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Aspects of noncanonical inflation

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Abstract. This paper reviews recent work by the authors on inflation in single-field models with noncanonical (higher-order) kinetic terms. Such terms arise naturally in an EFT approach to modelling inflation, and also from certain string theory embeddings, such as in DBI inflation. Theories containing such terms can support a period of noncanonical inflation which is smoothly connected to the usual slow-roll regime. We give a general formalism for finding solutions of the equations of motion corresponding to noncanonical inflation. However, not every Lagrangian with higher-power kinetic operators supports noncanonical inflation, and not every set of initial conditions in a suitable theory will give rise to an inflationary trajectory. We give some sufficient conditions required for a Lagrangian to support inflation in the non-canonical regime. Further, we investigate the problem of fine-tuning in these models. We find that noncanonical kinetic terms tend to reduce the severity of the fine-tuning problem in general, as long as a regime of noncanonical inflation is present.

1. Noncanonical inflation

1.1. Inflation as an effective field theory

Inflation is an appealing solution to the homogeneity, isotropy, and flatness problems, and is well supported by observation as providing the dominant mechanism for structure formation. For these reasons inflation is widely believed to have taken place , and has been the subject of intense research. However, because the UV-complete theory in which it should take place is as yet unknown, all meaningful discussion is necessarily at the effective-field-theory (EFT) level.

The EFT description is valid only up to some energy cutoff Λ, with the physics above this scale being integrated out. The resulting higher-dimensional operators are suppressed by powers of Λ, and can usually be safely neglected. However, the EFT can be sensitive to the UV physics, since in inflationary theories the self-coupling of the inflaton is small.

One well-known example is the η problem: dimension 6 corrections of the form $\frac{\mathcal{O}_6}{M_p^2} \sim \frac{\mathcal{O}_4}{M_p^2}\phi^2$, where ϕ is the inflaton, can spoil the flatness of the potential, leading to $\mathcal{O}(1)$ corrections to the slow-roll parameter $\eta = M_p^2 \frac{V''}{V}$ when $\mathcal{O}_4$ has a VEV comparable to the inflationary energy density. Inflation, which depends on η being small, can be spoilt by these corrections. This is a generic problem for supergravity models of inflation in which the F-term potential is nonzero (see e.g. [1]).

1.2. Noncanonical kinetic terms

We saw above that the underlying UV theory can affect the potential; it can also lead to important corrections to the kinetic terms. Consider the following two-field toy model:

$$\mathcal{L} \;=\; \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\rho)^2 + \frac{\rho}{M}(\partial\phi)^2 - \frac{1}{2}M^2\rho^2.$$

At energy scales $H << M$, the heavy field ρ can be integrated out. Because of its coupling to ϕ, a higher-order kinetic term for ϕ results: $\mathcal{L}_{eff} = \frac{1}{2}(\partial\phi)^2 + \frac{(\partial\phi)^4}{M^4}$. The DBI action, which contains exactly such higher-order kinetic terms in a closed-form expression, can be analogously understood as arising when massive degrees of freedom (W-bosons) are integrated out [2].

Such couplings and massive degrees of freedom will be generically present in any UV-complete description of inflation. Thus noncanonical kinetic terms are generically expected to be present. While remaining completely ignorant of the correct UV-complete description, we can ask what the effect of these noncanonical kinetic terms will be. We consider general single-field noncanonical Lagrangians, by which we mean Lagrangians of the form

$$S \;=\; \int d^4x\sqrt{-g_4}\left[\frac{M_p^2}{2}\mathcal{R}_4 + p(X,\phi)\right], \tag{1.1}$$

where $p(X,\phi)$ is a general function of ϕ and the canonical kinetic term $X = \frac{1}{2}(\dot{\phi})^2$. It is consistent to ignore higher derivative corrections of the form $\partial^n\phi$, as shown in [3]. Lagrangians of the form $\mathcal{L}_{eff} = p(X,\phi)$, with $p(X,\phi) \to 0$ as $X \to 0$ and zero potential energy have been shown to support non-slow roll inflation, and are known as k-inflation models [4]. These have been studied before [5, 6, 7, 8], but no general formalism for finding inflationary solutions to the equations of motion was given. In the following section we outline the formalism for finding general inflationary solutions to Lagrangians of the form (1.1), in which both kinetic and potential energy terms enter and play a role, and which encompass "noncanonical" inflation (NCI) as well as the canonical slow-roll solutions. In Section 1.4 we give conditions which a given Lagrangian must satisfy in order to guarantee that it supports a period of NCI. The work in Sections 1.3 and 1.4 was published in [3].[1]

Models of NCI can give rise to an observable amount of non-gaussianity in the CMB, making them especially interesting. The leading order non-gaussianity from noncanonical single-field models is of the equilateral type [6], with $f_{NL}^{equil} \sim c_s^{-2} = (1 + 2Xp_{X}X/p_X)$ a function of how far we are from the canonical limit. Current constraints are fairly weak ($|f_{NL}| \lesssim \mathcal{O}(100)$; $c_s \gtrsim 0.1$)[9, 10, 11], but are expected to improve with more sensitive data in the near future.

1.3. Noncanonical inflationary solutions

To find solutions of noncanonical inflation, we first generalise the canonical slow-roll parameters and then invert the equations of motion in the limit that our new generalised inflationary parameters are small. This procedure reproduces the inflationary solutions of canonical slow-roll inflation in the canonical limit (see [3] for details). By definition, H must be approximately constant on inflationary trajectories. This condition defines ϵ, as in the canonical case:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \;=\; -\frac{d}{dN}\log H << 1 \tag{1.2}$$

[1] We consider only Lagrangians which obey the null-energy condition, i.e. $\frac{\partial p}{\partial X} \geq 0$, but our formalism can also be extended to Lagrangians with negative coefficients for the higher-dimensional operators in some region of the phase space. In this case we reproduce the results of [4] (along with subleading corrections) and furthermore find an additional inflationary parameter which must be small and is not seen in the analysis of [4].

Furthermore, in order for inflation to last long enough, the time variation of ϵ must be small over several e-foldings:

$$\eta = \frac{\dot{\epsilon}}{H\epsilon} \;=\; 4\epsilon - \eta_X - \eta_\Pi << 1, \tag{1.3}$$

where

$$\eta_X \;\equiv\; \epsilon - \frac{\dot{X}}{2HX;} \quad \eta_\Pi \;\equiv\; \epsilon - \frac{\dot{\Pi}}{H\Pi}.$$

Rewriting the EOM for $\epsilon, \eta_X, \eta_\Pi$ small, we find the **general non-canonical inflationary solutions**

$$p_{inf} \;=\; -\rho_{inf}\left(1 + \frac{2}{3}\epsilon\right) \approx -\rho_{inf} \tag{1.4}$$

$$\Pi_{inf}(\phi) \;=\; \frac{\partial p}{\partial \phi}\frac{1}{3H}\left(1 + \frac{\epsilon - \eta_\Pi}{3}\right)^{-1} \approx \frac{\partial p}{\partial \phi}\frac{1}{3H}. \tag{1.5}$$

Here p_X indicates a derivative with respect to X, $\rho \equiv 2Xp_X - p$, and $\Pi \equiv \frac{\partial p}{\partial \dot{\phi}} = \dot{\phi}\frac{\partial p}{\partial X}$. The inflationary trajectories described by these solutions can be shown to be attractors in both the small and large field case for all models of noncanonical inflation, including the canonical limit of small and large field inflation [3]. Note that (1.4) and (1.5) reduce to the canonical solution in that limit, while $\epsilon \to M_p^2/2(V'/V)^2$ and $\eta_X, \eta_\Pi \to M_p^2 V''/V$. Whether we are in the canonical or non-canonical regime depends on the parameter

$$A \;=\; \frac{V'}{3H}\frac{1}{\Lambda^2} \;=\left(\frac{2}{3}\epsilon_{SR}\frac{V}{\Lambda^4}\right)^{\frac{1}{2}}; \tag{1.6}$$

the limit $A \ll 1 \Rightarrow \epsilon_{SR} \ll 1$ corresponds to canonical inflation, while in the case $A \gg 1 \Rightarrow \epsilon_{SR} \geq 1$ inflation will be highly non-canonical. The inflationary parameters are suppressed by powers of A compared to the usual slow-roll results: $\epsilon, \eta_X \approx \frac{\epsilon_{SR}}{A^m}$ and $\eta_\Pi \approx \frac{\eta_{SR}}{A^m}$.

1.4. Conditions for noncanonical inflation
Not all Lagrangians of the form (1.1) support NCI. A detailed discussion of the conditions sufficient for a Lagrangian to have non-canonical inflationary solutions is given in [3]; here we give a brief summary. First, we impose two physicality constraints, requiring that the action satisfy the null-energy condition $\frac{\partial p}{\partial X} \geq 0$, and that superluminal propagation of perturbations be disallowed, i.e. $\frac{\partial^2 p}{\partial X^2} > 0$. We begin by assuming that the Lagrangian can be written in the separable form

$$p(X, \phi) \;\approx \Lambda^4 q\left(\tfrac{X}{\Lambda^4}\right) - V(\phi). \tag{1.7}$$

For power series Lagrangians with a nonzero radius of convergence $X = R$, NCI will be supported within the radius of convergence if A and V/Λ^4 are large and $\partial p/\partial X$ diverges at R. For a closed-form Lagrangian we must have $A, V/\Lambda^4 \gg 1$ as well as a positively curved kinetic term $\partial^2 q/\partial X^2 > 0$. This condition is just the condition hat the speed of propagation of perturbations be real and subluminal, which we have already imposed. If a Lagrangian is not separable, noncanonical inflationary solutions exist only in the limit where the coefficients of X^n do not depend strongly on ϕ. These conditions are illuminating, since they make it clear that the existence of a speed limit on ϕ, seen in DBI inflation for instance, is neither necessary nor sufficient for the existence of NCI solutions [3].

2. Initial Conditions

2.1. The Initial Conditions Fine Tuning Problem

We now know how to tell if a given Lagrangian supports NCI, how to find the general inflationary solutions, and that inflationary trajectories are attractors in phase space [3]. However, not all initial conditions give rise to trajectories that reach the inflationary attractors. Trajectories with an arbitrarily large initial momentum can miss the attractor altogether, a situation known as the initial conditions fine-tuning (ICFT) or overshoot problem. Typically, if $\Delta\phi$ is the distance the inflaton travels during inflation, small-field ($\Delta\phi \ll M_p$) models have an ICFT problem while large-field ($\Delta\phi \gg M_p$) models do not [12]. In this section we summarise the effect of a period of NCI on the ICFTP, finding that trajectories generically decay towards the inflationary solution at a steeper angle for noncanonical kinetic terms, compared to canonical kinetic terms, so that a larger fraction of the initial-coniditons space leads to inflation. Thus noncanonical kinetic terms can reduce the ICFT problem in small-field inflation models, when NCI is relevant in the allowed phase space. This work was published in [13].

2.2. Regional Analysis

To begin with, we divide phase space into regions of interest, depending on the dominant terms in the EOM: for

$$\rho = K.E + P.E; \qquad \dot{\Pi} = -3H\Pi + V'(\phi), \tag{2.8}$$

we define

- **Region A**: $H\Pi \gg V'$ and K.E. $\gg V(\phi)$.
- **Region B**: $H\Pi \ll V'$ and K.E. $\ll V(\phi)$
- **Region C**: $H\Pi \gg V'$ and K.E. $\ll V(\phi)$
- **Region D**: $H\Pi \ll V'$ and K.E. $\gg V(\phi)$

as shown in Figure 1. [2] The inflationary trajectory, which exists for only part of the phase space range, is indicated by the dashed line Γ. Notice how Regions A and B shrink when going to the non-canonical case. The dot-dashed line $\Pi = \Lambda^2$ corresponds to $A = 1$; inflation below this line is non-canonical while inflation above this line is noncanonical. Arrows denote the general direction of flow of trajectories, and are generally steeper for non-canonical kinetic terms. The trend is clearly visible for the specific example, shown in Figure 2, of an inflection-point potential (with parameters chosen to be consistent with current observations of the power spectrum and spectral index). Previously overshooting trajectories for a canonical kinetic term are attracted strongly to the inflationary solution. In Section 2.3 we will study this behaviour analytically.

2.3. Parametrising the dynamical behaviour

We can analytically calculate the angle the trajectory makes in phase space:

$$\tan\theta = \tfrac{1}{M_p}\tfrac{d\Pi}{d\phi} = \frac{1}{M_p}\frac{\dot{\Pi}}{\dot{\phi}}; \tag{2.9}$$

$$\tan\theta_{canon} = -\tfrac{\dot{\Pi}}{\Pi M_p} = \frac{3H\Pi - V'}{\Pi M_p}; \tag{2.10}$$

$$\tan\theta_{non-canon} = -\tfrac{\dot{\Pi}}{M_p\sqrt{2X}} = \frac{3H\Pi - V'}{\sqrt{2R}M_p\Lambda^2}\,. \tag{2.11}$$

[2] Note that in this section, Π has an additional minus sign compared to the usual definition because we work in only one quadrant of phase space where $\phi \geq 0$ and $\dot{\phi} \leq 0$ and it is convenient to choose this to be the purely positive (first) quadrant where $\phi, \Pi \geq 0$.

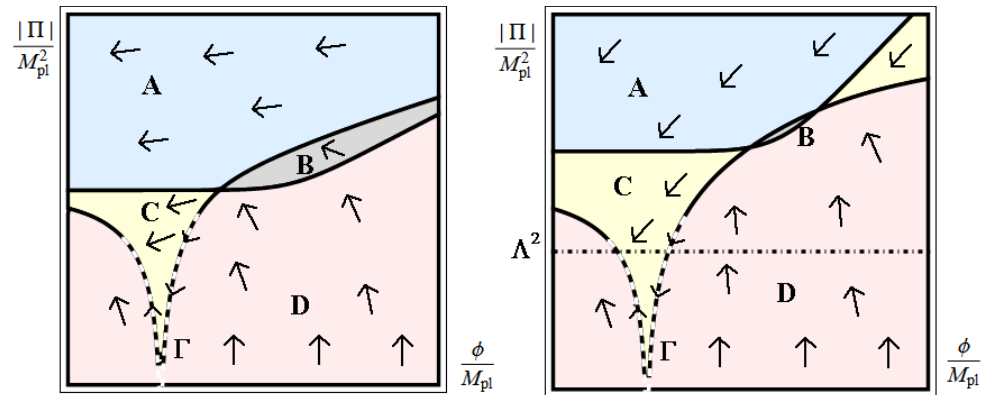

Figure 1. Regions A-D of (ϕ, Π) phase space for an inflection-point type potential with canonical (left) and non-canonical (right) kinetic terms.

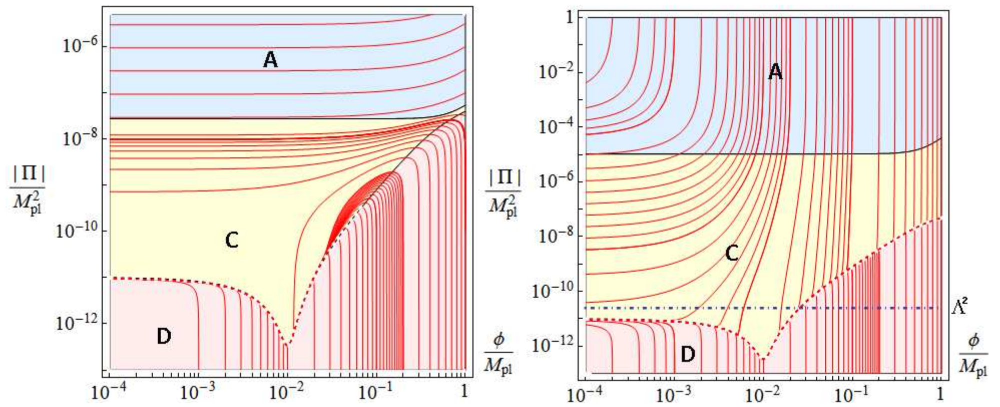

Figure 2. Left: Phase-space plot for a canonical Lagrangian with an inflection-point potential. Right: Phase-space plot for the non-canonical Lagrangian with the same inflection-point potential.

By this definition we find, as expected, that trajectories in Region A are approximately horizontal in the canonical case. These are the overshoot trajectories. In all regions it follows from (2.9)-(2.11) that **non-canonical trajectories are steeper**:

$$\frac{\tan\theta_{non-can}}{\tan\theta_{canon}} \gg 1. \qquad (2.12)$$

Furthermore, we can parametrise the amount of overshoot using the overshoot parameter α, defined as the ratio of the typical distance a trajectory must travel in order to exit Region A, $(\Delta\phi)_A$, to the size of field space in which inflation occurs, $(\Delta\phi)_{inf}$:

$$\alpha \equiv \frac{(\Delta\phi)_A}{(\Delta\phi)_{inf}}. \qquad (2.13)$$

When $\alpha \ll 1$, there is no fine-tuning problem, while for $\alpha \gg 1$, fine tuning will be necessary to avoid overshoot. Advantages of using α to parametrise the fine-tuning are that it can be calculated analytically and is independent of potential. Calculating $(\Delta\phi)^{canon}_A$ (by integrating $\frac{d\Pi}{d\phi}\Big|_A = \sqrt{\frac{3}{2}\frac{\Pi}{M_p}}$) gives $(\Delta\phi)^{canon}_A \sim M_p$ for canonical models, independent of the potential. This makes explicit why small-field models, for which $(\Delta\phi)_{inf} \ll M_p \sim (\Delta\phi)_A$, have a fine-tuning problem while large-field models, for which $(\Delta\phi)_{inf} \sim \mathcal{O}(10-100) \times M_p$, do not. Similarly, $(\Delta\phi)_A^{noncanon}$ can be computed for noncanonical inflationary models. Comparing to canonical kinetic terms, the ratio $\alpha_{non-canon}/\alpha_{canon}$ is found to be

$$\frac{\alpha_{non-canon}}{\alpha_{canon}} \sim \frac{(2R)^{1/2}}{2\sqrt{3}}\sqrt{\frac{\Lambda^4}{V}}, \tag{2.14}$$

confirming that the overshoot problem is dramatically reduced by the presence of noncanonical kinetic terms exactly when $A \sim \sqrt{V/\Lambda^4}$ is large, i.e. the theory is strongly noncanonical. This confirms our intuition from the plots above.

3. Conclusion

In this article we have reviewed the results of [3] and [13] on the nature of noncanonical inflationary solutions. The results can be summarised as follows: we have given a general formalism for constructing NCI soliutions in terms of the generalised inflationary parameters ϵ, η_X and η_Π, which smoothly reduce to the usual SR parameters in the canonical limit. We have given (sufficient) conditions for constructing Lagrangians which support NCI. We have also analysed the effect of noncanonical kinetic terms on the dynamical behaviour in (homogeneous) phase space, finding that trajectories decay towards the inflationary attractor at a steeper angle in the noncanonical case. Thus noncanonical kinetic terms tend to reduce the severity of the overshoot problem, parametrised by the overshoot parameter α, as long as some period of NCI occurs in the allowed phase space of the theory.

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