

One-Loop β Functions for Mirror Noncommutative Euclidean Yukawa Sector within Electroweak-Scale Right-Handed Neutrinos Model

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Received September 27, 2024; Accepted January 17, 2025; Published January 27, 2025

In this paper, we derive the renormalization scale dependence of noncommutative mirror Yukawa couplings. To achieve this, we first formulate a Euclidean noncommutative version of the Yukawa sector within the electroweak-scale mirror right-handed neutrinos model. Then, we calculate the noncommutative one-loop β functions of Yukawa couplings for mirror fermions involved in this model, by taking advantage of the Slavnov–Taylor identities for the universal mirror Yukawa couplings, and by using the noncommutative vulcanized scalar and spinor propagators that prevent the UV/IR mixing. This leads us to a system of six cubic coupled first-order differential equations that depend only on the mirror Yukawa couplings and not on the noncommutative deformation and vulcanized parameters. We solve this system numerically for different initial conditions to get the evolution of the mirror Yukawa couplings in terms of the renormalization scale. Furthermore, we discuss the link to the commutative case and analyze the occurrence of the Landau pole for some specific sets of initial conditions.

Subject Index B32, B40, B46, B82

1. Introduction

The Standard Model (SM) of particle physics is the current well experimentally established model that describes the electroweak and strong fundamental interactions and the mass generation of most fundamental particles [1,2]. However, some observations such as the neutrino masses and mixing and the baryon asymmetry, and also some expected features like the anticipated unification of the SM with the gravitational theory at the Planck scale, cannot be accommodated within the SM. This makes it widely believed that the SM is an effective theory valid until the TeV scale [3]. Beyond this energy scale, a more general theory should hold. This paves the way to a wide range of beyond standard model (BSM) extensions that extend the particle sector, the underlying dynamic, or the spacetime structure [3].

Among these extensions, the electroweak-scale mirror right-handed neutrinos model (EW ν_R^M) [4] is of particular interest. This model, built on the commutative Minkowski spacetime, preserves the same gauge group as in the SM, but extends the scalar sector and adds mirror

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fermions. Moreover, it allows the nonsterile right-handed neutrinos to get mass in the electroweak energy scale [4]. This feature makes it important to deepen the investigation of the Yukawa coupling behavior of these right-handed neutrinos in terms of the energy scale. To shed more light on the electroweak symmetry breaking mechanism involved in the mirror fermion sector within this commutative model, Le and Hung in Ref. [5] have calculated the one-loop β functions of the Yukawa coupling of the right-handed neutrinos, as well as of the other mirror fermions embroiled in this model. This allowed them to introduce mirror fermion condensate states, at an energy scale close to that of the Landau pole presented by the Yukawa couplings of mirror fermions, which manifests itself on the TeV scale [6]. Furthermore, it is important to note that this model fits well with the observed Large Hadron Collider (LHC) Higgs particle [7] and the observed Large Hadron Collider beauty (LHCb) muon transmutation [8,9]. This makes it a good candidate to be efficient at the energies explored by the LHC.

However, the prospect of a noncommutative version of the Yukawa mirror sector of this model becomes relevant in the context of future colliders [10,11]. These colliders will probe the electroweak symmetry breaking mechanism more deeply, starting to explore the low sector in the intermediate range between the LHC scale and the Planck scale, where new physics correlated with the structure of spacetime should manifest itself. Furthermore, a noncommutative version holds promise in shedding more light on challenging nonperturbative physical phenomena, such as the dynamical electroweak breaking symmetry. This adds to the fact that during the last two decades, looking for the renormalization group equations in the noncommutative spacetime era has gained importance. In particular, some noncommutative β functions have been investigated especially for scalar quantum electrodynamics (QED) in Ref. [12], fermionic QED in Ref. [13], the scalar Gurau model in Ref. [14], the Gross–Neveu model in Ref. [15], and ϕ^4 theory in Ref. [16]. Recently special attention has been paid to the noncommutative extension of the Yukawa interaction [17]. Hence, to contribute to dealing with renormalization group equations in the noncommutative spacetime, we are especially interested in studying, to the leading radiative corrections, the effect of the noncommutative spacetime on the behavior of the $\text{EW}\nu_R^M$ mirror's Yukawa couplings as a function of the renormalization scale.

First, we construct the corresponding Euclidean Yukawa noncommutative Lagrangian, by replacing, in the Euclidean version of the $\text{EW}\nu_R^M$, the usual fields product by the Moyal star product [18,19] that accounts properly for the noncommutative spacetime effect, and by taking into account all involved field permutations.

Moreover, it is well known that loop calculations in a noncommutative spacetime are confronted with the existence of a correlation between the ultraviolet (UV) and infrared (IR) regions of momentum space induced by the nonplanar Feynman diagrams due to the nonlocal character inherent to the star product [20–22]. To tackle this mixing problem without spoiling translation invariance, Gurau et al. [23] and subsequently Bouchachia et al. [24] introduced nonlocal counter terms in the vein of the vulcanization method [25–28], which leads to a vulcanized Euclidean scalar and spinor propagators that prevent the UV/IR mixing in a ϕ^4 theory supplemented by a Yukawa interaction, and consequently restores the renormalizability of the theory.

Therefore, making use of these vulcanized propagators, we are well equipped to undertake the calculation of the noncommutative β functions of the mirror Yukawa couplings to the one-loop order within the proposed mirror Yukawa sector of the Euclidean noncommutative electroweak-scale mirror right-handed neutrinos model (ENC- $\text{EW}\nu_R^M$).

This paper is structured as follows. In the next section, we briefly review the $\text{EW}\nu_R^M$ model in Minkowski spacetime, then switch to Euclidean spacetime and formulate the noncommutative Euclidean Yukawa Lagrangian of the electroweak-scale right-handed neutrinos model. In Section 3, we express the mirror Yukawa Lagrangian in terms of renormalization constants and renormalized couplings and fields, then establish the Slavnov–Taylor identities for mirror Yukawa couplings, and formulate the relation that connects the β function to the counter terms involved in the mirror Yukawa interaction. Section 4 establishes the expression of the necessary one-loop order UV counter terms, by applying the noncommutative vulcanized Feynman rules to the selected processes and calculating the corresponding radiative corrections, the computational details of which are reported in the Appendix. In Section 5, we derive in terms of the mirror Yukawa couplings the corresponding expressions of the β function. This leads us to the establishment of a system of six first-order coupled nonlinear differential equations. Furthermore, the connection with the commutative case is discussed. Section 6 is devoted to the extraction and the discussion of the numerical solutions of the differential equation system. We draw the dependence of the mirror Yukawa couplings on the renormalization scale. We also report a link to the commutative results of Le–Hung [5,6] and discuss the position of the occurrence of the Landau pole. Finally, we draw our concluding remarks.

2. Model formulation

2.1. A brief review of the Minkowski commutative $\text{EW}\nu_R^M$ model

The $\text{EW}\nu_R^M$ model is an extension of the SM that preserves the gauge sector but extends the spinor sector by right-handed mirror leptons l_R^M and right-handed mirror quarks q_R^M doublets, and left-handed mirror leptons e_L^M and left-handed mirror quarks u_L^M and d_L^M singlets. The scalar sector is also extended by a singlet φ_s , a new doublet Φ_{2M} , and two triplets χ and ξ fields as depicted in Table 1.

The corresponding $\mathcal{L}_{\text{EW}\nu_R^M}$ Lagrangian constructed by Hung [4] on the Minkowski spacetime is composed of the SM Lagrangian supplemented by the kinetic term for the mirror fermions $\mathcal{L}_{\text{Fermions}}^M$, the kinetic terms and the potential for the new scalar fields expressed in $\mathcal{L}_{\text{Higgs}}^M$, the interaction between the SM scalar and the new scalar fields in $\mathcal{L}_{\text{Higgs}}^{M-\text{SM}}$, the Yukawa interactions for the mirror fermions $\mathcal{L}_{\text{Yukawa}}^M$, and the combined mirrors and SM fermions Yukawa interactions $\mathcal{L}_{\text{Yukawa}}^{M-\text{SM}}$. Hence we can write:

$$\mathcal{L}_{\text{EW}\nu_R^M} = \mathcal{L}_{\text{Gauge}}^{\text{SM}} + \mathcal{L}_{\text{Fermions}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}, \quad (1)$$

where the kinetic Lagrangian for the fermions is:

$$\mathcal{L}_{\text{Fermions}} = \mathcal{L}_{\text{Fermions}}^{\text{SM}} + \mathcal{L}_{\text{Fermions}}^M, \quad (2)$$

with $\mathcal{L}_{\text{Fermions}}^M$ having, for the mirror fermions, an expression similar to that of the Lagrangian $\mathcal{L}_{\text{Fermions}}^{\text{SM}}$ for the SM fermions with the difference that the SM left-handed doublet fermions are replaced by the right-handed doublet mirror fermions and the SM right-handed singlet fermions are replaced by the left-handed singlet mirror fermions.

Table 1. Elementary fields in the commutative $\text{EW}v_R^M$ model. Each symbol v_e , e , u , or d stands for the three corresponding SM flavors. There is no mixing of mirror leptons and no mixing of mirror quarks.

SM fermion fields	$SU(3)_c \otimes SU(2)_W \otimes U(1)_Y$	Mirror fermion fields	$SU(3)_c \otimes SU(2)_W \otimes U(1)_Y$
$l_L = \begin{pmatrix} v_{e_L} \\ e_L \end{pmatrix}$	$\left(1, 2, -\frac{1}{2}\right)$	$l_R^M = \begin{pmatrix} v_{e_R^M} \\ e_R^M \end{pmatrix}$	$\left(1, 2, -\frac{1}{2}\right)$
$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\left(3, 2, \frac{1}{6}\right)$	$q_R^M = \begin{pmatrix} u_R^M \\ d_R^M \end{pmatrix}$	$\left(3, 2, \frac{1}{6}\right)$
e_R	$(1, 1, -1)$	e_L^M	$(1, 1, -1)$
u_R	$\left(3, 1, \frac{2}{3}\right)$	u_L^M	$\left(3, 1, \frac{2}{3}\right)$
d_R	$\left(3, 1, -\frac{1}{3}\right)$	d_L^M	$\left(3, 1, -\frac{1}{3}\right)$
SM scalar fields	$SU(3)_c \otimes SU(2)_W \otimes U(1)_Y$	Extra-SM scalar fields	$SU(3)_c \otimes SU(2)_W \otimes U(1)_Y$
$\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$	$\left(1, 2, \frac{1}{2}\right)$	φ_s $\Phi_{2M} = \begin{pmatrix} \phi_{2M}^+ \\ \phi_{2M}^0 \end{pmatrix}$	$(1, 1, 0)$ $\left(1, 2, \frac{1}{2}\right)$ $(1, 3, 1)$
		$\tilde{\chi} = \frac{1}{\sqrt{2}} \vec{\tau} \cdot \vec{\chi} = \begin{pmatrix} \frac{1}{\sqrt{2}} \chi^+ & \chi^{++} \\ \chi^0 & -\frac{1}{\sqrt{2}} \chi^+ \end{pmatrix}$ $\xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$	
SM and $\text{EW}v_R^M$ gauge fields	$SU(3)_c \otimes SU(2)_W \otimes U(1)_Y$		$(1, 3, 0)$
W_μ^a	$(1, 3, 0)$		
B_μ	$(1, 1, 0)$		
g_μ^b	$(8, 1, 0)$		

The scalar sector is governed by the Lagrangian [6]:

$$\begin{aligned}
 \mathcal{L}_{\text{Higgs}} = & \frac{1}{2} \text{Tr} \left[(D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) \right] \\
 & + \frac{1}{2} \left\{ |\partial_\mu \varphi_s|^2 + \text{Tr} \left[(D_\mu \Phi_{2M})^\dagger (D^\mu \Phi_{2M}) \right] + \text{Tr} \left[(D_\mu X)^\dagger (D^\mu X) \right] \right\} \\
 & + V(\varphi_s, \Phi_2, \Phi_{2M}, X) = \mathcal{L}_{\text{Higgs}}^{\text{SM}} + \mathcal{L}_{\text{Higgs}}^M + \mathcal{L}_{\text{Higgs}}^{M-\text{SM}}, \tag{3}
 \end{aligned}$$

with $V(\varphi_s, \Phi_2, \Phi_{2M}, X)$ being the potential interaction between the scalar fields [6], and X is the 3×3 matrix representation of the two scalar triplets written as follows [29]:

$$X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^{--} & \xi^- & \chi^0 \end{pmatrix}, \tag{4}$$

with $\chi^{--} = \chi^{++*}$, $\chi^- = -\chi^{+*}$, $\xi^- = -\xi^{+*}$, and the covariant derivatives acting on the scalar sector are given by [6,29,30]:

$$\begin{cases} D_\mu \Phi_2 \equiv \partial_\mu \Phi_2 + i \frac{g}{2} (\vec{W}_\mu \cdot \vec{\tau}) \Phi_2 + i \frac{g'}{2} B_\mu \Phi_2, \\ D_\mu \Phi_{2M} \equiv \partial_\mu \Phi_{2M} + i \frac{g}{2} (\vec{W}_\mu \cdot \vec{\tau}) \Phi_{2M} + i \frac{g'}{2} B_\mu \Phi_{2M}, \\ D_\mu X \equiv \partial_\mu X + ig (\vec{W}_\mu \cdot \vec{T}) X - ig' B_\mu X T_3, \end{cases} \quad (5)$$

where τ_i are the Pauli matrices, and the T_i are the following 3×3 matrix representation of the $SU(2)$ generators:

$$T_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (6)$$

The remaining $\mathcal{L}_{\text{Yukawa}}$ that enables fermions to acquire masses through electroweak symmetry breaking is:

$$\mathcal{L}_{\text{Yukawa}} = \mathcal{L}_{\text{Yukawa}}^{\text{SM}} + \mathcal{L}_{\text{Yukawa}}^M + \mathcal{L}_{\text{Yukawa}}^{M-\text{SM}}, \quad (7)$$

where $\mathcal{L}_{\text{Yukawa}}^{\text{SM}}$ is the SM Yukawa Lagrangian, and

$$\mathcal{L}_{\text{Yukawa}}^M = \mathcal{L}_{e^M} + \mathcal{L}_{q^M} + \mathcal{L}_{\nu_{e_R}} \quad (8)$$

is the pure mirror Yukawa's interaction with [5]:

$$\mathcal{L}_{e^M} = -g_{e^M} \bar{l}_R^M \Phi_{2M} e_L^M - g_{e^M} \bar{e}_L^M \Phi_{2M}^\dagger l_R^M, \quad (9)$$

$$\mathcal{L}_{q^M} = -g_{d^M} \bar{q}_R^M \Phi_{2M} d_L^M - g_{d^M} \bar{d}_L^M \Phi_{2M}^\dagger q_R^M - g_{u^M} \bar{q}_R^M \tilde{\Phi}_{2M} u_L^M - g_{u^M} \bar{u}_L^M \tilde{\Phi}_{2M}^\dagger q_R^M, \quad (10)$$

$$\mathcal{L}_{\nu_{e_R}} = g_{\nu_{e_R}} l_R^{M,T} \sigma_2 \tau_2 \tilde{\chi} l_R^M. \quad (11)$$

In the previous Eqs. (9–11) we set $\tilde{\Phi}_{2M} = i\tau_2 \Phi_{2M}^*$, and we assume universal couplings g_{e^M} , g_{d^M} , and $g_{q^M} = g_{u^M} = g_{d^M}$ whatever is the flavor. Moreover, a global symmetry is introduced to prevent a Majorana mass term for left-handed neutrinos [4]. Besides, we also have the Yukawa interaction that mixes between mirror and SM fermions:

$$\mathcal{L}_{\text{Yukawa}}^{M-\text{SM}} = \mathcal{L}_{S_l} + \mathcal{L}_{S_q}, \quad (12)$$

where:

$$\mathcal{L}_{S_l} = -g_{S_l} \left(\bar{l}_L l_R^M + \bar{l}_R^M l_L \right) \varphi_s, \quad (13)$$

$$\mathcal{L}_{S_q} = -g_{S_q} \left(\bar{q}_R^M q_L + \bar{q}_L q_R^M \right) \varphi_s - g'_{S_q} \left(\bar{q}_L^M q_R + \bar{q}_R q_L^M \right) \varphi_s. \quad (14)$$

Finally, we draw the reader's attention to the fact that the gauge-fixing Lagrangian \mathcal{L}_{GF} and the ghosts Lagrangian \mathcal{L}_{FP} keep the same expressions as those of the SM [31,32] but with the three Goldstone fields expressed as a combination of the components of all the doublets and triplets scalar fields involved in this extension as in Refs. [29,30].

2.2. The mirror Yukawa Euclidean noncommutative $EW\nu_R^M$ Lagrangian

Starting from the Minkowski commutative Yukawa Lagrangian $\mathcal{L}_{\text{Yukawa}}^M$, to construct the mirror Yukawa Euclidean noncommutative $EW\nu_R^M$ Lagrangian $\mathcal{L}_{\text{Yukawa}}^{M,\text{ENC}}$, we follow the following two steps:¹

¹The construction of the whole $EW\nu_R^M$ theory on noncommutative spacetime will require in addition deforming the covariant derivatives [33,34] and is left for future work.

(i) First, we transform the $\mathcal{L}_{\text{Yukawa}}^M$ Lagrangian from the Minkowski to the Euclidean commutative spacetime, by applying the Wick rotation to get $\mathcal{L}_{\text{Yukawa}}^{M,(\text{Eucl.})}$ which simply includes a relative minus sign.

(ii) The second step consists of two actions:

- First, we replace, in the commutative $\mathcal{L}_{\text{Yukawa}}^{M,(\text{Eucl.})}$, the ordinary product between fields by the star Weyl–Moyal product [18,35,36] that enables us to properly incorporate the effect of the noncommutative spacetime, and which is defined as follows ($\hbar = c = 1$):

$$f_1(x) \star f_2(x) = f_1(x) \left[e^{\frac{i}{2}\theta^{\mu\nu} \frac{\overleftarrow{\partial}}{\partial x_\mu} \frac{\overrightarrow{\partial}}{\partial y_\nu}} \right] f_2(y)|_{x=y}, \quad (15)$$

where f_1 and f_2 stand for fields and x and y are the spacetime positions with:

$$x^\mu \star x^\nu - x^\nu \star x^\mu = [x^\mu, x^\nu]_\star = i\theta^{\mu\nu}, \quad (16)$$

and in the Euclidean spacetime the $\theta^{\mu\nu}$ are the components of the following totally antisymmetric tensor:

$$[\theta^{\mu\nu}] = \begin{bmatrix} 0 & \theta & 0 & 0 \\ -\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta \\ 0 & 0 & -\theta & 0 \end{bmatrix}, \quad (17)$$

in which θ is the noncommutative small real deformation parameter and has a dimension inverse to the square of the energy.

- Second, we add all permutations between fields and use the trace property of the star product [24], which ultimately leads to the following substitution when going from the commutative Yukawa interaction between a spinor field $\psi(x)$ and a scalar field $\phi(x)$ to the corresponding noncommutative ones:

$$g^{(c)}\bar{\psi}(x)\psi(x)\phi(x) \rightarrow g^{(nc)}\bar{\psi}(x) \star \psi(x) \star \phi(x) + \tilde{g}^{(nc)}\bar{\psi}(x) \star \phi(x) \star \psi(x), \quad (18)$$

where superscript c stands for commutative and superscript nc for noncommutative. Since in what follows, we are dealing with noncommutative cases, these indices will be dropped, except in a few possibly confusing cases.

We can summarize our procedure as follows:

$$\mathcal{L}_{\text{Yukawa}}^M \xrightarrow{\text{Wick rotation}} \mathcal{L}_{\text{Yukawa}}^{M,(\text{Eucl.})} \xrightarrow{\text{Moyal product}} \mathcal{L}_{\text{Yukawa}}^{M,\text{ENC}}. \quad (19)$$

Applying this procedure, and assuming universal noncommutative mirror Yukawa couplings respectively for all mirror down leptons' (e^M), all mirror quarks' (q^M) and all mirror up leptons' (ν_{e_R}), enables us to write explicitly the Yukawa parts of the Lagrangian corresponding to each mirror fermionic sector as follows:

Mirror down leptons' Yukawa sector: $\mathcal{L}_{e^M} \rightarrow \mathcal{L}_{e^M}^{\text{ENC}}$

$$\begin{aligned} \mathcal{L}_{e^M}^{\text{ENC}} = & g_{e^M} \sum_{e.\text{flavors}} \left[\bar{\nu}_{e_R}^M \star \phi_{2M}^+ \star e_L^M + \bar{e}_R^M \star \phi_{2M}^0 \star e_L^M - \bar{e}_L^M \star \phi_{2M}^- \star \nu_{e_R}^M + \bar{e}_L^M \star \phi_{2M}^{0*} \star e_R^M \right] \\ & + \tilde{g}_{e^M} \sum_{e.\text{flavors}} \left[\bar{\nu}_{e_R}^M \star e_L^M \star \phi_{2M}^+ + \bar{e}_R^M \star e_L^M \star \phi_{2M}^0 - \bar{e}_L^M \star \nu_{e_R}^M \star \phi_{2M}^- + \bar{e}_L^M \star e_R^M \star \phi_{2M}^{0*} \right], \end{aligned} \quad (20)$$

with $\phi_{2M}^- = -\phi_{2M}^{+*}$.

Mirror quarks' Yukawa sector: $\mathcal{L}_{q^M} \rightarrow \mathcal{L}_{q^M}^{\text{ENC}}$

$$\begin{aligned}
 \mathcal{L}_{q^M}^{\text{ENC}} = & g_{q^M} \sum_{q.\text{flavors}} \left[\bar{u}_R^M \star \phi_{2M}^+ \star d_L^M + \bar{d}_R^M \star \phi_{2M}^0 \star d_L^M - \bar{d}_L^M \star \phi_{2M}^- \star u_R^M + \bar{d}_L^M \star \phi_{2M}^{0*} \star d_R^M \right. \\
 & + \bar{u}_R^M \star \phi_{2M}^{0*} \star u_L^M + \bar{d}_R^M \star \phi_{2M}^- \star u_L^M + \bar{u}_L^M \star \phi_{2M}^0 \star u_R^M - \bar{u}_L^M \star \phi_{2M}^+ \star d_R^M \Big] \\
 & + \tilde{g}_{q^M} \sum_{q.\text{flavors}} \left[\bar{u}_R^M \star d_L^M \star \phi_{2M}^+ + \bar{d}_R^M \star d_L^M \star \phi_{2M}^0 - \bar{d}_L^M \star u_R^M \star \phi_{2M}^- + \bar{d}_L^M \star d_R^M \star \phi_{2M}^{0*} \right. \\
 & \left. + \bar{u}_R^M \star u_L^M \star \phi_{2M}^{0*} + \bar{d}_R^M \star u_L^M \star \phi_{2M}^- + \bar{u}_L^M \star u_R^M \star \phi_{2M}^0 - \bar{u}_L^M \star d_R^M \star \phi_{2M}^+ \right]. \tag{21}
 \end{aligned}$$

Majorana right-handed neutrinos' Yukawa sector: $\mathcal{L}_{\nu_{e_R}} \rightarrow \mathcal{L}_{\nu_{e_R}}^{\text{ENC}}$

$$\begin{aligned}
 \mathcal{L}_{\nu_{e_R}}^{\text{ENC}} = & g_M \sum_{e.\text{flavors}} \left(\nu_{e_R}^{M,T} \star \chi^0 \star i\sigma_2 \nu_{e_R}^M - \frac{1}{\sqrt{2}} \nu_{e_R}^{M,T} \star \chi^+ \star i\sigma_2 e_R^M - \frac{1}{\sqrt{2}} e_R^{M,T} \star \chi^+ \star i\sigma_2 \nu_{e_R}^M \right. \\
 & - e_R^{M,T} \star \chi^{++} \star i\sigma_2 e_R^M \Big) + \tilde{g}_M \sum_{e.\text{flavors}} \left(\nu_{e_R}^{M,T} \star i\sigma_2 \nu_{e_R}^M \star \chi^0 - \frac{1}{\sqrt{2}} \nu_{e_R}^{M,T} \star i\sigma_2 e_R^M \star \chi^+ \right. \\
 & \left. - \frac{1}{\sqrt{2}} e_R^{M,T} \star i\sigma_2 \nu_{e_R}^M \star \chi^+ - e_R^{M,T} \star i\sigma_2 e_R^M \star \chi^{++} \right). \tag{22}
 \end{aligned}$$

It is worth noting here that in this transition from commutative to noncommutative formulations, the Euclidean two points scalar and spinor functions have to undergo the vulcanization procedure [23,24] to prevent subsequent UV/IR mixing and gain renormalizability.

3. Renormalization of the mirror Yukawa sector

3.1. Renormalization constants

The bare quantities² ψ_0 , ϕ_0 , $g_{0,X}$, and $\tilde{g}_{0,X}$, and renormalized quantities ψ_r , ϕ_r , $g_{X,r}$, and $\tilde{g}_{X,r}$, are related through the renormalization constants Z_ψ , Z_ϕ , Z_{g_X} , and $Z_{\tilde{g}_X}$ as follows:

$$\psi_0 = \sqrt{Z_\psi} \psi_r, \quad \phi_0 = \sqrt{Z_\phi} \phi_r, \quad g_{0,X} = Z_{g_X} g_{X,r}, \quad \text{and} \quad \tilde{g}_{0,X} = Z_{\tilde{g}_X} \tilde{g}_{X,r}, \tag{23}$$

where, depending on the case, the index X refers to M , e^M , or q^M . The spinor field ψ stands for $\psi \equiv \nu_{e_R}^M, e_L^M, e_R^M, u_L^M, u_R^M, d_L^M, d_R^M$; with e , u , and d each representing the three mirror down lepton flavors, the three mirror up quark flavors, and the three mirror down quark flavors, respectively. Whereas the scalar field ϕ stands for: $\phi \equiv \chi^0, \chi^+, \chi^{++}, \phi_{2M}^0, \phi_{2M}^+, \phi_{2M}^-$.³

For simplicity, we assume that the scalar field's renormalization constants fulfill the following relations:

$$\begin{aligned}
 \sqrt{Z_\chi} & \equiv \sqrt{Z_{\chi^0}} = \sqrt{Z_{\chi^+}} = \sqrt{Z_{\chi^{++}}}, \\
 \sqrt{Z_\phi} & \equiv \sqrt{Z_{\phi_{2M}^0}} = \sqrt{Z_{\phi_{2M}^+}} = \sqrt{Z_{\phi_{2M}^-}}. \tag{24}
 \end{aligned}$$

Furthermore, for the spinor fields' renormalization constants, we assume that $Z_{\nu_{e_R}^M}, Z_{e_L^M}, Z_{e_R^M}, Z_{u_R^M}, Z_{u_L^M}, Z_{d_R^M}$, and $Z_{d_L^M}$ are invariant under flavor changing.

²In the previous section all couplings and fields were bare ones.

³We do not consider here the ξ and φ_s fields since they are not involved in the purely mirror Yukawa sector with which we are dealing.

3.2. Slavnov–Taylor identities

For the mirror down leptons' Yukawa sector, replacing the bare quantities in terms of the corresponding renormalized ones leads to the following form of the Lagrangian $\mathcal{L}_{e^M}^{\text{ENC}}$:

$$\begin{aligned} \mathcal{L}_{e^M}^{\text{ENC}} = & g_{e^M} \left[Z_{v_R e_L \phi} \sum_{e.\text{flavors}} (\bar{v}_{e_R}^M \star \phi_{2M}^+ \star e_L^M - \bar{e}_L^M \star \phi_{2M}^- \star v_{e_R}^M) + Z_{e_R e_L \phi} \sum_{e.\text{flavors}} (\bar{e}_R^M \star \phi_{2M}^0 \star e_L^M \right. \\ & \left. + \bar{e}_L^M \star \phi_{2M}^{0*} \star e_R^M) \right] + \tilde{g}_{e^M} \left[\tilde{Z}_{v_R e_L \phi} \sum_{e.\text{flavors}} (\bar{v}_{e_R}^M \star e_L^M \star \phi_{2M}^+ - \bar{e}_L^M \star v_{e_R}^M \star \phi_{2M}^-) \right. \\ & \left. + \tilde{Z}_{e_R e_L \phi} \sum_{e.\text{flavors}} (\bar{e}_R^M \star e_L^M \star \phi_{2M}^0 + \bar{e}_L^M \star e_R^M \star \phi_{2M}^{0*}) \right], \end{aligned} \quad (25)$$

where here the fields and couplings are the renormalized ones even if in all that follows, we drop the r index so as not to clutter the writing. The vertex renormalization constants $Z_{v_R e_L \phi}$, $Z_{e_R e_L \phi}$, $\tilde{Z}_{v_R e_L \phi}$, and $\tilde{Z}_{e_R e_L \phi}$ are connected to the fields' and couplings' renormalization constants through the following relations:

$$\begin{aligned} Z_{v_R e_L \phi} &= Z_{g_{e^M}} \sqrt{Z_{v_{e_R}^M}} \sqrt{Z_{e_L^M}} \sqrt{Z_\phi}, \quad Z_{e_R e_L \phi} = Z_{e_L e_R \phi} = Z_{g_{e^M}} \sqrt{Z_{e_R^M}} \sqrt{Z_{e_L^M}} \sqrt{Z_\phi}, \\ \tilde{Z}_{v_R e_L \phi} &= \tilde{Z}_{\tilde{g}_{e^M}} \sqrt{Z_{v_{e_R}^M}} \sqrt{Z_{e_L^M}} \sqrt{Z_\phi}, \quad \tilde{Z}_{e_R e_L \phi} = \tilde{Z}_{e_L e_R \phi} = \tilde{Z}_{\tilde{g}_{e^M}} \sqrt{Z_{e_R^M}} \sqrt{Z_{e_L^M}} \sqrt{Z_\phi}. \end{aligned} \quad (26)$$

The universality of the mirror down leptonic Yukawa coupling constants leads to the following Slavnov–Taylor identities:

$$\begin{aligned} Z_{g_{e^M}} &= \frac{Z_{v_R e_L \phi}}{\sqrt{Z_{v_{e_R}^M}} \sqrt{Z_{e_L^M}} \sqrt{Z_\phi}} = \frac{Z_{e_R e_L \phi}}{\sqrt{Z_{e_R^M}} \sqrt{Z_{e_L^M}} \sqrt{Z_\phi}}, \\ \tilde{Z}_{\tilde{g}_{e^M}} &= \frac{\tilde{Z}_{v_R e_L \phi}}{\sqrt{Z_{v_{e_R}^M}} \sqrt{Z_{e_L^M}} \sqrt{Z_\phi}} = \frac{\tilde{Z}_{e_R e_L \phi}}{\sqrt{Z_{e_R^M}} \sqrt{Z_{e_L^M}} \sqrt{Z_\phi}}. \end{aligned} \quad (27)$$

Besides, for the mirror quarks' Yukawa sector, the Lagrangian $\mathcal{L}_{q^M}^{\text{ENC}}$ expressed in terms of the renormalized quantities takes the form:

$$\begin{aligned} \mathcal{L}_{q^M}^{\text{ENC}} = & g_{q^M} \sum_{q.\text{flavors}} \left[Z_{u_R d_L \phi} \left(\bar{u}_R^M \star \phi_{2M}^+ \star d_L^M - \bar{d}_L^M \star \phi_{2M}^- \star u_R^M \right) + Z_{u_L d_R \phi} \left(\bar{d}_R^M \star \phi_{2M}^- \star u_L^M \right. \right. \\ & \left. - \bar{u}_L^M \star \phi_{2M}^+ \star d_R^M \right) + Z_{d_R d_L \phi} \left(\bar{d}_R^M \star \phi_{2M}^0 \star d_L^M + \bar{d}_L^M \star \phi_{2M}^{0*} \star d_R^M \right) \\ & \left. + Z_{u_R u_L \phi} \left(\bar{u}_R^M \star \phi_{2M}^{0*} \star u_L^M + \bar{u}_L^M \star \phi_{2M}^0 \star u_R^M \right) \right] \\ & + \tilde{g}_{q^M} \sum_{q.\text{flavors}} \left[\tilde{Z}_{u_R d_L \phi} \left(\bar{u}_R^M \star d_L^M \star \phi_{2M}^+ - \bar{d}_L^M \star u_R^M \star \phi_{2M}^- \right) + \tilde{Z}_{u_L d_R \phi} \left(\bar{d}_R^M \star u_L^M \star \phi_{2M}^- \right. \right. \\ & \left. - \bar{u}_L^M \star d_R^M \star \phi_{2M}^+ \right) + \tilde{Z}_{d_R d_L \phi} \left(\bar{d}_R^M \star d_L^M \star \phi_{2M}^0 + \bar{d}_L^M \star d_R^M \star \phi_{2M}^{0*} \right) \\ & \left. + \tilde{Z}_{u_R u_L \phi} \left(\bar{u}_R^M \star u_L^M \star \phi_{2M}^{0*} + \bar{u}_L^M \star u_R^M \star \phi_{2M}^0 \right) \right]. \end{aligned} \quad (28)$$

The corresponding vertex renormalization constants $Z_{u_R d_L \phi}$, $Z_{u_L d_R \phi}$, $Z_{d_R d_L \phi}$, $Z_{u_R u_L \phi}$, $\tilde{Z}_{u_R d_L \phi}$, $\tilde{Z}_{u_L d_R \phi}$, and $\tilde{Z}_{u_R u_L \phi}$ are related to the renormalization constants of fields and couplings

as follows:

$$\begin{aligned}
 Z_{u_R d_L \phi} &= Z_{g_{qM}} \sqrt{Z_{u_R^M}} \sqrt{Z_{d_L^M}} \sqrt{Z_\phi}, & Z_{d_R d_L \phi} &= Z_{d_L d_R \phi} = Z_{g_{qM}} \sqrt{Z_{d_R^M}} \sqrt{Z_{d_L^M}} \sqrt{Z_\phi}, \\
 Z_{u_L d_R \phi} &= Z_{g_{qM}} \sqrt{Z_{u_L^M}} \sqrt{Z_{d_R^M}} \sqrt{Z_\phi}, & Z_{u_R u_L \phi} &= Z_{u_L u_R \phi} = Z_{g_{qM}} \sqrt{Z_{u_R^M}} \sqrt{Z_{u_L^M}} \sqrt{Z_\phi}, \\
 \tilde{Z}_{u_R d_L \phi} &= Z_{\tilde{g}_{qM}} \sqrt{Z_{u_R^M}} \sqrt{Z_{d_L^M}} \sqrt{Z_\phi}, & \tilde{Z}_{d_R d_L \phi} &= \tilde{Z}_{d_L d_R \phi} = Z_{\tilde{g}_{qM}} \sqrt{Z_{d_R^M}} \sqrt{Z_{d_L^M}} \sqrt{Z_\phi}, \\
 \tilde{Z}_{u_L d_R \phi} &= Z_{\tilde{g}_{qM}} \sqrt{Z_{u_L^M}} \sqrt{Z_{d_R^M}} \sqrt{Z_\phi}, & \tilde{Z}_{u_R u_L \phi} &= \tilde{Z}_{u_L u_R \phi} = Z_{\tilde{g}_{qM}} \sqrt{Z_{u_R^M}} \sqrt{Z_{u_L^M}} \sqrt{Z_\phi},
 \end{aligned} \tag{29}$$

with the corresponding Slavnov–Taylor identities given by:

$$\begin{aligned}
 Z_{g_{qM}} &= \frac{Z_{u_R d_L \phi}}{\sqrt{Z_{u_R^M}} \sqrt{Z_{d_L^M}} \sqrt{Z_\phi}} = \frac{Z_{u_L d_R \phi}}{\sqrt{Z_{u_L^M}} \sqrt{Z_{d_R^M}} \sqrt{Z_\phi}} = \frac{Z_{d_R d_L \phi}}{\sqrt{Z_{d_R^M}} \sqrt{Z_{d_L^M}} \sqrt{Z_\phi}} = \frac{Z_{u_R u_L \phi}}{\sqrt{Z_{u_R^M}} \sqrt{Z_{u_L^M}} \sqrt{Z_\phi}}, \\
 Z_{\tilde{g}_{qM}} &= \frac{Z_{u_R d_L \phi}}{\sqrt{Z_{u_R^M}} \sqrt{Z_{d_L^M}} \sqrt{Z_\phi}} = \frac{Z_{u_L d_R \phi}}{\sqrt{Z_{u_L^M}} \sqrt{Z_{d_R^M}} \sqrt{Z_\phi}} = \frac{Z_{d_R d_L \phi}}{\sqrt{Z_{d_R^M}} \sqrt{Z_{d_L^M}} \sqrt{Z_\phi}} = \frac{Z_{u_R u_L \phi}}{\sqrt{Z_{u_R^M}} \sqrt{Z_{u_L^M}} \sqrt{Z_\phi}}.
 \end{aligned} \tag{30}$$

Furthermore, for the Majorana right-handed neutrinos' Yukawa sector, in terms of renormalized quantities the Lagrangian $\mathcal{L}_{\nu_{e_R}}^{\text{ENC}}$ takes the form:

$$\begin{aligned}
 \mathcal{L}_{\nu_{e_R}}^{\text{ENC}} &= g_M \left[Z_{\nu_R^T \nu_R \chi} \sum_{e.\text{flavors}} (\nu_{e_R}^{M,T} \star \chi^0 \star i\sigma_2 \nu_{e_R}) - Z_{e_R^T e_R \chi} \sum_{e.\text{flavors}} (e_R^{M,T} \star \chi^{++} \star i\sigma_2 e_R^M) \right. \\
 &\quad - \frac{1}{\sqrt{2}} Z_{\nu_R^T e_R \chi} \sum_{e.\text{flavors}} (\nu_{e_R}^{M,T} \star \chi^+ \star i\sigma_2 e_R^M) - \frac{1}{\sqrt{2}} Z_{\nu_R e_R^T \chi} \sum_{e.\text{flavors}} (e_R^{M,T} \star \chi^+ \star i\sigma_2 \nu_{e_R}^M) \\
 &\quad \left. + \tilde{g}_M \left[\tilde{Z}_{\nu_R^T \nu_R \chi} \sum_{e.\text{flavors}} (\nu_{e_R}^{M,T} \star i\sigma_2 \nu_{e_R}^M \star \chi^0) - \tilde{Z}_{e_R^T e_R \chi} \sum_{e.\text{flavors}} (e_R^{M,T} \star i\sigma_2 e_R^M \star \chi^{++}) \right. \right. \\
 &\quad \left. \left. - \frac{1}{\sqrt{2}} \tilde{Z}_{\nu_R^T e_R \chi} \sum_{e.\text{flavors}} (\nu_{e_R}^{M,T} \star i\sigma_2 e_R^M \star \chi^+) - \frac{1}{\sqrt{2}} \tilde{Z}_{\nu_R e_R^T \chi} \sum_{e.\text{flavors}} (e_R^{M,T} \star i\sigma_2 \nu_{e_R}^M \star \chi^+) \right] \right].
 \end{aligned} \tag{31}$$

In terms of the fields' and couplings' renormalization constants, the vertex renormalization constants $Z_{\nu_R^T \nu_R \chi}$, $Z_{e_R^T e_R \chi}$, $Z_{\nu_R^T e_R \chi}$, $Z_{\nu_R e_R^T \chi}$, $\tilde{Z}_{\nu_R^T \nu_R \chi}$, $\tilde{Z}_{e_R^T e_R \chi}$, $\tilde{Z}_{\nu_R e_R^T \chi}$, and $\tilde{Z}_{\nu_R e_R^T \chi}$ can be written as:

$$\begin{aligned}
 Z_{\nu_R^T \nu_R \chi} &= Z_{g_M} \sqrt{Z_{\nu_{e_R}^{M,T}}} \sqrt{Z_{\nu_{e_R}^M}} \sqrt{Z_\chi}, & \tilde{Z}_{\nu_R^T \nu_R \chi} &= Z_{\tilde{g}_M} \sqrt{Z_{\nu_{e_R}^{M,T}}} \sqrt{Z_{\nu_{e_R}^M}} \sqrt{Z_\chi}, \\
 Z_{e_R^T e_R \chi} &= Z_{g_M} \sqrt{Z_{e_R^{M,T}}} \sqrt{Z_{e_R^M}} \sqrt{Z_\chi}, & \tilde{Z}_{e_R^T e_R \chi} &= Z_{\tilde{g}_M} \sqrt{Z_{e_R^{M,T}}} \sqrt{Z_{e_R^M}} \sqrt{Z_\chi}, \\
 Z_{\nu_R^T e_R \chi} &= Z_{g_M} \sqrt{Z_{\nu_{e_R}^{M,T}}} \sqrt{Z_{e_R^M}} \sqrt{Z_\chi}, & \tilde{Z}_{\nu_R^T e_R \chi} &= Z_{\tilde{g}_M} \sqrt{Z_{\nu_{e_R}^{M,T}}} \sqrt{Z_{e_R^M}} \sqrt{Z_\chi}, \\
 Z_{\nu_R e_R^T \chi} &= Z_{g_M} \sqrt{Z_{\nu_{e_R}^M}} \sqrt{Z_{e_R^{M,T}}} \sqrt{Z_\chi}, & \tilde{Z}_{\nu_R e_R^T \chi} &= Z_{\tilde{g}_M} \sqrt{Z_{\nu_{e_R}^M}} \sqrt{Z_{e_R^{M,T}}} \sqrt{Z_\chi},
 \end{aligned} \tag{32}$$

with the corresponding Slavnov–Taylor identities expressed as:

$$Z_{g_M} = \frac{Z_{v_R^T v_R^\chi}}{\sqrt{Z_{v_{e_R}^M, T}} \sqrt{Z_{v_{e_R}^M}} \sqrt{Z_\chi}} = \frac{Z_{e_R^T e_R^\chi}}{\sqrt{Z_{e_R^M, T}} \sqrt{Z_{e_R^M}} \sqrt{Z_\chi}} = \frac{Z_{v_R^T e_R^\chi}}{\sqrt{Z_{v_{e_R}^M, T}} \sqrt{Z_{e_R^M}} \sqrt{Z_\chi}} = \frac{Z_{v_R e_R^T \chi}}{\sqrt{Z_{v_{e_R}^M}} \sqrt{Z_{e_R^M, T}} \sqrt{Z_\chi}} , \quad (33)$$

$$Z_{\tilde{g}_M} = \frac{\tilde{Z}_{v_R^T v_R^\chi}}{\sqrt{Z_{v_{e_R}^M, T}} \sqrt{Z_{v_{e_R}^M}} \sqrt{Z_\chi}} = \frac{\tilde{Z}_{e_R^T e_R^\chi}}{\sqrt{Z_{e_R^M, T}} \sqrt{Z_{e_R^M}} \sqrt{Z_\chi}} = \frac{\tilde{Z}_{v_R^T e_R^\chi}}{\sqrt{Z_{v_{e_R}^M, T}} \sqrt{Z_{e_R^M}} \sqrt{Z_\chi}} = \frac{\tilde{Z}_{v_R e_R^T \chi}}{\sqrt{Z_{v_{e_R}^M}} \sqrt{Z_{e_R^M, T}} \sqrt{Z_\chi}} .$$

Equipped with the previous Slavnov–Taylor identities in Eqs. (27), (30), and (33), to determine for each mirror Yukawa coupling the corresponding expression of the renormalization constant, it suffices to choose from the right-hand side of these identities one of the given expressions and to use it as a basis for calculation. Thus, under this choice, for each mirror Yukawa coupling, we keep only one term in the underlying renormalized mirror Yukawa Lagrangian, draw the Feynman diagrams of the corresponding process at the loop level, and then compute the necessary radiative corrections that lead to the desired β functions.

3.3. β function

The dependence on the renormalization scale μ of a renormalized dimensionless coupling constant g is given through the β function defined by:

$$\beta_g = \mu \frac{dg}{d\mu} = \frac{dg}{dt} , \quad (34)$$

where t is the dimensionless quantity that we relate to the renormalization scale μ through the relation $t = \ln(\mu/m_t)$, with m_t being the mass of the top quark.

In this work, we are interested in looking for the dependence of the mirror Yukawa couplings on the renormalization scale in a Euclidean noncommutative spacetime of dimension D which we have to bring to four at the end of the calculation. Hence, to derive the corresponding mirror Yukawa β functions from the belonging renormalization constants, we use the following defining relation:

$$\beta_{g^{(k)}} = \lim_{\varepsilon_{\text{UV}} \rightarrow 0} \mu \frac{\partial}{\partial \mu} g^{(k)} = \lim_{\varepsilon_{\text{UV}} \rightarrow 0} \left\{ -\frac{\varepsilon_{\text{UV}}}{2} g^{(k)} - g^{(k)} \frac{\mu}{Z_g^{(k)}} \frac{\partial Z_g^{(k)}}{\partial \mu} \right\} = -g^{(k)} \frac{\partial \ln Z_g^{(k)}}{\partial \ln \mu} , \quad (35)$$

with:

$$Z_g^{(k)} = \frac{Z_{\psi \psi' \phi}^{(k)}}{Z_\psi^{1/2} Z_{\psi'}^{1/2} Z_\phi^{1/2}} , \quad g^{(k)} = g, \tilde{g}, \quad \text{and} \quad \varepsilon_{\text{UV}} = 4 - D , \quad (36)$$

and:

$$Z_{\psi \psi' \phi}^{(k)} = 1 + \frac{\delta_g^{(k)}}{g^{(k)}}, \quad Z_\psi = 1 + \delta_\psi, \quad Z_{\psi'} = 1 + \delta_{\psi'}, \quad Z_\phi = 1 + \delta_\phi, \quad (37)$$

where $\delta_g^{(k)}$ is the vertex counter term, and δ_ψ , $\delta_{\psi'}$, and δ_ϕ are spinor and scalar self-energies counter terms, respectively.

Hence, we can write:

$$\beta_{g^{(k)}} = g^{(k)} \frac{\partial}{\partial \ln \mu} \left[-\frac{\delta_g^{(k)}}{g^{(k)}} + \frac{1}{2} (\delta_\psi + \delta_{\psi'}) + \frac{1}{2} \delta_\phi \right] . \quad (38)$$

So, to get the β function that governs each of the mirror Yukawa couplings, we have first to select, in respect of the Slavnov–Taylor identities, one term from the corresponding mirror Yukawa Lagrangian. Then for a process that embodies this chosen term, we determine through radiative corrections calculation the counter terms $\delta_g^{(k)}$, δ_ψ , $\delta_{\psi'}$, and δ_ϕ , needed for Eq. (38).

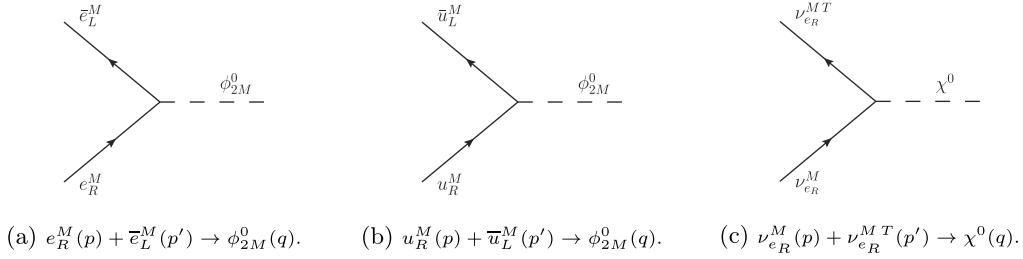


Fig. 1. Tree-level Feynman diagrams for the processes used in the computation of the mirror Yukawa β functions.⁴

For better clarity, and before getting into the details of the loop calculations, it is worthwhile to recall here that at the high energy scale in which we are interested, our computational assumptions are:

- The mirror Yukawa couplings g_M , g_{e^M} , and g_{q^M} are flavor independent.
- As in Ref. [4], relative to the mirror Yukawa couplings g_M , g_{e^M} , and g_{q^M} , we neglect:
 - the gauge couplings g , g' , and g_s ,
 - the g_{S_l} , g_{S_q} , and g'_{S_q} couplings between mirror and SM fermions,
 - the λ_i couplings occurring in the $V(\varphi_s, \Phi_2, \Phi_{2M}, X)$ scalar potential.
- There is no mixing of mirror leptons and no mixing of mirror quarks.

4. One-loop counter terms for the mirror Yukawa sector

To obtain the leading contributions to the β functions for the mirror Yukawa couplings, we consider respectively the one-loop radiative corrections of the process $\nu_{eR}(p) + \nu_{eR}^T(p') \rightarrow \chi^0(q)$ for the g_M , \tilde{g}_M couplings, then of the process $e_R^M(p) + \bar{e}_L^M(p') \rightarrow \phi_{2M}^0(q)$ for the g_{e^M} , \tilde{g}_{e^M} couplings, and then of the process $u_R^M(p) + \bar{u}_L^M(p') \rightarrow \phi_{2M}^0(q)$ for the g_{q^M} and \tilde{g}_{q^M} couplings. The tree-level Feynman diagrams belonging to these processes are depicted in Fig. 1. The corresponding one-loop Feynman diagrams are shown in the Appendix and are consistent with the assumptions highlighted at the end of the previous section.

To achieve the calculation of the radiative corrections, we harness the work of Gurau et al. [23] and Bouchachia et al. [24] and adjust their results to our case of massless scalar and fermionic fields, to comply with the fact that we are dealing with a Lagrangian form formulated at energies before the occurrence of the electroweak symmetry breaking. Hence, in the Euclidean momentum space, for the scalar massless vulcanized propagator $\Delta(p)$, we use:

$$\phi \xrightarrow{p} \phi = \Delta(p) = \frac{1}{p^2 + \frac{a^2}{\theta^2 p^2}}. \quad (39)$$

And, for the spinor massless vulcanized propagator $S(p)$, we use:

$$\bar{\psi} \xrightarrow{p} \psi = S(p) = \frac{i\cancel{p} + ib\frac{\tilde{\cancel{p}}}{\theta^2 p^2}}{p^2 + \frac{b^2}{\theta^2 p^2}}, \quad (40)$$

where θ is the dimensionful deformation parameter, a and b are real dimensionless vulcanized parameters, and $\tilde{\cancel{p}} = \gamma_\mu \tilde{p}^\mu = \gamma_\mu \theta^{\mu\nu} p_\nu$.

⁴Feynman diagrams are drawn using Jaxodraw [37].

For the Yukawa vertex $V_g(p', p)$, we use:

$$V_g(p', p) = - \left[g e^{i/2 p' \tilde{p}} + \tilde{g} e^{-i/2 p' \tilde{p}} \right], \quad (41)$$

where p and p' stand for momenta, $p' \tilde{p} = p'_\mu \theta^{\mu\nu} p_\nu \equiv p' \wedge p$, $q = p - p'$, and g and \tilde{g} are the noncommutative Yukawa couplings related to the same vertex. ψ and ψ' are spinor fields, and ϕ is a scalar field.

To identify our needed counter terms, we use the minimal subtraction defining relations [24] that connect the divergent part of the radiative corrections to their corresponding counter terms.

Hence:

- The relation between δ_ϕ and δ_{a^2} counter terms, and the $\Pi_\phi^{(\text{div})}$ divergent part of the self-energy of the vulcanized massless scalar propagator, observes the formula:⁵

$$\Pi_\phi^{(\text{div})} - \delta_\phi p^2 - \delta_{a^2} \frac{1}{\theta^2 p^2} = \Pi_\phi^{(\text{div})} - (Z_\phi - 1)p^2 - (a_0^2 Z_\phi - a^2) \frac{1}{\theta^2 p^2} = 0. \quad (42)$$

- The relation between δ_ψ and δ_b counter terms, and the $\Sigma_\psi^{(\text{div})}$ divergent part of the self-energy of the vulcanized massless fermionic propagator, fulfills the constraint:

$$\Sigma_\psi^{(\text{div})} + i\delta_\psi \not{p} - i\delta_b \frac{\not{p}}{\theta^2 p^2} = \Sigma_\psi^{(\text{div})} + i(Z_\psi - 1) \not{p} - i(b_0 Z_\psi - b) \frac{\not{p}}{\theta^2 p^2} = 0. \quad (43)$$

- The relation between δ_g and $\delta_{\tilde{g}}$ counter terms, and the $\Gamma_g^{(\text{div})}$ divergent part of the noncommutative Yukawa vertex, is governed by:

$$\Gamma_g^{(\text{div})} - \delta_g e^{i/2 p' \tilde{p}} - \delta_{\tilde{g}} e^{-i/2 p' \tilde{p}} = \Gamma_g^{(\text{div})} + (Z_{\psi\psi'\phi} - 1) e^{i/2 p' \tilde{p}} + (\tilde{Z}_{\psi\psi'\phi} - 1) e^{-i/2 p' \tilde{p}} = 0. \quad (44)$$

So, to get the needed counter terms, we first use the one-loop Feynman diagrams of Figs. A1, A2, and A3 drawn in the Appendix for each of the tree-level processes depicted in Fig. 1. Then, we apply the previous vulcanized Feynman rules and perform the integrals to get the corresponding radiative corrections (see the Appendix for calculation details). Finally, we use Eqs. (42–44) to extract the following UV divergent parts of the counter terms:

(i) From the process $e_R^M(p) + \bar{e}_L^M(p') \rightarrow \phi_{2M}^0(q)$, we obtain for the mirror down leptons' Yukawa sector:

- the vertex $\delta_{g_{e^M}}$ and $\delta_{\tilde{g}_{e^M}}$ counter terms:

$$\frac{\delta_{g_{e^M}}}{g_{e^M}} = \frac{\delta_{\tilde{g}_{e^M}}}{\tilde{g}_{e^M}} = - \frac{2 g_{e^M} \tilde{g}_{e^M}}{(4\pi)^2} \frac{1}{\varepsilon_{\text{UV}}}, \quad (45)$$

- the fermionic self-energy δ_e counter term:

$$\delta_e = \delta_{e_R^M} + \delta_{\bar{e}_L^M} = \frac{3(g_{e^M}^2 + \tilde{g}_{e^M}^2) + \frac{3}{2}(g_M^2 + \tilde{g}_M^2)}{(4\pi)^2} \frac{1}{\varepsilon_{\text{UV}}}, \quad (46)$$

- and the scalar self-energy $\delta_{\phi^0}^{(e)}$ counter term:

$$\delta_{\phi^0}^{(e)} = 3 \times \frac{8(g_{e^M}^2 + \tilde{g}_{e^M}^2) + 16(g_{q^M}^2 + \tilde{g}_{q^M}^2)}{(4\pi)^2} \frac{1}{\varepsilon_{\text{UV}}}. \quad (47)$$

⁵ a_0 and b_0 are bare parameters, while a and b are the corresponding renormalized ones.

(ii) From the process $u_R^M(p) + \bar{u}_L^M(p') \rightarrow \phi_{2M}^0(q)$ and by ignoring color charges, we infer for the mirror quarks' Yukawa sector:

- the vertex $\delta_{g_{qM}}$ and $\delta_{\tilde{g}_{qM}}$ counter terms:

$$\frac{\delta_{g_{qM}}}{g_{qM}} = \frac{\delta_{\tilde{g}_{qM}}}{\tilde{g}_{qM}} = -\frac{2 g_{qM} \tilde{g}_{qM}}{(4\pi)^2} \frac{1}{\varepsilon_{\text{UV}}}, \quad (48)$$

- the fermionic self-energy δ_q counter term:

$$\delta_q = \delta_{u_R^M} + \delta_{\bar{u}_L^M} = \frac{4 \left[g_{qM}^2 + \tilde{g}_{qM}^2 \right]}{(4\pi)^2} \frac{1}{\varepsilon_{\text{UV}}}, \quad (49)$$

- and the scalar self-energy $\delta_{\phi^0}^{(q)}$ counter term:

$$\delta_{\phi^0}^{(q)} = \delta_{\phi^0}^{(e)} = 3 \times \frac{8 (g_{eM}^2 + \tilde{g}_{eM}^2) + 16 (g_{qM}^2 + \tilde{g}_{qM}^2)}{(4\pi)^2} \frac{1}{\varepsilon_{\text{UV}}}. \quad (50)$$

(iii) From the process $\nu_{eR}(p) + \nu_{eR}^T(p') \rightarrow \chi^0(q)$, we get for the Majorana right-handed neutrinos' Yukawa sector:

- the vertex δ_{g_M} and $\delta_{\tilde{g}_M}$ counter terms:

$$\frac{\delta_{g_M}}{g_M} = \frac{\delta_{\tilde{g}_M}}{\tilde{g}_M} = -\frac{2 g_M \tilde{g}_M}{(4\pi)^2} \frac{1}{\varepsilon_{\text{UV}}}, \quad (51)$$

- the fermionic self-energy δ_ν counter term:

$$\delta_\nu = \delta_{\nu_R} + \delta_{\nu_R^T} = \frac{3 (g_M^2 + \tilde{g}_M^2) + g_{eM}^2 + \tilde{g}_{eM}^2}{(4\pi)^2} \frac{1}{\varepsilon_{\text{UV}}}, \quad (52)$$

- and the scalar self-energy $\delta_{\chi^{(0)}}^{(\nu)}$ counter term:

$$\delta_{\chi^{(0)}}^{(\nu)} = 3 \times \frac{4 (g_M^2 + \tilde{g}_M^2)}{(4\pi)^2} \frac{1}{\varepsilon_{\text{UV}}}. \quad (53)$$

It is worth drawing the reader's attention here to the fact that, for all our previously found UV divergent parts of the counter terms, there is no explicit dependence on the noncommutative spacetime deformation parameter θ , in agreement with the results found in Ref. [12] for the noncommutative scalar QED. Also, there is no dependence on the vulcanization parameters a and b . This feature can be traced to the following decomposition of the vulcanized scalar and spinor propagators:⁶

$$\Delta(p) = \frac{1}{p^2 + \frac{a^2}{\theta^2 p^2}} = \frac{1}{p^2} - \frac{1}{p^2} \frac{a^2}{(\theta^2 p^4 + a^2)}, \quad (54)$$

and

$$S(p) = \left(i \not{p} + i b \frac{\not{p}}{\theta^2 p^2} \right) \frac{1}{p^2 + \frac{b^2}{\theta^2 p^2}} = i \frac{\not{p}}{p^2} + \frac{i b \not{p}}{\theta^2 p^4} - \frac{i b^2 \not{p}}{p^2 (\theta^2 p^4 + b^2)} - \frac{i b^3 \not{p}}{\theta^2 p^4 (\theta^2 p^4 + b^2)}. \quad (55)$$

When these propagators occur in the one-loop integrals, only the first term of each of the corresponding decompositions contributes to the logarithmic UV divergent part. This term is free from the deformation parameter θ and the vulcanized parameters a and b . All the remaining terms that depend on θ , a , and b have finite contributions to the loop integrals. Therefore, they

⁶We use here as in Ref. [14] the relationship:

$$\frac{1}{A+B} = \frac{1}{A} - \frac{1}{A} B \frac{1}{A+B}.$$

do not contribute to the UV divergent parts of the counter terms in the minimal subtraction scheme of interest.

Lastly, it is worthwhile to point out the following:

- (i) First, when trying to get the commutative limit starting from the noncommutative expressions by letting $\theta \rightarrow 0$ in a direct, “naive” way, the vulcanized propagators no longer make sense. In fact, to correctly obtain the commutative limit one should proceed as indicated by Magnen et al. [38]. Nevertheless, in this study, we are not concerned with the commutative limit; we instead focus on getting the most suitable link between the commutative and the noncommutative β functions at the one-loop order.
- (ii) Subsequently, by examining the formula (38) from which the β functions are established, we notice that it depends on a sum involving the counter term $\delta_g^{(k)}$ of the Yukawa vertex and the counter terms δ_ψ , $\delta_{\psi'}$, and δ_ϕ of spinor and scalar propagators. As pointed out in Ref. [24], at the one-loop order the link between commutative and noncommutative versions of the counter terms $\delta_g^{(k)}$ of Yukawa vertices expressed in terms of couplings follows a different formula than the link between commutative and noncommutative versions of the counter terms δ_ψ , $\delta_{\psi'}$, and δ_ϕ of the propagators expressed in terms of couplings. In fact, at the one-loop order, starting from the commutative version, the $\beta_{g_X^{(c)}}$ function, expressed in the form of the formula (38), i.e. in terms of the counter terms $\delta_g^{(k)}$, δ_ψ , $\delta_{\psi'}$, and δ_ϕ and the couplings $g^{(k)}$, to recover the corresponding noncommutative version $\beta_{g_X^{(nc)}}$ (or respectively : $\beta_{\tilde{g}_X^{(nc)}}$), we have to perform the following substitutions:
 - For $g^{(k)}$, replace $g_X^{(c)}$ by g_X (resp. \tilde{g}_X).
 - In the $\frac{\delta_g^{(k)}}{g^{(k)}}$ expression, replace $[g_X^{(c)}]^2$ by $g_X \tilde{g}_X$ (resp. $\tilde{g}_X g_X$).
 - In $\delta_\psi + \delta_{\psi'}$ and in δ_ϕ expressions, replace $[g_X^{(c)}]^2$ by $g_X^2 + \tilde{g}_X^2$ (resp. $\tilde{g}_X^2 + g_X^2$).

Therefore, given this substitution relation, there can be no exact proportionality between commutative and noncommutative β functions for Yukawa mirror couplings. Moreover, since spinor and scalar propagators fulfill the same substitution formula, the more loops are involved in the radiative corrections of the propagators, the more the predominance of the propagator counter terms over the vertex counter terms becomes in favor of the propagator counter terms. The latter case, if fulfilled, induces an approximate proportionality rule between the commutative and noncommutative cases which would be governed by the counter terms of the propagators.

5. One-loop noncommutative mirror Yukawa β functions

Since in the β function evaluation, we are concerned with the renormalized dimensionless couplings $g_{X,\text{Ren.}}$, while our counter terms are expressed in terms of couplings $g_{X,\text{ren.}}$ carrying dimension, we first incorporate the connecting relation $g_{X,\text{ren.}} = g_{X,\text{Ren.}} \mu^{\varepsilon_{\text{UV}}/2}$ to obtain the μ -dependence of the counter terms. Then, we introduce the corresponding counter terms in the formula (38) of the β function to get the following system of six autonomous coupled cubic

ordinary differential equations.⁷

$$\left\{
 \begin{aligned}
 \beta_{g_M}^{(nc),\text{dif}} &= \frac{dg_M}{dt} = \frac{1}{32\pi^2} \left[15g_M^3(t) + 15g_M(t)\tilde{g}_M^2(t) + 4g_M^2(t)\tilde{g}_M(t) + g_M(t)g_{e^M}^2(t) \right. \\
 &\quad \left. + g_M(t)\tilde{g}_{e^M}^2(t) \right], \\
 \beta_{\tilde{g}_M}^{(nc),\text{dif}} &= \frac{d\tilde{g}_M}{dt} = \frac{1}{32\pi^2} \left[15\tilde{g}_M^3(t) + 15g_M^2(t)\tilde{g}_M(t) + 4g_M(t)\tilde{g}_M^2(t) + \tilde{g}_M(t)g_{e^M}^2(t) \right. \\
 &\quad \left. + \tilde{g}_M(t)\tilde{g}_{e^M}^2(t) \right], \\
 \beta_{g_{e^M}}^{(nc),\text{dif}} &= \frac{dg_{e^M}}{dt} = \frac{1}{32\pi^2} \left[27g_{e^M}^3(t) + 27g_{e^M}(t)\tilde{g}_{e^M}^2(t) + 4g_{e^M}^2(t)\tilde{g}_{e^M}(t) \right. \\
 &\quad \left. + \frac{3}{2}g_{e^M}(t)g_M^2(t) + \frac{3}{2}g_{e^M}(t)\tilde{g}_M^2(t) + 48g_{q^M}^2(t)g_{e^M}(t) + 48\tilde{g}_{q^M}^2(t)g_{e^M}(t) \right], \\
 \beta_{\tilde{g}_{e^M}}^{(nc),\text{dif}} &= \frac{d\tilde{g}_{e^M}}{dt} = \frac{1}{32\pi^2} \left[27\tilde{g}_{e^M}^3(t) + 27g_{e^M}^2(t)\tilde{g}_{e^M}(t) + 4g_{e^M}(t)\tilde{g}_{e^M}^2(t) \right. \\
 &\quad \left. + \frac{3}{2}\tilde{g}_{e^M}(t)g_M^2(t) + \frac{3}{2}\tilde{g}_{e^M}(t)\tilde{g}_M^2(t) + 48g_{q^M}^2(t)\tilde{g}_{e^M}(t) + 48\tilde{g}_{q^M}^2(t)\tilde{g}_{e^M}(t) \right], \\
 \beta_{g_{q^M}}^{(nc),\text{dif}} &= \frac{dg_{q^M}}{dt} = \frac{1}{8\pi^2} \left[13g_{q^M}^3(t) + 13g_{q^M}(t)\tilde{g}_{q^M}^2(t) + g_{q^M}^2(t)\tilde{g}_{q^M}(t) \right. \\
 &\quad \left. + 6g_{q^M}(t)g_{e^M}^2(t) + 6g_{q^M}(t)\tilde{g}_{e^M}^2(t) \right], \\
 \beta_{\tilde{g}_{q^M}}^{(nc),\text{dif}} &= \frac{d\tilde{g}_{q^M}}{dt} = \frac{1}{8\pi^2} \left[13\tilde{g}_{q^M}^3(t) + 13\tilde{g}_{q^M}(t)g_{q^M}^2(t) + g_{q^M}(t)\tilde{g}_{q^M}^2(t) \right. \\
 &\quad \left. + 6\tilde{g}_{q^M}(t)g_{e^M}^2(t) + 6\tilde{g}_{q^M}(t)\tilde{g}_{e^M}^2(t) \right].
 \end{aligned}
 \right. \tag{56}$$

These equations are symmetric in the swap between g_X and \tilde{g}_X couplings belonging to the same vertex. The index X stands for M , e^M , or q^M . Hence, to get the dependence of the non-commutative mirror Yukawa coupling constants on the renormalization energy scale, we need to solve these coupled differential equations numerically, for given sets of initial values.

Furthermore, it is worth pointing out here that for the special case where $g_X(t) = \tilde{g}_X(t)$, our noncommutative system of the differential equations becomes:

$$\left\{
 \begin{aligned}
 \beta_{g_M}^{(nc),\text{equ}} &= \beta_{\tilde{g}_M}^{(nc),\text{equ}} = \frac{dg_M}{dt} = \frac{1}{16\pi^2} \left[17g_M^3(t) + g_M(t)g_{e^M}^2(t) \right], \\
 \beta_{g_{e^M}}^{(nc),\text{equ}} &= \beta_{\tilde{g}_{e^M}}^{(nc),\text{equ}} = \frac{dg_{e^M}}{dt} = \frac{1}{32\pi^2} \left[58g_{e^M}^3(t) + 3g_{e^M}(t)g_M^2(t) + 96g_{e^M}(t)g_{q^M}^2(t) \right], \\
 \beta_{g_{q^M}}^{(nc),\text{equ}} &= \beta_{\tilde{g}_{q^M}}^{(nc),\text{equ}} = \frac{dg_{q^M}}{dt} = \frac{1}{4\pi^2} \left[13.5g_{q^M}^3(t) + 6g_{q^M}(t)g_{e^M}^2(t) \right].
 \end{aligned}
 \right. \tag{57}$$

⁷In what follows, all couplings are dimensionless although we drop the index Ren .

A similar computation in the commutative case⁸ enables us to get the following β functions for the corresponding commutative mirror Yukawa couplings $g_X^{(c)}$.⁹

$$\left\{ \begin{array}{l} \beta_{g_M^{(c)}} = \frac{dg_M^{(c)}}{dt} = \frac{1}{2} \times \frac{1}{16\pi^2} \left\{ 19 \left[g_M^{(c)}(t) \right]^3 + g_M^{(c)}(t) \left[g_{e^M}^{(c)}(t) \right]^2 \right\}, \\ \beta_{g_{e^M}^{(c)}} = \frac{dg_{e^M}^{(c)}}{dt} = \frac{1}{2} \times \frac{1}{32\pi^2} \left\{ 62 \left[g_{e^M}^{(c)}(t) \right]^3 + 3 g_{e^M}^{(c)}(t) \left[g_M^{(c)}(t) \right]^2 + 96 g_{e^M}^{(c)}(t) \left[g_{q^M}^{(c)}(t) \right]^2 \right\}, \\ \beta_{g_{q^M}^{(c)}} = \frac{dg_{q^M}^{(c)}}{dt} = \frac{1}{2} \times \frac{1}{4\pi^2} \left\{ 14 \left[g_{q^M}^{(c)}(t) \right]^3 + 6 g_{q^M}^{(c)}(t) \left[g_{e^M}^{(c)}(t) \right]^2 \right\}. \end{array} \right. \quad (58)$$

These results lead us to the following remarks:

- (i) There are six independent noncommutative mirror Yukawa couplings that are coordinated through the $\beta_{g_X^{(nc),\text{dif}}}$ functions. This number of independent couplings reduces to three for the special case of $g_X(t) = \tilde{g}_X(t)$ as in the commutative case.
- (ii) The special case where $g_X(t) = \tilde{g}_X(t)$ does not correspond to the commutative case since it always carries information about noncommutative spacetime. This difference seems natural since it can be partly attributed to the fact that even for $g_X(t) = \tilde{g}_X(t)$ the dependence of the noncommutative vertex coupling (41) on the small noncommutative finite parameter θ persists and is given by:

$$g_X e^{\frac{i}{2}p'\tilde{p}} + \tilde{g}_X e^{-\frac{i}{2}p'\tilde{p}} \rightarrow 2g_X \cos\left(\frac{1}{2}p'_\mu \theta^{\mu\nu} p_\nu\right). \quad (59)$$

In addition, in the loop radiative calculations of interest, the involved vulcanized propagators [(39), (40)] contain the noncommutative parameters, which can give rise to slightly modified β functions compared to the commutative case.

- (iii) The commutative system of Eqs. (58) has coefficients that are roughly half of the corresponding ones in the noncommutative system (57), i.e. for $g_X(t) = \tilde{g}_X(t)$, which means that:

$$\beta_{g_X^{(c)}} = \frac{dg_X^{(c)}(t')}{dt'} \simeq \frac{1}{2} \beta_{g_X^{(nc),\text{equ}}} = \frac{1}{2} \frac{dg_X^{(nc),\text{equ}}(t)}{dt}. \quad (60)$$

This property can be easily understood by inspecting the formula (38) of the β function, and simultaneously by taking into account the substitution formulas at the end of the previous section and the one-loop Feynman diagrams of Figs. A1, A2, and A3 presented in the Appendix.

- (a) In fact, out of respect for the loop order substitution relations presented at the end of the previous section, when passing from commutative to noncommutative functions

⁸For the Yukawa interaction of interest, Feynman diagrams for the commutative case are the same as those for the noncommutative one since no new vertex is introduced by the noncommutativity. All that happens in the noncommutative case is that the Yukawa vertex splits into two orthogonal parts, and the propagators get vulcanized to cure the UV/IR mixing.

⁹We fall back into the Le-Hung commutative result [5] if we add the Feynman diagram depicted in the right side of Fig. 1 in Ref. [5], and if in the scalar self-energy we consider only one diagram with a mirror leptonic loop for the case of $\beta_{g_{e^M}}$ and a mirror quark loop for the case of $\beta_{g_{q^M}}$, hence inducing the decoupling of the mirror quarks' Yukawa β function as revealed by Eq. (23) in Ref. [5].

β for $g_x(t) = \tilde{g}_x(t)$, we must perform the following substitutions:

$$\begin{cases} [g_x^{(c)}]^2 \rightarrow [g_x^{(nc),\text{equ}}]^2, & \text{in the vertex counter term,} \\ [g_x^{(c)}]^2 \rightarrow 2 [g_x^{(nc),\text{equ}}]^2, & \text{in the propagator counter terms.} \end{cases} \quad (61)$$

(b) Next, looking at the one-loop Feynman diagrams in Figs. A1, A2, and A3 in the [Appendix](#), we notice that for each process of interest, there is only one vertex Feynman diagram for about ten self-energy Feynman diagrams involved. Thus, in our expressions for the β functions, the propagators that contribute a factor of two outnumber the vertices that contribute only a factor of one. This roughly translates to a factor of two of the noncommutative β functions relative to the corresponding commutative functions, which appears in the relation (60).

(iv) Moreover, by setting $\theta = 0$ in the tree-level noncommutative Yukawa vertex relationship (41), we end up with the following correspondence rules between the commutative and noncommutative mirror Yukawa couplings:¹⁰

$$\begin{cases} g_x^{(c)} = g_{x,c} + \tilde{g}_{x,c} \rightarrow g_x + \tilde{g}_x, & \text{for } g_x(t) \neq \tilde{g}_x(t). \\ g_x^{(c)} = 2g_{x,c} \rightarrow 2g_x, & \text{for } g_x(t) = \tilde{g}_x(t). \end{cases} \quad (62)$$

Hence, to look for the discrepancy between the noncommutative and the commutative mirror Yukawa couplings as a function of the renormalization scale, we will compare the combined noncommutative Yukawa mirror fine structure $\alpha_X^{(nc)} = \frac{[g_x + \tilde{g}_x]^2}{4\pi}$ to the commutative one: $\alpha_X^{(c)} = \frac{[g_x^{(c)}]^2}{4\pi}$.

6. Numerical solutions of the β functions systems

Solving numerically the previously obtained β systems of equations¹¹ enables us to display the dependence of the mirror Yukawa couplings in terms of the t renormalization scale. Figure 2 shows this dependence, in the noncommutative case both for $g_x^{(nc),\text{dif}}$ and for $g_x^{(nc),\text{equ}}$,¹² respectively for Set 1 and Set 2 of the values listed in Table 2, and which are related to the initial mirror Yukawa coupling values through the following relations:

$$\begin{cases} g_M^{(nc),\text{dif}}(0) = x, & g_{e^M}^{(nc),\text{dif}}(0) = y, & g_{q^M}^{(nc),\text{dif}}(0) = z, \\ \tilde{g}_M^{(nc),\text{dif}}(0) = \tilde{x}, & \tilde{g}_{e^M}^{(nc),\text{dif}}(0) = \tilde{y}, & \tilde{g}_{q^M}^{(nc),\text{dif}}(0) = \tilde{z}, \end{cases} \quad (63)$$

$$\begin{cases} g_M^{(nc),\text{equ}}(0) = \tilde{g}_M^{(nc),\text{equ}}(0) = (x + \tilde{x})/2, & g_{e^M}^{(nc),\text{equ}}(0) = \tilde{g}_{e^M}^{(nc),\text{equ}}(0) = (y + \tilde{y})/2, \\ g_{q^M}^{(nc),\text{equ}}(0) = \tilde{g}_{q^M}^{(nc),\text{equ}}(0) = (z + \tilde{z})/2, \end{cases} \quad (64)$$

¹⁰Here, we must pay attention to the fact that $g_x^{(c)} = g_{x,c} + \tilde{g}_{x,c}$, whereas $g_x^{(c)} \neq g_x + \tilde{g}_x$ since the former are commutative quantities whereas the latter are noncommutative ones. Hence, we will just compare the commutative couplings $g_x^{(c)}(t)$ evolution in terms of t , with the corresponding noncommutative $g_x + \tilde{g}_x$ ones (i.e. $g_x^{(c)} \rightarrow g_x + \tilde{g}_x$), in order to identify discrepancies between them. Also, one should not confuse the factors of 2 that appear in Eqs. (61) and (62). The first originates from a loop-level computation whereas the second comes from a tree-level formula.

¹¹The numerical calculations and the drawing of the corresponding plots are performed through Mathematica [39].

¹²Where $g_x^{(nc),\text{dif}}$ stands for g_x or \tilde{g}_x with $g_x \neq \tilde{g}_x$, and $g_x^{(nc),\text{equ}}$ also for g_x or \tilde{g}_x but with $g_x = \tilde{g}_x$.

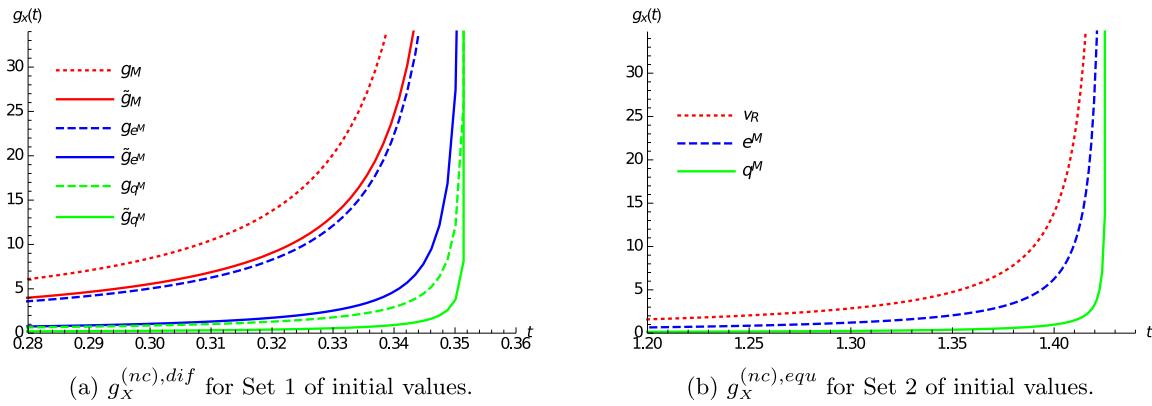


Fig. 2. Evolution of the mirror Yukawa couplings as a function of the renormalization scale t .

Table 2. Used constant sets to construct the corresponding initial mirror Yukawa coupling values.¹³

	x	\tilde{x}	y	\tilde{y}	z	\tilde{z}
Set 1	3.95	3.2	2.95	1.35	1.41	0.82
Set 2	1.784	1.784	1.1	1.1	0.617	0.617
Set 3	3.9	1.6	1.26	2.2	1.25	0.8
Set 4	2.3	0.2	2.0	0.46	1.7	1.03
Set 5	2.32	0.5	1.1	0.48	1.9	1.06
Set 6	0.92	0.25	1.02	0.35	1.12	0.45

$$g_M^{(c)}(0) = x + \tilde{x}, \quad g_{e^M}^{(c)}(0) = y + \tilde{y}, \quad g_{q^M}^{(c)}(0) = z + \tilde{z}. \quad (65)$$

Moreover, to smooth the comparison between the commutative and the noncommutative behaviors, it is more convenient to deal with the mirror fine structure constants $\alpha_X^{(nc)}(t)$ and $\alpha_X^{(c)}(t)$, for which the previous initial mirror Yukawa coupling values' relationship to the constant sets of Table 2 has been set to ensure that:

$$\alpha_X^{(nc),dif}(0) = \alpha_X^{(nc),equ}(0) = \alpha_X^{(c)}(0), \quad (66)$$

as shown in Fig. 3(b–d) for the plots' starting point at $t = 0$.

Our results presented in Figs. 2 and 3, and Table 3 highlight the following remarks:

- (i) The noncommutative mirror Yukawa couplings preserve the behavior of the commutative ones pointed out in Ref. [5], i.e. a plateau for low t values, then a Landau pole at some higher t_p scale, as shown in Fig. 2 for the mirror Yukawa couplings and in Fig. 3 in terms of the corresponding mirror Yukawa fine structure constants.
- (ii) The position of the Landau poles in the noncommutative and commutative calculation approaches depends on the chosen set of initial values as shown in Table 3. In particular, columns 7, 11, and 12 of this table show that we can set the initial values of the mirror Yukawa coupling so that the noncommutative Landau poles (namely $t_p^{(nc),dif} = 1.50$ and $t_p^{(nc),equ} = 1.78$ for Set 6 of the initial values) are the same as the commutative one (i.e.

¹³In choosing these sets of constants, we ensured that the initial values of all commutative Yukawa couplings are greater than the Yukawa top quark coupling which approaches unity. Moreover, Set 4 and Set 5 correspond, in the commutative case, to the initial values taken by Le–Hung in Refs. [5] and [6], respectively.

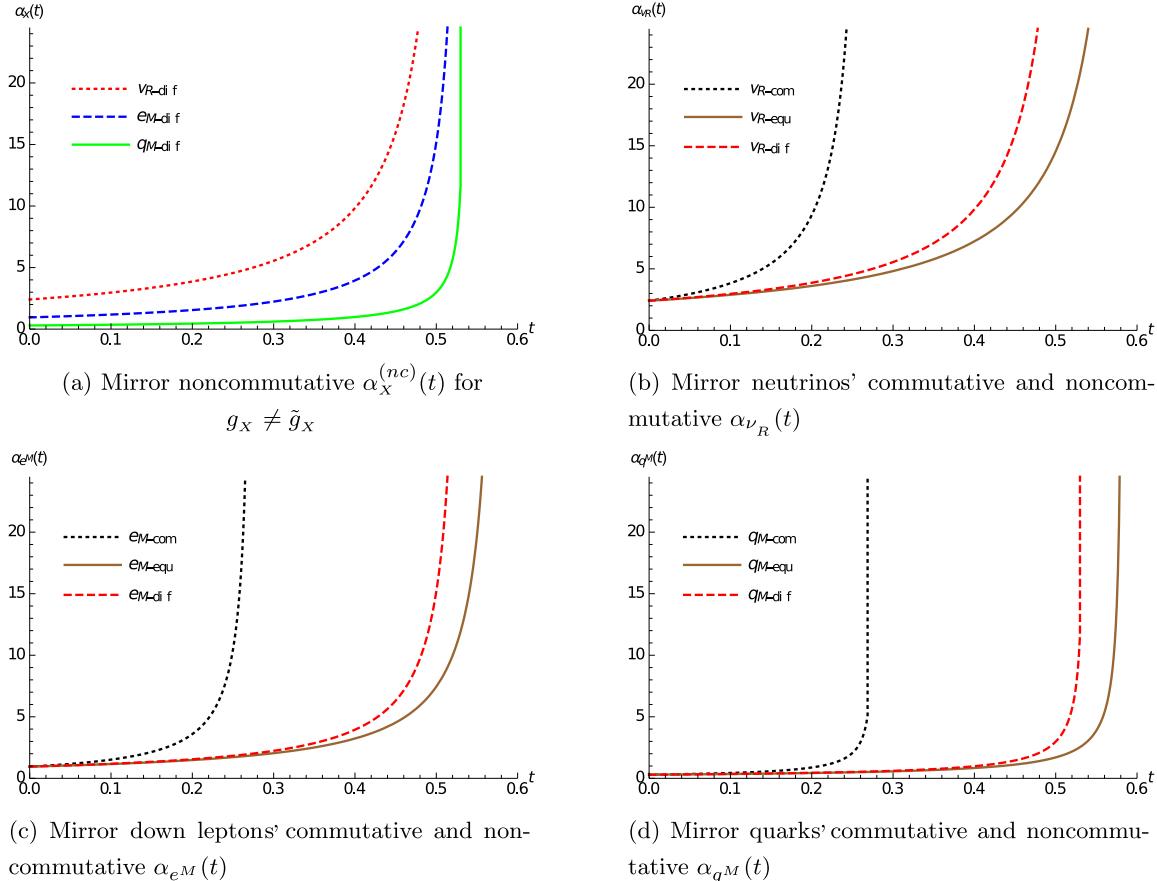


Fig. 3. Evolution of the mirror Yukawa fine structure constants in terms of the renormalization scale t for Set 3 of initial values.

Table 3. Landau pole positions for different sets of initial values of mirror Yukawa couplings.

Initial values	All involved loops and flavor families are considered in the scalar propagator						Only one special loop is considered in the scalar propagator as in Ref. [5]					
	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6
$t_p^{(nc),\text{dif}}$	0.35	1.43	0.53	0.50	0.55	1.50	0.66	2.68	1.03	3.68	3.44	11.55
$t_p^{(nc),\text{equ}}$	0.36	1.43	0.58	0.58	0.60	1.78	0.67	2.68	1.13	4.24	3.60	12.81
$t_p^{(c)}$	0.16	0.64	0.27	0.28	0.29	0.87	0.27	1.10	0.46	1.77	1.50	5.34

$t_p^{(c)} = 1.50$ for Set 5 of the initial values and $t_p^{(c)} = 1.77$ for Set 4 of the initial values). The above Landau pole values are of special interest because they would correspond to an energy scale in the TeV domain, and besides they refer to the values pointed out in Refs. [5] and [6].

- (iii) Comparing the right side part to the left side part of Table 3 shows that for the same set of initial values, the fewer loops taken into account in the scalar propagator, the higher the values of the Landau poles.
- (iv) The swap symmetry between g_X and \tilde{g}_X couplings in the differential equations manifests itself in the corresponding solutions through the three following ways:
 - (a) In Fig. 2(a) belonging to Set 1 of the initial values, the plots for g_X and \tilde{g}_X are interchanged when we swap the corresponding initial values.

(b) When we adopt Set 2 of the initial values in Eqs. (56) and (57), their corresponding solutions become superimposed and give the plots drawn in Fig. 2(b).

(c) For Set 2 of the initial values, the Landau poles $t_p^{(nc),\text{dif}}$ and $t_p^{(nc),\text{equ}}$ arising from Eqs. (56) and (57), respectively, become identical as indicated by the corresponding columns in Table 3.

(v) Table 3 and Fig. 3(b–d) show that the noncommutative Landau poles' positions are roughly twice the corresponding commutative ones for the same initial values. This can be seen as follows.

For simplicity's sake, we consider the right-hand side of the noncommutative equation system (57) and take into account the relationship (62) that connects the commutative and the noncommutative couplings, i.e. by plugging in $\frac{g_x^{(c)}(t')}{2}$ for each $g_x(t)$. We end up with an additional $\frac{1}{8}$ global factor on the right-hand side due to cubic terms, and a factor $\frac{1}{2}$ on the left-hand side. Which, for the first equation in Eq. (57), leads to:

$$\frac{dg_M^{(nc),\text{equ}}(t)}{dt} \rightarrow \frac{1}{2} \frac{dg_M^{(c)}(t')}{dt} = \frac{1}{16\pi^2} \left[\frac{17}{8} \left\{ g_M^{(c)} \right\}^3(t') + \frac{1}{8} g_M^{(c)}(t') \left\{ g_{e^M}^{(c)} \right\}^2(t') \right]. \quad (67)$$

Or equivalently:

$$\frac{dg_M^{(c)}(t')}{d(t/2)} = \frac{1}{16\pi^2} \left[\frac{17}{2} \left\{ g_M^{(c)} \right\}^3(t') + \frac{1}{2} g_M^{(c)}(t') \left\{ g_{e^M}^{(c)} \right\}^2(t') \right]. \quad (68)$$

Which must be brought closer to the right-hand side of the first equation in Eqs. (58), i.e.

$$\frac{dg_M^{(c)}(t')}{dt'} = \frac{1}{16\pi^2} \left\{ \frac{19}{2} \left[g_M^{(c)}(t') \right]^3 + \frac{1}{2} g_M^{(c)}(t') \left[g_{e^M}^{(c)}(t') \right]^2 \right\}. \quad (69)$$

Hence, if in Eq. (68) we set:

$$t = 2t', \quad (70)$$

we end up with an equation that has a similar form to Eq. (69) with the coefficient 19 replaced by the coefficient 17, and with t linked to the noncommutative case and t' to the commutative case. Thus, by modeling a simple commutative Landau pole as:

$$\lim_{t' \rightarrow t_p^{(c)}} \frac{A}{B - Ct'} \rightarrow +\infty, \quad \text{with } A > 0, B > 0, \text{ and } C > 0, \quad (71)$$

we obtain:

$$t_p^{(c)} = \frac{B}{C}. \quad (72)$$

Then, by relaxing the constraint (70) so that the equality becomes approximate rather than exact, the corresponding noncommutative Landau pole manifests at:

$$t_p^{(nc)} \approx 2t_p^{(c)}, \quad (73)$$

which is in complete agreement with the results cited above.

(vi) Table 3 and Fig. 3(b–d) show that for each of the sets 1, 3, 4, 5, and 6 of the initial values that fulfill for $t = 0$ the relation $g_x^{(nc),\text{dif}}(0) \neq \tilde{g}_x^{(nc),\text{dif}}(0)$, the noncommutative Landau

poles $t_p^{(nc),\text{dif}}$ and $t_p^{(nc),\text{equ}}$ belonging to Eqs. (56) and (57), respectively, have distinct values as expected, but with $t_p^{(nc),\text{equ}}$ always greater than $t_p^{(nc),\text{dif}}$, while for Set 2 of the initial values for which at $t = 0$ we have $g_x^{(nc),\text{dif}}(0) = \tilde{g}_x^{(nc),\text{dif}}(0)$, the pole positions $t_p^{(nc),\text{dif}}$ and $t_p^{(nc),\text{equ}}$ are the same.

To understand this property, we focus on the Landau pole region for which t is in the interval $]t_0, t_p[$ with t_0 having a value lower than but close to t_p . In this region, all the Yukawa coupling functions $g_x(t)$ and $\tilde{g}_x(t)$ carry large values. Let us therefore consider two functions $\delta(t)$ and $\eta(t)$ that carry large values and have behavior that mimics the mirror Yukawa couplings in this region. Furthermore, keeping in mind the above swap symmetry, we can write:

$$\left. \begin{array}{l} g_x^{(nc),\text{dif}}(t) = \delta(t) + \epsilon, \\ \tilde{g}_x^{(nc),\text{dif}}(t) = \delta(t) - \epsilon, \\ g_x^{(nc),\text{equ}}(t) = \tilde{g}_x^{(nc),\text{equ}}(t) = \eta(t), \end{array} \right\} \text{for } t \in]t_0, t_p[, \quad (74)$$

regardless of X being M , e_M , or q_M , and with ϵ being an infinitesimal positive parameter. In addition, using the first equation in the equations system (56) and the first equation in the equations system (57), we obtain the following:

$$\left. \begin{array}{l} (56) \rightarrow \frac{d\delta(t)}{dt} = \frac{1}{8\pi^2} [4[\delta(t) + \epsilon]^3 + 4[\delta(t) + \epsilon][\delta(t) - \epsilon]^2 + [\delta(t) + \epsilon]^2[\delta(t) - \epsilon]], \\ (57) \rightarrow \frac{d\eta(t)}{dt} = \frac{9}{8\pi^2} \eta^3(t). \end{array} \right. \quad (75)$$

So, to first order in the infinitesimal parameter ϵ , the above equations take the form:

$$\left. \begin{array}{l} \frac{d\delta(t)}{dt} = \left(1 + \frac{\epsilon}{\delta}\right) \frac{9}{8\pi^2} \delta^3(t) = (1 + \tilde{\epsilon}) \frac{9}{8\pi^2} \delta^3(t), \\ \frac{d\eta(t)}{dt} = \frac{9}{8\pi^2} \eta^3(t), \end{array} \right. \quad (76)$$

with $\tilde{\epsilon} = \epsilon/\delta$ being a new infinitesimal positive parameter.

Thus, for a given t in $]t_0, t_p[$, the slope $\frac{d\delta(t)}{dt}$ of the function $\delta(t)$ is slightly greater than the slope $\frac{d\eta(t)}{dt}$ of the function $\eta(t)$. Therefore, assuming $\delta(t_0) = \eta(t_0)$ we get that $\delta(t)$ will reach its asymptote, i.e. $t_p^{(nc),\text{dif}}$, sooner than $\eta(t)$ does, i.e. $t_p^{(nc),\text{equ}}$.

We should then have:

$$t_p^{(nc),\text{equ}} \geq t_p^{(nc),\text{dif}}, \quad (77)$$

which is what we obtain numerically.

7. Concluding remarks

In this work, we have first used the Moyal product to construct a noncommutative Euclidean mirror Yukawa Lagrangian within the electroweak-scale right-handed neutrinos model.

Second, we established the Slavnov–Taylor identities for the noncommutative universal mirror Yukawa couplings. We utilized them to select from several possibilities the processes for which we had to calculate the one-loop order radiative corrections involved in determining the

counter terms for the noncommutative mirror Yukawa β functions. Through this calculation, the noncommutative vulcanized scalar and spinor propagators have provided valuable support to avoid the mixing between the UV and IR momenta.

The obtained counter term UV divergent parts result only from planar integrals and thus have the same divergent structure as the commutative case but with different factors. Moreover, they do not depend explicitly on the deformation parameter θ , nor on the vulcanized parameters a and b . These parameters only affect the finite part for which both planar and nonplanar integrals contribute. This means that the commutative limit cannot be obtained simply by making the noncommutative parameters tend to zero, in which case this leads us to expressions which no longer make sense.

However, analyzing the expressions of these counter terms in parallel with their implication in the expression of the β function allows us to state a simple relationship that enables a direct link between the commutative and the noncommutative one-loop β function expressions in terms of occurring couplings. These correspondence rules act differently for propagators and vertices and thus reflect the fact that there can be no strict proportionality between the considered commutative and noncommutative β functions, but rather a possible approximate proportionality depending on the relative weights between the involved counter terms.

Our procedure results in a system of six coupled nonlinear first-order differential equations, which we solved numerically. The obtained numerical solutions allowed us to state that, at the one-loop level within the EW_R^M and in terms of the renormalization scale, the noncommutative geometry preserves the behavior of a plateau followed by a Landau pole for the mirror Yukawa couplings as is the case in the commutative formulation, with the notable difference that the position of the noncommutative Landau pole is roughly twice that of the commutative one. This would indicate that in noncommutative spacetime, new phenomena can be triggered by a phase transition at higher energy scales than would be expected by the commutative case.

Moreover, the exact behavior of the obtained solutions and the position of the corresponding Landau pole depend on the choice of the initial values that may be related to low energy masses of the mirror fermions. Therefore, it seems interesting to probe the inverse problem in a multidimensional coupling space to shed more light on the flow between high and low energy scales. This is left for future work.

Finally, exploring the effects of the λ_i couplings occurring in the scalar potential $V(\varphi_s, \Phi_2, \Phi_{2M}, X)$, on the running of the noncommutative mirror Yukawa couplings g_X and \tilde{g}_X , is a natural extension of this work since the Yukawa interaction is involved in the one-loop Feynman diagrams corresponding to this potential. This is currently under investigation and will be discussed separately in the future.

Acknowledgements

K.A.B. would like to acknowledge fruitful discussions with V. Rivassau and with M. Schweda at the beginning of this work. N.B. would like to thank M. Mamou for helpful discussions about inverse problems and numerical solutions of differential equations.

The authors would like to thank P. Aurenche, B. Si Lakh, and A. Yanallah for reading the manuscript and giving fruitful comments and suggestions, and A.M. Bouayed for proofreading the manuscript and revealing some grammatical errors and misspellings.

Funding

Open Access funding: SCOAP³.

Appendix. Radiative corrections leading to the counter terms (45–53)

Applying noncommutative vulcanized Feynman rules to the Feynman diagrams depicted in Figs. A1, A2, and A3 belonging to the processes $e_R^M(p) + \bar{e}_L^M(p') \rightarrow \phi_{2M}^0(q)$, $u_R^M(p) + \bar{u}_L^M(p') \rightarrow \phi_{2M}^0(q)$, and $\nu_{e_R}(p) + \bar{\nu}_{e_R}^T(p') \rightarrow \chi^0(q)$, respectively, leads to the following radiative corrections.

- The Γ_e , Γ_q , and Γ_ν vertex one-loop corrections:

$$\Gamma_e(p, p', \theta, a, b, g_{e^M}, \tilde{g}_{e^M}) = I(p, p', \theta, a, b, g_{e^M}, \tilde{g}_{e^M}), \quad (\text{A.1})$$

$$\Gamma_q(p, p', \theta, a, b, g_{q^M}, \tilde{g}_{q^M}) = I(p, p', \theta, a, b, g_{q^M}, \tilde{g}_{q^M}), \quad (\text{A.2})$$

$$\Gamma_\nu(p, p', \theta, a, b, g_{\nu}, \tilde{g}_{\nu}) = I(p, p', \theta, a, b, g_{\nu}, \tilde{g}_{\nu}), \quad (\text{A.3})$$

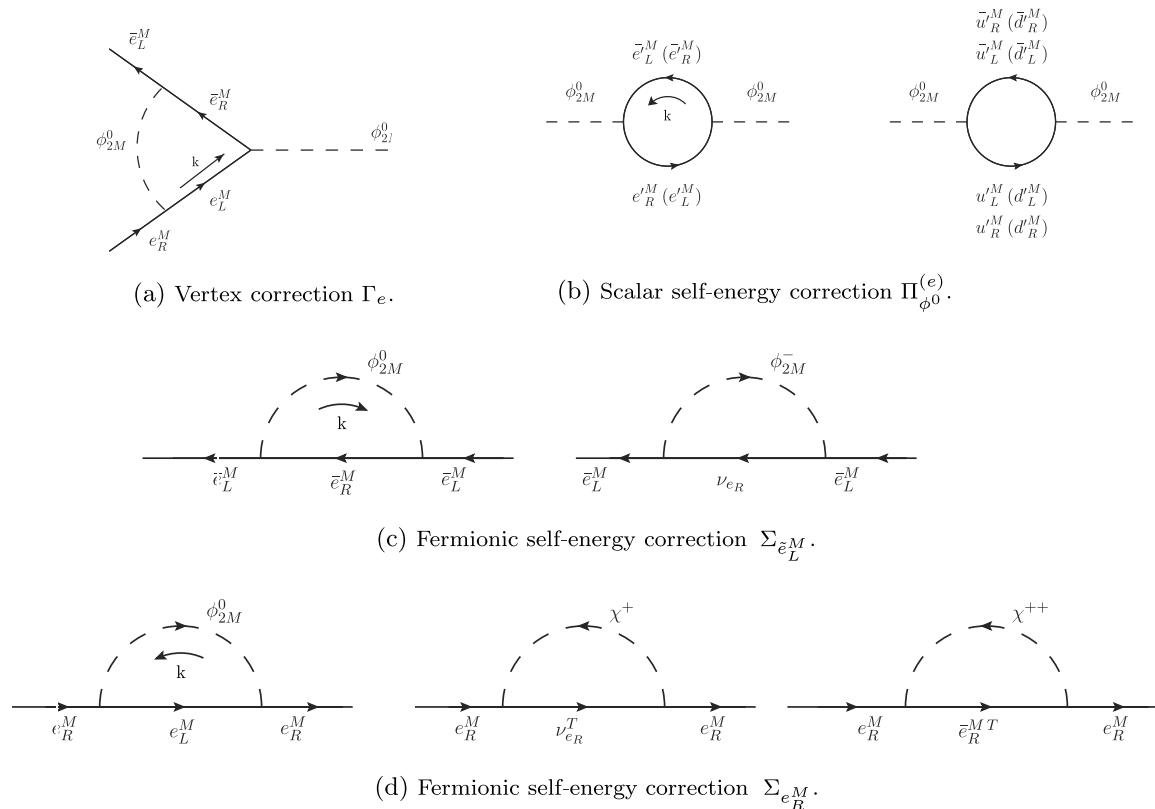


Fig. A1. One-loop radiative corrections to the process $e_R^M(p) + \bar{e}_L^M(p') \rightarrow \phi_{2M}^0(q)$, where e stands for a specific mirror lepton flavor, and u' , d' , and e' each stand for the three mirror up quark, three mirror down quark, and three mirror lepton flavors, respectively.

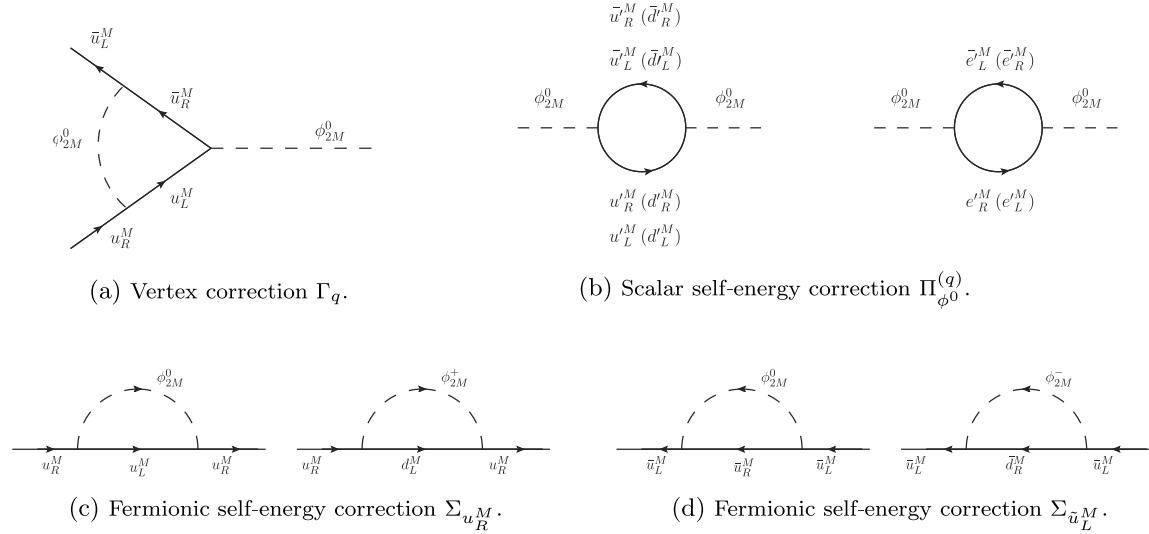


Fig. A2. One-loop radiative corrections to the process $u_R^M(p) + \bar{u}_L^M(p') \rightarrow \phi_{2M}^0(q)$, where u stands for a specific mirror up quark flavor, and u' , d' , and e' each stand for the three mirror up quark, three mirror down quark, and three mirror lepton flavors, respectively.

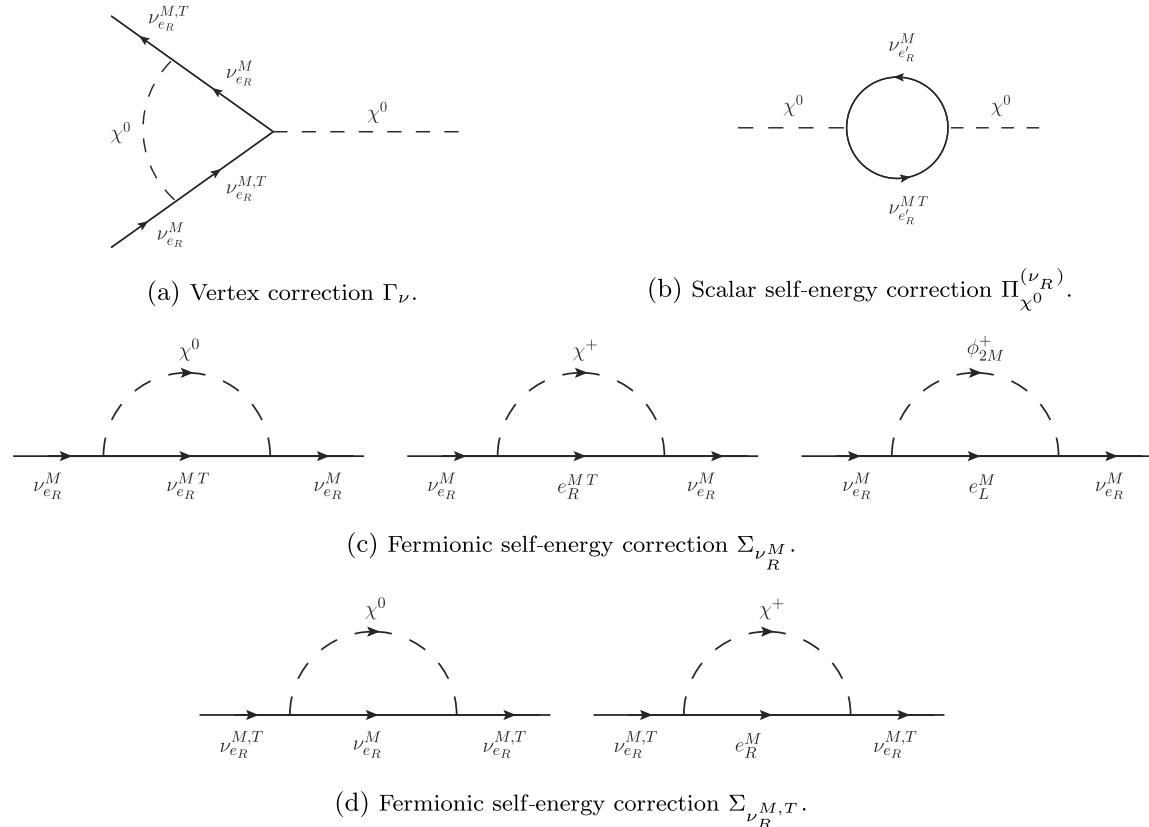


Fig. A3. One-loop radiative corrections to the process $\nu_{e_R}(p) + \nu_{e_R}^T(p') \rightarrow \chi^0(q)$, where e stands for a specific mirror lepton flavor and e' stands for the three mirror lepton flavors.

where:

$$\begin{aligned}
I(p, p', \theta, a, b, g_x, \tilde{g}_x) = & \int \frac{d^D k}{(2\pi)^D} (-1) \left(g_x e^{-\frac{i}{2}(p-p')\tilde{k}} + \tilde{g}_x e^{+\frac{i}{2}(p-p')\tilde{k}} \right) \left(g_x e^{\frac{i}{2}p'(\tilde{k}-\tilde{p})} \right. \\
& \left. + \tilde{g}_x e^{-\frac{i}{2}p'(\tilde{k}-\tilde{p})} \right) \times \left(g_x e^{\frac{i}{2}k\tilde{p}} + \tilde{g}_x e^{-\frac{i}{2}k\tilde{p}} \right) \times \frac{i k + ib \frac{\tilde{k}}{\theta^2 k^2}}{k^2 + \frac{b^2}{\theta^2 k^2}} \\
& \times \frac{i(k - \not{p} + \not{p}') + ib \frac{\tilde{k} - \not{p} + \not{p}'}{\theta^2(k-p+p')^2}}{(k-p+p')^2 + \frac{b^2}{\theta^2(k-p+p')^2}} \times \frac{1}{(k-p)^2 + \frac{a^2}{\theta^2(k-p)^2}}. \quad (\text{A.4})
\end{aligned}$$

- The Σ_e mirror down leptonic self-energy one-loop correction:

$$\begin{aligned}
\Sigma_e(p, p', \theta, a, b, g_{e^M}, \tilde{g}_{e^M}, g_M, \tilde{g}_M) = & \Sigma_{e_R^M}(p, \theta, a, b, g_{e^M}, \tilde{g}_{e^M}, g_M, \tilde{g}_M) \\
& + \Sigma_{\tilde{e}_L^M}(p', \theta, a, b, g_{e^M}, \tilde{g}_{e^M}), \quad (\text{A.5})
\end{aligned}$$

with:

$$\Sigma_{e_R^M}(p, \theta, a, b, g_{e^M}, \tilde{g}_{e^M}, g_M, \tilde{g}_M) = J(p, \theta, a, b, g_{e^M}, \tilde{g}_{e^M}) + \frac{3}{2} J(p, \theta, a, b, g_M, \tilde{g}_M), \quad (\text{A.6})$$

and

$$\Sigma_{\tilde{e}_L^M}(p', \theta, a, b, g_{e^M}, \tilde{g}_{e^M}) = 2 J(p', \theta, a, b, g_{e^M}, \tilde{g}_{e^M}), \quad (\text{A.7})$$

where:

$$\begin{aligned}
J(l, \theta, a, b, g_x, \tilde{g}_x) = & \int \frac{d^D k}{(2\pi)^D} \left[g_x e^{-\frac{i}{2}l\tilde{k}} + \tilde{g}_x e^{+\frac{i}{2}l\tilde{k}} \right] \left[g_x e^{-\frac{i}{2}k\tilde{l}} + \tilde{g}_x e^{+\frac{i}{2}k\tilde{l}} \right] \\
& \times \left(\frac{i(l+k) + ib \frac{\tilde{k} + \tilde{l}}{\theta^2(l+k)^2}}{(l+k)^2 + \frac{b^2}{\theta^2(l+k)^2}} \right) \frac{1}{k^2 + \frac{a^2}{\theta^2 k^2}}. \quad (\text{A.8})
\end{aligned}$$

- The Σ_q mirror quark self-energy one-loop correction:

$$\Sigma_q(p, p', \theta, a, b, g_{q^M}, \tilde{g}_{q^M}) = \Sigma_{q_R^M}(p, \theta, a, b, g_{q^M}, \tilde{g}_{q^M}) + \Sigma_{\tilde{e}_L^M}(p', \theta, a, b, g_{q^M}, \tilde{g}_{q^M}), \quad (\text{A.9})$$

with

$$\Sigma_{q_R^M}(p, \theta, a, b, g_{q^M}, \tilde{g}_{q^M}) = 2 J(p, \theta, a, b, g_{q^M}, \tilde{g}_{q^M}), \quad (\text{A.10})$$

and

$$\Sigma_{\tilde{e}_L^M}(p', \theta, a, b, g_{q^M}, \tilde{g}_{q^M}) = 2 J(p', \theta, a, b, g_{q^M}, \tilde{g}_{q^M}). \quad (\text{A.11})$$

- The Σ_ν right-handed neutrinos' self-energy one-loop corrections:

$$\begin{aligned}
\Sigma_\nu(p, p', \theta, a, b, g_M, \tilde{g}_M, g_{e^M}, \tilde{g}_{e^M}) = & \Sigma_{\nu_R^M}(p, \theta, a, b, g_M, \tilde{g}_M, g_{e^M}, \tilde{g}_{e^M}) \\
& + \Sigma_{\nu_R^{M,T}}(p', \theta, a, b, g_M, \tilde{g}_M), \quad (\text{A.12})
\end{aligned}$$

with:

$$\Sigma_{\nu_R^M}(p, \theta, a, b, g_M, \tilde{g}_M, g_{e^M}, \tilde{g}_{e^M}) = \frac{3}{2} J(p, \theta, a, b, g_M, \tilde{g}_M) + J(p, \theta, a, b, g_{e^M}, \tilde{g}_{e^M}), \quad (\text{A.13})$$

and

$$\Sigma_{v_R^{M,T}}(p', \theta, a, b, g_M, \tilde{g}_M) = \frac{3}{2} J(p', \theta, a, b, g_M, \tilde{g}_M). \quad (\text{A.14})$$

- And the $\Pi_{\phi^0}^{(e)}$, $\Pi_{\phi^0}^{(q)}$, and $\Pi_{\chi^0}^{(v_R)}$ scalar self-energy one-loop correction:

$$\begin{aligned} \Pi_{\phi^0}^{(e)}(p - p', \theta, b, g_{e^M}, \tilde{g}_{e^M}, g_{q^M}, \tilde{g}_{q^M}) &= \Pi_{\phi^0}^{(q)}(p - p', \theta, b, g_{e^M}, \tilde{g}_{e^M}, g_{q^M}, \tilde{g}_{q^M}) \\ &= 6 K(p - p', \theta, b, g_{e^M}, \tilde{g}_{e^M}) + 12 K(p - p', \theta, b, g_{q^M}, \tilde{g}_{q^M}), \end{aligned} \quad (\text{A.15})$$

$$\Pi_{\chi^0}^{(v_R)}(p - p', \theta, b, g_M, \tilde{g}_M) = 3 K(p - p', \theta, b, g_M, \tilde{g}_M), \quad (\text{A.16})$$

where

$$\begin{aligned} K(l, \theta, b, g_x, \tilde{g}_x) &= \int \frac{d^D k}{(2\pi)^D} (-1) \left[g_x e^{-\frac{i}{2} l \tilde{k}} + \tilde{g}_x e^{+\frac{i}{2} l \tilde{k}} \right] \left[g_x e^{-\frac{i}{2} k \tilde{l}} + \tilde{g}_x e^{+\frac{i}{2} k \tilde{l}} \right] \\ &\times \text{tr} \left[\left(\frac{i k + i b \frac{\tilde{k}}{\theta^2 k^2}}{k^2 + \frac{b^2}{\theta^2 k^2}} \right) \left(\frac{i(l+k) + i b \frac{\tilde{k} + \tilde{l}}{\theta^2 (l+k)^2}}{(l+k)^2 + \frac{b^2}{\theta^2 (l+k)^2}} \right) \right]. \end{aligned} \quad (\text{A.17})$$

Each of these D-dimension Euclidean momentum space integrals can be sundered into planar and nonplanar integrals. Hence, performing this calculation using the dimensional regularization method and in the way described in Refs. [24, 40] leads us, for our special case of massless particles, to the following UV divergent part:

$$\begin{aligned} \Gamma_e(p, p', \theta, a, b, g_{e^M}, \tilde{g}_{e^M}) &= -\frac{2 g_{e^M} \tilde{g}_{e^M}}{(4\pi)^2 \varepsilon_{\text{UV}}} \left\{ g_{e^M} e^{-\frac{i}{2} p' \tilde{p}} + \tilde{g}_{e^M} e^{+\frac{i}{2} p' \tilde{p}} \right\} \\ &\quad + \text{UV finite part}(p, p', \theta, a, b, g_{e^M}, \tilde{g}_{e^M}), \end{aligned} \quad (\text{A.18})$$

$$\begin{aligned} \Gamma_q(p, p', \theta, a, b, g_{q^M}, \tilde{g}_{q^M}) &= -\frac{2 g_{q^M} \tilde{g}_{q^M}}{(4\pi)^2 \varepsilon_{\text{UV}}} \left\{ g_{q^M} e^{-\frac{i}{2} p' \tilde{p}} + \tilde{g}_{q^M} e^{+\frac{i}{2} p' \tilde{p}} \right\} \\ &\quad + \text{UV finite part}(p, p', \theta, a, b, g_{q^M}, \tilde{g}_{q^M}), \end{aligned} \quad (\text{A.19})$$

$$\begin{aligned} \Gamma_v(p, p', \theta, a, b, g_M, \tilde{g}_M) &= -\frac{2 g_M \tilde{g}_M}{(4\pi)^2 \varepsilon_{\text{UV}}} \left\{ g_M e^{-\frac{i}{2} p' \tilde{p}} + \tilde{g}_M e^{+\frac{i}{2} p' \tilde{p}} \right\} \\ &\quad + \text{UV finite part}(p, p', \theta, a, b, g_M, \tilde{g}_M), \end{aligned} \quad (\text{A.20})$$

$$\begin{aligned} \Sigma_e(p, p', \theta, a, b, g_{e^M}, \tilde{g}_{e^M}, g_M, \tilde{g}_M) &= \frac{-i}{(4\pi)^2 \varepsilon_{\text{UV}}} \left\{ \left[(g_{e^M}^2 + \tilde{g}_{e^M}^2) + \frac{3}{2} (g_M^2 + \tilde{g}_M^2) \right] \not{p} \right. \\ &\quad \left. + 2 \left[g_{e^M}^2 + \tilde{g}_{e^M}^2 \right] \not{p}' \right\} + \text{UV finite part}(p, p', \theta, a, b, g_{e^M}, \tilde{g}_{e^M}, g_M, \tilde{g}_M), \end{aligned} \quad (\text{A.21})$$

$$\Sigma_q(p, p', \theta, a, b, g_{q^M}, \tilde{g}_{q^M}) = -\frac{2i \left[g_{q^M}^2 + \tilde{g}_{q^M}^2 \right]}{(4\pi)^2 \varepsilon_{\text{UV}}} (\not{p} + \not{p}') + \text{UV finite part}(p, p', \theta, a, b, g_{q^M}, \tilde{g}_{q^M}), \quad (\text{A.22})$$

$$\Sigma_v(p, p', \theta, a, b, g_M, \tilde{g}_M, g_{e^M}, \tilde{g}_{e^M}) = \frac{-i}{(4\pi)^2 \varepsilon_{\text{UV}}} \left\{ \left[\frac{3}{2} (g_M^2 + \tilde{g}_M^2) + g_{e^M}^2 + \tilde{g}_{e^M}^2 \right] \not{p} \right. \\ \left. + \left[\frac{3}{2} (g_M^2 + \tilde{g}_M^2) \right] \not{p}' \right\} + \text{UV finite part}(p, p', \theta, a, b, g_M, \tilde{g}_M, g_{e^M}, \tilde{g}_{e^M}), \quad (\text{A.23})$$

$$\Pi_{\phi^0}^{(e)}(p - p', \theta, b, g_{e^M}, \tilde{g}_{e^M}, g_{q^M}, \tilde{g}_{q^M}) = \Pi_{\phi^0}^{(q)}(p - p', \theta, b, g_{e^M}, \tilde{g}_{e^M}, g_{q^M}, \tilde{g}_{q^M}) \\ = 3 \times \frac{8(g_{e^M}^2 + \tilde{g}_{e^M}^2) + 16(g_{q^M}^2 + \tilde{g}_{q^M}^2)}{(4\pi)^2 \varepsilon_{\text{UV}}} (p - p')^2 \\ + \text{UV finite part}(p - p', \theta, b, g_{e^M}, \tilde{g}_{e^M}, g_{q^M}, \tilde{g}_{q^M}), \quad (\text{A.24})$$

$$\Pi_{\chi^0}^{(v)}(p - p', \theta, b, g_M, \tilde{g}_M) = 3 \times \frac{4(g_M^2 + \tilde{g}_M^2)}{(4\pi)^2 \varepsilon_{\text{UV}}} (p - p')^2 + \text{UV finite part}(p - p', \theta, b, g_M, \tilde{g}_M). \quad (\text{A.25})$$

We then introduce the divergent parts of these radiative corrections into Eqs. (42–44), to infer directly the corresponding counter terms given by Eqs. (45–53).

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