

# Massive hot white dwarfs: Consequences of finite temperature in the structure and on the onset of instabilities

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In this work, we investigate the observable white dwarfs with high-surface gravity in the framework of general relativity. We consider the stellar fluid composed of nucleons and electrons confined in a Wigner-Seitz cell surrounded by free photons. Besides, we implement a temperature depending on the mass density with the presence of an isothermal core. The impact of temperature on the equilibrium and stability of white dwarfs is observed. We compare our results with massive white dwarfs estimated from the Extreme Ultraviolet Explorer Survey and Sloan Digital Sky Survey. We find that the high-surface gravity white dwarfs are well described by our curves with higher central temperatures. Our results suggest that these hot massive stars detected are within the range of white dwarfs with more radial stability. Moreover, we note the radial instability is attained before the pycnonuclear reaction for  $T_c \geq 1.0 \times 10^8$  [K].

*Keywords:* White dwarfs; charged white dwarfs; type Ia supernovae explosions

## Introduction

Observable white dwarfs' structure has been reported in works using the Extreme Ultraviolet Explorer Survey (EUVE) and Sloan Digital Sky Survey (SDSS).<sup>1-4</sup> Some of these stars have a great surface gravity ( $\log(g/g_\odot) \geq 4.4$ ) and a high effective temperature ( $T_{\text{eff}} \gtrsim 10^4$  [K]).<sup>1,2</sup> Due to their high surface gravity, it is important to investigate their instabilities against small radial perturbation and pycnonuclear reactions in general relativity scope considering temperature.

The studies concerning instabilities in white dwarfs<sup>5,6</sup> consider a completely degenerate star. To compare results with observable white dwarfs, we include temperature in the model. Thus, following recent white dwarfs models with finite temperature,<sup>7,8</sup> we analyze the structure and stability of hot white dwarf' and compare

the results with some observable data. In this article, we detail some conclusions of the previous work.<sup>9</sup>

We study the structure and stability of hot massive white dwarfs and we compare our results with those ones reported by EUVE and SDSS. In addition, we made an instability analysis against the pycnonuclear reactions and radial perturbations. We analyze the stellar fluid by considering nucleons and electrons confined in a Wigner-Seitz cell with surrounding free photons. For this case, the pressure and energy density are respectively considered as following

$$P = P_e + P_N + P_\gamma + P_L, \quad (1)$$

$$\varepsilon = \varepsilon_e + \varepsilon_N + \varepsilon_\gamma + \varepsilon_L, \quad (2)$$

with the subscripts e, N, and  $\gamma$  representing the respective contributions of electrons, nucleons, and photons. Due to the partially degeneracy, the electron contributions are obtained with an adaptive quadrature method.<sup>8</sup>

## 1. The stellar structure equations and boundary conditions

We consider the unperturbed white dwarf is made of by a perfect fluid, whose energy-momentum tensor is expressed of the form

$$T_{\mu\beta} = (P + \varepsilon)u_\mu u_\beta + P g_{\mu\beta}. \quad (3)$$

The background metric employed to describe the spherically symmetric white dwarf, in Schwarzschild-like coordinates, is given by

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (4)$$

where the functions  $\nu$  and  $\lambda$  dependent on the radial coordinate only. The nonzero Einstein field equation components, for the line element assumed, lead to obtain the stellar structure equations:

$$\frac{dm}{dr} = 4\pi\varepsilon^2, \quad (5)$$

$$\frac{dP}{dr} = -(P + \varepsilon) \left[ 4\pi r p + \frac{m}{r^2} \right] e^\lambda, \quad (6)$$

$$\frac{d\nu}{dr} = -\frac{2}{(p + \varepsilon)} \frac{dP}{dr}, \quad (7)$$

being  $m$  the mass inside the radius  $r$ . The potential metric  $e^\lambda$  is described as

$$e^\lambda = \left[ 1 - \frac{2m}{r} \right]^{-1}. \quad (8)$$

Since we are considering a temperature distribution  $T$  dependent on mass density  $\rho$  with the presence of an isothermal core, we assume that the parameters  $T$  and  $\rho$  are related by the equality

$$T/\rho^{2/3} = \text{constant}. \quad (9)$$

The process starts integrating the stellar structure equations from the center ( $r = 0$ ) until the surface of the star ( $r = R$ ). At the center of the star are considered the initial conditions:

$$m(0) = 0, \quad \varepsilon(0) = \varepsilon_0, \quad \text{and} \quad \nu(0) = \nu_0. \quad (10)$$

The surface of the star is determined at  $P(r = R) = 0$ .

## 2. Results

Due to our analysis be restricted to very massive WD, we can use a simple equation for the temperature distribution as Eq. (9). To ensure this affirmation, we compare the mass and radius obtained with the temperature distribution in Eq. (9) and the one found by Kritcher and collaborators<sup>10</sup> which considers a more complete processes as radiation and convection. To analyze massive WDs, we use the temperature distribution stated Eq. (9). By using this relation, we obtain similar masses to those reported by Kritcher's and collaborators<sup>10</sup> for stars near the maximum-mass limit. This is due to the more contribution of the WD mass coming from the degenerate core, which has a constant temperature. The envelope with a different temperature distribution should only modify the stellar radius. In this case, the percentage between the difference obtained by Eq. (9) and Kritcher's envelope, for  $g \geq 10^9 [\text{cm s}^{-2}]$ , is around 2%.

The surface gravity as a function of radial coordinate is plotted in Fig. 1 for several central temperatures and  $M = 1.37M_\odot$ . From the panel, we note that the surface gravity increases with the radial coordinate until it reaches the maximum gravity point, hereafter, gravity decays monotonically with the increment of the radial coordinate.

The change of surface's gravity and total radius of the star with the increment of the central temperature is also observed in Fig. 1. For larger temperature, greater total stellar radius and lower surface's gravity are found. In this sense, WDs with smaller surface gravity -than those ones predicted with the cold catalyzed matter- are expected to have high central temperatures.

The mass against the total radius is shown in Fig. 2 for different values of central temperature. The full triangles in pink over the curves indicate the maximum mass points. The blue dots represent the threshold instability due to pycnonuclear reactions. In addition, in figure, some observational results obtained from the catalogs Nalezyty<sup>2</sup> in green diamond, and in Vennes<sup>1</sup> in brown squares. In all curves of the panel, we note that the mass grows with the surface's gravity until reaches the maximum mass point, thereafter, the mass decreases with the grows of gravity.

In Fig. 2, we observe that the analysis of the pycnonuclear reaction has to be considered for central temperatures below  $T_c < 10^8 [\text{K}]$ . For central temperatures above  $10^8 [\text{K}]$ , the secular stability point is reached before the pycnonuclear reaction. It is important to mention that the secular stability point is marked by the maximum mass point.

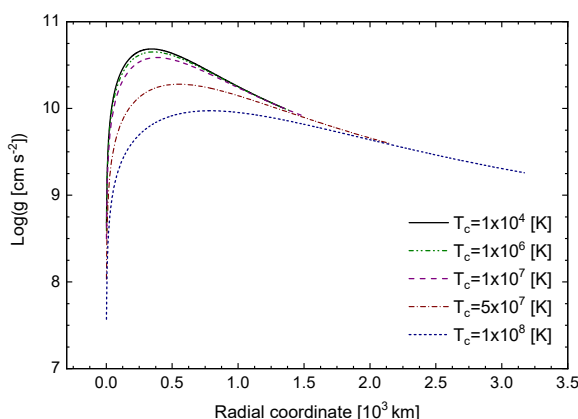


Fig. 1. Surface gravity as a function of radial coordinate for several central temperatures with a fixed  $M = 1.37M_{\odot}$ .

Vennes,<sup>1</sup> and Nalezty<sup>2</sup> have reported some very massive white dwarfs with very high surface gravity. In Fig. 2, these white dwarfs are matching with the curves of very high central temperatures and are very close to the instabilities thresholds. Indeed, one of them, with a mass of  $M = 1.41M_{\odot}$  can have a central temperature higher than  $10^8$  [K]. From this, we could understand that these massive white dwarfs could be treated differently. Using the surface gravity and effective temperature reported by Vennes, we could investigate these stars by employing the values of effective temperature and surface gravity. Moreover, considering the general relativity effects, we found that the masses of these stars decrease, thus highlighting the importance of general relativity in the study of massive white dwarfs.

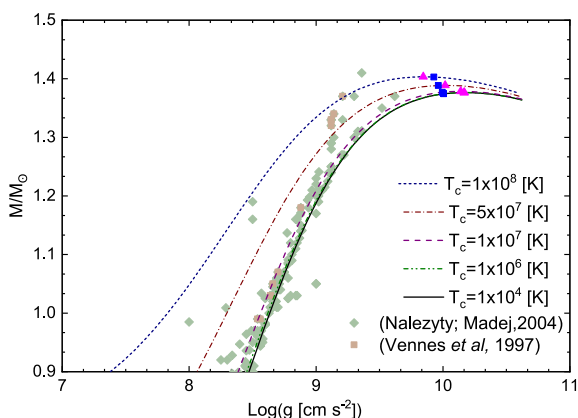


Fig. 2. Mass as a function of surface gravity for several central temperatures. The green diamonds<sup>2</sup> and the brown squares<sup>1</sup> represents observational data.

### 3. Conclusion

In this work, we investigate the structure and stability of hot white dwarfs. We regard the stellar fluid as composed of nucleons and electrons confined in a Wigner-Seitz cell, with free photons. We consider the temperature profile depending on the mass density with the presence of an isothermal core. We use the stellar structure equations to analyze the equilibrium configurations of white dwarfs and the stability against the pycnonuclear reactions and small radial perturbations.

Comparing our results with observational data, we found that the high masses white dwarfs provided by Vennes<sup>1</sup> could be described by our model. Moreover, we discussed about the importance of considering central temperature and general relativity in the study of massive white dwarfs.

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