

Introduction to the Decay of Common Particles

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Abstract. One of the most important scientific research results in decades is the discovery of a boson particle that has the properties of the Higgs boson, which reiterates the importance of studying particle decay. In this paper, an introduction to common particle decays is given. It contains theoretical and experimental information. On the theoretical side, the paper discusses the concept of decay rate and branching fraction and dives into specific particle decays—the decays of the muon, neutral pion, and neutron. The theoretical results are then correspondingly compared with experimental results. The last section on the decay of the Higgs boson takes an experimental perspective, exploring the details of the data produced at the Large Hadron Collider (LHC).

1. Introduction

Particle physics, an important subfield in physics, focuses on finding properties of elementary particles, such as leptons, quarks, force-carrying bosons, and their interactions with one another^[1]. From a practical point of view, the development of particle physics fosters the development of all subfields in physics, as elementary particles make up all known matter. Technologies developed for particle physics also have practical applications. For example, particle accelerators can produce medical isotopes^[2]. Conseil Européen pour la Recherche Nucléaire (CERN) developed the touchscreen technology initially for the Super Proton Synchrotron (SPS) control system^[3]. From a philosophical point of view, understanding the most fundamental constitution of the material world has always been the purest pursuit of humanity. The earliest attempts from the Jains and the Greeks can date back to the 5th century BCE^[4]. Nevertheless, the early ideas are rather philosophical; the physical particle theory derived from experimentation commenced from Dalton's atomic theory in the 19th century^[5]. Later, from J. J. Thomson's discovery of electrons to the discovery of the Higgs boson, physicists built an elegant model describing all the known particles and interactions to high precision, known as the Standard Model (SM)^[6]. Common particles encountered the most in daily life, such as electrons and protons, are rather stable. They may still decay, but the mean lifetime can be extraordinarily long. Electrons, for instance, have a mean lifetime of around 6.6×10^{28} years^[7]; protons stay stable for about 10^{34} years^[8]. However, in SM, a large portion of the particles are unstable, decaying soon after production and possessing a short mean lifetime. Taking muon, a second-generation lepton, as an example, it has a mean lifetime of 2.2×10^{-6} seconds^[9]. To capture properties of unstable particles, studying particle decay is essential as it allows physicists to reconstruct the properties, such as mass, of the original particle by collecting information on newly created particles. A particle often has multiple decay channels with their own probabilities, or branching fraction, which is associated with the decay rate of each decay channel^[10]. The branching fraction of a decay channel can be both calculated theoretically and measured experimentally; these results are often compared to test the accuracy of the theory^[11]. Studies of particle decay explore physics beyond the SM. Ongoing research on observed decay into invisible particles may



open a door for comprehending dark matter. Exotic particles that are not described in the SM sometimes arise in decay. To help achieve a better understanding of current research, this paper is dedicated to giving an introduction to particle decays. It will start with a discussion of the decay rate and branching fraction, including the definitions, calculations, and experimental interpretations. Next, decays of several common particles will be introduced. Finally, the paper will focus on the decay of one specific and important particle, the Higgs boson.

2. Decay rate and branching fraction

2.1 Decay rate

The decay rate of a specific decay mode of a particle, usually denoted as Γ , is described by the following decay equation:

$$N(t) = N(0)e^{-\Gamma t} \quad (1)$$

where $N(0)$ is the initial number of particles, and $N(t)$ is the number of particles at time t . Note that the decay rate is decay mode specific, rather than particle specific. The total decay rate of a particle is the sum of the decay rates of all decay modes^[1]:

$$\Gamma_{total} = \sum_{i=1}^n \Gamma_i \quad (2)$$

The mean lifetime of a particle is simply the reciprocal of the total decay rate. For an arbitrary decay, where particle 1 decays into particle 2 ~ n, the decay rate can be found by the following equation^[1]:

$$\Gamma = \frac{S}{2\hbar m_1} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3 - \dots - \mathbf{p}_n) \times \prod_{j=2}^n 2\pi \delta(p_j^2 - m_j^2 c^2) \theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4} \quad (3)$$

where S is a statistical factor, for the same s particles, a factor of $1/s!$ is multiplied; \mathcal{M} is the amplitude, which is a function of various momenta and can be calculated by evaluating the Feynman diagrams; all the momenta represented in the equation are four-momenta. The decay rate calculation contains two factors, amplitude and phase space, the former depending solely on the Feynman diagram, and the latter being kinematic depending on the masses, energies and momenta of all particles participating in the decay^[1].

The general calculation of the amplitude \mathcal{M} is rather complicated and beyond the scope of this paper. Instead, a hypothetical and special decay mode is discussed here: the lowest-order decay mode of $A \rightarrow B + C$, where A , B , and C are all spin-zero particles and each their own antiparticles. In this case, the amplitude \mathcal{M} is simply a constant:

$$\mathcal{M} = g \quad (4)$$

where g is the coupling constant, which describes the strength of the force in an interaction^[11]. The statistical factor S is 1. Plugging in the equation, an elegant expression is obtained^[1]:

$$\Gamma = \frac{g^2 |p|}{8\pi \hbar m_A^2 c} \quad (5)$$

where $|p|$ is the magnitude of the outgoing momentum^[1].

2.2 Branching fraction

Branching fraction, also known as branching ratio, describes the ratio of particles that decay in a specific decay mode over all particles that decay. The calculation is rather simple; the branching fraction can be calculated by dividing the decay rate of an individual decay mode by the total decay rate^[1]:

$$\text{branching ratio for } i\text{th decay mode} = \Gamma_i / \Gamma_{total} \quad (6)$$

2.3 Experimental Aspect

While the branching fraction of an individual decay mode can be calculated theoretically from the SM, it can also be measured experimentally. These two results are often compared to test the accuracy of the

SM prediction. For example, for the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$, CMS collaboration has analyzed 2011 Compact Muon Solenoid (CMS) experiment data collected at the Large Hadron Collider (LHC) and concluded that the measurements correspond to the SM prediction well^[12].

3. Decays of common particles

In this section, decays of several common particles (not necessarily elementary particles), such as muon, neutral pion, and neutron, are introduced.

3.1 Muon decay

Muons are second-generation leptons, or “heavier electrons”, as the muon mass is 207 times as large as the electron mass^[13]. Same as electrons, muons can have a positive or negative charge and have spin $1/2$ ^[13]. Muons are towards the stable side of all unstable particles, possessing a comparatively long lifetime, $2.2 \mu\text{s}$. Mediated by W bosons, muons almost exclusively decay through the mode $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$, also known as Michel decay, which has a branching fraction of nearly 100% and whose Feynman diagram is included in Figure 1^[9]. Some other observed decay modes and their corresponding branching fractions are listed in Table 1:

Table 1. Muon decay modes^[9].

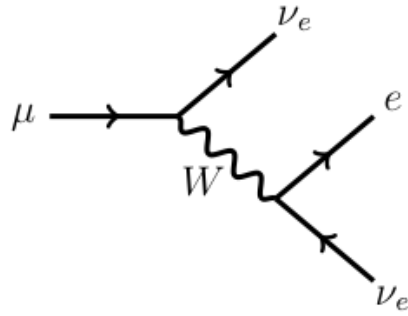
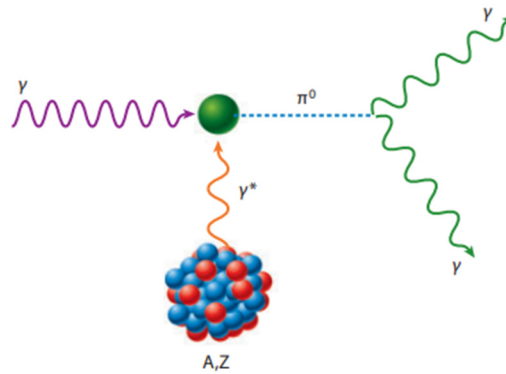
Decay modes	Branching ratio
$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$	$\sim 100\%$
$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \gamma$	$1.4 \pm 0.4\%$ (for $E_\gamma > 10 \text{ MeV}$)
$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu e^+ e^-$	$(3.4 \pm 0.4) \times 10^{-5}$
$\mu^- \rightarrow e^- \bar{\nu}_\mu \nu_e$	$< 1.2\%$
$\mu^- \rightarrow e^- \gamma$	$< 1.2 \times 10^{-11}$
$\mu^- \rightarrow e^- e^- e^+$	$< 1.0 \times 10^{-12}$
$\mu^- \rightarrow e^- \gamma \gamma$	$< 7.2 \times 10^{-11}$

In the normal muon decay mode, $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$, the decay rate is given by the following differential equation^[9]:

$$\frac{d^2\Gamma}{dx d\cos\theta_e} = \frac{m_\mu}{4\pi^3} W_{e\mu}^4 G_F^2 \sqrt{x^2 - x_0^2} (F_{IS}(x) \pm P_\mu \cos\theta_e F_{AS}(x)) (1 + \vec{P}_e(x, \theta_e) \cdot \hat{\xi}) \quad (7)$$

where $W_{e\mu} = (m_\mu^2 + m_e^2)/(2m_\mu)$, $x = E_e/W_{e\mu}$, and $x_0 = m_e/W_{e\mu}$. E_e here represents the energy of the electron. m_μ and m_e represent the muon and electron mass, respectively. P_μ and P_e are muon and electron polarization vectors. θ_e is the angle between the electron momentum and muon polarization, containing kinetic information. F_{IS} and F_{AS} are bound by Michel parameters ρ , η , ξ , and δ . In the SM, $\rho = \delta = \frac{3}{4}$, $\eta = 0$, and $\xi = 1$ ^[9]. These parameters are also experimentally determined, showing high correspondence with the SM. For instance, ρ is measured to be 0.7518 ± 0.0026 and δ is measured to be 0.7486 ± 0.0038 ^[14].

Interestingly, lepton-flavor-violating decays, such as $\mu^+ \rightarrow e^+ \gamma$ and $\mu^+ \rightarrow e^+ e^+ e^-$, have been observed^[9]. Extensive studies have been carried out on these decays, which may point to physics outside of the SM.

Figure 1. Normal muon decay Feynman diagram^[15].Figure 2. Diagram of Primakoff reaction^[16].

3.2 Neutral pion decay

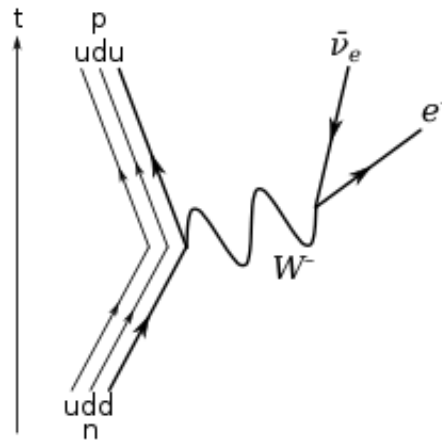
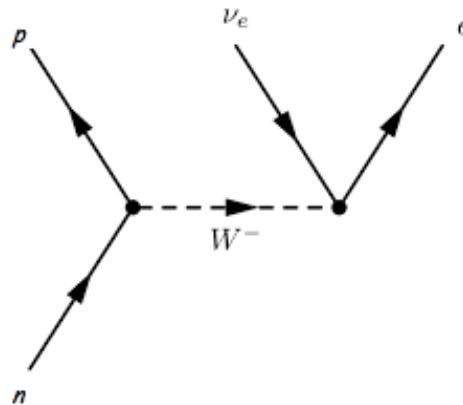
The neutral pion, composed of either $u\bar{u}$ or $d\bar{d}$, is the lightest naturally-occurring strongly-interacting particle, for which its research has great importance. Quantum chromodynamics (QCD) chiral symmetry plays a major role in π^0 decay and occurs in the limit where the masses of up and down quarks approach zero^[16]. In this limit, the decay rate of the principal decay mode $\pi^0 \rightarrow \gamma\gamma$ is simple enough^[16]:

$$\Gamma = \frac{\alpha^2 m_\pi^3}{576 \pi^3 F_\pi^2} N_C^2 = 7.73 \text{ eV} \quad (8)$$

where α is the fine-structure constant, F_π is the pion decay constant, and N_C is the number of colors in QCD, which is universally accepted as 3^[16]. Experimentally, Primakoff first proposed measuring the decay rate through the photoproduction of the neutral pion. This reaction is named after Primakoff and its diagram is shown in Figure 2. A more accurate decay width has been determined at Jefferson Laboratory, bearing two results: $7.79 \pm 0.18 \text{ eV}$ from 12C target and $7.85 \pm 0.23 \text{ eV}$ from 208Pb target, according to the SM result, and it confirms that the number of QCD colors is three.

3.3 Neutron decay

Neutron decay is the fundamental mechanism of β^- decay, a type of nuclear decay that is taught in high school. However, theoretically calculating the lifetime of the neutron is not as easy as one may expect because the neutron is a composite particle. The Feynman diagram of the free neutron decay is shown in Figure 3. A simpler estimate can be employed by treating neutrons and protons as point particles, as shown in Figure 4. By employing this method, one can get $\tau = 1318 \text{ s}^{[1]}$. It is, nevertheless, an estimate, as the lifetime is measured to be $887.7 \pm 1.2 \pm 1.9 \text{ s}^{[17]}$.

Figure 3. Free neutron decay Feynman diagram^[1].Figure 4. Simplified schematic diagram of neutron decay^[1].

4. Decay of the Higgs boson

In 2012, an announcement from LHC excited the physics community; a subatomic particle with all the expected properties of the Higgs boson was discovered by the ATLAS and CMS experiments^[18]. Higgs's theory is a fundamental constitution of the Standard Model (SM) of elementary particles. The mechanism associated with the Higgs boson, the Higgs mechanism, provides mass for some particles that are otherwise required to be massless to be described by gauge invariance theory which is against experimental observations^[19]. The Higgs boson has a zero spin, no electric charge, and no color charge. It is extremely unstable and it decays immediately after it is generated, making it difficult to capture the traces of the Higgs boson. There have, however, been extensive studies on various decay modes of the Higgs boson. This paper focuses on the principal $H \rightarrow \gamma\gamma$ and $H \rightarrow b\bar{b}$ decay modes from an experimental point of view.

$$H \rightarrow \gamma\gamma$$

The $H \rightarrow \gamma\gamma$ has a relatively low branching ratio, about 2.27×10^{-3} , yet it is still the most extensively studied channel because it has a clear background and final state particles^[18]. In this case, photons can be measured and used to reconstruct the mass of the Higgs boson. The main background comes from prompt diphoton production and $pp \rightarrow \gamma + jet$, $pp \rightarrow jet + jet$ ^[20]. The former is irreducible while the latter is reducible. CMS collaboration has used photon identification requirements that greatly reduce the background caused by non-prompt photons^[20]. Moreover, both CMS and ATLAS have calibrated the diphoton energy response using $Z \rightarrow e^+e^-$ and $Z \rightarrow \mu^+\mu^- \gamma$ ^[18]. Consequently, great

results have been produced. CMS collaboration measured the signal strength to be $1.18^{+0.26}_{-0.23}$ and $1.18^{+0.17}_{-0.14}$ in Run 1 and Run 2, respectively, where the signal strength is defined to be the ratio of the measured decay rate to the SM prediction^[19].

$$H \rightarrow b\bar{b}$$

$H \rightarrow b\bar{b}$ decay mode has been notoriously difficult to study for its large QCD background^[19]. Nonetheless, the difficulty was overcome in 2018^[20]. The graph below presents data collected by ATLAS Collaboration of the decay to bottom quark pair through the production of vector boson fusion (VBF). The X-axis represents the invariant mass of the bottom quark pair and the y-axis represents the relative number of events taking place, which is modified with statistical methods. Red bars represent the Higgs boson decay and the gray bars represent the Z boson decay. The events of the Higgs boson decay peak at an invariant mass of 125 GeV, corresponding to the mass of the Higgs boson; the events of the Z boson decay peak at an invariant mass of 91 GeV, corresponding to the mass of the Z boson. Comparing experimental results to theoretical predictions, signal strength can be obtained. In this case, the signal strength is $0.95 \pm 0.35 \pm 0.37$, strongly supporting SM^[19]. Similarly, the signal strengths of other production modes can be calculated and are given in Table 2 below, where VH represents the associated production of vector bosons and the Higgs bosons and $t\bar{t}H$ represents the associated production of top quark pairs and the Higgs bosons. It can be observed that all datasets so far do not violate SM prediction.

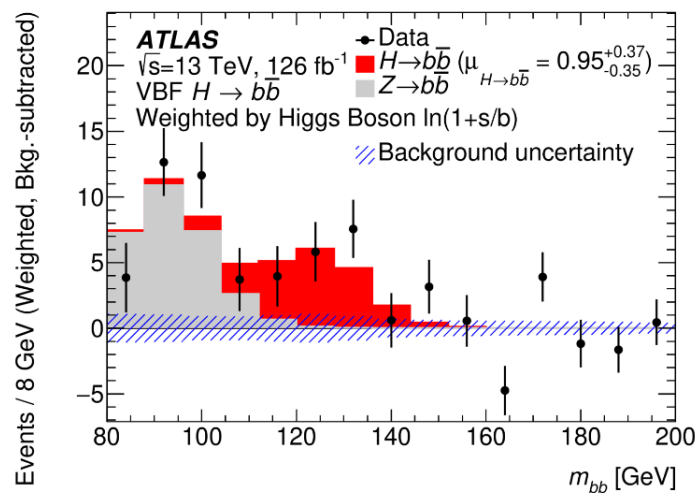


Figure 5. The reconstructed mass distribution of bottom quark pairs^[21].

Table 2. The signal strength of various decay modes^[18].

$0.99 H \rightarrow b\bar{b}$	Tevatron	ATLAS Run 1	CMS Run 1	ATLAS Run 2	CMS Run 2
VH	1.6 ± 0.7	$0.52 \pm 0.32 \pm 0.24$	1.0 ± 0.5	$1.16 \pm 0.16^{+0.21}_{-0.19}$	1.01 ± 0.22
VBF (γ)	—	-0.8 ± 2.3	$2.8 \pm 1.4 \pm 0.8$	2.5 ± 1.3	1.3 ± 1.2
$t\bar{t}H$	—	$1.4 \pm 0.6 \pm 0.8$	0.7 ± 1.9	$0.79 \pm 0.29 \pm 0.53$	$1.49 \pm 0.21 \pm 0.39$
Inclusive	—	—	—	$5.8 \pm 3.1 \pm 2.5$	2.3 ± 1.7

5. Conclusion

In particle physics, the vast majorities of the particles are unstable. Their decays, therefore, possess significance. The decays of different particles are studied at different levels. This paper aims to give a general introduction to the studies of particle decay. It includes the concept of the decay rate, which is the inverse of the mean lifetime of a particle. The general equation to theoretically formulate the decay rate is provided. Then, decays of muons, neutral pions, and neutrons are taken as examples for basic

applications of the theoretical approach to particle decays. The theoretical results are compared with experimental results, showing high precision of the SM prediction. Then, the decay of the Higgs boson is introduced with its experimental details. Nevertheless, the study of particle decay is much more intricate than what this paper can describe. Meanwhile, the study of particle decays itself is also far from reaching the end. As more and more exotic and SM-violating decays surface, research is carried out aiming to discover physics beyond the SM. In the future, the understanding of particle decay will no doubt improve to a great extent both theoretically and experimentally.

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