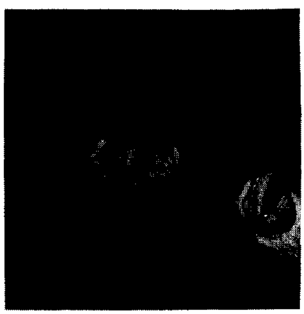
UNIVERSALITY OF QUARKS AND LEPTONS

H. FRITZSCH

CERN -- Geneva



ABSTRACT

The unification of QCD and flavour dynamics of quarks and leptons within a unified theory based on a simple gauge group is discussed. Two possibilities ("orthogonal" unification, "exceptional" unification) are especially studied.

1. INTRODUCTION

Now it is almost ten years since the first results of the electroproduction experiments at SLAC became available for theoreticians to think about. Those experimental results, and the results of the subsequent neutrino production experiments, have made it clear that quarks do not only play a fundamental role in hadron spectroscopy but that they also behave as nearly point-like objects inside hadrons, just like leptons inside atoms. Thus quarks and leptons are very much alike, and one might speculate if in some sense quarks and leptons are different editions of the same basic entity. In this talk I shall discuss some of the ideas theoreticians have discussed in order to realize such a universality of leptons and quarks.

Let me first describe the present picture of particle physics which has emerged during the last few years. We have reasons to believe that all interactions observed thus far belong to the following three clases of gauge interactions:

I	Gravity	(gauge group: Poincaré group)
II	QCD	[gauge group: $SU(3)^{colour}$]
III	QFD	(quantum flavourdynamics: weak, electromagnetic and possibly other interactions, gauge group yet unknown).

In particular we are dealing with three types of unbroken gauge interactions (gravity, QCD, and electromagnetism). On the other hand, QFD is described by a spontaneously broken gauge theory. The smallest possible gauge group of QFD is SU(2) × U(1), and the minimal set-up of elementary fermions is described by the following scheme:

$$\begin{pmatrix} u_\nu & u_g & u_b \\ d_\nu & d_g & d_b \end{pmatrix} \quad ; \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \tag{1.1}$$

where the indices r, g, b refer to the quark colour (red, green, and blue). The eight basic fermions denoted above are those fermions which constitute the observed stable particles in the world. The scheme (1.1) is not self-consistent, due to the existence of the Cabibbo angle. The latter provides a bridge to the next layer of heavier basic fermions, and the minimal set-up of elementary fermions consistent with present observations is given by the familiar four-quark/four-lepton scheme

$$\begin{pmatrix} u_\nu & u_g & u_b \\ d'_\nu & d'_g & d'_b \end{pmatrix} \qquad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$$

$$\begin{pmatrix} c_\nu & c_g & c_b \\ s'_\nu & s'_g & s'_b \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \qquad \begin{aligned} d' &= d\cos\theta_c + s\sin\theta_c \\ s' &= \perp \end{aligned} \tag{1.2}$$

Until recently it looked as if this scheme describes all basic fermions in the world. However, as we heard, for example, in last week's session on lepton physics, the existence of another heavy charged lepton with mass just below 2 GeV seems rather certain; thus one should be prepared to expand the system of elementary fermions even further.

2. QUESTIONS

What are the puzzling questions theorists come across by looking at the emerging picture of basic interactions? One question which came up first, with the discovery of the proton, is the question of charge quantization. Within the conventional SU(2) × U(1) framework the experimental fact that the proton and positron charge are equal, or the charges of the u and d quarks are $^2/_3$ and $-^1/_3$ respectively, is a complete mystery.

Another puzzling question is the question which constraint dictates the colour and flavour structure of the elementary fermions. Why $SU(3)^{colour}$, and not $SU(2)^{colour}$, or $SU(4)^{colour}$?

In the conventional picture (see above) the gauge groups of the three basic interactions commute with each other, certainly not a very appealing situation. Can we envisage a situation where those three groups emerge as subgroups of a large simple group?

Another important question is: What kind of physics dictates the mass pattern of leptons, quarks, and bosons? In the conventional gauge theory framework all masses are generated by spontaneous symmetry breaking. However, in all schemes discussed thus far by theorists the number of free parameters is comparable to the number of mass parameters; no mass relations exist. The various masses have to be adjusted; clearly a highly unsatisfactory situation. Who can do better?

3. ATTEMPTS TO QUANTIZE ELECTRICITY

3.1 Quantization within QFD

The most attractive way to obtain charge quantization would be to realize it within QFD, for example within the system of the observed weak and electromagnetic interactions. Unfortunately the pattern of those interactions as observed in nature does not allow us to realize such a possibility: The weak and electromagnetic interactions observed seem to follow an SU(2) × U(1) pattern. Can we embed the group SU(2) × U(1) in a larger simple group, in which case the charges would be quantized? The simplest way to do it is to use the group SU(3). However, in the conventional picture of QFD based on the scheme (1.2) the fermions do transform under SU(2) × U(1) as SU(2)-doublets or SU(2)-singlets in a way which does not allow an extension to

SU(3). An expansion of the fermion system is necessary. In Ref. 1 the minimal expansion of the fermion system was discussed. What is needed are two new quark flavours of charge $-1/3$ (b,h), two new charged leptons (E^-,M^-), and four new neutral leptons. The scheme is vector-like, the $SU(2)^{weak}$ representations are:

Quarks

$$\begin{pmatrix} u & c \\ d' & s' \end{pmatrix}_L , \begin{pmatrix} u & c \\ b'' & h'' \end{pmatrix}_R \quad (b''_L \,, d'_R), \; (h''_L \,, s'_R) \tag{3.1}$$

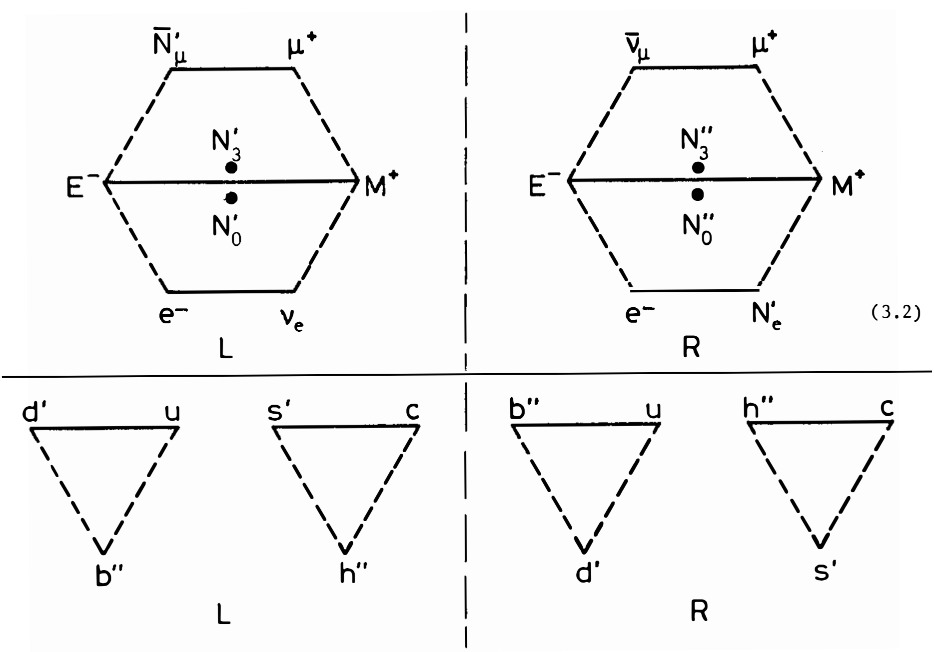
($b'' = b \cos\theta'' + h \sin\theta''$, $h'' = \perp$, θ'': right-handed analog of the Cabibbo angle)
(L: left-handed, R: right-handed)

Leptons

$$\begin{pmatrix} \nu_e & \nu_\mu \\ e^- & \mu^- \end{pmatrix}_L \begin{pmatrix} M^+ \\ N'_3 \\ E^- \end{pmatrix}_L N'_{oL} \; ; \; \begin{pmatrix} N'_e & N'_\mu \\ e^- & \mu^- \end{pmatrix}_R , \begin{pmatrix} M^+ \\ N''_3 \\ E^- \end{pmatrix}_R N''_{oR}$$

($N'_e = N_e \cos\alpha + N_\mu \sin\alpha$, $N'_\mu = \perp$; α: leptonic weak mixing angle, N_e, N_μ: massive neutral leptons, $m_{N_e}, m_{N_\mu} \gtrsim m_K$; $N'_3 = N_3 \cos\beta' + N_0 \sin\beta'$, $N'_0 = \perp$; $N''_3 = N_3 \cos\beta'' + N_0 \sin\beta''$; β', β''; triplet-singlet weak mixing angles, N_3, N_0: neutral leptons, masses unconstrained.)

The fermion scheme (3.1) can be arranged in an SU(3) scheme as follows:



(3.2)

The gauge group of the observed weak and electromagnetic interactions is regarded as the subgroup SU(2) × U(1) of SU(3); the fermions transform under $SU(3)^{colour} \times SU(3)^{flavour}$ as follows:

$$\begin{array}{ll} \text{quarks} : & 2 \times (3^c, 3) \\ \text{leptons} : & (1^c, 8) \end{array} \tag{3.3}$$

Within the SU(3) scheme the charges are quantized: The integral charges of the leptons and the fractional charges of the quarks are obtained as a consequence of the octet structure of leptons and the triplet structure of the quarks. The SU(2) × U(1) mixing angle θ_W (unrenormalized) is 60° ($\sin^2 \theta = 3/4$), i.e. the neutral current is a pure axial vector. This is excluded by experiment; thus sizeable renormalization effects must exist such as to give $\sin^2 \theta_W \lesssim 0.6$ which is required by experiment (for a detailed discussion of the phenomenological constraints see Ref. 2). The masses of the $SU(3)^{flavour}$ octet bosons, except the ones belonging to the SU(2) × U(1) subgroup of the conventional weak and electromagnetic interactions, must be very heavy implying that the interactions caused by them are negligible for present day phenomenology.

3.2 Leptons as the fourth colour

Another way to obtain charge quantization is to enlarge QCD by including leptons as "the fourth colour", following an idea of Pati and Salam[3]. [Note, however, that those authors base themselves on unconfined colour [$SU(3)^{colour}$ is a broken gauge group], and the quarks are assumed to have integral charges. I shall follow the route described in Ref. 4.] Of course, the leptons are not really a fourth colour, in which case they would be confined like the quarks. What is meant is that $SU(3)^{colour}$ is an unbroken subgroup of $SU(4)^c$, the latter being a broken gauge group. The fermion scheme can be described as follows:

$$\underbrace{\begin{pmatrix} u_\nu & u_g & u_b & \vdots & \nu_e \\ d'_\nu & d'_g & d'_b & \vdots & e^- \end{pmatrix}_L}_{SU_4} \Big\} SU_2^{weak} \tag{3.4}$$

[and an anologous set-up for the (c-s'; ν_μ-μ^-) system].

Suppose we start out from the gauge group $SU(4)^c \times SU(2)^L \times SU(2)^R$, where $SU(2)^{L(R)}$ are groups under which the quark and lepton flavours (left-handed, right-handed respectively) transform as doublets. (Since no right-handed charged weak currents are observed, the boson $W_R^\pm$ must aquire relatively high masses). The electric charge can be written as

$$Q^e = \begin{pmatrix} 2/3 & 2/3 & 2/3 & \vdots & 0 \\ -1/3 & -1/3 & -1/3 & \vdots & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & \vdots & 1 \\ -1 & -1 & -1 & \vdots & -1 \end{pmatrix} + \frac{1}{6}\begin{pmatrix} 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & -3 \end{pmatrix} = T_3^{L+R} + Y \quad (3.5)$$

where Y is the so-called lepton-quark hypercharge [it is the 15-generator of SU(4)]. The interesting feature of Eq. (3.5) is that the electric charge is the sum of two generators of non-Abelian groups; thus the charges are quantized. Note, however, that there is still one flaw in Eq. (3.5). Since we are dealing with the semi-simple group $SU(4) \times SU(2)^L \times SU(2)^R$, which allows two independent coupling constants, if we impose "parity invariance" of the Lagrangian, there is in general an arbitrary parameter in the expression of the electric charge. In general we have

$$Q^e = T_3^{L+R} + \alpha \cdot Y ; \quad (3.6)$$

the ratio $Q^e(\text{u-quark})/Q^e(e^-)$ is not $(2/3)/-1$, but $[(3 + \alpha)/(3 + 3\alpha)]/-1$, and the neutrino charge is not zero, but $\frac{1}{2}(1 - \alpha)$. Thus within the $SU(4) \times SU(2)^L \times SU(2)^R$ scheme the charges come out correctly only if we impose in addition $Q(\nu_e) = Q(\nu_\mu) = 0$ $(\alpha = 1)$.

The fifteen gauge bosons belonging to $SU(4)^c$ transform under $SU(3)^{\text{colour}}$ as $(\underset{\sim}{15}) = (\underset{\sim}{8}) + (\underset{\sim}{3}) + (\bar{\underset{\sim}{3}}) + 1$; $(\underset{\sim}{8})$: gluons, $(\underset{\sim}{3})$, $(\bar{\underset{\sim}{3}})$: leptoquark bosons, $(\underset{\sim}{1})$: Y. The present phenomenological constraints on the leptoquark boson masses are not very strong ($\gtrsim$ 100 GeV). Leptoquark exchange can lead to proton decay[3], in which case they have to be slightly more massive as indicated above. However, proton decay is *not* a necessary consequence of the $SU(4)^c$ scheme. There exist ways to break the gauge symmetry such that baryon number is exactly conserved (see Ref. 4 and Ref. 5).

4. UNIFICATION OF QCD AND QFD

The ultimate goal of physics would be to unify all interactions by some perhaps relatively large simple gauge group. Today not even an ansatz in this direction exists; all attempts made in this direction have failed due to the enormous conceptual difficulties to incorporate gravity. Thus the best we can do at the moment is to leave out the gravitational interaction, reserving to include it at some later time, and try to construct theories which unite QCD and QFD within a simple group. Many models have already been constructed. There is no point to review all of them here. What I will do instead is to describe two canonical ways to achieve a unification, both based on the two different schemes discussed in the previous section.

Let me first mention one feature of unified theories which is of phenomenological importance and does not depend on the specific group. If we choose a subgroup SU(2) × U(1) of a simple group and declare it to be the gauge group of the observed weak and electromagnetic interactions, the SU(2) × U(1) mixing angle is fixed to[4)]

$$\sin^2 \theta_W = \frac{\mathrm{tr}\ T_3^2}{\mathrm{tr}\ Q_e^2} \tag{4.1}$$

where the summation is to be carried out over all left-handed and right-handed fermion fields (including colour). For example, in the scheme (1.1) and (1.2) one obtains $\sin^2 \theta_W = 3/8$. The result (4.1) describes the SU(2) × U(1) mixing angle in the energy region where the unification becomes relevant. Thus it cannot be applied blindly for phenomenological purposes, since relatively big renormalization effects can occur. There exist theories where the latter are really substantial, while in other theories essentially no renormalization is present[4,5)].

4.1 Orthogonal unification

Let us return to the $SU(4)^c \times SU(2)^L \times SU(2)^R$ scheme discussed in Section 3. How can we unify this direct product of three groups such that the correct fermion pattern results?

Let me use the identification SU(4) ∿ SO(6); SU(2) × SU(2) ∿ SO(4). Thus we realize one way of embedding $SU(4) \times SU(2)^2$ in a simple group[4)]

$$SU_4^c \times SU_2^L \times SU_2^R \sim SO_6 \times SO_4 \subset SO_{10} \tag{4.2}$$

Do we obtain the desired fermion content? Yes, if we choose the complex $(\underset{\sim}{16})$ representation of SO(10), which decomposes under the subgroup $SU(4)^c \times SU(2)^L \times SU(2)^R$ as follows

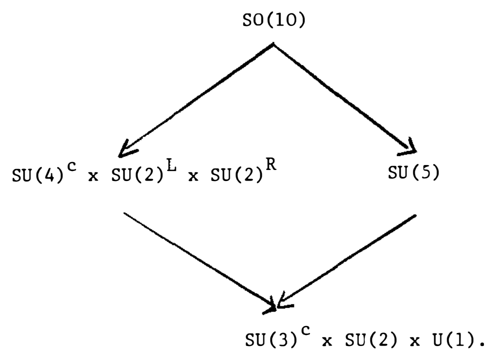
$$(16) = (4, 2, 1) + (\bar{4}, 1, 2) \tag{4.3}$$

These are one quark-lepton doublet and the corresponding antiparticles. The fermion scheme (1.2) can be described by two $(\underset{\sim}{16})$ representations of SO(10)

$$\begin{pmatrix} u & u & u & \nu_e & \nu_e & \bar{u} & \bar{u} & \bar{u} \\ d' & d' & d' & e^- & e^+ & \bar{d}' & \bar{d}' & \bar{d}' \end{pmatrix}, \begin{pmatrix} c & c & c & \nu_\mu & \nu_\mu & \bar{c} & \bar{c} & \bar{c} \\ s' & s' & s' & \mu^- & \mu^+ & \bar{s}' & \bar{s}' & \bar{s}' \end{pmatrix}$$

Note that right-handed neutrinos (left-handed antineutrinos) have to be added.

The breaking of the SO(10) gauge symmetry can proceed in two different ways, given by the following diagram



The subgroup SU(5) results, if we go along the chain on the right-handed side; the (16) decomposes under SU(5) into: $(\underset{\sim}{16}) = (\underset{\sim}{10}) + (\bar{\underset{\sim}{5}}) + (\underset{\sim}{1})$. In this way we arrive at the SU(5) theory of Ref. 7 as a subtheory of the SO(10) scheme.

4.2 Exceptional unification

What kind of unification can one discuss for the $SU(3)^c \times SU(3)^{flavour}$ scheme discussed in Section 3? We must look for gauge groups which provide "natural" embeddings of SU(3) groups. Such groups are the exceptional groups, and we shall study below various lepton-quark schemes based on the exceptional groups (G_2, F_4, E_6, E_7, E_8). The pioneering work in this direction has been done by the Yale group[8]. For details on the group theory aspects we refer to the mathematical literature[9].

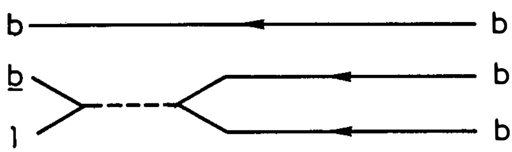
The group G_2

This group being of rank 2 is, of course, too small in order to be useful as a gauge group for a unified theory of the strong, electromagnetic, and weak interactions. Nevertheless a gauge theory based on G_2 is an interesting example since one can display here in a simplified manner some of the problems one is dealing with in case of the other exceptional groups.

The group G_2 has as its maximal subgroup of maximal rank the group SU(3). Let us identify this group with the colour group $SU(3)^c$. We suppose that the gauge group G_2 is broken such that $SU(3)^c$ is left as an unbroken subgroup

$$\begin{array}{c} G_2 \\ \downarrow \\ SU_3^c \end{array}$$

The basic representation 7 transforms under $SU(3)^c$ as $7 = 3 + \bar{3} + 1$, i.e. it consists of a quark triplet q, antiquark triplet $\bar{q}$ and a lepton. (Majorana lepton.). The adjoint representation decomposes as $14 = 8 + 3 + \bar{3}$. Besides the colour generators (coupled to the gluons) one has six generators which have both the quantum numbers of a diquark (qq) and a leptoquark ($\bar{q}\ell$), i.e. a diquark can annihilate into an antiquark and a lepton via the diagram displayed below:



This feature (which is a general feature of the exceptional groups) implies not only that there exists no baryon number generator in the G_2 scheme, but that a three quark colour singlet system like the proton can decay in second order of the gauge coupling. The only way to suppress this decay is to choose a very high mass M for the corresponding gauge boson; the experimental limit on the proton lifetime of $\sim 10^{30}$ years implies $M \gtrsim 10^{18}$ GeV.

Higher exceptional groups

All exceptional groups contain the subgroup G_2, and therefore the colour group $SU(3)^c$. In the following table we display the decompositions of the various exceptional groups into $SU(3)^c$ × flavour group as well as indicate the transformation properties of the basic and adjoint representations.

In the previous section we have argued that a possible description of the strong, electromagnetic and weak interactions is one based on the group $SU(3)^c \times SU(3)^{flavour}$. Needed for such a description were six quark flavours and eight leptons:

$$2 \times (3^c, 3) + (1,8) + 2 \times (\bar{3}^c, \bar{3}) + (1,8)$$

Table 1

Group G	$G \supset SU(3)^c \times ?$	Basic representation and decomposition under subgroup	Adjoint representation and decomposition under subgroup
G_2	$SU(3)^c$	$7 = 3^c + \bar{3}^c + 1$	$14 = 8^c + 3^c + \bar{3}^c$
F_4	$SU(3)^c \times SU(3)$	$26 = (3^c,3) + (\bar{3}^c,\bar{3}) + (1^c,8)$	$52 = (8^c,1) + (1^c,8) + (3^c,\bar{6}) + (\bar{3}^c,6)$
E_6	$SU(3) \times SU(3) \times SU(3)$	$27 = (3^c,3,1) + (\bar{3}^c,1,\bar{3}) + (1^c,\bar{3},3)$	$78 = (8^c,1,1) + (1^c,8,1) + (1^c,1,8) + (3^c,\bar{3},\bar{3}) + (\bar{3}^c,3,3)$
E_7	$SU(3)^c \times SU(6)$	$56 = (3^c,6) + (\bar{3}^c,\bar{6}) + (1^c,20)$	$133 = (1^c,35) + (8^c,1) + (3^c,15) + (\bar{3}^c,\overline{15})$
E_8	$SU(3)^c \times E(6)$	$248 = (1^c,78) + (8^c,1) + (3^c,27) + (3^c,\overline{27})$	

Below we shall discuss the embedding of the SU(3) × SU(3) scheme into the various schemes based on the exceptional groups. We emphasize that all gauge theories based on the exceptional groups are anomaly free. This is easy to see for all exceptional groups except E_6, since those have only real representations. Some of the representations of E_6 are complex (e.g. 27, 351); however, also if these representations are involved as fermion representations, the corresponding theory is anomaly free[10].

The group F_4: F_4 is the smallest exceptional group which contains the desired flavour group SU(3). Since the basic representation 26 contains three quarks and antiquarks and a lepton octet, we need two 26 representations for a minimal realistic scheme (altogether six quarks and antiquarks, one lepton and one antilepton octet). Thus we obtain just the fermion content as required

$$\begin{aligned} \underset{\sim}{26} + \underset{\sim}{26} &= (3^c, 3) + (3^c, 3) + 1{,}8 + \text{antiparticles} \\ &= (u, d', b'') + (c, s', h'') + \text{leptons} \\ &\qquad + \text{antiparticles} \end{aligned} \tag{4.4}$$

As in the G_2 scheme, the unifying interactions in the F_4 scheme cause the decay of the proton into leptons in second order of the gauge coupling, thus superheavy bosons are necessary.

The group E_6: Here the basic representation is 27 dimensional. The minimal scheme one can construct is essentially equivalent to the F_4 scheme discussed above, if we interpret the direct sum SU(3) of the two SU(3) groups in the flavour group SU(3) × SU(3) as the gauge group for the weak and electromagnetic interactions. In this case the 27 representation decomposes into $(1^c,8) + (1^c,1) + (3^c,3) + (\bar{3}^c,\bar{3}^c)$; {one new neutral lepton [SU(3)-singlet] has to be added to the fermion content of the F_4 scheme}. Two 27 representations of E_6 provide us with six quarks and one lepton octet.

The group E_7: The fermion representation is 56 dimensional, the flavour group is SU(6). We interpret the group $SU(3)^{\text{flavour}}$ as the SU(3) subgroup of the flavour group, in which case the 56 representation decomposes under $SU(3)^c \times SU(3)^{\text{flavour}}$ as follows:

$$56 = 2 \times \left[(3^c, 3) + (\bar{3}^c, \bar{3}) + (1^c, 8) + 2\,(1^c, 1)\right]$$

We obtain the desired fermion content (six quarks, one lepton octet) plus four neutral lepton states. The remarkable feature of the E_7 scheme is that here we need only one irreducible fermion representation. Of course, within this scheme superheavy gauge bosons are required in order to cure the problems of proton decay.

<u>The group E_8</u>: E_8 is the only simple group, for which the basic and the adjoint representation coincide. Thus it is required that the boson and fermion representation of the E_8 gauge theory have the same structure with respect to the internal symmetry group, and *E_8 is therefore especially suited for the formulation of a supersymmetric theory of quarks and leptons.*

In case of E_8 the flavour group is E_6. The fermion representation decomposes under $SU(3)^c \times E(6)$ as follows:

$$248 = (1^c, 78) + (8^c, 1) + (3^c, 27) + (\bar{3}, \overline{27})$$

thus the fermion content of the theory consists of 78 leptons, 27 quark flavours and eight colour octet "quarks". The latter are the fermion counterparts of the colour octet gluons. Note that these colour octet quarks are singlets under the flavour group, i.e. they would not participate in the electromagnetic and weak interactions.

The colour octet quarks as well as 23 of the "normal" quark flavours must be relatively heavy: the observed hadron spectrum can be described in terms of only four quark flavours.

Since the flavour group E_6 is of rank 6, there are many possibilities to associate the generators of the electric charge with one of the self-adjoint generators of E_6. Consequently there exists a considerable amount of freedom to assign the electric charges to the various quarks and leptons. In order to be specific, let us consider a particular scheme for the electromagnetic and weak interactions. We start from the subgroup $SU(6) \times SU(2)$ of E_6. The decomposition of the basic representation of E_8 under the subgroup $SU(3) \times SU(6) \times SU(2)$ is

$$\begin{aligned} 248 &= (3^c, 6, 2) + (\bar{3}^c, 6, 2) + (1^c, 1, 3) + (1^c, 35, 1) \\ &= (8^c, 1, 1) + (1^c, 20, 2) + (3^c, 15, 1) + (\bar{3}^c, \overline{15}, 1) \end{aligned}$$

Let us further assume that the $SU(3)^{flavour}$ subgroup needed for the description of the weak and electromagnetic interactions is an SU(3) subgroup of SU(6). The 248 representation can be decomposed under $SU(3)^c \times SU(3)^{flavour}$ as follows

$$\begin{aligned} 248 = {} & 7 \cdot \left[(3^c, 3) + (\bar{3}^c, \bar{3})\right] + 8\,(1^c, 8) + 14\,(1^c, 1) \\ & + (8^c, 1) + (3^c, \bar{6}) + (\bar{3}^c, 6) \end{aligned}$$

Thus we obtain:

21 quark flavours with the charges $2/3$, $-1/3$, $-1/3$,

6 quark flavours transforming as the 6 representation of $SU(3)^{flavour}$. These quarks have unconventional electromagnetic and weak charges -- in particular the charge $4/3$ appears.

14 SU(3)-singlet leptons: they are neutral and uncoupled from the "normal" weak interaction.

8 $SU(3)^{flavour}$ octets of leptons, among those 32 leptons of charge -1.

Obviously the E_8 scheme is a very complex system, containing about eight times more fermions as the 32 fermions needed for the contemporary phenomenology of hadrons and leptons. At the present time it might look ridiculous to view such a scheme as a realistic one. However, if future experiments should reveal the existence of many new quark flavours and leptons as well as of new interactions, the E_8 scheme might have to be regarded as a serious possibility to describe the pattern of leptons and quarks in nature.

5. OUTLOOK

In this lecture I could only discuss a few aspects of a possible understanding of QCD and QFD within a larger scheme of interactions. One of those aspects I tried to emphasize was the proliferation of the number of quarks and leptons, which is a feature of many schemes. How can a physicist who is trying to reduce the number of elementary constituents of nature to a minimal number live with this fact? One way to understand the large number of leptons and quarks would be to interpret them in some sense as composite objects. However at present there exists no evidence for a substructure of the fermions: no excitations of leptons and quarks with higher angular momenta and different parity have been observed. The fact that the anomalous magnetic moments of the electron and muon agree to very high accuracy with the values predicted in QED suggests that the electron and muon have no internal structure. If a substructure of quarks and leptons exists, it is presumably one occurring at a more sophisticated level than the kind of substructures studied in atomic, nuclear and hadron physics.

Another, and perhaps more convincing way to live with a large number of "elementary" fermions is to change our attitude towards the physics of leptons and quarks. It could well be, as emphasized by Abdus Salam, that even on the level of the "elementary" constituents nature does the same as anywhere else: It is rich in structure, but based on very few principles. Thus, if we regard principles, e.g. the gauge principle, a particular gauge group, etc. as the "elementary" constituents of nature, we need not worry about how many leptons and quarks exist -- be it 32 or 248.

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APPENDIX

BASIC PROPERTIES OF THE EXCEPTIONAL GROUPS

Group G_2: The group G_2 is the group of all automorphisms of the octonion algebra (see, for example, Ref. 9). Below we summarize some properties of G_2.

Fundamental representations: 7, 14 (adjoint representation 14).
Rank: 2

$$7 \times 7 = 1 + 7 + 7 + 14 + 20$$

The other exceptional groups (F_4, E_6, E_7, E_8) are the automorphism groups of the Jordan algebras one can construct using the matrices of the type

$$\Omega = \begin{pmatrix} \alpha & \omega_3 & \omega_2 \\ \overline{\omega}_3 & \beta & \omega_1 \\ \overline{\omega}_2 & \overline{\omega}_1 & \gamma \end{pmatrix}$$

(α, β, γ: real; $\omega_1, \omega_2, \omega_3$: octonions).

We mention the following properties of these groups.

Group F_4: Fundamental representations: 26, 52, 273, 1274 (adjoint representation 52).
Rank: 4

$26 \times 26 = 1 + 26 + 52 + 273 + 324$.

Group E_6: Fundamental representation 27, 78, 351, 351′, 2925 (adjoint representation: 78). Note: E_6 is the only exceptional group which has complex representations, e.g. 27, 351
Rank: 6

$27 \times 27 = \overline{27} + 351 + 351'$
$27 \times \overline{27} = 1 + 78 + 650$

Group E_7: Fundamental representations: 56, 133, 912, 1539, 8645, 27664, 355750 (adjoint representation: 133).
Rank: 7

$56 \times 56 = 1 + 133 + 1539 + 1463$
$E_7 \supset E_6 \times U_1$: $56 = 1 + 1 + 27 + \overline{27}$

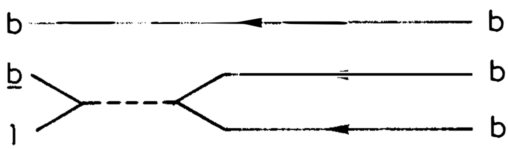
Group E_8: Fundamental representations: 248, 3875, 30, 380, 147, 250, 2 450 240, 6 696 000, 146 325 270, 6 899 079 264 (adjoint representation: 248).

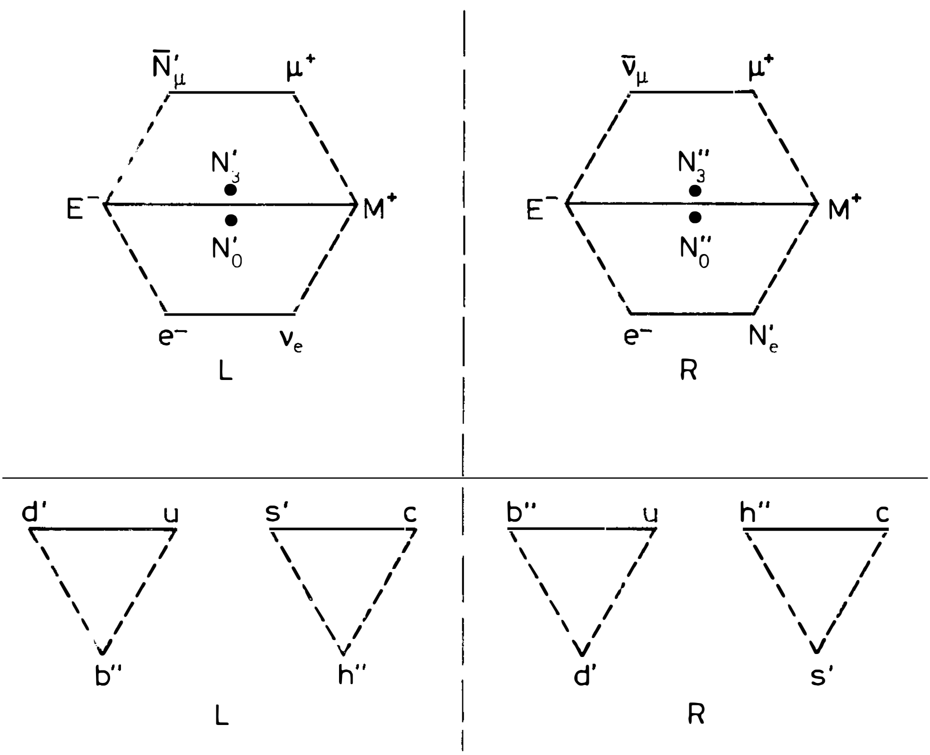
Note: Basic and adjoint representation coincide.

Rank: 8

$$248 \times 248 = 1 + 248 + 3875 + 30\ 380 + 27\ 000$$

$$E_8 \supset E_7 \times SU(2):\quad 248 = (56,2) + (133,1) + (1,3)$$





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