curvature scale, and it becomes unstable when it is larger than that scale.

This time, we do a similar analysis on the KR model. One of the differences from the RS-II model is the behavior of warp factor in the bulk; in the RS-II model, it just decreases down to zero as we move into the bulk from the brane; in KR model, it first decreases, but it begins to grow again after we passed over a critical point which is at a distant of the bulk curvature scale. The effective potential for a small particle behaves in a similar manner to the warp factor. If we put a small black hole at that minimum point of the effective potential, it will stay there stably. At this stage, no black object is visible from an observer on the brane; radiation field of nonzero temperature is induced on the brane by that BH, and the brane observer will see only a lump of this thermal radiation.

Now, let us gradually increase the size of that BH which sits at that potential minimum. It will keep to stay there as long as its size is smaller than the bulk curvature scale. A curious phase transition occurs when its size become comparable to the bulk curavature scale: the BH touches the brane, and it becomes observable from an observer on the brane. By the way, in the four-dimensional AdS spacetime, it is known that such a phase transition occurs between thermal AdS phase and AdS BH phase. This phenomenon is known as the Hawking-Page transition [16]. We conjecture that the transition of the bulk BH shape to be a holographic dual of the Hawking-Page transition on the brane [11]. In order to check this conjecture, we construct initial data of bulk BHs and brane-localized BHs in the KR model, and test the thermodynamic property of those BHs.

2 Initial data construction method

In this section, we introduce a construction method of time-symmetric initial data with a BH in the KR model. This model is composed of two copies of five-dimensional empty bulk with negative cosmological constant Λ, and they are separated by a $\mathbb{Z}_2$-symmetric positive tension brane. The tension of the brane is given by $\ = 3k(1+\delta)/4\pi G_5$ with $k = \sqrt{-\Lambda/6}$, where G_5 is the five-dimensional gravitational constant. The parameter δ describes how a brane tension is detuned from that of the RS-II model; when $\delta = 0$, this model reduce to the RS-II model with a Minkowski brane. Four-dimensional cosmological constant is given by $\Lambda_{4\mathrm{D}} = 3k^2\left(2\delta + \delta^2\right)$, thus a negative cosmological constant appears when $\delta < 0$. In this work, we assume $\delta < 0$. The initial data we consider have $O(3)$-symmetry in the spacelike dimension as well as the symmetry with respect to time reversal. These symmetries are property shared with static brane-localized BH solutions. Hence, we think it appropriate to restrict our attention to this class of initial data.

The starting point of our construction procedure is to choose an asymptotically AdS vacuum solution of the Einstein equations with negative cosmological constant Λ. In this study, we use the well-known AdS-Schwarzschild solution. The metric is given by

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2\left(d\chi^2 + \sin^2\chi d\Omega_{\mathrm{II}}\right), \quad U(r) = 1 + k^2r^2 - \frac{\ }{r^2}, \tag{1}$$

where $d\Omega_{\mathrm{II}}$ is the line element on a unit S^2, and is the mass parameter of the AdS-Schwarzschild BH. The spacetime described by this metric is asymptotically AdS, and has an spherical event horizon at $r = r_g$ where $U(r)$ vanishes. In the following discussion, we set k to unity by rescaling the unit of length. In this sense, this background spacetime has only one free parameter , which becomes one of free parameters of the initial data.

We put a vacuum brane with $\mathbb{Z}_2$-symmetry in this AdS BH bulk. We denote the unit normal of the brane by $\tilde{s}$, and take this $\tilde{s}$ in the direction toward the bulk from the brane. We introduce the induced metric $\tilde{\gamma} \equiv g - \tilde{s}\ \tilde{s}$ on the brane. The extrinsic curvature $\tilde{K}_{ab}$ on the brane is defined by $\tilde{K}_{ab} = -\tilde{\gamma}_a\ \tilde{\gamma}_b\ \nabla\ \tilde{s}$. Here Latin indices starting from the beginning of the alphabet $(a, b, \cdots)$ run the four-dimensional coordinates on the brane. A vacuum brane has the four-dimensional energy-momentum tensor localized on the brane given by $T_{ab} = -\ \tilde{\gamma}_{ab}$. Israel's junction condition [17] on the brane is given by $\tilde{K}_{ab} - \tilde{K}\tilde{\gamma}_{ab} = \frac{1}{2} \cdot 8\pi G_5 T_{ab}$, where we used $\mathbb{Z}_2$-symmetry across the brane. At the moment of the time-reversal symmetry, we only have to solve the Hamiltonian constraint, which is the (t, t)-component of the junction condition. Using the normal vector $\tilde{s}$, this equation is written as

$$D_i \tilde{s}^i = -3k\,(1 + \delta)\,, \tag{2}$$

where D_i is a covariant differenciation with respect to the induced metric $\tilde{\gamma}_{ij}$. Assuming $O(3)$-symmetry of the brane, we specify the brane trajectory by $(r, \chi) = (r_b(\xi), \chi_b(\xi))$, where ξ is the proper radial length along the brane. The spacelike unit normal $\tilde{s}$ is given by

$$\tilde{s} = \left(\sqrt{U} r_{\rm b} \chi_{\rm b}', -\frac{r_{\rm b}'}{\sqrt{U} r_{\rm b}} \right). \tag{3}$$

Then the Hamiltonian constraint (2) becomes a second order ODE of $r_b(\xi)$ and $\chi_b(\xi)$. We solve Eq. (2) numerically to obtain the brane trajectory. In this way, we can construct a system of a brane and a BH which floats on it. We can change the brane position with respect to the bulk BH, and also we can change the mass parameter of the bulk BH as well as the detune paramter δ of the brane tension. Namely, we can construct a three-parameter family of the initial data. We note here that we can construct an initial data which have a BH localized on the brane, not one floating in the bulk.

3 Analysis method

In order to study thermodynamic properties of these BHs in the KR model, we have to calculate mass and entropy of them. When we studied the system with a flat brane, we calculated four-dimensional ADM mass measured on the brane and five-dimensional BH horizon area, and conducted a thermodynamic analysis using them. We obtained some pieces of evidence that brane-localized BHs larger than the bulk curvature scale are unstable. We would like to do a similar analysis using BH initial data on the KR model, but there is a difficulty in the calculation of mass. In the RS-II model, the localization of the four-dimensional graviton is perfect, and the four-dimensional ADM mass is expected to be an appropriate quantity to characterize the system. On the other hand, the bulk of the KR model largely opens up to the AdS boundary, and then the localization of the graviton is not perfect; the lowest Kaluza-Klein mode have very small mass, and since it behaves differently from zero mass graviton when it propagates a long length. In order to calculate mass in this system with imperfectly localized gravitation, we have to calculate five-dimensionally defined mass, not the mass which is four-dimensionally defined on the brane. However, there are no definite way to define five-dimensional mass in the KR model.

In [18], a way to define mass in AdS spacetime is proposed. They construct a conserved quantity which is expressed as a surface integral of metric perturbation. Our proposal is to use this definition to calculate mass in the KR model. We regard pure AdS_5 spacetime as background spacetime, calculate metric perturbation which is induced by the brane and the BH, and then calculate mass by integrating the metric perturbation on some surface which is located near spatial infinity.

When we applied this Abbott-Deser definition of mass to the KR model, we found that the mass defined in this way is conserved only when we keep to take the Gaussian-Normal coordinates. Thus, the mass defined in this way is coordinate dependent. It is not so clear that this mass satisfies the laws of thermodynamics, but we expect that we can obtain some information of the system using mass defined in this way.

Consider a family of initial data for a fixed value of mass. There are initial data of two types in this family: one with a BH floating in the bulk, and one with a BH localized on the brane. In this two-parameter family of the initial data for a fixed mass, we can find one which extremizes the entropy. We expect this initial data to be one with a floating BH in the bulk when the temperature on the brane is low, and to be one with a brane-localized BH when the temperature is above a critical temperature which is determined by the curvature scale on the brane.

From this analysis, we will obtain useful information about the holography in RS-II and KR models. Based on this result, we would like to tackle more difficult problems toward proof of the classical evaporation conjecture. Especially, we would like to simulate a time-evolving brane-localized black hole using numerical relativity technique. This analysis will clarify the behavior of BH solutions on these models, and will provide essential knowledge about the holography on these braneworld models.

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