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# Landauer's Princple for Fermionic Fields in One-Dimensional Bags

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**Abstract:** In recent years, growing interest has been paid to the exploration of the concepts of entropy, heat and information, which are closely related to the symmetry properties of the physical systems in quantum theory. In this paper, we follow this line of research on the the validity of the concepts in quantum field theory by studying Landauer's principle for a Dirac field interacting perturbatively with an Unruh–DeWitt detector in a $1+1$-dimensional MIT bag cavity. When the field is initially prepared in the vacuum state, we find that the field always absorbs heat, while the Unruh–DeWitt detector can either gain or lose entropy, depending on its motion status, as a result of the Unruh effect. When the field is initially prepared in the thermal state and the detector remains still, the heat transfer and entropy change can be obtained under two additional but reasonable approximations: (i) one is where the duration of the interaction is turned on for a sufficiently long period, and (ii) the other is where the Unruh–DeWitt detector is in resonance with one of the field modes. Landauer's principle is verified for both considered cases. Compared to the results of a real scalar field, we find that the formulas of the vacuum initial state differ solely in the internal degree of freedom of the Dirac field, and the distinguishability of the fermion and anti-fermion comes into play when the initial state of the Dirac field is thermal. We also point out that the results for a massless fermionic field can be obtained by taking the particle mass $m \to 0$ straightforwardly.

**Keywords:** Landauer's principle; fermionic field; quantum thermodynamics



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## 1. Introduction

One of the pillars of the current knowledge about the building blocks of the universe is quantum field theory, which gives an almost perfect description of the interaction of microscopic particles and is the theoretical foundation of many branches of modern physics, such as particle physics, condensed-matter physics and quantum optics. Conventionally, the study of quantum field theory is concentrated on the computation of scattering spectra and the exploration of the symmetries of the fundamental forces, while less attention is paid to quantum states. However, some of the conceptual puzzles that lie at the heart of quantum physics require us to examine quantum information concepts like measurement theory and entanglement at the quantum field theory level [1–6] requiring the knowledge of the explicit forms of the quantum states of the quantum fields with which the validity of concepts like entropy, heat and information in quantum field theory can be examined, which are also related to the symmetry properties of the theory. One of the most important applications of this direction is the exploration of the nature of black hole entropy, whose corresponding micro-states are believed to be quantum in nature, thus calling for a deeper and thorough discussion about the relationship between gravity, quantum field theory, information theory and thermodynamics.

The research on the information aspects of quantum field theory is diverse and has already led to many astonishing results, such as the violation of Bell's inequality by the quantum vacuum [7–10] and the lack of a self-consistent theory of measurement of quantum fields, resulting in the establishment of the entanglement harvesting protocol [11–15]. Landauer's principle, considered to be a bridge that connects the concepts of information theory and thermodynamics, states that at least $k_B T \ln 2$ energy is required to erase one bit of information [16,17], providing a new perspective for exploring the black hole information paradox [18,19]. Unfortunately, there has been no convincing conception of Landauer's principle to date. In 2013, in their work [20], Reeb and Wolf proposed a tightened Landauer's principle in the framework of the quantum statistical physics approach. They stated that the tightened Landauer's principle takes the following form of the inequality equation:

$$\Delta Q \geq T_R \Delta S, \quad (1)$$

where $\Delta Q$ is the heat transferred to the reservoir, $T_R$ is the initial temperature of the reservoir, and $\Delta S$ is the von Newmann entropy change of the system interacting with the reservoir when four assumptions are met [20]. Equation (1) looks very different from the conventional statement of Landauer's principle and the textbook definition of heat transfer. This is because Equation (1) is a law of quantum thermodynamics based on quantum information concepts.

The implication of Landauer's principle in quantum field theory has recently emerged as a topic of discussion. In [21,22], the authors discussed and verified Landauer's principle in a qubit–cavity system composed of an Unruh–DeWitt detector linearly coupled to a real scalar field in the $1+1$ dimension through monopole momentum. Theoretically speaking, the applicability of Landauer's principle should be irrelevant to the specific type of field. And it is well known that, in quantum field theory, physical fields can be classified into two kinds, namely, fermion fields and boson fields, where the former are considered to be the building blocks of the matter world, while the latter are the carriers of interaction forces. Thus, it is natural to think that Landauer's principle must also hold for fermionic fields; even the techniques for this generalization may not be as trivial as one may think. Experimentally speaking, the realization and control of the cavity boundary, as well as the interaction between a two-level detector and an electromagnetic field, can be conducted in a precise and mature manner; replacing the electromagnetic field with a real scalar field in theoretical models captures all the physical essentials and greatly simplifies the calculation. For a fermionic field, the baryon boundary and four-fermion interaction are possible candidates for realizing the boundary condition of a cavity and linear coupling with detectors, which provides a platform for studying Landauer's principle for fermion fields [11,23–26].

In this paper, we study an Unruh–DeWitt detector traveling through the Dirac field in a $1+1$-dimensional cavity, where the cavity boundaries are modeled using the MIT bag boundary condition [25,26]. The Dirac field is linearly coupled to the Unruh–DeWitt detector, and the coupling strength is set to be small, thus allowing for the use of perturbation theory. We obtain the heat transfer and entropy change when the Dirac field is initially prepared in vacuum and thermal states. The results of the thermal states are computed under two additional assumptions: one is where the interaction is turned on for a sufficiently long period, and the other is where the detector is tuned in resonance with one of the field modes. By making a comparison with the results obtained for a real scalar field [21], we find that, for the vacuum initial state, the difference in heat transfer and entropy change only comes from the internal degree of freedom of the Dirac field, and the detector can either gain or lose entropy, where the former is a result of the fermion version of the Unruh effect [27]. For the thermal initial state, the difference between a real scalar field and the Dirac field also comes from the distinguishability of the fermion and anti-fermion. We point out that the results for the massless fermionic field can be obtained by taking the $m \to 0$ limit of our results straightforwardly. Landauer's principle is verified in all the cases considered in this paper. This paper is organized as follows: In Section 2, we demonstrate the setup of our system and provide the basic formula that is later used. The heat transfer and entropy change when the Dirac field is initially prepared in the vacuum state are given in Section 3, where we also obtain the results for an accelerating detector and discuss the role played by the Unruh effect. In Section 4, when the Dirac field is initially prepared in the thermal state, we obtain the heat transfer and entropy change under the assumption that the interaction is turned on for a sufficiently long period and the detector is in resonance with one of the field modes. The conclusion and discussion are presented in Section 5.

Throughout this paper, the units are chosen as $\hbar = c = k_B = 1$.

## 2. Model Setup

In this work, the field equation that we consider is the Dirac equation in the $1+1$ dimension:

$$(i\gamma_\mu \partial^\mu - m)\psi(t,x) = 0, \tag{2}$$

where $m$ is the fermion mass, and $\mu = 0, 1$. The spacetime metric is $g_{\mu\nu} = diag(1, -1)$, and the representation $\gamma_0 = \sigma_1, \gamma_1 = i\sigma_3, \gamma_5 = \sigma_2$ is used, where $\sigma_\alpha, \alpha = 1, 2, 3$ denotes the Pauli matrices.

Unlike the boson field, the Dirac field cannot be perfectly confined by the Dirichlet boundary condition, resulting from the famous Klein paradox [26]. As such, we apply the so-called MIT bag boundary condition [28]:

$$(e^{i\theta\gamma_5} + in^\mu \gamma_\mu)\psi(t,x)|_\Sigma = 0, \tag{3}$$

where $\Sigma$ represents the boundaries of the cavity, which, in this case, are the points $x = 0$ and $x = L$, with $L$ being the length of the cavity. $n_\mu$ is the normal vector of the boundary, and $\theta$ is an arbitrary real number. Under the MIT bag boundary condition of Equation (3), the Dirac field can be mode-expanded as

$$\psi(t,x) = \sum_n b(k_n) e^{-i\omega_n t} f_n(x) + d^\dagger(k_n) e^{i\omega_n t} h_n(x), \tag{4}$$

where $n$ is the mode number, and $b(k_n)$ and $d^\dagger(k_n)$ are the annihilation operator of the fermion and the creation operator of the anti-fermion of mode $n$, respectively. And $\{f_n(x), h_n(x)\}$ are the normalized bases of the solution space of the Dirac Equation (3), which is given as [28]

$$\begin{aligned} f_n(x) &= N_n \begin{pmatrix} \frac{\omega_n}{k_n} \sin(k_n x) \\ \cos(k_n x) + \frac{m}{k_n} \sin(k_n x) \end{pmatrix}, \\ h_n(x) &= N_n \begin{pmatrix} -\frac{\omega_n}{k_n} \sin(k_n x) \\ \cos(k_n x) + \frac{m}{k_n} \sin(k_n x) \end{pmatrix}, \\ N_n &= \sqrt{2} k_n^2 [k_n^2 (m + 2L\omega_n^2) + m\omega_n^2 \sin^2(k_n L)]^{-\frac{1}{2}}, \end{aligned} \tag{5}$$

where $N_n$ is the normalization constant, and the mode frequency is defined as

$$\omega_n = \sqrt{k_n^2 + m^2}, \tag{6}$$

with which the field modes can be obtained from the transcendental equation [28]

$$\frac{m}{k_n} \sin(k_n L) + \cos(k_n L) = 0. \tag{7}$$

Note that, by taking $m \to 0$, we obtain the spectra of the massless fermionic field straightforwardly.

The total Hamiltonian of our system consists of three parts, namely, $H = H_f + H_D + H_{int}$, where $H_f$, $H_D$ and $H_{int}$ denote the Hamiltonian of the free Dirac field inside the cavity, the free Unruh–DeWitt detector and the interaction between the detector and the Dirac field, respectively. Note that the linear interaction term $H_{int}$ takes the same form as the one used in the entanglement harvest setup in [11] and, in some sense, can be regarded as an effective theory of four-fermion interactions [23,24]. In the interaction picture, $H_f$, $H_D$ and $H_{int}$ take the forms of

$$\begin{aligned} H_f &= \sum_n \omega_n [b^\dagger(k_n)b(k_n) + d^\dagger(k_n)d(k_n)], \\ H_D &= \Omega\sigma_3, \\ H_{int} &= \lambda(e^{i\Omega t}\sigma_+ \bar{\psi}[x(t)]\Lambda(t) + e^{-i\Omega t}\sigma_- \bar{\Lambda}(t)\psi[x(t)]), \end{aligned} \tag{8}$$

where $\bar{\psi}(t,x) = \psi^\dagger(t,x)\gamma^0$, $\Omega$ is the frequency of the detector, and $\lambda$ is the coupling strength. $\sigma_3$ and $\sigma_\pm$ are the Pauli matrix and corresponding ladder operators acting on the Hilbert space of the detector. The smear function is given by $\Lambda(t) = \eta\chi(t)$, where $\eta$ is a spinor, which is taken as $\eta = (\Gamma, 0)^T$ in the sequel. $\Gamma$ is a parameter determined by the energy scale of the fermion [11], which is set to 1 in the following. And the switching function $\chi(t)$ turns the interaction on and off; specifically, $\chi(t) = 1$ when $0 < t < T$ and vanishes otherwise. The trajectory of the Unruh–DeWitt detector is given by $x(t)$.

The evolution operator can be expressed as

$$U(t) = \mathcal{T} \exp\left(-i \int_0^t dt' H_{int}(t')\right), \tag{9}$$

where $\mathcal{T}$ is the time-ordering symbol. Then, (9) is expanded in the Dyson series

$$\begin{aligned} U(T) = I &+ (-i)\int_0^T dt H_{int}(t) + (-i)^2 \int_0^T dt \int_0^t dt' H_{int}(t)H_{int}(t') \\ &+ (-i)^3 \int_0^T dt \int_0^t dt' \int_0^{t'} dt'' H_{int}(t)H_{int}(t')H_{int}(t'') + \cdots, \end{aligned} \tag{10}$$

where $I$ denotes the unitary matrix, and (10) can be organized by the order of $\lambda$ as $U(t) = U^{(0)}(t) + U^{(1)}(t) + U^{(2)}(t) + \mathcal{O}(\lambda^3)$, where $U^{(i)}(t)$ denotes the $i$-th order of the expansion. The initial density matrix $\rho_0$ evolves according to

$$\rho(t) = U(t)\rho_0 U^\dagger(t). \tag{11}$$

Thus, the final density matrix $\rho(T)$ can be expanded with respect to $\lambda$ as $\rho(T) = \rho^{(0)}(T) + \rho^{(1)}(T) + \rho^{(2)}(T) + \mathcal{O}(\lambda^3)$, where

$$\begin{aligned} \rho^{(0)}(T) &= \rho_0, \\ \rho^{(1)}(T) &= U^{(1)}(T)\rho_0 + \rho_0 U^{(1)\dagger}(T), \\ \rho^{(2)}(T) &= U^{(2)}(T)\rho_0 + U^{(1)}(T)\rho_0 U^{(1)\dagger}(T) + \rho_0 U^{(2)\dagger}(T). \end{aligned} \tag{12}$$

In general, the $n$-th order term $\rho^{(n)}(T)$ is a summation of terms such as $U^{(p)}(T)\rho_0 U^{(q)\dagger}(t)$ with $p + q = n$.

## 3. Vacuum State

In this section, we study the situation where the Dirac field is initially prepared in the vacuum state $|0\rangle$; this is also a thermal state corresponding to the condition $T_R = 0$, where $T_R$ denotes the temperature of the field, which acts as the reservoir in our case. The initial state of the whole system can be written as $\rho_0 = [(1-p)|-\rangle\langle-| + p|+\rangle\langle+|] \otimes |0\rangle\langle 0|$, where $|+\rangle$ and $|-\rangle$ correspond to the excited and ground states of the Unruh–DeWitt detector, respectively. $p$ is a real number, and $0 \le p \le 1$. Thus, the initial state $\rho_0$ is a product state of the Dirac field and detector, where the Dirac field is in a pure vacuum state, while the detector is in a mixed state, with the possibility of being $p$ in the excited state and the possibility of being $1-p$ in the ground state. The heat transferred to the Dirac field and the entropy change of the detector are given by [20]

$$\begin{aligned}\Delta Q &= tr[H_f(\rho_{T,f}-\rho_{0,f})],\\ \Delta S &= S(\rho_{0,D})-S(\rho_{T,D}),\end{aligned} \tag{13}$$

where $\Delta Q$ is the heat transferred to the Dirac field, and $\rho_{T,f}$ and $\rho_{0,f}$ are the final and initial density matrices of the Dirac field, respectively. $\Delta S$ is the von Newmann entropy change of the Unruh–DeWitt detector, with $\rho_{0,D}$ and $\rho_{T,D}$ being the initial and final density matrices of the detector. From (8), we can see that only the diagonal elements of the corresponding reduced density matrix of the Unruh–DeWitt detector and Dirac field contribute to $\Delta Q$ and $\Delta S$. Since $U^{(1)}(t)$ is made up of terms like $\sigma_+ d, \sigma_+ b^\dagger, \sigma_- b$ and $\sigma_- d^\dagger$, its action upon $\rho_0$ only gives rise to non-diagonal elements in the reduced density matrices of the detector and Dirac field, which means that $\rho^{(1)}(t)$ does not contribute to the heat transfer $\Delta Q$ and entropy change $\Delta S$.

Now, we consider the second-order contribution $\rho_T^{(2)}$. With (8) and (10), we obtain

$$\begin{aligned}U^{(1)}(T)|-\rangle|0\rangle &= \lambda\int_0^T dt e^{i(\Omega+\omega_n)t}\bar{f}_n[x(t)]\eta|+\rangle|1_n\rangle,\\ U^{(1)}(T)|+\rangle|0\rangle &= \lambda\int_0^T dt e^{i(\omega_n-\Omega)t}\bar{f}_n[x(t)]\eta|-\rangle|\bar{1}_n\rangle,\end{aligned} \tag{14}$$

where $|1_n\rangle, |\bar{1}_n\rangle$ denotes one fermion and the anti-fermion state of mode $n$, respectively. Thus,

$$\begin{aligned}U^{(1)}(T)\rho_0 U^{(1)\dagger}(T) &= \lambda^2\sum_n\Big((1-p)|W_n|^2|+\rangle\langle+|\otimes|1_n\rangle\langle 1_n| + p|V_n|^2|-\rangle\langle-|\otimes|\bar{1}_n\rangle\langle\bar{1}_n|\Big),\\ U^{(2)}(T)\rho_0 = \rho_0 U^{(2)\dagger}(T) &= -\frac{\lambda^2}{2}\sum_n\Big((1-p)|W_n|^2|-\rangle\langle-| + p|V_n|^2|+\rangle\langle+|\Big)\otimes|0\rangle\langle 0|.\end{aligned} \tag{15}$$

where the notations

$$\begin{aligned}W_n &= \int_0^T dt e^{-i(\Omega+\omega_n)t}\bar{\eta} f_n[x(t)],\\ V_n &= \int_0^T dt e^{i(\Omega-\omega_n)t}\bar{h}_n[x(t)]\eta\end{aligned} \tag{16}$$

are used. Thus, the density matrices of the field are obtained,

$$\begin{aligned}\rho_{T,f} = &\Big(1-\lambda^2\sum_n[(1-p)|W_n|^2+p|V_n|^2]\Big)|0\rangle\langle 0|\\ &+\lambda^2(1-p)\sum_n|W_n|^2|1_n\rangle\langle 1_n| + \lambda^2 p\sum_n|V_n|^2|\bar{1}_n\rangle\langle\bar{1}_n|,\end{aligned} \tag{17}$$

as well as the reduced density matrix of the detector

$$\rho_{T,D} = (1-p-\delta p)|-\rangle\langle-| + (p+\delta p)|+\rangle\langle+|, \tag{18}$$

where

$$\delta p = \lambda^2\sum_n[(1-p)|W_n|^2 - p|V_n|^2]. \tag{19}$$

Then, we obtain the heat transferred to the Dirac field and the entropy change of the detector as

$$\Delta S = \lambda^2 \ln \frac{1-p}{p} \sum_n [p|V_n|^2 - (1-p)|W_n|^2],$$
$$\Delta Q = \lambda^2 \sum_n [(1-p)|W_n|^2 + p|V_n|^2]\omega_n. \quad (20)$$

From (20), we can see that, in this case, the difference between the heat transfer and entropy growth of the real scalar field and Dirac field solely comes from the internal degree of freedom of the Dirac field from the factors $\bar{\eta} f_n[x(t)]$ and $\bar{h}_n[x(t)]\eta$ in the integrand (16). And from (20), we can see that the heat transfer is always non-negative, which is in agreement with the intuition that, when in the ground state of $H_f$, the Dirac vacuum can only absorb energy, while the entropy of the detector can either grow or decrease. In this case, Landauer's principle is always fulfilled since $T_R = 0$.

For an accelerating detector with a constant proper acceleration $a$ in its proper frame, the trajectory is given by

$$t(\tau) = \frac{1}{a} \sinh(a\tau),$$
$$x(\tau) = \frac{1}{a}(\cosh(a\tau) - 1), \quad (21)$$

where $\tau$ is the proper time of the detector. $\Delta Q$ and $\Delta S$ are plotted in Figure 1. The decrease in $\Delta S$ can be understood with the following picture. From the perspective of the detector, it is immersed in the thermal state of the quantum field, as indicated by the Unruh effect. Thus, it is possible for the detector to drain energy from the Dirac field. Together with (20), we can see that both the Dirac field vacuum and the detector absorb energy. From the perspective of an observer in the lab, this phenomenon comes from the process of the excition of the detector accompanied by the emission of the particle, which is intuitively thought to be banned by the energy conservation law. However, this does not occur in our considered case because the emitted particles cause the recoil of the detector. So to keep the acceleration (21), an external force must be applied to the detector, thus pouring energy into the system. To see this in a mathematical form, we can substitute (21) into (8), and it is easy to check that the Hamiltonian is time-dependent, i.e., $\frac{dH}{dt} \neq 0$. Note that this does not violate the assumptions introduced in [20], where the Hilbert space of the whole system is still the direct product of the Dirac field and the detector, and the evolution of the system is still described by unitary evolution, because the external force is applied in a classical manner.

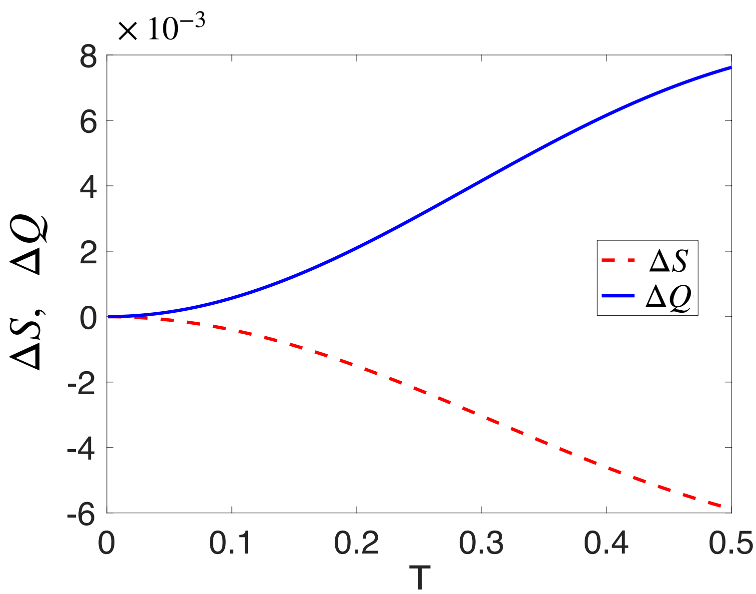


**Figure 1.** Accelerating detector in the vacuum state. We set $\lambda = 0.1$, $m = 1$, $\Omega = \omega_3$, $p = 0.05$, $a = 0.5$ and $L = 10$.

## 4. Thermal State

In this section, we consider the case where the Dirac field is initially prepared in the thermal state. The Hilbert space of the Dirac field can be factorized as $\mathcal{H}_f = \otimes_n \mathcal{H}_n$, where $\mathcal{H}_n$ denotes the Hilbert subspace of mode $n$, spanned by the basis $\{|0_n, \bar{0}_n\rangle, |1_n, \bar{0}_n\rangle, |0_n, \bar{1}_n\rangle, |1_n, \bar{1}_n\rangle\}$, where the numbers of the fermion and anti-fermion in mode $n$ are given by $j_n$ and $\bar{j}_n$, respectively, with $j = 0, 1$. Thus the partition function can also be factorized by mode numbers as $Z = \prod_n Z_n$, where $Z_n = 1 + 2e^{-\beta\omega_n} + e^{-2\beta\omega_n}$, and $\beta = \frac{1}{T_R}$. Thus, the thermal state of the field can be written as

$$\rho_{0,f} = \bigotimes_n \sum_{j_n=0}^{1} \sum_{\bar{j}_n=0}^{1} P_{\{j_n,\bar{j}_n\}} |j_n, \bar{j}_n\rangle\langle j_n, \bar{j}_n|, \tag{22}$$

which is also a diagonal matrix, and the possibility of the occupation configuration $\{j_n, \bar{j}_n\}$ is given by $P_{\{j_n,\bar{j}_n\}} = \frac{1}{Z}e^{-\beta(j_n+\bar{j}_n)\omega_n}$. It is easy to verify that the trace of $\rho_{0,f}$ equals 1. By taking $T_R \to 0$, only $P_{\{0_n,\bar{0}_n\}}$ survives for each $n$; thus, (22) becomes the vacuum state. From Equation (6), it is easy to see that, for each mode $n$, the frequency $\omega_n > m$; thus, the possibility of having a single fermion or anti-fermion in the initial state is

$$P_{0_1,\bar{0}_1,0_2,\bar{0}_2\cdots 1_n,\bar{0}_n\cdots 0_Q,\bar{0}_Q} = P_{0_1,\bar{0}_1,0_2,\bar{0}_2\cdots 0_n,\bar{1}_n\cdots 0_Q,\bar{0}_Q} < \frac{1}{Z}e^{-\beta m}, \tag{23}$$

where $Q$ is the largest field number corresponding to the cut-off field frequency $\omega_Q$. From Equation (23), we can see that the possibility increases when the temperature $T_R$ grows or the mass $m$ decreases. The right-hand side of Equation (23) provides us with a simple way to estimate the possibility of having a single fermion or anti-fermion in the initial state. If we assume the mass to be the electron mass $m = m_e = 0.511$ MeV and the temperature to be room temperature $T = 273$ K, then we can obtain a very small value from the right-hand side of Equation (23). Thus, it is quite straightforward to see that the temperature of the field must be extremely high if we want to observe the existence of particles with a reasonable probability when the mass $m$ of the particles is relatively large. This can be understood in a picturized manner: when preparing the initial state, the particles are boiled out of vacuum.

The anticommutation relation of the fermion operators $\{c_n, c_m^\dagger\} = \delta_{nm}, c = b, d$ makes it impossible to write $c_n^\dagger|0_n\rangle = |1_n\rangle, c_n^\dagger|1_n\rangle = 0$ in a unified manner; thus, the formula of heat transfer and entropy growth will be too complicated. Fortunately, things are not as hopeless as it seems. By observing (16), we find that, under certain conditions, we can obtain an elegant result by introducing two additional but reasonable assumptions. When the Unruh–DeWitt detector stays still in the thermal bath during the whole process, one of the field modes $B$ is in resonance with the detector (i.e., $\omega_B = \Omega$) and the interacting time $T$ is taken to be large enough, working out the integrals in (16), we can see that $V_B$ is the dominant part growing linearly with the interaction time $T$, while the exponential part $e^{\pm i(\Omega+\omega_n)t}$ in other terms gives the upper bound, which is less than 1.

The initial state of the Unruh–DeWitt detector is also chosen as $\rho_{0,D} = [(1-p)|-\rangle\langle -| + p|+\rangle\langle +|]$, whose direct product with (22) gives the initial state of the whole system. By using the same arguments as in the last section, we can see that $\rho^{(1)}(t)$ does not contribute to the heat transfer and entropy growth in this case either. The second order of $\lambda$ and the contribution are dominated by resonance mode $B$; thus, we have the relevant contributions

$$\begin{aligned}
U^{(1)}(T)(|1_B, \bar{1}_B\rangle\langle 1_B, \bar{1}_B| \otimes |-\rangle\langle -|)U^{(1)\dagger}(T) &\to \lambda^2|V_B|^2|1_B, \bar{0}_B\rangle\langle 1_B, \bar{0}_B| \otimes |+\rangle\langle +|, \\
U^{(1)}(T)(|0_B, \bar{0}_B\rangle\langle 0_B, \bar{0}_B| \otimes |+\rangle\langle +|)U^{(1)\dagger}(T) &\to \lambda^2|V_B|^2|0_B, \bar{1}_B\rangle\langle 0_B, \bar{1}_B| \otimes |-\rangle\langle -|. \\
U^{(1)}(T)(|1_B, \bar{0}_B\rangle\langle 1_B, \bar{0}_B| \otimes |+\rangle\langle +|)U^{(1)\dagger}(T) &\to \lambda^2|V_B|^2|1_B, \bar{1}_B\rangle\langle 1_B, \bar{1}_B| \otimes |-\rangle\langle -|, \\
U^{(1)}(T)(|0_B, \bar{1}_B\rangle\langle 0_B, \bar{1}_B| \otimes |-\rangle\langle -|)U^{(1)\dagger}(T) &\to \lambda^2|V_B|^2|0_B, \bar{0}_B\rangle\langle 0_B, \bar{0}_B| \otimes |+\rangle\langle +|,
\end{aligned} \tag{24}$$

and

$$\begin{aligned}
&U^{(2)}(T)(|1_B,\bar{0}_B\rangle\langle 1_B,\bar{0}_B|\otimes|-\rangle\langle -|) = (|1_B,\bar{0}_B\rangle\langle 1_B,\bar{0}_B|\otimes|-\rangle\langle -|)U^{(2)\dagger}(T)\\
&= -\frac{\lambda^2}{2}|V_B|^2|1_B,\bar{0}_B\rangle\langle 1_B,\bar{0}_B|\otimes|-\rangle\langle -|,\\
&U^{(2)}(T)(|0_B,\bar{0}_B\rangle\langle 0_B,\bar{0}_B|\otimes|+\rangle\langle +|) = (|0_B,\bar{0}_B\rangle\langle 0_B,\bar{0}_B|\otimes|+\rangle\langle +|)U^{(2)\dagger}(T)\\
&= -\frac{\lambda^2}{2}|V_B|^2|0_B,\bar{0}_B\rangle\langle 0_B,\bar{0}_B|\otimes|+\rangle\langle +|,\\
&U^{(2)}(T)(|1_B,\bar{0}_B\rangle\langle 1_B,\bar{0}_B|\otimes|+\rangle\langle +|) = (|1_B,\bar{0}_B\rangle\langle 1_B,\bar{0}_B|\otimes|+\rangle\langle +|)U^{(2)\dagger}(T)\\
&= -\frac{\lambda^2}{2}|V_B|^2|1_B,\bar{0}_B\rangle\langle 1_B,\bar{0}_B|\otimes|+\rangle\langle +|,\\
&U^{(2)}(T)(|0_B,\bar{1}_B\rangle\langle 0_B,\bar{1}_B|\otimes|+\rangle\langle +|) = (|0_B,\bar{1}_B\rangle\langle 0_B,\bar{1}_B|\otimes|+\rangle\langle +|)U^{(2)\dagger}(T)\\
&= -\frac{\lambda^2}{2}|V_B|^2|0_B,\bar{1}_B\rangle\langle 0_B,\bar{1}_B|\otimes|+\rangle\langle +|.
\end{aligned} \tag{25}$$

By tracing out the degrees of freedom of the Dirac field and using the same notation of $\delta p$ in (18), we obtain

$$\delta p = \lambda^2|V_B|^2\big((1-p)P_2 + (1-2p)P_1 - pP_0\big), \tag{26}$$

where

$$\begin{aligned}
P_0 &= \sum_{\{j_{n'}\},\{\bar{j}_{n'}\};n'\neq B} P_{0_B,\bar{0}_B},\\
P_1 &= \sum_{\{j_{n'}\},\{\bar{j}_{n'}\};n'\neq B} P_{1_B,\bar{0}_B} = \sum_{\{j_{n'}\},\{\bar{j}_{n'}\};n'\neq B} P_{0_B,\bar{1}_B},\\
P_2 &= \sum_{\{j_{n'}\},\{\bar{j}_{n'}\};n'\neq B} P_{1_B,\bar{1}_B},
\end{aligned} \tag{27}$$

where $P_{i_B,\bar{j}_B}, i,j = 0,1$ denotes the probabilities of distribution with the $i$ fermion and $j$ anti-fermion of mode $B$, and the numbers of the particles of other modes are summed over here. Then, we obtain the heat transfer and entropy growth as

$$\begin{aligned}
\Delta S &= \lambda^2 \ln\frac{1-p}{p}|V_B|^2\Big(pP_0 + (2p-1)P_1 - (1-p)P_2\Big),\\
\Delta Q &= \lambda^2\omega_B|V_B|^2\Big(pP_0 + (2p-1)P_1 - (1-p)P_2\Big).
\end{aligned} \tag{28}$$

In this case, we can see that the distinguishability of the fermion and anti-fermion comes into play by shaping the structure of $\mathcal{H}_n$. By taking $T_R \to 0$, the results reduce to (20) if we only keep the $V_B$ term. By substituting (27) into (28), and after calculating some algebra, we can check that Landauer's principle $\Delta Q \geq T_R\Delta S$ is satisfied. The initial state of the detector can also be assigned with a temperature $T_D$ with $p = \frac{e^{\frac{\omega_B}{T_D}}}{e^{\frac{\omega_B}{T_D}}+1}$. From (28), we can see that the Dirac field absorbs heat when $T_D > T_R$ and emits heat when $T_R > T_D$, which is in agreement with classical thermodynamics. Note that there is no stronger condition for heat transfer like the one obtained in [21] for a real scalar field. A possible reason for this is the resonance approximation introduced in our case. In Figures 2 and 3, we present the results by setting $T_R = 10$ and $T_R = 0.1$ with $p = 0.05$, which correspond to the conditions $T_R > T_D$ and $T_R < T_D$, respectively. In Figure 2, we can see that the energy flows from the Dirac field towards the detector, while in Figure 3, the energy flow direction is inverted.

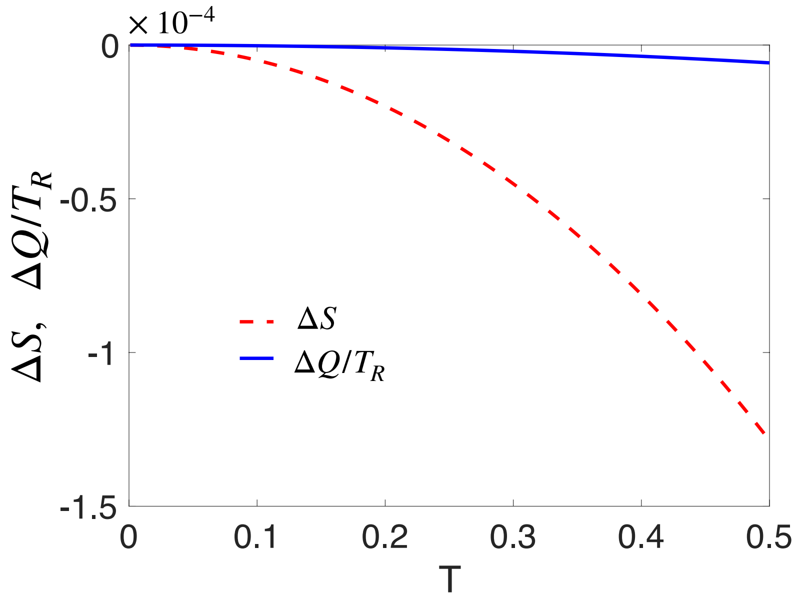


**Figure 2.** The case of $T_R = 10$. We set $\lambda = 0.1$, $m = 1$, $\Omega = \omega_3$, $p = 0.05$ and $L = 10$.

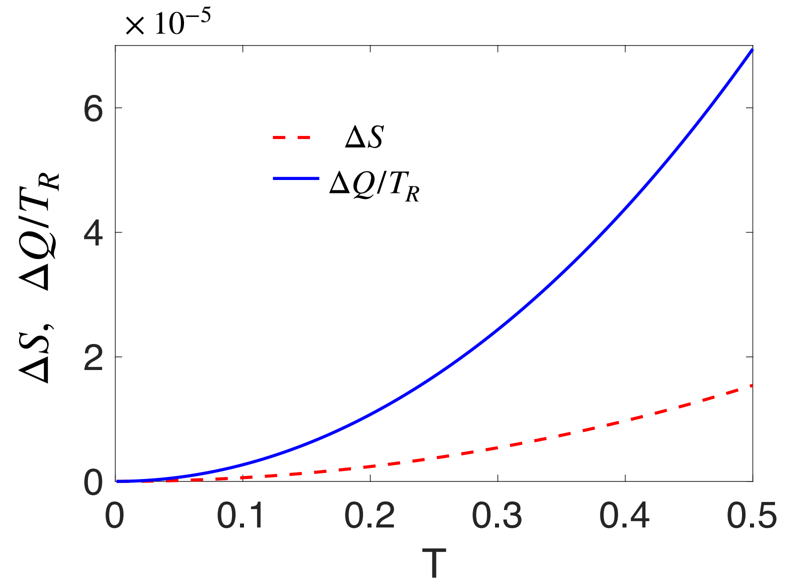


**Figure 3.** The case of $T_R = 0.1$. We set $\lambda = 0.1$, $m = 1$, $\Omega = \omega_3$, $p = 0.05$ and $L = 10$.

## 5. Conclusions

In this work, we studied Landauer's principle in a qubit–cavity system composed of an Unruh–DeWitt detector linearly coupled to a Dirac field in a $1 + 1$-dimensional cavity with MIT bag boundary condition. We showed that, when the Dirac field is initially prepared in the vacuum state, the heat transferred to the Dirac field is always non-negative, thus preserving Landauer's principle. In the meantime, the entropy of the detector can either grow or decrease, where the former can be viewed as the result of the Unruh effect. We found that, when the initial state of the Dirac field is thermal, heat transfer and entropy growth can be obtained under two additional assumptions, and we verified the validity of Landauer's principle. We found that, compared to the case of a real scalar field [21], the heat transfer and entropy growth of the vacuum initial state differ only due to the internal degree of freedom of the Dirac field, while for the thermal initial state, the distinguishability of the fermion and anti-fermion also plays a part. Our work has the potential of being applied to the neutrino–baryon system, where the internal structure of the baryon provides the MIT bag boundary, and the interaction between the nucleon and neutrino models the interaction between the Unruh–DeWitt detector and Dirac field [23,24,29], which may be experimentally feasible.

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