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On Planetary Orbits, Ungravity and Entropic Gravity

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Abstract: In previous works, entropic gravity and ungravity have been considered as possible solutions to the dark energy and dark matter problems. To test the viability of these models, modifications to planetary orbits are calculated for ungravity and different models of entropic gravity. Using the gravitational sector of unparticles, an equation for the contribution to the effect of orbital precession is obtained. We conclude that the estimated values for the ungravity parameters from planetary orbits are inconsistent with the values needed for the cosmological constant. The same ideas are explored for entropic gravity arising from a modified entropy–area relationship.

Keywords: ungravity; entropic gravity; RG classical tests

1. Introduction

General Relativity (GR) has been a successful theory since it was postulated by Einstein in 1915. The first solution of the Einstein field equations by Schwarzschild and the first experimental proof of the effects described by this theory, such as the light deflection observation by Eddington, came soon after it was published. Since then, GR has been successfully tested in several phenomena in nature, from simple low-energy systems, such as the orbital precession in the solar system, to more complex high-energy systems, such as neutron stars or black holes. However, some effects remain unexplained, such as the case of dark matter and dark energy [1,2], which we have concluded to be essential components of the universe, but until now we have been unable to describe the mechanisms involved in observations or find an appropriate frame to describe them. Also, the recent experimental confirmation of the existence of black holes [3] forces us to understand the most fundamental aspects of gravity. Moreover, black holes are gravitational systems in which quantum effects can be important, and due to our ignorance of quantum gravity, alternative approaches must be considered. For example, in the semiclassical approach, some macroscopic effects with information about the hidden quantum degrees of freedom exist. Such proposals were made by Bekenstein, Hawking and Unruh in the 1960s. Then, following the analogy between gravity and thermodynamics, Jacobson wrote Einstein's equations as an equation of state [4]. The consideration of gravity as an emergent phenomenon allows the use of the statistical mechanics framework to study this interaction. This idea was revived in [5], where it was proposed that Newtonian gravity is an entropic force, analogous to emergent forces in the study of polymers. The motivation is based on the idea of holography and its relation to black holes. In this formulation, one can propose modifications to Newtonian gravity by analyzing modifications to the entropy–area relationship. This approach to gravity has been used to study several gravitational phenomena in connection to anomalous galactic rotation curves [6], late time acceleration of the universe and dark energy [7,8], or black hole quasinormal modes [9]; different modifications to the Hawking–Bekenstein entropy–area relationship are used to modify either the Newtonian equation for gravity or the Friedmann equations and therefore study the effects and modifications to GR.

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In a completely different context from entropic gravity and using different physical principles, one can derive another type of modified gravity. We can start by considering higher-energy extensions to the Standard Model. That is the case of unparticles, a scale-invariant hidden sector of the Standard Model proposed in [10]. The components of this sector, called unparticles (unlike particles), have a continuous mass spectrum and a characteristic energy for interactions with SM particles [11]. In the context of gravity, one can understand ungravity as the result of ungraviton interactions. To introduce the effects of unparticle physics on gravity, one adds an unparticle term to the Hilbert–Einstein action. In [12], the authors studied the ungravity counterpart of the Schwarzschild black hole. Moreover, using ungravity's temperature and entropy, the ungravity sector effects have been studied in cosmology, allowing to relate ungravity parameters with the cosmological constant value [13]. In order to see if this result is consistent with other gravitational scenarios, we can compare it to planetary motion. Therefore, the main goal of this paper is to determine if ungravity and entropic gravity are consistent with observations.

The paper is arranged as follows: In Section 2, we obtain orbital precession from a modified Schwarzschild ungravity metric. In Section 3, a brief review of Newtonian entropic gravity is presented, and modifications to orbital precession are calculated. Corrections are obtained from a generalized entropy–area relationship. As in the ungravity case, the corrections are evaluated using solar system data. Finally, Section 4 is devoted to discussion and final remarks.

2. Ungravity Contributions to the Orbital Precession

In this section, we consider the unparticle generalization. This theory is known as ungravity and is constructed by considering ungraviton interactions. The action is constructed as the sum of the Einstein–Hilbert action, the matter action and the effective action S_U for the ungravitons. The ungravity action [14] is given by

$$S_U = \frac{1}{2k^2}\int d^4x\sqrt{g}R\left[1+\frac{A_{d_U}}{(2d_U-1)\sin\pi d_U}\frac{\kappa_*^2}{\kappa^2}\left(\frac{-D^2}{\Lambda_U^2}\right)^{1-d_U}\right]^{-1}, \quad (1)$$

where D^2 is the D'Alemberteian, κ_* is the ungravitational Newton constant, $\kappa = 16\pi G_N$ and A_{d_U} is a constant.[1] The modified Einstein equations are

$$G_{\mu\nu} = \kappa^2 T^{\mu}{}_{\nu} + \kappa_*^2\frac{A_{d_U}}{\sin\pi d_U}. \quad (2)$$

Solving for the Schwarzschild black hole, one obtains the ungravity Schwarzschild metric [12]

$$g_{rr}^{-1} = -g_{00} = 1 - \frac{2MG_N}{r}\left[1+\kappa_*^2\frac{A_{d_U}}{\sin(\pi d_U)}\frac{M(r)}{2MG_N}\right] \quad (3)$$

where

$$M(r) = \frac{2^{2d_U-2}}{4\pi^{1/2}}\frac{\Gamma(d_U-1/2)}{\Gamma(2-d_U)}M\Lambda_U^{2-2d_U}\left(\frac{1}{r}\right)^{2d_U-2}. \quad (4)$$

This leads to the following modified metric:

$$g_{rr}^{-1} = -g_{00} = 1 - \frac{2G_N M}{r}\left[1-\left(\frac{R_G}{r}\right)^{2d_u-2}\right], \quad (5)$$

where R_G is related to the ungravity parameters [10,12] as

$$R_G = \frac{1}{\pi\Lambda_u}\left(\frac{M_{pl}}{2M_u}\right)^{1/(d_u-1)}\left(\frac{1}{2\pi}\frac{\Gamma(d_u+\frac{1}{2})\Gamma(d_u-\frac{1}{2})}{\Gamma(2d_u)}\right)^{1/(2du-2)}. \quad (6)$$

This term can be understood as the length scale at which unparticle effects will be relevant and is a free parameter of the model, together with Λ_u, the characteristic energy of the model; M_u, the characteristic mass; d_u, the dimension of the extra operators in the action; and Planck's mass M_{pl}.

Let us now calculate the orbital precession using the ungravity Schwarzschild metric. Considering a particle near a spherically symmetric gravitational field, the geodesic equation is

$$\begin{aligned} 2K &= \left(1 - \frac{2GM}{r}\left[1 - \left(\frac{R_G}{r}\right)^{2d_u-2}\right]\right)\dot{t}^2 \\ &- \left(1 - \frac{2GM}{r}\left[1 - \left(\frac{R_G}{r}\right)^{2d_u-2}\right]\right)^{-1}\dot{r}^2 - r^2\dot{\theta}^2 - r^2\sin^2\theta\dot{\phi}^2, \end{aligned} \tag{7}$$

where M is the gravitational mass, and the dots denote a derivative with respect to the proper time (also, we set $c = 1$). The above equation is solved taking $2K = 1$, as we are considering time-like geodesics. The Euler–Lagrange equation leads to the following conserved quantities:

$$q \equiv \left(1 - \frac{2GM}{r}\left[1 - \left(\frac{R_G}{r}\right)^{2d_u-2}\right]\right)\dot{t}, \qquad h \equiv r^2\sin^2\theta\dot{\phi}. \tag{8}$$

When considering planar motion $\theta_0 = \frac{\pi}{2}$, the conserved quantity h can be identified with angular momentum per unit mass, in analogy with the usual Kepler problem. An equation of motion is obtained in terms of the constant q, which is related to the conservation of energy. Using the change of variable $u = 1/r$ in this θ_0 plane, we obtain

$$\left(\frac{du}{d\phi}\right)^2 + u^2 = \frac{q^2-1}{h^2} + 2GM\left(\frac{u}{h^2} + u^3\right)\left[1 - (uR_G)^{2d_u-2}\right]. \tag{9}$$

Finally, differentiation with respect to ϕ together with $\mu \equiv GM$ gives rise to a modified Binet equation,

$$\frac{d^2u}{d\phi^2} + u = \frac{\mu}{h^2}\left[1 + 3u^2h^2 - (2d_u-1)(uR_G)^{2d_u-2} - (2d_u+1)h^2R_G^{2d_u-2}u^{2d_u}\right]. \tag{10}$$

Using the parameter $\varepsilon \equiv 3\mu^2/h^2$ to solve perturbatively, we propose the solution[2] $u = u_0 + \varepsilon\, u_1$. The zeroth-order equation $u_0'' + u_0 = \mu/h^2$ has the usual conic section solution, and the first-order differential equation is

$$\begin{aligned} u_1'' + u_1 =& \frac{\mu}{h^2}(1 + e\cos\phi)^2 - \frac{\mu^{2d_u-3}}{3h^{4d_u-4}}(2d_u-1)R_G^{2d_u-2}(1 + e\cos\phi)^{2d_u-2} \\ &- \frac{\mu^{2d_u-1}}{3h^{4d_u-2}}(2d_u+1)R_G^{2d_u-2}(1 + e\cos\phi)^{2d_u}, \end{aligned} \tag{11}$$

where u_1'' denotes derivatives with respect to ϕ, and e denotes the orbital eccentricity. The first term in the right-hand side can be identified as the usual GR contribution to the Kepler problem. After setting[3] $d_u = \frac{3}{2}$, we solve the equation above by considering only linear contributions in ϕ. Then, the solution to the first-order differential equation is

$$u_1 = \left(\frac{\mu}{h^2} - \frac{R_G}{3h^2} - \frac{2R_G\mu^2}{h^4} - \frac{R_G\mu^2}{2h^4}e^2\right)e\phi\sin\phi. \tag{12}$$

Finally, the complete solution $u = u_0 + \varepsilon u_1$ is

$$u = \frac{\mu}{h^2}\{1 + e\cos[\phi(1-\delta)]\}, \tag{13}$$

where

$$\delta \equiv \frac{3\mu^2}{h^2} - \frac{R_G\mu}{h^2} - \frac{6R_G\mu^3}{h^4} - \frac{3R_G\mu^3 e^2}{2h^4} \ll 1. \tag{14}$$

The orbital precession is calculated by taking a complete period $\phi_T(1-\delta) = 2\pi$ such that the extra contribution represents the precessed angle. In this case, the ungravity contribution to the precession on each revolution is

$$|\Delta\phi_u| = 2\pi\left(\frac{R_G\mu}{h^2} + \frac{6R_G\mu^3}{h^4} + \frac{3R_G\mu^3e^2}{2h^4}\right), \tag{15}$$

which can be rewritten in terms of astronomical variables $h^2 = GMa(1-e^2)$

$$|\Delta\phi_u| = 2\pi\left[\frac{R_G}{a(1-e^2)} + \frac{6GMR_G}{a^2(1-e^2)^2}\left(1+\frac{e^2}{4}\right)\right], \tag{16}$$

where a denotes the orbital semi major axis. This new contribution must be less than the difference between the precession predicted by general relativity and the observed value so that R_G can be inferred using planetary data. Using Mercury's data [16], we obtain an estimated value $|R_G| \lesssim 0.005$ m. It is important to emphasize that the values calculated are not constraints of a new theory since we are contrasting our results with derived quantities [17,18], calculated from other measured parameters in a particular GR framework. Real constraints should come from calculating all the solar system parameters in the appropriate framework of that theory.

For other classical GR tests, we derive the ungravity contributions using Equation (5). For light deflection, we take $2K = 0$ in Equation (7), as well as the conserved quantity h in Equation (8), and in terms of $u = R^{-1}$, we obtain

$$u'' + u = 3GMu^2 - (2d_U + 1)2GMR_G^{2d_U - 2}u^{2d_U}. \tag{17}$$

The above equation is solved for $d_U = 3/2$ (as it is our case of interest) using a perturbation method in terms of $GMu/c^2 \ll 1$. We obtain the deviation from the straight line solution $u = \sin\phi/D$, where D is the closest distance from the light ray trajectory to the gravitational source. In the limit for large R, the asymptotic trajectory and the apparent trajectory form the deflection angle. Considering that $R_G \ll D^2$, the deflection angle is

$$\delta_U \approx \frac{4GM}{D}\left[1 - \frac{3\pi R_G}{4D}\right]. \tag{18}$$

We can constrain the parameter R_G by using the deflection caused by the Sun [19]; this gives the bound $|R_G| \lesssim 1.035 \times 10^7$ m.

The ungravity correction for the gravitational redshift is calculated with the modified g_{00} term of the metric and the weak field approximation $g_{00} \approx 1 + 2\Phi/c^2$, where Φ is the classical potential such that

$$\frac{\Delta_\nu}{\nu} = GM\left[\frac{1}{r_1} - \frac{1}{r_2} - R_G\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)\right], \tag{19}$$

where r_1 is the radius of the emitter, and r_2 is the radius of the detector of a shifted photon. Experimental data from [20] lead to the relation $|R_G| \lesssim 3.32 \times 10^2$ m.

For completeness, the ungravity contribution to the Shapiro time delay is calculated

$$dt_U = \pm\left(\frac{2GMR_G}{r\sqrt{r^2-D^2}} - \frac{D^2GMR_G}{r^3\sqrt{r^2-D^2}}\right)dr, \tag{20}$$

and using data from [21], the free parameter is constrained as $|R_G| \lesssim 1.1\times10^5$ m.

3. Entropic Contributions to the Orbital Precession

Based on Verlinde's derivation of classical gravity as an entropic force by employing a holographic surface [5] and considering the thermodynamics relation $F\Delta x = T\Delta S$, one can calculate modifications to Newtonian gravity by adding corrections to the entropy–area relationship. The modified Newtonian force is given by

$$\mathbf{F} = -\frac{GMm}{R^2}\left[1+4l_P^2\frac{\partial \mathcal{S}}{\partial A}\right]\Bigg|_{A=4\pi R^2}\hat{R}, \tag{21}$$

where $\mathcal{S}$ is a modified Bekenstein–Hawking entropy as a function of A, the area of a holographic closed surface between a system formed by a rest mass M and a test particle m. If a volumetric correction[4] to the entropy is considered

$$\frac{S[A]}{k_B} = \frac{A}{4l_P^2} + \epsilon\left(\frac{A}{2l_P^2}\right)^{\frac{3}{2}}, \tag{22}$$

using Equation (21), the modified Newtonian force is

$$\mathbf{F}_M = -\frac{GMm}{R^2}\left(1+\frac{3\sqrt{2\pi}}{l_P}\epsilon R\right)\hat{R}. \tag{23}$$

As in the usual Kepler problem, the angular momentum is conserved, and the orbits are restricted to a plane, so we identify $R^2\dot{\phi} = h$ with the magnitude of the angular momentum. Taking the radial equation with the change of variable $u = R^{-1}$ and after defining $\mu \equiv GM$, we obtain

$$\frac{d^2u}{d\phi^2} + u = \frac{\mu}{h^2}\left(1+\frac{3\sqrt{2\pi}}{l_P}\epsilon u^{-1}\right). \tag{24}$$

Solving perturbatively, we can calculate the perihelion shift. In terms of the constant free parameter ϵ, the orbital precession contribution is [22]

$$\Delta_\phi \approx -\frac{3\pi\sqrt{2\pi}a(1-e^2)}{l_P}\epsilon. \tag{25}$$

This extra contribution to the orbital precession must be less than the difference between the observed precession and the one predicted by GR. The bound is calculated using the data for Mercury, resulting in $|\epsilon| \leq 2.8\times10^{-58}$.

Other modifications to the entropy–area relationship can be considered. One interesting option is the general entropy presented in [23,24]. This entropy reproduces several generalizations to Shannon's entropy for particular values of the parameters. It has been studied in the context of cosmology, more precisely, to understand the dark energy sector. The generalized entropy is given by

$$S_g = \frac{1}{\gamma}\left[\left(1+\frac{\alpha_+}{\beta}S_{BH}\right)^{\beta} - \left(1+\frac{\alpha_-}{\beta}S_{BH}\right)^{-\beta}\right], \tag{26}$$

where S_{BH} is the Bekenstein–Hawking entropy. This reduces to various known entropies (see Table 1) by fixing the free parameters α_+, α_-, β and γ, which are constrained to be positive.

Table 1. Different entropies can be derived by fixing the positive free parameters in Equation (26). These are written as a function of the Bekenstein–Hawking entropy S_{BH}.

Entropy	Parameters	Entropy–Area Relationship
Tsallis–Barrow	$\alpha_+ \to \infty, \alpha_- = 0, \gamma = (\alpha_+/\beta)^\beta$	$S = S_{BH}^\beta$
Rényi	$\alpha_- = 0, \alpha_+ = \gamma, \beta \to 0, \alpha = \frac{\alpha_+}{\beta} \to$ finite	$S = \frac{1}{\alpha}\ln(1 + \alpha S_{BH})$
Sharma–Mittal	$\alpha_- = 0, \gamma = k = \alpha_+, \beta = k/\delta$	$S = (1 + \delta S_{BH})^{k/\delta} - 1$
Kaniadakis	$\beta \to \infty, \alpha_+ = \alpha_- = \gamma/2 = K$	$S = \sinh(K S_{BH})$
LQG	$\alpha_- = 0, \beta \to \infty, \gamma = \alpha_+ = (1 - q)$	$S = e^{(1-q)S_{BH}} - 1$

Following [5], the modified Newtonian force is

$$\mathbf{F} = -\frac{GMm}{R^2}\frac{1}{\gamma}\left[\alpha_+\left(1 + \frac{\alpha_+\pi}{\beta l_p^2}R^2\right)^{\beta-1} + \alpha_-\left(1 + \frac{\alpha_-\pi R^2}{\beta l_p^2}\right)^{-\beta-1}\right]\hat{R}. \tag{27}$$

For planetary orbits, the modified Binet equation is obtained from the generalized entropy in analogy with the procedure shown in [22]. Using the conservation of angular momentum and the change in variable $u = R^{-1}$, we obtain

$$u'' + u = \frac{\mu}{h^2}\left[\frac{\alpha_+}{\gamma}\left(1 + \frac{\alpha_+\pi}{\beta l p^2 u^2}\right)^{\beta-1} + \frac{\alpha_-}{\gamma}\left(1 + \frac{\alpha_-\pi}{\beta l_p^2 u^2}\right)^{-\beta-1}\right], \tag{28}$$

which can be compared with experimental data by fixing the free parameters.[5] It is noticed from Table 1 that Tsallis–Barrow and Sharma–Mittal entropies only recover a Newtonian force term if $\beta = 1$, which is the case for the Bekenstein–Hawking entropy. For Kaniadakis and LQG entropy, the limit $\beta \to \infty$ is used, inconsistently with the values of β, which lead to an asymptotically null force; the resulting forces only converge if $K = 0$ and $q = 1$, respectively, and in this case, the force is reduced to Newton's law. For Rényi entropy, the contribution to the orbital precession is

$$\Delta = 2\pi\left(1 - \frac{\mu^2 l_p^2}{\alpha\pi h^4}\right), \tag{29}$$

and the parameter α is bounded as $\alpha \leq 2.7 \times 10^{-92}$, comparing with data for Mercury. Notice that fixing the parameters as $\alpha_- = 0$, $\beta = \frac{3}{2}$ and $R \longrightarrow \infty$ in Equation (27) leads to

$$F = -\frac{GMm\alpha_+}{\gamma l_p R}\sqrt{\frac{2}{3}\pi\alpha_+}. \tag{30}$$

This equation can be used to describe stars far from the galaxy center, then used to analyze galaxy rotation curves, with the orbital velocity

$$v^2 = \frac{GM\alpha_+}{\gamma l_p}\sqrt{\frac{2}{3}\pi\alpha_+}. \tag{31}$$

Comparing with MOND, for $(\alpha_+)^{3/2}/\gamma \simeq 10^{-56}$, the model is consistent with galactic rotation curves. Unfortunately, for small R, it is inconsistent with Newton's gravitational law.

We can also consider that the generalized entropy is a correction of the form $S = S_{BH} + S_g$, then the modified Newtonian gravitational force is

$$F = -\frac{GMm}{R^2}\left[1 + \frac{\alpha_+}{\gamma}\left(1 + \frac{\alpha_+ \pi}{\beta l_p^2} R^2\right)^{\beta-1} + \frac{\alpha_-}{\gamma}\left(1 + \frac{\alpha_- \pi R^2}{\beta l_p^2}\right)^{-\beta-1}\right]. \quad (32)$$

Of particular interest is the behavior for large R and $\beta = \frac{3}{2}$. In this case,

$$F = -\frac{GMm}{R^2}\left[1 + \frac{\alpha+}{\gamma}\sqrt{\frac{2\pi\alpha_+}{3}}\frac{R}{l_p} + \sqrt{\frac{2\pi\alpha_+}{3}}\frac{3l_p}{4\gamma\pi R}\right] + \mathcal{O}\left(\frac{1}{R^5}\right). \quad (33)$$

Taking the first correction term in the brackets (which is R^{-1}) and with the considerations that lead to Equation (28), the differential equation for $u(\phi)$ is

$$u'' + u = \frac{\mu}{h^2}\left[1 + \frac{\alpha+}{\gamma}\sqrt{\frac{2\pi\alpha_+}{3}}\frac{1}{l_p u}\right]. \quad (34)$$

In analogy with the procedure described before, this modified Binet equation is solved and the shift of the perihelion, constrained by the GR contribution and experimental data, is

$$|\Delta| = 2\pi\left|\frac{h^2}{\mu}\frac{\alpha_+}{\gamma}\sqrt{\frac{2\pi\alpha_+}{3}}\frac{1}{2l_p}\right| \leq 2\pi \times 10^{-12}\frac{\text{rad}}{\text{rev}}, \quad (35)$$

and the entropy free parameters are bounded by $\frac{(\alpha_+)^{3/2}}{\gamma} \leq 4.026 \times 10^{-58}$.

If we consider the circular motion of a star far from the galactic center, the velocity obtained in our model is a constant; this is the same behavior one obtains from MOND. After comparing with MOND, we obtain

$$\frac{\alpha_+^{3/2}}{\gamma} = \sqrt{\frac{3\hbar a_0}{2\pi c^3 M}}, \quad (36)$$

where $l_p = \sqrt{\hbar G/c^3}$ and $a_0 = 1.2 \times 10^{-10}$ m/s^2. For our galaxy, we obtain $(\alpha_+)^{3/2}/\gamma = 1.12 \times 10^{-56}$. Comparing with the bounds from the perihelion shift, this model is discarded as an explanation of the anomalous galactic rotation curve.

4. Discussion and Final Remarks

In this paper, we considered the effects of ungravity and entropic gravity on planetary orbits, with the goal of establishing the bounds to the parameters of these theories

Ungravity corrections were previously studied in the cosmological context [13], providing an ungravity origin to the cosmological constant. In this model, the effective cosmological constant

$$\Lambda_{eff} \sim \Lambda_u^2\left(\frac{M_u}{M_{pl}}\right)^{\frac{2}{d_u-1}}, \quad (37)$$

is given in terms of the ungravity scale Λ_u, the ungravity coupling constant M_u and the scaling parameter[6] d_u. For $d_u = \frac{3}{2}$, the effective cosmological constant [13] can be written in terms of R_G using Equation (6) as

$$\Lambda_{eff} \sim \frac{1}{R_G^2}. \quad (38)$$

Using the bounds for R_G obtained from different ungravity and unparticle effects, the value of the effective cosmological constant can be calculated.

If we assume that the free parameters of ungravity and unparticles are the same (although this is not necessarily true), particle observations can also be considered to bound ungravity parameters. Assuming that ungravity and unparticle parameters (M_u, Λ_u, d_u) have the same values, we can make predictions using both gravity and particle experiments. Using the unparticle contributions to the hydrogen atom's ground state, we can fix the remaining parameters and calculate the perihelion shift. From [25], the parameters are related as follows

$$\lambda = c_u \frac{\Lambda_u}{M_u}^k, \tag{39}$$

where λ is the coupling constant, and C_u together with k are dimensionless constants related to unparticle operators. A modification term V_u to the potential is added and, using first-order Rayleigh–Schrodinger perturbation theory, the contribution to the ground state is $\Delta^{(1)}_{100} = \langle 100^0 | V_u | 100^0 \rangle$, and it is related to the parameters as

$$\Delta^{(1)}_{100} = -2\frac{\lambda^2 \alpha^2 \mu^2}{2\Lambda_u^{2d_u-2}}, \tag{40}$$

where μ is the reduced mass, and α is the square of the electron charge. This new contribution to the energy can be bounded by experimental and theoretical errors as $|\Delta^{(1)}_{100}| = \delta E_{th} + \delta E_{exp}$ with the maximum error $\delta_{max} = (\delta E_{th} + \delta E_{exp})/|E^s_{th}| \approx 1.1 \times 10^{-5}$. We can write the relation between parameters as

$$\left| \frac{\Delta^{(1)}_{100}}{E^s_{th}} \right| = \frac{\lambda^2 \mu}{\Lambda_u} < \delta_{max}, \tag{41}$$

and write the orbital precession contribution by introducing R_G in terms of these parameters. The perihelion shift is

$$|\Delta\phi_u| = \frac{2M_{pl}^2}{\pi \Lambda_u^3} \left(\frac{\lambda}{c_u}\right)^{2/k} \left[\frac{1}{a(1-e^2)} + \frac{6GM}{a^2(1-e^2)^2}\left(1 + \frac{e^2}{4}\right)\right]. \tag{42}$$

We can see that the bounds for λ and c_u derived from atomic physics will give insignificant orbital precession contributions, emphasizing that unparticle parameters are not necessarily the same as the ungravity ones.

From the gravitational classical tests, we obtain the following bounds for R_G: for light bending $|R_G| \lesssim 1.035 \times 10^7$ m, for the Shapiro time delay $R_G \leq 1.1 \times 10^5$ m, for gravitational redshift $R_G \lesssim 3.32 \times 10^2$ m and for precession $|R_G| \lesssim 0.005$ m. As stated before, these are not constraints but estimated values for the parameters of the theory. In the non-gravitational sector and assuming that the parameters of ungravity and unparticles are the same, Λ_u and M_u can be calculated from bounds in [25]. From these results, the value derived for the cosmological constant is incompatible with the cosmological observations.

Another modification to gravity that we have considered is derived from modified entropy–area relationships. In particular, we use a general expression for the entropy that in particular limits the reproduction of several non-additive entropies. This general entropy has been used in the context of cosmology, more precisely in connection to the dark energy sector. As in the case of ungravity, we use the perihelion shift in order to verify the validity of the resulting entropic gravitational force. We find that the contribution to the perihelion shift is negligible. We also study an entropy–area relationship constructed as the sum of the Hawking–Bekenstein entropy and this general entropy. In particular, for $\beta = 3/2$ and large R, flat rotation curves are predicted. Furthermore, we can fix the value of the remaining parameters by comparing with MOND and obtain $(\alpha_+)^{3/2}/\gamma = 1.12 \times 10^{-56}$. Unfortunately, this value is inconsistent with the bounds obtained from the perihelion shift.

In summary, using the perihelion shift and the solar system data, we can obtain maximal values for the parameters of ungravity as well as for different models of entropic gravity. In the case of ungravity, we conclude that, with this methodology, the bounds on the ungravity parameters are incompatible with the cosmological observations for Λ, discarding ungravity as an origin for the cosmological constant. For entropic gravity, one can have a modified entropy–area relationship that is consistent with the bounds of dark energy and planetary motion, but when also considering galactic rotation curves, the solar system bound on the parameters favors an interpretation where the volumetric contribution is relevant at the cosmological scale but not at the galactic scale.

Finally, we would like to emphasize that combining solar system and galaxy rotation curve data is a useful tool to discard modifications to gravity [26–30].

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Notes

1. This constant depends on the ungravity parameter d_U and gamma functions $A_{d_U} = \frac{16\pi^{5/2}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U+1/2)}{\Gamma(d_U-1)\Gamma(2d_U)}$.
2. The assumption is that the ungravity contribution is of the same order as for GR, for the purpose of comparing with experimental data. This is consistent with $1 < d_U < 2$ [15], which are usually the stated values for this parameter.
3. This value for β is phenomenologically relevant. It gives an entropy proportional to a volumetric contribution, and in the cosmological scenario gives an effective cosmological constant [12,13].
4. In [6], the authors study a volumetric correction to the entropy and show that it is the most relevant contribution for galactic rotation curves.
5. We will restrict to $0 > \beta > 2$, as for other values of β we have positive powers of R.
6. In particular, for $d_u = 3/2$ the resulting theory is consistent with an entropy that has a volumetric correction.

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