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Automatic Resummation of Logarithmic Enhanced Multiloop Contributions to the lightest CP-even Higgs Boson Mass in FlexibleSUSY

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Abstract

In this thesis, the computation of the lightest CP-even Higgs mass in the FlexibleSUSY framework was augmented by resumming large logarithmic contributions via an effective field theory approach in arbitrary models. Several algorithms were developed, analyzed and discussed, especially in the MSSM. Comparisons to other codes have been performed, with focus on the SUSYHD package, which was found to be in good agreement with our codes.

Kurzdarstellung

Die die Berechnung der leichtesten, CP-geraden Higgsmasse für allgemeine Modelle im Programmpacket FlexibleSUSY wurde durch die Resummation von logarithmisch verstärkten Termen mittels eines effektiven Feldtheorieansatzes verfeinert. Mehrere Algorithmen wurden dazu entwickelt, analysiert und diskutiert, zu meist im MSSM. Weiterhin wurden Vergleiche zu anderen Programmen durchgeführt, besonderes Augenmerk wurde dabei auf das Paket SUSYHD gelegt, welches gute Übereinstimmung mit dem hier implementierten Code aufzeigt.

Contents

Motivation

After the discovery of a Higgs boson at the LHC in 2012 [1, 2], its mass is a crucial observable to constrain physics beyond the Standard Model. Nevertheless, the search for new physics continues, but so far without any findings comparable to the former in its notability [3]. However, one possibility to reconcile these results with theories extending the Standard Model is that additional particles need to have very large masses. Unfortunately, the precision of ordinary fixed-order mass spectrum generators for the Higgs mass calculation is limited in those scenarios, due to large logarithms contributions to the self-energies, which extenuates the convergence of the perturbation series. Consequently, while the Higgs mass is measured with 300 MeV accuracy [4], spectrum generators may differ by several GeV even with the same input data [5]. This thesis is dedicated to the improvement of this calculation by resumming large logarithmic contributions.

Driven by this motivation, the outline of this thesis will start with a brief introduction to models and formalisms in chapter 1, followed by a short descriptive overview of the FlexibleSUSY framework in chapter 2. In chapter 3, the theoretical background of the algorithms will be discussed. The implementations and benchmarks are documented in the chapters 4, 5 and 6. Finally, conclusions will follow in the last chapter, 7.

1 Introduction

This chapter is dedicated to introduce some formalisms used in this thesis. Since the algorithm is based on the Standard Model (SM) as an effective field theory (EFT) of an extended theory, there will be a short review of the SM gauge group, particle content as well as the Higgs mechanism. The same will be done for the Minimal Supersymmetric Standard Model, since it is used extensively for crosschecking the code. The Next-to-Minimal Supersymmetric Standard Model is also briefly mentioned. Additionally, a general review of effective field theories will follow.

1.1 The Standard Model

The Standard Model of particle physics was formulated by Weinberg, Salam and Glashow [6]. It is by design the minimal formulation compliant with both: the axioms of Quantum Field Theory and the observed particle and interaction content.

	symbol/generations	spin	Q	$Y\,(l,r)$	$I_3\,(l,r)$	$SU(3)$ multiplicity
up-type quarks	u(up), c(charm), t(top)	$\frac{1}{2}$	$+\frac{2}{3}$	$\left(+\frac{1}{3},+\frac{4}{3}\right)$	$\left(+\frac{1}{2},0\right)$	3
down-type quarks	d(down), s(strange), b(bottom)	$\frac{1}{2}$	$-\frac{1}{3}$	$\left(+\frac{2}{3},-\frac{2}{3}\right)$	$\left(-\frac{1}{2},0\right)$	3
charged leptons	e(electron), μ (muon) τ (tauon)	$\frac{1}{2}$	-1	$(-1,-2)$	$\left(-\frac{1}{2},0\right)$	1
neutrinos	ν_e, ν_μ, ν_τ	$\frac{1}{2}$	0	$(-1,-)$	$\left(\frac{1}{2},-\right)$	1
gauge bosons	γ (photon)	1	0	0	0	1
	Z^0	1	0	0	0	1
	$W^\mp$	1	∓1	0	$\mp\frac{1}{2}$	1
	g (gluon)	1	0	0	1	8
Higgs boson	H	0	0	1	$-\frac{1}{2}$	1

Table 1: Particle content of the Standard Model, Q is the electric, $Y(l,r)$ the hypercharge of left- and right-chiral part, I_3 is the 3-component of the weak isospin. Right-chiral neutrinos are not included in the SM.

The bare Lagrangian looks as follows:

$$\mathcal{L} = \mathcal{L}_{Gauge} + \mathcal{L}_{Fermion} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs} + \mathcal{L}_{GF} + \mathcal{L}_{Ghost} \tag{1.1.1}$$

Where the last two terms correspond to gauge fixing and ghost interactions and shall not be of our concern here. The fermionic part $\mathcal{L}_{Fermion}$ contains propagators as well as minimally coupled interactions of the Dirac fermions, featuring the (gauge-)covariant derivative ∇_μ.

$$\mathcal{L}_{Fermion} = i\overline{\psi}\gamma^\mu\nabla_\mu\psi \tag{1.1.2}$$

$$\nabla_\mu = \partial_\mu - igA^a_\mu T^a \tag{1.1.3}$$

Where A_μ^a is a general Yang-Mills gauge field, g its coupling constant and T^a the generator corresponding to the representation of the $SU(N)$ gauge group. ψ are the spinor representation for fermions. The local gauge symmetry transformation is defined as

$$\psi \to \mathrm{e}^{ig\,\theta^a(x)T^a}\,\psi \tag{1.1.4}$$
$$A_\mu^a T^a \to \mathrm{e}^{ig\,\theta^b(x)T^b}\left[A_\mu^a + \partial_\mu \theta^a(x)\right] T^a \mathrm{e}^{-ig\,\theta^c(x)T^c} \tag{1.1.5}$$

which leaves the Lagrangian invariant. The kinetic part, featuring the propagators and self-interactions of the gauge fields, is:

$$\mathcal{L}_{Gauge} = -\frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu} \tag{1.1.6}$$
$$F_{\mu\nu} = \frac{i}{g}\left[\nabla_\mu, \nabla_\nu\right] = \partial_\mu A_\nu^a T^a - \partial_\nu A_\mu^a T^a - igA_\mu^a A_\nu^b \left[T^a, T^b\right] \tag{1.1.7}$$
$$= F_{\mu\nu}^a T^a \qquad = \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c\right) T^a \tag{1.1.8}$$

The SM gauge group in this form is $U(1) \otimes SU(2) \otimes SU(3)$, where the charges associated with these gauge symmetries are called hypercharge, weak isospin and color. However, for the Lagrange density to be invariant under gauge transformations, mass terms of gauge fields are strictly forbidden, and since the $SU(2)$ gauge group affects only left chiral modes, also for fermions. To resolve this issue, the Englert–Brout–Higgs–Guralnik–Hagen–Kibble mechanism, or shorter just Higgs mechanism, was introduced via an additional complex, colorless, hypercharged scalar $SU(2)$ doublet [7–10].

$$\Phi = \left(\phi^u, \phi^d\right)^T \tag{1.1.9}$$
$$\nabla_\mu \Phi = \left(\partial_\mu - \frac{i}{2} g_Y B_\mu \mathbb{I} - ig_2 W_\mu^a T^a\right) \tag{1.1.10}$$
$$\mathcal{L}_{Higgs} = \left(\nabla_\mu \Phi\right)^\dagger \left(\nabla^\mu \Phi\right) + \mu^2 \left|\Phi\right|^2 - \frac{\lambda}{2}\left|\Phi\right|^4 \tag{1.1.11}$$
$$\begin{aligned}\mathcal{L}_{Yukawa} = &- y_d \left(\overline{\psi}_L^u \phi^u + \overline{\psi}_L^d \phi^d\right)\psi_R^d \; - \; y_d^* \overline{\psi}_R^d \left(\phi^u \psi_L^u + \phi^d \psi_L^d\right) \\ &- y_u \left(\overline{\psi}_L^u \phi^{d\,*} - \overline{\psi}_L^d \phi^{u\,*}\right)\psi_R^u \; - y_u^* \overline{\psi}_R^u \left(\phi^{d\,*}\psi_L^u - \phi^{u\,*}\psi_L^d\right)\end{aligned} \tag{1.1.12}$$

Where $\psi^{u,d}$ are general (up,down)-type fermions, and $y_{u,d}$ their respective Yukawa couplings. Due to the potential term in $\mathcal{L}_{Higgs}$, the vacuum expectation value (VEV) of the Φ field is non-zero if μ^2 is positive. Hence, the theory undergoes spontaneous symmetry breaking, in this case dubbed electroweak symmetry breaking (EWSB). One possible expansion of ϕ around its VEV may look like this:

$$\Phi = \frac{1}{\sqrt{2}}\left(0, v + H(x)\right)^T \mathrm{e}^{i\theta^a(x)\,T^a} \tag{1.1.13}$$
$$v^2 = \frac{2\mu^2}{\lambda} + \mathcal{O}\left(1\,\text{loop}\right) \tag{1.1.14}$$

Where the θ^a are called Nambu-Golstone bosons. However, these are unphysical, since they vanish for a certain choice of gauge. More consistently, they may be interpreted as longitudinal modes of new vector fields composed of the $U(1)$ and $SU(2)$ gauge fields. By doing so, these three new eigenstates gain mass terms by the VEV, while a massive scalar excitation remains: the Higgs boson H. Since each massive field exhibits one additional degree of freedom compared to massless ones, all four real degrees of freedom of Φ are conserved.

$$\begin{aligned}\mathcal{L}_{Higgs} \to & -\frac{1}{4}g_2^2\, v^2 W_\mu^+ W^{-\,\mu} - \frac{1}{8}\left(g_Y^2+g_2^2\right) v^2\, Z_\mu Z^\mu - \frac{1}{4}g_2^2\,\left(H^2+2vH\right) W_\mu^+ W^{-\,\mu} \\ & -\frac{1}{8}\left(g_Y^2+g_2^2\right)\left(H^2+2vH\right)\, Z_\mu Z^\mu \\ & +\frac{1}{2}\partial_\mu H \partial^\mu H + \frac{1}{2}\left(\mu^2-\frac{3}{2}\lambda v^2\right) H^2 - \frac{\lambda v}{2}H^3 - \frac{\lambda}{8}H^4 \end{aligned} \quad (1.1.15)$$

$$\mathcal{L}_{Yukawa} \to -\frac{v}{\sqrt{2}}\overline{\psi}^u\, y_u \psi^u - \frac{v}{\sqrt{2}}\overline{\psi}^d\, y_d \psi^d - \frac{1}{\sqrt{2}}\overline{\psi}^u\, y_u H \psi^u - \frac{1}{\sqrt{2}}\overline{\psi}^d\, y_d H \psi^d \quad (1.1.16)$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right) \quad (1.1.17)$$

$$Z_\mu = \cos\theta_W\, W_\mu^3 - \sin\theta_W\, B_\mu \quad (1.1.18)$$

$$A_\mu = \sin\theta_W\, W_\mu^3 + \cos\theta_W\, B_\mu \quad (1.1.19)$$

$$\tan\theta_W = \frac{g_Y}{g_2} \quad (1.1.20)$$

Where the mass eigenstates $W_\mu^\pm$, Z_μ, A_μ are interpreted as W, Z and photon fields. One may see that the electric charge Q number is connected to the hypercharge Y as well as to some component of the weak isospin I_3: $\frac{1}{2}Y + I_3 = Q$. By this mechanism, W and Z retain mass terms while the photon stays massless, but the initial gauge group is broken in the electroweak sector:
$U(1)_{g_Y} \otimes SU(2)_{g_2} \otimes SU(3)_{g_3} \to U(1)_e \otimes SU(3)_{g_3}$

1.2 The Minimal Supersymmetric Standard Model

Supersymmetry transformations relate bosonic and fermionic fields in a theory with each other [11, 12]. The Minimal Supersymmetric Standard Model (MSSM) contains, true to its name, the SM gauge group and particle content. Furthermore, a new field - called superpartner - is related by a supersymmetry transformation to each SM particle [13]. In the MSSM, these superpartners are regarded as additional, fundamental fields, with a spin difference of $\frac{1}{2}$. Hence, a new scalar is introduced for each left- and right-chiral fermion, called sfermions, and fermionic fields corresponding to the gauge and Higgs bosons, called gauginos and higgsinos respectively. These will mix in electrically neutral or charged mass eigenstates after elecroweak symmetry breaking, called neutralinos and charginos. There has to be another Higgs doublet in order to make the Yukawa terms supersymmetric. In order to explain why no fundamental superpartner (sparticle) has been discovered yet, supersymmetry breaking is required to results in a mass splitting of SM particles and their superpartners, which is only possible if supersymmetry is softly broken by non-supersymmetric terms.
As before, the structure of the theory will be introduced using general fields, the Lagrange density may be written as:

$$\mathcal{L} = \mathcal{L}_{Gauge} + \mathcal{L}_{Free} + \mathcal{L}_{Pot} + \mathcal{L}_{Soft} + \mathcal{L}_{GF} + \mathcal{L}_{Ghost} \tag{1.2.1}$$

Disregarding the last two terms corresponding to gauge fixing and Fadeev-Popov ghosts, we will start introducing free SM fermions in the second term altogether with their superpartners, closely following the argumentation in [13], see there for a more elaborated introduction. Since there are superpartner for each left- and right-chiral modes of every fermion respectively, we will consider ψ to be a general (left-) chiral fermion, and ϕ its superpartner.
The Supersymmetry transformation changes the theory by expanding each field in the Lagrangian by terms containing a global fermionic field ϵ:

$$\delta\phi = \epsilon\psi \tag{1.2.2}$$

$$\delta\psi = i\sigma^\mu \epsilon^\dagger \nabla_\mu \phi + \epsilon F \tag{1.2.3}$$

$$\delta F = -i\epsilon\overline{\sigma}^\mu \nabla_\mu \psi + \sqrt{2} g \epsilon^\dagger {\lambda^a}^\dagger T^a \phi \tag{1.2.4}$$

Where F is an auxiliary field needed to close the supersymmetry algebra off-shell, and ∇_μ denotes the gauge covariant derivative:

$$\nabla_\mu \phi = \partial_\mu \phi - igA^a_\mu T^a \phi \tag{1.2.5}$$

$$\nabla_\mu \psi = \partial_\mu \phi - igA^a_\mu T^a \psi \tag{1.2.6}$$

$$\nabla_\mu \lambda^a = \partial_\mu \lambda^a + gf^{abc} A^b_\mu \lambda^c \tag{1.2.7}$$

Where λ^a are the gauginos of A^a_μ. The identity matrix in $\mathbf{C}^2$ appears in $\sigma^0 = \overline{\sigma}^0 = I_2$ and the spatial components $\sigma^a = -\overline{\sigma}^a$ are the Pauli matrices. Obviously, the last term in δF emerges due to the transformation of the gauge field in the covariant derivative, and may be dropped when these are regarded as usual partial derivatives, which decouples the gauge sector. Then, following these transformation rules, it can be shown that the free (s)fermionic Lagrangian is invariant up to a total derivative.

$$\mathcal{L}_{Free} = \left(\nabla_\mu \phi\right)^\dagger \left(\nabla^\mu \phi\right) + i\psi^\dagger \overline{\sigma}^\mu \nabla_\mu \psi + F^* F \tag{1.2.8}$$

The inclusion of right-chiral fields is analogue. Without decoupling the gauge field A_μ, the superalgebra only closes under consideration of a gauge sector, containing, besides the propagator and self-interaction terms of the gauge field and its superpartner, additional terms featuring another auxiliary field.

$$\begin{aligned}\mathcal{L}_{Gauge} =& -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + i\lambda^{a\dagger}\overline{\sigma}^\mu\nabla_\mu\lambda^a + \frac{1}{2}D^aD^a \\ & -\sqrt{2}g\,\phi^*T^a\psi\lambda^a - \sqrt{2}g\,\lambda^{a\dagger}\psi^\dagger T^a\phi + g\,\phi^*T^a\phi D^a \end{aligned} \tag{1.2.9}$$

With transformation rules:

$$\sqrt{2}\,\delta A^a_\mu = -\epsilon^\dagger\overline{\sigma}_\mu\lambda^a - \lambda^{a\dagger}\overline{\sigma}_\mu\epsilon \tag{1.2.10}$$

$$\sqrt{2}\,\delta\lambda^a = \frac{i}{2}\sigma^\mu\overline{\sigma}^\nu\epsilon F^a_{\mu\nu} + \epsilon D^a \tag{1.2.11}$$

$$\sqrt{2}\,\delta D^a = -i\epsilon^\dagger\overline{\sigma}^\mu\nabla_\mu\lambda^a + i\nabla_\mu\lambda^{a\dagger}\overline{\sigma}^\mu\epsilon \tag{1.2.12}$$

and infinitesimal gauge transformations:

$$A^a_\mu \to A^a_\mu + \partial_\mu\Lambda^a + gf^{abc}A^b_\mu\Lambda^c \tag{1.2.13}$$

$$\lambda^a \to \lambda^a + gf^{abc}\lambda^b\Lambda^c \tag{1.2.14}$$

Integrating out the auxiliary field D^a by substituting it with its equation of motion generates a scalar potential term from those terms containing D^a, called D-term:

$$\mathcal{L}_{D-term} = \frac{1}{2}D^aD^a = \frac{g^2}{2}\left(\phi^*T^a\phi\right)^2 \tag{1.2.15}$$

For including additional scalar potential terms, it was argued in [13] that, excluding tadpole terms compliant with supersymmetry, these are bound to take the following structure:

$$\mathcal{L}_{Pot} = -\frac{1}{2}\frac{\delta^2 W}{\delta\phi^i\,\delta\phi^j}\psi^i\psi^j + \frac{\delta W}{\delta\phi^i}F^i + h.\,c. \tag{1.2.16}$$

$$W = \frac{1}{2}M_{ij}\phi^i\phi^j + \frac{1}{3!}y_{ijk}\phi^i\phi^j\phi^k \tag{1.2.17}$$

Collecting terms containing auxiliary left- or right-chiral fields together with this potential and integrating out the F^i fields, one obtains the F-term potential.

$$\begin{aligned}\mathcal{L}_{Pot} + F^{i*}F^i =& \left(-\frac{1}{2}M_{ij}\psi^i\psi^j - \frac{1}{2}y_{ijk}\phi^i\psi^j\psi^k + h.\,c.\right) - M^*_{ik}M_{kj}\phi^{i*}\phi^j - \\ & \frac{1}{2}\left(M_{il}\,y^*_{jkl}\,\phi^i\phi^{j*}\psi^{k*} + h.\,c.\right) - \frac{1}{4}y_{ijh}y^*_{klh}\phi^i\phi^j\phi^{k*}\phi^{l*}\end{aligned} \tag{1.2.18}$$

Where y_{ijk} and M_{ij} are considered total symmetric here. In the MSSM, the quantity 1.2.17 is defined as:

$$W = \mu H_u H_d + \mathbf{y_u}\overline{\phi_u}\,H_u\,\phi_Q + \mathbf{y_d}\overline{\phi_d}\,H_d\,\phi_Q + \mathbf{y_e}\overline{\phi_e}\,H_d\,\phi_L \tag{1.2.19}$$

The matrices y_{ijk} are considered as Yukawa couplings, and chosen in a way that the SM Yukawa sector emerges:

$$\mathcal{L}_{Yukawa} = -\left(\mathbf{y_u}\overline{u}_RQH_u + \mathbf{y_d}\overline{d}_RQH_d + \mathbf{y_e}\overline{e}_RLH_d + h.\,c.\right) \tag{1.2.20}$$

Where Q, L are the $SU(2)$ doublets of quarks and leptons and ϕ_Q, ϕ_L their respective superpartner doublets, while H_u, H_d are up- and down-type Higgs doublets. Additionally, these gain a mass by $M_{ij} = \mu$, $i \neq j$:

$$\mathcal{L}_{Higgs} = -\left|\mu\right|^2\left(\left|H_u\right|^2 + \left|H_d\right|^2\right) \tag{1.2.21}$$

Additionally possible terms violating lepton- or baryon number conservation are not considered, for these would e.g. contradict experimental bounds on proton decay [13]. Furthermore, it can be inferred from the Lagrangian that particles and their respective superpartners share the same mass, which is clearly ruled out by observation. Hence, supersymmetry must be broken, which is done softly to guarantee renormalizability by including positive mass power couplings [14]. The most general approach to this (see [13]) is:

$$
\begin{aligned}
\mathcal{L}_{Soft} = &-\frac{1}{2}\sum_{i=1}^{3}\left(M_i\,\lambda_i^a\lambda_i^a + h.\,c.\right)\\
&-\left(\mathbf{A_u}\mathbf{y_u}\,\widetilde{\overline{u}}_R\,\widetilde{Q}\,H_u + \mathbf{A_d}\mathbf{y_d}\,\widetilde{\overline{d}}_R\,\widetilde{Q}\,H_d + \mathbf{A_e}\mathbf{y_e}\,\widetilde{\overline{e}}_R\,\widetilde{L}\,H_d + h.\,c.\right)\\
&-\widetilde{Q}^\dagger\mathbf{m^2_Q}\widetilde{Q} - \widetilde{L}^\dagger\mathbf{m^2_L}\widetilde{L} - \widetilde{\overline{u}}_R\,\mathbf{m^2_u}\widetilde{\overline{u}}^\dagger_R - \widetilde{\overline{d}}_R\mathbf{m^2_d}\,\widetilde{\overline{d}}^\dagger_R - \widetilde{\overline{e}}_R\mathbf{m^2_e}\,\widetilde{\overline{e}}^\dagger_R\\
&-m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - B_\mu H_u H_d - B_\mu^* H_u^* H_d^* \qquad (1.2.22)
\end{aligned}
$$

The first line introduces gaugino mass terms, the second line adds trilinear couplings in sfermions and Higgs scalars, mixing into different generations of each family. The third line introduces independent masses for left- and right-chiral sfermions, and the last line individual masses as well as mixing terms for both Higgs scalars. Both Higgs fields can be parametrized and inserted into the Higgs potential:

$$H_u = \left(H_u^+, H_u^0\right)^T \qquad (1.2.23)$$

$$H_d = \left(H_d^0, H_d^-\right)^T \qquad (1.2.24)$$

$$
\begin{aligned}
\mathcal{L}_{Higgs} = &-\left(|\mu|^2 + m^2_{H_u}\right)\left(\left|H_u^0\right|^2 + \left|H_u^+\right|^2\right) - \left(|\mu|^2 + m^2_{H_d}\right)\left(\left|H_d^0\right|^2 + \left|H_d^-\right|^2\right)\\
&-\left(B_\mu\left(H_u^+H_d^- - H_u^0H_d^0\right) + h.\,c.\right) - \frac{g_2^2}{2}\left|H_u^+H_d^{0*} + H_u^0H_d^{-*}\right|^2\\
&-\frac{1}{8}\left(g_Y^2 + g_2^2\right)\left(\left|H_u^0\right|^2 - \left|H_d^0\right|^2 + \left|H_u^+\right|^2 - \left|H_d^-\right|^2\right)^2 \qquad (1.2.25)
\end{aligned}
$$

The real parts of $H^0_{u,d}$ can then be expanded around their respective VEVs $v_{u,d}/\sqrt{2}$. The relation between those can be parametrized by $\tan\beta = \frac{v_u}{v_d}$. After electroweak symmetry breaking, these Higgs fields mix into 2 CP-even, 2 CP-odd neutral, and 2×2 electrically charged real scalars. The lightest CP-odd and two lightest charged fields are then the Nambu-Goldstone bosons corresponding to longitudinal modes of Z and $W^\pm$, respectively. In the scenarios considered in this thesis, the lightest CP-even field is equivalent to the Higgs boson encountered in the SM, while the rest are additional, heavier fields. Integrating out these fields as well as all sparticles yields the SM as effective field theory, where at tree level the following identities relate the parameters:

$$g_i^{SM} = g_i^{MSSM} \qquad (1.2.26)$$

$$y_u^{SM} = y_u^{MSSM}\sin\beta \qquad (1.2.27)$$

$$y_{d,e}^{SM} = y_{d,e}^{MSSM}\cos\beta \qquad (1.2.28)$$

$$v^2 = v_u^2 + v_d^2 \qquad (1.2.29)$$

$$\lambda = \frac{1}{4}\left(g_Y^2 + g_2^2\right)\cos 2\beta \qquad (1.2.30)$$

It is remarkable that λ in the SM is at tree level related to gauge couplings and β from D-term contributions in the MSSM.

1.3 Next-to-Minimal Supersymmetric Standard Model

In the Next-to-Minimal Supersymmetric Standard Model (NMSSM) [15], the MSSM has been extended by introducing another gauge singlet, a scalar S and its superpartner. Most commonly, an additional $\mathbb{Z}_3$ symmetry on the fields in the potential 1.2.17 is assumed, allowing only dimensionless coupling terms therein. One reason for considering the NMSSM is motivated by the hierarchy problem: instead of introducing a Higgs mass parameter μ and hence the SUSY scale by hand, this term is generated dynamically from the VEV of the new scalar S. Therefore, the SUSY scale is completely determined by the EWSB and hence the mechanism of SUSY-breaking, see [15]. For the NMSSM, equation 1.2.17 is modified to:

$$W = \mathbf{y_u} H_u \overline{\phi_u}\, \phi_Q + \mathbf{y_d} H_d \overline{\phi_d}\, \phi_Q + \mathbf{y_e} H_d \overline{\phi_e}\, \phi_L + \lambda S H_u H_d + \frac{\kappa}{3} S^3 \tag{1.3.1}$$

Indeed, only dimensionless couplings appear, and the MSSM parameter μ from 1.2.19 is then determined by:

$$\mu_{eff} = \frac{v_S}{\sqrt{2}} \lambda \tag{1.3.2}$$

In the soft breaking sector, the MSSM Lagrangian is extended by terms including the new scalar field. However, in order to comply with the mentioned $\mathbb{Z}_3$ symmetry, $B_\mu = 0$ must be applied.

$$\mathcal{L}_{soft}^{\text{NMSSM}} = \mathcal{L}_{soft}^{\text{MSSM}}\Big|_{B_\mu = 0} - m_S^2\, S^* S - \left(A_\lambda\, \lambda\, S\, H_u\, H_d + \frac{1}{3} A_\kappa\, \kappa\, S^3 + h.c. \right) \tag{1.3.3}$$

Since the NMSSM is only used briefly, this stub shall suffice as an introduction to its basic parameters, see [15] for a more detailed review.

1.4 Effective Field Theories

Effective field theories (EFTs) are a technique that allows the study of the low energy limit of a theory, where the in- and outgoing four-momentum squares in the amplitudes are considered to be small compared to the mass scale of some heavy particles included. Hence, such particles cannot be produced on-shell, and do only appear as internal lines in each amplitude. For such particles, the Appelquist-Carazzone theorem [16] states that their contribution is suppressed by terms proportional to momenta over heavy masses. The heavy particles therefore decouple from the theory if their masses approach infinity. In the path integral formulation, this can be seen by integrating out heavy particles Φ_i while keeping light modes ϕ_i. Since no external field Φ_i is allowed, the heavy current J can be set to zero in the partition function and the expression partially solved.

$$\begin{aligned} Z\left(j\right) &= \int \prod_{k,l} \mathcal{D}\Phi_k \, \mathcal{D}\phi_l \;\; \mathrm{e}^{i\int \mathrm{d}^4x\, \mathcal{L}(\phi_l,\Phi_k)+J_k\Phi_k+j_l\phi_l}\Big|_{J_k=0} \\ &= \int \prod_{l} \mathcal{D}\phi_l \;\; \mathrm{e}^{i\int \mathrm{d}^4x\, \mathcal{L}_{eff}(\phi_l)+j_l\phi_l} \end{aligned} \tag{1.4.1}$$

$\mathcal{L}_{eff}$ is the Lagrangian of the EFT. Diagrammatically, this corresponds to a theory containing all one-particle-irreducible amplitudes of the full theory, where each heavy loop is contracted into one spacetime point and vertices connected by heavy lines are joined. Consequently, each heavy interaction is considered to be local in spacetime, which reflects that the momenta of the heavy fields being far off-shell $p^2 \ll M^2$ so their propagation can be neglected. While external light fields are kept in each term, the heavy ones are replaced by some coefficient adjusted to yield the same amplitude as the full theory. Since these one-particle irreducible graphs include all loop orders, there is an infinite amount of terms to be considered for each EFT. However, since each heavy propagator can be expanded around zero momentum in a fashion like $\frac{1}{p^2-M^2} \approx -\frac{1}{M^2} + \frac{p^2}{M^4} + \mathcal{O}\left(\frac{p^4}{M^6}\right)$, the terms contained in $\mathcal{L}_{eff}$ are local, and indeed suppressed by powers of the momenta over the heavy masses. Consequently, this means that the effective field theory is not renormalizable by power counting, since terms with mass dimension greater than 4 appear in this theory, with constant coefficients of negative mass dimension. Nevertheless, this does not pose a problem, since higher order terms can always be considered to renormalize the EFT to a fixed order of mass dimensions.

In this thesis, the Standard Model is regarded as an EFT in the low energy limit of various models. These are defined in the $\overline{\text{DR}}$ or $\overline{\text{MS}}$ scheme, which do not include the decoupling theorem by design, because the beta functions are not explicitly scale dependent [17]. Nevertheless, such a decoupling occurs, and an EFT can be constructed by matching amplitudes, thus the theory parameters, to yield the same result. To reduce loop contributions, this is done at the scale of magnitude of the heavy masses. However, the Standard Model is always considered as effective theory, excluding non-renormalizable parts. Hence, calculations in this model might differ by terms suppressed by inverse powers of heavy masses, which decouple when these approach infinity.

2 About FlexibleSUSY

FlexibleSUSY [18] is a spectrum generator generator written in C++, Wolfram Language [19] and Bourne shell. This program package is able to calculate the mass spectrum and additional observables for any arbitrary extension of the Standard Model (SM), provided that such models can be defined in the external Mathematica based package SARAH [20]. This creates, from specified symmetries, gauge groups and particle contents, symbolic expressions for the vertices, beta functions, self-energies and tadpole diagrams. This raw input model is converted to a C++ spectrum generator by the FlexibleSUSY meta code written in Wolfram Language, including user specified constraints for all model parameters at different scales. An important example of these constraints are the conditions imposed by the electro-weak symmetry breaking (EWSB) at one-loop order. FlexibleSUSY calculates these automatically and leaves the choice which parameters to adapt in order to fulfill these conditions to the user.

By default, two to three scales will be created automatically at the meta code level: a low scale for constraints involving light particles, e.g. from the Standard Model, an intermediate scale for constraints containing heavy particles where the EWSB conditions are solved at, and optionally a very high scale to impose GUT constraints. While the low scale is fixed to be at the Z boson pole mass, the other scales can be specified by the user. For the intermediate scale, it is highly recommended to choose it at some mean between the heavy masses, to make large logarithms from loop contributions in the constraints become small. Additionally, FlexibleSUSY calculates the SM gauge couplings automatically, including thresholds at the low scale, and is also able to do so with the Yukawa couplings. Anyway, the solver algorithms for the constraints are able to handle an arbitrary number of scales registered for each model, so the user has freedom to add as many constraint objects as he/she prefers, however, this insertion is not automated yet and has to be performed at the C++ level. In the same manner, one may also create matching classes for different FlexibleSUSY models in order to glue these together as a tower of effective field theories. This work is indeed partially motivated as an exercise for the automatization of this process.

The freedom of choice of the constraints, and hence the control over the degrees of freedom of each theory is one of the greatest assets of FlexibleSUSY, together with its performance speed, precision and modularity which allows easy modification of the code. These and additional options, essential for the structure and functionality of the spectrum generator are specified in a Mathematica meta code file for each model, which controls the creation of C++ code.

Input of softer options, not altering the structure of the code, like external input values for observables referenced in the constraints, is possible by command line or in the SUSY Les Houches Accord file format (SLHA [21, 22]). The same is also true for the output of the calculation.

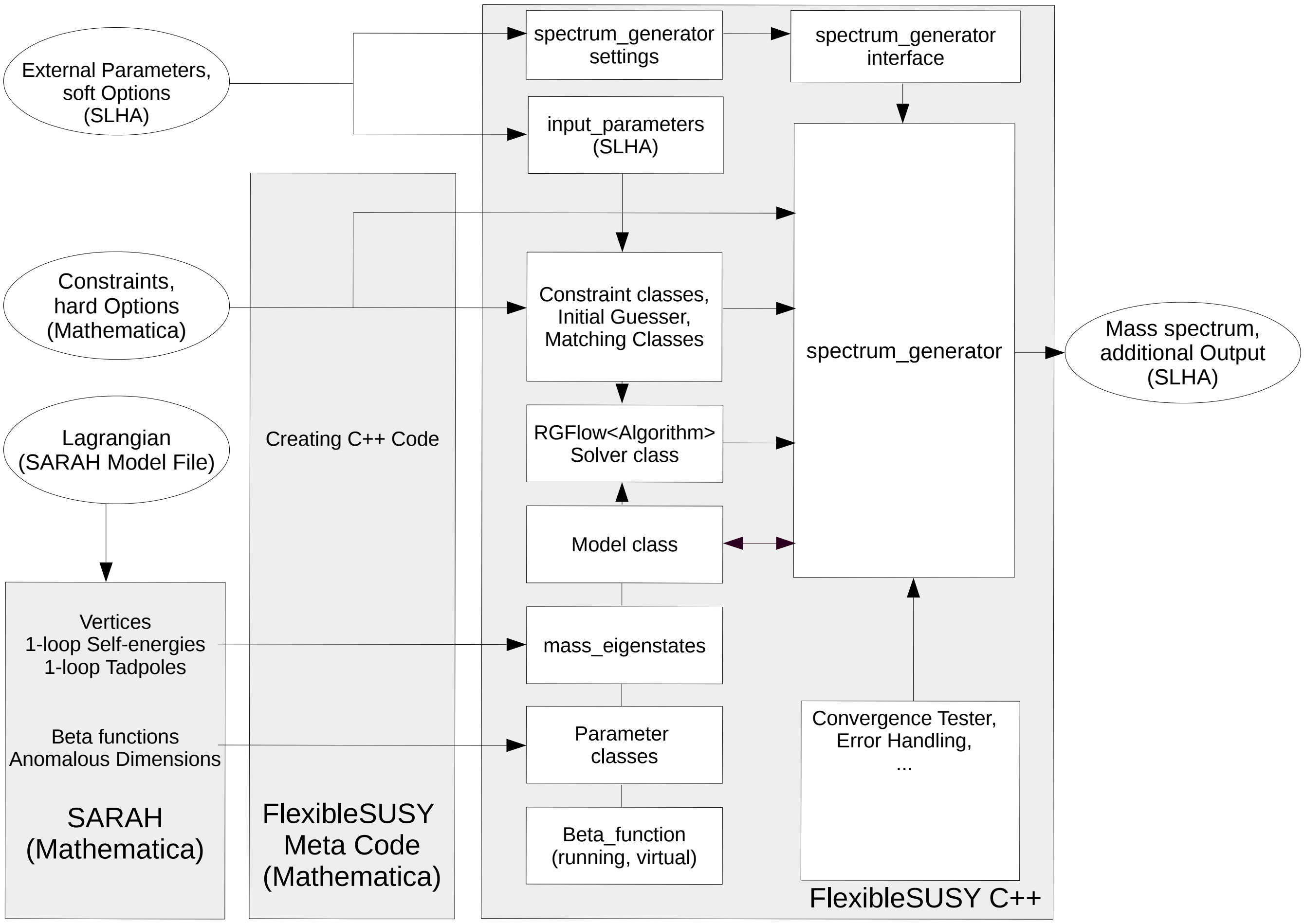


Figure 1: Schematic functionality of FlexibleSUSY.

In spite of the name, FlexibleSUSY is capable of dealing with all kinds of theories, supersymmetric or not, which include the gauge group and field content of the Standard Model. However, the model parameters are bound to be defined in the $\overline{\text{MS}}$ or $\overline{\text{DR}}$ scheme. The generated source code can be compiled and run, predefined models exist in order to work without a Mathematica license.

FlexibleSUSY will try to resolve all constraints to find numerical values for each model parameter, using beta functions for running between the scales. These are calculated automatically for arbitrary models at two-loop order, while self-energies and tadpoles are available at one-loop order, unless the user adds additional contributions to one of these quantities. After this is finished, the pole mass spectrum is calculated.

Although higher order corrections are available for models of common interest, diagrammatic exact and complete expressions for pole masses of a general theory are hence only available up to one-loop order. For the lightest, CP-even Higgs boson, considered the SM Higgs, the pole mass calculation will be enhanced by the algorithms considered in this thesis.

3 Theoretical considerations

3.1 Problem and task of the thesis

As mentioned before, FlexibleSUSY only uses one-loop self-energies to calculate pole masses in arbitrary theories. However, since these are defined in the $\overline{\text{MS}}$ or $\overline{\text{DR}}$ renormalization schemes, self-energies contain terms $\sim \ln\frac{\mu}{m_i}$, where m_i denote running masses of the particles involved in loop corrections and μ is the renormalization scale. In the following, it will be ideally assumed that two coarse mass scales exist: a low Standard Model scale m and a higher scale M which is due to extensions of the SM with heavy particles. No matter how the renormalization scale is then chosen between these scales, loop corrections will always include terms logarithmically enhanced by $\sim \ln^n\left(\frac{m}{M}\right)$. This weakens the convergence of the perturbation series for each loop correction, especially if the scale difference is large. However, including additional contributions from higher loop orders in each calculation for compensation is difficult due to the increasing complexity of the diagrammatic expressions. Hence, an Effective Field Theory algorithm will be deployed to include logarithmic enhanced terms from all loop orders. It is task of the thesis to apply this to the pole mass calculation of the lightest CP-even Higgs boson, and enhance the self-energy additionally to the full one-loop expression provided.

This is done by matching the arbitrary model at a heavy mass scale to an Effective Field Theory, which is fixed to be the Standard Model, and running all SM parameters from the high scale M to the low scale m. The loop order l of the beta function used for this determines the contributions included in the calculation, since in each loop order n, all terms $\sim \ln^n\left(\frac{m}{M}\right)$, ..., $\ln^{n-l+1}\left(\frac{m}{M}\right)$ are fully considered. At the low scale, the physical Higgs mass is calculated in the EFT, containing logarithmic contributions resummed to all loop orders. Tree-level and one-loop order of the self-energy expression can be substituted by the full diagrammatic terms provided by FlexibleSUSY via SARAH, which will be discussed in the following sections.

This algorithm is applicable to arbitrary high scale models extending the SM, but only effective when heavy masses are approximately degenerate and the scale difference is large (especially relative to the heavy mass differences). All light fields are content of the SM, so that this is the right Effective Field Theory. Otherwise, large logarithmic terms are not resummed properly, which may destroy the precision of the algorithm.

3.2 EFT-based resummation

The Higgs pole mass at the low-energy scale $\mu = m$ (e.g. M_Z) in the Standard Model as an Effective Field Theory can be calculated via:

$$M_{Pole}^2 = \left(M_{tree}^{SM}\right)^2 - \Sigma^{SM}(p = M_{Pole}) \tag{3.2.1}$$

This expression can be expanded in loop orders as a power series in the coupling constants α_i^{EFT}. Since the low energy scale is considered the mass scale of all Standard Model particles, it does not contain large logarithms.

$$\begin{aligned} M_{Pole}^2 = {} & \alpha_i^{EFT}(m)\; M_i^{SM} + \alpha_i^{EFT}(m)\; \alpha_j^{EFT}(m)\; \Sigma_{ij}^{SM,1L} \\ & + \alpha_i^{EFT}(m)\; \alpha_j^{EFT}(m)\; \alpha_k^{EFT}(m)\; \Sigma_{ijk}^{SM,2L} + ... \end{aligned} \tag{3.2.2}$$

Where the M_i are expansion parameters for the tree-level Higgs mass, and Σ for the self-energies. The latter one might be interpreted, with respect of the SM self-energy expression at arbitrary scale, as the term not proportional to logarithmic contributions, since these are small compared to logarithms containing low-energy and heavy scale. The running of the scale-dependend coupling constants depends on the beta-functions, which might as well be expanded in powers of coupling constants, yielding Renormalization Group Equations (RGEs).

$$\beta_i = \frac{\partial \alpha_i}{\partial \ln(\mu)} = \alpha_j\,\alpha_k\,\beta^{1L}_{ijk} + \alpha_j\,\alpha_k\,\alpha_l\,\beta^{2L}_{ijkl} + ... \tag{3.2.3}$$

$$\alpha_i(m) = \exp\left(\ln\left(\frac{m}{M}\right)\sum_j \beta_j\,\frac{\partial}{\partial \alpha_j(M)}\right)\,\alpha_i(M) \tag{3.2.4}$$

$$\begin{aligned} &= \alpha_i(M) + \alpha_j(M)\,\alpha_k(M)\,\beta^{1L}_{ijk}\ln\left(\frac{m}{M}\right) \\ &\quad + \alpha_j(M)\,\alpha_k(M)\,\alpha_l(M)\left(\beta^{2L}_{ijkl}\ln\left(\frac{m}{M}\right) + \beta^{1L}_{ijn}\,\beta^{1L}_{nkl}\ln^2\left(\frac{m}{M}\right)\right) + ... \end{aligned} \tag{3.2.5}$$

Where M is assumed to be a heavy scale that roughly fits all heavy masses. Hence the pole mass can be expressed by the EFT coupling at this scale.

$$\begin{aligned} M^2_{Pole} &= \alpha^{EFT}_i(M)\,M^{SM}_i + \alpha^{EFT}_i(M)\,\alpha^{EFT}_j(M)\,\left[\Sigma^{SM,1L}_{ij} + \beta^{SM,1L}_{kij}\,M^{SM}_k\ln\left(\frac{m}{M}\right)\right] \\ &+ \alpha^{EFT}_i(M)\,\alpha^{EFT}_j(M)\,\alpha^{EFT}_k(M)\,\Big[\Sigma^{SM,2L}_{ijk} + \\ &\quad \left(2\,\Sigma^{SM,1L}_{il}\beta^{SM,1L}_{ljk} + M^{SM}_l\beta^{SM,2L}_{lijk}\right)\ln\left(\frac{m}{M}\right) + M^{SM}_l\beta^{SM,1L}_{lin}\,\beta^{SM,1L}_{njk}\ln^2\left(\frac{m}{M}\right)\Big] \\ &+ ... \end{aligned} \tag{3.2.6}$$

Finally, all the couplings in the Standard Model are defined by matching an extended theory to it at the heavy mass scale M.

$$\alpha^{EFT}_i(M) = \alpha^{Full}_i(M) + a_{ijk}\,\alpha^{Full}_j(M)\,\alpha^{Full}_k(M) + b_{ijkl}\,\alpha^{Full}_j(M)\,\alpha^{Full}_k(M)\,\alpha^{Full}_l(M) + ... \tag{3.2.7}$$

While parameter redefinitions by tree-level matching might be absorbed into other variables, the loop corrections proportional to a_{ijk}, b_{ijkl} arise from integrating out heavy fields at one- and two-loop order, i.e. sparticles in supersymmetric theories. These conditions might now be plugged into the pole mass formula.

$$\begin{aligned} M^2_{Pole} &= \alpha^{Full}_i(M)\,M^{SM}_i \\ &+ \alpha^{Full}_i(M)\,\alpha^{Full}_j(M)\,\left[\Sigma^{SM,1L}_{ij} + \beta^{SM,1L}_{kij}\,M^{SM}_k\ln\left(\frac{m}{M}\right) + M^{SM}_k a_{kij}\right] \\ &+ \alpha^{Full}_i(M)\,\alpha^{Full}_j(M)\,\alpha^{Full}_k(M)\,\Big[M^{SM}_l b_{lijk} + 2\,\Sigma^{SM,1L}_{il}a_{ljk} + \Sigma^{SM,2L}_{ijk} \\ &\quad + \left(2\,\Sigma^{SM,1L}_{il}\beta^{SM,1L}_{ljk} + M^{SM}_l\beta^{SM,2L}_{lijk} + 2\,M^{SM}_k\beta^{SM,1L}_{kni}a_{njk}\right)\ln\left(\frac{m}{M}\right) \\ &\quad + M^{SM}_l\beta^{SM,1L}_{lin}\,\beta^{SM,1L}_{njk}\ln^2\left(\frac{m}{M}\right)\Big] + ... \end{aligned} \tag{3.2.8}$$

This result resembles the pole mass formula in the full theory at the scale M: The terms proportional to Σ^{SM} are, as argued above, roughly the non-logarithmic terms from the SM self-energy, while the terms proportional to β^{SM} are accurate contributions from SM particles proportional to large logarithms, if the loop orders of the beta functions used for running are at least as large as the order of the self-energy. All terms proportional to a, b are non-logarithmic contributions from heavy particle loops. Applying the same argument as above, there are no (large) logarithmic contributions from heavy particles, since their masses resemble the scale M. Mixed terms at two-loop level or higher result from combining heavy and soft loops in the self-energy. With large differences between the scales m and M, the logarithmic terms are the major contributions, and high order-observables of the full theory can be approximated conveniently with knowledge of the beta functions in the Effective Field Theory. To obtain further accuracy, one may wish to integrate e.g. the known one-loop self-energy into this result, and therefore needs to filter out tree-level and one-loop terms, which is non-trivial in a pure numerical calculation. This can be achieved by subtracting the pole mass corrections according to the same formula, but with cuts at one-loop level introduced for running and matching, defining the functions $\mathcal{M}_1^2$, $\mathcal{M}_2^2$ and $\mathcal{M}_3^2$ as follows:

$\mathcal{M}_1^2$ will undergo running and matching only at tree level, but the one-loop pole mass formula is used.

$$\begin{aligned}\mathcal{M}_1^2 : \quad & \alpha_i^{EFT}(m)\, M_i^{SM} + \alpha_i^{EFT}(m)\, \alpha_j^{EFT}(m)\, \Sigma_{ij}^{SM,1L} \\ & \Rightarrow \alpha_i^{EFT}(M)\, M_i^{SM} + \alpha_i^{EFT}(M)\, \alpha_j^{EFT}(M)\, \Sigma_{ij}^{SM,1L} \\ & \Rightarrow \alpha_i^{Full}(M)\, M_i^{SM} + \alpha_i^{Full}(M)\, \alpha_j^{Full}(M)\, \Sigma_{ij}^{SM,1L}\end{aligned} \tag{3.2.9}$$

$\mathcal{M}_2^2$ runs with one-loop part only, which is easily implemented when the beta function is known. Matching and pole mass only contribute at tree level:

$$\begin{aligned}\mathcal{M}_2^2 : \quad & \alpha_i^{EFT}(m)\, M_i^{SM} \Rightarrow \alpha_i^{EFT}(M)\, \alpha_j^{EFT}(M)\, M_k^{SM} \beta_{kij}^{SM,1L} \ln\left(\frac{m}{M}\right) \\ & \Rightarrow \alpha_i^{Full}(M)\, \alpha_j^{Full}(M)\, M_k^{SM} \beta_{kij}^{SM,1L} \ln\left(\frac{m}{M}\right)\end{aligned} \tag{3.2.10}$$

Finally there is $\mathcal{M}_3^2$, where only one-loop matching is taken into account and the other contributions are kept at tree level only.

$$\mathcal{M}_3^2 : \quad \alpha_i^{EFT}(m)\, M_i^{SM} \Rightarrow \alpha_i^{EFT}(M)\, M_i^{SM} \Rightarrow \alpha_i^{Full}(M)\, \alpha_j^{Full}(M)\, M_k^{SM}\, a_{kij} \tag{3.2.11}$$

The sum of these contributions is exactly tree level and one-loop order of the result obtained above.

$$M_{Pole}^2\Big|_{TL+1L} = \mathcal{M}_1^2 + \mathcal{M}_2^2 + \mathcal{M}_3^2 \tag{3.2.12}$$

3.3 Pole mass matching

In the EFT framework, matching of theory parameters is properly defined by identifying Greens functions in both theories. However, additional ambiguities exist since the matching scale might not be definite in general theories, and there is a momentum dependence of these parameters. In the FlexibleSUSY framework, gauge couplings are matched at $p = 0$, but all other parameters are fixed by identifying pole masses of both theories as matching condition, with a strict cut at each loop order.

$$\begin{aligned}M_1^{pole} &= M_2^{pole} \\ M_1^{tree} &= M_2^{tree} \\ \left(M_1^{tree}\right)^2 - \Sigma_1^{1L}\left(M_1^{tree}\right) &= \left(M_2^{1Lm}\right)^2 - \Sigma_2^{1L}\left(M_1^{tree}\right)\end{aligned}$$

Where the index $1Lm$ denotes one-loop matching. The matching condition for the tree-level masses can be determined by the self-energy difference at the momentum of the pole mass, without the necessity to subtract some field-renormalization matching:

$$\delta\Sigma(p) = -\delta m^2 + \left(p^2 - m^2\right)\delta Z \tag{3.3.1}$$

When plugging in $p^2 = M^2_{Pole}|_{TL} = m^2$ one obtains:

$$\delta\Sigma(p) = -\delta m^2 \tag{3.3.2}$$

The parameters matched with this method are tree-level masses, and arbitrary theory parameters can be retrieved by using tree-level relation to these masses in the matched theory. Thus, when neglecting two-loop matching corrections, both self-energies contain only tree-level matched parameters, so the matching can be done loop order by loop order, without an iteration.
Additionally, it has been observed that by using this matching condition, the Higgs pole mass will contain the same expression at tree-level and one-loop order in the full and effective theory. Thus, the substitution of the terms $\mathcal{M}^2_{1\ldots3}$, as defined in the previous section, with the tree-level and one-loop expression from the full theory is redundant, which will be proven below.
Since each (physical) pole mass cannot be affected by (unphysical) running of the tree-level mass parameter, which is merely a change in the renormalization scale, the corresponding gamma function at one-loop level must contain the scale dependent part of the one-loop self-energy:

$$\frac{\partial}{\partial\mu}\left(-\mathrm{Re}\,\Sigma^{1L}_i(M_i,\mu) + \gamma^{1L}_{M_i}\ln\mu\right) = 0 \tag{3.3.3}$$

At one-loop order, this Gamma function can be derived with ease from the tree-level mass and the one-loop beta function of the model parameters, as done in our example for the SM Higgs boson:

$$\left(M_H^{SM,\,TL}\right)^2 = -\mu^2 + \frac{3}{2}\lambda v^2 \quad\Rightarrow\quad \gamma_{M^2_H} = -\beta_{\mu^2} + \frac{3}{2}\beta_\lambda v^2 + 3\lambda v\beta_v \tag{3.3.4}$$

Thus, the one-loop term from of the tree level compensates the explicit renormalization scale dependence in the one-loop self-energy. This does indeed resemble the contribution from equation 3.2.10, and one obtains:

$$\mathcal{M}^2_2 := \gamma^{1L}_{M^2_H}\,\ln\left(\frac{m}{M}\right) \tag{3.3.5}$$

Using the relation to the self-energy (3.3.3), one obtains:

$$-\mathrm{Re}\,\Sigma_H^{SM,\,1L}\left(M_H^{SM,\,TL}, m\right) + \mathcal{M}^2_2 = -\mathrm{Re}\,\Sigma_H^{SM,\,1L}\left(M_H^{SM,\,TL}, M\right) \tag{3.3.6}$$

By demanding pole mass matching at tree level and one-loop order for the Higgs mass, $\mathcal{M}_3$ from equation (3.2.11) reads:

$$\mathcal{M}^2_3 = -\mathrm{Re}\left[\Sigma_H^{Full,\,1L}\left(M_H^{Full,\,TL}(M), M\right) - \Sigma_H^{EFT,\,1L}\left(M_H^{Full,\,TL}, M\right)\right] \tag{3.3.7}$$

Finally, $\mathcal{M}_1$ from (3.2.9) was matched and run by tree-level relation, where the later is equivalent to only change the explicit scale dependence in the self energy, but not in any model parameter:

$$\mathcal{M}^2_1 = \left(M^2_H(M)\right)^{Full,\,TL} - \mathrm{Re}\left[\Sigma_H^{1L,\,EFT}\left(M_H^{Full,\,TL}(M), m\right)\right] \tag{3.3.8}$$

Plugging the definition of $\mathcal{M}_{1\ldots3}$ in numerical order into our resummation and substitution formula yields the promised relation:

$$\begin{aligned}
\left(M_H^2\right)^{Full+EFT} &= \mathrm{Re}\Big[\left(M_H^2\right)^{EFT,\,pole} - \mathcal{M}_1^2 - \mathcal{M}_2^2 - \mathcal{M}_3^2 + \left(M_H^2\right)^{Full,\,TL+1L}\Big] \\
&= \mathrm{Re}\Big[\left(M_H^2\right)^{EFT,\,pole} - \left(M_H^2(M)\right)^{Full,\,TL} + \Sigma_H^{1L,\,EFT}\left(M_H^{Full,\,TL}(M), m\right) \\
&\qquad - \mathcal{M}_2^2 - \mathcal{M}_3^2 + \left(M_H^2\right)^{Full,\,TL+1L}\Big] \\
&= \mathrm{Re}\Big[\left(M_H^2\right)^{EFT,\,pole} - \left(M_H^2(M)\right)^{Full,\,TL} + \Sigma_H^{1L,\,EFT}\left(M_H^{Full,\,TL}(M), M\right) \\
&\qquad - \mathcal{M}_3^2 + \left(M_H^2\right)^{Full,\,TL+1L}\Big] \\
&= \mathrm{Re}\Big[\left(M_H^2\right)^{EFT,\,pole} + \Sigma_H^{1L,\,EFT}\left(M_H^{Full,\,TL}(M), M\right) \\
&\qquad - \mathcal{M}_3^2 - \Sigma_H^{Full,\,1L}\left(M_H^{Full,\,TL}(M), M\right)\Big] \\
&= \left(M_H^2\right)^{EFT,\,pole}
\end{aligned} \tag{3.3.9}$$

Hence, this approach has the advantage not to differ from the full theory up to one-loop order, which may be the case for other matching algorithms. In the following, an algorithm is developed which uses pole mass matching by default, and still substitutes the terms $\sum_i \mathcal{M}_i^2$ with the one-loop pole mass from the full theory, which could have been omitted, but was nevertheless included to enable the user to modify the matching conditions at will.

In a more general aspect, this proof can be extended to arbitrary pole mass loop orders N in the effective theory, including exact n-loop terms from the full theory and looks like the following, presuming $n \leq N$:

$$\left(M_{Full+EFT}^{n,N}\right)^2 = \left(M_{EFT}^{N\,\mathrm{loop}}\right)^2 + \left(M_{Full}^{n\,\mathrm{loop}}\right)^2 - \left(M_{EFT}^{N\,\mathrm{loop}}\right)^2\Big|_{n\,\mathrm{loop}} \tag{3.3.10}$$

Regarding all terms as pole masses, which are scale independent. Choosing the scale to be of order of the heavy fields included in the full - but not the effective - field theory, and assuming the matching of pole masses at the same loop level for the full theory at this scale:

$$\left(M_{EFT}^{pole}\right)^2\Big|_{n\,\mathrm{loop}} \stackrel{!}{=} \left(M_{Full}^{pole}\right)^2\Big|_{n\,\mathrm{loop}} = \left(M_{Full}^{n\,\mathrm{loop}}\right)^2 \tag{3.3.11}$$

$$\Rightarrow \quad \left(M_{EFT}^{N\,\mathrm{loop}}\right)^2\Big|_{n\,\mathrm{loop}} = \left(M_{EFT}^{pole}\right)^2\Big|_{n\,\mathrm{loop}} = \left(M_{Full}^{n\,\mathrm{loop}}\right)^2 \tag{3.3.12}$$

Which gives:

$$\left(M_{Full+EFT}^{n,N}\right)^2 = \left(M_{EFT}^{N\,\mathrm{loop}}\right)^2 \tag{3.3.13}$$

If running is now applied to all scale dependent parameters by solving the renormalization group equations at i-loop level, assuming $i \geq N \geq n$, and this pole mass is recalculated at the scale matching the masses of the EFT fields, loop corrections will have a minimal contribution to the physical mass. Not only are all full theory diagrams up to n-loop order then still included due to the invariance of the fixed order part of the pole mass to scale redefinition, but also large logarithms at $N^{(i-1)}LL$ order are resummed.

4 Algorithm matching all SM parameters

In this chapter, the implementation of an algorithm in FlexibleSUSY is discussed, which enhances the one-loop Higgs self-energy by terms including large logarithms, resummed into all loop orders. The summarized procedure in this feature is matching an arbitrary FlexibleSUSY model at the scale of the heavy masses to the Standard Model by matching all SM gauge couplings and SM particle pole masses. These SM parameters are run to the low energy scale, internally fixed at M_Z^{pole}. The Higgs mass is calculated in the SM while replacing terms corresponding to tree level and one-loop order in the full theory at the matching scale with terms obtained from the exact diagrammatic calculation previously done by FlexibleSUSY. To do so, more than one SM model class is defined in this algorithm to serve as helper functions in filtering out the mentioned terms. Therefore the matching is done both at tree level and one-loop level, discussed in the next two sections, and defines all SM parameters by the full theory at the matching scale. The rest of the algorithm will be discussed in a third section. Additionally, some comparisons and consistency tests are included. Note that the algorithm is designed to work flawlessly with any kind of model properly implemented in FlexibleSUSY, but has to be manually adapted, if the SM class is modified.

4.1 Tree-level matching procedure

Since there are no momentum dependencies or $\overline{\text{MS}}$-$\overline{\text{DR}}$-conversion terms at tree level, the matching procedure is trivially done identifying tree-level parameters shared by both models. In the following notation, the index TL will be used to emphasize that a parameter is used at tree-level order in the corresponding theory.

1. Identifying scale parameters

$$\mu^{SM} := \mu^{full} \tag{4.1.1}$$

2. Identifying gauge couplings from $U(1) \times SU(2) \times SU(3)$ group, taking possibly differing GUT-normalizations in both models into account:

$$g_i^{SM,\,TL} := N_i \; g_i^{full,\,TL} \tag{4.1.2}$$

3. Matching VEV by using the tree-level relation:

$$v^{SM,\,TL} := 2M_W^{full,\,TL}/g_2^{SM,\,TL} \tag{4.1.3}$$

4. Defining Yukawa couplings from tree-level relation with VEV (using correct sign convention):

$$y_f^{SM,\,TL} := \pm\sqrt{2}\, m_f^{full,\,TL}/v^{SM,\,TL} \tag{4.1.4}$$

5. Defining quartic Higgs coupling by the tree-level relation:

$$\lambda^{SM,\,TL} := \left(M_H^{full,\,TL}/v^{SM,\,TL}\right)^2 \tag{4.1.5}$$

6. Defining bilinear Higgs coupling at tree level using the definition of Higgs mass in the SM at tree level:

$$\left(\mu^2\right)^{SM,\,TL} := \frac{3}{2}\lambda^{SM,\,TL}\left(v^{SM,\,TL}\right)^2 - \left(M_H^2\right)^{full,\,TL} \tag{4.1.6}$$

7. Calculate all $\overline{\text{MS}}$ parameter masses in the SM-object via `calculate_DRbar_masses()`

Any physical meaning assigned to the the parameters in the SM-class is inherited from the parameters in the model class. An important example is the VEV, which is, as any other VEV in FlexibleSUSY, defined by minimization of the effective potential at loop level, here at one-loop order. Contrary to this fact, the VEV is treated like a tree-level parameter by FlexibleSUSY, while μ^2 is expressed in terms of it and λ at tree level. Further discussion concerning tadpole contributions at one-loop order are attached in the next section.

4.2 One-loop matching procedure

The idea behind this routine is to match the gauge couplings in a fashion similar to the method used in FlexibleSUSY's threshold corrections [18, 23], from the renormalization scheme used in the full theory to the $\overline{\text{MS}}$-scheme used in the EFT model at momentum $p = 0$. The other parameters are gained from tree-level relations between these gauge couplings and $\overline{\text{MS}}$-masses regarded as tree level as well in the EFT, but obtained from integrating out heavy fields at one-loop order in the full theory. The matching condition is:

$$M^{SM}_{Pole} = M^{full}_{Pole} \tag{4.2.1}$$

Evaluated strictly at one-loop order in the full theory, which is identical to the matching at $p = M^{TL}_{full}$. Another subtlety is that the self-energies generated by SARAH do not contain tadpole contributions, which is no problem since the VEV is defined to minimize the effective potential at loop level, thus absorbing the tadpoles.
Anyway, when VEV and tree-level Higgs mass are known, λ and μ^2 can be fixed by the Higgs mass definition and the one-loop effective potential minimization constraint, but μ^2 in this case is then defined by a one-loop relation containing the VEV, λ and tadpoles. The tree-level matching routine cuts the one-loop term, which will be restored in this procedure.
In spite of containing loop corrections by definition, the VEV is regarded as a tree-level parameter by FlexibleSUSY, while μ^2 is adapted to the EWSB-condition at the desired loop level. Hence, what is denoted as tree or loop level in FlexibleSUSY, is rather a notion of powers in a polynomial of model parameters, which is identical to a diagrammatic loop order for the gauge- and Yukawa couplings as well as λ, but not for the VEV and μ^2. Nevertheless, we will cling to this notation in the following. Some parameters will now be tagged with $TL(1)$ in reference to be a product of one-loop matching. The algorithm looks like this:

1. Invoke one-loop matching routine to obtain tree-level parameters

2. Calculate e^{full} from model definition

3. Calculate one-loop corrections δe^2, δg_3^2:

$$\left(\delta g_3^2\right)^{1L} = -\frac{(g_3^4)^{full}}{8\pi^2} \sum_i \left(F_i\, T_i \ln\frac{m_i}{\mu}\right) \left[+ \frac{(g_3^4)^{full}}{16\pi^2} \right] \tag{4.2.2}$$

$$\left(\delta e^2\right)^{1L} = -\frac{(e^4)^{full}}{8\pi^2} \sum_i \left(F_i\, T_i \ln\frac{m_i}{\mu}\right) \left[+ \frac{(e^4)^{full}}{24\pi^2} \right] \tag{4.2.3}$$

Where i runs over all heavy particles, with Dynkin-indices T_i for the respective gauge group and the variables F_i which is $\frac{1}{3}$ for each real scalar degree of freedom [18]. The last term is only added if the full theory is defined in the $\overline{\text{DR}}$ scheme; μ denotes the matching scale.

4. Obtain one-loop matched SM particle masses:

$$\left(M_i^2\right)^{SM,\,TL(1)} := \left(M_i^2\right)^{full,\,TL} - \mathrm{Re}\left[\Sigma_i^{full,\,1L}\left(M_i^{full,\,TL}\right) - \Sigma_i^{SM,\,1L}\left(M_i^{full,\,TL}\right)\right] \tag{4.2.4}$$

where tree-level matched parameters are used in the SM self-energy.

5. Calculate gauge couplings at one-loop order:

$$g_1^{SM,\,TL(1)} = e^{full}\frac{M_Z^{full,\,TL}}{M_W^{full,\,TL}}\sqrt{1+\frac{(\delta e^2)^{1L}}{(e^2)^{full}}+\frac{(M_Z^2)^{SM,\,TL(1)}}{(M_Z^2)^{full,\,TL}}-\frac{(M_W^2)^{SM,\,TL(1)}}{(M_W^2)^{full,\,TL}}} \tag{4.2.5}$$

$$g_2^{SM,\,TL(1)} = \frac{e^{full}}{\sqrt{1-\left(\frac{M_W^{full,\,TL}}{M_Z^{full,\,TL}}\right)^2}}\sqrt{1+\frac{(\delta e^2)^{1L}}{(e^2)^{full}}+\frac{\frac{\left(M_Z^2\right)^{SM,\,TL(1)}}{\left(M_Z^2\right)^{full,\,TL}}-\frac{\left(M_W^2\right)^{SM,\,TL(1)}}{\left(M_W^2\right)^{full,\,TL}}}{\sqrt{1-\left(\frac{M_W^{full,\,TL}}{M_Z^{full,\,TL}}\right)^2}}} \tag{4.2.6}$$

$$g_3^{SM,\,TL(1)} = \sqrt{(g_3^2)^{full}+(\delta g_3^2)^{1L}} \tag{4.2.7}$$

6. Matching VEV by using tree-level relation:

$$v^{SM,\,TL} := \frac{2M_W^{full,\,TL}}{g_2^{full}} \tag{4.2.8}$$

$$v^{SM,\,TL(1)} := v^{SM,\,TL}\sqrt{1+\frac{(M_W^2)^{SM,\,TL(1)}}{(M_W^2)^{full,\,TL}}-\frac{(g_2^2)^{SM,\,TL(1)}}{(g_2^2)^{full}}} \tag{4.2.9}$$

7. Defining Yukawa couplings from tree-level relation with VEV (using right sign convention):

$$y_f^{SM,\,TL(1)} := \pm\sqrt{2}\frac{M_f^{full,\,TL}}{v^{SM,\,TL}}\sqrt{1+\left(\frac{M_f^{SM,\,TL(1)}}{M_f^{full,\,TL}}\right)^2-\left(\frac{v^{SM,\,TL(1)}}{v^{SM,\,TL}}\right)^2} \tag{4.2.10}$$

8. Defining quartic Higgs coupling by tree-level relation:

$$\lambda^{SM,\,TL} := \left(\frac{M_H^{full,\,TL}}{v^{full,\,TL}}\right)^2 \tag{4.2.11}$$

$$\lambda^{SM,\,TL(1)} := \lambda^{SM,\,TL}\sqrt{1+\frac{(M_H^2)^{SM,\,TL(1)}}{(M_H^2)^{full,\,TL}}-\left(\frac{v^{SM,\,TL(1)}}{v^{SM,\,TL}}\right)^2} \tag{4.2.12}$$

9. Defining bilinear Higgs coupling at tree level using the definition of Higgs mass in the SM at tree level:

$$\left(\mu^2\right)^{SM,\,TL(1)} := \frac{3}{2}\lambda^{SM,\,TL}\left(v^{SM,\,TL}\right)^2\left(\frac{\lambda^{SM,\,TL(1)}}{\lambda^{SM,\,TL}}+\left(\frac{v^{SM,\,TL(1)}}{v^{SM,\,TL}}\right)^2-1\right)-\left(M_H^2\right)^{SM,\,TL(1)} \tag{4.2.13}$$

10. Calculate all $\overline{\text{MS}}$-parameter masses in the SM-object via `calculate_DRbar_masses()`

As in the tree-level matching procedure, the parameter μ^2 is now defined by the tree-level EWSB relation from the VEV and λ.

4.3 Higgs mass calculation

The actual computation is invoked right after the spectrum is calculated at the EWSB scale. Four SM-class objects are defined in order to calculate the EFT contribution to the Higgs mass, without tree-level and one-loop terms:

`SM1`: This class contributes the full EFT Higgs mass and self-energy, by undergoing matching at one-loop and running at two-loop level, while the EWSB equations are solved to be valid at one-loop order as well at the low scale. The other objects do only subtract one-loop and tree-level terms (with respect to the full theory at matching scale) from the pole mass expression of this class.

`SM2`: This class undergoes matching and running only at tree level, but the EWSB equations are solved at one-loop order. Tree-level mass and one-loop self-energy expressions are used to subtract the entire tree level and those one-loop terms from tree-level matching and running of the self-energy in the full theory at matching scale.

`SM3`: This class undergoes matching at tree level, and only the one-loop term is retained from running. The EWSB-equations are fulfilled at tree level. Variables contained in the tree-level expression of the Higgs pole mass are used to subtract contributions from one-loop running in the full theory at matching scale.

`SM4`: This class undergoes matching at one-loop order (of the full theory at matching scale), but running at tree level, while the EWSB-equations are valid at tree level. Since only the tree-level part of the Higgs mass is required to subtract one-loop terms from the pole mass in the full theory at matching scale, and tree-level running means adjusting the scale variable (which is not contained in tree-level expressions) while leaving the parameters untouched. Hence, this contribution can be calculated after matching, without any running.

Special attention is given to the parameter μ^2 and the corresponding EWSB parameters in the full model: Since the VEVs in FlexibleSUSY are minimizing the effective potential at loop level, there are parameters, like λ in the SM, regarded as fundamental, and others, like μ^2 adapted to the EWSB-equations at a loop order depending on the needed order in the other parameters, including VEVs. Therefore the index $TL + Tad$ is introduced to denote that the tree-level expression with respect to all parameters is used, but some parameters are defined by a one-loop expression of others (tadpoles). This is how the algorithm works:

1. Matching: `SM1` and `SM4` are matched at one-loop order, `SM2` and `SM3` at tree level from the full theory.

2. Solving EWSB-constraint for μ^2 at one-loop order for `SM2` and tree level for `SM3`.

3. Running from matching to M_Z scale: `SM1` runs with full two-loop beta functions (three-loop parts may be included by the user), `SM2` runs at tree level, meaning only the scale parameter is changed, `SM3` runs in a way that the parameters are redefined as their one-loop part from this running with respect to the EFT at the matching scale:

$$\delta\alpha_i^{SM3,\,1L}(M_Z) := \ln\left(\frac{M_Z}{\mu}\right)\ \beta_i^{SM3,\,1L}(\mu) \tag{4.3.1}$$

4. The tree-level masses in `SM1` and `SM2` are recalculated at the new scale.

5. Solving EWSB constraint for μ^2 at one-loop order for `SM1`.

6. The Higgs mass contribution from `SM3` is calculated via:

$$\left(\delta M_H^2\right)^{SM3,\,TL(1)} := -\left(\delta\left(\mu^2\right)\right)^{SM3,\,1L} + \frac{3}{2}\delta\lambda^{SM3,\,1L}\left(v^{SM2,\,TL}\right)^2 + 3\lambda^{SM2,\,TL}v^{SM2,\,TL}\left(\delta v^{SM3,\,1L}\right) \tag{4.3.2}$$

   Where the tree-level parameter $\alpha^{SM2,\,TL}$, run at tree level as well, are taken from `SM2`.

7. The pure tree-level part of the Higgs mass from `SM2`, with respect to the EFT at matching scale is subtracted from the contribution of `SM4`:

$$\left(\delta M_H^2\right)^{SM4,\,TL(1)} := \left(M_H^2\right)^{SM4,\,TL(1)} - \left(M_H^2\right)^{SM2,\,TL} \tag{4.3.3}$$

8. Read p from the `physical` structure of the full model, containing the previously calculated Higgs pole mass.

9. If this is not the first iteration, recalculate the Higgs pole mass in the full theory by the heavy model at momentum p, otherwise this is already done. Note that the Index $TL+Tad$ states that a parameter is defined at tree level, but EWSB-parameters are defined by minimizing the one-loop effective potential, hence containing one-loop contributions of the model parameter.

$$\left(M_H^2\right)^{full,\,TL+1L} := \left(M_H^2\right)^{full,\,TL+Tad} - \mathrm{Re}\,\Sigma_H^{full,\,1L}(p) \tag{4.3.4}$$

10. After diagonalizing the last result and selecting the lowest eigenvalue, the EFT corrections are added:

$$\left(M_H^2\right)^{EFT,Pole} := \mathrm{Re}\Big[\left(M_H^2\right)^{full,\,TL+1L} + \left(M_H^2\right)^{SM1,\,TL+Tad} - \Sigma_H^{SM1,\,1L}(p) - \left(M_H^2\right)^{SM2,\,TL+Tad} + \Sigma_H^{SM2,\,1L}(p) - \left(\delta M_H^2\right)^{SM3,\,TL(1)} - \left(\delta M_H^2\right)^{SM4,\,TL(1)}\Big] \tag{4.3.5}$$

11. This is written into the `physical` structure, and compared to the momentum p. If this is greater than the precision of the full model, and the iteration number does not exceed the variable `number_of_mass_iterations` of the full model, go back to the beginning of the momentum loop (8.).

After this algorithm is finished, the corrected pole mass is stored in the `physical` structure as intended for every pole mass. Special attention must be paid to choosing the scale to adapt μ^2 to the EWSB-conditions: Since the full theory contains heavy as well as light particles contributing to tadpole diagrams in the Higgs sector, there is no trivial proper choice of scale to minimize loop corrections for the effective potential. For the matching procedure at the high scale, large logarithms in the tadpole contributions of the full theory arise from SM particles in loop diagrams. However, none of these terms enter the one-loop matched parameters, since the one-loop tadpoles from the SM are subtracted from the pole mass of the full theory as part of the matching procedure. For the matched SM instances, a proper choice of scale to fulfill EWSB relations is some light, electroweak scale, instead of the heavy scale. Since `SM2` is bound to subtract some one-loop contribution from `SM1`, the EWSB relation must be fulfilled at the high scale, otherwise double counting of the one-loop terms from `SM3` occurs. Nevertheless, regarding `SM1` it is essential for the accuracy of this algorithms, if loop corrections to the tadpoles are minimized, and thus, the EWSB is solved at the low scale. It shall be denoted that the fixing of μ^2 in FlexibleSUSY is quite convenient since no beta function other than its own depends on this parameter. Hence, no iteration between the scales of fixing μ^2 and matching of other parameters is necessary.

4.4 Benchmark: Matching SM to SM

In order to check the functionality of the previously described algorithm, it is useful to examine the running and substitution of the tree-level and one-loop part from the effective field theory apart from the matching. To do so, the Standard Model class in FlexibleSUSY was matched to itself, without integrating out any fields. In this scenario, λ was given as input parameter at the high scale $\mu_{high} = v\sqrt{\lambda}$, thus controlling the scale difference between high and low scale, the latter one to be chosen as the Z boson pole mass. Since the high scale happens to be the tree-level Higgs mass, the plot also shows the contribution of loop corrections to the physical mass as deviation from the symmetric diagonal line.

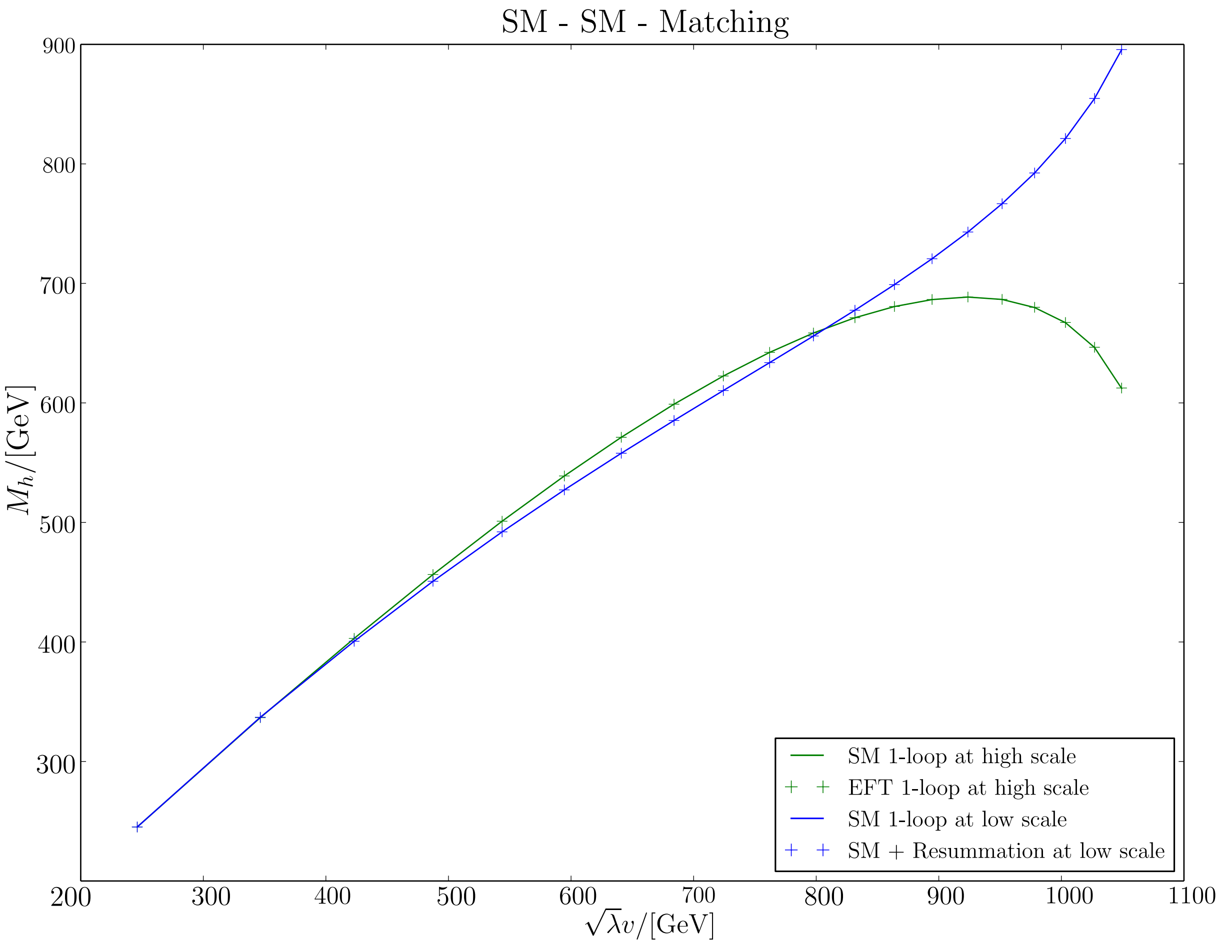


Figure 2: Standard Model Higgs pole masses, with the Standard Model as effective field theory

The solid green line depicts the Higgs pole mass calculated via one-loop self-energy at the high scale, while the green +-shaped points mark the pole mass calculated from a matched theory at the same scale, proving that the SM-SM matching at one-loop order does indeed yield the same results. The solid blue line shows the pole mass calculated from the initial SM class run to the low scale, while the blue markers were calculated using the replacement algorithm to extract tree-level and one-loop self-energy from the effective theory and replace it with the exact terms. The equality of both graphs suggests that the replacement is working properly, at least in cases where the matching is trivial. Another observation is the convergence of all depicted curves in the limit of low λ, as the high scale approximates the low scale. This behavior is expected since both matching and running have no effect in this limit; a non-vanishing difference of the blue crosses from any other of the graphs would have been an indicator for bugs in the implementation.

4.5 Benchmark: Comparison to SUSYHD

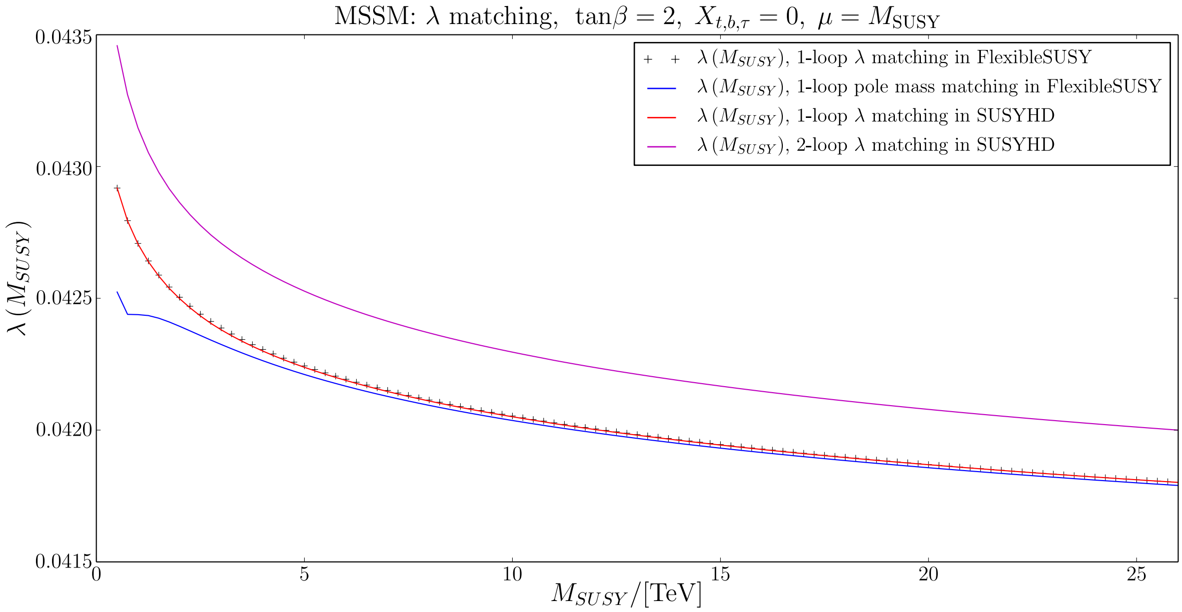
For the sake of further comparison, the algorithm was applied to the Minimal Supersymmetric Standard Model (MSSM). To achieve quasi-degeneracy of all heavy particle masses, soft sfermion and gaugino masses, the CP-odd Higgs A^0 tree-level mass and the parameter μ are chosen to be and at some heavy SUSY scale M_{SUSY}. Furthermore, the sfermion mixing parameters $X_{t,b,\tau}$ have been set to zero by adjusting the trilinear couplings. Two scenarios have been considered: $\tan\beta = 2$ and $\tan\beta = 20$, both renormalized at the SUSY scale.
For cross checking, SUSYHD [24], another EFT-based package specialized to the MSSM was taken into account. Since SUSYHD v1.0.1 uses hard coded Yukawa- and gauge couplings at the low scale chosen to be the top pole mass, these parameters were constrained in the MSSM in FlexibleSUSY such that the effective theory would yield the same values at the top pole mass. SUSYHD also makes use of an approximated Higgs pole mass formula taken from [25]:

$$M_h^{Pole} = \frac{1}{2^{1/4}\sqrt{G_F}}\,\sqrt{1.0075\;\lambda + \delta\lambda_{2L}} \qquad (4.5.1)$$

including two-loop contributions from QCD and QED, taken from [26], while FlexibleSUSY uses the complete one-loop expression for the self-energy, which complicates the comparability between the two codes.

Additionally, SUSYHD implements a direct one-loop matching formula for λ, supplemented by two-loop terms $\sim \mathcal{O}\left(\alpha_t^2, \alpha_s\alpha_t\right)$ and resummed contributions $\sim y_{b,\tau}\,\tan\beta$, both taken from [27] and based on their own work [24], neglecting all Yukawa couplings of the first two generations and terms proportional to inverse orders of the SUSY masses. FlexibleSUSY on the other hand matches by the condition that the Higgs pole masses are equal at one-loop order, using the full one-loop self-energy.

The figures 3 and 4 depict the values of λ obtained with each code. For both choices of $\tan\beta$, every λ at the low scale is numerically very close to each other, even adding two-loop contributions provided by SUSYHD does not make any difference. As a measure for relativization, λ at the low scale was also plotted for mere tree-level matching at the high scale. At the SUSY scale, the relative difference between all curves is much larger, the value at the low scale is mainly determined by additive running corrections from parts of the beta functions not proportional to λ itself. This diminishes the influence of different matching methods on the Higgs mass calculation. The one-loop SUSYHD output exhibits good agreement with matching using expressions from [27] implemented in FlexibleSUSY, as expected since both matching algorithms are equal. Nevertheless, this proves the equivalence of the running in both packages. The pole mass matching in FlexibleSUSY exhibits differences to the matching in SUSYHD especially for smaller SUSY scales, but this difference diminishes for increasing scales to a small remaining limit. That might be due to terms suppressed by inverse powers of the SUSY masses included in the pole mass matching but not in SUSYHD. To introduce a notion of loop order effects, additional matching contributions included in SUSYHD have been plotted, but tree-level matching is not, since it would be situated outside the scale. Overall, both packages show satisfactory agreement in the matching and running of λ, which motivates further discussions on the pole mass itself, in spite of the mentioned difficulties regarding comparability.



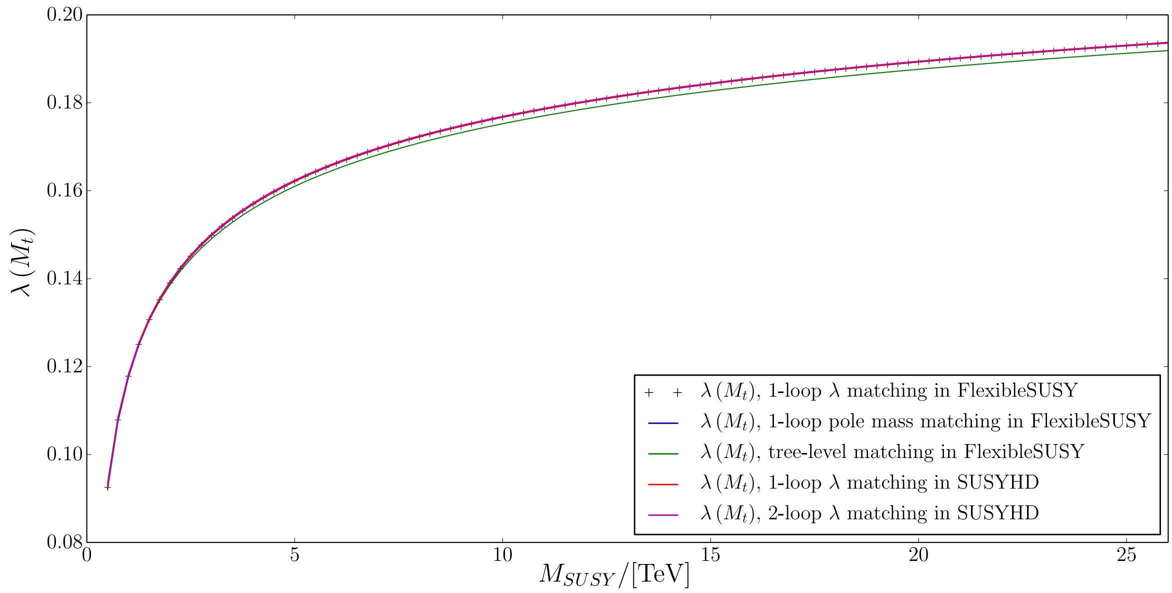


Figure 3: λ matched from MSSM with $\tan\beta = 2$, $X_{t,b,\tau} = 0$, $\mu = M_{SUSY}$, blue line: λ from matching one-loop Higgs pole masses in FlexibleSUSY (default algorithm), green line: tree-level matching for λ in FlexibleSUSY, black marker: one-loop matching for λ using formula provided by [27], red line: one-loop matching implemented in SUSYHD, purple line: one-loop matching + 2-loop $\mathcal{O}\left(\alpha_t^2,\, \alpha_s\alpha_t\right)$ + resummed $\mathcal{O}\left(\alpha_{b,\tau}\tan\beta\right)$ contributions implemented in SUSYHD

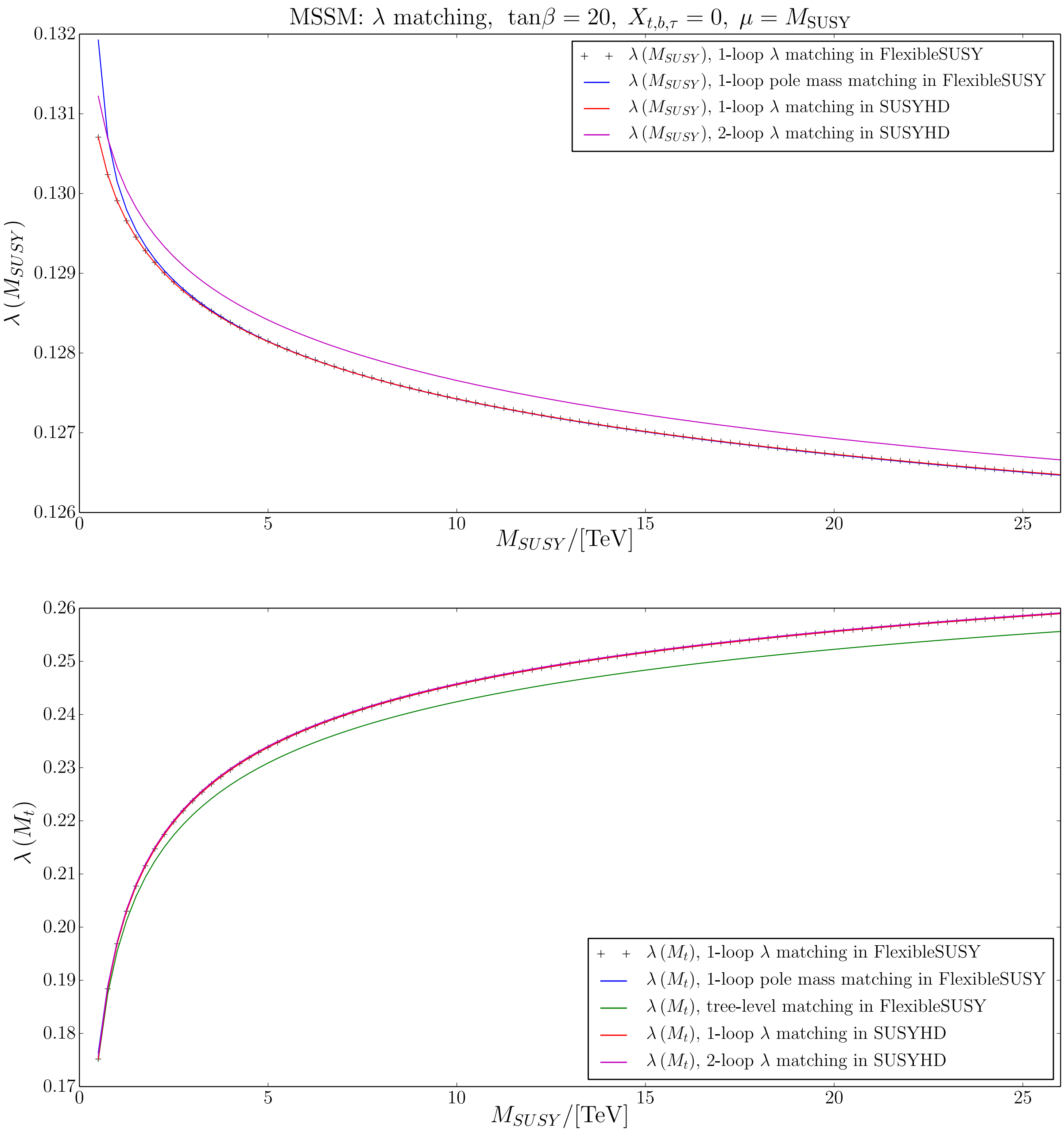


Figure 4: λ matched from MSSM with $\tan\beta = 20$, $X_{t,b,\tau} = 0$, $\mu = M_{SUSY}$, blue line: λ from matching one-loop Higgs pole masses in FlexibleSUSY (default algorithm), green line: tree-level matching for λ in FlexibleSUSY, black marker: one-loop matching for λ using formula provided by [27], red line: one-loop matching implemented in SUSYHD, purple line: one-loop matching + 2-loop $\mathcal{O}(\alpha_t^2,\ \alpha_s\alpha_t)$ + resummed $\mathcal{O}(\alpha_{b,\tau}\tan\beta)$ contributions implemented in SUSYHD

Higgs pole masses are depicted in figures 5 and 6 over the SUSY scale. Comparison between the one- and two-loop fixed order calculations in FlexibleSUSY and the resummed one exhibits significant differences between these two computations within the same package, especially for large SUSY masses. However, comparing the RGE enhanced computations between FlexibleSUSY and SUSYHD, the discrepancy is $< 1\,\text{GeV}$ for the SUSY mass spectrum 0.5 ... 26 TeV in the investigated cases of $\tan\beta = 2,\ 20$. Plotting the pole mass obtained by FlexibleSUSY using the same matching algorithm as SUSYHD does, shows excellent agreement with the default algorithm matching pole masses. Since it has been shown earlier that this reproduces the model parameters in accord to SUSYHD, we deduce that the discrepancy between the latter and FlexibleSUSY is mainly caused by the method to calculate the Higgs pole mass at the low scale. Additionally, the substitution of tree-level and one-loop term in the Higgs pole mass calculated using the EFT approach in FlexibleSUSY with the exact terms from the MSSM tree level and self-energy has been plotted to ensure again that this procedure does not alter the result. It might be difficult to resolve from the graph, but this is indeed the case up to small numerical uncertainties, and it still holds for smaller SUSY scale regions, where this equivalence is not shadowed by the λ matching curve.

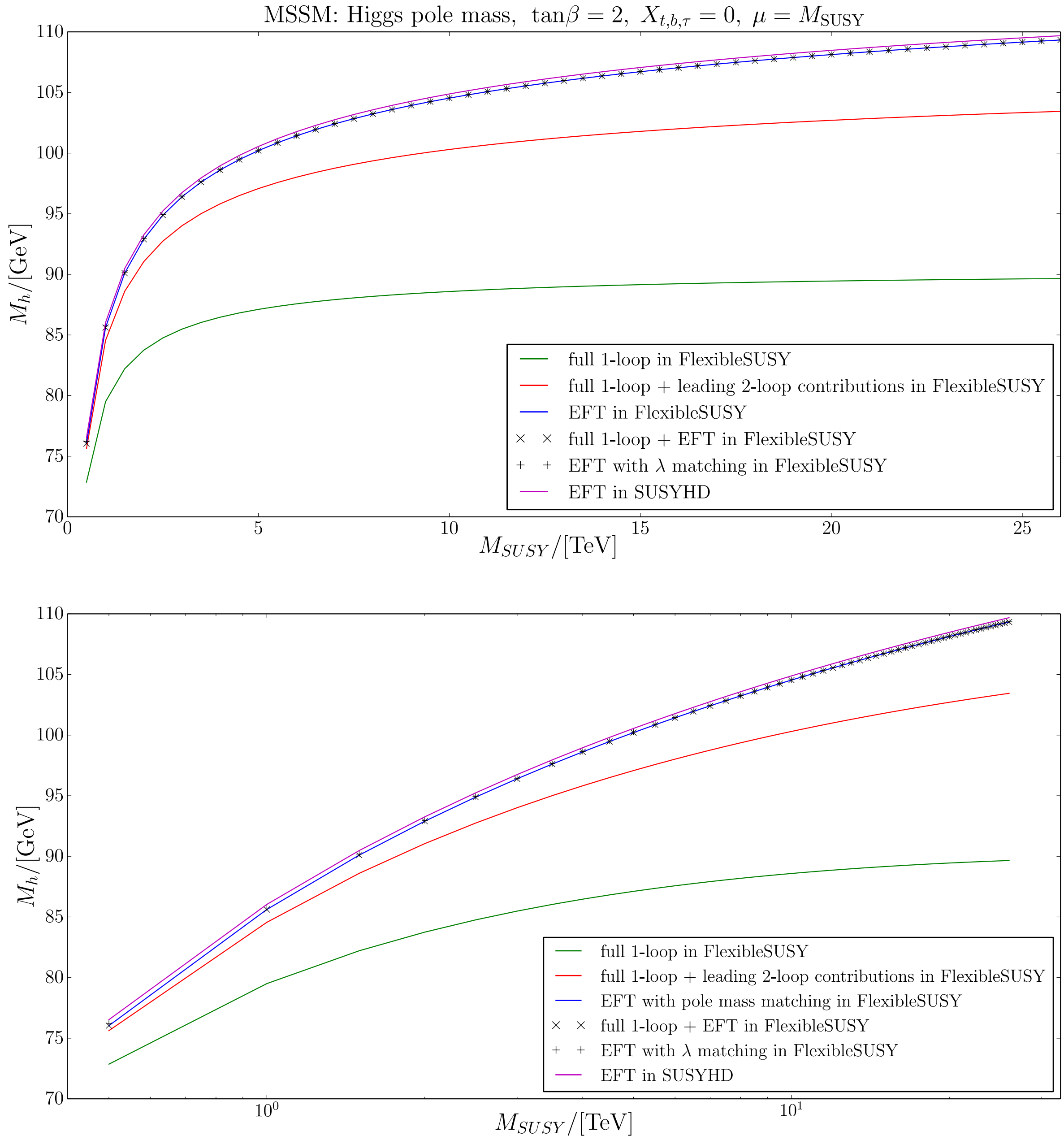


Figure 5: Higgs pole masses in the MSSM with $\tan\beta = 2$, $X_{t,b,\tau} = 0$, $\mu = M_{SUSY}$, green line: complete one-loop MSSM Higgs pole mass calculated by FlexibleSUSY, red line: like green line, but with two-loop contributions $\mathcal{O}\left(\alpha_{t,b,\tau}^2,\ \alpha_t\alpha_b,\ \alpha_{t,b}\alpha_s\right)$, blue line: pole mass in effective field theory with default pole mass matching in FlexibleSUSY, black '+'-marker: pole mass in effective field theory with λ matching in FlexibleSUSY, black '×'-marker: full one-loop order in MSSM and resummed contributions from (default) EFT matching in FlexibleSUSY, purple line: pole mass in effective theory in SUSYHD

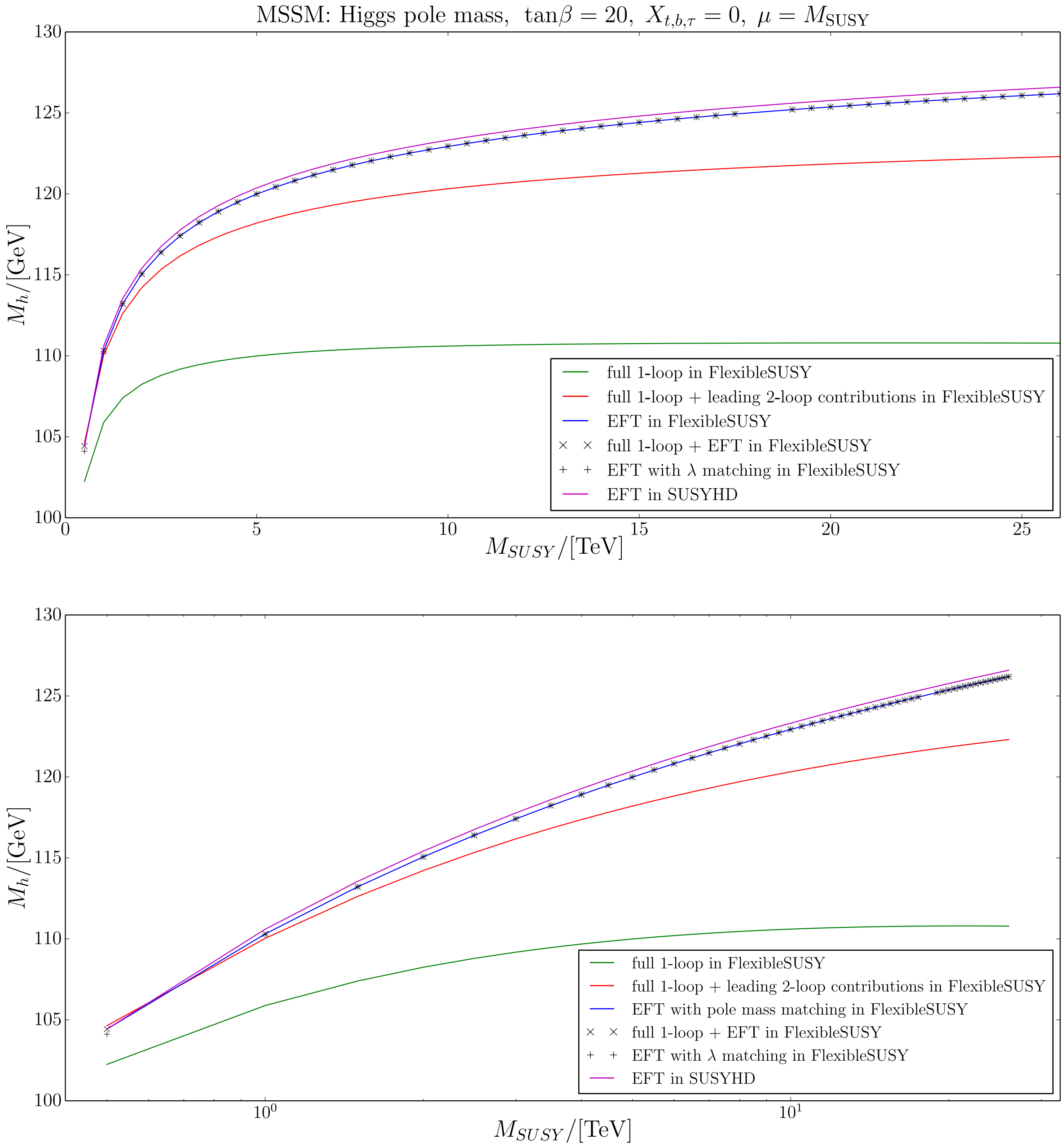


Figure 6: Higgs pole masses in the MSSM with tan$\beta = 20$, $X_{t,b,\tau} = 0$, $\mu = M_{SUSY}$, green line: complete one-loop MSSM Higgs pole mass calculated by FlexibleSUSY, red line: like green line, but with two-loop contributions $\mathcal{O}\left(\alpha^2_{t,b,\tau},\ \alpha_t\alpha_b,\ \alpha_{t,b}\alpha_s\right)$, blue line: pole mass in effective field theory with default pole mass matching in FlexibleSUSY, black $'+'$-marker: pole mass in effective field theory with λ matching in FlexibleSUSY, black $'\times'$-marker: full one-loop order in MSSM and resummed contributions from (default) EFT matching in FlexibleSUSY, purple line: pole mass in effective theory in SUSYHD

Summarizing this chapter, the algorithm described, including the matching, running and substitution of the one-loop order and tree level with terms from the effective theory seems to work flawlessly, and results from SUSYHD are reproducible. However, the last point was achieved by adapting gauge and Yukawa couplings in the MSSM, that after matching to the SM, would reproduce the hard-coded SUSYHD input. But this is a quite artificial scenario, since it bypasses the functionality of FlexibleSUSY to constraint these parameters, and also circumvents a critical issue discussed in the next chapter, which spoils the precision observed in this section and suggests to design another algorithm to cope.

5 Algorithm matching λ only

5.1 Motivation: Dependence on input parameters and threshold corrections

In this section, the implementation of another algorithm, matching only λ to the effective model, and fixing VEV, gauge and Yukawa couplings at the low scale will be motivated, which abandons the goal to retain the tree-level and one-loop terms from the full theory in the effective Higgs pole mass for precision, since the full theory parameters are not calculated with optimal precision in the first place. The necessity of this is due to a design flaw of FlexibleSUSY: the automated computation of threshold corrections for Yukawa- and gauge couplings in FlexibleSUSY are done in two steps: considering top corrections to gain Standard Model values from the effective 5-flavor input-values, and adding contributions from beyond the Standard Model. The second ones are one-loop contributions from fields that are, in a scenario assumed ideally for our purpose of using an effective field theory approach, heavy compared to all SM particles. However, both kinds of thresholds are calculated at the lowest scale, fixed to be the Z boson pole mass. Unfortunately, if the heavy mass scale is large, so are their one-loop thresholds, due to the appearance of large logarithms. These contributions could be resummed by implementing a running between the low scale, where the top contributions are calculated, and another scale where the heavy particles are integrated out from the full theory, but this is not implemented. The procedure is equivalent to constraining the effective theory by data obtained from electroweak observables in the SM at the low scale, instead by matching from the full theory. A consequence of this omission is that couplings in the full theory, taken for matching to an effective one, are missing thresholds containing large logarithms resummed in all loop orders. The matched theory, although consisting of the Standard Model gauge group and particle content, then differs in numerical values for model parameters and consequently also observables compared to those taken as input, since thresholds are calculated from the SM to the full theory at the low scale first, but the matching back is done at the heavy mass scale. Consequently, the resummed Higgs mass will differ by resummed large logarithms leading at two-loop order and higher in the effective theory at the low scale.

Moreover, this imprecision is persistent and does not vanish by increasing the heavy mass scale, on the contrary, it becomes more severe, since this is an resummation error. SUSYHD for instance does only match λ from the full to the effective theory, since all other SM parameters are fixed by electroweak observables. Since this is done at a convenient scale, no precision issues due to thresholds arise in this code. Indeed, the discrepancy to the algorithm from the last chapter might be, depending on the scenario, several percents in the Higgs pole mass. It will be shown that this is mainly due to inaccuracies of the top Yukawa coupling, which, as discussed before, dominates the running of λ in non-multiplicative RGE terms. Caused by this mechanism, the impact of inaccuracies of the matched λ at the SUSY scale diminishes with increasing matching scale, as discussed in the previous chapter. Thus the idea to amend the algorithm is only to match λ from the full theory, taking threshold induced inaccuracies into account, but fixing VEV, gauge and Yukawa coupling in the effective field theory at the low scale, not to produce inaccuracies in these quantities enhanced by large logarithms.

5.2 Implementation

As stated before, λ is the only parameter input from the full theory, but also options like loop orders for considered thresholds and beta functions as well as numerical values for Z, W and top pole masses are passed from the FlexibleSUSY SLHA input to this algorithm, but low scale constraints imposed have no effects. The algorithm commences after the solver in FlexibleSUSY has finished.
The gauge and top coupling is fixed in this algorithm at the top pole mass scale by the conditions taken from [26], the VEV is determined at the same scale by [28]:

$$g_Y\left(M_t\right) := 0.35830 + 0.00011\left(\frac{M_t}{\text{GeV}} - 173.34\right) - 0.00020\left(\frac{M_W}{\text{GeV}} - 80.384\right)/0.014 \quad (5.2.1)$$

$$g_2\left(M_t\right) := 0.64779 + 0.00004\left(\frac{M_t}{\text{GeV}} - 173.34\right) + 0.00011\left(\frac{M_W}{\text{GeV}} - 80.384\right)/0.014 \quad (5.2.2)$$

$$g_3\left(M_t\right) := 1.1666 - 0.00046\left(\frac{M_t}{\text{GeV}} - 173.34\right) + 0.00314\left(\frac{\alpha_3^{5\,\text{fl}}\left(M_Z\right) - 0.1184}{0.0007}\right) \quad (5.2.3)$$

$$\left|y_t\left(M_t\right)\right| := 0.93690 - 0.00556\left(\frac{M_t}{\text{GeV}} - 173.34\right) - 0.00042\left(\frac{\alpha_3^{5\,\text{fl}}\left(M_Z\right) - 0.1184}{0.0007}\right) \quad (5.2.4)$$

$$\begin{aligned} v^2\left(M_t\right) := \frac{1}{\sqrt{2}G_F} + \frac{1}{(4\pi)^2}\Bigg[& -3M_t^2 + M_W^2\left(5 - 12\ln\frac{M_W}{M_t}\right) + M_Z^2\left(\frac{5}{2} - 6\ln\frac{M_Z}{M_t}\right) \\ & + \frac{3}{2}\frac{M_W^2 M_Z^2}{M_Z^2 - M_W^2}\ln\frac{M_Z}{M_W} - 6\frac{M_W^2 M_h^2}{M_h^2 - M_W^2}\ln\frac{M_h}{M_W} - \frac{M_h^2}{2}\Bigg] \end{aligned} \quad (5.2.5)$$

Other Yukawa couplings are calculated using this VEV and the $\overline{MS}$ masses defined by the `QedQcd` object provided by the low scale constraint, calculating those from SLHA input. The top Yukawa definition includes 3-loop QCD contributions, which accounts for a constant difference

$$y_t^{2L} - y_t^{2L+3L_{QCD}} = 0.00328$$

that are included for comparison in this chapter, but can in general be controlled by the threshold loop order of the full theory, same is true for the running. It shall be denoted that the definitions of the gauge and top coupling are those also used in SUSYHD, when adjusting the SLHA input parameter:

$$M_t := 173.34\,\text{GeV} \qquad M_h := 125.09\,\text{GeV} \qquad \alpha_3^{5\,\text{fl}}\left(M_Z\right) := 0.1184$$
$$M_W := 80.384\,\text{GeV} \qquad M_Z := 91.1876\,\text{GeV} \qquad G_\mu := 1.1663787 \times 10^{-5}\,\text{GeV}^{-2}$$

The running in the SM includes full two-loop beta functions with g_3, y_t three-loop terms for and with gauge and top couplings, the Higgs mass calculation at the low scale is done at one-loop order, but can be adjusted in general. Thus, only λ remains a free parameter, while all others are fixed at the top mass scale. At the heavy scale, λ is calculated using the following expression, which is equivalent to the λ matching in the algorithm described earlier:

$$\lambda^{EFT} := \left(\frac{e^{(1L)}\,M_h^{(1L)}\,M_Z^{(1L)}}{2\,M_W^{(1L)}}\right)^2 \frac{1}{\left(M_Z^{(1L)}\right)^2 - \left(M_W^{(1L)}\right)^2} \quad (5.2.6)$$

Where the masses are matched from the full theory by pole mass matching, as described before. Since $\lambda^{EFT}\left(M_{SUSY}\right)$ merely depends on parameters from the full model, it is only calculated once. The iterative running between both scales continues until λ at the light scale and the VEV at the heavy scale have converged, or a maximum number of iterations has been reached.

5.3 Comparison to SUSYHD

In the scenario considered in the previous chapter, the MSSM, regarded as the full theory, was constrained in the SUSY parameters to yield sparticle masses degenerate around the SUSY scale, as well as gauge and Yukawa couplings in the effective theory matched by the algorithm described previously, that are compliant with SM input data. Hence, the thresholds have automatically been resummed in the full theory.
This was, for the sake of comparison, replotted in Fig. 7 as blue crossed markers. Anyway, implementation of such an algorithm, readapting parameters of the full model, invoked after the solver in FlexibleSUSY has finished is neither feasible for general theories nor reasonable, since convergence and compliance of all model parameters with the constraints are violated. The new algorithm, matching only λ is marked as a solid black line. The previously implemented algorithm (solid blue line), was matching all SM parameters from the full theory, is also depicted in a scenario where the MSSM is not adapted to yield fixed parameters after matching, but are determined by using the default FlexibleSUSY SLHA input via observables. This is a more convenient example of out-of-the-box usage of FlexibleSUSY, but there is no proper resummation of thresholds using the old algorithm. It may be denoted that the difference between both algorithms in the pole masses calculated by FlexibleSUSY in this plot is even around 10 GeV, and therefore the SUSYHD result is now rather overestimated than underestimated by the calculation using two-loop contributions, since the fixed order calculation do also shift because the MSSM parameters are now determined by default FlexibleSUSY procedures, see [18]. In spite of that, the output produced by SUSYHD does not shift due to the input consisting of non-SM parameters only, fixed at the SUSY scale by definition.

The comparison between the computations considering the MSSM with resummed thresholds at the heavy scale and the new algorithm merely using properly resummed thresholds in the effective theory exhibits only very small deviations. This raises both: hope to have found a viable solution for dealing with the inaccuracies occurring due to threshold issues, and the question why this discrepancy is so tiny compared to the deviation to the algorithm without resummed heavy thresholds. This will be the topic of further investigations.
Additionally, the remaining difference between SUSYHD and the calculations including resummed thresholds is once more < 1 GeV and probably due to the differences in the pole mass calculation, following the argumentation from the previous chapter.

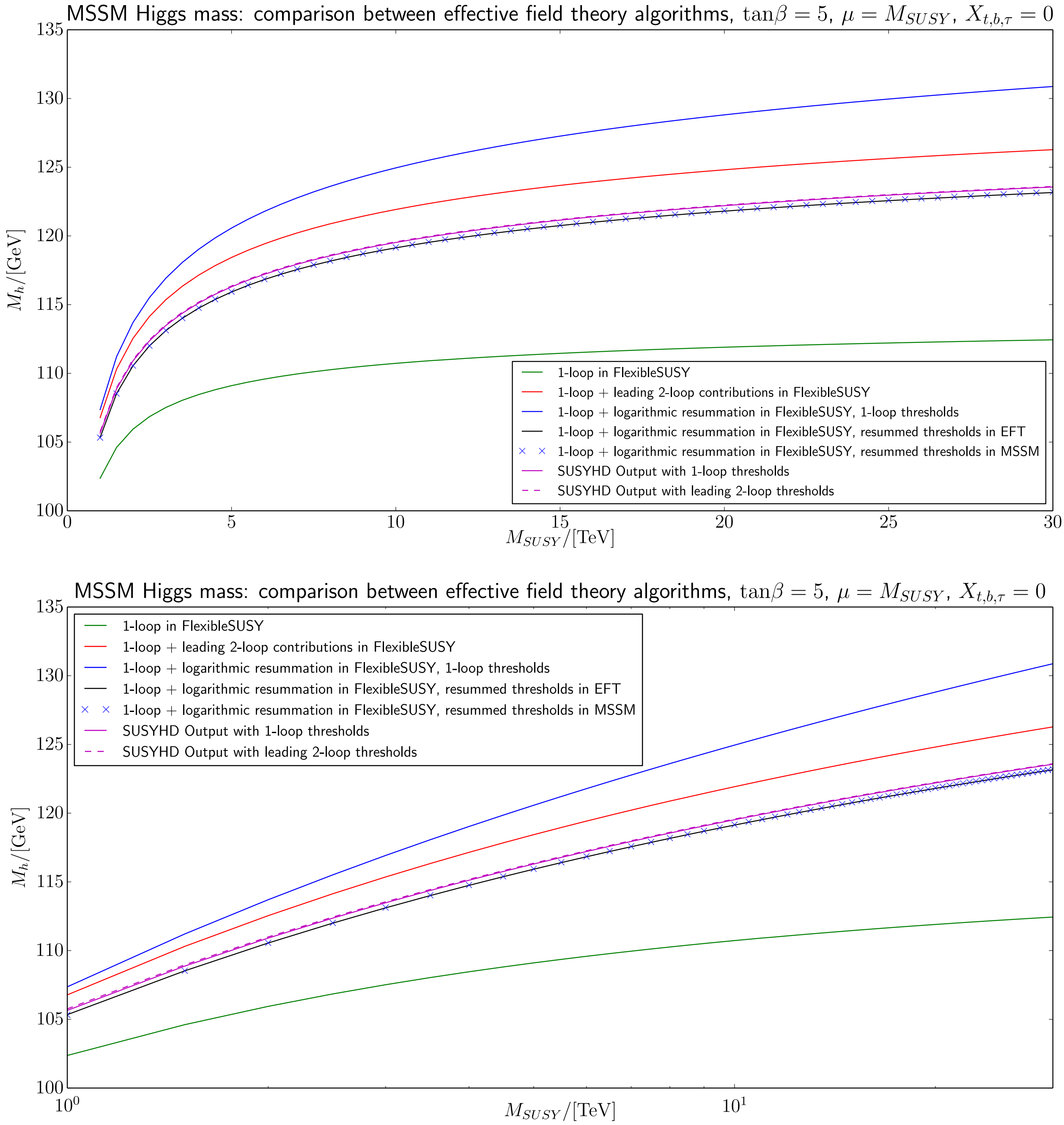


Figure 7: Lightest Higgs pole mass in the MSSM, $\tan\beta = 5$, soft squark and gaugino masses at the SUSY scale, no squark mixing. Green line: one-loop FlexibleSUSY result with heavy thresholds at the low scale; red line: like the green one, but with leading two-loop corrections; blue line: heavy thresholds at the low scale and matching to the effective field theory; black line: effective field theory with gauge and Yukawa couplings from electro-weak input, but λ from pole mass matching; blue marker: heavy thresholds at the SUSY scale and matching to the effective field theory; purple lines: SUSYHD output using one- and two-loop threshold corrections for matching of λ

For additional examinations, λ parameters matched from the MSSM were plotted at the SUSY scale as well as the top pole mass scale in Fig. 8. Once more, SUSYHD output was plotted (red and purple lines) as well as FlexibleSUSY computations with heavy thresholds at the low scale (solid blue line), the effective theory with only using λ as input and all other SM parameters fixed (solid black line), and properly resummed heavy thresholds in the MSSM, displayed as blue dots.

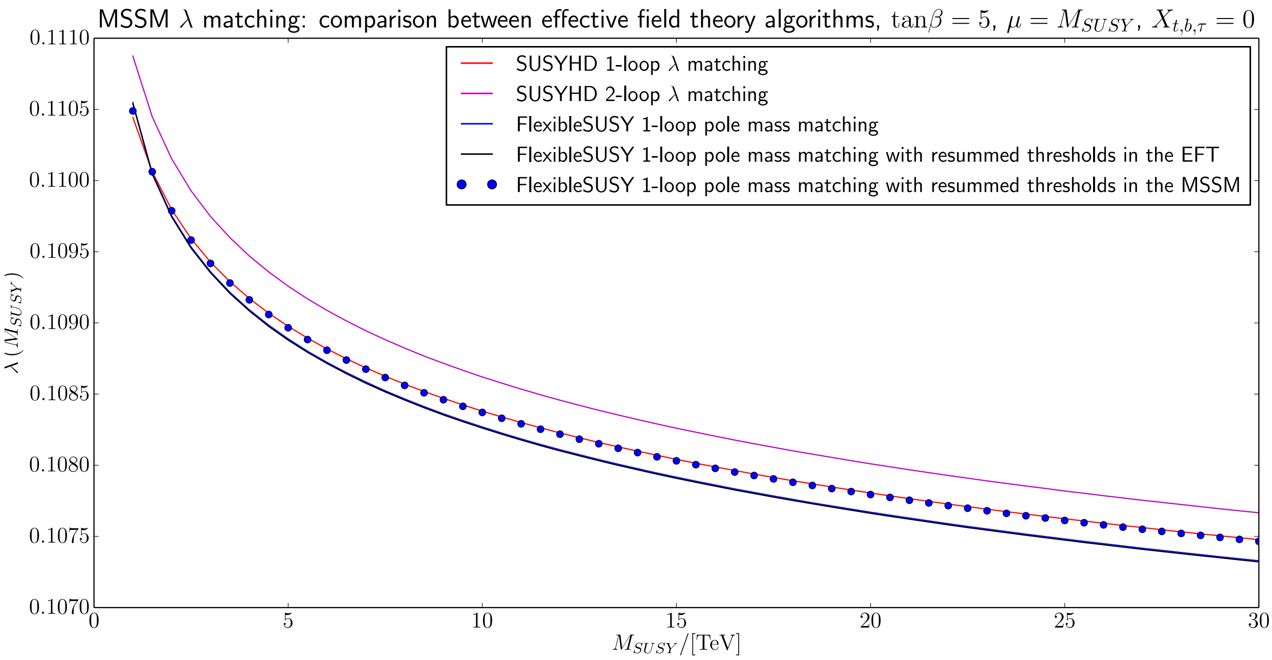
For λ at the SUSY scale, both algorithms in FlexibleSUSY computing heavy thresholds at the low scale yield the same curve, since the matching algorithm is identical. Furthermore, the one-loop matched SUSYHD output and the FlexibleSUSY calculation involving the MSSM with resummed SUSY thresholds at the heavy scale yield equal results, which was already an outcome in the last chapter. Interesting is the relative difference between both pairs of lines, which is $< 1\%$ in the plotted SUSY scale region, and hence cannot account for the 10 GeV difference in the previous plot. Beholding $\lambda\left(M_t\right)$, it becomes obvious that the primary reason of this discrepancy is rather caused by the running: on the one hand, the deviation of SUSYHD from the new algorithm with fixed gauge and Yukawa couplings in the effective theory has diminished, but on the other hand, in spite of still being equal to the new algorithm at the SUSY scale, the full matching of all parameters from the MSSM with heavy thresholds at the low scale gains a large deviation from all other λ's, increasing with the SUSY scale. Since the running routines have been checked and this is the only deviating curve at the low scale while there is none at the high scale, it is most likely caused by other parameters than λ itself at the latter scale.

Indeed, low scale values of parameters like the top Yukawa coupling have critical influence on the precision, since the beta function (obtained using [20]) for λ contains terms not multiplicative to itself:

$$\begin{aligned}(4\pi)^2\beta_\lambda = {}& 12\lambda^2 - \frac{9}{5}g_1^2\lambda - 9g_2^2\lambda + 12\,\mathrm{Tr}\left\{\mathbf{y_u}\,\mathbf{y_u^\dagger}\right\}\lambda + 12\,\mathrm{Tr}\left\{\mathbf{y_d}\,\mathbf{y_d^\dagger}\right\}\lambda + 4\,\mathrm{Tr}\left\{\mathbf{y_e}\,\mathbf{y_e^\dagger}\right\}\lambda \\ &+ \frac{27}{100}g_1^4 + \frac{9}{10}g_1^2g_2^2 + \frac{9}{4}g_2^4 - 12\,\mathrm{Tr}\left\{\mathbf{y_u}\,\mathbf{y_u^\dagger}\,\mathbf{y_u}\,\mathbf{y_u^\dagger}\right\} - 12\,\mathrm{Tr}\left\{\mathbf{y_d}\,\mathbf{y_d^\dagger}\,\mathbf{y_d}\,\mathbf{y_d^\dagger}\right\} \\ &- 4\,\mathrm{Tr}\left\{\mathbf{y_e}\,\mathbf{y_e^\dagger}\,\mathbf{y_e}\,\mathbf{y_e^\dagger}\right\} + \mathcal{O}\left(2\text{ loop}\right)\end{aligned} \tag{5.3.1}$$

These terms not proportional to λ, but to couplings larger that itself, like the top Yukawa, are dominating in the beta function and therefore influence the running. These are contributions enhanced by large logarithms at the low scale, especially if the SUSY scale is high. Tree level and logarithmic enhanced part of the one-loop Higgs self-energy does, besides depending on the quartic coupling $\tilde{\lambda}$ at the heavy scale $\tilde{\mu}$, also gain larger contributions from top and gauge couplings at the low scale μ, enhanced by a large logarithm:

$$m_h^2\left(\mu\right) = v^2\tilde{\lambda} + \frac{v^2}{\left(4\pi\right)^2}\ln\left(\frac{\mu}{\tilde{\mu}}\right)\left(-12\,y_t^4 + \frac{27}{100}g_1^4 + \frac{9}{10}g_1^2g_2^2 + \frac{9}{4}g_2^4\right) + \ldots \tag{5.3.2}$$



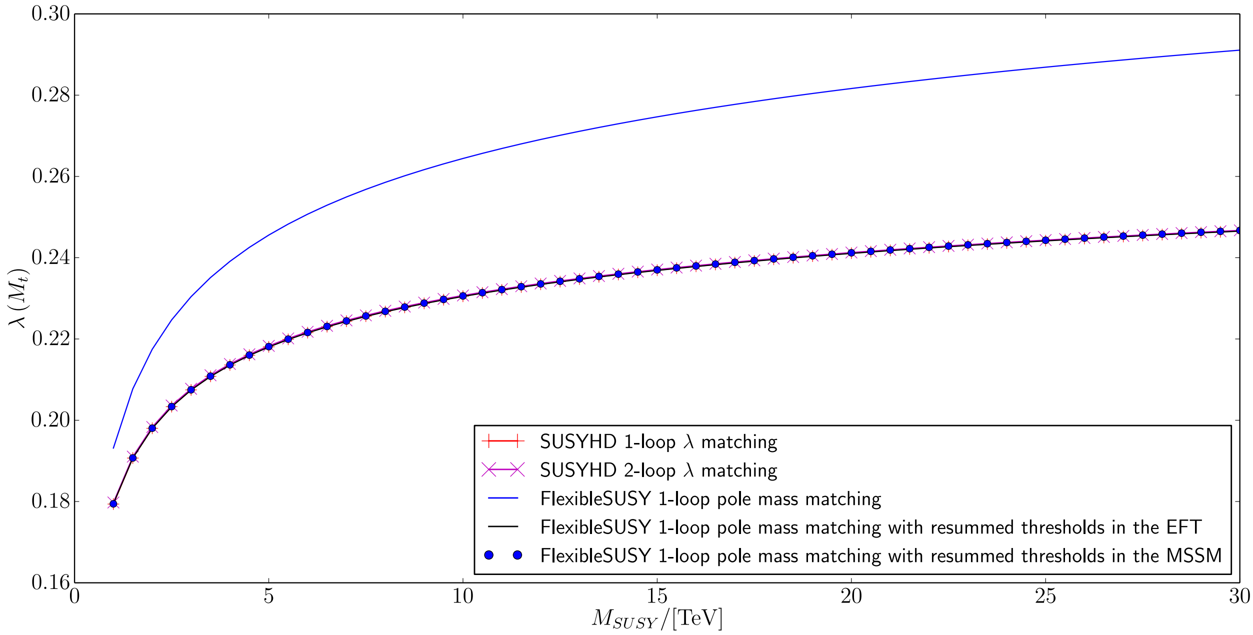


Figure 8: λ matched from MSSM, $\tan\beta = 5$, soft squark and gaugino masses at the SUSY scale, no squark mixing. Blue line: pole mass matching from MSSM with heavy thresholds at the low scale in FlexibleSUSY; black line: pole mass matching for λ, other couplings fixed by electroweak input; red and purple lines/markers: matching in SUSYHD, using leading one- and two-loop thresholds; blue markers: λ matched from an MSSM with gauge and Yukawa couplings fixed to yield the low energy values used in SUSYHD after full pole mass matching in FlexibleSUSY

Concerning this specific test, it shall be denoted that comparing the SM parameters in e.g. SUSYHD and those from the full matching of all parameters from the MSSM without resummed heavy thresholds at the SUSY scale, the discrepancies for the gauge couplings are indeed one order of magnitude smaller than those of the Yukawas, where the top coupling, due to its largeness, has the most influence. Main causes of this deviation can be traced back to a difference of roughly 5% of the Yukawa couplings in the effective theory, plotted in Fig. 9.

Running back to the top pole mass scale, the algorithms considering fixed couplings at the low scale exhibit equivalence as expected, while the one using matching for all effective parameters deviates with logarithmically increasing magnitude.

Fig. 10 shows the heavy one-loop top Yukawa threshold (including sparticles but also top

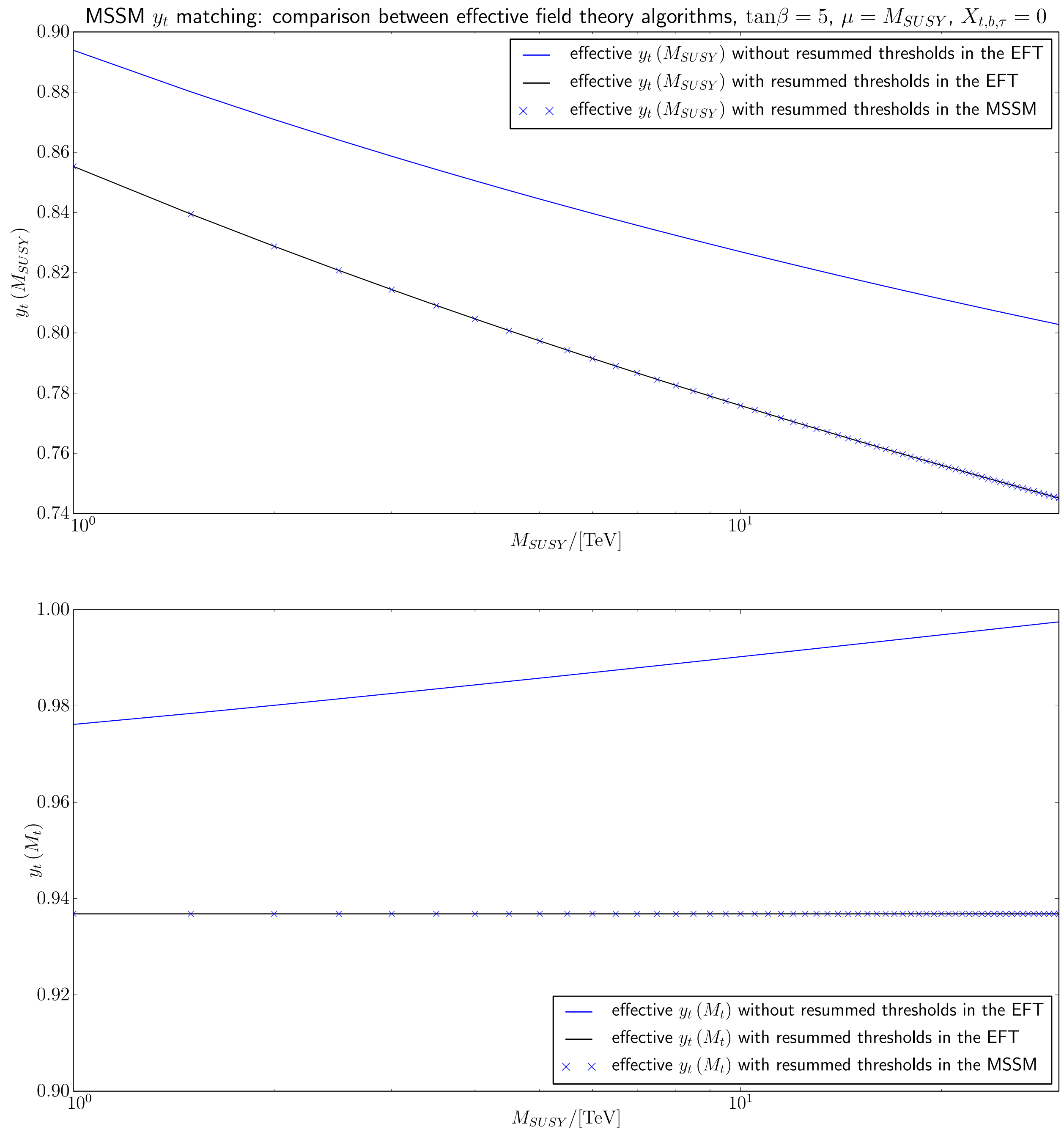


Figure 9: Top Yukawa coupling in the SM, matched from MSSM: $\tan\beta = 5$, soft squark and gaugino masses at the SUSY scale, no squark mixing. Blue line: pole mass matched coupling from MSSM using FlexibleSUSY input parameter and one-loop thresholds at the low scale; black line: same as blue, but only λ is matched and other parameters are fixed by electroweak observables; blue marker: full matching of all parameters, but MSSM was modified to yield fixed gauge and Yukawa coupling values after matching

contributions) assumed by FlexibleSUSY, calculated at different scales from the top pole mass and the one-loop self-energy. Obviously, the correction determined at the M_Z scale (on the very left side) as it is default in the framework, is twice as large as calculated at the SUSY scale. This difference is of the same magnitude as the deviation discussed before, and hence may indeed explain the different top couplings and consequently Higgs pole masses.

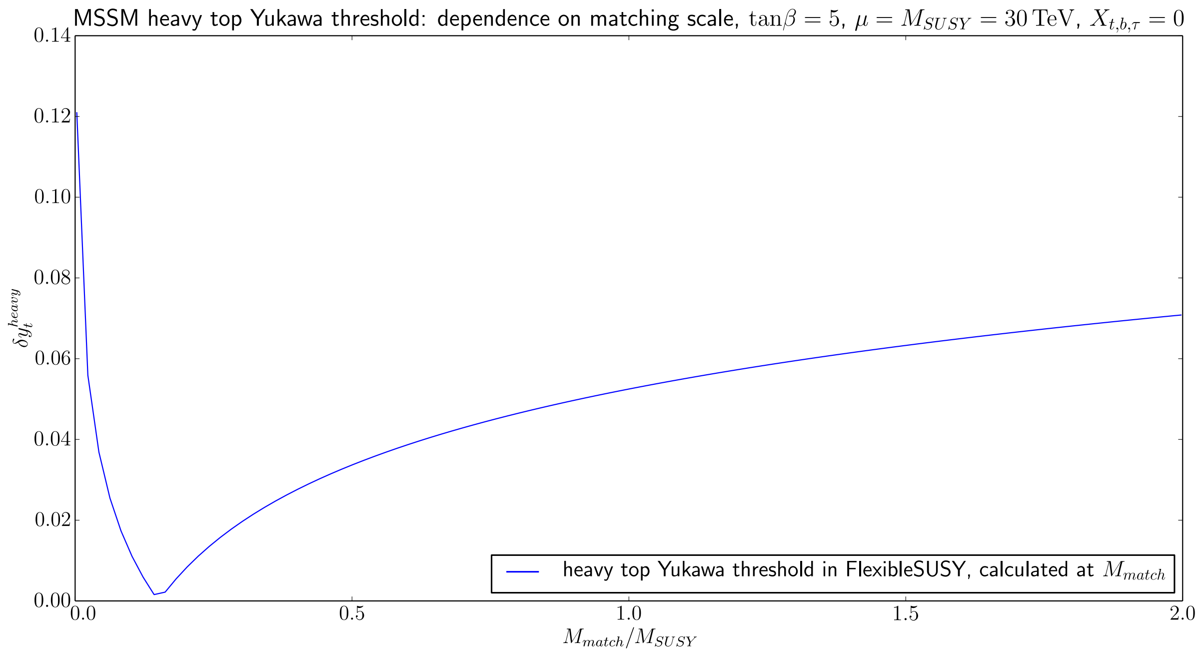


Figure 10: Top Yukawa coupling in the MSSM: $\tan\beta = 5$, $M_{SUSY} = 30$ TeV, soft squark and gaugino masses at the SUSY scale, no squark mixing. Shown is the one-loop threshold for the top Yukawa coupling calculated by FlexibleSUSY as a function of the scale the threshold is defined over the SUSY scale. Default is on the very left side.

Conclusively, precision in the low energy values for Yukawas, gauge coupling and the VEV in the SM as effective field theory are as essential for the overall accuracy of the Higgs pole mass as the quartic coupling itself. In the next section, examinations of the choice of the low scale will follow.

5.4 Dependence on low energy scale

In this section, the stability of the algorithm when modifying the low scale in the effective field theory is examined. Again, the MSSM with soft squark and gaugino masses as well as μ and the A^0 $\overline{\mathrm{DR}}$ mass at the SUSY scale is considered, with vanishing squark mixings $X_{t,b,\tau}$.

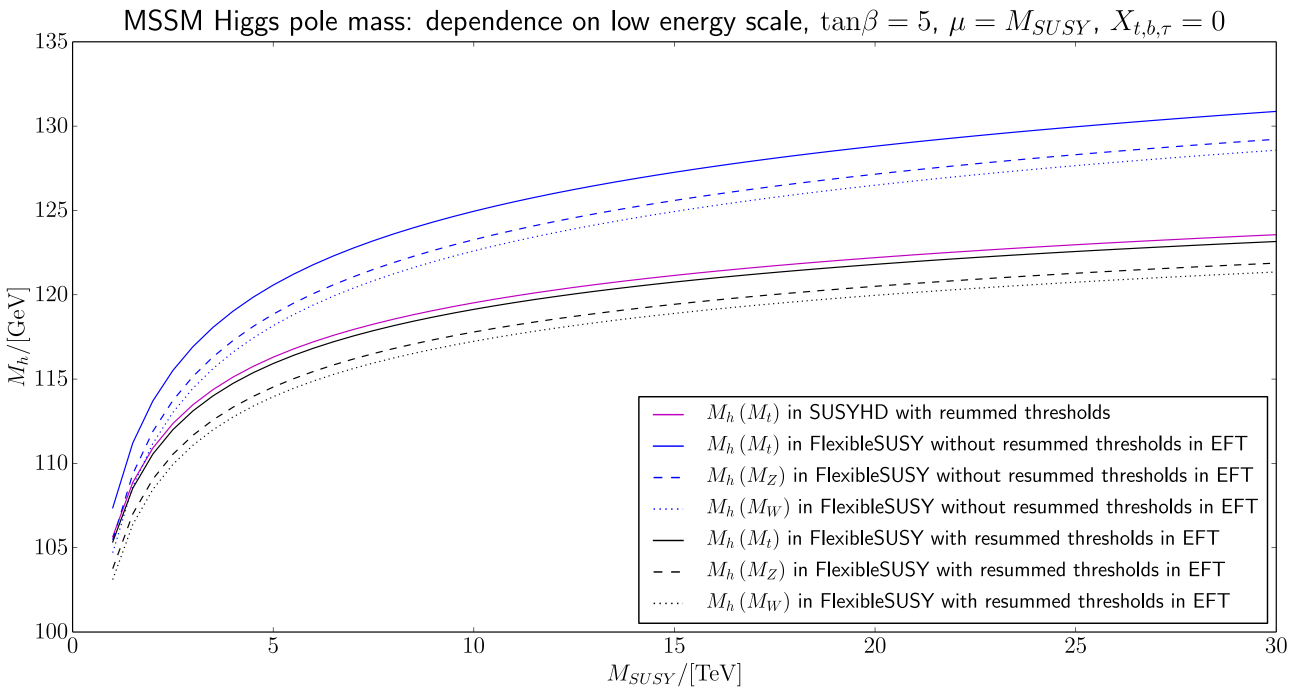


Figure 11: Higgs pole mass in effective field theory matched from MSSM: $\tan\beta = 5$, soft squark and gaugino masses at the SUSY scale, no squark mixing. The pole mass calculation was done at different scales in the EFT: M_W, M_Z and M_t. Purple line: SUSYHD pole mass; blue lines: FlexibleSUSY pole masses after full matching of all parameters; black lines: FlexibleSUSY pole mass after matching of λ and assuming fixed values for gauge and Yukawa couplings in the EFT

Varying the scale of the Higgs mass calculation in the effective SM cannot alter its value at one-loop order since the full one-loop self-energy is used in the calculation, differences occur at two-loop order and higher. These can be approximated by collecting all terms proportional to only g_3 and/or y_t at two-loop order, which are the largest contributions. The following beta functions, provided by SARAH [20], determine the running, using the convention $\beta_X = \frac{\partial X}{\partial \ln\mu}$.

$$\beta_\lambda = \frac{12y_t^2}{(4\pi)^2}\left(\lambda - y_t^2\right) + \frac{4y_t^4}{(4\pi)^4}\left(15y_t^2 - 16g_3^2\right) \tag{5.4.1}$$

$$\beta_v = -\frac{3\,v\,y_t^2}{(4\pi)^2} + \frac{v\,y_t^2}{(4\pi)^4}\left(\frac{27}{4}y_t^2 - 20g_3^2\right) \tag{5.4.2}$$

$$\beta_{y_t} = \frac{y_t}{(4\pi)^2}\left(\frac{9}{2}y_t^2 - 8g_3^2\right) + \frac{y_t}{(4\pi)^4}\left(36\,g_3^2\,y_t^2 - 108\,g_3^4 - 12\,y_t^4\right) \tag{5.4.3}$$

$$\beta_{g_3} = -\frac{7\,g_3^3}{(4\pi)^2} - \frac{2\,g_3^3}{(4\pi)^4}\left(y_t^2 + 13\,g_3^2\right) \tag{5.4.4}$$

The relevant part of the one-loop self-energy looks like this:

$$\delta\Sigma_h^{1L} = -\frac{3\,v^2}{(4\pi)^2}\,y_t^2\left(2\,y_t^2 - \lambda\right)B_0\left(m_h, m_t, m_t\right) - \frac{6\,y_t^2}{(4\pi)^2}A_0\left(m_t\right) \tag{5.4.5}$$

Where A_0 and B_0 are Passarino-Veltman integrals. The EWSB condition at one-loop order must be fulfilled by the Higgs potential parameter m^2 in order to avoid the necessity to include tadpole terms in the self-energy:

$$0 = -m^2 + \frac{1}{2}\lambda\, v^2 - \delta T_h^{1L} \tag{5.4.6}$$

$$\delta T_h^{1L} = \frac{6\, y_t^2}{(4\pi)^2}\, A_0\,(m_t) \tag{5.4.7}$$

The Passarino-Veltman functions are defined by:

$$A_0\,(M) := M^2 \left(\Delta + 1 - \ln\frac{M^2}{\mu^2} + \mathcal{O}\,(\epsilon)\right) \tag{5.4.8}$$

$$B_0\,(M, m, m) := \quad \left(\Delta + 2 - \ln\frac{m^2}{\mu^2} + \mathcal{O}\left(\frac{m^2}{M^2}, \epsilon\right)\right) \tag{5.4.9}$$

Combining everything to one expression:

$$\left(M_h^{pole}\right)^2 = v^2\lambda + \delta T_h^{1L} - \delta\Sigma_h^{1L} \tag{5.4.10}$$

And running this from one low scale μ to another one μ‘, one extracts the difference:

$$\left(M_h^{pole}\right)^2\Big|_{\mu‘} - \left(M_h^{pole}\right)^2\Big|_{\mu} = \frac{v^2}{(4\pi)^4}\left[\ln\frac{\mu‘}{\mu}\,\left(312\, y_t^6 - 448\, y_t^4\, g_3^2\right) + \ln^2\frac{\mu‘}{\mu}\,\left(432\, y_t^6 - 384\, y_t^4\, g_3^2\right)\right] \tag{5.4.11}$$

Figure 11 displays the uncertainty to be considered beholding Fig. 7, imposed by the ambiguity of choice of the scale of Higgs mass calculation. It is shown that especially the difference between pole masses calculated at the $M_{W,Z}$ and the M_t scale are of order of several GeV, although both scale choices seem appropriate compared to the SUSY scales considered in this scenario. However, this effect is of smaller magnitude than the influences discussed in the previous section.
Figure 12 is plotted for a fixed choice of the SUSY scale, but a variable low energy scale. It can be observed that a maximum exists for the low scale near the top pole mass, with steep slopes on both sides which causes the rather large pole mass differences in Fig. 11.

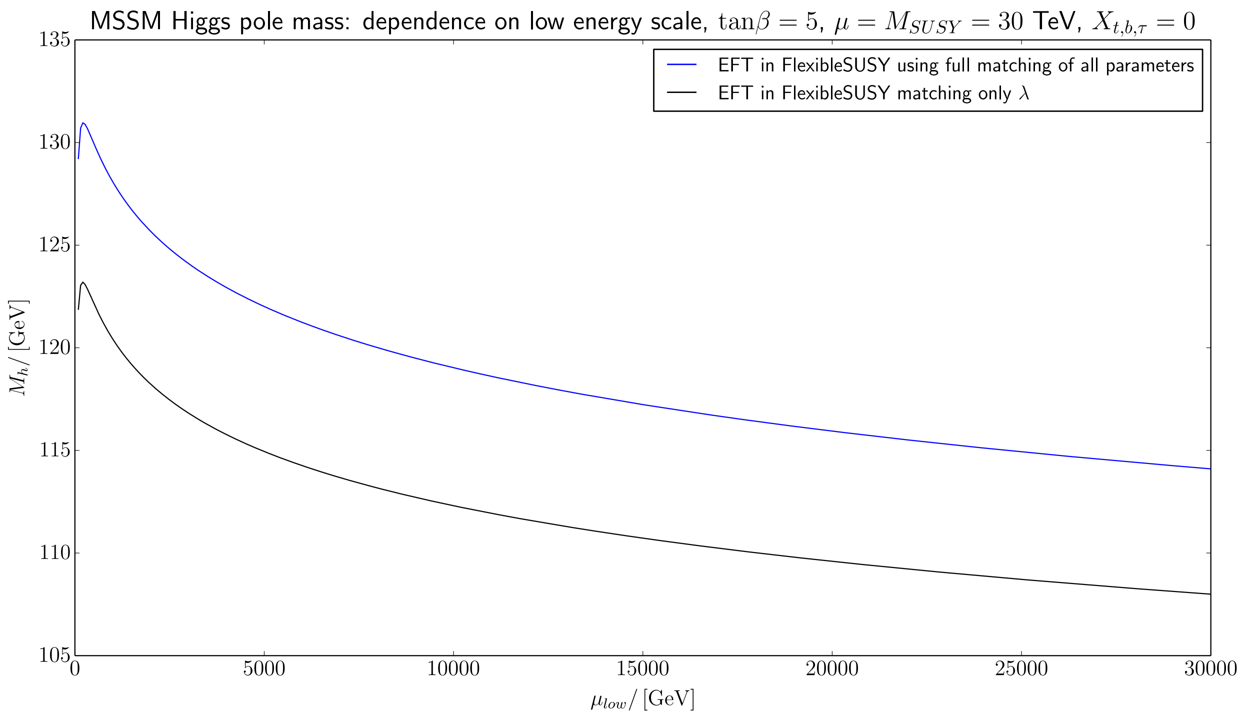


Figure 12: Higgs pole mass in effective field theory matched from MSSM: $\tan\beta = 5$, $M_{SUSY} =$ 30 TeV, soft squark and gaugino masses at the SUSY scale, no squark mixing. The pole mass calculation was done at different scales μ_{low} in the EFT. Blue lines: FlexibleSUSY pole masses after full matching of all parameters at the SUSY scale; black lines: FlexibleSUSY pole mass after matching of λ at the SUSY scale and assuming fixed values for gauge and Yukawa couplings in the EFT

As a consequence of this behavior, leading two-loop contributions in the effective field theory will be considered in future calculations to reduce the impact of the low scale choice. Conclusively for the entire chapter, it has been shown that the λ matching algorithm reproduces the SUSYHD results in this scenario, while the full SM-parameter matching suffers from heavy inaccuracies, and is, although working as desired, in practice unusable. However, the greatest asset of the new algorithm is also its greatest flaw - at least from a paradigmatic point of view - the ignorance to the low scale constraint in FlexibleSUSY. Although this is a necessary evil, the gained precision does only effect the lightest CP-even Higgs pole mass - the model parameters in the full theory nevertheless suffer from improperly resummed heavy thresholds, and there is still an error in the matched λ. An alternative ansatz will be implemented in the next chapter, which may resolve these issues altogether: the construction of a model tower in FlexibleSUSY.

6 Tower of models in FlexibleSUSY

6.1 Overview

As mentioned earlier, it is possible in FlexibleSUSY to stack different models together into a tower of effective field theories, and implement matching conditions for all parameters manually at the C++ level. Such a setup is especially useful for the scenario considered in the previous chapters, where a Standard Model class can be stacked below some extended model, providing beta functions and self-energies for the running and matching in the effective field theory, respectively. Therefore, heavy threshold corrections for gauge and Yukawa couplings, which have been observed to be the cause of inaccuracies, will automatically be calculated at the heavy scale by matching from the SM to the full theory, avoiding the inclusions of large logarithms in these fixed order calculations.
As part of this thesis, the setup of such an aforementioned tower was implemented on Mathematica level, featuring SM and an arbitrary high scale theory, including full SM parameter matching to the effective theory, and matching of gauge and Yukawa couplings to the full one.

6.2 Comparison to SUSYHD

For the sake of a final comparison to SUSYHD, systematical differences have been switched off and analyzed gradually, which is depicted in Fig. 14 and 13:

- The SUSYHD result in full precision, including QCD-contributions up to to three-loop level for y_t at the top scale and two-loop matching conditions for λ at the SUSY scale, have been plotted as black pluses. These results have been reproduced successfully by adapting boundary conditions and pole mass calculation from SUSYHD in the FlexibleSUSY-tower, and are depicted as solid red line.

- Substituting the two-loop λ matching condition taken from [27] with one-loop pole mass matching (yellow line), does only produce seizable effects for non-vanishing stop mixing. However, Higgs masses now differ not only by two-loop, but also one-loop matching terms in λ, suppressed by higher orders of electroweak over SUSY-scale. Since these effects are small for degenerate SUSY spectra in the TeV range, the Higgs masses are in accord with one-loop λ matching performed by SUSYHD (black 'x').

- Same is true if three-loop QCD thresholds to y_t are switched off in both frameworks (solid purple line and black three-armed crossed), although this causes a mass shift of roughly 0.5 GeV for degenerate sfermion masses.

- The next step is to replace the SUSY-like low scale conditions with constraints native to the FlexibleSUSY SM class (blue line), described in the appendix 8. The shift induced is rather small ($\approx$ 100 MeV), and mostly due to non-leading two-loop differences in the determination of y_t.

- By now switching the Higgs mass calculation to the FlexibleSUSY formula, the last remaining piece of SUSYHD code has been removed from the tower for the price of another 0.6 GeV shift (green line). Since SUSYHD v1.0.2 has been used, which expands the approximate Higgs one-loop self-energy (4.5.1) to the full expression, see e.g. equation (90) in [26], the deviation occur at two-loop order. FlexibleSUSY merely uses contributions $\sim g_3^2\, y_t^2,\, y_t^4$ from equation (20) in [28], while SUSYHD takes into account the approximated equations (34) and (35) from [26], not only including additional terms in the gaugeless limit, but also electroweak contributions. Moreover, SUSYHD considers the VEV as scale-independend input by the Fermi constant, but SUSYHD calculates the running VEV from the Z mass at one-loop order, which renders a direct comparison more difficult.

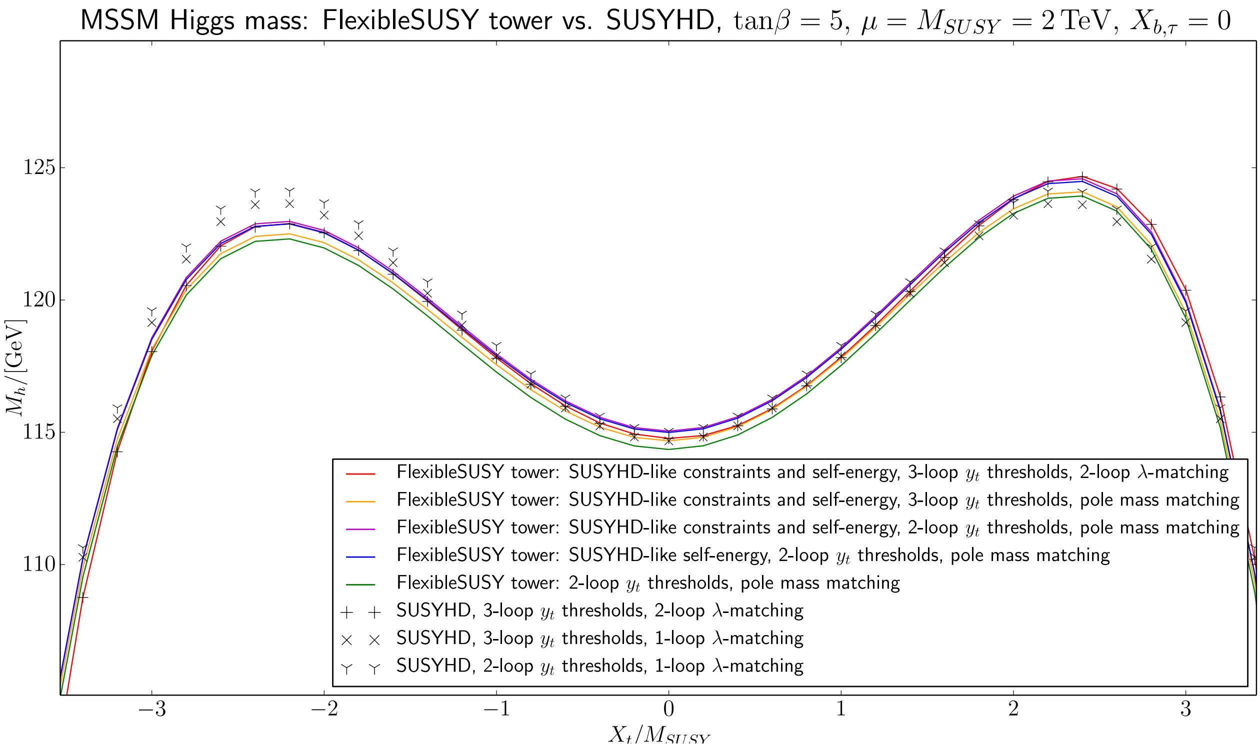


Figure 13: Lightest Higgs pole mass in a model tower of MSSM and SM, $\tan\beta = 5$, soft squark and gaugino masses at the SUSY scale, except for stop mixing.

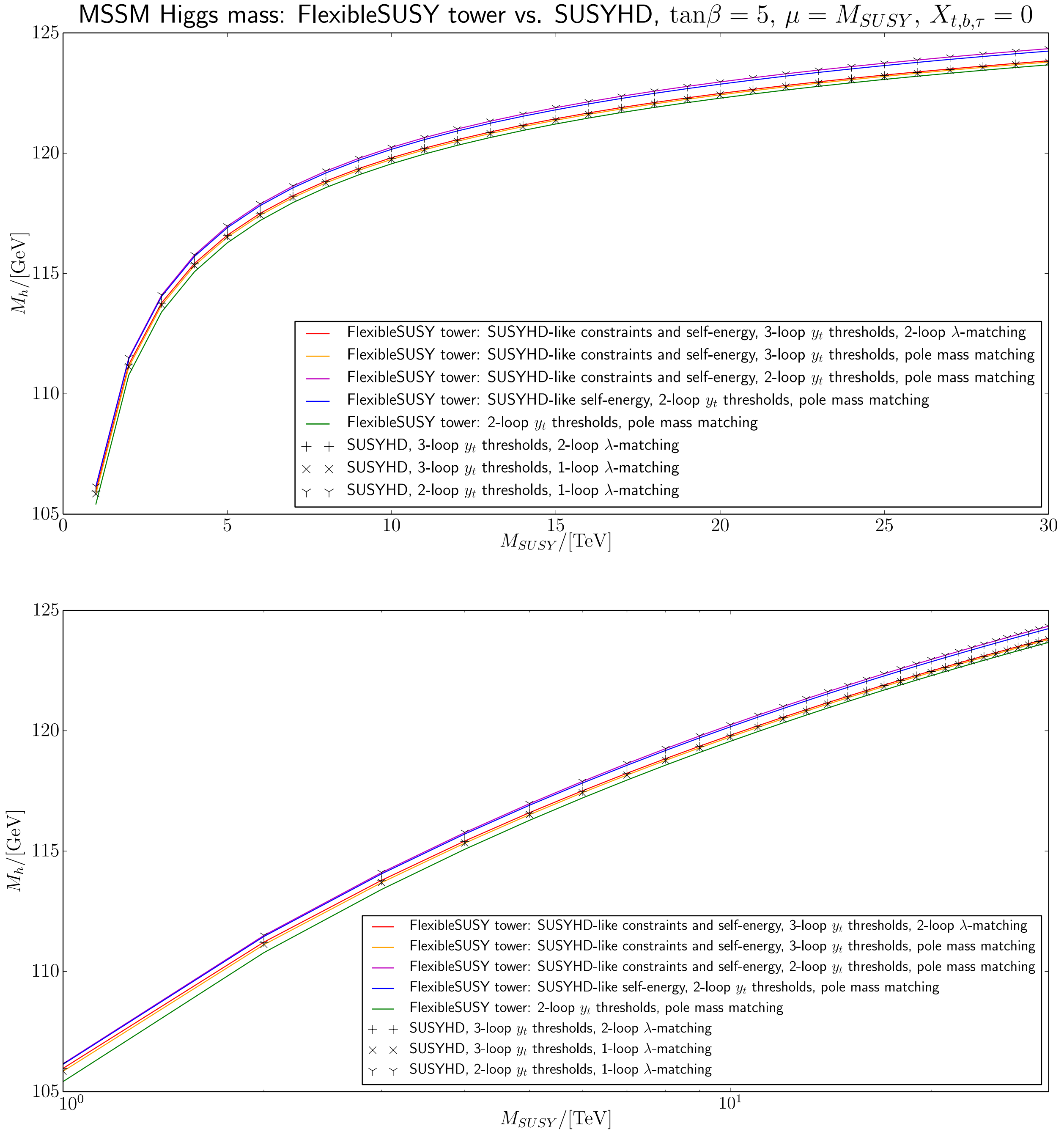


Figure 14: Lightest Higgs pole mass in a model tower of MSSM and SM, $\tan\beta = 5$, soft squark and gaugino masses at the SUSY scale, no squark mixing.

6.3 Benchmark within FlexibleSUSY framework

In this section, the tower will be discussed in the context of fixed order calculations in FlexibleSUSY and the algorithms developed in the previous chapters. This has been done in Fig. 15 and 16.

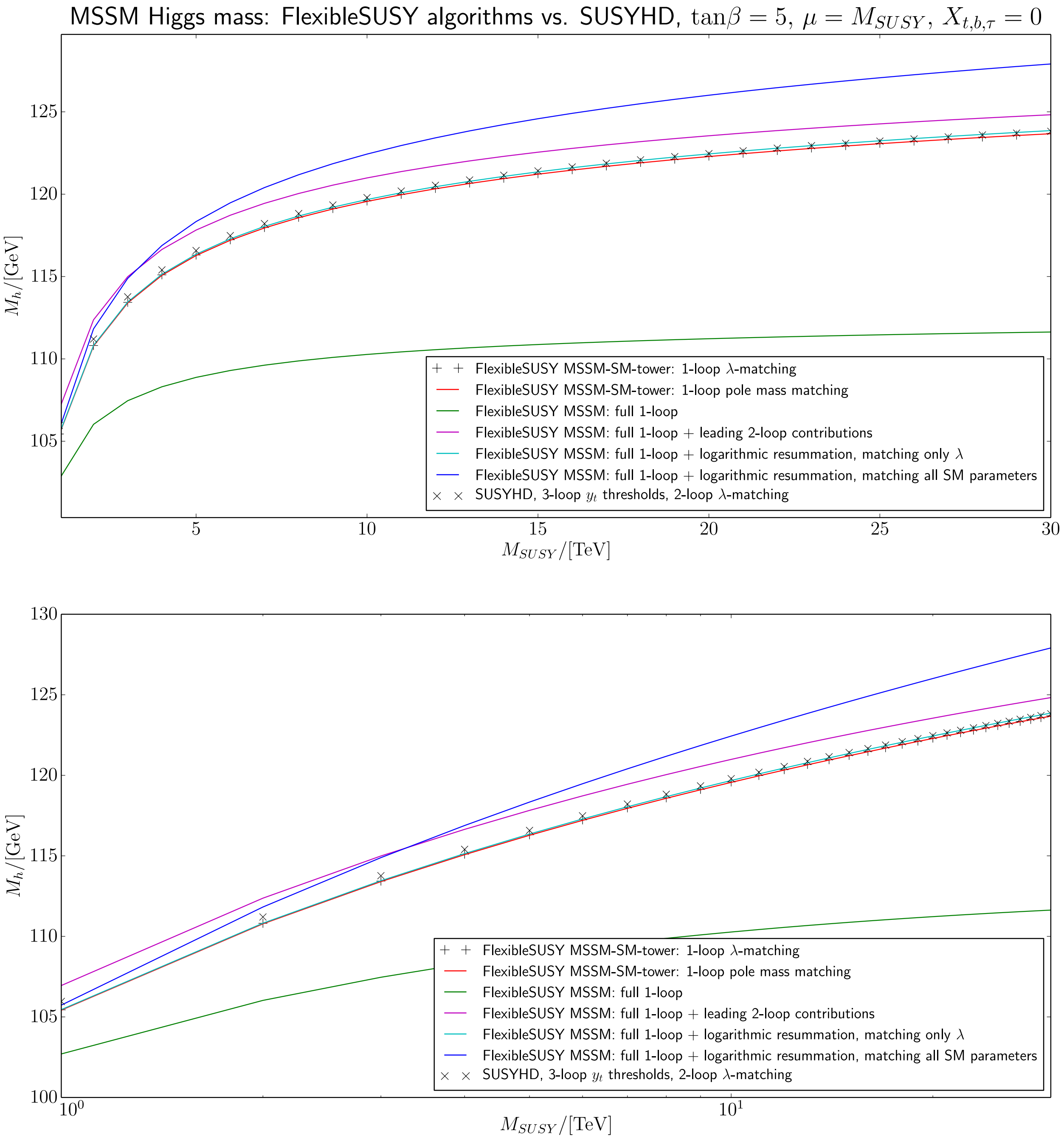


Figure 15: Lightest Higgs pole mass in the MSSM, $\tan\beta = 5$, soft squark and gaugino masses at the SUSY scale, no squark mixing.

- The MSSM-SM-tower (red line) and SUSYHD with full precision (black 'x's) have been plotted for reference.
- The MSSM-SM-tower has also been plotted using the explicit λ matching formula from [27] at one-loop level (black '+'). For small stop mixing, this is equivalent to λ matching via identification of pole masses, which is the default. As discussed before, this approach

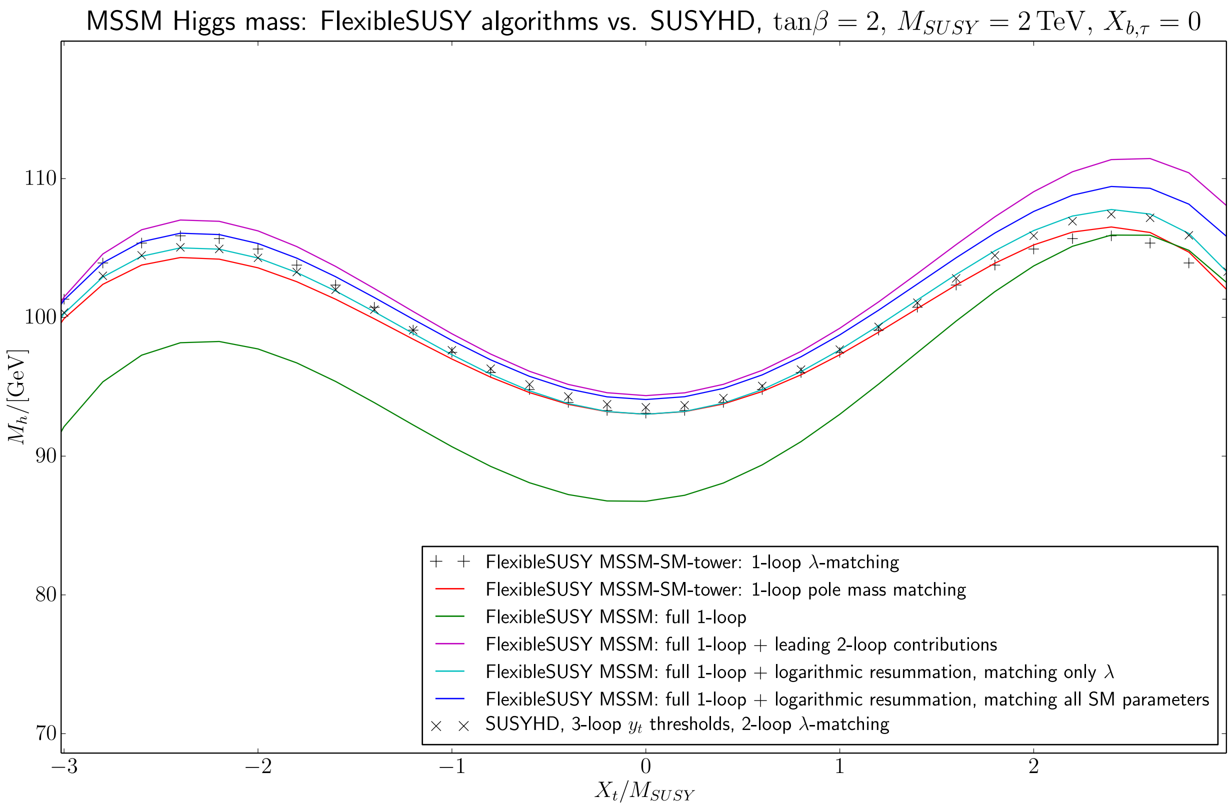


Figure 16: Lightest Higgs pole mass in a model tower of MSSM and SM, $\tan\beta = 5$, soft squark and gaugino masses at the SUSY scale, except for stop mixing.

is neither viable in arbitrary theories, nor avoiding double-counting with respect to the MSSM.

- The default MSSM class in FlexibleSUSY has been used to calculate the Higgs pole mass at one-loop order (green) and with two-loop contribution $\sim \alpha_s \alpha_{t,b}$, $\alpha_{t,b}\alpha_{t,b}$, α_τ^2 (purple), both suffering from large logarithmic contributions not resummed in the pole mass calculation as well as thresholds considered for gauge and Yukawa couplings.

- The algorithm extending the one-loop fixed-order calculation by resumming logarithmic contributions in all loop orders by matching all SM parameters via identification of pole masses is depicted (blue). However, due to matching gauge and Yukawa couplings at the low scale, this algorithm deviates by a logarithmic enhanced two-loop term from the tower.

- The algorithm matching λ only to the effective theory via identification of pole masses is also plotted (cyan). As discussed thoroughly, the effect of large logarithms in the thresholds is extenuated by the non-multiplicative running of λ. Hence, this algorithm is, in spite of bearing systematical errors, in good agreement with the tower. For large scales, the accord with SUSYHD gets even better, since both are using the same input data for gauge- and top coupling, which dominate the running.

Furthermore, using the tower and two-loop contributions for the Higgs mass in the effective theory, scale dependence of the calculation has been improved, featured in figure 17.

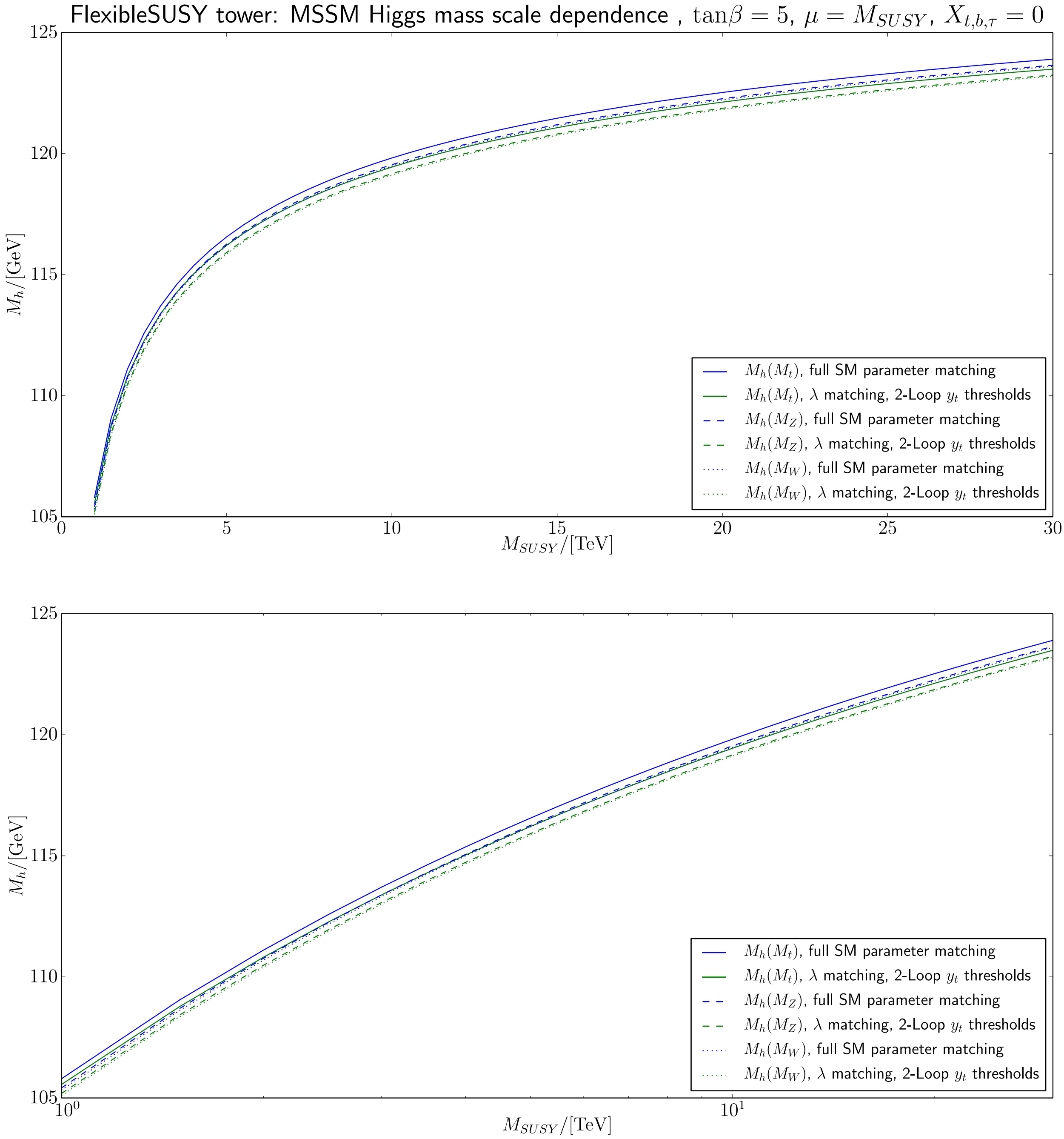


Figure 17: Lightest Higgs pole mass in a model tower of MSSM and SM, $\tan\beta = 5$, soft squark and gaugino masses at the SUSY scale, no squark mixing. Calculated at different scales: 173.34 GeV (solid lines), 91.1876 GeV (dashed) and 80.384 GeV (dotted). Blue lines mark full SM parameter matching in FlexibleSUSY, using default one-loop + two-loop QCD top coupling thresholds, green lines denote λ matching only, using two-loop y_t thresholds. All self-energies consist of a full one-loop part and leading two-loop terms.

6.4 Comparison to external codes

In this section, besides FlexibleSUSY 1.2.2 and SUSYHD 1.0.2, FeynHiggs 2.11.2 (see [29–33]), and SPheno 3.3.7 (see [34, 35]), generated by SARAH 4.5.8 [36] have been used to compute Higgs masses. Additionally, the HSSUSY model in FlexibleSUSY is depicted, which implements a Standard Model class with matching conditions for λ from [27]. Hence, the result is quite close to SUSYHD, small deviations arise with large SUSY scales from small numerical differences in the low scale constraints. The latter one was modified to enable the calculation for SUSY-scales smaller than top mass. In spite of discrepancies discussed in the previous sections, the tower produces results similar to these codes, even if not as similar as they are for SUSYHD and HSSUSY.

SPheno uses diagrammatic two-loop terms in the gaugeless limit provided by SARAH to do a fixed order calculation of the Higgs mass. The MSSM-SM-tower is used to determine tanβ at M_Z scale as input, since SPheno does not implement RGE running for VEVs. FeynHiggs calculates mainly in the on-shell scheme, but resums logarithmic contributions. The MSSM-SM-tower is used to input tanβ in the $\overline{\text{DR}}$ scheme at M_t, as well as the lightest, CP-odd Higgs mass. For small SUSY-scales, FeynHiggs, SPheno and the pure MSSM in FlexibleSUSY agree well on the Higgs mass, while the tower is off by roughly 2 GeV. Since this effect remains while the heavy approaches the light scale, it must be a two-loop effect due to the one-loop matching conditions between effective and full theory. For HSSUSY and SUSYHD, this deviation is even more fatal, since one-loop terms in the λ matching not considered due to suppression by powers of heavy masses cannot be neglected in this limit.

The FlexibleSUSY-MSSM deviates sooner with increasing SUSY masses from FeynHiggs than SPheno, and predicts masses right between latter ones and the other codes. FeynHiggs disagrees with any other code when considering large stop mixing, especially regarding the X_t value maximizing the Higgs mass. SPheno and the MSSM in FlexibleSUSY are plotted only in a region where the MSSM Higgs mass is not tachyonic at tree level.

SPheno and MSSM in FlexibleSUSY are deviating for large Higgs masses by logarithmic enhanced terms, this is due to a difference in the top Yukawa coupling, which is calculated from the $\overline{\text{DR}}$ mass. In both cases, this is acquired from the physical mass and self-energy expressions at one-loop order with two-loop QCD contributions from the SM sector, however, SPheno calculates the running mass iteratively:

$$m_t^{\overline{\text{DR}}}(M_Z) = M_t + \text{Re}\left[\widetilde{\Sigma}_t^{1L} + m_t^{\overline{\text{DR}}}\left(\Sigma_{t,QCD}^{1L} + \Sigma_{t,QCD}^{2L}\right)\right] \tag{6.4.1}$$

While FlexibleSUSY uses the pole mass directly:

$$m_t^{\overline{\text{DR}}}(M_Z) = M_t + \text{Re}\left[\widetilde{\Sigma}_t^{1L} + M_t\left(\Sigma_{t,QCD}^{1L} + \Sigma_{t,QCD}^{2L} + \left(\Sigma_{t,QCD}^{1L}\right)^2\right)\right] \tag{6.4.2}$$

Where $\widetilde{\Sigma}_t^{1L}$ denotes the one-loop self-energy without contributions $\sim \alpha_s$ from the Standard Model, which are included as $m_t^{\overline{\text{DR}}}\Sigma_{t,QCD}^{1L}$. Moreover, $m_t^{\overline{\text{DR}}}\Sigma_{t,QCD}^{2L}$ is a pure QCD contribution as well. Hence, double counting is avoided, but the algorithms differ by a term $m_t^{\overline{\text{DR}}}\Sigma_{t,QCD}^{1L}\,\widetilde{\Sigma}_t^{1L}$ at two-loop order.

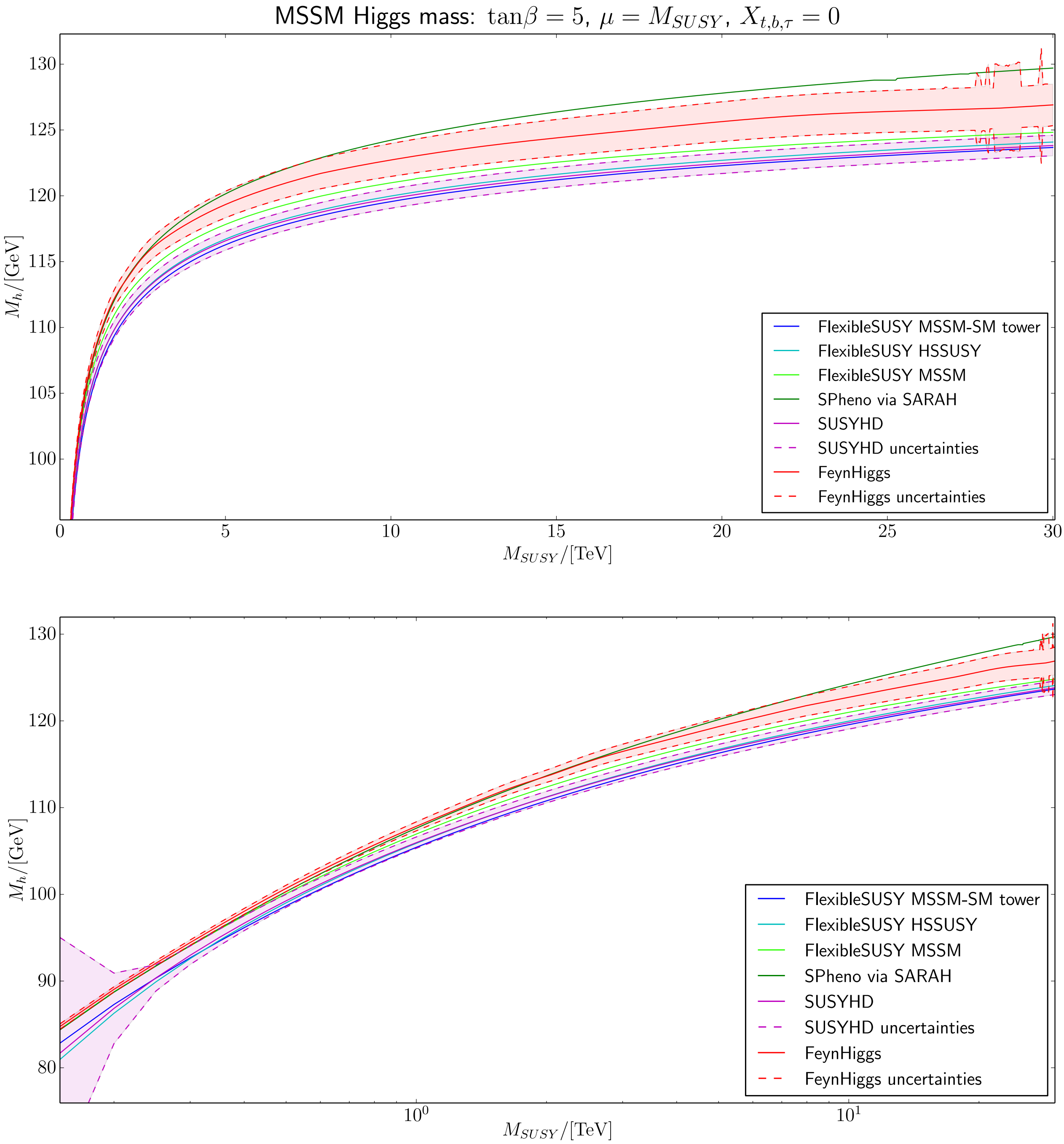
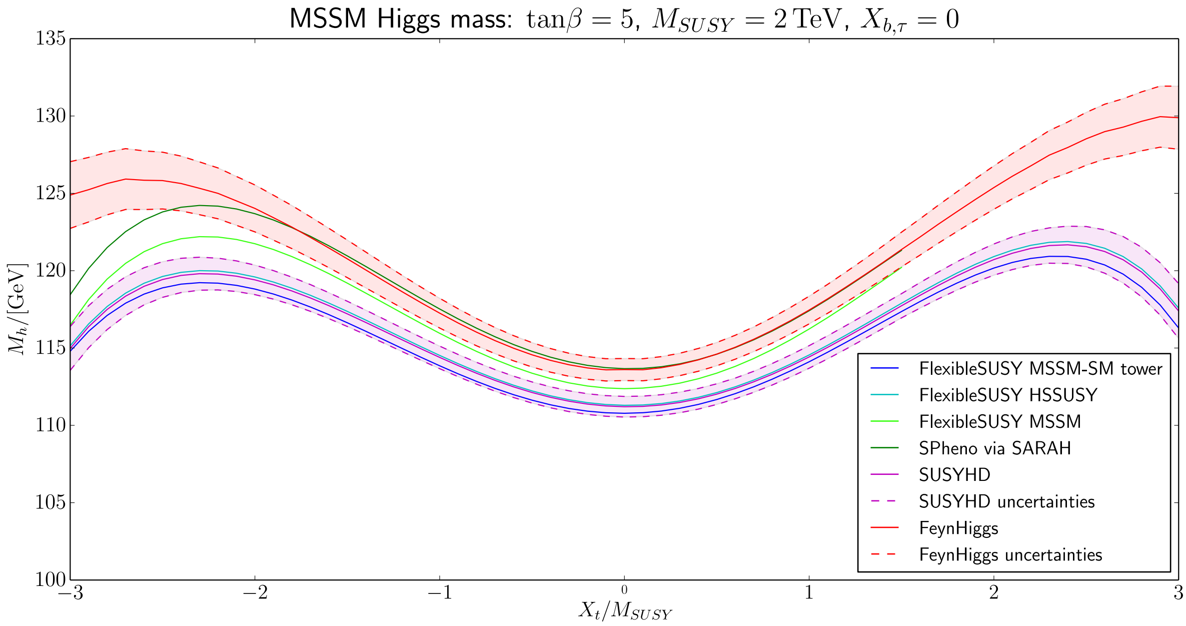


Figure 18: MSSM lightest CP-even Higgs mass: soft masses degenerate at SUSY scale, no stop mixing, $\tan\beta = 5$. Comparison between different codes.



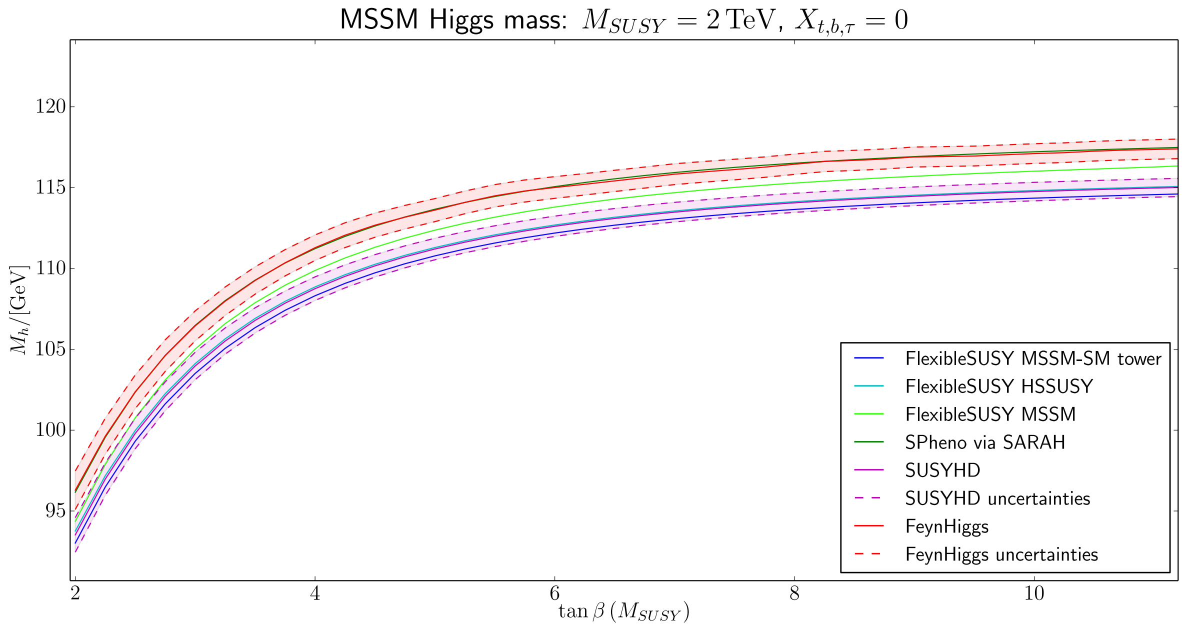


Figure 19: MSSM lightest CP-even Higgs mass: soft masses degenerate at 2 TeV. Comparison between different codes.

6.5 NMSSM test points

In this section, the tower will be tested once more using non-ideal scenarios in the NMSSM. These points are chosen according to the paper [5], where different public NMSSM spectrum generators have been compared, including FlexibleSUSY [18], SPheno via SARAH [34–36], SOFTSUSY [37–39], NMSSMCALC [40–43] and NMSSMTools [44–48].

	Q	$\tan\beta$	λ	κ	A_λ	A_κ	μ_{eff}	M_1	M_2	M_3	A_t	A_b	$m_{\tilde{t}_L}$	$m_{\tilde{t}_R}$
1	1.5	10	0.1	0.1	-0.01	-0.01	0.9	0.5	1	3	3	0	1.5	1.5
2	1.5	10	0.05	0.1	-0.2	-0.2	1.5	1	2	2.5	-2.9	0	2.5	0.5
3	1	3	0.67	0.1	0.65	-0.01	0.2	0.2	0.4	2	1	1	1	1
4	0.75	2	0.67	0.2	0.405	0	0.2	0.12	0.2	1.5	1	1	0.75	0.75
5	1.5	3	0.67	0.2	0.57	-0.025	0.2	0.135	0.2	1.4	0	0	1.5	1.5
6	1.5	3	1.6	1.61	0.375	-1.605	0.614	0.2	0.4	2	0	0	1.5	1.5

Table 2: Input parameter for NMSSM test points. Dimensionful couplings are given in TeV. For each point, $A_\tau = 0$ and, with exception of stop, the soft sfermion masses were considered $m_{\tilde{f}} = 1.5\,\text{TeV}$. Other parameters are defined at the scale Q, while $\tan\beta$ is defined at M_Z.

	point 1	point 2	point 3	point 4	point 5	point 6
FlexibleSUSY	123.55	122.84	91.11	127.62	120.86	126.46
NMSSMCALC	120.34	118.57	90.88	126.37	120.32	123.45
NMSSMTools	123.52	121.83	90.78	127.30	119.31	126.63
SOFTSUSY	123.84	123.08	90.99	127.52	120.81	126.67
SPheno	124.84	124.74	89.54	126.62	119.11	131.29
FS tower at M_{SUSY}	122.00	120.44	90.90	126.58	120.57	124.16
FS tower at M_Z	121.38	119.97	-	125.24	-	127.43

Table 3: Lightest CP-even Higgs masses for the test points in table 2. First five lines are taken from [5]. Remaining results are obtained by using an NMSSM-SM tower in FlexibleSUSY, where pole masses have been calculated in the NMSSM at M_{SUSY} and the effective SM at M_Z. Since the SM Higgs is not the lightest scalar in point 3 and 5, the SM is not the correct EFT, and the Higgs mass calculation there is omitted. All values are displayed in GeV.

Results for the tower are given in two different versions: calculated at the SUSY scale in the full theory, using leading two-loop contributions from the MSSM, and in the effective theory at M_Z. The former one deviates from the pure NMSSM version of FlexibleSUSY by the fact that SUSY thresholds are calculated at the SUSY scale. This renders the Higgs masses computed for these six points slightly smaller. However, except for point 4, the NMSSMCALC result is still smaller, suffering from one-loop differences in the top-thresholds.

Points 1 and 2 are MSSM-like, while the latter one features large stop mixing. As already experienced in the former section, the tower yields smaller masses than the fixed-order FlexibleSUSY code, but not smaller than the NMSSMCALC result. For point 2, the tower suffers a bit from the large stop splitting. Points 3 and 5 exhibit a light scalar singlet not regarded as SM Higgs. Hence, the SM is not a suitable low-energy limit of this theory which renders results from the tower questionable. Hence, the computation via matching to the Standard Model is omitted. Nevertheless, the MSSM calculation in the tower does only shift the scale of SUSY thresholds, and exhibits good agreement to the pure MSSM in FlexibleSUSY. In point 4, the RGE improved computation suffers from a quite light second Higgs. For point 6, SPheno excels by using NMSSM two-loop corrections not implemented in any other package, but the result from the tower is the closest to it.

7 Conclusions

Conclusively, it has shown that the algorithm expanding the full one-loop self-energy of arbitrary models by logarithmic enhanced contributions is working, but for gauge and Yukawa couplings fixed at the low scale, a loss of accuracy occurs due to a systematic resummation error. However, this code can still be useful when considering GUT scenarios.

A second algorithm matching only λ resolves these issues and does reproduce SUSYHD quite well when deployed in the MSSM. But furthermore, this code is also applicable to arbitrary extensions of the SM. However, systematical inaccuracies are still present in the matched scalar quartic coupling, as well as every parameter in the full theory. In fact, no observable except for the Higgs mass does actually benefit from this algorithm.

By constructing a tower of models in FlexibleSUSY, the aforementioned algorithm is rendered obsolete by taking over its assets while resolving the disadvantages. Large logarithmic contributions in the threshold corrections for all gauge and Yukawa couplings are by design resummed without systematical errors, and every SM quantity benefits from the EFT approach. Furthermore, due to the implementation of Higgs pole mass matching, the goals to extend the exact one-loop self-energy of arbitrary models by RGE enhanced terms without double-counting and to constrain this high-energy theory by low-scale observables properly, have been reconciled in this approach.
However, this does not mean that all large logarithms are avoided by design - there are still a number of parameters in the full theory adapted to fulfill the EWSB conditions at loop order. These calculations involve tadpole diagrams of both, heavy and light particles involved, and hence large logarithms. To avoid this, one would have to match these parameters as well at the SUSY scale, which is technically difficult for general theories and parameter choices. In spite of that, if one is interested in the mass spectrum, these large logarithms drop out in the matching procedure and are not present any more in the mass calculation of Standard model particles. For the mass computation of non-SM particles, such logarithms are still present in the EWSB-adapted parameters, but of course also in the tadpole free self-energy contributions from light loops, thus the accuracy is not spoiled by the neglect of matching of the EWSB-adapted parameters alone.

For the MSSM, SUSYHD has been reproduced with acceptable accuracy in the tower, and all sources of deviation have been broken down. The accordance is especially good for high SUSY masses. However, it became obvious that in this limit all codes yield a vast range of Higgs masses, caused by small numerical deviations of mainly the top Yukawa coupling determined at the electroweak scale, since this dominates the running of the scalar quartic coupling. To reconcile these computations, a systematic consideration of two-loop contributions would be required.
For smaller heavy masses and less ideal scenarios with e.g. an non-degenerate spectrum, the extension of the matching algorithm to two-loop order should be considered, which could aid to resolve discrepancies between fixed-order and RGE-codes in this limit.
However, last points remain a prospect to future work.

8 Appendix: SM low scale constraint in FlexibleSUSY

In this appendix, the matching of $\overline{\text{MS}}$ parameters of the Standard Model class from electroweak observables is documented. This resembles the algorithm from [18], except that no heavy thresholds are included, since these are calculated at the matching scale.
The following parameters are input from the SLHA file:
$G_F,\ \alpha_e^{5\,\text{fl},\,\overline{\text{MS}}}\left(M_Z^{pole}\right),\ \alpha_s^{5\,\text{fl},\,\overline{\text{MS}}}\left(M_Z^{pole}\right),\ M_Z^{pole},\ M_W^{pole},\ M_t^{pole},\ M_{\tau,\,\mu,\,e}^{pole},\ M_{\nu_{1\ldots3}}^{pole},$
$m_b^{\overline{\text{MS}}}\left(m_b^{\overline{\text{MS}}}\right),\ m_c^{\overline{\text{MS}}}\left(m_c^{\overline{\text{MS}}}\right),\ m_{u,\,d,\,s}^{\overline{\text{MS}}}\left(2\,\text{GeV}\right)$
The remaining $\overline{\text{MS}}$ parameters are calculated iteratively at the Z pole mass scale M_Z [5, 18, 23, 49]:

$$\alpha_e^{\overline{\text{MS}}}\left(M_Z\right) = \frac{\alpha_e^{5\,\text{fl},\,\overline{\text{MS}}}\left(M_Z\right)}{1+\frac{8}{9\pi}\alpha_e^{5\,\text{fl},\,\overline{\text{MS}}}\ln\frac{m_t^{\overline{\text{MS}}}}{M_Z}} \tag{8.0.1}$$

$$\alpha_s^{\overline{\text{MS}}}\left(M_Z\right) = \frac{\alpha_s^{5\,\text{fl},\,\overline{\text{MS}}}\left(M_Z\right)}{1+\frac{1}{3\pi}\alpha_s^{5\,\text{fl},\,\overline{\text{MS}}}\ln\frac{m_t^{\overline{\text{MS}}}}{M_Z}} \tag{8.0.2}$$

$$g_1^{\overline{\text{MS}}}\left(M_Z\right) = \sqrt{\frac{5}{3}}\frac{\sqrt{4\pi\alpha_e^{\overline{\text{MS}}}}}{\cos\theta_W^{\overline{\text{MS}}}} \tag{8.0.3}$$

$$g_2^{\overline{\text{MS}}}\left(M_Z\right) = \frac{\sqrt{4\pi\alpha_e^{\overline{\text{MS}}}}}{\sin\theta_W^{\overline{\text{MS}}}} \tag{8.0.4}$$

$$g_3^{\overline{\text{MS}}}\left(M_Z\right) = \sqrt{4\pi\alpha_s^{\overline{\text{MS}}}} \tag{8.0.5}$$

$$\left(m_Z^{\overline{\text{MS}}}\left(M_Z\right)\right)^2 = \left(M_Z\right)^2 + \text{Re}\,\Sigma_Z^{1L,\,\overline{\text{MS}}}\left(M_Z\right) \tag{8.0.6}$$

$$\left(m_W^{\overline{\text{MS}}}\left(M_Z\right)\right)^2 = \left(M_W\right)^2 + \text{Re}\,\Sigma_W^{1L,\,\overline{\text{MS}}}\left(M_W\right) \tag{8.0.7}$$

$$m_{t,\,heavy}^{\overline{\text{MS}}}\left(M_Z\right) = M_t + \text{Re}\left[\Sigma_{t,\,S}^{1L,\,\overline{\text{MS}}}\left(M_t\right) + \ M_t\left(\Sigma_{t,\,L}^{1L,\,\overline{\text{MS}}}\left(M_t\right) + \Sigma_{t,\,R}^{1L,\,\overline{\text{MS}}}\left(M_t\right)\right)\right] + \tag{8.0.8}$$

$$M_t\left[-\frac{\alpha_s^{\overline{\text{MS}}}}{6\pi}\left(2-3\ln\frac{m_t^{\overline{\text{MS}}}}{M_Z}\right)+\left(\frac{\alpha_s^{\overline{\text{MS}}}}{24\pi}\right)^2\left(315\ln\left(\frac{m_t^{\overline{\text{MS}}}}{M_Z}\right)\right.\right.$$
$$\left.\left.-54\ln^2\left(\frac{m_t^{\overline{\text{MS}}}}{M_Z}\right)-4618+96\zeta\left(3\right)-32\pi^2\left(1+\ln 4\right)\right)\right] \tag{8.0.9}$$

$$m_{b,\,heavy}^{\overline{\text{MS}}}\left(M_Z\right) = \frac{m_b^{\overline{\text{MS}}}\left(M_Z\right)}{1-\text{Re}\left[\Sigma_{b,\,S}^{1L,\,heavy}\left(m_b^{\overline{\text{MS}}}\right)/m_b^{\overline{\text{MS}}}+\Sigma_{b,\,L}^{1L,\,heavy}\left(m_b^{\overline{\text{MS}}}\right)+\Sigma_{b,\,R}^{1L,\,heavy}\left(m_b^{\overline{\text{MS}}}\right)\right]} \tag{8.0.10}$$

$$m_{l,\,heavy}^{\overline{\text{MS}}}\left(M_Z\right) = m_l^{\overline{\text{MS}}} + \text{Re}\left[\Sigma_{l,\,S}^{1L,\,heavy}\left(m_l^{\overline{\text{MS}}}\right)+m_l^{\overline{\text{MS}}}\left(\Sigma_{f,\,L}^{1L,\,heavy}\left(m_l^{\overline{\text{MS}}}\right)+\Sigma_{l,\,R}^{1L,\,heavy}\left(m_l^{\overline{\text{MS}}}\right)\right)\right] \tag{8.0.11}$$

$$v^{\overline{\text{MS}}}\left(M_Z\right) = \frac{2\,m_Z^{\overline{\text{MS}}}\left(M_Z\right)}{\sqrt{\frac{3}{5}\left(g_1^{\overline{\text{MS}}}\right)^2+\left(g_2^{\overline{\text{MS}}}\right)^2}} \tag{8.0.12}$$

$$\left|y_f^{\overline{\text{MS}}}\left(M_Z\right)\right| = \sqrt{2}\,\frac{m_{f,\,heavy}^{\overline{\text{MS}}}\left(M_Z\right)}{v^{\overline{\text{MS}}}\left(M_Z\right)} \tag{8.0.13}$$

Where the self-energies for fermions are divided into non-polarized, left- and right chiral part, indicated by S, L and R. For the top quark, QCD contributions are included non-diagrammatically at loop level. For leptons and the bottom quark, $\Sigma^{1L,heavy}$ denote contributions including top and heavy gauge boson loops, matching the running masses from the low-energy sector to the full SM. This is done at tree level for the quarks of the first two generations. Below the M_Z scale, $\overline{\mathrm{MS}}$ masses and gauge couplings are run with beta functions containing one-loop QED and 3-loop QCD contributions, gradually adapting to include each particle from its mass scale on. The electroweak mixing angle $\theta_W^{\overline{\mathrm{MS}}}$ is determined iteratively:

$$\left(\sin\theta_W^{\overline{\mathrm{MS}}}\cos\theta_W^{\overline{\mathrm{MS}}}\right)^2 = \frac{\pi\alpha_e^{\overline{\mathrm{MS}}}\left(M_Z\right)}{\sqrt{2}G_F M_Z^2\left(1-\delta_r\right)} \tag{8.0.14}$$

$$\delta_r = \hat{\rho}\frac{\Sigma_W^{1L}\left(0\right)}{M_W^2} - \mathrm{Re}\frac{\Sigma_Z^{1L}\left(M_Z\right)}{M_Z^2} + \delta_{VB} + \delta_r^{2L} \tag{8.0.15}$$

$$\hat{\rho} = \left[1 - \frac{\mathrm{Re}\Sigma_Z^{1L}\left(M_Z\right)}{\hat{\rho}\,M_Z^2} + \frac{\mathrm{Re}\Sigma_W^{1L}\left(M_W\right)}{M_W^2} + \Delta\hat{\rho}^{(2L)}\right]^{-1} \tag{8.0.16}$$

Where δ_{VB}, δ_r^{2L} and $\Delta\hat{\rho}^{(2L)}$ are calculated according to [50, 51].

Erklärung

Hiermit erkläre ich, dass ich diese Arbeit im Rahmen der Betreuung am Institut für Kern- und Teilchenphysik ohne unzulässige Hilfe Dritter verfasst und alle Quellen als solche gekennzeichnet habe. Diese Arbeit wurde bisher weder im Inland noch im Ausland in gleicher oder ähnlicher Form einer Prüfungsbehörde vorgelegt.

Tom Steudtner,
02. November 2015

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