Modern Physics Letters A
Vol. 35, No. 17 (2020) 2050144 (17 pages)

DOI: 10.1142/S0217732320501448

Quintessences compact star with Durgapal potential

Gabino Estevez-Delgado

Facultad de Químico Farmacobiología de la Universidad Michoacana de San Nicolás de Hidalgo, Tzintzuntzan No. 173, Col. Matamoros, CP 58240, Morelia Michoacán, México
gestevez.ge@gmail.com

Joaquin Estevez-Delgado*

Facultad de Ciencias Físico Matemáticas de la Universidad Michoacana de San Nicolás de Hidalgo, Edificio B Ciudad Universitaria, Morelia Michoacán, México
joaquin@fismat.umich.mx

Aurelio Tamez Murguía

Facultad de Ciencias, Universidad Autónoma del Estado de México, Instituto Literario 100, Colonia Centro, CP 50000, Toluca, Estado de México, México
atm@uaemex.mx

Rafael Soto-Espitia

Facultad de Ingenieria Civil de la Universidad Michoacana de San Nicolás de Hidalgo Edificio A, Ciudad Universitaria, CP 58030, Morelia Michoacán, México
rsoto@umich.mx

Arthur Cleary-Balderas

Facultad de Ingeniería Eléctrica de la Universidad Michoacana de San Nicolás de Hidalgo, División de Estudios de Posgrado, Edificio Ω, Ciudad Universitaria, CP 58030, Morelia Michoacán, México
luisarthur@fie.umich.mx

Received 10 December 2019
Revised 17 February 2020
Accepted 6 March 2020
Published 23 March 2020

A compact star model formed by quintessence and ordinary matter is presented, both sources have anisotropic pressures and are described by linear state equations, also the state equation of the tangential pressure for the ordinary matter incorporates the effect of

*Corresponding author.

the quintessence. It is shown that depending on the compactness of the star $u = GM/c^2R$ the constant of proportionality μ between the density of the ordinary matter and the radial pressure, $P_r = \mu c^2(\rho - \rho_b)$, has an interval of values which is consistent with the possibility that the matter is formed by a mixture of particles like quarks, neutrons and electrons and not only by one type of them. The geometry is described by the Durgapal metric for $n = 5$ and each one of the pressures and densities is positive, finite and monotonic decreasing, as well as satisfying the condition of causality and of stability $v_t^2 - v_r^2 < 0$, which makes our model physically acceptable. The maximum compactness that we have is $u \leq 0.28551$, so we can apply our solution considering the observational data of mass and radii $M = (2.01 \pm 0.04)\ M_\odot$, $R \in [12.062, 12.957]$ km which generate a compactness $0.22448 \leq u \leq 0.25448$ associated to the star PSR J0348 + 0432. In this case, the interval of $\mu \in [0.78055, 1]$ and its maximum central density ρ_c and in the surface ρ_b of the star are $\rho_c = 7.0387 \times 10^{17}$ kg/m^3 and $\rho_b = 4.6807 \times 10^{17}$ kg/m^3, respectively, meanwhile the central density of the quintessence $\rho_{qc} = 3.4792 \times 10^{16}$ kg/m^3.

Keywords: Compact stars; exact solutions; quintessence anisotropic fluid.

1. Introduction

The internal composition of the stars is still an unresolved question, although there are considerable advancements on what type of matter composes stars and their behavior. For example, it is known that when the internal density is greater than the nuclear density, there is the possibility that the radial and tangential pressures are different from each other, which would indicate that its interior is not necessarily formed by a perfect fluid, but instead by an anisotropic fluid.[1] The description of the stars formed by anisotropic fluids has been studied since the last century[2–18] and one of the consequences is that it would allow to describe stars with a compactness value greater than the Buchdahl limit $u < 4/9$,[19–22] this presents itself even in the case of the anisotropic model with constant density.[23] The generalization of the Buchdahl limit for the anisotropic case only requires conditions on the behavior of the function of density, the pressure and its coupling with the exterior geometry described by the Schwarzschild solution[20] and not on a specific form of the state equation. In different works, the possible behavior of the interior of some stars has been described through models with anisotropic pressures and this has shown that the observational results match those expected and obtained by a theoretical models, it has also been shown the importance that the anisotropy has in the description of physically acceptable models.[7] The type of matter that could conform the stars with density greater than the nuclear density is expected to contain neutrons, quarks as well as electrons, although not only one of these components. The stars where their matter is mainly formed by quarks may be more compact than the stars constituted mainly by neutrons, in addition to this, it has been speculated on the possibility that the quintessence matter is also part of the star's interior, this type of matter could counteract the effect of the gravitational attraction because, as it is known, it generates negative pressures. In cosmology, the quintessence fields are associated to matter described by a perfect fluid with positive density and negative pressure and its state equation is given by $P_q = wc^2\rho_q$ with $-1 < w < -1/3$.[24] This has been presented to give an explanation to the observations on the accelerated expansion of the universe, since it requires the existence of fields or matter with negative pressure.[25,26] Some works about compact stars have already approached the possibility of quintessence and ordinary matter, the quintessence is described mainly by a fluid with radial and tangential pressures linked to the quintessence described by $P_{rq} = -c^2\rho_q$ and $P_{tq} = (1+3w)c^2\rho_q/2$ with $-1 < w < -1/3$ where ρ_q represents the density of the quintessence matter, these equations were proposed initially for the case of black holes with quintessence,[27] meanwhile for the ordinary matter, there have been different proposed sources.[28,29] Within the variety of compact stars with quintessence,[30–32] one of the most approached is that associated to the MIT Bag model,[33,34] for this case, starting from a state equation associated to the ordinary matter and considering the effect of the quintessence, it has been shown that the geometry of the solutions for Einstein's equations with perfect fluid works for describing compact stars with quintessence.[35] However, it is to be expected that the ordinary matter that conforms a star is not only constituted by quarks and that depending on its mass, radius, density or compactness its state equation may differ. Taking this into account, and as to generalize from previous works,[35] in this work, we present a compact star constituted by quintessence and ordinary matter with radial and tangential pressures described by the state equations $P_r = \mu c^2(\rho - \rho_b)$ and $P_t = \mu c^2(\rho - \rho_b) - 3(1+w)c^2/2\rho_q$, respectively, where $\rho_b \equiv \rho(R)$ is the density of ordinary matter on the surface. The work is organized as follows: in Sec. 2, we obtain the equations for the quintessence model and we list the conditions that must be met for the system to be physically acceptable, Sec. 3 is focused in obtaining the solution, starting from the Durgapal geometry, for $n = 5$.[36] Starting from the solution obtained in Sec. 4, we apply the model for a possible representation of the star PSR J0348+0432 and we finalize with conclusions in Sec. 5.

2. The Field Equations and Physical Conditions

We assume that the geometry of the interior of the compact star is static and spherically symmetrical as it can be described by the metric[37]

$$ds^2 = -y(r)^2\,dt + \frac{dr^2}{B(r)} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2), \quad r \le R. \tag{1}$$

We assume that the source of matter is formed by a quintessence field and normal matter with anisotropic pressures. So, Einstein's equations are given by

$$G_{\alpha\beta} = k[T^{(q)}{}_{\alpha\beta} + (c^2\rho + P_t)u_\alpha u_\beta + P_t g_{\alpha\beta} + (P_r - P_t)\chi_\alpha\chi_\beta].$$

These vectors are satisfying where $k = \frac{8\pi G}{c^4}$, G is the universal gravitational constant, c is the speed of light; ρ is the density, (P_r, P_t) are the radial and tangential pressures, respectively. Further, u^μ is the fluid 4-velocity and χ^μ is the unit vector in the radial direction, which under comoving reference frame are defined as $u^\mu = \frac{1}{y(r)}\delta_0^\mu$ and $\chi^\mu = \sqrt{B(r)}\delta_r^\mu$. While $T^{(q)}{}_{\alpha\beta}$ represents the energy–momentum tensor of the quintessence-like field, with components given by[27]

$$T^{(q)t}{}_t = -c^2\rho_q, \quad T^{(q)r}{}_r = -c^2\rho_q, \quad T^{(q)\theta}{}_\theta = T^{(q)\phi}{}_\phi = -\frac{1+3w}{2}c^2\rho_q, \tag{2}$$

with the quintessences parameter w such that $-1 < w < -\frac{1}{3}$. In our case, we assume state equations P_r and P_t are linear and are given by

$$P_r = \mu c^2(\rho - \rho_b), \quad P_t = \mu c^2(\rho - \rho_b) - \frac{3}{2}c^2(w+1)\rho_q, \tag{3}$$

where ρ_b is the density on the surface of the star. The state equation for the radial pressure is a generalization of the MIT Bag state equation, $P = (c^2\rho - 4B_g)/3$ which describes the interaction of confined quarks in a bag of finite dimension as result of the balance of the bag pressure B_g, which is directed inward, and the stress arising from the kinetic energy of the quarks.[38,39] In our case, the parameter $\mu \in (0, 1]$ has an interval of values, which is consistent with the possibility that the matter is formed by a mixture of particles like quarks, neutrons and electrons and not only by one type of them. On the other hand, as it has been argued in a previous investigation report[35] and following this idea, other investigation works have been written,[32,34] the structure of the state equation for the tangential pressure is proposed to describe how the quintessence could disturb a perfect fluid. As it can be observed, in the absence of quintessence, the radial and tangential pressures would be equal, so our suggestion is that the quintessence density may cause anisotropy in the pressures. The anisotropy in the pressures is important when trying to describe compact objects with density greater than the nuclear density, in which case it is possible to have objects in equilibrium with a greater density and compactness than in the case of perfect fluid.[1,20–22] Considering the metric given by Eq. (1) and the sources of matter mentioned, Einstein's equations generate the following set of equations:

$$kc^2(\rho + \rho_q) = -\frac{B'}{r} + \frac{1-B}{r^2}, \tag{4}$$

$$k\mu c^2(\rho - \rho_b) - kc^2\rho_q = \frac{2By'}{ry} - \frac{1-B}{r^2}, \tag{5}$$

$$k\mu c^2(\rho - \rho_b) - kc^2\rho_q = \frac{(ry'' + y')B}{ry} + \frac{(ry' + y)B'}{2ry}, \tag{6}$$

"$'$" denotes the derivative with respect to the coordinate r. It is important to note that the parameter w in not present on these equations and it is only reflected on the density of the quintessence. However, it is present in the state equation for the tangential pressure and as consequence, as we will see ahead, in the tangential speed of sound v_t. It is this characteristic which allows that a geometry associated to a perfect fluid in absence of quintessence is applicable to the anisotropic case with quintessence.[32,34,35] Systems (4)–(6), after algebra, can be rewritten in the equivalent form as follows[35]:

$$kc^2\rho = -\frac{B'}{(1+\mu)r} + \frac{2By'}{(1+\mu)ry} + \frac{kc^2\mu\rho_b}{1+\mu}, \tag{7}$$

$$kc^2\rho_q = -\frac{\mu B'}{(1+\mu)r} - \frac{2By'}{(1+\mu)ry} + \frac{1-B}{r^2} - \frac{kc^2\mu\rho_b}{1+\mu}, \tag{8}$$

$$2r(ry''-y')B + r(ry'+y)B' + 2y(1-B), \tag{9}$$

that it is a generalization to the case of MIT Bag state equation $\mu = 1/3$.[35] So, to build a solution of a star with quintessence field and state equations of ordinary matter given by (3), it is enough with solving systems (7)–(9), a one way of integrating this system is solving the differential equation (9) which relates the metrical coefficients and afterwards, by means of substitution, obtains the density functions. Equation (9) is the same as that in the case of the perfect fluid, and in this work, we consider the geometry associated to the Durgapal solution with $n = 5$.

2.1. *Physical conditions*

The conditions that must be satisfied by the functions $(\rho, P_r, P_t, \rho_q, y, B)$, so a solution to Eqs. (4)–(6) is physically acceptable, particularly when there is no presence of quintessence, have been discussed in different works, we listed these and proposed the requirements for the quintessence.[7,35,40–42]

(1) There must be a region $r = R$, identified as the surface of the star in which the radial pressure is nullified, i.e. $P_r(R) = 0$. The exterior geometry, $r \geq R$, is described by the Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2GM}{c^2r}\right)dt^2 + \left(1 - \frac{2GM}{c^2r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{10}$$

where M represents the total mass inside the fluid sphere and the interior geometry, given by the solution of (4)–(6), over the surface must be continuous.

(2) The solution must be regular, which means that the density and pressures must be bounded for $0 \leq r \leq R$ and the geometry must be non-singular inside the star, i.e. the Kretschmann scalar:

$$R^{\alpha\beta\sigma\delta}R_{\alpha\beta\sigma\delta} = 4\left[\frac{1-B}{r^2}\right]^2 + \frac{2B'^2}{r^2} + \frac{8B^2}{y^2}\frac{y'^2}{r^2} + \left[\frac{B'y'}{y} + \frac{2y''}{y}B\right]^2, \tag{11}$$

must be regular $\forall r \leq R$.[41]

(3) The density ρ and the pressures (P_r, P_t) must be positive functions. Furthermore, the density and radial pressure must be monotonic decreasing functions with a maximum value in the center, that is to say

$$\rho(0) > 0, \quad \rho'(r)|_{r=0} = 0, \quad \rho''(r)|_{r=0} < 0, \quad P_r(0) > 0,$$
$$P_r'(r)|_{r=0} = 0, \quad P_r''(r)|_{r=0} < 0, \quad \text{and for } r > 0,\ \rho' < 0 \text{ and } P_r' < 0.$$

(4) The density of the quintessences matter ρ_q must be a regular function. Although the positive or negative behavior inside is not entirely determined, we consider a $\rho_q \geq 0$ which implies $P_q \leq 0$ in concordance with the cosmological case. In some works discussed, ρ_q has been reported as positive and in some other cases, it has regions where it is positive and in other regions negative.[43]

(5) The conditions of causality must be satisfied, which means that the magnitude of the radial and tangential speed of sound v_r and v_t, respectively, must not be greater than the speed of light

$$0 \leq v_r^2 \equiv \frac{\partial}{\partial \rho} P_r(\rho) = \frac{dP_r}{dr} \Big/ \frac{d\rho}{dr} \leq c^2, \quad 0 \leq v_t^2 \equiv \frac{\partial}{\partial \rho} P_t(\rho) = \frac{dP_t}{dr} \Big/ \frac{d\rho}{dr} \leq c^2.$$

(6) For the anisotropic fluid configuration, the energy condition like null energy condition (NEC), weak energy condition (WEC), the dominant energy condition (DEC), the strong energy condition (SEC) must be satisfied throughout the interior region, i.e.[42]

$$\rho \geq 0, \quad c^2\rho \pm P_r \geq 0, \quad c^2\rho \pm P_t \geq 0, \quad c^2\rho + P_r + 2P_t \geq 0. \tag{12}$$

(7) Additionally, for the solution to be potentially stable, we require that the Herrera cracking condition $v_t^2 - v_r^2 < 0$ is satisfied.[44]

These properties allow to set the integration constants and determine when a model is physically acceptable.

3. The Solution and Its Analysis

The structure of Eqs. (7)–(9) does not impose restrictions on the choice of the type of geometry (y, B) associated to the case with perfect fluid. The choice is based on the properties which are required to adequately represent a star and these were mentioned in the previous section, in addition to this, we have the compactness value. As such, we have chosen one of the solutions analyzed in the frame of the general theory of relativity.[36,40] Durgapal analyzes one form for the gravitational potential $y(r)^2 = (1 + ar^2)^n$ with $n = 1, 2, 3, 4, 5$ known as Durgapal's solution, our choice of $n = 5$ is because in the case of perfect fluid the solution is regular and its compactness rate is $u = 0.265$. Furthermore, in previous works, it has already been shown that the compactness rate of the initial model with perfect fluid and its respective case for the anisotropic fluid with quintessence has the same function of the compactness rate and its maximum value is of the same order.[32,34,35] The form of the metric coefficients of the Durgapal metric for $n = 5$, is

$$\begin{aligned} y(r) &= (1 + ar^2)^{\frac{5}{2}}, \\ B(r) &= \frac{\sqrt[3]{1 + 6ar^2}(112 - 309ar^2 - 54a^2r^4 - 8a^3r^6) - Aar^2}{112(1 + ar^2)^3 \sqrt[3]{1 + 6ar^2}}. \end{aligned} \tag{13}$$

Starting from these functions by means of substitution in (7) and (8), we obtain the density of the matter and of the quintessence

$$\rho(r) = \frac{5(241 - 411ar^2 - 60a^2r^4 - 8a^3r^6)a}{56(1+\mu)(1+ar^2)^4kc^2} + \frac{Aa(1 - 3ar^2 - 44a^2r^4)}{56(1+\mu)(1+ar^2)^4(1+6ar^2)^{\frac{4}{3}}kc^2} + \frac{\mu\rho_b}{(1+\mu)}, \tag{14}$$

$$\rho_q(r) = \frac{5[387\mu - 95 + 3(275+\mu)ar^2 + 30(7+3\mu)a^2r^4 + 8(5+3\mu)a^3r^6]a}{112(1+\mu)(1+ar^2)^4kc^2} + \frac{a[1 + 3\mu + (17+11\mu)ar^2 + 22(3-\mu)a^2r^4]A}{112(1+\mu)(1+ar^2)^4(1+6ar^2)^{4/3}kc^2} - \frac{\mu\rho_b}{(1+\mu)}. \tag{15}$$

The pressures, given by the state equation (3), are a linear combination of the densities. So, we will only give the expressions of the first derivative of these functions and the values of the second derivative in the center of the star.

$$\rho'(r) = \frac{P_r'(r)}{\mu c^2} = -\frac{5a^2r(3 + 21ar^2 + 2a^2r^4 - 176a^3r^6)A}{28(1+\mu)(1+ar^2)^5(1+6ar^2)^{\frac{7}{3}}kc^2} - \frac{5(1375 - 1113ar^2 - 96a^2r^4 - 8a^3r^6)ra^2}{28(1+\mu)(1+ar^2)^5kc^2}, \tag{16}$$

$$\rho_q'(r) = \frac{5[5(241 - 411ar^2 - 60a^2r^4 - 8a^3r^6) - 3(515 - 57ar^2 + S(r))\mu]a^2r}{56(1+\mu)(1+ar^2)^5kc^2} + \frac{5[(1+6ar^2)(1 - 3ar^2 - 44a^2r^4) - (5 + 39ar^2 + 66a^2r^4 - 88a^3r^6)\mu]Aa^2r}{56(1+\mu)(1+ar^2)^5(1+6ar^2)^{7/3}kc^2} \tag{17}$$

where $S(r) = 36a^2r^4 + 8a^3r^6$.

From the derivatives, we have that these are nullified in the center, but it is not clear that these correspond to monotonic decreasing functions. The second derivatives evaluated in the center are

$$\rho''(0) = -\frac{5a^2(3A + 1375)}{28kc^2(1+\mu)} \le 0, \quad \rho_q''(0) = -\frac{5a^2(5A\mu - A + 1545\mu - 1205)}{56kc^2(1+\mu)} \le 0,$$

and these impose restrictions on the constant of integration A. To know the explicit form of the constants A and ρ_b, we apply the conditions of continuity of the metric, the second fundamental form and that the pressure at the surface is nullified, these conditions also imply $\rho_q(R) = 0$, after solving the system we arrive at

$$A = \frac{25(19 - 165s - 42s^2 - 8s^3)\sqrt[3]{1+6s}}{1+11s}, \tag{18}$$

$$\rho_b = \frac{10(3+11s)s}{kc^2R^2(1+11s)(1+6s)}, \tag{19}$$

$$u(s) \equiv \frac{GM}{c^2R} = \frac{5s}{1+11s}, \tag{20}$$

where $s = aR^2 > 0$ and $u(s)$ represent the compactness ratio. One property that helps us to determine the interval of μ is the condition of stability in the surface of the star:

$$v_t^2(0) - \mu c^2 = -\frac{3(1+w)[5(5\mu-1)S_1 + (309\mu-241)(1+11s)]}{20[3S_1 + 55(1+11s)]}c^2 < 0, \quad (21)$$

with $S_1 = [19 - 165s - 42s^2 - 8s^3]\sqrt[3]{1+6s}$. In obtaining (21), we have taken into account the state equation for the radial pressure, which implies that $v_r^2 = \mu c^2$. Equation (21) imposes a restriction on the range of values of μ, the minimum value is $\mu = 3/7$ and its exact value depends on the compactness. For values lower that this, we have that $v_t^2(0) - v_r^2 > 0$ which would imply that the system is not potentially stable. In a similar manner, it happens that for $\mu < 3/7$, $v_t^2(R) - v_r^2 > 0$, although in some cases the expression is more simple, for example for a model with MIT Bag state equation, $\mu = 1/3$ results are as follows:

$$v_t^2(R) - \frac{1}{3}c^2 = -\frac{(1+w)(374s^2 + 139s + 5)c^2}{2(25 + 151s + 286s^2)} > 0, \quad (22)$$

which implies that with this source of matter and Durgapal geometry, the solution with MIT Bag state equation is not stable. On the other hand, for values of μ slightly greater than 3/7, the validity range of s is a small vicinity surrounding $s = 0$ and as such, according to Eq. (20), the compactness also approaches zero. As we increase the value of μ, the interval of s is greater and as such allows us to represent objects with a greater compactness rate u. The maximum interval of s occurs for $\mu = 1$ and we can determine it from the condition of stability

$$v_t^2(0) - c^2 = -\frac{3(1+w)[5\sqrt[3]{1+6s}S_1 + 17(1+11s)]}{5[3\sqrt[3]{1+6s}S_1 + 55(1+11s)]}c^2 < 0, \quad (23)$$

as such the admissible values of s are $0 < s < s_{\max} = 0.1535462$, which implies that it is possible to represent stellar objects with a compactness rate $u \leq u(s_{\max}) \equiv u_{\max} = 0.28551$. Objects with a compactness value close to $u_{\max}$ and for them to be stable they would require a state equation with $\mu \to 1$. From this analysis, we can note that although the state equation allows us to represent compact objects, the value of the compactness obtained starting from the mass and the radius influences the possible value of μ. In the following section, we will show the graphic behavior with the observational data of the star PSR J0348 + 0432.

4. The PSR J0348 + 0432 Star

The results of the previous section show that the solution is applicable for stars with a compactness rate $u \leq 0.28551$, this being the result of imposing the stability. According to the observational data, the star PSR J0348 + 0432 has a compactness $0.22448 \leq u \leq 0.25448$, this is because of its mass $M = (2.01 \pm 0.04)\ M_\odot$ and its radius $12.062\ \text{km} \leq R \leq 12.957\ \text{km}$.[45,46] Inside the possible interval for the compactness, we consider the combinations of the values of the mass and maximum

Table 1. Values of the predicted densities for observational mass and radius. In the table, four columns are reported, each one corresponds with a value of possible mass and radius with their respective associated values of densities. The central density of the ordinary matter ρ_c and the one of quintessence ρ_{qc} depend of the μ value, here we only report the ones corresponding to the extremes. Meanwhile, the density of matter on the surface does not depend on the value of μ.

	$M\,(M_\odot)$	2.05	1.97	2.05	1.97
	R (km)	12.062	12.062	12.957	12.957
	u (s)	0.2509	0.2411	0.2336	0.2245
	s	0.1120	0.1027	0.0961	0.0887
$\mu = 0.78055$	$\rho_c(10^{17}\,\frac{\text{kg}}{\text{m}^3})$	7.0387	6.6474	5.5111	5.2206
	$\rho_{qc}(10^{16}\,\frac{\text{kg}}{\text{m}^3})$	0.8905	1.1422	1.1029	1.1865
$\mu = 1$	$\rho_c(10^{17}\,\frac{kg}{m^3})$	6.7800	6.4168	5.3275	5.0561
	$\rho_{qc}(10^{16}\,\frac{\text{kg}}{\text{m}^3})$	3.4792	3.4549	2.9385	2.8335
$0.78055 \leq \mu \leq 1$	$\rho_b(10^{17}\,\frac{\text{kg}}{\text{m}^3})$	4.6807	4.5403	3.8389	3.7198

and minimum radii. The values of the densities in the center and on the surface are reported in Table 1. The generation of values of Table 1 is as follows: Given the values of mass and radius, we obtain the compactness $u = GM/c^2R$, for Eq. (20) determining the corresponding value of s. With the value of s, replacing it in Eq. (21) for the condition of stability, we find the validity interval of μ. For each value of compactness, the interval μ is different, for a greater compactness the interval is lower, with $u = 0.25093$ we have $\mu \in [0.78055, 1]$, meanwhile, with $u = 0.22448$, we associate $\mu \in [0.71519, 1]$. To be able to realize a comparison between the values of the densities in the center and on the surface, the values that we take for μ are $0.78055 \leq \mu \leq 1$. From Table 1, we can see that, for a fixed mass and radius, the values of the density of normal matter are greater for $\mu = 0.78055$ contrary to what happens for the quintessence density. Also, as the compactness increases, the density of ordinary matter also increases, the same happens for the density of the quintessence on the surface although its value in the center does not behave in such a manner. As part of the properties that the model has, there is the density of the quintessence which is zero on the surface and in Eq. (19), we have the density of the ordinary matter on the surface that does not depend on the value of μ.

The behavior of the model for the values of masses and radii considered in Table 1 are presented graphically in this part. We identify the graphs by their compactness value, so $u = 0.2509$ means that this represents the behavior of the star with mass 2.05 $M_\odot$ and radius 12.062 km, while $u = 0.2411$ means that this represents the behavior of the star with mass 1.97 $M_\odot$ and radius 12.062 km, and likewise for the other values of mass and radius presented in Table 1. From Figs. 1 and 2, the monotonically decreasing value of the density, as well as the effect of the

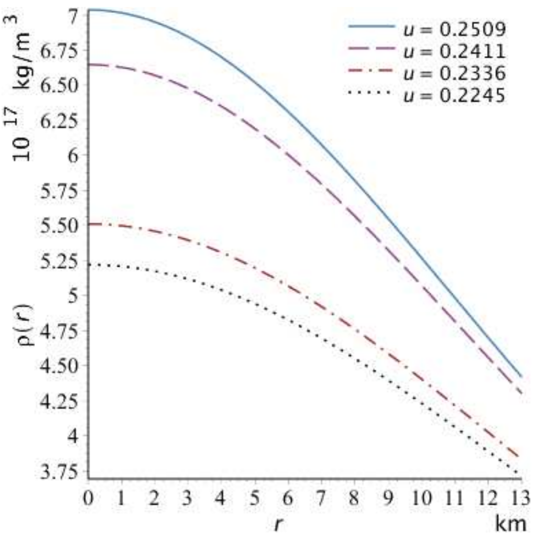

Fig. 1. The density of ordinary matter for the star PSR J0348 + 043 with $\mu = 0.78055$ considering different compactness values.

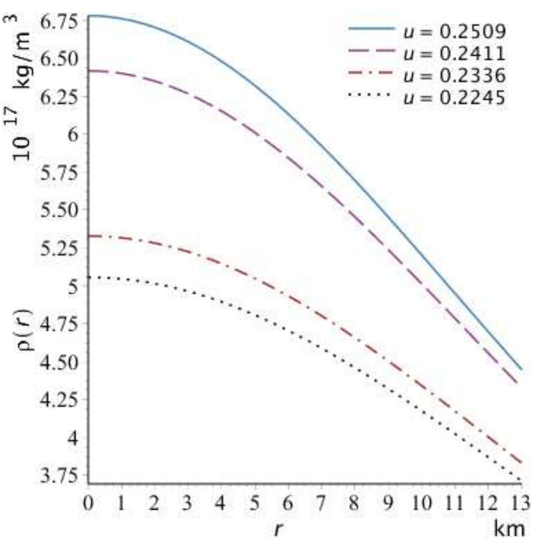

Fig. 2. The density of ordinary matter for the star PSR J0348 + 043 with $\mu = 1$ considering different compactness values.

parameter μ can be observed. As it can be seen, the difference in the density for the values of μ are reduced as the compactness lowers. The graphs in Figs. 3 and 4 show how the densities of quintessence are positive, monotonically decreasing and are zero on the surface, we also have that their values are one of the two orders of magnitude for the ordinary density. Also, for $\mu = 1$, the values remain same

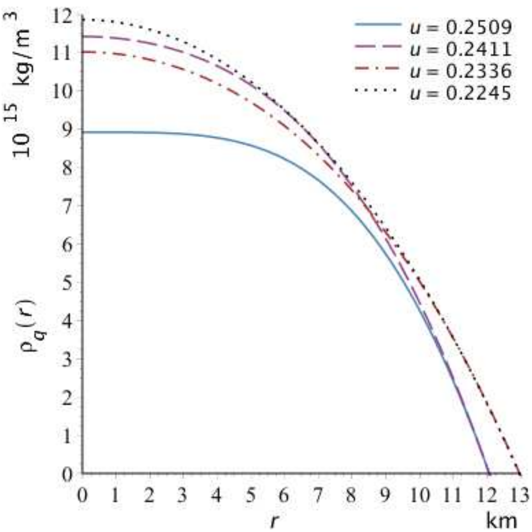

Fig. 3. In this graph, the behavior of the quintessences density for the star PSR J0348 + 043 with $\mu = 0.78055$ is represented for different values of compactness.

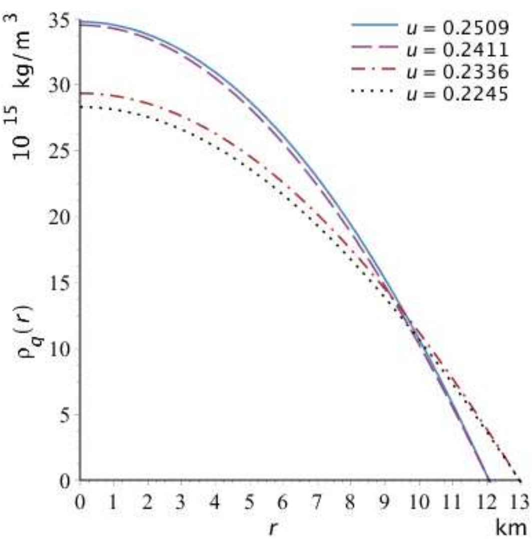

Fig. 4. In this graph, the behavior of the quintessences density for the star PSR J0348 + 043 with $\mu = 1$ is represented for different values of compactness.

for the ordinary density while for the quintessence density, it is greater when the compactness is greater, meanwhile for $\mu = 0.78055$, the opposite occurs.

In the graphs of Figs. 5 and 6, we can observe the monotonically decreasing behavior of the radial pressure and its similar behavior to the behavior of the density but with lower values than this one.

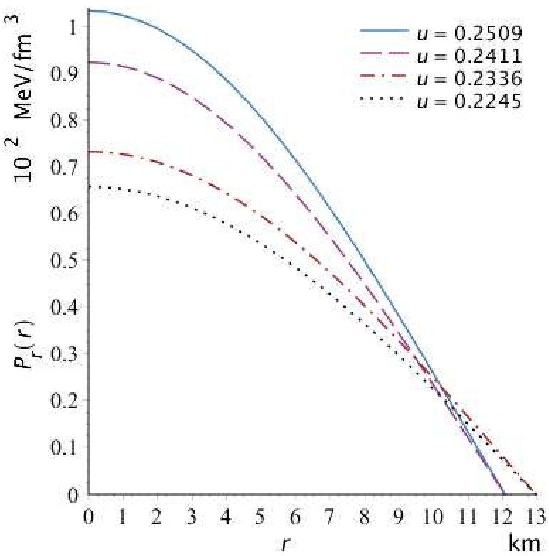

Fig. 5. Radial pressure for possible values of the compactness of the star PSR J0348 + 043 with $\mu = 0.78055$ considering different values of compactness.

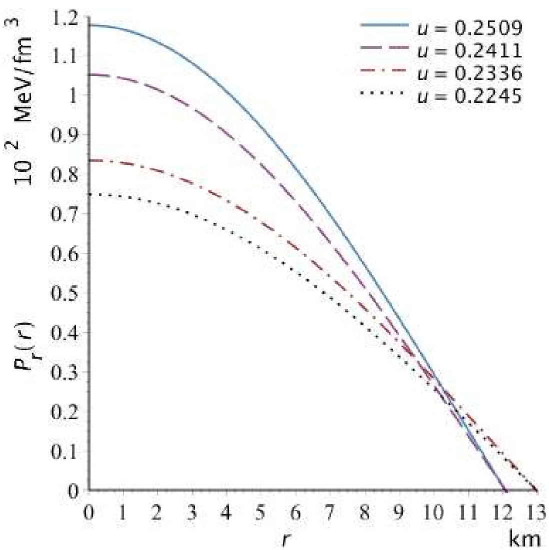

Fig. 6. Radial pressure for possible values of the mass and radius of the star PSR J0348 + 043 with $\mu = 1$ considering different values of compactness.

As it can be seen from Figs. 7 and 8, the tangential pressure is also nullified in the surface and its behavior is monotonically decreasing. For $w = -0.4$ and $\mu = 0.78055$, the values of the radial pressure are very close, meanwhile for $w = -0.4$ and $\mu = 1$, there is a notable difference in their values.

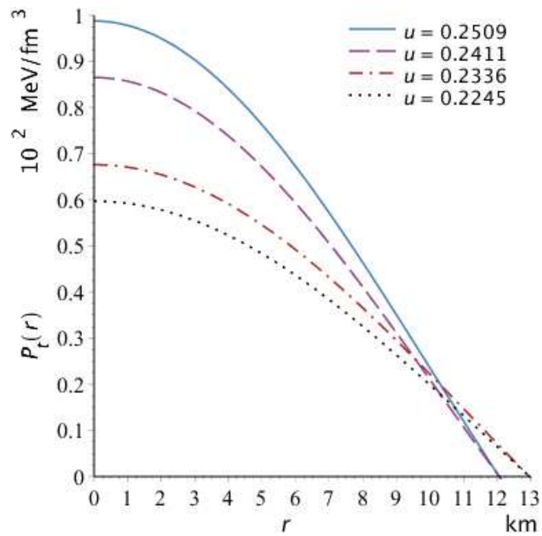

Fig. 7. Tangential pressure for the observational dates of the star PSR J0348 + 043, considering $\mu = 0.78055$ and $w = -0.4$.

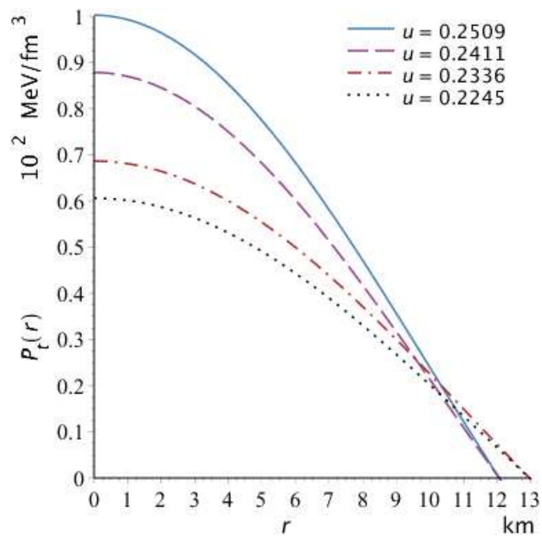

Fig. 8. Tangential pressure for the observational dates of the star PSR J0348 + 043, considering $\mu = 1$ and $w = -0.4$ considering different values of compactness.

In Figs. 9 and 10, the tangential speed of sound was graphed in dimensionless units, that is to say $v_t(r)^2/c^2 \to v_t^2(x)$ and as it can be seen, the tangential speed of sound is positive and lower than the speed of light, also it is monotonically decreasing. From this graphs and since $\mu \leq 1$, the radial speed of sound $v_r(r)^2 \leq c^2$, then the condition of $v_t^2 - v_r^2 < 0$, so that the model is potentially stable.

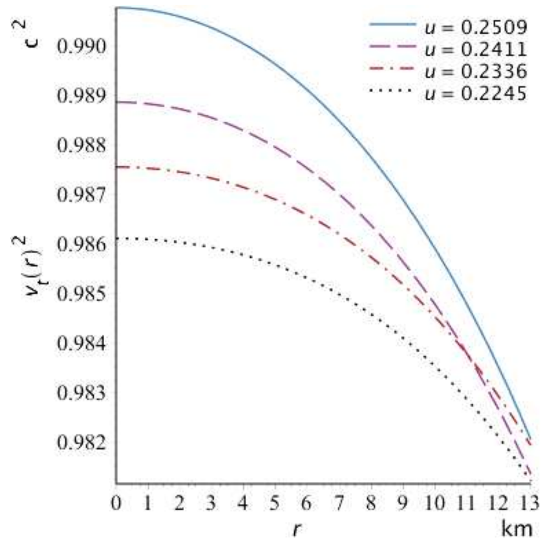

Fig. 9. Tangential speed of sound for possible values of mass and radius of the star with $\mu = 1$ and $w = -0.95$ for different values of compactness for different values of compactness.

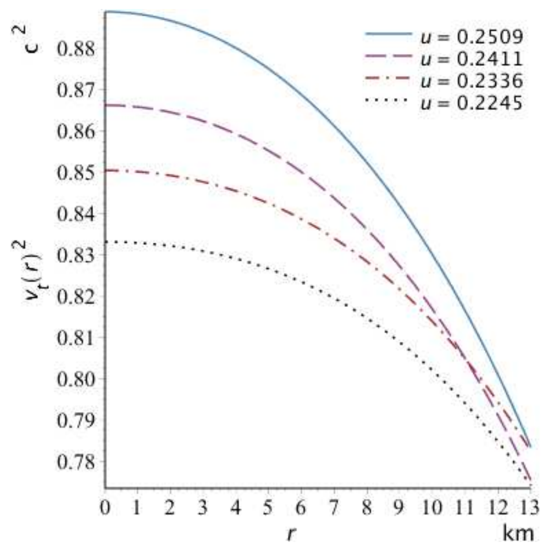

Fig. 10. Tangential speed of sound for possible values of the mass and radius of the star with $\mu = 1$ and $w = -0.4$ for different values of compactness.

5. Conclusions

In this work, we proposed and discussed a compact star of quintessence for strange stars with a compactness rate $u \leq 0.28551$ in which the state equation for the radial pressure associated to the normal matter is $P_r(\rho) = \mu c^2(\rho - \rho_b)$ and the tangential state equation $P_r(\rho) = \mu c^2(\rho - \rho_b) - \frac{3}{2}(1 + w)c^2\rho_q$ in such a way that this last one considers the quintessence effect. It is shown that for our model, each one of the functions (ρ, ρ_q, P_r, P_t) is monotonically decreasing that they are regular with their maximum value in the center of the star and each one of them is nullified on the surface with the exception of the density of the ordinary matter. The solution is obtained starting from the application, of a variant, of a theorem presented in a previous work. The geometry that we have adopted for the interior of the star is the geometry of Durgapal for $n = 5$. Through the solution, we have shown that the compactness of the star is linked to the possible values of the constants of proportionality μ in the state equation. So, this shows that the internal structure of the stars is influenced by the compactness of the star, since as we know the state equations are a consequence of the type of matter and the interaction between them, and in our case, of the specific value of μ, this is a considerable difference of our work with respect to other.[32,35] As well as that, in our model, objects with a greater compactness are characterized because μ should be close to 1, due to this, the speed of sound would be very close to the speed of light. This generates some questions on the effect of the quintessence when it interacts with the matter. Is it possible to give some physical conclusions on the properties of the quintessence and matter mixture inside the star? The presence of the quintessence elevates the speed of propagation of the sound? How is the propagation of sound modified in the presence of the quintessence? What is the relevance of the quintessence on the stability of the stellar solutions? Considering another geometry with the same state equation, is the parameter μ also limited to an interval? Also, the application of our model for the star J0348 + 0432 was obtained with the maximum value of the density of the matter which matches both at the center and on the surface to the maximum compactness with the minimum value of μ. On the other hand, for the density of quintessence, with the maximum compactness and minimum of μ, we have the minimum value. Moreover, the values of the density of ordinary matter obtained are of the expected order for stars with this compactness value, meanwhile, the values of the density of the quintessence are two orders of magnitude lower, which shows a possible difficulty for their detection, so it is necessary to propose theoretical models that will allow to directly or indirectly show its existence or its lack of it in the interior of the stars. The ease with which the solution was obtained, by the use of the theorem, gives us the opportunity of being able to build other solutions with a similar state equation or other state equations and starting from them not only representing the interior of the known stars, but also show that, according to the expectations, the parameters of the state equation are determined by the compactness or equivalently associated to the density of the star.

Acknowledgments

We thank Universidad Michoacana de San Nicolás de Hidalgo for the facilities provided during the realization of this research and we acknowledge financial support through CIC-UMSNH and PRODEP. The authors thank Dra. Tatjana Vukasinac and Itauhki Solìs Mendoza for their observations regarding this work. We also thank the reviewers for the suggestions realized, which helped to improve this paper.

References

1. M. Ruderman, *Annu. Rev. Astron. Astrophys.* **10**, 427 (1972).
2. L. Herrera and N. O. Santos, *Phys. Rep.* **286**, 53 (1997).
3. S. K. Maurya, Y. K. Gupta, S. Ray and B. Dayanandan, *Eur. Phys. J. C* **75**, 225 (2015).
4. G. Estevez-Delgado and J. Estevez-Delgado, *Mod. Phys. Lett. A* **33**, 1850081 (2018).
5. S. K. Maurya, Y. K. Gupta and M. K. Jasim, *Rep. Math. Phys.* **76**, 21 (2015).
6. M. K. Mak and T. Harko, *Proc. R. Soc. Lond. A* **459**, 393 (2003).
7. G. Estevez-Delgado and J. Estevez-Delgado, *Eur. Phys. J. C* **78**, 673 (2018).
8. P. Fuloria, *Eur. Phys. J. A* **54**, 179 (2018).
9. J. Estevez-Delgado, A. Cleary-Balderas, A. Tamez Murguía and R. Soto-Espitia, *Eur. Phys. J. Plus* **134**, 600 (2019).
10. D. Deb, S. R. Chowdhury, S. Ray, F. Rahaman and B. K. Guha, *Ann. Phys.* **387**, 239 (2015).
11. S. K. Maurya, Y. K. Gupta, T. T. Smitha and F. Rahaman, *Eur. Phys. J. A* **52**, 191 (2016).
12. K. Dev and M. Gleiser, *Gen. Relat. Gravit.* **34**, 1793 (2002).
13. S. Gedela *et al.*, *Eur. Phys. J. A* **55**, 95 (2019).
14. S. K. Maurya and S. D. Maharaj, *Eur. Phys. J. A* **54**, 68 (2018).
15. D. Deb, S. R. Chowdhury, S. Ray, F. Rahaman, *Gen. Relat. Gravit.* **50** 112 (2018).
16. S. K. Maurya, *Eur. Phys. J. A* **53**, 89 (2017).
17. J. Estevez-Delgado, J. Vega Cabrera, J. A. Rodriguez Ceballos and M. Paulin-Fuentes, *Mod. Phys. Lett. A*, accepted (2020).
18. J. Estevez-Delgado, R. Soto-Espitia, J. A. Rodriguez Ceballos, A. Cleary-Balderas and J. Vega Cabrera, *Mod. Phys. Lett. A*, doi:10.1142/S02177323205013334.
19. H. A. Buchdahl, *Phys. Rev.* **116**, 1027 (1959).
20. C. G. Boehmer and T. Harko, *Class. Quantum Grav.* **23**, 6479 (2006).
21. H. Andréasson, *J. Differ. Equations* **245**, 2243 (2008).
22. M. Mars, M. Martín-Prats and J. M. M. Senovilla, *Phys. Lett. A* **218**, 147 (1996).
23. R. L. Bowers and E. P. T. Liang, *Astrophys. J.* **188**, 657 (1974).
24. S. Tsujikawa, *Class. Quantum Grav.* **30**, 214003 (2013).
25. J. P. E. Peebles, *Astrophys. J.* **284**, 439 (1984).
26. L. Wang, R. Caldwell, J. P. Ostriker and P. J. Steinhardt, *Astrophys. J.* **530**, 17 (2000).
27. V. V. Kiselev, *Class. Quantum Grav.* **20**, 1187 (2003).
28. P. Saha and U. Debnath, *Eur. Phys. J. C* **79**, 919 (2019).
29. E. Witten, *Phys. Rev. D* **30**, 272 (1984).
30. F. Rahaman *et al.*, *Gen. Relat. Gravit.* **44**, 107 (2012).
31. M. Kalam, F. Rahaman, S. Molla and M. Hossein, *Astrophys. Space Sci.* **349**, 865 (2014).
32. J. Estevez-Delgado *et al.*, *Eur. Phys. J. Plus* **135**, 204 (2020).
33. M. Kalam *et al.*, *Astrophys. Space Sci.* **349**, 865 (2014).
34. G. Estevez Delgado *et al.*, *Eur. Phys. J. Plus* **135**, 143 (2020).
35. G. Estevez-Delgado and J. Estevez-Delgado, submitted to *Eur. Phys. J. C* (2019).
36. M. C. Durgapal, *J. Phys. A* **15**, 2637 (1982).

37. R. Wald, *General Relativity* (The University of Chicago Press, 1984).
38. A. Chodos *et al.*, *Phys. Rev. D* **9**, 3471 (1974).
39. E. Witten, *Phys. Rev. D* **30**, 272 (1984).
40. M. S. R. Delgaty and K. Lake, *Comput. Phys. Commun.* **115**, 395 (1998).
41. G. Estevez-Delgado and J. Estevez-Delgado, *Mod. Phys. Lett. A* **33**, 1850081 (2018).
42. V. Varela, F. Rahaman, S. Ray, K. Chakraborty and M. Kalam, *Phys. Rev. D* **82**, 044052 (2010).
43. G. Abbas, S. Qaisar, W. Javed and W. Ibrahima, *Can. J. Phys.* **97**, 374 (2019).
44. H. Abreu, H. Hernández and L. A. Núẽz, *Class. Quantum Grav.* **24**, 4631 (2007).
45. J. Antoniadis *et al.*, *Nature* **340**, 448 (2013).
46. X.-F. Zhao and H.-Y. Jia, *Rev. Mex. Astron. Astr.* **50**, 103 (2014).