DUAL DESCRIPTION OF A CONFINED COLOUR FIELD

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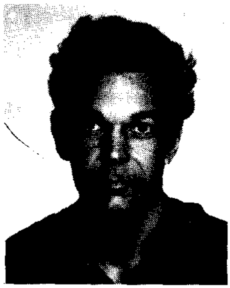


ABSTRACT

ıe result of perturbative QCD can be formulated in two dual or complementary ıys, either in terms of quarks and gluons or in terms of colour dipoles. The :lation between the two descriptions is similar to that between a lattice and :s dual lattice. If the confined colour field behaves like a vortex line in a ıperconductor, then the dipoles form a chain along the vortex line.

In this talk I want to show that the results of perturbative QCD (in the case of e.g. an e^+e^- annihilation event) can be formulated in two dual or complementary ways, either in terms of quarks and gluons or in terms of colour dipoles [1].

Due to the confinement property a colour field does not spread out as an ordinary electromagnetic field, when the charges are separated. It was early suggested that it may be compressed into vortex lines like the magnetic field in a type II superconductor [2]. Such a vortex line consists of a thin core, which is kept together by currents circulating around it, and surrounded by a more extended magnetic field, which at large distances is exponentially damped. The field of such a vortex line is the same as that of a chain of dipoles lined up along the vortex (see fig. 1). (Due to the damping in the superconducting medium this field is not just the sum of the fields from the first and last charges in the dipole chain.)

The dynamics of a vortex line is that of the massless relativistic string as long as it is not very strongly curled up, where the scale is determined by the thickness of the core. In the Lund model gluons emitted in an e^+e^- annihilation event are assumed to act as excitations or kinks on a stringlike field [3]. Thus the field is stretched from the quark via the gluons to the antiquark. This picture implies a set of observable asymmetries and correlations which now have strong experimental support [4]. The gluons are ordered along the string and e.g. a red charge of one gluon and an antired charge of the next one form together one dipole in the vortex line chain, as shown in fig. 2.

The stringlike behaviour can also be understood from perturbative QCD [5]. If one gluon has been emitted with a given momentum, then the radiation of further soft gluons is that given by two dipoles, one formed by the quark and the gluon and the other by the gluon and the antiquark. There should also be subtracted, with the relative weight $1/N_c^2$ the emission corresponding to one dipole formed by the quark and the antiquark. Thus soft gluons are emitted in those regions where the string model and the experiments give an excess of hadrons. If this scheme is generalized to more gluons we find that a system with n gluons a quark and an antiquark radiates like n+1 dipoles (with an $O(1/N_c^2)$ correction from the quark charges).

Fig. 1

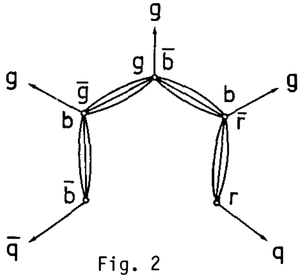


Fig. 2

When one dipole emits a gluon this dipole is split into two dipoles (see fig. 3). If each dipole could be given a definite colour (e.g. red-antired) it would have been possible to determine which dipole end emitted the soft gluon. For the situation in fig. 3 the red charge in the dipole may be carried e.g. by a $r\bar{g}$ gluon which emits a $r\bar{b}$ gluon and turns into $b\bar{g}$. This is not possible however. Because each dipole is a coherent combination of different colours we are not able to associate the emitted gluon with one of the dipole ends.

The physical state can be described in two alternative ways. Either one specifies the energy-momentum and polarization of all the gluons, quarks and antiquarks (and their colour ordering) or one specifies the energy-momentum and orientation (polarization) of all the dipoles. The dipoles are links which connect the gluons and the gluons are links which join the dipoles (cf fig. 2). Thus the relation between the two ways to describe the state is similar to the relation between a lattice and its dual lattice.

We now want to study the gluon emission from one dipole. If we go to the rest frame of one of the dipoles we have two gluons (or e.g. a gluon and a quark) which move in opposite directions with momenta $\bar{p}_1$ and $\bar{p}_2 = -\bar{p}_1$. The emission of gluons which are soft compared to these momenta, is given by

$$d\sigma \approx \frac{\alpha_s}{4\pi^2} N_c \frac{dq_\perp^2}{q_\perp^2} dy\, d\phi \tag{1}$$

Here $q_\perp$, y and ϕ denote the transverse momentum, rapidity, and azimuth angle for the emitted gluon.

We may (somewhat inadequately) regard gluons with $y>0$ ($y<0$) as emitted by the rightmoving (leftmoving) charge. We now make a (large) Lorenz transformation to e.g. the original total cms. For simplicity we assume that this boost is perpendicular to the dipole moment (and in the direction corresponding to $\phi=0$). The momenta after the emission are denoted

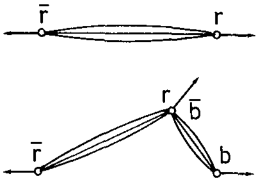
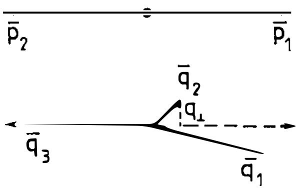


Fig. 3



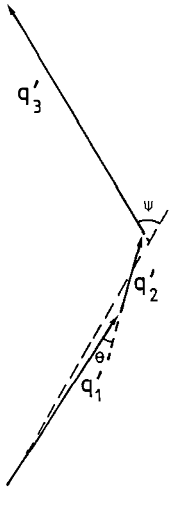


Fig. 4

q_i and q'_i in the old and the new Lorentz frame respectively (cf. fig.4), and we introduce the variables z and Q^2 by the relations

$$q'_2 \approx z\, q'_1 \quad (\approx z p'_1)$$
$$Q^2 = (q_1+q_2)^2 = (q'_1+q'_2)^2 \qquad (2)$$

For large enough z-values the distribution in eq. (1) now takes the form

$$d\sigma \approx \frac{\alpha_s}{2\pi} N_c \frac{dQ^2}{Q^2} \cdot \frac{dz}{z} \;, \quad z >> Q^2/W^2 \qquad (3)$$

This corresponds to one half of the wellknown expression for gluon splitting. The other half corresponds of course to the adjacent dipole. However, the kinematical constraint y>0 implies a cut-off for lower z-values. This cut-off corresponds just to the situation where the angel θ in fig 4 equals the angle ψ.

$$y>0 \Rightarrow \theta<\psi \qquad (4)$$

We conclude that the result in eqs (3) and (4) corresponds exactly to the splitting of massive gluons into two gluons including the angular cut-offs caused by soft gluon interference, as discussed by Mueller, Marchesini, Webber and others [6].

In the same way gluons with y<0 can be regarded as emitted by the other charge in the dipole. It is very natural that the formula for dipole radiation automatically reproduces the soft gluon cut-off caused by interference. A charge and anticharge do not radiate independently when the wavelength of the radiation is large compared to the relative distance. Of course the choice y=0 as the borderline between the two regions is quite arbitrary. The distribution is smooth in y and the choice of another boundary, e.g. $y=y_0$, only means that some gluons, previously regarded as emitted by one of the gluons, will instead be regarded as emitted by the other.

When a dipole emits a gluon it is split into two dipoles. Thus the production of a final state can be treated as a branching process where dipoles are split into smaller and smaller pieces. In the region where the virtualities are strongly ordered the results of perturbative QCD can therefore be formulated as a branching process in two different equivalent ways, either in terms of gluons which are split into two gluons at each branching, or in terms of dipoles which are split into two dipoles.

The formulation in terms of dipoles has however the great advantage that each branching is completely independent of the rest of the tree. In the gluon formulation the cut-off for low z-values depends on the other gluons. Also the emission is not azimuthaly symmetric around the gluon direction.

There will however be differences in the results if the branching processes are terminated at a fixed gluon mass or at a fixed dipole mass. Here we want to argue that a fixed dipole mass is a more relevant termination point. Two opposite charges which move along each other (i.e. with low invariant mass) do not radiate. On the other hand, if they move apart they can emit dipole radiation, even if they are not highly virtual. In Monte Carlo generation programs based on gluon splitting the termination often produces large masses for the colourless clusters (i.e. large dipole masses).

We note however that the termination point is less essential if the final system, after termination of the perturbative development, is treated like a string when it fragments into hadrons. In this case the emission of more and more very soft gluons does not modify the string state.

It is obviously also possible to include the break of the vortex line in two pieces by the process $g \to q\bar{q}$. In this case the charges with momenta q_1 and q_2 in fig. 4 correspond to a quark and an antiquark, and there is no dipole which connects these charges. This process is suppressed relative to the normal one, first by a factor $1/N_c$, and second because there is no z-pole in the (Q^2,z)-distribution.

Our conclusions are that the results of perturbative QCD can be formulated in two dual or complementary ways, either in terms of quarks and gluons or in terms of colour dipoles. The dipole formulation is "natural" if the colour field behaves like a vortex line in a superconductor. It is convenient because in the branching process each branching is independent of the rest of the tree. It can provide a link between perturbative QCD and the string picture.

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