

Two Micron-Size Dark Dimensions

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Two extra dimensions of micron scale might simultaneously address the gauge and cosmological hierarchy problems. In this paper various observational bounds in scenarios with one and two large extra dimensions are examined, to see if they are compatible with the micron scale. The authors show that consistency with astrophysical observations requires that two extra dimensions of micron scale must not admit isometries, whereby conservation of the extra dimensional momentum is violated, allowing the massive Kaluza–Klein modes of the graviton to decay to other lighter graviton modes. However, to remain consistent with cosmological observations two extra dimensions of micron scale require a delicately fine tuning of the temperature at which the universe enters the radiation dominated epoch. Diving into this fine-tuned scenario the authors also show that primordial black holes with masses in the range $10^8 \lesssim M_{\text{BH}}/g \lesssim 10^{21}$ could make all cosmological dark matter.

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1. General Idea

Extra dimensions are in general thought to be compact, since a four-dimensional (4D) description fits the world we perceive and measure (so far) very well. It is this compactness that sets a new length scale. A particularly attractive framework has as its main feature large extra dimensions along which only gravity propagates, because Standard Model (SM) fields are localized on a brane extended in the non-compact dimensions.^[1] Most notably, this construct can be embedded into string theory.^[2] Within this framework the usual 4D graviton is complemented by a tower of Kaluza–Klein (KK) modes, corresponding to the new available phase space in the bulk. In the

canonical example, spacetime is a direct product of a non-compact 4D spacetime manifold and a flat spatial n -torus of common linear size $2\pi R_{\text{KK}}$ and volume $V_n = (2\pi R_{\text{KK}})^n$. The $(4+n)$ dimensional Planck scale (or more generally the species scale Λ_{sp} where gravity becomes strong^[3–7]) is related to the 4D reduced Planck mass M_p according to

$$\Lambda_{\text{sp}} = M_p^{2/(2+n)} V_n^{1/(2+n)} \quad (1)$$

Besides, by combining this idea with conjectures of the Swampland program^[8–11] a deep interplay between physics in the UV and the IR becomes exposed. For example, the distance conjecture states that infinite distance limits $\Delta\phi \rightarrow \infty$ in the moduli space of massless scalar fields are accompanied by an infinite tower of exponentially light states $m_{\text{KK}} \sim e^{-\alpha\Delta\phi}$, where distance and masses are measured in Planck units.^[12] Connected to the distance conjecture is the anti-de Sitter (AdS) distance conjecture, which correlates the dark energy density to the mass scale m_{KK} characterizing the infinite tower of states,

$$m_{\text{KK}} \sim |\Lambda|^\alpha \quad (2)$$

as the negative AdS vacuum energy $\Lambda \rightarrow 0$, where α is an $\mathcal{O}(1)$ positive constant.^[13] Besides, under the hypothesis that this scaling behavior (2) holds in de Sitter (or quasi de Sitter) space, an unbounded number of massless modes should also emerge in the limit $\Lambda \rightarrow 0$.

Note that the KK (or string) tower contains massive spin-2 bosons, and thus for positive $\Lambda \rightarrow 0$ the Higuchi bound^[14] places an absolute upper limit to the exponent: $\alpha = 1/2$. In addition, explicit string calculations of the vacuum energy set a lower bound: $\alpha = 1/4$ ^[15] (see e.g., also refs. [16–25]). Now, the combination of the theoretically accessible parameter range $1/4 \leq \alpha \leq 1/2$ with

the experimental constraints on deviations from Newton's gravitational inverse-square law,^[26] namely

$$R_{\text{KK}} \lesssim 30 \mu\text{m} \quad (3)$$

selects $\alpha = 1/4$ as the preferred value and gives in this way a procedure to connect R_{KK} to the dark energy scale $\Lambda/M_p^4 \sim 10^{-120}$ (i.e., $\Lambda^{1/4} \sim 2.39 \text{ meV}^{[27]}$) as follows^[15]

$$R_{\text{KK}} \sim \lambda \Lambda^{-1/4} \simeq \mathcal{O}(\mu\text{m}) \quad (4)$$

where the proportionality factor is estimated to be within the range $10^{-1} < \lambda < 10^{-4}$.

Actually, the relation (4) is independent of the number n of dark dimensions of micron size. Taking into account the SNe and neutron star constraints (see the discussion in the next section), the authors of ref. [15] have argued in favor of a single dark dimension in the micron range. In this case, one is led to a species scale of order^[15]

$$\Lambda_{\text{sp}} \sim M_p^{2/3} R_{\text{KK}}^{-1/3} \sim M_p^{2/3} \Lambda^{1/12} \sim 10^9 \text{ GeV} \quad (5)$$

In this paper we explore the possibility of a compact space with two dark dimensions of micron scale. In this case, the species scale

$$\Lambda_{\text{sp}} \sim (M_p/R_{\text{KK}})^{1/2} \sim M_p^{1/2} \Lambda^{1/8} \quad (6)$$

would be in the 10 TeV regime, within reach of the Future Circular Collider in the modality of hadron–hadron interactions (FCC-hh).^[28] Moreover, this scenario could simultaneously account for the cosmological and gauge hierarchy problems. The goal of this article is to establish the required conditions for (6) to be consistent with experiment.

The layout of the paper is as follows. In Section 2 we confront (6) to null results from collider searches, in Section 3 we confront it to astrophysical data, and in Section 4 we confront it to cosmological observations. In Section 5 we reexamine whether neutrino masses and mixings could exclusively occur due to physics in the bulk. In Section 6 we show that an all-dark-matter interpretation in terms of 6D primordial black holes (PBH) should be feasible for masses in the range $10^8 \lesssim M_{\text{BH}}/g \lesssim 10^{21}$. In Section 7 we take a look at the possibility of probing Λ_{sp} with ultra-high-energy cosmic rays. Conclusions are given in Section 8.

2. Collider Constraints

ATLAS and CMS are the two primary, general-purpose experiments, at the Large Hadron Collider (LHC). Since the early days of the LHC the ATLAS and CMS collaborations developed analysis strategies to search for signals of large extra dimensions.

A particular signal of these searches is the production of a single very energetic “mono-object” that does not balance the transverse momentum carried by anything else emerging from the collision (as would be required by momentum and energy conservation). Examples of such objects are very energetic photons or particle jets. Collisions producing these mono-objects only appear to be imbalanced because the emerging photon or jet is balanced by a graviton that escapes detection. As a consequence, SM

processes such as the production of a jet plus a Z boson which decays into neutrinos can mimic a graviton production signal. The absence of any excess in the mono-photon or mono-jet event channels at ATLAS and CMS has led to stringent limits on the species scale, see e.g., refs. [29–36]. Null results from searches of mono-objects were also reported by the Tevatron D0^[37] and CDF^[38] collaborations.

One can also search for virtual graviton effects, which manifest themselves as a new contribution to the continuum in the invariant mass spectrum of two energetic photons or fermions (dileptons or dijets). As of today no signals have been observed at ATLAS and CMS, excluding such contributions for Λ_{sp} up to several TeV.^[39–42]

Today, the most restrictive bounds on the species scale come from searches in events with an energetic jet and missing transverse momentum. For $n = 2$, the 95% CL lower bound reported by the ATLAS Collaboration is^[35]

$$\Lambda_{\text{sp,min}} = 4.5 \text{ TeV} \quad (7)$$

whereas the CMS Collaboration reported^[36]

$$\Lambda_{\text{sp,min}} = 4.5 \text{ TeV} \quad (8)$$

We note that we have reduced the limits as given in in Table 10 of ref. [35] and in Table 3 of ref. [36] by a factor of $(2\pi)^{n/(n+2)}$. This is because both the ATLAS^[34] and CMS^[33] collaborations define the volume of the compact space as introduced in ref. [43], i.e., volume equal to R_{KK}^n rather than $(2\pi R_{\text{KK}})^n$.

If the large extra dimension scenario is embedded in a string theory at the TeV scale,^[2] we expect the string scale $M_s \lesssim \Lambda_{\text{sp}}$, and thus production of string resonances at the LHC.^[44–49] Only one assumption is necessary in order to set up a solid framework: the string coupling must be small in order to rely on perturbation theory in the computations of scattering amplitudes. In this case, black hole production and other strong gravity effects occur at energies above the string scale; therefore at least a few lowest Regge recurrences are available for examination, free from interference with some complex quantum gravitational phenomena. Certain amplitudes to leading order in string coupling (but including all string α' corrections) are universal. These amplitudes, which include $2 \rightarrow 2$ scattering processes involving 4 gluons or 2 gluons and 2 quarks, are independent of the details of the compactification, such as the configuration of branes, the geometry of the extra dimensions, and whether SUSY is broken or not. This model independence makes it possible to compute the string corrections to dijet signals at the LHC.^[47]

In addition, the mixing between hypercharge and the gauge baryon number symmetry implies that tree level gluon-gluon scattering processes such as $gg \rightarrow g\gamma$ and $gg \rightarrow \gamma\gamma$ are allowed even though they can only appear at loop order in the SM.^[44] Unlike the dijet case, these amplitudes are mildly model dependent, i.e., they depend on a mixing parameter which is fixed by the D-brane model but are otherwise independent of the compactification scheme.

The ATLAS and CMS collaborations have searched for signals of Regge recurrences analyzing the dijet and $\gamma + \text{jet}$ invariant mass distributions.^[50–59] Null results of these searches lead to a lower bound $M_s > 8 \text{ TeV}$ at the 95% CL. Putting all this together,

we conclude that collider constraints are consistent with two dark dimensions of micron size and $\Lambda_{sp} \sim 10$ TeV.

3. Astrophysical Constraints

Supernova (SN) explosions have long been considered to be powerful probes of large extra dimensions.^[60] This is because SN cores could emit sizable fluxes of KK gravitons, with masses up to about 100 MeV. The KK emission process could then compete with neutrino cooling, shortening the observable signal. This implies that the size of the extra dimensions could be constrained by requiring that SN 1987A did not emit more KK gravitons than is compatible with the observed neutrino signal duration.^[61–64] This reasoning leads to a bound

$$R_{KK} < \begin{cases} 490 \text{ m} & \text{for } n = 1 \\ 0.96 \text{ } \mu\text{m} & \text{for } n = 2 \end{cases} \quad (9)$$

corresponding to $\Lambda_{sp} > 740$ TeV and $\Lambda_{sp} > 8.9$ TeV, respectively.^[65]

A more restrictive constraint emerges by considering the radiative decay of KK gravitons, which could produce potentially observable gamma rays.^[66] The partial decay width of this process is estimated to be

$$\Gamma_{\gamma\gamma}^{\vec{l}} \sim \tilde{\lambda}^2 \frac{m_{KK}^3 \vec{l}^3}{80\pi M_p^2} \quad (10)$$

where we have considered a tower of equally spaced gravitons, indexed by an integer \vec{l} and with mass $m_l = \vec{l} m_{KK}$, and where the parameter $\tilde{\lambda}$ measures the value of the dark graviton wave function at the SM brane and is expected to be $\mathcal{O}(1)$.^[67,68] Following^[65] we assume $\tilde{\lambda} = 1$. The non-observation of these gamma rays in data collected by the Energetic Gamma Ray Experiment Telescope (EGRET) leads to the following limits on the compactification scale:

- The diffuse γ -ray flux measured by the EGRET instrument constrains the number of KK-gravitons that may have been emitted by all cosmic SNe. This constraints can be translated into the bound

$$R_{KK} < \begin{cases} 4.9 \times 10^2 \text{ m} & \text{for } n = 1 \\ 1 \text{ } \mu\text{m} & \text{for } n = 2 \end{cases} \quad (11)$$

corresponding to limits of $\Lambda_{sp} > 3.4 \times 10^3$ TeV and $\Lambda_{sp} > 28$ TeV, respectively.^[65]

- Cas A is a young SN remnant. Based on its age, a cloud of emitted KK-gravitons should still appear as a point source to EGRET. However, EGRET does not observe a photon flux at the expected point in space. The difference between the theoretically expected flux and observation restricts the emissivity of KK gravitons. This restriction can be translated into the bound

$$R_{KK} < \begin{cases} 5.3 \times 10^3 \text{ m} & \text{for } n = 1 \\ 1.3 \text{ } \mu\text{m} & \text{for } n = 2 \end{cases} \quad (12)$$

corresponding to limits of $\Lambda_{sp} > 330$ TeV and $\Lambda_{sp} > 7.7$ TeV, respectively.^[65]

- In a core collapse SN most KK gravitons are produced close to the kinematic threshold. For SN core temperature ~ 30 MeV, the threshold condition implies that the typical mass of KK gravitons is $\mathcal{O}(100$ MeV). Thus, most gravitons leave the SN core with rather non-relativistic velocities such that a large fraction of them ends up being gravitationally retained in a cloud around the neutron star remnant. Trapped in this cloud the KK gravitons would subsequently decay to SM particles on a time scale comparable to the age of the universe. We would then expect that neutron stars should shine brightly in 100 MeV gamma-rays. However, this is not the case relative to EGRET data, yielding

$$R_{KK} < \begin{cases} 16 \text{ m} & \text{for } n = 1 \\ 67 \text{ nm} & \text{for } n = 2 \end{cases} \quad (13)$$

corresponding to limits of $\Lambda_{sp} > 2.2 \times 10^3$ TeV and $\Lambda_{sp} > 34$ TeV, respectively.^[65]

The decay of KK gravitons also leads to an excess heating of neutron stars.^[69] This heating effect should have been seen by the Hubble Space Telescope which is able to observe the thermal emission from the surface of neutron stars. The lack of such an excess heat leads to the most stringent bounds on the size of the compact space,

$$R_{KK} < \begin{cases} 8.3 \text{ m} & \text{for } n = 1 \\ 59 \text{ nm} & \text{for } n = 2 \end{cases} \quad (14)$$

corresponding to $\Lambda_{sp} > 2.8 \times 10^3$ TeV and $\Lambda_{sp} > 36$ TeV, respectively.^[65]

Notwithstanding, the constraints coming from the decays of KK gravitons into photons are model dependent and can be completely evaded. Note that the constraints from EGRET data and neutron-star heating rely on the assumption that KK modes can only decay into SM fields localized on the brane, but cannot decay into other KK modes with smaller bulk momenta. If large extra dimensions do not admit isometries this assumption is in general not valid and the bounds in (11), (12), (13), and (14) can be evaded.^[70] Indeed, if the KK momentum of the graviton tower is not conserved a given KK mode of the tower could decay into final states that include other, lighter KK excitations.

Following,^[71] we set $n = 1$ and assume that a parent particle with mass m_l can decay to two daughter particles with masses $m_{l'}$ and $m_{l''}$ such that $m_l = m_{l'} + m_{l''} + \epsilon$, with $\epsilon \leq m_{KK} \delta$ and $\delta \sim \mathcal{O}(1)$. As might be expected, the KK mode l decays with gravitational strength to lighter elements of the tower and so the partial decay width is shown to be

$$\Gamma_{l'l''}^l \sim m_l^3 / M_p^2 \quad (15)$$

In principle, there are m_l/m_{KK} possibilities for the $l'l''$ pair of KK modes that l can decay to, but because of a small violation of KK quantum number, the choice effectively becomes $m_l \delta / m_{KK}$.

Taking into account the phase space factor which is roughly the velocity of decay products at threshold,

$$v \sim \sqrt{m_{\text{KK}} \delta / m_1} \quad (16)$$

the total decay width of graviton l is found to be,

$$\begin{aligned} \Gamma_{\text{tot}}^l &\sim \sum_{l' < l} \sum_{0 < l'' < l-l'} \Gamma_{l''}^l \\ &\sim \beta^2 \frac{m_1^3}{M_p^2} \times \frac{m_1}{m_{\text{KK}}} \delta \times \sqrt{\frac{m_{\text{KK}} \delta}{m_1}} \\ &\sim \beta^2 \delta^{3/2} \frac{m_1^{7/2}}{M_p^2 m_{\text{KK}}^{1/2}} \end{aligned} \quad (17)$$

where β is parameter that controls the strength of the intra-tower decay amplitudes which correlates with the amplitudes on inhomogeneities in the dark dimension.^[71] Assuming that for times larger than $1/\Gamma_{\text{tot}}^l$ dark matter which is heavier than the corresponding m_1 has already decayed, it is straightforward to see that

$$m_1(t) \sim \left(\frac{M_p^4 m_{\text{KK}}}{\beta^4 \delta^3} \right)^{1/7} t^{-2/7} \quad (18)$$

where t indicates the time elapsed since the big bang.

For $n = 2$, conservation of energy forces the the momenta of the decay products to be almost parallel and so the number of decay channels available is still effectively 1D, up to a width that will be set by δ . The evolution of the KK mass in the graviton tower is also given by (18).

We now turn to discuss how dark-to-dark decays provide a path to evade the lower bounds on the species scale given in (11), (13), and (14). Following Hannestad and Raffelt (HR), we parametrize the bound on the species scale in the absence of intra-tower decays by

$$\Lambda_{\text{sp,min}}^{\text{HR}} = \left(\frac{f_{\text{KK}}^{\text{HR}}}{0.5} \right)^{-1/(n+2)} \Lambda_{\text{sp},n}^{\text{SN 1987A}} \quad (19)$$

where $f_{\text{KK}}^{\text{HR}}$ is the fraction of the total energy emitted by the SN in the form of KK gravitons and $\Lambda_{\text{sp},n}^{\text{SN 1987A}}$ is the bound on the species scale derived from observations of SN 1987A.^[69] In the absence of dark-to-dark decays all of the $f_{\text{KK}}^{\text{HR}}$ fraction would decay dominantly into photons, and so the null results on searches for these photons further reduce the fraction of allowed emitted KK modes, i.e. $f_{\text{KK}}^{\text{HR}} < 0.5$. For example, for $n = 2$, the bound in (19) can be recast as

$$\Lambda_{\text{sp,min}}^{\text{HR}} = \left(\frac{f_{\text{KK}}^{\text{HR}}}{0.5} \right)^{-1/4} 8.9 \text{ TeV} \quad (20)$$

Due to the intra-tower decays the fraction of KK gravitons that could decay into photons gets a multiplicative correction to accommodate the evolution of dark-to-dark decays,

$$f_{\text{KK}}^{\text{new}} = f_{\text{KK}}^{\text{HR}} \times \overline{f_{\text{KK}}} \quad (21)$$

where $\overline{f_{\text{KK}}}$ is determined by the ratio of the initial decay rate Γ_0 of the KK gravitons to photons to the final decay rate $\Gamma(t)$ of the KK gravitons to photons,

$$\overline{f_{\text{KK}}} = \int_0^{\tau_{\text{NS}}} \frac{\Gamma_0}{\Gamma(t)} \frac{dt}{\tau_{\text{NS}}} \quad (22)$$

and where $\tau_{\text{NS}} \sim 17$ Myr is the lifetime of the neutron star,^[65]

$$\Gamma_0 \sim m_0^3 / M_p^2 \quad (23)$$

$m_0 \sim 100$ MeV is the mass of the KK modes at production, and

$$\Gamma(t) = m_1^3(t) / M_p^2 \quad (24)$$

Substituting (23) and (24) into (22) while using (18) we obtain

$$\overline{f_{\text{KK}}} \sim \frac{13}{7} \left[\frac{m_0}{(M_p^4 m_{\text{KK}})^{1/7}} \right]^3 \tau_{\text{NS}}^{6/7} \sim 881 \quad (25)$$

where we have taken $\delta = \beta = 1$. The corrected lower limit on the species scale is then

$$\Lambda_{\text{sp,min}}^{\text{new}} = \left(\frac{f_{\text{KK}}^{\text{HR}}}{0.5} \right)^{-\frac{1}{n+2}} \times \overline{f_{\text{KK}}}^{\frac{1}{n+2}} \Lambda_{\text{sp},n}^{\text{SN 1987A}} \quad (26)$$

All in all, for $n = 2$, the new bounds on the species scale associated to (13) and (14) become

$$\Lambda_{\text{sp,min}}^{\text{new}} = \overline{f_{\text{KK}}}^{-1/4} 34 \text{ TeV} = 6.2 \text{ TeV} \quad (27)$$

and

$$\Lambda_{\text{sp,min}}^{\text{new}} = \overline{f_{\text{KK}}}^{-1/4} 36 \text{ TeV} = 6.6 \text{ TeV} \quad (28)$$

respectively.

Duplicating this procedure, but substituting τ_{NS} by the age of the universe in the limit of integration of (22), we obtain the $\overline{f_{\text{KK}}}$ to rescale the bound on the species scale associated to (11). It is straightforward to see that the corresponding value of $\overline{f_{\text{KK}}} > 881$ and therefore the bound on (11) can be evaded.

We conclude that in the absence of isometries the SN constraints are given by (9) and one or two extra dimensions with $R_{\text{KK}} \sim 1 \mu\text{m}$ are consistent with observations.

4. Cosmological Constraints

Cosmological observations lead to additional model independent and model dependent constraints. We begin discussing constraints on models in which KK intra-tower decay is not allowed, and then we relax this condition by considering extra dimensions which do not admit isometries.

Model independent constraints originate again in the emission of KK modes. Expansion dominated cooling gives a model independent bound. Indeed, an upper limit on the “normalcy” temperature T_* at which the universe must be free of bulk modes

can be derived by demanding that the rate at which radiation energy density on the brane evaporates into KK modes in the bulk remains below the normal cooling due to cosmological expansion,

$$T_* \lesssim 10^{\frac{7n-7}{n+1}} \left(\frac{\Lambda_{\text{sp}}}{10 \text{ TeV}} \right)^{\frac{n+2}{n+1}} \text{ MeV} \quad (29)$$

which for $n = 2$, leads to $T_* \lesssim 200 \text{ MeV}$.^[60] Big Bang nucleosynthesis (BBN) set a more restrictive (but model dependent) bound on T_* . From the 4D perspective, the gravitons produced at temperature T are massive KK modes with mass $\sim T$. This is because the emission rate of each KK mode $\sim T^3/M_p$. It goes without saying that for given T , all KK modes up to T are produced, but most particles have a mass $\sim T$, because the emission rate goes down rapidly with decreasing T . Putting all this together, the energy density in KK modes redshifts away as a^3 rather than a^4 , where a is the cosmic scale factor. Then, to insure normal expansion rate during BBN the bound on the normalcy temperature is slightly stronger,

$$T_* \lesssim 10^{\frac{7n-7}{n+2}} \left(\frac{\Lambda_{\text{sp}}}{10 \text{ TeV}} \right) \text{ MeV} \quad (30)$$

which for $n = 2$, leads to $T_* \lesssim 50 \text{ MeV}$.^[60]

Even more restrictive bounds emerge by requiring that the energy density of KK gravitons today does not overclose the universe. Indeed, to overcome overproduction of KK modes at early times the temperature at which the universe enters the radiation dominated epoch must be delicately fine-tuned.^[60,68,72] The contribution of thermally produced KK to the energy density today is estimated to be

$$\frac{\rho_{\text{KK}_0}}{T_0^3} \sim \frac{2}{3} \frac{M_p T_*^4}{\sqrt{g_*} \Lambda_{\text{sp}}^4} \quad (31)$$

where g_* is the effective number of relativistic degrees of freedom and where we have used the fact that the ratio ρ_{KK}/T^3 is invariant. The critical density of the universe today corresponds to $(\rho_{\text{crit}}/T_0^3) \sim 3 \times 10^{-9} \text{ GeV}$. Setting $\rho_{\text{KK}} \leq \rho_{\text{crit}}$, with $\Lambda_{\text{sp}} = 10 \text{ TeV}$ and $g_* = 10.75$ leads $T_* \sim 2.8 \text{ MeV}$.^[60] This normalcy temperature derived more carefully is closer to the decoupling temperature for the muon and tau neutrinos. Indeed, for $T_* = 2.15 \text{ MeV}$, it follows that

$$R_{\text{KK}} < 3.3 h \mu\text{m} \sim 2.3 \mu\text{m} \quad (32)$$

which corresponds to

$$\Lambda_{\text{sp}} > 5 \text{ TeV}/\sqrt{h} \sim 6 \text{ TeV} \quad (33)$$

where $H_0 = 100 h \text{ km Mpc}^{-1} \text{ s}^{-1}$ is the Hubble constant, with $h \simeq 0.7$.^[68] Needless to say, the bounds in (32) and (33) only take into account thermal production of the KK modes of gravitons in the early universe. Successful inflation and reheating, as well

¹ Note that $M = 2^{1/(n+2)} (2\pi)^{n/(n+2)} \Lambda_{\text{sp}}$,^[65] where M is the underlying $(n+4)$ -dimensional scale adopted in ref. [68].

as baryogenesis, usually stand in need for the existence of fields in the bulk, most notably the inflaton. The non-thermal production of KK modes accompanying the inflaton decay would further constrain R_{KK} and exclude the possibility of two extra dimensions of micron size.^[72]

If we are prepared to accept that KK gravitons are only produced thermally, it is of interest to further investigate whether low reheating temperature universes, with $T_* \sim 2 \text{ MeV}$, remain consistent with cosmological observations. Lower bounds on the reheating temperature have been derived assuming the late-time entropy production near BBN is dominated by radiative decays. The incomplete thermalization of neutrinos could modify the production of ${}^4\text{He}$ and this leads to a bound $T_* > 0.5 \text{ MeV}$ at 95%CL.^[75] When hadronic decays are also considered $T_* > 2.4 \text{ MeV}$ at 95%CL.^[76] In addition, the effect of flavor neutrino oscillations turns out to be quite relevant to accommodate BBN, shifting the lower bound to T_* higher temperatures. Indeed, the thermalization process of two- and three-flavor neutrino oscillations have been analyzed yielding the following 95% CL bounds: $T_* > 2 \text{ MeV}$ and $T_* > 4.1 \text{ MeV}$ assuming radiative decay of the massive particles.^[77,78] More recently, it was shown that when both neutrino oscillations and self-interactions are considered in the thermalization process the 95% CL lower bounds on the normalcy temperature are less restrictive: $T_* > 1.8 \text{ MeV}$ and $T_* > 4 \text{ MeV}$, for radiative decays and hadronic decays; respectively.^[79] CMB data and large scale structure lead to comparable bounds on the normalcy temperature, $T_* \gtrsim 2 \text{ MeV}$.^[80–82]

Recently, cosmologies with $\mathcal{O}(\text{MeV})$ normalcy temperatures have become the focus of a great deal of interest because they can accommodate several new physics scenarios that would normally be constrained by high-temperature reheating models, including massive sterile neutrinos.^[84] For example, if $T_* \sim 1.8 \text{ MeV}$ sterile neutrinos (of mass $m_s \sim \text{keV}$) with large mixing angle θ and a dark decay to radiation may ameliorate the H_0 tension.^[85] Several neutrino experiments are scanning the 0.1 to 100 keV mass range to search for these sterile neutrinos, which may be visible in the lab but may be invisible to cosmological observations; for details see Figure 1.

For n extra dimensions, the constraint from cosmological overproduction of bulk graviton modes can be written as:

$$T_* \sim 10^{3+(6n-15)/(n+2)} \left(\frac{\Lambda_{\text{sp}}}{10^9 \text{ GeV}} \right) \text{ GeV} \quad (34)$$

which implies that for a single dark dimension in the micron range the universe must be free of bulk modes at $T_* \sim 1 \text{ GeV}$.^[60]

Dark gravitons (KK modes) provide a promising dark matter candidate.^[71] As noted in the previous section, the cosmological

² For values of T_* above the QCD phase transition further considerations are necessary to constrain R_{KK} .^[73,74]

³ We note in passing that a recent study combining Planck CMB data and DESI DR1 results with $N_{\text{eff}} = 2.58$ gives $T_* > 3.79 \text{ MeV}$ at 95% CL, and when BBN predictions are included in the likelihood analysis using $N_{\text{eff}} = 2.98$ yields $T_* > 5.96 \text{ MeV}$.^[83] However, it should also be noted that because both the primordial helium abundance Y_p and the relativistic degrees of freedom affect the CMB damping tail, they are partially degenerate. Allowing N_{eff} and Y_p to vary in the likelihood analysis leads to weaker lower bound of $N_{\text{eff}} = 2.32$.^[27] This will naturally shift the value of the minimum allowed T_* to lower temperatures.

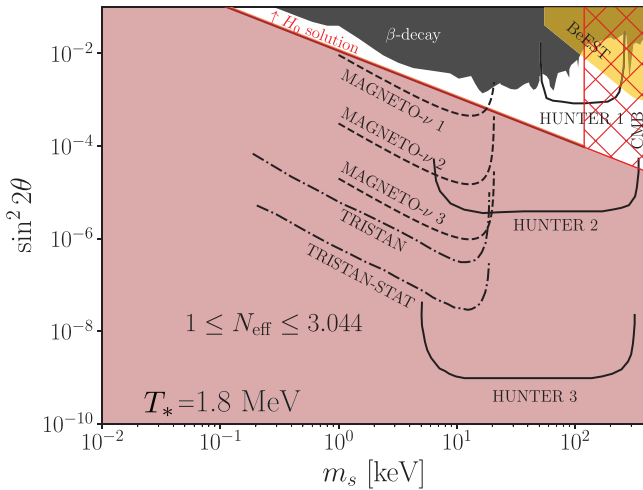


Figure 1. The (m_s, θ) parameter space for $T_* = 1.8$ MeV. The black region shows constraints from a collection of nuclear beta decay searches^[86] and the region in golden yellow constraints from β -decay searches at BeEST.^[87] The thermal nature of the CMB constrains the red hatched portion in the upper right.^[88] In the pink region $1 \leq N_{\text{eff}} \leq 3.044$. The diagonal red line correspond to the scenario when the decay happens at the temperature of matter-radiation equality. Above the diagonal red line, $N_{\text{eff}} > 3.044$ and the parameter space can accommodate an alleviation of the H_0 tension. The experimental sensitivity of HUNTER,^[89] TRISTAN,^[90] MAGNETO- ν ,^[91] and PTOLEMY^[92] for nuclear decay searches of keV-scale sterile neutrinos is also shown. Taken from ref. [84].

evolution of the dark sector is mostly driven by dark-to-dark decay processes, which regulate the decay of KK gravitons within the dark tower, realizing a particular version of the dynamical dark matter framework.^[93] CMB and Lyman α data set constraints on $\tilde{\lambda}$.^[94] In addition, KK decays impart momentum to the daughter particles, injecting kinetic energy into virialized self-gravitating dark-matter halos. These momentum “kicks” – which have been shown in (16) to depend on δ – gradually reduce the central density of cuspy dark matter halos and deplete the amount of mass in the halos.^[95,96] This process also makes sub-halos that have a virial velocity smaller than the velocity kick more susceptible to tidal disruption as they orbit within a host halo, leading to the suppression of the abundance of low-mass halos and sub-halos at late times. To preserve the structure and stability of galaxy halos, the analysis of [97] severely constrains the $(\tilde{\lambda}, \delta, \beta)$ parameter space. Using Figure 4 in [97] it is easily seen that the allowed region of the $(\tilde{\lambda}, \delta, \beta)$ parameter space leads to larger values of $\overline{f_{\text{KK}}}$.

In summary, two extra dimensions of micron-size require an upper limit on the normalcy temperature at which the universe must be free of bulk modes, which is estimated to be $T_* \sim 2.15$ MeV.^[68] This is comparable to the lowest possible reheating temperature consistent with CMB data and abundances predicted by BBN, which is estimated to be $T_* \sim 1.8$ MeV.^[79]

5. Constraints on Neutrino Towers

The smallness of the neutrino mass may be ascribed to the fact that right-handed (R) neutrinos could live in the bulk.^[98–100] The coupling of R -neutrinos to the left-handed SM neutrinos living

on the brane is inversely proportional to the square-root of the bulk volume. The Dirac neutrino mass can be shown to be

$$m_\nu \sim \gamma \langle H \rangle \Lambda_{\text{sp}} / M_p \quad (35)$$

where γ is the Yukawa coupling and $\langle H \rangle = 246$ GeV is the Higgs vacuum expectation value. Note that for $\Lambda_{\text{sp}} / M_p \ll 1$, the neutrino mass scale is well below the electroweak symmetry breaking scale $\langle H \rangle$, even for order-one Yukawa couplings.

If R -neutrinos live in the bulk their KK modes would mix with SM neutrinos modifying their oscillation pattern. The phenomenology is similar to that of light sterile neutrinos, which induce observable short-baseline neutrino oscillations and small perturbations to the neutrino oscillations in solar, atmospheric, and long-baseline experiments that are well-described by standard three-neutrino mixing. The non-observation of these effects in neutrino-detection facilities set 90% C.L. bounds on the size of the largest extra dimension

$$R_{\text{KK}} < \begin{cases} 0.2 \mu\text{m} & \text{NO} \\ 0.1 \mu\text{m} & \text{IO} \end{cases} \quad (36)$$

for normal (NO) and inverted (IO) ordering of neutrino masses.^[101–103] However, these bounds could be significantly relaxed in the presence of bulk neutrino masses, because the mixing of the first KK modes to active SM neutrinos becomes suppressed, making the contribution of heavier KK modes to oscillations relatively more important.^[104,105]

If R -neutrinos live in the bulk, the limit (9) would be stronger because neutrino towers could offer more modes into which energy may be lost. The SN energy loss rate because of nucleon-nucleon gravi-strahlung $\dot{\epsilon}_{\text{SN}} \propto \Lambda_{\text{sp}}^{-(n+2)}$.^[61] A rough estimate of the Λ_{sp} lower bound can be established by scaling the graviton energy loss rate of the total number of degrees of freedom, N , in the bulk whose couplings allow them to be emitted singly starting only from brane states

$$\Lambda_{\text{sp}} \gtrsim 8.9 \left(\frac{N}{3} \right)^{1/4} \text{TeV} \quad (37)$$

with $n = 2$.^[106] Obviously, if only gravitons could be emitted, then we have $N = 3$ because out of the 9 6D graviton polarizations only 3 can couple to purely 4D stress energy. Each Weyl R -neutrino contributes with two polarizations, yielding $\Lambda_{\text{sp}} \gtrsim 12$ TeV. For $n = 1$, the cosmological evolution of R -neutrino towers was discussed elsewhere.^[107]

6. Six-Dimensional Black Holes as All Dark Matter

It is well-known that black holes are expected to have formed when very large density perturbations collapse.^[108–110] An attractive idea is that these over densities are of inflationary origin.^[111–113] Once the over dense regions come back into causal contact after inflation, they instantly collapse if they exceed a medium-specific threshold. These PBHs fulfill all of the necessary requirements to be a captivating dark matter candidate: they are cold, non-baryonic, quasi-stable, and can be formed in the right abundance to be the dark matter. Actually, if PBHs contribute to more than 10% of the dark matter density,

then their energy density today is of the same order as that of the baryons.^[114] This cosmic coincidence might hint at a mutual origin for PBHs and the baryon asymmetry of the Universe. Strikingly, PBHs fostered by a transition from a slow-roll to ultraslow-roll during single field inflation can contribute as a significant dark matter component,^[115] while such a transition of the inflationary background can trigger successful baryogenesis via the Affleck-Dine mechanism.^[116]

PBHs evaporate by emitting Hawking radiation, with a temperature inversely proportional to its mass.^[117,118] Nevertheless, a PBH could be stable on cosmological scales and its lifetime could even be longer than the age of the Universe. For example, if the initial mass of a 4D PBH is $M_{\text{BH}} \gtrsim 10^{15}$ g it can survive until today.^[119] Even so, an all-dark-matter interpretation in terms of PBHs have been ruled out through observations across most mass ranges.^[120–122] Constraints mostly come from microlensing surveys and non-observation of Hawking radiation. PBHs in the mass range $10^{17.5} \lesssim M_{\text{BH}}/g \lesssim 10^{21}$ remain a viable explanation for all dark matter.

Higher-dimensional black holes of Schwarzschild radii smaller than a micron are: bigger, colder, and longer-lived than a usual 4D black hole of the same mass.^[123] In a series of recent publications we have used these black hole properties to extend the PBH range as the entirety of dark matter compared to that in the 4D theory by several orders of magnitude in the low mass window.^[124–127] In what follows we particularize our investigation to 6D black holes.

Shortly after being formed a higher-dimensional PBH would emit SM particles and gravitons on the brane as well as gravitons into the bulk. However, the recoil effect due to graviton emission imparts the black hole a relative kick velocity with respect to the brane, allowing the PBH to wander off into the bulk.^[128–132] The Hawking evaporation time characterizing the lifetime of such a bulk PBH is estimated to be

$$\tau_{\text{BH}} \sim r_s S_{\text{BH}} \sim \frac{1}{\Lambda_{\text{sp}}} \left(\frac{M_{\text{BH}}}{\Lambda_{\text{sp}}} \right)^{(n+3)/(n+1)} \quad (38)$$

where

$$r_s \sim \frac{1}{\Lambda_{\text{sp}}} \left(\frac{M_{\text{BH}}}{\Lambda_{\text{sp}}} \right)^{1/(n+1)} \quad (39)$$

is the $(4+n)$ -dimensional Schwarzschild radius^[133,134] and

$$S_{\text{BH}} \sim (M_{\text{BH}}/\Lambda_{\text{sp}})^{(n+2)/(n+1)} \quad (40)$$

is the black hole entropy.^[135] All in all, taking $n = 2$ and $\Lambda_{\text{sp}} \sim 10$ TeV we obtain

$$\tau_{\text{BH}} \sim 13.7 \left(\frac{M_{\text{BH}}}{10^{7.8} \text{ g}} \right)^{5/3} \text{ Gyr} \quad (41)$$

We conclude that the mass range for 6D PBHs to make all the cosmological dark matter is then

$$10^8 \lesssim M_{\text{BH}}/g \lesssim 10^{21} \quad (42)$$

However, one caveat of the PBH all dark-matter interpretation is that for $R_{\text{KK}} \sim 1 \mu\text{m}$, thermal production of KK gravitons almost

overcloses the universe. Therefore, a significant fraction of dark matter in the form of PBHs would exacerbate the required fine tuning on the reheating temperature.

In closing, we stress that if there were 6D primordial near-extremal black holes in nature, then it would be possible to lower the minimum mass allowing a PBH all-dark-matter interpretation, because near-extremal black holes are colder and longer-lived.^[126] Note, however, that near-extremal black holes cannot escape from the brane into the bulk if they carry e.g., electromagnetic charge.

7. Ultra-High-Energy Cosmic Rays as Probes of Two Dark Dimensions

Extensive air showers induced from ultra-high-energy cosmic rays (UHECRs) provide a window into understanding the most energetic interactions.^[136] To be specific, the highest energy cosmic rays interact in the Earth's atmosphere through collisions with center-of-mass energies $\sqrt{s} \sim 300$ TeV, *viz.* not only well above possibilities at the LHC, but even beyond the projections for the FCC-hh.

The initial interaction of a cosmic ray in the atmosphere produces a set of secondary particles (mostly π 's) carrying a fraction of the primary energy. The π^0 's produced in the first interaction ($\sim 1/3$ of all pions in accord with isospin invariance) promptly decay to a pair of gamma rays. The gamma rays produce e^\pm pairs when passing near nuclei. The electrons and positrons re-generate gamma rays via bremsstrahlung, thereby building an electromagnetic cascade. The π^\pm , which carry $2/3$ of the energy lost by the cosmic ray, begin to move through the atmosphere and either decay or interact generating new sets of secondaries. In these interactions the π^\pm again carry $2/3$ of the parent energy and $1/3$ goes into electromagnetic cascade. After j hadronic cascade generations, only $(2/3)^j$ of the total energy remains in the hadronic cascade. The π^\pm produce air shower muons when they decay. The number of shower muons depends on the amount of energy i.e., left in the hadronic cascade when pion energies have dropped to the level where decay is more likely than collision. If this happens after relatively few cascade generations, then copious muon production occurs. If the reduction of pion energies takes relatively many generations, then more of the energy will have been lost from the hadronic cascade to the electromagnetic cascade, and meager muon production occurs. Eventually, the energy of the shower particles is degraded to the point where ionization losses dominate, and their number starts to decline. The electromagnetic cascade dissipates about 90% of the primary cosmic ray energy and the remaining 10% is carried by muons and neutrinos. The total number of muons produced by a cosmic ray nucleus scales roughly as:

$$N_\mu \propto A (E/A)^{0.93} \quad (43)$$

where A is the nucleus baryon number.^[136] The expectation is then that a cosmic ray nucleus to produce about $A^{0.07}$ more muons than a proton. This means that a shower initiated by an iron nucleus produces about 30% more muons than a proton shower.

Measurements from several cosmic-ray experiments seem to indicate there is a significant, yet unexplained, discrepancy

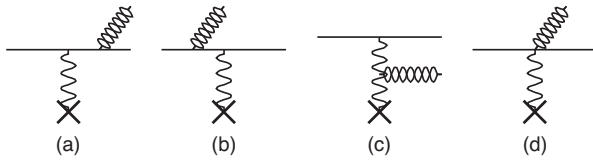


Figure 2. Feynman diagrams for a bremsstrahlung emission of KK gravitons by electrons in the static electric field generated by atmospheric nuclei.

between the observed muon content in cosmic ray showers and that predicted by state-of-the-art interaction models, suggesting a need for refinements in our understanding of fundamental physics. In particular, Auger data suggest that showers induced by cosmic rays of energy $10^{9.8} < E/\text{GeV} < 10^{10.2}$ contain about 30% to 60% more muons than expected.^[137,138] The significance of the discrepancy between data and model prediction has been experimentally established at 8σ .^[139] The onset of the discrepancy is seen in showers with $E \sim 10^{7.7}$ GeV, which corresponds to a first interaction at $\sqrt{s} \sim 10$ TeV in the nucleon-nucleon system.

A thought-provoking observation is that scattering processes above the species scale necessarily have internal structure in the compact space and to a certain extent would lead to the production of KK modes. Indeed the phase space seen by the emitted graviton grows quite quickly with energy on very general grounds, yielding prolific production of KK modes in particle interactions.⁴ Along this line, air shower simulations seem to indicate that production of KK gravitons by nucleon-nucleon bremsstrahlung (at large impact parameter) could lead to a sizable fraction of shower missing energy.^[143,144]

Nevertheless, herein we conjecture that KK graviton emission takes a stand against processes driving the electromagnetic component of the shower (viz. Bethe-Heitler pair production and electron bremsstrahlung) rather than those generating the hadronic cascade; see e.g., Figure 2. Note that the number of electromagnetic interactions surpasses the number of hadronic collisions in the shower by orders of magnitude. Now, if KK production would only alter the evolution of the electromagnetic component of the shower, then current simulations of extensive air showers would underestimate the ratio of the *visible* energy dissipated by the hadronic cascade E_{had} to that in the electromagnetic cascade E_{EM} . Obviously, an underestimation of the ratio $E_{\text{had}}/E_{\text{EM}}$ would alter the energy reconstruction of the shower.^[145] To accommodate the imbalance between E_{had} and E_{EM} , the cosmic ray energy E must be slightly increased, and via (43) this implies an increase in the number of muons that could account for the discrepancy between simulations and experiment.

Note that in our conjecture the production of KK modes does not alter the evolution of the hadronic shower. This is in agreement with Auger data which indicate that the event-to-event fluctuations in the muon number are well described by existing air shower simulations.^[146] Furthermore, a recent study shows that

⁴ Moreover, black holes are expected to be produced in collisions with $\sqrt{s} \gg \Lambda_{\text{sp}}$.^[140] However, one caveat is that QCD cross sections dominate over the black hole production cross section by a factor of roughly 10^9 .^[141] This implies that black holes produced in cosmic ray air showers seem effectively unobservable. This is also the case for the production of string massive modes.^[142]

the distribution of atmospheric depth in which a cosmic ray shower reaches its maximum number of particles (a.k.a. X_{max}) is systematically off in all interaction models.^[147] The best description of the data is achieved if the model predictions are shifted to deeper X_{max} values. This new result is also in agreement with predictions of our conjecture. While it is difficult to be precise, if there were two micron-size dark dimensions gravity would become strong around 10 TeV, an energy scale where discrepancies between model building and cosmic-ray experimental data have been observed. Ultimately, the impact of new scattering process on cosmic ray air showers, including those that spread out into a large internal space, can be best studied using a full-scale high-fidelity simulation.

8. Conclusions

The motivation for our work is the observation that two extra dimensions on the micron scale can simultaneously address the cosmological and gauge hierarchy problems. To be precise, the dark dimension scenario is not solving the problem of why there is a small cosmological constant, but it finds a model independent consequence, and if $n = 2$ it is also connected to the electroweak hierarchy by low scale gravity. Consistency with astrophysical observations requires two extra dimensions which do not admit isometries, whereby conservation of the extra dimensional momentum is violated, allowing the massive KK modes of the graviton to decay to other lighter graviton modes. Within this scenario dark gravitons or else PBHs with masses in the range $10^8 \lesssim M_{\text{BH}}/g \lesssim 10^{21}$ could make all cosmological dark matter, though the PBH all dark-matter interpretation requires some fine tuning. To this conclusion, however, we must add an important caveat: to remain consistent with cosmological observations two extra dimensions of micron scale require a delicately fine tuning of the temperature at which the universe enters the radiation dominated epoch. Indeed, to overcome overproduction of KK modes at early times the normalcy temperature is estimated to be $T_* \sim 2.15$ MeV.^[68] We note that such a value of T_* is comparable to the lowest possible reheating temperature consistent with CMB data and abundances predicted by BBN, which is estimated to be $T_* \sim 1.8$ MeV.^[79]

Above and beyond searches of KK signals at future particle colliders, a variety of astrophysical observations will be able to probe the ideas discussed in this paper. For example, the asteroid-mass window for PBHs to be all dark matter will be tested in the near future by microlensing of X-ray pulsars.^[148,149] This window include the interesting range ($10^{16.5} \lesssim M_{\text{BH}}/g \lesssim 10^{17.5}$) where an all dark matter interpretation in terms of 4D Schwarzschild PBHs is excluded by the non-observation of their Hawking radiation.^[150] In addition, the long-baseline Deep Underground Neutrino Experiment (DUNE) will probe relevant aspects of neutrino mixing in scenarios with large extra dimensions.^[151,152] Finally, Auger-Prime full efficiency data taking (a.k.a. Auger Phase II) started in 2024 and is foreseen to add ten more years of data.^[153] These data might help elucidate whether KK production is at the center of the cosmic-ray *muon puzzle*.

In conclusion, the dark dimension scenario does not provide a theoretical prediction of why the cosmological constant is so small, but it connects it to other scales, namely the size of the extra dimensions, and for the case of $n = 2$ to TeV scale physics.

Actually, the 6D Planck scale in the 10 TeV regime is on the one hand already quite high in order for being a “natural” solution of the electroweak hierarchy problem and would already need some fine tuning. On the other hand, it is not yet excluded by LHC data, however the next high luminosity LHC run could possibly exclude it. As shown in our paper, the more severe tension of the two extra dark dimension scenario comes from cosmology. A further argument in favor of only one dark dimension comes from gauge unification, which apparently needs a high species scale.^[154] Yet another argument favoring $n = 1$ comes from axion physics. On the one hand, for a single dark dimension, the QCD axion can be naturally localized on the SM brane, where observational constraints force a narrow range for the axion decay constant $10^9 \lesssim f_a/\text{GeV} \lesssim 10^{10}$ while the axion mass, $1 \lesssim m_a/\text{meV} \lesssim 10$, is surprisingly near the mass scale for the dark energy, the dark matter tower, and the neutrino.^[155] On the other hand, for two dark dimensions of micron size, a large value of f_a consistent with observations^[156] could come from the suppression of the wave function if the axion lives in the bulk, but for $T_* \sim 2.15 \text{ MeV}$, such an axion would be severely constrained through its contribution to N_{eff} .

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are openly available in arXiv at <https://arxiv.org/abs/2501.11690>, reference number arXiv:2501.11690.

Keywords

dark energy, large extra dimensions, string theory

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