

Superallowed $0^+ \rightarrow 0^+$ beta-decay from $T_z = -1$ *sd*-shell nuclei

J.C. Hardy and I.S. Towner

Cyclotron Institute, Texas A&M University, College Station, TX 77843-3366

E-mail: hardy@comp.tamu.edu

Abstract. Superallowed nuclear beta-decay between 0^+ analogue states probes the vector part of the weak interaction, with the measured *ft*-value of each such transition leading to a value for the vector coupling constant, G_V . To date, the *ft*-values for thirteen $0^+ \rightarrow 0^+$ transitions have been measured with $\sim 0.1\%$ precision or better. The results yield fully consistent values for G_V and an experimental value for V_{ud} , which is the leading diagonal element of the quark mixing matrix, the Cabibbo-Kobayashi-Maskawa (CKM) matrix. With a precise value for V_{ud} established, the unitarity of the CKM matrix can be tested and limits set on the possibility of new physics lying beyond the Standard Model. This work demands high experimental precision and a high degree of reliability in the small theoretical correction terms required to extract G_V and V_{ud} from the experimental data. One of the correction terms must account for isospin symmetry-breaking between the parent and daughter nuclei, and its associated uncertainty contributes significantly to the uncertainty in V_{ud} . The superallowed decays of $T_z = -1$ *sd*-shell nuclei, such as ^{22}Mg , ^{26}Si , ^{34}Ar and ^{38}Ca , have so far not played a significant role in the determination of V_{ud} because experimental challenges have made high precision unattainable for these transitions. However, if they were to be measured precisely, they would enable important tests of the calculated isospin symmetry-breaking corrections and potentially reduce the latter's uncertainties. Experiments aimed at characterizing these transitions are described.

1. Introduction

Superallowed beta decay between nuclear analogue states with $T = 1$ and $J^\pi = 0^+$ occurs only via the vector current of the weak interaction: angular momentum conservation completely rules out the axial-vector current, which must carry off a spin of one and cannot connect two states that both have spin zero. Furthermore, since the parent and daughter states are analogues of one another, the strength of the transition is affected only by the small difference between the parent and daughter configurations resulting from isospin symmetry breaking, not by the dominant nuclear structure common to them both.

The measured strength of such a transition – expressed as an “*ft* value” – can then be related directly to the vector coupling constant, G_V with the intervention of only a few small ($\sim 1\%$) calculated terms to account for radiative and isospin symmetry-breaking effects. Once G_V has been determined in this way, it is only another short step to obtain a value for V_{ud} , the up-down mixing element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, with which it is possible to test the top-row unitarity of that matrix. Since the unitary CKM matrix is a central pillar of the three-generation Standard Model, any experimentally determined deviation from CKM unitarity would be a signature of new physics beyond the Model; and even uncertainty limits

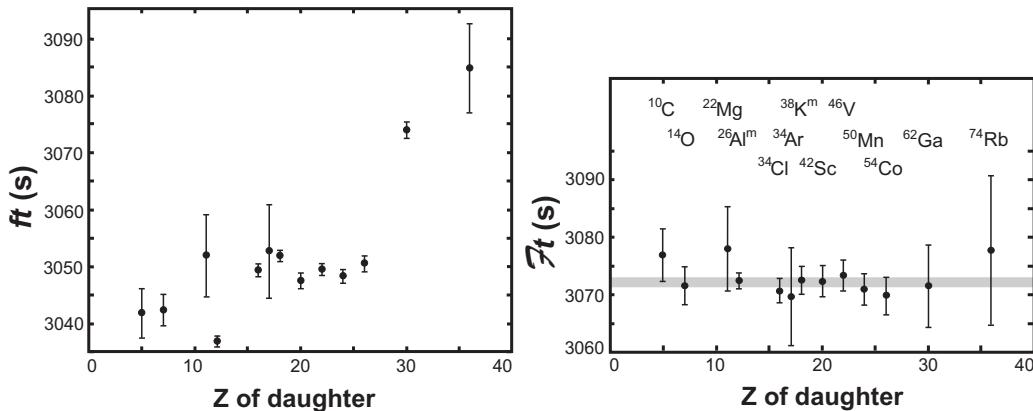


Figure 1. Results from the 2009 survey [1]: uncorrected ft values for the 13 best known superallowed decays on the left; the same results but incorporating the δ'_R , δ_C and δ_{NS} correction terms on the right. The grey band in the right panel is the average $\mathcal{F}t$ value and its uncertainty.

on a sum that agrees with unitarity can serve as a constraint on possible candidates for new physics.

Currently superallowed β -decay yields the most precise value for V_{ud} and the most exacting test of CKM unitarity, with a precision of 0.06% on the latter. This precision can be expected to improve further as a result of decay measurements that focus specifically on defining the effects of isospin-symmetry breaking between the analogue parent and daughter states in each superallowed transition. Previously uncharacterized superallowed transitions from $T_z = -1$ nuclei in the sd -shell can play an important role in this development.

2. Present status of superallowed decays

In dealing with superallowed decays, it is convenient to combine some of the small correction terms with the measured ft -value and define a “corrected” $\mathcal{F}t$ -value. Thus, we write [1]

$$\mathcal{F}t \equiv ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G_V^2(1 + \Delta_R^V)}, \quad (1)$$

where $K = 8120.2787(11) \times 10^{-10}$ GeV $^{-4}$ s; δ_C is the isospin-symmetry-breaking correction and Δ_R^V is the transition-independent part of the radiative correction. The terms δ'_R and δ_{NS} comprise the transition-dependent part of the radiative correction, the former being a function only of the electron’s energy and the Z of the daughter nucleus, while the latter, like δ_C , depends in its evaluation on the details of nuclear structure. From this equation, it can be seen that a measurement of any one superallowed transition establishes an individual value for G_V . A measurement of several of them tests the Conserved Vector Current (CVC) hypothesis that G_V is not renormalized in the nuclear medium. If indeed G_V is constant – *i.e.* all the $\mathcal{F}t$ -values are the same – then an average value for G_V can be determined and V_{ud} obtained from the relation $V_{ud} = G_V/G_F$, where G_F is the well known [2] weak-interaction constant for purely leptonic muon decay.

The ft -value that characterizes any β -transition depends on three measured quantities: the total transition energy, Q_{EC} , the half-life, $t_{1/2}$, of the parent state and the branching ratio, R , for the particular transition of interest. The most recent critical survey of world data on superallowed $0^+ \rightarrow 0^+$ beta decays was published [1] in 2009. It lists the ft values for 13 transitions, which have been precisely determined from a very robust data set with more than 150 independent measurements contributing to the various input quantities. The results were then used to obtain the corrected $\mathcal{F}t$ values, with the outcome shown in Fig. 1.

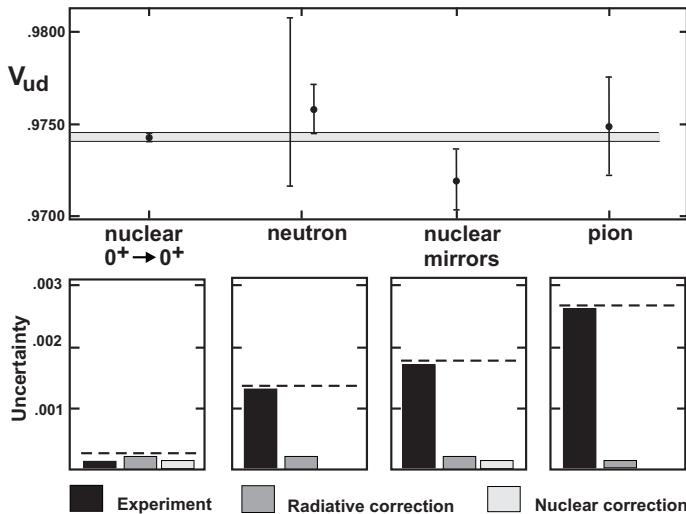


Figure 2. (top) Values for V_{ud} derived from four different types of measurement, the grey band being the average value. The first of the neutron values includes all data; the second includes just the most recent measurements.

(bottom) The four panels show the error budgets for the four results shown in the top panel with points and error bars. The three contributors to the uncertainties – experiment, radiative correction and nuclear correction – are separately identified.

It is immediately evident from the figure that the $\mathcal{F}t$ values are all consistent with one another from $A=10$ to $A=74$. This simultaneously confirms the CVC expectation of a constant value for G_V and demonstrates the absence of any significant scalar current, which would introduce an upward or downward curve into the $\mathcal{F}t$ -value locus at low Z . It also goes a long way towards validating the calculated isospin symmetry-breaking corrections: The calculations of δ_C and δ_{NS} for each transition in this analysis [3] employed the best available shell-model wave functions, which had been based on a wide range of spectroscopic data for nuclei in the same mass region. They were further tuned to agree with measured binding energies, charge radii and coefficients of the isobaric multiplet mass equation for the specific states involved. This means that the origins of these correction terms are completely independent of the superallowed decay data, so consistency in the corrected $\mathcal{F}t$ values gives powerful support to the calculated corrections used in the derivation of those $\mathcal{F}t$ values.

With a mutually consistent set of $\mathcal{F}t$ values, one is then justified in proceeding to determine the value of G_V and, from it, V_{ud} . The result we obtained in Ref. [1] was $V_{ud} = 0.97425(22)$, which, when combined with current values for V_{us} and V_{ub} , yielded a CKM unitarity sum of $0.99995(61)$, in remarkable agreement with Standard Model expectations. Note that this analysis was only possible because the $\mathcal{F}t$ values formed a consistent set. Without demonstrated consistency with CVC, there can be no justification for extracting a unique value for G_V from the data, let alone one for V_{ud} .

In the top panel of Fig. 2, the value for V_{ud} obtained from superallowed $0^+ \rightarrow 0^+$ β -decays is compared with the values obtained from neutron decay, mirror $T=\frac{1}{2}$ nuclear β decays, and pion decay, all three of which are much less precise but still agree within their quoted uncertainties. The individual contributions to the overall uncertainties for each method are displayed in the bottom panels of the figure. The $0^+ \rightarrow 0^+$ result is the only one not dominated by the experimental uncertainty. For it, the radiative correction – principally Δ_R^V – is the largest contributor to the overall uncertainty, with the “nuclear correction” – δ_C and δ_{NS} – a close second.

Although from this analysis of uncertainties one might conclude that experiment has no further role to play in improving the CKM unitarity test via $0^+ \rightarrow 0^+$ superallowed decays, this would not be correct. Since the correction terms δ_C and δ_{NS} exhibit very pronounced differences from transition to transition (compare the two panels in Fig. 1, which differ principally by the application of these correction terms) their veracity can be tested and possibly improved by new measurements that either reduce the experimental uncertainties on the currently measured $\mathcal{F}t$

values, or else increase the number of precisely measured transitions, particularly by including cases with much larger calculated correction terms. Depending on whether these new results confirm the transition-to-transition variations obtained from the calculated corrections or not, the calculations may be validated or alternatively refined to restore agreement. Either way, it will likely be possible to reduce the associated theoretical uncertainties.

3. Testing δ_C calculations

We have devised a test [5] that can be applied to any set of isospin symmetry-breaking corrections, δ_C . Our test is based upon the premise that CVC is valid at least to the level of precision attained by the best ft -value measurements. Under that condition, a valid set of structure-dependent correction terms should produce a statistically consistent set of $\mathcal{F}t$ values, the average of which we can write as $\overline{\mathcal{F}t}$. It then follows from Eq. (1) that, for each individual transition in the set, we can write

$$\delta_C - \delta_{NS} = 1 - \frac{\overline{\mathcal{F}t}}{ft(1 + \delta'_R)}. \quad (2)$$

For any set of corrections to be acceptable, the calculated value of $\delta_C - \delta_{NS}$ for each superallowed transition must satisfy this equation, where ft is the measured result for that transition and $\overline{\mathcal{F}t}$ has the same value for all of them. Thus, to test a set of correction terms for n superallowed transitions, one can treat $\overline{\mathcal{F}t}$ as a single adjustable parameter and use it to bring the n results from the right side of Eq. (2), which are based predominantly on experiment, into the best possible agreement with the corresponding n calculated values for $\delta_C - \delta_{NS}$. The normalized χ^2 , minimized by this process, then provides a figure of merit for that set of calculations.

As it happens, there is only one set of calculations available for δ_{NS} [3, 4] but many for the isospin-symmetry-breaking term δ_C . It therefore becomes more useful to rearrange Eq. (2) to read:

$$\delta_C = 1 + \delta_{NS} - \frac{\overline{\mathcal{F}t}}{ft(1 + \delta'_R)}. \quad (3)$$

The same least-squares minimization process can of course be used in the application of this equation.

This test was applied to a number of sets of calculated δ_C correction terms in Ref. [5]. A sample of the results is given here in Fig. 3. Only one theoretical model – the “Shell-model Saxon-Woods” (SM-SW) model [3] illustrated in panel “a” of the figure – produces fully satisfactory agreement with CVC, having a normalized χ^2 of 0.40. A second – the “Shell-model Hartree-Fock” (SM-HF) model [1] shown in panel “b” – is also reasonably acceptable, with a normalized χ^2 of 2.0. It was the first of these, the SM-SW model, that was employed to calculate the δ_C values used in the extraction of V_{ud} from the $0^+ \rightarrow 0^+$ ft -value data [1]. As already described, this is a semi-phenomenological model tuned to match a wide variety of experimental data that is independent of the ft values themselves. The SM-HF model is similarly semi-phenomenological in character, and it was used in a second analysis of the data, in order to assess the extent to which the final result differed between two models that both gave reasonable agreement with CVC. The final uncertainty quoted for V_{ud} in Ref. [1] included a systematic component that reflected the small difference obtained.

A representative of the unacceptable models, the isovector monopole resonance model [6], is presented in panel “c” of the figure; it completely fails to reproduce the data. However, a completely new calculation based on nuclear density functional theory has appeared [7] since our test was published. Its results are given in panel “d”. Although its normalized χ^2 is relatively high at 5.2, this is almost entirely due to the disagreement with the ft value for the decay of ^{62}Ga to ^{62}Zn , which is plotted at $Z = 30$. If that transition is removed from consideration, the

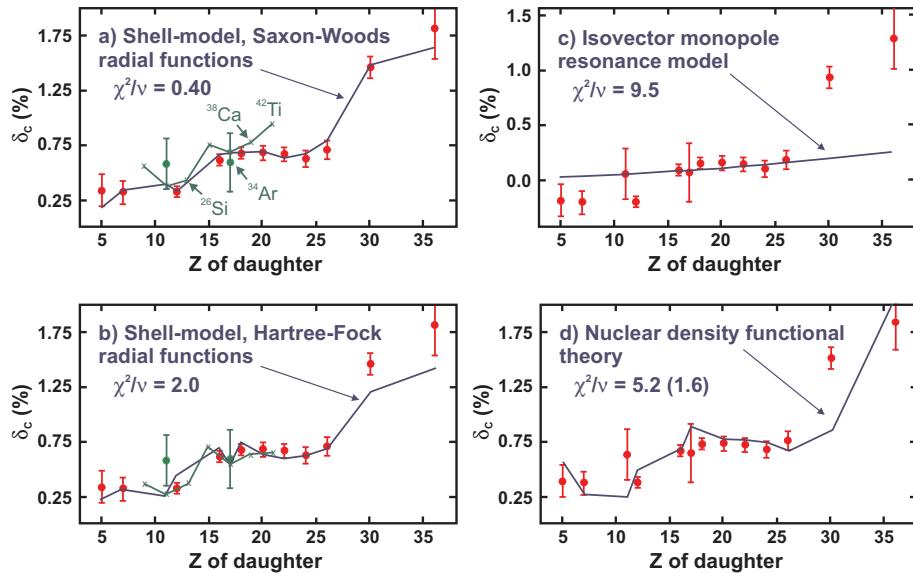


Figure 3. Isospin-symmetry-breaking correction, δ_C , in percent units plotted as a function of atomic number, Z , of the daughter nucleus. The solid circular points with error bars are the values of δ_C obtained from Eq. (3), with the experimental ft values and the values of δ'_R and δ_{NS} (and their uncertainties) all taken from Refs. [1, 5]. In effect, we treat these as the “experimental” δ_C values. The blue lines represent the δ_C values for the well-measured ft values as calculated by the various models described in the text and identified in the upper left of each graph. The value of $\bar{f}t$ in Eq. (3) has been adjusted in each case by least-squares fitting to optimize the agreement between the “experimental” δ_C values and the calculated ones. The corresponding values of χ^2/n_d are also shown. The green lines in panels “a” and “b” represent the calculated δ_C values for $T_z = -1$ nuclei between ^{18}Ne and ^{42}Ti .

normalized χ^2 becomes 1.6. It is particularly gratifying that the value of V_{ud} extracted from the ft -value data by use of this model [7] agrees well with the result obtained earlier from the SM-SW model [1].

Panels “a” and “b” of Fig. 3 include lines in green that represent the calculated δ_C values for $T_z = -1$ nuclei between ^{18}Ne and ^{42}Ti . Of particular interest are the four parent nuclei specifically identified in panel “a”. These are the parents of superallowed transitions from $T_z = -1$ to $T_z = 0$ nuclei, corresponding to each of which there is another superallowed decay from the $T_z = 0$ to $T_z = +1$ nuclei with the same value of A : for example $^{34}\text{Ar} \rightarrow ^{34}\text{Cl}$ and $^{34}\text{Cl} \rightarrow ^{34}\text{S}$. By comparing the calculations for these “mirror” transitions in Fig. 3 one can see an interesting difference between the SM-SW calculations and the SM-HF ones. The former consistently predict that the δ_C value for the $T_z = -1$ parent is larger than the δ_C value for the $T_z = 0$ parent, while the SM-HF calculation predicts just the opposite.

In fact, we find for the SM-HF model that the difference between the mirror δ_C values predicted by that model depends systematically on the value of the x_0 asymmetry parameter in the Skyrme potential used in the Hartree-Fock calculation of the radial wave functions. We have calculated the mirror δ_C values using the SM-HF model with 15 different Skyrme potentials taken from the literature, and find a convincing linear dependence between the predicted δ_C difference and the x_0 parameter in the potential [8]. It is even possible to find a potential that predicts very nearly the same result as does the SM-SW model.

As of now, none of the superallowed transitions from the $T_z = -1$ parents identified in Fig. 3 has been measured with sufficient precision to distinguish among these various δ_C predictions.

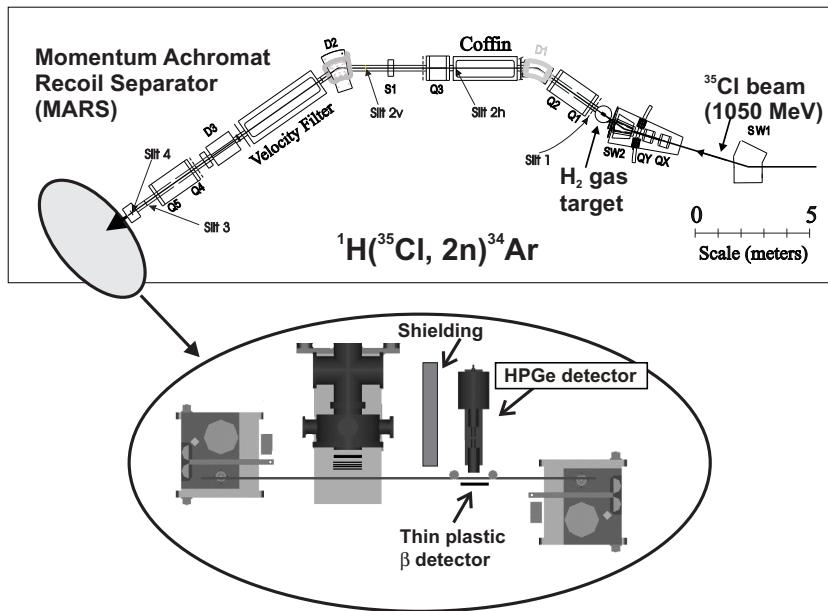


Figure 4. Experimental arrangement used for the ^{34}Ar branching-ratio measurement. For the half-life measurements, the HPGe detector and thin plastic β detector were replaced by a 4π gas proportional counter; the tape passed through the center of that detector.

However, if this deficiency could be rectified it would have several important benefits. It could distinguish definitively between the SM-SW and SM-HF models, thus removing the need to include both when determining the uncertainty on V_{ud} . It might also provide a means for constraining the asymmetry parameter x_0 in the Skyrme potential.

4. Measurements of decays from $T_z = -1$ nuclei

To determine any superallowed transition with useful precision, it is necessary to measure its Q_{EC} -value to ± 100 eV, its half-life to $\pm 0.02\%$ and its branching ratio to $\pm 0.1\%$. For most of the well known $T_z = 0$ parent decays, the branching ratio for the superallowed branch is greater than 99% so it has been determined very precisely via measurement of the small competing Gamow-Teller branches. If the latter can be measured even to $\pm 10\%$, when the result is subtracted from 100% it leads to an uncertainty of less than 0.1% in the superallowed branching ratio.

For $T_z = -1$ parent decays, the situation is much more complicated. First, the branching ratio for the superallowed branch is smaller, and the Gamow-Teller branches are correspondingly larger: for ^{26}Si the superallowed branch is $\sim 76\%$; for ^{34}Ar it is $\sim 94\%$; and for ^{38}Ca it is $\sim 77\%$. Second, each of these decays is immediately followed by the decay of its daughter, which in each case has a half-life about twice that of the decay that feeds it. This means that the total beta-decay curve is composite, with the daughter's half-life predominating. Third, these $T_z = -1$ nuclei are farther from stability than the $T_z = 0$ ones, so are more difficult to produce copiously and free of impurities.

At Texas A&M, we are currently embarked on a program to measure the $T_z = -1$ parent superallowed decays, particularly those like ^{26}Si , ^{34}Ar and ^{38}Ca that feed mirror superallowed decays from their daughters. We have completed and published our half-life results for these three cases [9, 10, 11] and are currently at work on the branching ratios.

Taking ^{34}Ar as an example, we achieve the goal of purity by using a production reaction with inverse kinematics, $^1\text{H}(^{35}\text{Cl}, 2n)^{34}\text{Ar}$, and selecting the desired reaction product with the Momentum Achromatic Recoil Separator (MARS). See Fig. 4. A primary beam of 30-AMeV ^{35}Cl from the Texas A&M superconducting cyclotron impinges on a liquid-nitrogen-cooled hydrogen gas target operated at 1.6-atm. The resultant 26-AMeV beam of ^{34}Ar separated by MARS exits the vacuum chamber through a thin Kapton window and then passes through a 0.3-mm-thick plastic scintillator and a series of Al degraders, which are adjusted to ensure the

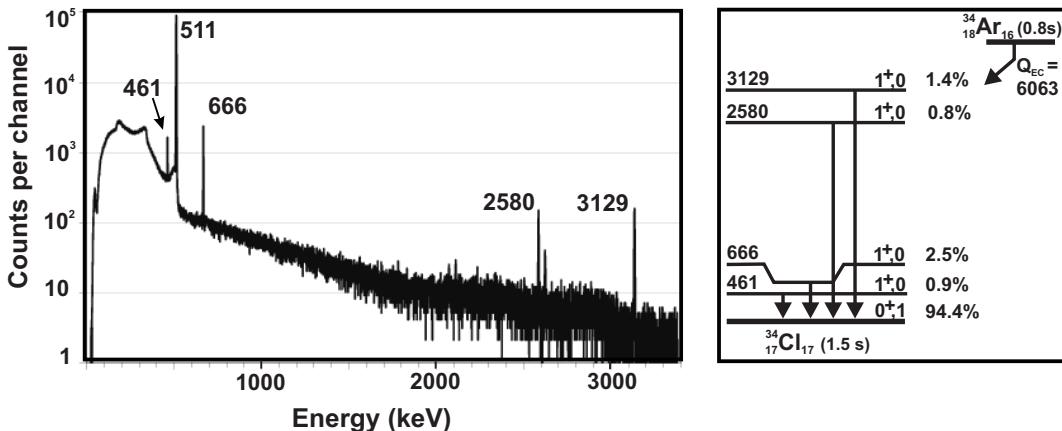


Figure 5. Spectrum of γ rays observed in coincidence with positrons from collected ^{34}Ar sources. Also shown is the β -decay scheme for ^{34}Ar .

implantation of the ^{34}Ar nuclei at the center of a 76- μm -thick aluminized Mylar tape, part of our fast tape-transport system. With an ^{34}Ar beam intensity of about 3×10^4 particles/s, we collect a radioactive sample, $>99.8\%$ pure, for typically 1s, then turn off the beam and transport the sample in 180 ms to a shielded counting location where data are collected for a preset length of time, after which the cycle is repeated. This sequence is repeated until sufficient statistics have been collected.

What detection equipment is placed at the counting location depends on whether a branching-ratio or a half-life is being measured. Figure 4 shows the arrangement used for the former, in which a spectrum of γ -rays was recorded for those events in the HPGe detector that were observed to be in coincidence with positrons in the thin plastic β detector. For the latter, a 4π gas proportional counter split into two halves was used, with the tape passing between the halves and the collected sample being stopped exactly at the center for each measurement period [9].

The spectrum of recorded β -delayed γ rays is shown in left panel of Fig. 5, where no impurity peaks can be detected. The β -decay branching ratios were determined from the corresponding intensities of the β -delayed γ -ray peaks, which all correspond to γ -transitions to the ground-state (see the right panel in Fig. 5). If the γ ray de-exciting state i in the daughter is denoted by γ_i , then the β -branching ratio, R_i , for the β -transition populating that state can be written:

$$R_i = \frac{N_{\beta\gamma_i}}{N_\beta} k, \quad (4)$$

where $N_{\beta\gamma_i}$ is the total number of β - γ coincidences measured in the γ_i peak, N_β is the total number of β singles, ϵ_{γ_i} is the detector efficiency for γ ray, γ_i , and k is a small correction factor (*i.e.* $k \sim 1$) that, among other things, takes into account the differences in the β -detector efficiency for the different transitions participating in ^{34}Ar decay. (See Ref. [12] for a detailed description of the factors that contribute to k .) This relation highlights the importance of a precise absolute efficiency calibration for the γ -ray detector and a reasonable knowledge of relative efficiencies in the beta detector. Our HPGe detector's absolute efficiency is accurately known (to $\pm 0.2\%$ for 50-1400 keV γ rays and $\pm 0.4\%$ up to 3500 keV) from source measurements and Monte Carlo calculations [13]. The relative efficiency as a function of β energy in the plastic scintillator was determined by Monte Carlo calculations and checked by comparison with measurements on conversion-electron sources.

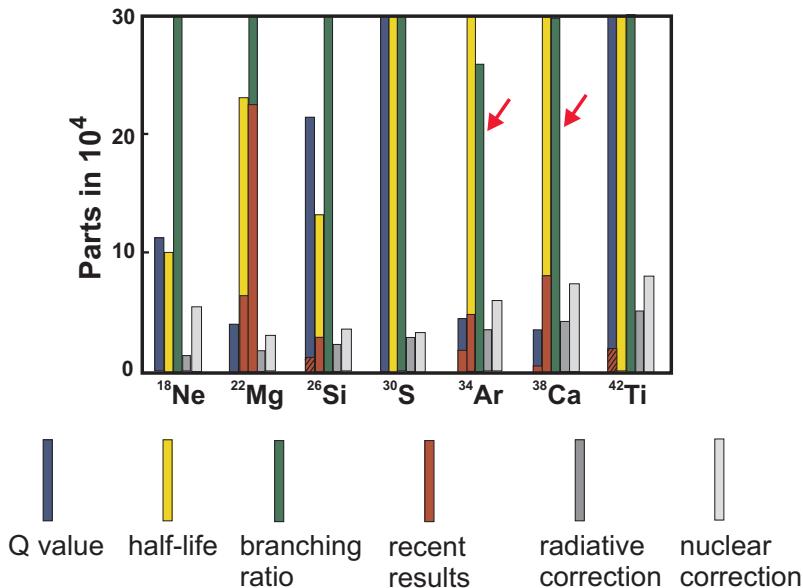


Figure 6. Error budgets for superallowed $0^+ \rightarrow 0^+$ transitions from $T_z = -1$ parent nuclei between ^{18}Ne and ^{42}Ti . The theory contributions are shown in two shades of grey; the three experimental contributions are identified by the color code give at the bottom. Red bars indicate recent improvements in any of the three. The red arrows indicate the two branching ratios we expect to improve significantly in the near future.

Branching-ratio measurements for both ^{34}Ar and ^{38}Ca have been made but are still being evaluated, with special attention directed to dead times and other potential sources of error. We anticipate having final results this year with a precision on the superallowed branch that reaches $\pm 0.2\%$ and perhaps better. Figure 6 presents an overview of the current status of error budgets for the superallowed $0^+ \rightarrow 0^+$ transitions from $T_z = -1$ parent nuclei between ^{18}Ne and ^{42}Ti . The red bars show the improved results that have appeared over the last few years, and the red arrows point to the ^{34}Ar and ^{38}Ca branching ratios we expect to improve soon.

5. Conclusions

Measurements of superallowed $0^+ \rightarrow 0^+$ nuclear beta decays have already achieved remarkable precision and are playing an important role in delimiting the electroweak standard model. Most significantly, they have demonstrated the unitarity of the CKM matrix to within 0.06%. Current activity in the field is now focused on improving this precision even further, with special emphasis on reducing, through experiment, the uncertainties currently attached to the theoretical correction terms that must be applied to the experimental results before fundamental weak interaction parameters can be extracted from them. The isospin-symmetry-breaking corrections are particularly important, but we can test the validity of any set of such calculations by requiring that the resulting corrected $\mathcal{F}t$ values satisfy the conservation of the vector current by being statistically consistent with one another from transition to transition.

We have described here a potentially powerful way to apply this test to mirror pairs of $0^+ \rightarrow 0^+$ transitions: for example $^{34}\text{Ar} \rightarrow ^{34}\text{Cl}$ and $^{34}\text{Cl} \rightarrow ^{34}\text{S}$. All experimental data required for a precise test with this pair of transitions is in hand with the exception of the branching ratio for the decay from ^{34}Ar . An experiment has been described, which is designed to precisely measure this branching ratio, and those like it from other $T_z = -1$ superallowed parent nuclei. Results can be expected within the next year.

We are optimistic that the value of V_{ud} and the CKM unitarity test can still be improved by nuclear measurements.

5.1. Acknowledgments

The experimental work referred to here is being done in collaboration with V.E. Iacob, H.I. Park and the other members of the Cyclotron Institute weak-interactions group. The work was

supported by the U.S. Department of Energy under contract no. DE-FG02-93ER-40773 and by the Robert A. Welch Foundation under grant no. A-1397.

References

- [1] Hardy J C and Towner I S 2009 *Phys. Rev. C* **79** 055502.
- [2] Nakamura K *et. al.* 2010 [Particle Data Group] *J. Phys. G* **37** 075021 and 2011 partial update for the 2012 edition.
- [3] Towner I S and Hardy J C 2008 *Phys. Rev. C* **77** 025501.
- [4] Towner I S 1994 *Phys. Lett. B* **333** 13.
- [5] Towner I S and Hardy J C 2010 *Phys. Rev. C* **82** 065501.
- [6] Auerbach N 2009 *Phys. Rev. C* **79** 035502.
- [7] Satula W, Dobaczewski J, Nazarewicz W and Rafalski M 2011 *Phys. Rev. Lett.* **106** 132502.
- [8] Towner I S and Hardy J C 2012 To be published.
- [9] Iacob V E, Hardy J C, Brinkley J F, Gagliardi C A, Mayes V E, Nica N, Sanchez-Vega M, Tabacaru, G, Trache L and Tribble R E 2006 *Phys. Rev. C* **74** 055502.
- [10] Iacob V E, Hardy J C, Banu A, Chen L, Golovko V V, Goodwin J, Horvat V, Nica N, Park H I, Trache L and Tribble R E 2010 *Phys. Rev. C* **82** 035502.
- [11] Park H I, Hardy J C, Iacob V E, Banu A, Chen L, Golovko V V, Goodwin J, Horvat V, Nica N, Simmons E, Trache L and Tribble R E 2011 *Phys. Rev. C* **84** 065502.
- [12] Iacob V E, Hardy J C, Gagliardi C A, Goodwin J, Nica N, Park H I, Tabacaru G, Trache L, Tribble R E, Zhai Y and Towner I S, *Phys. Rev. C* **74**, 015501 (2006).
- [13] Hardy J C, Iacob V E, Sanchez-Vega M, Effinger R T, Lipnik P, Mayes V E, Willis D K and Helmer R G, *Appl. Radiat. Isot.* **56**, 65 (2002); Helmer R G, Hardy J C, Iacob V E, Sanchez-Vega M, Neilson R G and Nelson J, *Nucl. Instr. Meth. Phys. Res. A* **511**, 360 (2003); and Helmer R G, Nica N, Hardy J C and Iacob V E, *Appl. Radiat. Isot.* **60**, 173 (2004).