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# רקעים של תורת המיתר עם שטפים וייחודיויות
# String Theory backgrounds with fluxes and singularities

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**For Yarden**

Who might read this one day...

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# Contents

**1 Introduction** **9**

1.1 General Introducion . . . 9

1.2 Dimensionality of Space-Time – Fluxes in String Theory . . . 12

1.2.1 Open String Moduli . . . 14

1.2.2 AdS/CFT Correspondence with Fluxes . . . 17

1.3 The Particles in Our World – Singularities in String Theory . 19

1.3.1 Flavor Parameters and Neutrino Physics from String Theory . . . 22

1.3.2 A Simple Model for Breaking Supersymmetry . . . 25

**2 Papers** **31**

2.1 Open String Moduli in Kachru-Kallosh-Linde-Trivedi Compactifications . . . 33

2.2 Solving Flavor Puzzles with Quiver Gauge Theories . . . 45

2.3 Dynamical Supersymmetry Breaking from Simple Quivers . . 59

2.4 On the Conformal Field Theory Duals of Type IIA $AdS_4$ Flux Compactifications . . . 65

# Chapter 1

# Introduction

In this thesis we present the work published during the Ph.D. and work that was sent to the arXiv and is pending publication. It is based on four papers in the field of string theory and particle physics [1, 2, 3, 4].

Below, we provide a brief introduction to the research done during the Ph.D. studies in which we studied the role of fluxes and singularities in realizing phenomenological features of string theory. The reader is referred to the abstract, introduction and conclusions of each of the papers presented in chapter 2 for a more technical description of the work.

## 1.1 General Introducion

There are currently two generally accepted physical frameworks describing the behavior of our world. The first, quantum mechanics, describes the behavior of the small constituents of matter. This framework, using the formalism of quantum fields, can be applied to describe processes that take place at particle accelerators from a theoretical point of view. This framework has become known as the Standard Model of particle physics and its predictions agree perfectly with the values measured in experiment. The second framework, general relativity, deals with very large and massive objects such as stars and galaxies. It describes the deformations of space and time caused by such masses and their mutual influence. General relativity is used also to understand the cosmological evolution of the universe. This theory as well is tested to a high degree of accuracy.

Evidently, the two theories describe very different physical situations. However, there are physical circumstances in which the large and the small are incorporated together. An example of this is the astrophysical object known as a black hole. A black holes is an objects whose mass is large enough to attract anything in its surroundings, such that objects drawn into it cannot escape. At the same time, due to their enormous gravitational force, these objects are confined in a small enough space such that the energy density is larger than some mass scale, known as the Planck scale which is of order of $10^{19}$GeV. At this scale, quantum effects are significant, and thus the physics inside a black hole must be described by both frameworks.

Another example is physics during the early stages of the universe. According to the cosmological theory of the evolution of our universe, the universe began by expanding from a point size singularity to the huge size it is today. When the universe was just born, all of its energy and mass were localized in a very small and energetic volume of space. Here, as before, the physics of quantum fields and of general relativity live side by side.

In order to better understand such examples, one needs a new theory that incorporates both quantum mechanics and general relativity into some larger framework. A simple attempt at combining these two theories aims at representing gravity by a quantum field, whose fluctuations describe deformations in the geometry of space and time. This attempt leads to an inconsistent theory since fluctuations over very small scales can be very energetic and can create singularities, which are manifested by divergences in the calculations. As of today there is no known theory that fully resolves this problem even at a theoretical level except string theory. It is hoped that in the future string theory could be verified experimentally.

Perturbative string theory is the quantum theory of extended objects propagating in space-time, meaning the basic object is a string instead of a point. There may be either open strings with two endpoints or strings that close on themselves, whose spectrum of excitations describes the different matter fields. In any consistent formulation of string theory one of the closed string excitations is always a spin two graviton, the quantum manifestation of general relativity. The finite size of the strings is the key ingredient that solves the problem that arises at infinitely small distances when one tries to quantize gravity.

However it is still unclear how to use the formalism of string theory to describe the physics of our world, as the extended nature of the strings sets many restrictions on the consistency of the theory. In the simplest formulation of string theory, one such requirement is that the space-time in which the objects propagate must be ten dimensional, which is in conflict with the four dimensions we see around us. The spectrum of the string excitations in these models contains a graviton, fermions, and vector fields as we indeed have in Standard Model, however it is not known how to reduce the spectrum to match the details of the Standard Model, nor is it known how to generate the exact interactions between them with the correct parameters we measure.

These difficulties, however, can be amended by including additional structures into the model. It is possible to reduce the dimensionality of space-time to the known four dimensions by taking six of them to span a compact space. It turns out that it is then necessary to include also non vanishing fluxes for the fields in our theory. A viable spectrum of particles and their interactions arises when we consider singular points of the geometry. This thesis studies these structures in string theory with the hope that by incorporating them it will be possible to generate a completely viable description of physics at low energies.

## 1.2 Dimensionality of Space-Time – Fluxes in String Theory

Although string theory can be formulated as a consistent theory of quantum gravity, it still has to obey the equivalence principle and to reduce to the Standard Model when we consider its low energy limit. In order to describe string theory at low energies one can integrate out the massive excitations of the string. The resulting limit leads to a supergravity theory whose action can be calculated from string amplitudes. Solving the supergravity equations of motion, one gets backgrounds on which the full string theory can be constructed.

Since string theory can be consistent only when the number of space-time dimensions is exactly ten, its low energy is described by a ten dimensional background geometry. This seems to disqualify string theory, as we know that there are only four space-time dimensions in the world we see. However, it turns out that there are ways to make a ten dimensional world seem to be only four dimensional. One such way is known as "compactification" and can be described as follows. Consider the extra dimensions to span a very small compact geometry. In order to probe it we need very energetic excitations. As we take the extra dimensions to be smaller, we will need more energetic particles in order to probe them. However, if the energy required is much larger than what can be achieved currently in particle accelerators, the extra dimensions cannot be observed directly.

In order to end up with a four dimensional theory we thus take the background to be of the form $\mathbb{R}^{1,3} \times Y$, where Y is a compact six dimensional space. By following the Kaluza-Klein reduction we can then dimensionally reduce the theory to a four dimensional one. Many of the properties of the four dimensional physics are determined from properties of the compact space $Y$. For example, the symmetries of the four dimensional theory are determined by isometries of $Y$. The size and shape of this manifold also appear in the four dimensional theory as massless scalar fields, moduli. Phenomenologically these must receive a non vanishing mass that will stabilize them at some value and make them non dynamical in the low energy limit. Therefore there should be a mechanism in string theory that generates a moduli dependent potential. One such mechanism is the use of supergravity backgrounds with additional non trivial fluxes for the fields [5, 6, 7, 8, 9, 10, 11, 12]. The action then includes a potential that generically lifts all moduli.

Consequently, it is important to better understand string theory in backgrounds that include fluxes. In the next subsections I will describe our work carried out towards this goal. In subsection 1.2.1 we consider a different type of moduli that are not stabilized by the fluxes and we consider a different mechanism to stabilize them in some specific context. In subsection 1.2.2, we study how such a string background with fluxes turned on can be described by a quantum field theory with no gravity using different degrees of freedom.

### 1.2.1 Open String Moduli

Type II string compactifications on generic Calabi-Yau three-folds lead to hundreds of massless scalar moduli fields, causing various phenomenological problems since no such light scalar fields have been observed in nature. By turning on some background value for the Neveu-Schwarz-Neveu-Schwarz (NSNS) and Ramond-Ramond (RR) fluxes on cycles of the Calabi-Yau, a potential develops that stabilizes these moduli at some fixed value and generates a mass for the scalar fields.

In type IIB string theory, the classical supergravity action generates a potential for the complex structure moduli of the Calabi-Yau manifold but not for its Kähler structure moduli. Since the total volume of the compact manifold is a Kähler modulus, it is not possible to fix all moduli by fluxes in the type IIB supergravity approximation. However, it has been argued [13] that non-perturbative effects in type IIB string theory introduce a potential that depends also on the Kähler moduli. Including these non-perturbative effects leads to a potential with a negative minimum, describing a supersymmetric

Anti de-Sitter (AdS) background. This vacuum can be furthermore lifted to a positive value of the energy density for which one gets a meta-stable de Sitter (dS) background, in agreement with recent observations suggesting a positive cosmological constant. The modification involves introducing a space-filling anti-D3-brane (which we will denote as a $\overline{\text{D}}$3-brane) that raises the potential energy. This breaks all the supersymmetry, and using some fine tuning it was argued that it is possible to obtain a positive yet small cosmological constant. Following the work of [13], various other suggestions for constructing meta-stable dS vacua have also appeared.

In addition to changing the potential, the addition of the $\overline{\text{D}}$3-brane has implications regarding the moduli in the theory. In the presence of the $\overline{\text{D}}$3-brane there is also an open string sector that includes some moduli that can be interpreted as the location of the $\overline{\text{D}}$3-brane in the compact space. In our research we studied these moduli.

In the Kachru-Kallosh-Linde-Trivedi (KKLT) construction the moduli are stabilized near a conifold singularity and the fluxes generate a warped Klebanov-Strassler (KS) like "throat" with a warp factor $a_0$. The $\overline{\text{D}}$3-brane is added at the tip of this "throat". In [1] we studied the mass of the open string moduli corresponding to the position of the $\overline{\text{D}}$3-brane. In the limit of an infinite "throat" these moduli are massless since they are Goldstone bosons. However, when the "throat" is finite the background is changed and the moduli obtain a mass. We studied in detail the deviation of the finite "throat" theory from the infinite "throat" theory of [14], and we identified the leading deviation that contributes to the mass of the open string moduli. The approximate conformal symmetry of the "throat" theory is used to classify the deviations, and we found that the leading deviation corresponds to an operator of dimension $\Delta = \sqrt{28} \simeq 5.29$, and that it leads to a mass-squared for the open string moduli scaling as $a_0^{\Delta-2} \simeq a_0^{3.29}$. In the interesting limit of large warping, $a_0 \ll 1$, this mass is exponentially lighter than the other mass scales appearing in the warped compactification, implying that the KKLT scenario generally leads to light scalars, which could cause phenomenological problems.

We also studied a possible way to resolve this problem and increase the mass of the moduli, by positioning two of the orientifold 3-planes (which must be present anyway in KKLT-type compactifications) at the tip of the "throat", and adding to them half-D3-branes so that they become $O3^+$-planes rather than $O3^-$-planes. The $\overline{\text{D}}$3-brane is then attracted to these $O3^+$-planes, increasing the mass of the open string moduli. The mass-squared is still smaller than the typical mass scales, but only by a factor of the string coupling $g_s$ which does not have to be very small. Therefore, this can be used to avoid phenomenological problems (especially if the standard model fields live at a different position in the Calabi-Yau and couple very weakly to the $\overline{\text{D}}$3-brane fields). Our scenario has the added advantage that by adding two half D3-branes in addition to the $\overline{\text{D}}$3-brane we do not generate a tadpole for the D3-brane charge. This is in contrast to the original KKLT scenario where such a tadpole exists and leads to subtleties in using the probe approximation for describing the $\overline{\text{D}}$3-brane (due to the necessity to change the background elsewhere to compensate for the $\overline{\text{D}}$3-brane charge).

### 1.2.2 AdS/CFT Correspondence with Fluxes

An important concept in string theory is that of holography [17, 18]. According to holography, quantum theories of gravity in d space-time dimensions can be described by dual quantum field theories, which do not include gravity, in d-1 dimensions. The most well studied example is when the gravitational theory is formulated on a space with geometry known as Anti de-Sitter (AdS). The dual theory can be written in terms of a conformal field theory (CFT) that exhibits a special type of symmetry called conformal symmetry. This is known as the AdS/CFT correspondence [19, 20, 21]. This duality is a strong/weak duality; it relates a theory of weakly interacting particles to a strongly coupled theory, and vice versa. Such a duality allows us to calculate things in a weakly coupled theory and deduce results for the dual theory in a strong coupling regime where the results cannot be calculated directly.

In the previous subsection we saw that before lifting the potential by adding $\overline{\text{D}}$-branes, we have an $AdS_4$ background. This turns out to be a common feature in flux backgrounds. Using the holographic arguments we are led to conjecture that a dual three dimensional conformal field theory exists for the $AdS_4$ flux compactifications.

For a generic type IIB flux compactification the complex structure moduli as well as the axio-dilaton are stabilized by the flux induced potential. However there is always at least one Kähler modulus – the overall scale that is left unfixed. This results in the need for a non-perturbative analysis of these backgrounds. Motivated by dualities between type IIA and type IIB theories, such as mirror symmetry in which the role of the complex structure and Kähler moduli is interchanged between the two types, we can expect that in type IIA all Kähler moduli could easily be stabilized while the complex structure muduli would remain massless. It is then possible to look for a compact space without any complex structure moduli where all moduli will be fixed, without resorting to non-perturbative effects. A compactification of IIA string theory on backgrounds with fluxes was constructed in [15], where they found the form of the superpotential and Kähler potential. For a specific orientifold of $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$ this analysis was carried out by [16], where it was shown that indeed all moduli are stabilized by a generic flux configuration. However another novel feature was discovered. Unlike the type IIB case, not all flux parameters are constrained by tadpole cancellation conditions. Hence it is possible to take some flux parameters to be arbitrarily large, giving rise to a small dilaton and large volume where supergravity is under control. The non-compact space was shown to have a negative cosmological constant and thus was identified as an AdS space.

In [4] we revealed the features of the dual field theory of this background. We computed various basic properties of the dual field theory, like its central charges and the generic features of its operator spectrum. In order to find more clues about this mysterious field theory we investigated in some detail its moduli space, which can be described using configurations of domain walls in $AdS_4$. Of course, generic flux backgrounds preserve no supersymmetry so they would not be expected to have a moduli space. The flux backgrounds of [16] preserve a four dimensional $\mathcal{N} = 1$ supersymmetry, so they are dual to three dimensional $\mathcal{N} = 1$ superconformal field theories. This amount of supersymmetry is not enough to protect the moduli space from quantum corrections, since generic scalar potentials are consistent with three dimensional $\mathcal{N} = 1$ supersymmetry. Nevertheless, in our study (performed in the weak coupling weak curvature limit) we found a large moduli space in these backgrounds. Although we expect this moduli space to be lifted by quantum corrections (perhaps non-perturbative), they are small in the "large flux limit", and we expect the existence of an approximate moduli space in this limit to be a useful clue for the construction of the dual field theory. The moduli space turned out to be very complicated, with many different branches that might be interconnected. For each such branch we employed some mathematical theorems that count the number of solutions for polynomial equations, in order to enumerate the number of dimensions.

So far we have not been able to find a simple field theory model that would reproduce all the properties that we found. We hope that these properties will provide useful clues for the construction of such a field theory in the future.

## 1.3 The Particles in Our World – Singularities in String Theory

Physics that can be probed nowadays in particle accelerators is described in a very compact way, using the formalism of quantum field theory. Given a set of symmetries that are exhibited in nature and the different particles that exist, one can calculate to a high precision many experimental processes. The set of symmetries and particles that exist in our world is known as the Standard Model of particle physics. A requirement of any fundamental theory is that the Standard Model be obtained from it as a limiting case. Although many properties of the Standard Model can be realized in backgrounds of string theory, up to date no one has been able to construct it in full detail. The search for the way to relate string theory to observations is known as string phenomenology.

A crucial ingredient in the Standard Model that must also arise in string theory is the presence of non-abelian gauge groups, which describe the symmetries of the theory under which matter fields are charged. In string theory there is a natural way in which non-abelian gauge groups appear when we consider D-branes. These are extended objects in string theory spanning a p-dimensional submanifold on which open strings can end. The action of the open string degrees of freedom on $N$ coincident D-branes is given by a $U(N)$ supersymmetric Yang-Mills (SYM) theory. More realistic gauge groups appear when the D-branes span the four dimensional space-time and are located at singular points in the Calabi-Yau. There the $U(N)$ gauge group is broken and one can describe the resulting gauge groups and spectrum compactly using quiver diagrams.

As there are many types of singularities, each with many geometrical parameters, it seems that string theory can accommodate a vast amount of different solutions, each with different symmetries and spectrum of particles. The method of finding the string background that will exhibit the desired features known from experiments is known as the bottom-up approach. It is currently known how to construct singularities that, by adding the correct configuration of D-branes, will generate (almost) the exact low energy spectrum expected to be in the Standard Model. However it remains a standing problem to engineer the correct interactions between particles. In subsection 1.3.1 we employ such a bottom-up approach as a tool to test certain classes of string models by using restrictions arising from the structure of these interactions.

A different ingredient in string phenomenology constructions has to do with the extended symmetry of space-time that relates bosonic and fermionic degrees of freedom. This symmetry, called supersymmetry, is an intrinsic part of string theory at high energies that is needed for the consistency of the theory. This symmetry, if it exists, predicts that for any fermionic particle there will be a degenerate bosonic particle. However, in the low energy experiments such partners have never been observed. This ambiguity between the symmetries of the theory at high and low energies can be ameliorated by the mechanism of symmetry breaking.

There are other reasons to believe that supersymmetry is indeed a symmetry of physics at high energies. In the Standard Model there is a scalar particle called the Higgs, which has an important role in making the other particles massive. This particle has not yet been observed but there are bounds on its mass that restrict it to be no more than about 150GeV (with such a mass it should be detected soon in the experiments at the Large Hadron Collider commencing this year). However, quantum corrections generate additional contributions to the Higgs mass shifting it to many orders of magnitude above the bound (unless some fine tuning is introduced). The inclusion of the additional degrees of freedom predicted by supersymmetry controls the shift in the Higgs mass and pushes it back to the correct range.

There are several mechanisms for breaking supersymmetry at low energies. A highly motivated one is that of dynamical supersymmetry breaking (DSB), in which the breaking of supersymmetry is due only to non-perturbative effects. In the context of string theory, supersymmetry breaking usually occurs in a different “sector”, a gauge theory described by D-branes positioned on a different singularity than the one describing the Standard Model. In subsection 1.3.2 we were able to describe such a sector using very simple tools from string theory. Using this configuration one can then construct realistic models and study the properties of supersymmetry breaking in them.

### 1.3.1 Flavor Parameters and Neutrino Physics from String Theory

Recently it was realized that string theory has many different vacua, in which the low energy physics is different. Low energy physics puts constraints on the possible vacuum in which we live. In order to make contact with experiments it is necessary to find the background that generates the properties of the Standard Model. The interplay between string theory and phenomenology is bidirectional. Indeed, in many aspects string theory provides a stringent and predictive setting for phenomenology. This is due to the strict theoretical constraints that arise at high energy.

In [2] we employ a bottom-up approach as a tool to test certain classes of string models by using restrictions arising from low energy phenomenology. We study the incorporation of the hierarchy of the Yukawa coupling constants into string theory and find that it constrains the possible solutions of the theory. We also find that this constraint reflects back on the Standard Model and predicts restrictions on neutrino masses.

The charged fermion flavor parameters – quark masses and mixing angles and charged lepton masses – exhibit a structure that is not explained within the Standard Model. There are two puzzling features – the parameters are both small and hierarchical. This suggests that there is some approximate horizontal symmetry at work. The simplest framework that employs such a mechanism to explain the flavor puzzle is that of the Froggatt-Nielsen (FN) mechanism [22]. The various generations carry different charges under an Abelian symmetry. The symmetry is spontaneously broken, and the breaking is communicated to the Standard Model fermions via heavy fermions. The ratio between the scale of spontaneous symmetry breaking and the mass scale of the fermions provides a small symmetry-breaking parameter that explains the smallness of the flavor parameters. Yukawa couplings that break the FN symmetry are suppressed by powers of the breaking parameter, depending on their FN charge, and the difference between the charges explains the hierarchy. However, all FN predictions are subject to inherent limitations. The FN charges are not dictated by the theory and are chosen arbitrarily, the value of the small parameter is not predicted, and there is no information on the $\mathcal{O}(1)$ coefficients. The predictive power of the FN framework is thus limited.

To make further progress, one would like to embed the FN mechanism in the string theory framework. Gauge theories arise in string theory when D-branes are taken close to singular points of the geometry. There the low energy physics is described by a class of gauge theories known as quiver theories. In our research [2], we studied quiver gauge theories and their orientifold generalizations. These theories typically have numerous anomalous $U(1)$'s.

The anomalies are cancelled through the generalized Green-Schwarz mechanism. We considered FN models from quiver gauge theories by employing these anomalous $U(1)$'s as flavor symmetries and constructing FN models. We found that there are severe restrictions on the possible FN charges that lead to a generic constraint on the maximal hierarchy in this framework. Specifically, we constructed FN models in $SU(5)$-GUT theories with a single FN field. We found that there are only three possible FN charges for the $\mathbf{10}$-plets, and two for the $\bar{\mathbf{5}}$-plets. This situation makes the theory highly predictive. In particular, there is a unique configuration that gives rise to flavor structure for the quark masses in which the flavor structure of the lepton sector is fixed.

Additionally, there are more flavor parameters for the neutrinos. In the Standard Model neutrinos are massless, however recently indirect experiments have shown that this cannot be the case and that neutrinos must have some small mass that is yet to be measured explicitly. It is however plausible that the flavor structure of the neutrinos is different than that of the quarks and leptons. Specifically, the neutrinos seem to possess an anarchical flavor structure, with no hierarchy in their masses and with mixing angles of order one. By extending our quiver model to include neutrinos we predict that the only viable theory leads to neutrino masses that are anarchical.

The uniqueness of the model demonstrates the strong predictive power of quiver gauge theories and the possibility of constraining models in string theory through the bottom-up approach.

### 1.3.2 A Simple Model for Breaking Supersymmetry

Supersymmetry is an extension of the space-time symmetries that is required for the consistency of string theory. As this symmetry requires new particles that have not been found, it is necessarily broken at low energies and only exhibited at high energies. It has been argued that the breaking is accomplished through non-perturbative effects, a mechanism known as dynamical supersymmetry breaking. Several models of DSB have been found over the years; however they are non-generic and thus difficult to incorporate into string theory. The construction of such models involves very complex configurations in string theory that are very difficult to study.

As discussed in the previous subsection, the Standard Model is realized in string theory on D-branes located on singularities. Supersymmetry breaking then occurs in a gauge theory located on a different set of D-branes spatially separated from the Standard Model branes along the compact directions. The information on the breaking can then be transmitted to the Standard Model by several possible carriers such as a closed string (e.g. gravity) or an open string stretching between the two stacks of branes.

As a first step towards writing a complete compact solution, one must specify a local singularity on which one can construct the model that exhibits DSB. In [3] we formulated such a model of DSB using a very simple construction in string theory where the singularity is described by an orbifold projection of a smooth space. The model consists of an $SU(5)$ gauge group with one generation of fields in the antisymmetric and antifundamental representations, and is known to break supersymmetry due to non-perturbative effects.

The string construction is based on a quiver theory that includes D-branes that are fixed to the singular point of the geometry. The branes composing the quiver are located on additional singular six-dimensional spaces known as orientifolds. At the intersection of the orientifolds with the orbifold singularity we found a rich structure for the quiver diagram that allows us to generate the gauge group and matter content of our model. We were able to give the explicit construction of the local singularity that describes the desired four dimensional model.

We showed that the models described above offer new mechanisms to break supersymmetry while at the same time they stabilize various moduli (known as Kähler moduli) which otherwise require complicated and less understood stabilization mechanisms. Our example demonstrates the existence of a new class of quiver gauge theories located on orientifold planes. Using such a simple model one could do further calculations that will shed some light on the process of DSB in string theory.

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# Chapter 2

# Papers

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## 2.1 Open String Moduli in Kachru-Kallosh-Linde-Trivedi Compactifications

# Open string moduli in Kachru-Kallosh-Linde-Trivedi compactifications

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In the Kachru-Kallosh-Linde-Trivedi (KKLT) de-Sitter construction one introduces an anti-D3-brane that breaks the supersymmetry and leads to a positive cosmological constant. In this paper we investigate the open string moduli associated with this anti-D3-brane, corresponding to its position on the $S^3$ at the tip of the deformed conifold. We show that in the KKLT construction these moduli are very light, and we suggest a possible way to give these moduli a large mass by putting orientifold planes in the KKLT "throat."

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## I. INTRODUCTION

Type II string compactifications to four space-time dimensions with nontrivial Ramond-Ramond (RR) and Neveu-Schwarz-Neveu-Schwarz (NS-NS) background fluxes have been studied extensively in the literature in the past few years, as a way to stabilize moduli in string theory. Compactifications on generic Calabi-Yau (CY) three-folds without background fluxes lead to hundreds of massless scalar moduli fields, causing various phenomenological problems since no light scalar fields have been observed in nature. However, by turning on some background value for the fluxes on cycles of the Calabi-Yau manifold, a potential develops that stabilizes those moduli at some fixed value and generates a mass for the scalar fields (see [1] and references therein).

Several examples of this mechanism, involving orientifolds, have been studied in detail. In type IIA string theory there are several known examples of toroidal orientifolds in which all moduli are stabilized. In type IIB string theory, the classical supergravity action generates a potential for the complex structure moduli of the Calabi-Yau manifold but not for its Kähler structure moduli. Since the total volume of the compact manifold is a Kähler modulus, it is not possible to fix all moduli by fluxes in the type IIB supergravity approximation. However, it has been argued [2] that nonperturbative effects in type IIB string theory, such as gauge theory instantons or gaugino condensation in the world volume of D7-branes or wrapped Euclidean D3-branes, generate a potential which depends also on the Kähler moduli. Including these nonperturbative effects leads to a potential with a minimum with a negative cosmological constant, describing a supersymmetric anti de-Sitter (AdS) background.

The authors of [2] suggested that a slight modification of such a background could lead to a meta-stable de-Sitter (dS) background, in agreement with recent observations suggesting a positive cosmological constant. The modification involves introducing a space-filling anti-D3-brane (which we will denote as a $\overline{\mathrm{D3}}$-brane) which raises the potential energy. This breaks all the supersymmetry, and using some fine-tuning it was argued that it is possible to obtain a positive yet small cosmological constant. Following the work of [2], various other suggestions for constructing meta-stable dS vacua have also appeared.

In addition to changing the potential, the addition of the $\overline{\mathrm{D3}}$-brane has implications regarding the moduli in the theory. In the presence of the $\overline{\mathrm{D3}}$-brane there is also an open string sector, which includes some light scalar fields (moduli) that can be interpreted as the location of the $\overline{\mathrm{D3}}$-brane in the compact space. In this paper we study these moduli.

We begin in Section II by reviewing the KKLT construction, in which the moduli are stabilized near a conifold singularity such that the compactification includes a Klebanov-Strassler (KS) [3] type "throat," generating a hierarchy by a factor of the small warp factor $a_0$ at the tip of the throat [4], and a $\overline{\mathrm{D3}}$-brane is then added at the tip of the throat. In Section III we discuss the mass of the open string moduli corresponding to the position of the $\overline{\mathrm{D3}}$-brane. We argue that in the limit of an infinite throat these moduli are massless since they are Goldstone bosons, but when the throat is finite the background is changed and the moduli obtain a mass. We discuss in detail the deviation of the finite throat theory from the infinite throat theory of [3], and we identify the leading deviation which contributes to the mass of the open string moduli. We use the approximate conformal symmetry of the throat theory to classify the deviations, and we find that the leading deviation corresponds to an operator of dimension $\Delta = \sqrt{28} \simeq 5.29$, and that it leads to a mass squared for the open string moduli scaling as $a_0^{\Delta-2} \simeq a_0^{3.29}$. In the interesting limit of large warping, $a_0 \ll 1$, this mass is exponentially lighter than the other mass scales appearing in the warped compactification, implying that the KKLT scenario generally leads to light scalars which could cause phenomenological problems.

In Section IV we suggest a possible way to resolve this problem and increase the mass of the moduli, by position-

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ing two of the orientifold 3-planes (which must be present anyway in KKLT-type compactifications) at the tip of the throat, and adding to them half-D3-branes so that they become $O3^+$-planes rather than $O3^-$-planes. The $\overline{D3}$-brane is then attracted to these $O3^+$-planes, increasing the mass of the open string moduli. The mass squared is still smaller than the typical mass scales, but only by a factor of the string coupling $g_s$ which does not have to be very small, so this may not lead to phenomenological problems (especially if the standard model fields live in a different position in the Calabi-Yau manifold and couple very weakly to the $\overline{D3}$-brane fields). Our scenario has the added advantage that by adding two half D3-branes in addition to the $\overline{D3}$-brane we do not generate a tadpole for the D3-brane charge, unlike the original KKLT scenario where such a tadpole exists and leads to subtleties in using the probe approximation for describing the $\overline{D3}$-brane (due to the necessity to change the background elsewhere to compensate for the $\overline{D3}$-brane charge).

Finally, in two appendices we derive some results used in the text. In Appendix A we list the possible deformations of the $AdS_5 \times T^{1,1}$ background (which is a good approximation to the throat) which can appear as deformations of the throat in our background. In Appendix B we discuss the moduli space of the gauge theory dual to the throat region after deformations by superpotential operators, and we argue that any such deformations reduce the dimension of the moduli space.

## II. A REVIEW OF DS FLUX COMPACTIFICATIONS WITH $\overline{D3}$-BRANES

The setting for our analysis in the following sections is the dS background of KKLT [2]. We start with a brief overview of a general flux compactification and then proceed to describe the construction of the dS background. More details can be found in [2,4,5].

### A. Warped flux compactifications

We consider type IIB string theory in the supergravity approximation, described in the Einstein frame by the action

$$S_{IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big\{\mathcal{R} - \frac{\partial_M\tau\partial^M\bar{\tau}}{2(\mathrm{Im}\tau)^2} - \frac{G_3\cdot\bar{G}_3}{12\mathrm{Im}\tau} - \frac{\tilde{F}_5^2}{4\cdot 5!}\Big\} + \frac{1}{8i\kappa_{10}^2}\int\frac{C_4\wedge G_3\wedge G_3}{\mathrm{Im}\tau} + S_{\mathrm{local}}, \quad (2.1)$$

where $\tau = C_0 + ie^{-\phi}$ is the axio-dilaton field and we combine the RR and NS-NS threeform fields into the generalized complex threeform field $G_3 = F_3 - \tau H_3$. In addition one must impose a self duality condition on the fiveform $\tilde{F}_5 \equiv F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3$,

$$\tilde{F}_5 = *\tilde{F}_5. \quad (2.2)$$

The local action $S_{\mathrm{local}}$ includes the contributions from additional local objects such as D-branes or orientifold planes.

We begin by considering warped backgrounds, with a metric of the form

$$ds_{10}^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}(y)dy^m dy^n, \quad (2.3)$$

where $\mu, \nu = 0, 1, 2, 3$; $m, n = 4, \cdots, 9$, and the unwarped metric $\tilde{g}_{mn}$ scales as $\sigma^{1/2}$, where $\sigma$ is the imaginary component of the complex Kähler modulus related to the overall scale of the compact Calabi-Yau manifold. In addition, both the fiveform and threeform fields are turned on. Because of 4-dimensional Poincaré invariance only compact components of $G_3$ may be turned on, while for the fiveform, the Bianchi identity determines it to be of the form

$$\tilde{F}_5 = (1 + *)d\alpha(y)\wedge dx^0\wedge dx^1\wedge dx^2\wedge dx^3. \quad (2.4)$$

Finally, local objects extended in the four noncompact dimensions can be added wrapping cycles of the compact space. These must satisfy the tadpole cancellation condition

$$\frac{1}{2\kappa_{10}^2 T_3}\int_{\mathcal{M}_6} H_3\wedge F_3 + Q_3^{\mathrm{local}} = 0, \quad (2.5)$$

where $Q_3^{\mathrm{local}}$ is the D3-brane charge of the local objects.

The supergravity equations of motion for such a configuration of fields can be conveniently written in terms of the following combinations of the fiveform and warp factor

$$\Phi_\pm \equiv e^{4A} \pm \alpha. \quad (2.6)$$

The Einstein equation and the Bianchi identity for the fiveform field can be combined to give

$$\tilde{\nabla}^2\Phi_\pm = \frac{e^{2A}}{6\mathrm{Im}\tau}|G_\pm|^2 + e^{-6A}|\nabla\Phi_\pm|^2 + \mathrm{local}, \quad (2.7)$$

where we defined the imaginary self dual (ISD) and imaginary antiself dual (IASD) components of the generalized threeform flux,

$$G_\pm = iG \pm *_6 G \qquad \Rightarrow \qquad *_6 G_\pm = \pm iG_\pm. \quad (2.8)$$

The local objects act as sources for the fields $\Phi_\pm$. D3-branes and O-planes appear as sources only in the equation for $\Phi_+$, while $\overline{D3}$-branes appear only in the equation for $\Phi_-$. For a background with no $\overline{D3}$-branes there are no sources for $\Phi_-$, so we get using (2.7) and the compactness of the Calabi-Yau manifold

$$\Phi_- = 0 \Rightarrow \alpha = e^{4A}. \quad (2.9)$$

Since $|G_-|^2$ is positive definite it must vanish everywhere and so $G_3$ is ISD.

The equations of motion can be compactly summarized by a 4-dimensional superpotential [5]

$$W = \int \Omega \wedge G_3, \tag{2.10}$$

where $\Omega$ is the holomorphic (3,0) form, together with the standard supergravity Kähler potential. This notation makes explicit the fact that the equations give nontrivial restrictions on some of the moduli. The superpotential depends both on the axio-dilaton (through its appearance in $G_3$) and on the geometrical complex structure moduli that appear in $\Omega$. However, the resulting 4-dimensional supergravity theory is of the no-scale class. The Kähler moduli, including the global volume of the compact manifold, have no potential (in the supergravity approximation) and remain unfixed.

Consider probing this space with D3-branes. The D3-brane action in the Einstein frame without turning on any open string fields, including both the DBI and the Wess-Zumino term, is given by

$$S_{\rm D3} = -T_3 \int \sqrt{g_4} d^4 x \Phi_-. \tag{2.11}$$

For the type of solutions discussed above, obeying Eq. (2.9), we obtain that these probes feel no force, and their moduli space is the full compact manifold. For $\overline{\rm D3}$-brane probes, due to the opposite sign in the Wess-Zumino term, we find that the action is

$$S_{\overline{\rm D3}} = -T_3 \int \sqrt{g_4} d^4 x \Phi_+. \tag{2.12}$$

In our background where $\Phi_+ = 2e^{4A}$ there is thus a force on the $\overline{\rm D3}$-brane driving it towards smaller values of the warp factor.

### B. Getting a hierarchy from the conifold

It is phenomenologically interesting to find a background in which, in addition to fixing the moduli, there is a large warped throat. This can be used to realize the construction of Randall and Sundrum [6–8], giving a solution to the hierarchy problem. Such a background was found in [4] by considering a generic Calabi-Yau manifold near a special point in its moduli space where it develops a singularity. Generically such a singularity looks locally like the conifold singularity [9] which can be described by the submanifold of $\mathbb{C}^4$ defined by:

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0. \tag{2.13}$$

The conifold is a cone whose base is $T^{1,1} = (SU(2) \times SU(2))/U(1)$, a fibration of $S^3$ over $S^2$. The cone is singular at $(z_1, z_2, z_3, z_4) = (0, 0, 0, 0)$ where the spheres shrink to zero size. The isometry group of the base geometry is easily seen to be $SU(2) \times SU(2) \times U(1)$, where the $SU(2) \times SU(2) \simeq SO(4)$ rotates the $z_i$'s and the $U(1)$ adds a constant phase $z_i \to e^{i\alpha} z_i$.

The singularity of the conifold can be smoothed in two ways, by blowing up either of the spheres to a finite size. We will be interested in the deformation of the conifold, which is the submanifold given by

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = \mu, \tag{2.14}$$

where $\mu$ becomes a complex structure modulus for this manifold. Geometrically, in (2.14) the $S^2$ shrinks to zero size at the tip while the $S^3$ remains at some finite size. The minimal size $S^3$ at the "tip" is given by

$$|z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 = |\mu|. \tag{2.15}$$

This deformation breaks the symmetry group to $SU(2) \times SU(2) \times \mathbb{Z}_2$, where the $SO(4)$ can be understood geometrically as rotations of the $S^3$.

Placing $M$ fractional D3-branes at a conifold singularity, the background near the singularity is given, for large $g_s M$ and for some range of radial distances from the singularity, by the KS solution [3], where in the near horizon geometry one replaces the branes by fluxes. Such a configuration involves turning on $M$ units of $F_3$ flux on the $S^3$ at the tip of the conifold, and also $(-K)$ units of $H_3$ flux on the dual cycle (which is noncompact in [3] but is compact when we embed this into a compact Calabi-Yau manifold). It is customary to define $N = MK$. In [4] it was found that such fluxes generate a warped throat similar to [3] near the singularity. The superpotential stabilizes the complex structure modulus $\mu$ at a value for which the warp factor at the tip of the throat is given by

$$a_0 \equiv e^{A_0} = e^{-2\pi K/3Mg_s}, \tag{2.16}$$

which is exponentially small when $K \gg g_s M$ (the validity of the supergravity approximation in the throat requires also $g_s M \gg 1$).

In the throat region of the Calabi-Yau manifold the warp factor is given by

$$e^{-4A} = \frac{27\pi}{4u^4} \alpha'^2 g_s N\left(1 + \frac{g_s M}{K}\left(\frac{3}{8\pi} + \frac{3}{2\pi} \ln\left(\frac{u}{u_0}\right)\right)\right), \tag{2.17}$$

where $u$ is the radial coordinate along the throat. At the tip of the throat the redshift is minimal and given by (2.16). There we get $u \sim Ra_0$, where we defined $R^4 = \frac{27}{4}\pi\alpha'^2 g_s N$. The bulk of the Calabi-Yau manifold, where the warp factor is of order unity (and deviations from (2.17) are large) is at $u \sim R$.

### C. Lifting to a dS background

Although phenomenologically interesting, backgrounds of this type classically have at least one scalar modulus. The low-energy theories we arrive at are no-scale models, and the potential generated by the fluxes does not give any mass term for the Kähler modulus related to the volume of the compact space. This was mended in [2] by considering nonperturbative effects. Terms in the potential coming either from instantons in non-Abelian gauge groups on a stack of D7-branes or from Euclidean D3-branes wrapped on 4-cycles depend on the volume of the space, and stabilize it at some finite value.

The stabilization of the Kähler modulus leads to a vacuum with a negative cosmological constant, an AdS space. It was then argued that adding an $\overline{\mathrm{D3}}$-brane (which, as discussed above, should sit at the tip of the throat to be stable) results in a positive contribution to the scalar potential from (2.12), and with some tuning of the parameters it can lift the minimum of the potential to a small positive value. Thus it is possible to get a de-Sitter space with a small cosmological constant.

## III. THE $\overline{\mathrm{D3}}$-BRANE MODULI

A consequence of the introduction of an $\overline{\mathrm{D3}}$-brane to the warped background is the addition of new light scalar fields from the open strings ending on the $\overline{\mathrm{D3}}$-brane, corresponding to the position of the $\overline{\mathrm{D3}}$-brane on the compact space. In this section we analyze the potential for these moduli in the KKLT background. We first consider the background without the additional $\overline{\mathrm{D3}}$-brane, and estimate the deviation of the warped background with a compact Calabi-Yau manifold from the noncompact background of [3]. We then use this to estimate the masses for the position of the $\overline{\mathrm{D3}}$-brane, using the action (2.12) and considering the $\overline{\mathrm{D3}}$-brane as a probe (as in [2]). This approximation is valid when $g_s \ll 1 \ll g_s M$.

From (2.12) we see that the $\overline{\mathrm{D3}}$-branes are not free to move on the compact space since they have a nontrivial potential proportional to the warp factor. This potential drives them to the tip of the throat where the warp factor is minimal, giving a mass to the scalar field corresponding to the radial position of the $\overline{\mathrm{D3}}$-brane.

At the tip of the throat, the $\overline{\mathrm{D3}}$-brane can still move on the $S^3$. In the full infinite KS solution there is an exact $SO(4)$ symmetry corresponding to rotations in this 3-sphere, and placing the $\overline{\mathrm{D3}}$-brane breaks this symmetry as $SO(4) \rightarrow SO(3)$. This gives rise to three massless moduli, the three Goldstone bosons, which can also be interpreted as the three coordinates of the position of the $\overline{\mathrm{D3}}$-brane in the $S^3$.

In our background there are, however, corrections coming from the compactness of the Calabi-Yau manifold, as the background deviates from the KS solution away from the tip. From the point of view of the field theory dual of the KS background, these corrections are related to UV perturbations (changes in coupling constants). Some of these corrections explicitly break the $SO(4)$ symmetry, and thus generate a mass for the Goldstone bosons. We will first classify the possible perturbations that can be turned on in this class of backgrounds, and then go on to consider their effect on the mass for the three moduli of the $\overline{\mathrm{D3}}$-brane.

### A. UV corrections of the background

The deformation of our background away from the KS geometry, at large radial position away from the conifold, is easily described in the language of the dual field theory. The dual theory (at some cutoff scale) has an $SU(N) \times SU(N+M)$ gauge group, with gauge superfields $W_1$ and $W_2$ corresponding to the two gauge groups, and two doublets of chiral superfields $A_i$, $B_i$ ($i = 1, 2$) in the $(N, \overline{N+M})$ and $(\overline{N}, N+M)$ representations, respectively, of the $SU(N) \times SU(N+M)$ group, and in the $(0, \frac{1}{2})$ and $(0, \frac{1}{2})$ representations of the global $SU(2) \times SU(2)$ symmetry.

In the dual description the region near the singularity describes the low-energy physics of the field theory while the Calabi-Yau region end of the throat serves as a UV cutoff of the field theory. Deforming the solutions at large radial position is described by changing the theory at some large UV scale where the effective theory is some deformation of the KS theory,

$$\mathcal{L} = \mathcal{L}_{KS} + c_i \int \mathcal{O}_i. \tag{3.1}$$

Generally all possible operators might be turned on at this scale, and they could influence the $\overline{\mathrm{D3}}$-brane at the tip (the IR limit) and give a mass to the moduli. Because of the renormalization group flow the contribution to the mass of the $\overline{\mathrm{D3}}$-brane at the tip will be dominated by the most relevant operators at the IR, namely, the lowest dimension operators. Relevant and marginal operators will have a large effect, while that of the irrelevant operators will be suppressed.

It is sufficient to analyze the operators and their dimensions in the conformal case [10] where the gauge group is $SU(N) \times SU(N)$, since the cascading case is expected to behave similarly up to log corrections and operator mixings which should not change our conclusions. For this case the classification of all supergravity KK-modes on $T^{1,1}$ and the corresponding operators in the field theory was given in [11,12]. Since we are only interested in turning on operators that break neither 4-dimensional Lorentz invariance nor supersymmetry, we can restrict our attention to the highest components of the different superconformal multiplets and consider only those that are Lorentz scalars. The only possible operators come from vector multiplets of the 5-dimensional gauged supergravity which arises by KK reduction on $T^{1,1}$, either long multiplets or chiral multiplets.

The analysis of supergravity modes is carried out in Appendix A, where we find only one possible relevant operator

$$S_1 = \int d^2\theta \mathrm{Tr}(A^i B^j), \qquad i, j = 1, 2, \qquad \Delta_{S_1} = 2.5, \tag{3.2}$$

and three possible marginal operators

$$S_2 = \int d^2\theta \mathrm{Tr}(A^i B^j A^k B^l), \qquad \Delta_{S_2} = 4,$$
$$\Phi_0 = \int d^2\theta \mathrm{Tr}(W_1^2 + W_2^2), \qquad \Delta_{\Phi_0} = 4, \qquad (3.3)$$
$$\Psi_0 = \int d^2\theta \mathrm{Tr}(W_1^2 - W_2^2), \qquad \Delta_{\Psi_0} = 4.$$

The operator $S_2$ is symmetric in $(i, k)$ and $(j, l)$; the antisymmetric combination mixes with $\Phi_0$. There is also an infinite number of irrelevant operators, all of them with dimensions $\Delta \geq 5.29$.

In fact not all possible operators are turned on in the compact Calabi-Yau manifold background. As discussed above, a probe D-brane in this background must feel no force and its moduli space should describe the full 6-dimensional compact geometry. In Appendix B it is found that the addition of the operators $S_1$, $S_2$ changes the moduli space drastically and necessarily results in a force on the D3-brane. Thus, these operators are not turned on in the warped flux compactifications.

The two marginal operators, $\Phi_0$ and $\Psi_0$, can be turned on, but they are symmetric under the $SU(2) \times SU(2)$ and do not lead to symmetry breaking and to a mass for the $\overline{\mathrm{D3}}$-brane moduli. From the field theory perspective they correspond to changing the coupling constants that are already present in the nondeformed theory and do not generate new terms in the action.

### B. Masses from UV corrections

In the previous subsection we have seen that relevant operators are not turned on in the warped background, while the possible marginal operators do not break the symmetry and leave the moduli massless. Irrelevant operators, however, can be turned on, and we next discuss the masses generated by them. As discussed in Appendix A, the various operators which preserve SUSY and Lorentz invariance are related to Kaluza-Klein modes of the warped metric on the $T^{1,1}$ $\hat{g}_{ij}$, the field $\Phi_+$ defined in (2.6), the threeform field $G_3$ and the axio-dilaton $\tau$.

The operators are turned on at the UV cutoff, and in order to consider their effect on the IR physics we need to discuss their flow, or in the supergravity language their profile along the radial coordinate. We start by considering the profiles of the fields corresponding to these operators on $AdS_5 \times T^{1,1}$, using the metric $ds^2_{AdS} = u^2 dx^\mu dx_\mu + du^2/u^2$. There are two independent solutions for the field $\phi$ corresponding to an operator of dimension $\Delta > 2$, with the following $u$-dependence:

$$\phi(u) = au^{-\Delta} + bu^{\Delta-4}. \qquad (3.4)$$

In the KS background there are small logarithmic corrections to this, and in addition the behavior near the tip of the throat gives some IR boundary condition for the field equations. Generically this implies that at $u = Ra_0$ the two terms are of the same order. Then, at the UV cutoff $u \sim R$ (the Calabi-Yau), the second term will dominate so

$$\phi(u \sim R) \simeq bR^{\Delta-4}. \qquad (3.5)$$

Deforming the theory at the UV by some $\delta\phi(R) \sim \phi_0$ will then correspond in the IR to

$$\delta\phi(Ra_0) \sim \phi_0 \frac{(Ra_0)^{\Delta-4}}{R^{\Delta-4}} = \phi_0 a_0^{\Delta-4}. \qquad (3.6)$$

The deformation in the IR is suppressed for operators with higher dimension, as expected.

The largest contribution to a mass of an object localized near the tip will be from the operator with lowest dimension that breaks the $SU(2) \times SU(2)$ global symmetry. The analysis of the previous subsection and Appendix A implies that this is the lowest component of the vector multiplet I, with $j = l = 1$ and $r = 0$, whose dimension is $\Delta = \sqrt{28} \simeq 5.29$. This operator corresponds in the supergravity to a KK mode of the warped metric $\hat{g}_{ij}$. We do not see any reason why this operator should not appear in the CY compactification so we assume that it does.[1] At the UV we have $\hat{g}_{ij} \sim \sigma^{1/2}$, and we expect the deformation of the metric to be of the same order as the metric so we can approximate $\delta\hat{g}_{ij}|_{\mathrm{UV}} \sim \sigma^{1/2}$ and

$$\delta\hat{g}_{ij}|_{\mathrm{IR}} \sim \sigma^{1/2} a_0^{\Delta-4} = \sigma^{1/2} a_0^{1.29}. \qquad (3.7)$$

In order to evaluate the corresponding mass we need to write in more detail the action on a probe $\overline{\mathrm{D3}}$-brane. We consider a $\overline{\mathrm{D3}}$-brane filling the noncompact space-time and positioned at the point $X^m$ in the compact Calabi-Yau, which is near the tip of the throat. Expanding around this position, we get the action (2.12) with an additional kinetic term. Using the solution of the supergravity equations for the fiveform (2.9) we get

$$S_{\overline{\mathrm{D3}}} = -2T_3 \int \sqrt{g_4} d^4x e^{4A(X^m)} - T_3 \alpha' \int \sqrt{g_4} d^4x \frac{1}{2} g_4^{\mu\nu} \partial_\mu X^m \partial_\nu X^n \tilde{g}_{mn}, \qquad (3.8)$$

where $\tilde{g}_{mn}$ is the unwarped metric, $\tilde{g}_{mn} \simeq a_0^2 \hat{g}_{mn}$. Mass terms appear in this action only through the dependence of the warp factor, $e^{4A}$, on the position $X^m$. Since in the full noncompact case the warp factor has only radial dependence, for a mass to be generated in the $S^3$ directions we need to consider the change in the warp factor due to the deformed supergravity fields $\hat{g}_{ij}$.

Tracing the Einstein equation and using (2.9) we can write the equation of motion for the warp factor as

$$\hat{\nabla}^2 A = \frac{g_s}{48} |G|^2, \qquad (3.9)$$

where $\hat{\nabla}^2$ is the Laplacian on the warped compact space

[1] Note that this operator deforming the throat seems to be different from the one analyzed in the appendix of [13].

and we use the warped metric to raise and lower indices. The change in $A$ due to the deformation in $\hat{g}_{ij}$ will thus satisfy

$$\hat{\nabla}^2 \delta A = \frac{g_s}{48} G_{m_1 n_1 i} G^*_{m_2 n_2 j} \hat{g}^{m_1 m_2} \hat{g}^{n_1 n_2} \delta \hat{g}^{ij} \sim a_0^{\Delta - 4}. \quad (3.10)$$

In this equation we dropped a term $(\delta \hat{\nabla}^2)A$ since the change in the Laplacian due to the deformation in the compact metric $\hat{g}_{ij}$ will be proportional to derivatives in those directions, while the original $A$ has no such dependence and so this term vanishes.

The masses arise due to the variation of $A$ in the $S^3$ at the tip, where the $\overline{\rm D3}$-brane position is parametrized by $X^i$. Since in the warped metric we are using, this 3-sphere has constant size (with a radius $\sim\sqrt{g_s M}$), we can estimate

$$A \sim A_0 + (g_s M)^{-1} a_0^{\Delta - 4} \hat{g}_{ij} X^i X^j. \quad (3.11)$$

Plugging into (3.8) we find

$$S_{\overline{\rm D3}} \sim -T_3 \int \sqrt{g_4} d^4 x \bigg[ 2a_0^4 + 2(g_s M)^{-1} a_0^{\Delta - 2} \tilde{g}_{ij} X^i X^j + \frac{\alpha'}{2} g_4^{\mu\nu} \partial_\mu X^m \partial_\nu X^n \tilde{g}_{mn} \bigg] \quad (3.12)$$

where we changed the metric to the unwarped metric in both kinetic and mass terms.[2] We see that a mass term was generated with a mass of the order of $m^2 \sim (g_s M \alpha')^{-1} a_0^{\Delta - 2} = (g_s M \alpha')^{-1} a_0^{3.29}$.

We see that the deformation of the theory at the UV does indeed generate mass terms for the open string moduli. However, the highest contribution is of order $a_0^{3.29}$. In the warped background the typical IR mass scale is of order $a_0^2$, so the mass generated here is exponentially smaller (given (2.16)). In the Klebanov-Strassler background there are presumably subleading logarithmic corrections to this result, however it is still highly suppressed. Such light moduli would lead to phenomenological problems if we try to use such a scenario to describe the real world.

## IV. A LARGE OPEN STRING MODULI MASS FROM O-PLANES

It is possible to obtain a higher mass for the open string moduli by using O-planes. In this section we calculate this mass. Recall that integrating the supergravity equation of motion (2.7) on the compact space we get that the left-hand side vanishes since there are no boundaries. The right-hand side is positive definite, except for possible negative contributions in the local terms corresponding to orientifold planes. Hence in general we must have orientifold 3-planes in order to be able to solve the equations of motion. It is then natural to try and use these orientifolds for the purpose of stabilizing the moduli for the $\overline{\rm D3}$-brane, by choosing the position of these orientifolds to be at the tip of the throat.

For simplicity we consider an orientifold of the noncompact Klebanov-Strassler solution. Since the analysis is local, embedding this into the full background will not change the conclusions. The action of the orientifold is defined as in [15] by

$$(z_1, z_2, z_3, z_4) \to (z_1, -z_2, -z_3, -z_4). \quad (4.1)$$

This orientifold has two fixed points, both on the tip of the deformed conifold (2.15) at the poles of the $S^3$, $(z_1, z_2, z_3, z_4) = (\pm\sqrt{\mu}, 0, 0, 0)$. Physically there are two O3-planes at these points, which will interact with the $\overline{\rm D3}$-brane and generate a potential for its position on the 3-sphere. Note that the addition of the O3-planes has no effect on the supersymmetry of the model, since the O3-planes break the same supercharges as the fluxes.

The D3 charge of an orientifold plane as well as its tension is negative (equal to $-1/4$ that of a D3-brane), while for $\overline{\rm D3}$-branes the charge is negative and the tension is positive. We see that both effects result in a repulsive force, so that the $\overline{\rm D3}$-brane does not get stabilized but rather it would want to sit on the equator of the $S^3$. However one can use half-D3-branes to fix the situation. Putting a half-D3-brane on the orientifold singularity, we get an $O3^+$-plane with the opposite charge and tension. Since we have two singular points we can add two such half-D3-branes to make both O-planes positively charged. Note that the insertion of one additional unit of D3-brane charge is actually a necessity once we introduce the $\overline{\rm D3}$-brane, due to the tadpole cancellation condition. Assuming that without $\overline{\rm D3}$-branes the background with $O3^-$-planes is a solution of the supergravity, inserting the $\overline{\rm D3}$-brane will cause a deficiency in D3 charge, which can be resolved by the extra two half-D3-branes. Note that an $\overline{\rm D3}$-brane cannot annihilate with a half-D3 so the solution should still be (meta)-stable.

The potential between the $\overline{\rm D3}$-brane and the $O3^+$-plane can be calculated by world sheet methods [16]. The first contribution which depends on the distance between the $\overline{\rm D3}$-brane and the orientifold comes from the Möbius strip, and is equal to

$$\mathcal{M} = 2T_3 g_s \sum_{n=0}^{\infty} c_n r^{2n}, \quad (4.2)$$

where $r$ is the distance (in string units) between the $\overline{\rm D3}$-brane and the $O3^+$-plane, namely $r^2 = \hat{g}_{ij} X^i X^j$, and the coefficients are

$$c_n = (-1)^{n+1} k_{n-3} \bigg( \frac{2^{n-4} \pi^{(2-n)/2}}{n!} \bigg), \quad (4.3)$$

where $k_{n-3}$ is a positive number for $n = 0, 1$. This com-

[2]Note that the first term, even though naively it is independent of the volume $\sigma$ of the compact space, actually does give a potential for the volume factor [2] when we rescale the 4-dimensional metric canonically [14].

putation was done in flat space, but for large $g_sM$ the curvature is small and it is a good approximation.

The first term in the expansion is a correction to the energy which is independent of the distance. The second term is a quadratic potential for the position of the $\overline{\rm D3}$-brane which describes attraction between the $\overline{\rm D3}$-brane and the O3$^+$-plane. The contribution to the action is

$$-2T_3 \int \sqrt{g_4} d^4 x a_0^4 g_s c_1 \hat{g}_{ij} X^i X^j = -2T_3 \int \sqrt{g_4} d^4 x a_0^2 g_s c_1 \tilde{g}_{ij} X^i X^j, \quad (4.4)$$

so that the mass of the $X^i$ fields is

$$m^2 \sim \frac{2c_1}{\alpha'} g_s a_0^2. \quad (4.5)$$

We see that the orientifold gives these fields a mass of order $m^2 \sim g_s a_0^2$, which is the same scale as generic low mass scales in this background. Since this mass comes from the Möbius strip, it is suppressed by a factor of $g_s$ compared to other masses so these open string moduli are still light, but not exponentially as before, so hopefully they should not cause phenomenological problems.

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## APPENDIX A: LOW DIMENSIONAL OPERATORS FROM SUPERGRAVITY ANALYSIS

The Kaluza-Klein spectroscopy for the supergravity fields on $AdS_5 \times T^{1,1}$ was carried out in [11,12]. In this section we will review their results for the dimensions of the corresponding operators as a function of their $SU(2) \times SU(2) \times U(1)_r$ quantum numbers $j$, $l$, $r$. By considering the group theoretic restrictions on these quantum numbers for each multiplet, we will be able to find the operators with lowest dimension.

In general we expand the fields in spherical harmonics on the 5-dimensional compact space $T^{1,1} = \frac{SU(2)\times SU(2)}{U(1)}$, with fields of different spins on the compact space expanded using the corresponding $SO(5)$ harmonics. These harmonics furnish representations of the isometry (global symmetry) group, $SU(2) \times SU(2) \times U(1)_r$ in our case, but not all representations appear in the expansion. The specific participating representations depend on the Lorentz properties of the fields, but it turns out that all representations satisfy that either both $SU(2)$ spins $j$ and $l$ are integers or both are half integers.

All resulting modes can be arranged into multiplets of the $\mathcal{N} = 1$, $d = 4$ superconformal algebra. There are nine types of multiplets—one graviton multiplet, four gravitini, and four vector multiplets [11,12]. For specific values of the quantum numbers, some multiplets obey a shortening condition and become semilong, massless, or chiral multiplets.

For the current analysis we are interested in operators that can be turned on at some UV cutoff without breaking 4-dimensional Lorentz invariance or supersymmetry. Hence the relevant multiplets are only those with scalars as the highest component. These are only the vector multiplets, either with generic values of the quantum numbers or when they obey the condition for shortening to chiral multiplets. The top components of these multiplets are related to Kaluza-Klein modes of the warped 5D metric on the $T^{1,1}$ $\hat{g}_{ij}$, the field $\Phi_+$ defined in (2.6), the threeform field $G_3$, and the axio-dilaton $\tau$.

The first vector multiplet (vector multiplet I in the notations of [11,12]) has a top component related to the Kaluza-Klein modes of the warped 5D metric on the $T^{1,1}$ $\hat{g}_{ij}$, both when it is long and when it obeys the chiral shortening condition. The dimension of this multiplet, defined as the dimension of the lowest component, is given by

$$\Delta = \sqrt{H(j, l, r) + 4} - 2, \quad (A1)$$

where

$$H(j, l, r) \equiv 6\left(j(j+1) + l(l+1) - \frac{r^2}{8}\right), \quad (A2)$$

with $(j, l, r)$ the quantum numbers for the representation of the $SU(2) \times SU(2) \times U(1)_r$ symmetry group. The lowest component $b$, coming from a linear combination of the fiveform and the warp factor $\Phi_- = e^{4A} - \alpha$, is expanded in scalar harmonics that satisfy that $r$ is even (odd) for $j, l$ integers (half integers) and $|r| \leq 2\min(j, l)$.

Small dimensions arise when $H$ is small. Because of the $1/8$ factor in the third term, large values of $j$ and $l$ cannot be compensated by large values of $r$ and will give higher values. It is then enough to look at small values for $j$ and $l$. The lowest values and corresponding quantum numbers are written in Table I. In addition, for each multiplet one can check whether it obeys some shortening condition and what is the dimension of the operator corresponding to the top component. The $j = l = r = 0$ chiral operator can in fact be gauged away, so among the physical scalar operators we are left with one relevant operator and one marginal operator, and all others are irrelevant.

TABLE I. Lowest dimensional operators from vector multiplet I.

| $j$ | $l$ | $\|r\|$ | $H$ | $\Delta$ | Type | $\Delta_{top}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | chiral | 1 |
| 1/2 | 1/2 | 1 | 8.25 | 1.5 | chiral | 2.5 |
| 0 | 1 | 0 | 12 | 2 | semilong | ··· |
| 1 | 0 | 0 | 12 | 2 | semilong | ··· |
| 1 | 1 | 2 | 21 | 3 | chiral | 4 |
| 1 | 1 | 0 | 24 | 3.29 | none | 5.29 |
| 1/2 | 3/2 | 1 | 26.25 | 3.5 | semilong | ··· |
| 3/2 | 1/2 | 1 | 26.25 | 3.5 | semilong | ··· |

A similar analysis can be done for vector multiplet II for which the top component is related to $\Phi_+$. This multiplet does not satisfy any shortening condition. The dimension of the multiplet is given by a similar expression

$$\Delta = \sqrt{H(j, l, r) + 4} + 4. \qquad \text{(A3)}$$

In this case the top component is itself a mode of a ten-dimensional scalar field so the quantum numbers satisfy the same inequality as in the previous case. The lowest dimensional operator has $H = 0 \rightarrow \Delta = 6 \rightarrow \Delta_{top} = 8$ which is already irrelevant. Some of the low dimensional operators are described in Table II.

For the vector multiplet III, the top component (whether or not the multiplet obeys a shortening condition) is related to the threeform field $G_3$, and the dimension of the multiplet is

$$\Delta = \sqrt{H(j, l, r + 2) + 4} + 1. \qquad \text{(A4)}$$

For this multiplet none of the fields are expanded in scalar harmonics. Instead, the top component $a$ (which is the same for both the long and chiral multiplets) originating from the ten-dimensional twoform potential is expanded using the twoform harmonics. For these harmonics we again have that $r$ is even (odd) for $j$, $l$ integers (half integers), but now the restriction on the quantum numbers is $|r| \leq 2\min(j, l) + 2$.

TABLE II. Lowest dimensional operators from vector multiplet II.

| $j$ | $l$ | $\|r\|$ | $H$ | $\Delta$ | Type | $\Delta_{top}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 6 | none | 8 |
| 1/2 | 1/2 | 1 | 8.25 | 7.5 | none | 9.5 |
| 0 | 1 | 0 | 12 | 8 | none | 10 |
| 1 | 0 | 0 | 12 | 8 | none | 10 |
| 1 | 1 | 2 | 21 | 9 | none | 11 |
| 1 | 1 | 0 | 24 | 9.29 | none | 11.29 |
| 1/2 | 3/2 | 1 | 26.25 | 9.5 | none | 11.5 |
| 3/2 | 1/2 | 1 | 26.25 | 9.5 | none | 11.5 |

TABLE III. Lowest dimensional operators from vector multiplet III.

| $j$ | $l$ | $r$ | $H$ | $\Delta$ | Type | $\Delta_{top}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | −3 | 2 | chiral | 4 |
| 1/2 | 1/2 | 1 | 2.25 | 3.5 | chiral | 5.5 |
| 1/2 | 1/2 | −1 | 8.25 | 4.5 | none | 6.5 |
| 0 | 1 | 0 | 9 | 4.61 | none | 6.61 |
| 1 | 0 | 0 | 9 | 4.61 | none | 6.61 |

In this case we also have nontrivial restrictions from the unitarity bounds

$$2 - \Delta \leq \frac{3}{2} r \leq \Delta - 2. \qquad \text{(A5)}$$

The possible quantum numbers are given in Table III.

Finally, vector multiplet IV has dimension given by

$$\Delta = \sqrt{H(j, l, r - 2) + 4} + 1. \qquad \text{(A6)}$$

For long multiplets the top component is related to the threeform field $G_3$ while for multiplets satisfying a chiral shortening condition the top component is related to the axio-dilaton $\tau$. This last mode appears in all vector multiplets of this type (though it is not always the top component) and it is expanded in scalar harmonics. Its $r$-charge is equal to $r - 2$ so the quantum numbers obey $|r - 2| \leq 2\min(j, l)$. These are shown in Table IV. Since the dimension depends only on $r - 2$ we get a similar table to that of vector multiplet I but with the dimensions shifted.

To summarize this appendix we include in Table V the lowest dimension operators found above and their dimensions, as well as the form of the corresponding operators in the field theory when it is known. The first irrelevant operator is the top component of a long multiplet and contributes to the Kähler potential, but its exact form is not known.

## APPENDIX B: THE MODULI SPACE OF THE DEFORMED THEORY

In this appendix we consider deforming the superpotential of the Klebanov-Witten theory [10] by the relevant and marginal operators $S_1$ and $S_2$ given in (3.2) and (3.3). The resulting moduli space is analyzed and shown to be of lower dimension than the original symmetric product of $N$ copies of the conifold. In the dual gravity description this must be due to a deformation exerting a force on the D-branes. Such a deformation is forbidden by the equations of motion in our construction, so we conclude that these operators are not turned on. For the case of a Klebanov-Strassler background the conclusion should remain the same.

In the $SU(N) \times SU(N)$ theory with superpotential $W = h\epsilon_{ik}\epsilon_{jl}\mathrm{tr}(A^i B^j A^k B^l)$ the F-term equations require that the

TABLE IV. Lowest dimensional operators from vector multiplet IV.

| $j$ | $l$ | $r$ | $H$ | $E_0$ | Type | $\Delta_{top}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 3 | chiral | 4 |
| 1/2 | 1/2 | 1 | 8.25 | 4.5 | chiral | 5.5 |
| 0 | 1 | 0 | 12 | 5 | semilong | · · · |
| 1 | 0 | 0 | 12 | 5 | semilong | · · · |
| 1 | 1 | 2 | 21 | 6 | chiral | 7 |
| 1 | 1 | 0 | 24 | 6.29 | none | 8.29 |
| 1/2 | 3/2 | 1 | 26.25 | 6.5 | semilong | · · · |
| 3/2 | 1/2 | 1 | 26.25 | 6.5 | semilong | · · · |

chiral fields $A_i$, $B_i$, $i = 1, 2$ will commute, so that they can be simultaneously diagonalized by gauge transformations. The D-term equations then lead to the general solution being $N$ copies of the conifold. This branch describes D-branes moving separately on the 6-dimensional geometry. Such a branch must also exist for the deformed theory due to the no-force condition on the D-branes, and we expect that the subspace of diagonal matrices that solve the F-term and D-term equations should give us the $N$th symmetric product of the deformed 6-dimensional geometry.

For diagonal matrices the equations for the $N \times N$ matrices become decoupled so we can consider them as $N$ identical equations for single fields. In this case the moduli space will be the solution to the F-term equations divided by the complexification of the $U(1)$ gauge group, and the original superpotential can be ignored. Since all fields are charged under the $U(1)$ we get

$$\mathrm{Dim}\,(\text{moduli space}) = \mathrm{Dim}(\mathrm{F} - \text{term solutions}) - 2. \tag{B1}$$

Since the dimension should be six, we get that the solutions to the F-term equations should form an 8-dimensional space.

We begin by considering the relevant chiral operator $S_1$. The general deformation of the superpotential is given by:

$$\Delta W = \lambda_{ij}\mathrm{Tr}(A_i B_j), \tag{B2}$$

where $\lambda_{ij}$ is constant matrix. The F-term equations for $1 \times 1$ scalars are:

$$\lambda \cdot B = 0, \qquad \lambda^t \cdot A = 0, \tag{B3}$$

TABLE V. Lowest dimensional operators.

| $\Delta$ | $j$ | $l$ | $\lvert r\rvert$ | Multiplet | Type | Operator |
|---|---|---|---|---|---|---|
| 2.5 | 1/2 | 1/2 | 1 | I | chiral | $S_1 = \int d^2\theta \mathrm{Tr}(AB)$ |
| 4 | 1 | 1 | 2 | I | chiral | $S_2 = \int d^2\theta \mathrm{Tr}[(AB)^2]$ |
| 4 | 0 | 0 | 0 | IV | chiral | $\Phi_0 = \int d^2\theta \mathrm{Tr}(W_1^2 + W_2^2)$ |
| 4 | 0 | 0 | 0 | III | chiral | $\Phi_0 = \int d^2\theta \mathrm{Tr}(W_1^2 - W_2^2)$ |
| 5.29 | 1 | 1 | 0 | I | long | $\mathcal{O}_1 = \int d^4\theta (?)$ |

where we consider $A$, $B$ as 2-vectors and $\lambda$ as a $2 \times 2$ matrix. For $\det(\lambda) \neq 0$ there is no solution to the system of equations. For $\det(\lambda) = 0$, $\lambda \neq 0$ (so $\mathrm{rank}(\lambda) = 1$), there is a 2-dimensional space of complex solutions so the moduli space is 2 dimensional. Only for $\lambda = 0$ we get a moduli space large enough for describing free D-branes.

We now add also the marginal operator $S_2$. The deformed superpotential is

$$\Delta W = \lambda_{ij}\mathrm{Tr}(A_i B_j) + \frac{1}{2}\sigma_{ijkl}\mathrm{Tr}(A_i B_j A_k B_l), \tag{B4}$$

with $\sigma_{ijkl} = \sigma_{kjil} = \sigma_{ilkj}$. The symmetry condition for the indices comes from the fact that the chiral marginal operator is the $j = l = 1$ combination of the four fields. From the cyclicity of the trace we also get $\sigma_{ijkl} = \sigma_{klij}$.

The F-term equations for $1 \times 1$ scalars are now

$$\begin{aligned} \lambda_{ij}B_j + \frac{1}{2}\sigma_{ijkl}B_jA_kB_l + \frac{1}{2}\sigma_{kjil}B_jA_kB_l &= \lambda_{ij}B_j \\ &\quad + \sigma_{ijkl}B_jA_kB_l = 0, \\ \lambda_{ji}A_j + \sigma_{jikl}A_kB_lA_j &= 0, \end{aligned} \tag{B5}$$

where we used the symmetries of $\sigma$. There are 4 complex fields in the equations so the maximal dimension for the space of solutions is 8. If the solutions indeed form an 8-dimensional space then for a generic solution any change $A_i \rightarrow A_i + \delta A_i$ and $B_i \rightarrow B_i + \delta B_i$ will result in a new solution. Taking the first equation and shifting only the $A_i$ fields we find for any $\delta A_i$

$$\begin{aligned} \lambda_{ij}B_j + \sigma_{ijkl}B_jA_kB_l + \sigma_{ijkl}B_j\delta A_kB_l &= 0 \\ \Rightarrow \sigma_{ijkl}B_j\delta A_kB_l &= 0 \\ \Rightarrow \sigma_{ijkl}B_jB_l &= 0. \end{aligned} \tag{B6}$$

Again this holds for all solutions so we can now shift $B_i$ to find

$$\begin{aligned} \sigma_{ijkl}B_jB_l + \sigma_{ijkl}\delta B_jB_l + \sigma_{ijkl}B_j\delta B_l + \sigma_{ijkl}\delta B_j\delta B_l &= 0 \\ \Rightarrow (\sigma_{ijkl} + \sigma_{ilkj})B_j\delta B_l &= 0 \\ \Rightarrow \sigma_{ijkl} + \sigma_{ilkj} &= 0 \\ \Rightarrow \sigma_{ijkl} &= 0. \end{aligned} \tag{B7}$$

Where in the last step we used the symmetry properties of $\sigma_{ijkl}$. Hence we are left with only the relevant deformation which also vanishes by the previous argument.

This argument can be generalized to any higher deformation of this form. Consider the deformation $\mathrm{Tr}(A_{i_1}B_{j_1}\cdots A_{i_n}B_{j_n})$. Since the $SU(2) \times SU(2)$ representation is $j = l = \frac{n}{2}$ the coefficient of this term is symmetric under exchange of the $j$ indices and under exchange of the $i$ indices. We can then carry out a similar argument, where we take at each step another derivative with respect to $A$ or $B$. Because of the symmetry property, each time we will get the same coefficient with one lower power of the fields. After $2n$ steps we will be left with only this term and no lower terms, and will arrive to the conclusion that the coefficient must vanish.

We conclude that turning on these types of deformations the diagonal branch of the moduli space cannot have 6 dimensions. Since the geometry discussed in Section II accommodates D-branes on a 6-dimensional space, these operators are not turned on by deforming the Klebanov-Strassler solution to a compact Calabi-Yau manifold. In particular the relevant and marginal deformations ($n = 1$ and $n = 2$) are not turned on.

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## 2.2 Solving Flavor Puzzles with Quiver Gauge Theories

# Solving flavor puzzles with quiver gauge theories

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We consider a large class of models where the $SU(5)$ gauge symmetry and a Froggatt-Nielsen (FN) Abelian flavor symmetry arise from a $U(5) \times U(5)$ quiver gauge theory. An intriguing feature of these models is a relation between the gauge representation and the horizontal charge, leading to a restricted set of possible FN charges. Requiring that quark masses are hierarchical, the lepton flavor structure is uniquely determined. In particular, neutrino mass anarchy is predicted.

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## I. INTRODUCTION

The charged fermion flavor parameters—quark masses and mixing angles and charged lepton masses—exhibit a structure that is not explained within the standard model (SM). The two puzzling features—smallness and hierarchy—are very suggestive that an approximate horizontal symmetry is at work. The simplest framework that employs such a mechanism to explain the flavor puzzle is that of the Froggatt-Nielsen (FN) mechanism [1]. The various generations carry different charges under an Abelian symmetry. The symmetry is spontaneously broken, and the breaking is communicated to the SM fermions via heavy fermions in vectorlike representations. The ratio between the scale of spontaneous symmetry breaking and the mass scale of the vectorlike fermions provides a small symmetry-breaking parameter. Yukawa couplings that break the FN symmetry are suppressed by powers of the breaking parameter, depending on their FN charge.

Model building within the FN framework usually proceeds as follows. One chooses a value for the small symmetry-breaking parameter(s), and a set of FN charges for the fermion and Higgs fields. These choices determine the parametric suppression of masses and mixing angles. One then checks that the experimental data can be fitted with a reasonable choice of order-one coefficients for the various Yukawa couplings. Thus all FN predictions are subject to inherent limitations:

(i) The FN charges are not dictated by the theory.

(ii) The value of the small parameter is not predicted.

(iii) There is no information on the $\mathcal{O}(1)$ coefficients.

The predictive power of the FN framework is thus limited. There is one relation among the quark flavor parameters that is independent of the choice of horizontal charges [2], and there are three in the lepton sector [3]. The resulting predictions, that suffer from order-one uncertainties, are consistent with the data. Additional relations apply in the supersymmetric extension of the SM, but to provide new tests of the FN mechanism, supersymmetric contributions to flavor changing processes must be explored, and the universal effects of renormalization group equations running should be small [4]. The predictive power is sharply enhanced in the framework of GUT. With an $SU(5)$ gauge symmetry, the number of independent fermion charges is reduced from fifteen to six.

To make further progress, one would like to embed the FN mechanism in a framework where some or all of the inherent limitations described above are lifted. This may happen in string theory. While realistic constructions of the supersymmetric SM in string theory are still under study [5–14], much progress has been made in the search for string-inspired phenomenologically viable extensions of the SM such as the FN framework. The basic idea is that the FN symmetry is a pseudoanomalous $U(1)$ symmetry [15–19]. Then the small parameter depends on the FN charges and, furthermore, if one assumes gauge coupling unification, there is a single constraint on the FN charges that can be translated into a relation between the fermion masses and the $\mu$-term. This idea is based on ingredients of the heterotic string and has led to a detailed investigation of the resulting phenomenology (see, for example, Refs. [20–23]).

On the other hand, we are not aware of any attempt to-date to construct FN models which arise from D-brane configurations [24]. In this paper, we take a step in this direction and consider FN models from quiver gauge theories. These theories arise at low energy as the effective theories on D-branes placed at singular geometries (see [25–30] and references therein). As opposed to the heterotic case, these theories typically have numerous anomalous $U(1)$'s. The anomalies are canceled through the generalized Green-Schwartz (GS) mechanism [31–33]. We employ these anomalous $U(1)$'s as flavor symmetries and construct FN models.

As we show below, the structure of these theories tightly constrains model building and hence the realization of the FN mechanism. As a consequence, we will see that much can be said about the lepton sector. In particular, within the framework of the $SU(5)$ GUT model with a single FN-symmetry-breaking field, there is essentially a single viable model which predicts mass anarchy in the neutrino sector.

It is worth noting that, within the $SU(5)$ GUT model, it is *a priori* difficult to get the correct flavor hierarchy altogether. The reason for this is that **10** fields arise from open strings with both ends residing on the same set of $U(5)$

 

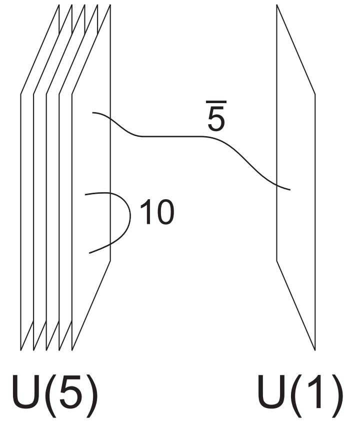


FIG. 1. A D-brane construction with an $SU(5)$ gauge group and a distinct $U(1)_{\rm FN}$. The fundamental fields are strings stretching between the two stacks and are thus charged under the FN group, while the antisymmetric fields connect only to the $U(5)$ stack and have no $U(1)_{\rm FN}$ charge.

stack of branes, while $\bar{\mathbf{5}}$ fields come from strings with one end on the $U(5)$ stack and the other on a different brane which may provide the necessary $U(1)_{\rm FN}$ symmetry. This situation is depicted in Fig. 1. In particular this means that the **10** fields are not charged under the FN symmetry and there is no hierarchy in the up sector. As we show below, one can overcome this problem by extending the gauge symmetry, and the obtained structure is sufficiently restrictive to provide prediction regarding the neutrino sector.

The paper is organized as follows. In Sec. II we discuss quiver gauge theories and their orientifold generalizations. We further discuss Higgsing and the role of anomalous $U(1)$'s (and the problems that accompany them) in such theories. In Sec. III we investigate the embedding of the FN mechanism within quiver gauge theories. We argue that it is difficult to construct models with nonrenormalizable superpotential terms. We focus on the case of a single $U(1)_{\rm FN}$ symmetry that arises from anomalous $U(1)$'s. We show that there are severe restrictions on the possible FN charges, which lead to a generic constraint on the maximal hierarchy in this framework. In Sec. IV we construct FN models in $SU(5)$-GUT theories with a single FN field. We show that there are only three possible FN charges for the **10**-plets, and two for the $\bar{\mathbf{5}}$-plets. This situation makes the theory highly predictive. In particular, requiring that quark masses are hierarchical, a single set of charges is singled out, leading to a unique flavor structure. The flavor structure of the lepton sector is fixed, and neutrino mass anarchy is predicted. We conclude in Sec. V. A full, consistent, quiver realization for the $SU(5)$ GUT theory, with emphasis on anomaly cancellation, is presented in the Appendix.

## II. QUIVER GAUGE THEORIES

A wide class of $\mathcal{N} = 1$ supersymmetric vacua is obtained by placing D-branes on singular manifolds of type II strings such as orbifolds and orientifolds [25–27,29,34–41]. Placing D-branes at such singularities produces at low energy conformal gauge theories [27], while adding fractional $D$-branes breaks the conformal symmetry, rendering a four dimensional chiral gauge theory.

A quiver diagram is an efficient way for describing the gauge theory obtained from the open string sector (for a review, see [30]). The degrees of freedom of oriented strings can be described as strings starting and ending on D-branes. Consequently, the fields in the theory transform in the fundamental of a $U(N_i)$ factor of the gauge group and in the antifundamental of another $U(N_j)$ factor. It is therefore possible to describe the field theory by a quiver diagram, where we denote each $U(N_i)$ factor by a node in the graph and the fields are represented by directed lines connecting two such nodes. The orientation of the line represents the orientation of the string: a line coming out of a node corresponding to a $U(N_i)$ gauge group factor stands for a field in the fundamental $\mathbf{N_i}$, while a line going into a node corresponding to $U(N_j)$ represents a field transforming in the antifundamental $\overline{\mathbf{N_j}}$. A line originating and ending on the same node describes a field in the adjoint representation of the corresponding $U(N)$ factor.

Gauge invariant field combinations, which may be present in the superpotential, can also be seen using the diagrammatic description. A field transforming in the fundamental of a given $U(N)$ must interact with a field in the antifundamental of the same $U(N)$ in order to get an invariant interaction. In other words, if there is a field coming into a given vertex, we must also have a field going out of that same vertex. This has to be the case for all the vertices, so an invariant interaction is described by a closed loop in the quiver diagram. In particular, a renormalizable cubic term in the superpotential is represented in the quiver by a closed triangle. In general, however, not every closed loop in the quiver which originates from a certain geometry appears in the superpotential. The general problem of extracting the spectrum and superpotential from a given geometry is still not solved, and only specific examples are known.

### A. Orientifold quivers

Strictly speaking, gauge theories arising from unoriented string theory are not quivers since the low energy field theory cannot be described by a directed graph. Nevertheless, these theories may also be described diagrammatically by ‘‘extended‘‘ quivers, where the lines representing the strings are no longer directed. Instead, the two ends of each string can independently be in either the fundamental or the antifundamental. Thus each line must be drawn with an arrow at each of the two ends, indicating what is the representation of the corresponding string under each of the two gauge group factors. Unoriented strings with both ends coming out of the same set of branes may reside in either the symmetric or the antisymmetric combination of $\mathbf{N} \times \mathbf{N}$. Which of these two options is realized is directly related to the orientifold projection in the original theory: it is the antisymmetric (symmetric) part for the $SO(N)$ $[Sp(N)]$ orientifold projection. Having said that, we stress that the effective field theories of unoriented strings on singular manifolds are only known for $\mathbb{Z}_n$ orbifolds. Nevertheless, it is expected that the unoriented nature of the string should lead to the same "extended" quiver type diagrams in more general singularities.

Again, invariant interactions follow from the (extended) quiver. It corresponds to a set of lines for which each node has the number of ingoing fields equal to the number of outgoing fields.

### B. Higgsing

Higgsing, that is the spontaneous breaking of the gauge group via vacuum expectation values (VEVs) of scalar fields, changes in general the low energy description of a given theory. In particular, by Higgsing a quiver gauge theory, one obtains a (not necessarily supersymmetric) new quiver gauge theory where the low energy degrees of freedom are bifundamentals of the conserved gauge groups. We can thus write an effective quiver by identifying or splitting the vertices and lines so that each new node represents an unbroken $U(N)$ gauge group.

As an example, consider string theory on a $\mathbb{Z}_3$ orbifold of the conifold (also known as $Y^{3,0}$), with $N$ branes at each representation of $\mathbb{Z}_3$. The resulting $[U(N)]^6$ gauge theory is described by the quiver of Fig. 2(a). If we Higgs the theory by giving $Z^1$ a VEV proportional to the unit matrix,

$$Z^1 = \begin{pmatrix} a & & & \\ & a & & \\ & & a & \\ & & & \ddots \end{pmatrix}, \tag{1}$$

the corresponding $U(N) \times U(N)$ group is broken to the diagonal $U(N)$, and the $[U(N)]^5$ gauge theory of Fig. 2(b) is obtained. The $Z^1$ field itself is eaten up by the longitudinal modes of the broken gauge fields. Higgsing $Z^2$ and $Z^3$ in the same way, one finds the $[U(N)]^3$ theory described by Fig. 2(c), which is exactly the quiver of the orbifold $\mathbb{C}^3/\mathbb{Z}_3$ [37].

If, instead, we break the $Y^{3,0}$ quiver by choosing a different form of the VEV for $Z^1$,

$$Z^1 = \begin{pmatrix} a & & & \\ & \ddots & & \\ & & b & \\ & & & \ddots \end{pmatrix}, \tag{2}$$

the symmetry breaking is $U(N) \times U(N) \to U(n) \times U(N-n)$. This breaking is no longer supersymmetric and $Z^1$ in not eaten up completely. The lines and the nodes split and the quiver takes a very different form, as can be seen in Fig. 3. Other breaking patterns of $U(N) \times U(M)$

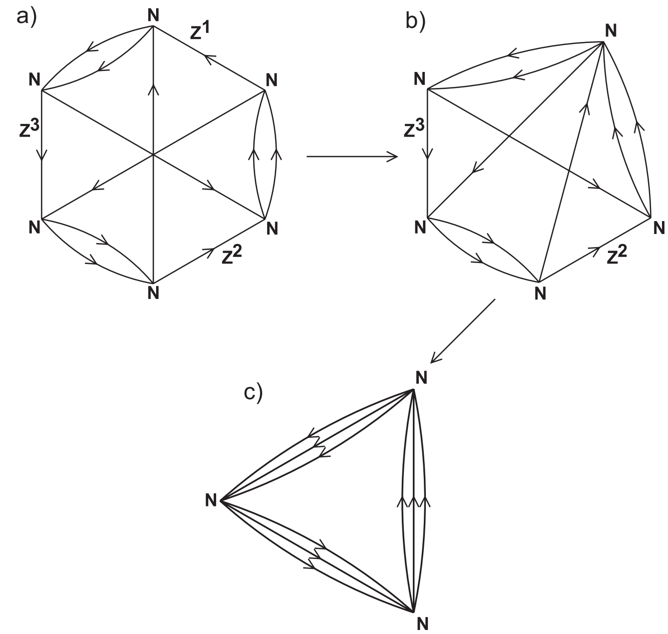


FIG. 2. Successive Higgsing of the $Y^{3,0}$ quiver diagram to a $\mathbb{C}^3$ quiver: (a) The $Y^{3,0}$ quiver diagram. (b) The effective quiver diagram induced by Higgsing $Z^1 \propto \mathbf{1}$. (c) The $\mathbb{C}^3/\mathbb{Z}_3$ quiver induced by Higgsing also $Z^2$, $Z^3 \propto \mathbf{1}$.

symmetries with bifundamentals can be obtained. The most general breaking is

$$U(N) \times U(M) \to U(N - M + n_0) \times \prod_{i=0}^{k} U(n_i) \tag{3}$$

with $\sum_{i=0}^{k} n_i = M$. It leads to more complicated quiver diagrams which, in general, will not be supersymmetric due to a nonvanishing $D$-term. However, by giving VEVs to a pair of fields in a vector representation, supersymmetry may easily be preserved.

Different patterns of breaking are generated when the Higgsed field is in the adjoint representation or, for "extended" quiver, in the fundamentals of both gauge groups

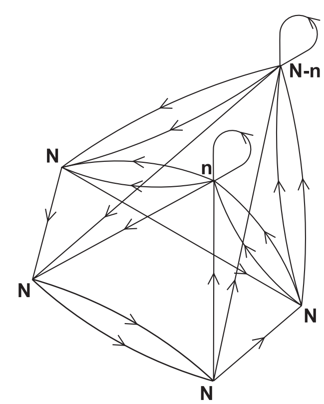


FIG. 3. Breaking the $Y^{3,0}$ quiver by a diagonal VEV with two different eigenvalues with multiplicities $n$ and $N - n$.

or in the symmetric (or antisymmetric) representation. In all cases, the resulting theories can be described by a new quiver diagram.

### C. Anomalous $U(1)$'s

Typically, many of the $U(1)$ factors associated with the $U(N)$ gauge groups are anomalous, with anomalies canceled by the generalized Green-Schwartz (GS) mechanism [25,31–33]. In contrast to the case of the heterotic string, in type II string theory there can be several such anomalous $U(1)$ factors. Furthermore, the corresponding gauge fields are massive independent of whether the symmetry is spontaneously broken [the spontaneous breaking is induced by nonvanishing Fayet-Illiopolous (FI) terms] [42,43]. In the case that the symmetry is not broken by scalar VEVs, these $U(1)$ factors remain as global symmetries [44] and the corresponding $SU(N)$ factors remain good gauge symmetries. Thus each node in the quiver has a corresponding (local) global (non)anomalous $U(1)$.

Phenomenologically, these $U(1)$ factors pose considerable problems to model building, essentially since the global $U(1)$ symmetries of the SM or its supersymmetric extensions (SSM) are not directly related to the local gauge groups. Consider, as an example, a supersymmetric quiver theory which contains the $SU(2)_W$ gauge group [5]. Under the corresponding $U(1)$, all the doublets ($Q_i$, $L_i$, $H_u$ and $H_d$) are charged $\pm 1$. The superpotential of the SSM has the following form:

$$W = Y^d_{ij} H_d Q_i d_j + Y^u_{ij} H_u Q_i u_j + Y^L_{ij} H_d L_i e_j + \mu H_u H_d. \tag{4}$$

It is clear that, to allow for $Y^u \neq 0$ and $Y^d \neq 0$, the $U(1)$ charges of $H_u$ and $H_d$ must be the same. But then the $\mu$ term is forbidden. A similar problem exists in the $SU(5)$ GUT models [8,9] and their extensions, as we discuss in Sec. IV.

There are three possible solutions to the above problem:

(1) The particle content of the low energy theory is extended in such a way that the symmetry is realized. For example, an extended Higgs sector [5] enlarges the global symmetry.

(2) The $U(1)$ is broken spontaneously. The only way to do this without breaking the associated $SU(N)$ gauge group is by letting a singlet composed of $N$ fundamentals to obtain a VEV. This breaks the $U(1)$ down to a $\mathbb{Z}_N$. Such composite singlets may exist if they are charged under a different, confining gauge group.

(3) The anomalous $U(1)$ is broken by nonperturbative effects. Depending on the matter content, such effects break the symmetry down to $\mathbb{Z}_{kN}$ for some integer $k$.

While difficult to analyze, these solutions allow realistic extensions of the SSM through quivers. We discuss these possibilities further in Sec. IV.

## III. THE FN MECHANISM FROM QUIVERS

### A. Renormalizable or nonrenormalizable

The Froggatt-Nielsen (FN) mechanism [1] provides an explanation to the flavor puzzle using a horizontal symmetry. Quarks and leptons of various generations are charged differently under a symmetry $\mathcal{H}$ which is spontaneously broken. The simplest realization is through the use of a single Abelian $U(1)$ which is broken by the VEV of a scalar field $S$. To allow for $\mathcal{H}$-invariant interaction terms involving the SM Higgs and fermion fields, powers of $S$ must be involved, depending on the $\mathcal{H}$-charge of the Yukawa interaction. Thus, nonrenormalizable interaction terms arise, suppressed by inverse powers of $M_V$, the scale at which the breaking of $\mathcal{H}$ is communicated to the SM. The effective Yukawa interactions are then suppressed by powers of $\langle S\rangle/M_V < 1$, and are characterized by smallness and hierarchy, as required phenomenologically.

One way of embedding the FN mechanism in string theory would be by identifying the scale $M_V$ with the string scale, and obtaining a superpotential that has most of its terms nonrenormalizable (the top, and perhaps other third generation Yukawa couplings, being the exception). This goal is difficult to achieve. Quiver gauge theories which arise from D-branes at singularities have a superpotential determined fully by the geometry. Geometrically engineering the required superpotential is hard. Most of the geometries which are under control arise from orbifold singularities [25] or toric geometries [37,38,40]. (These two classes are, of course, not separate: Abelian orbifolds are toric.) In the first type of geometry, the superpotential is obtained by truncating $\mathcal{N} = 4$ superconformal Yang-Mills which has cubic interactions. For the second type, in the case of $Y^{p,q}$ geometries, one obtains also quartic interactions. In any case, it is difficult to construct a quiver theory with a superpotential that has most of its terms nonrenormalizable.

A second approach is to construct a renormalizable model above the scale $M_V$ which, at low energy, produces the required interactions. Further motivation for such a construction comes from the possible identification of the FN symmetry with an anomalous $U(1)$, as discussed below.

A renormalizable model is easy to construct with the introduction of additional vectorlike massive fields. As an example, consider a nonrenormalizable superpotential term of the form

$$W = \left(\frac{S}{M_V}\right)^n \Phi_1 \Phi_2 \Phi_3. \tag{5}$$

To generate such interaction, we introduce additional vectorlike massive fields $V_k$, $\bar{V}_k$ ($k = 1, \ldots, n$), with masses at the scale $M_V$ and the following charges under the horizontal symmetry:

$$\mathcal{H}(\Phi_1) = n, \quad \mathcal{H}(\Phi_2) = 0, \quad \mathcal{H}(\Phi_3) = 0, \quad \mathcal{H}(S) = -1, \quad \mathcal{H}(V_k) = -\mathcal{H}(\bar{V}_k) = -k. \tag{6}$$

Taking the renormalizable superpotential to be

$$W = \Phi_1\Phi_2 V_n + \bar{V}_n V_{n-1} S + \cdots + \bar{V}_1 S \Phi_3 + \sum_i M_V \bar{V}_i V_i, \tag{7}$$

and integrating out $V_i$ and $\bar{V}_i$, one finds the required interaction, Eq. (5). Figure 4 shows the relevant diagrams which generate this low energy interaction for $n = 1$ and for general $n$.

### B. Identifying the FN symmetry

To relate the FN mechanism to a quiver field theory, one needs to identify a global symmetry that can play the role of a viable horizontal symmetry. We limit our search of viable models to theories with the following features:

(i) The FN symmetry arises from anomalous gauged $U(1)$'s in the quiver. (In particular, we do not consider global symmetries of the open string sector that are related to isometries in the dual gravitational theory.)

(ii) The spontaneous breaking of the symmetry comes from a VEV of a single field, $S$.

(iii) The ratio of the two relevant energy scales is small,

$$\epsilon \equiv \langle S\rangle / M_V \ll 1, \tag{8}$$

allowing for smallness and hierarchy in the effective Yukawa couplings.

We note several points that hold generically in a FN mechanism that arises from a quiver theory using an anomalous $U(1)$:

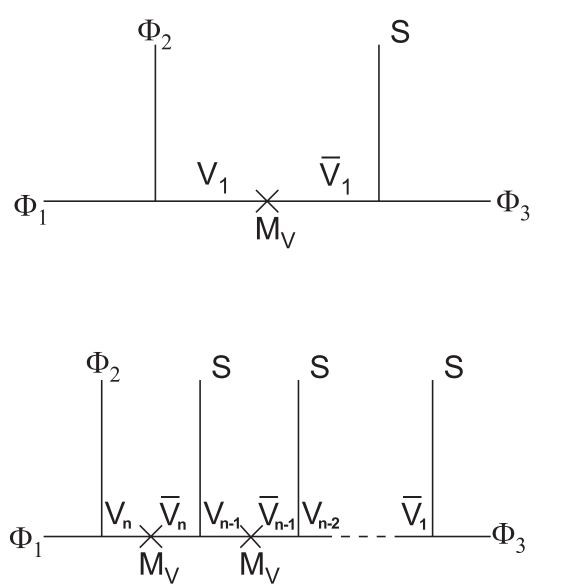


FIG. 4. Diagrams for generating interactions suppressed by different factors $\langle S\rangle/M_V$ using only cubic interactions.

(1) For $S$ to be charged under an anomalous $U(1)$, it must either be stretched between two distinct vertices, or have its two ends on the same vertex but with both ends sitting in the same representation (either fundamental or antifundamental). The second situation is possible only in the orientifold case. Since $S$ is generically charged under the full $U(N)[\times U(M)]$, and not just under the anomalous $U(1)$ factors, it necessarily breaks some of the gauge groups. This means that *the FN mechanism requires an extended group*.

(2) The charge of a given field under the FN symmetry is fixed by its representation under the corresponding non-Abelian gauge groups. For example, assume $S$ resides in the $(\mathbf{N_L}, \bar{\mathbf{N}}_\mathbf{R})$ representation of a $U(N_\mathrm{L}) \times U(N_\mathrm{R})$ gauge group. It is neutral under the sum of the two $U(1)$ factors, $U(1)_{\mathrm{L+R}}$, but has a charge $+2$ (in a specific normalization) under the difference, $U(1)_{\mathrm{L-R}}$. Thus we must identify $U(1)_{\mathrm{FN}}$ with $U(1)_{\mathrm{L-R}}$. Any field in the $(\mathbf{N_L}, \mathbf{1})$ representation has then a FN charge of $+1$, while a field in the $(\bar{\mathbf{N}}_\mathbf{L}, \bar{\mathbf{N}}_\mathbf{R})$ representation is neutral under the FN symmetry.

(3) There may be other fields in the theory obtaining VEVs that break additional gauge groups or $U(1)$'s. Our assumptions above mean that those VEVs are of the order of $M_V$ and do not contribute to the hierarchy. To distinguish between these fields and $S$, one may write the effective quiver after the Higgsing of all fields except $S$.

### C. FN charges

As explained above, the charges of the various fields under the FN symmetry are very restricted. This, in turn, affects the possible hierarchical structure within the quiver theory. In fact, the strongest suppression possible is by $\epsilon^3$ ($\epsilon \equiv \langle S\rangle/M_V$) and even this suppression is unlikely to actually appear, as we explain below.

Consider first oriented strings. As explained above, for directed quivers, $S$ must be stretched between two distinct branes and so its charge is $(+1, -1)$ under the corresponding $U(1)_\mathrm{L} \times U(1)_\mathrm{R}$. The FN symmetry is then $U(1)_{\mathrm{FN}} = U(1)_{\mathrm{L-R}}$ under which $S$ is charged $+2$. Any other field can be charged with one of the following: (0, 0), $(\pm 1, 0)$, $(0, \pm 1)$, $(\pm 1, \mp 1)$. Thus the strongest suppression of an effective Yukawa term of the form $\Phi_1\Phi_2\Phi_3$ is obtained when all three fields are charged $(-1, +1)$, giving an $\epsilon^3$ suppression. The corresponding quiver diagram is drawn in Fig. 5.

While an $\epsilon^3$-suppression can, in principle, be generated, it is unlikely to be relevant in practice, since it requires that all three fields $\Phi_i$ transform in the same way under the entire gauge group. In particular, there is no such case in $SU(5)$ GUT models, as we discuss further in Sec. IV.

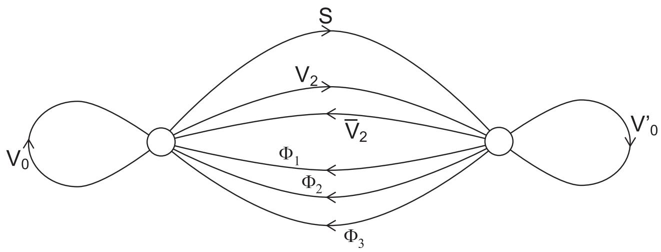


FIG. 5. A directed quiver diagram for an $\epsilon^3$-suppression of the effective $\Phi_1\Phi_2\Phi_3$ term in the superpotential.

To get an $\epsilon^2$-suppression, there are two possibilities for the charges: $(+1, -1)$, $(+1, -1)$, $(0, 0)$ or $(+1, -1)$, $(+1, 0)$, $(0, -1)$. Examples for the two sets of charges are drawn in Fig. 6.

There are five sets of charges that yield an $\epsilon$-suppression. These (and the other sets of charges that yield suppression) are presented in Table I. The quiver diagram that corresponds to the set $(0, 0)$, $(-1, 0)$, $(0, +1)$ is depicted in Fig. 7(b). Figure 7(a) shows the simplest triangle which produces a renormalizable (that is unsuppressed) Yukawa coupling.

For orientifolds, in addition to the above $U(1)_L \times U(1)_R$ charges, there could be fields with charges $(\pm 2, 0)$, $(0, \pm 2)$ and $(\pm 1, \pm 1)$. Furthermore, $S$ itself can have charges $(+1, -1)$ (similar to the oriented string), $(+1, +1)$ or $(\pm 2, 0)$. The breaking patterns of the gauge groups will be different for these three choices. The $U(1)_{FN}$ is then $U(1)_{L-R}$, $U(1)_{L+R}$ or $U(1)_L$, respectively. For the first case, Table II enumerates the possible suppression factors that arise in addition to the orbifold case of Table I. Note the additional configuration with an $\epsilon^3$ suppression, whose relevant quiver diagram is shown in Fig. 8. The second case, $U(1)_{L+R}$, can be easily obtained from the first one, $U(1)_{L-R}$, by multiplying the $U(1)_R$ charges by minus one. Finally, the third case, $U(1)_L$, again exhibits a configuration of $\epsilon^3$-suppression, as can be seen in Table III. Here, too, this configuration requires all three fields to be in the same representation of the non-Abelian gauge groups.

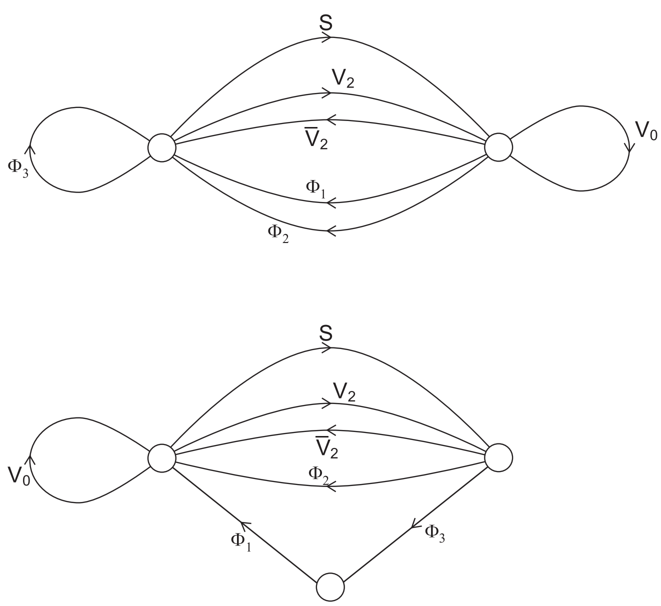


FIG. 6. Directed quiver diagrams for an $\epsilon^2$-suppression of the effective $\Phi_1\Phi_2\Phi_3$ term in the superpotential.

TABLE I. Possible $U(1)_L \times U(1)_R$ charges and the resulting suppression factors (namely the power in $\epsilon$) in oriented strings.

| Charges | Suppression |
|---|---|
| $(-1, +1)(-1, +1)(-1, +1)$ | 3 |
| $(0, 0)(-1, +1)(-1, +1)$ | 2 |
| $(-1, 0)(0, +1)(-1, +1)$ | 2 |
| $(0, 0)(0, 0)(-1, +1)$ | 1 |
| $(0, 0)(-1, 0)(0, +1)$ | 1 |
| $(0, +1)(0, -1)(-1, +1)$ | 1 |
| $(+1, 0)(-1, 0)(-1, +1)$ | 1 |
| $(+1, -1)(-1, +1)(-1, +1)$ | 1 |

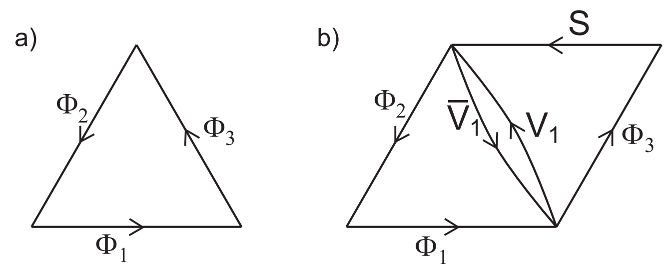


FIG. 7. Directed quiver diagrams which lead to (a) unsuppressed effective Yukawa coupling, and (b) $\epsilon$-suppression of the effective Yukawa coupling.

TABLE II. Possible charges and suppression factors (the power of $\epsilon$) with $S(+1, -1)$ that are specific to the unoriented case. The sets of Table I are also allowed.

| Charges | Suppression |
|---|---|
| $(-2, 0)(0, +2)(-1, +1)$ | 3 |
| $(-2, 0)(0, +2)(0, 0)$ | 2 |
| $(-2, 0)(+1, +1)(-1, +1)$ | 2 |
| $(-2, 0)(0, +1)(0, +1)$ | 2 |
| $(0, +2)(-1, -1)(-1, +1)$ | 2 |
| $(0, +2)(-1, 0)(-1, 0)$ | 2 |
| $(-2, 0)(0, +2)(+1, -1)$ | 1 |
| $(-2, 0)(+2, 0)(-1, +1)$ | 1 |
| $(-2, 0)(+1, +1)(0, 0)$ | 1 |
| $(-2, 0)(+1, 0)(0, +1)$ | 1 |
| $(0, +2)(0, -2)(-1, +1)$ | 1 |
| $(0, +2)(-1, -1)(0, 0)$ | 1 |
| $(0, +2)(-1, 0)(0, -1)$ | 1 |
| $(+1, +1)(-1, -1)(-1, +1)$ | 1 |
| $(+1, +1)(-1, 0)(-1, 0)$ | 1 |
| $(-1, -1)(0, +1)(0, +1)$ | 1 |

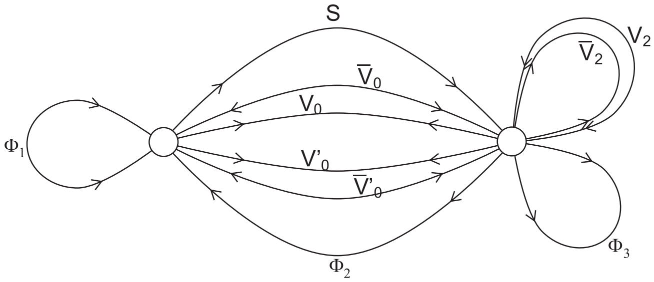


FIG. 8. A quiver diagram in the orientifold case that gives an $\epsilon^3$-suppression.

TABLE III. Possible charges and suppression factors (the power of $\epsilon$) in the unoriented case with $S(+2, 0)$. The $S(-2, 0)$ case is obtained by multiplying the $U(1)_L$-charge by $-1$.

| Charges | Suppression |
|---|---|
| $(-2, 0)(-2, 0)(-2, 0)$ | 3 |
| $(-2, 0)(-2, 0)(0, 0)$ | 2 |
| $(-2, 0)(-1, -1)(-1, 1)$ | 2 |
| $(-2, 0)(-1, 0)(-1, 0)$ | 2 |
| $(-2, 0)(-2, 0)(+2, 0)$ | 1 |
| $(-2, 0)(0, -2)(0, +2)$ | 1 |
| $(-2, 0)(-1, -1)(+1, +1)$ | 1 |
| $(-2, 0)(0, 0)(0, 0)$ | 1 |
| $(-2, 0)(-1, 0)(+1, 0)$ | 1 |
| $(-2, 0)(0, -1)(0, +1)$ | 1 |
| $(-2, 0)(-1, +1)(+1, -1)$ | 1 |
| $(0, -2)(-1, +1)(-1, +1)$ | 1 |
| $(0, 2)(-1, -1)(-1, -1)$ | 1 |
| $(-1, -1)(0, 0)(-1, +1)$ | 1 |
| $(-1, -1)(-1, 0)(0, +1)$ | 1 |
| $(0, 0)(-1, 0)(-1, 0)$ | 1 |
| $(-1, 0)(0, -1)(-1, +1)$ | 1 |

Our results in this section show the strong predictive power that is added to the generic Froggatt-Nielsen mechanism when embedded in string theory. In fact, the constraints are so strong—i.e. the strongest suppression of a Yukawa coupling is third order in a small parameter and, for practical purposes, probably only second order—that one may wonder whether our framework gives rise to any viable flavor model at all. Indeed, in the next section we show that, to construct viable models, one has to invoke (rather plausible) nonperturbative effects. These effects relax some of the constraints that we presented in this section and, in particular, allow an $\epsilon^4$-suppression of the Yukawa couplings in the case of $SU(5)$ GUT models.

## IV. $SU(5)$ GUT MODELS AND NEUTRINO MASS ANARCHY

In this section we consider $SU(5)$ GUT models with a FN flavor symmetry. We search for viable models that arise from quiver gauge theories. The theory turns out to have intriguing implications for the neutrino sector.

By an $SU(5)$ GUT model we mean that there is a range of energy scales where the gauge group is $SU(5)$, with matter fields that transform as **5**, $\bar{\mathbf{5}}$, and **10**. The presence of an antisymmetric multiplet of the gauge group requires that we consider orientifold theories and choose the appropriate projection. This projection is the one that results in an $SO(N)$ gauge group(s).

Our analysis focuses on the energy scale just above the GUT breaking scale. In general there can be many fields that break the various gauge groups that are present in the quiver theory. However, as discussed in the previous section, we consider a scale that is low enough so that the only field to play a relevant role in the breaking of the FN symmetry and possibly in breaking of a larger gauge group into $SU(5)$ is the FN field.

### A. General considerations and predictions

The strongest mass hierarchy in the various fermion mass matrices appears in the up sector. Thus, a minimal requirement that we put on viable models is that they produce an up mass hierarchy. This requirement significantly narrows down the possible configurations.

There are two options regarding the $SU(5)$ gauge group. First, it could be related to a single node, namely, it is a subgroup of a single $U(N)$ symmetry. In this scenario, the fields transforming as **10** must have both ends on the $SU(5)$-related node. Consequently, they all carry the same $U(1)_{\rm FN}$ charge, regardless of whether $U(1)_{\rm FN}$ is (i) a subgroup of the same $U(N)$, (ii) unrelated to this $U(N)$ or (iii) a subgroup of $U(N) \times U(M)$, where $U(M)$ is related to a different node. Thus, this scenario gives rise to up mass anarchy (that is, no special structure in the up mass matrix) and is therefore phenomenologically excluded.

The second scenario has the $SU(5)$ gauge group related to two nodes, namely, it is a subgroup of a $U(N_L) \times U(N_R)$ symmetry. The FN field must be in the bifundamental $(\mathbf{N_L}, \bar{\mathbf{N}}_\mathbf{R})$ of the two nodes. This case has a rich flavor structure and is the only one that can lead to phenomenologically viable models.

The simplest models have the following pattern of gauge symmetry breaking: $SU(5) \times SU(5) \to SU(5)_{\rm diag}$. We focus on this class of models. (More complicated breaking patterns have a similar hierarchical form, but involve extended particle content.) The $\bar{\mathbf{5}}$-plets then transform under the $SU(5) \times SU(5) \times U(1)_L \times U(1)_R$ as either $(\bar{\mathbf{5}}, 1)_{-1,0}$ or $(1, \bar{\mathbf{5}})_{0,-1}$. The **10**-plets transform as either $(\mathbf{10}, 1)_{+2,0}$ or $(1, \mathbf{10})_{0,+2}$ or $(\mathbf{5}, \mathbf{5})_{+1,+1}$. The $H_u(\mathbf{5})$ field transforms as either $(\mathbf{5}, 1)_{+1,0}$ or $(1, \mathbf{5})_{0,+1}$.

While $S$ breaks the $U(1)_{\rm FN} = U(1)_{\rm L-R}$ symmetry, it leaves $U(1)_{\rm L+R}$ as a global symmetry in the $SU(5)_{\rm diag}$ theory, under which the fields are charged as

$$\bar{\mathbf{5}}(-1), \qquad \mathbf{10}(+2), \qquad H_d(-1), \qquad H_u(+1). \qquad (9)$$

This symmetry is flavor diagonal, with charges that are determined solely through the $SU(5)$ representation. As mentioned in Sec. II C, such $U(1)$'s are generic and face strong phenomenological constraints. Clearly, the symmetry of Eq. (9) is not a symmetry of the $SU(5)$ GUT model. In particular, no up-type masses are allowed with the above symmetry [8,9]. One can try to overcome this problem by introducing a composite $H_u$ field of charge $-4$ [8]. This solution to the up mass(lessness) problem comes, however, at the cost of two new problems: First, the $H_u H_d$ term is now forbidden, rendering the Higgsinos massless. Second, the $\bar{\mathbf{5}}\,\bar{\mathbf{5}}\,H_u H_u$ terms are forbidden, rendering the neutrinos massless.

As discussed in Sec. II C, there are three possible solutions to this problem, which involve either extending the particle content and the symmetry of the low energy theory, or breaking the symmetry either spontaneously or nonperturbatively. Before going into details, we observe that, in fact, the quiver theory gives interesting predictions that are independent of which solution to the up mass problem is employed.

Indeed, since the $\bar{\mathbf{5}}$ fields carry $U(1)_{\rm L} \times U(1)_{\rm R}$ charges of either $(-1, 0)$ or $(0, -1)$, at least two of them have the same FN charge. Thus, there must be at least quasi-anarchy (that is two nonhierarchical masses and one mixing angle of order one) in the neutrino sector. In half of the models all three $\bar{\mathbf{5}}$ fields have the same FN charge, leading to complete neutrino mass anarchy (no hierarchy in the masses and all three angles of order one). In addition, this situation, where a maximum of two possible FN charges are available to the three $\bar{\mathbf{5}}$ fields, has implications for the down sector: either one or all three (in correspondence to quasi- or full-anarchy in the neutrinos) down mass ratios are of the same order as the corresponding mixing angles (e.g. $m_s/m_b \sim |V_{cb}|$).

A word of caution is, however, in order. The above restrictions on the possible FN charges can be evaded if the **10** and $\bar{\mathbf{5}}$-plets are composite fields.[1] In such a case, it is possible for all fields to carry various charges under additional $U(1)$ factors, which may play the role of the FN symmetry. Such a possibility, however, complicates the theory considerably and does not seem to be attractive, especially since the effective theory cannot be described by a quiver. We thus assume that the SM fermions are elementary fields, while condensates can either break the $U(1)_{\rm L+R}$ symmetry or generate effective Higgs fields.

We next discuss the possibilities for solving the $U(1)_{\rm L+R}$ problem. We do so in the specific context of $SU(5)$ GUT, but the solutions can be straightforwardly generalized to other gauge groups.

[1]We thank Micha Berkooz for drawing our attention to this point.

### B. Extending the Higgs sector

To allow for up-quark, Higgsino and neutrino masses in a model that has the $U(1)_{\rm L+R}$ symmetry, one needs to add matter fields. The simplest extension has a second pair of Higgs doublets [5]. The $U(1)_{\rm L+R}$ charges of the four Higgs fields are as follows:

$$H_u(-4), \qquad H_d(-1), \qquad h_d(+4), \qquad h_u(+1). \qquad (10)$$

Here $H_d$ and $h_u$ are fundamental fields, while $H_u$ and $h_d$ are composite.

One can now distinguish between models according to the FN charges of the matter fields, that is the three $\mathbf{10}_i$, the three $\bar{\mathbf{5}}_i$, the two elementary Higgs fields $H_d$ and $h_u$, and the two composite Higgs fields $H_u$ and $h_d$. There are 640 different sets of $U(1)_{\rm FN} = U(1)_{\rm L-R}$ charges. They are listed in Table IV.

We now impose phenomenological requirements to see, first, if there are viable models and, second, if these models make further predictions. It turns out that requiring that the quark masses are hierarchical is enough to select a *single* flavor structure for all fermions and, in particular, predict the flavor structure of the lepton sector.

We first consider the up sector. We require that the three up-type quarks have masses and that these masses are hierarchical. This means that no two **10**-plets are allowed

TABLE IV. All possible charge assignments for the model with additional fields and an unbroken $U(1)_{L+R}$ symmetry.

| $SU(5)$ | Model | $U(1)_L \times U(1)_R$ | $U(1)_{L-R}$ |
|---|---|---|---|
| $10_i$ ($i = 1, 2, 3$) | $T_1$ | (2, 0) (2, 0) (2, 0) | (+2, +2, +2) |
| | $T_2$ | (2, 0) (2, 0) (0, 2) | (+2, +2, −2) |
| | $T_3$ | (2, 0) (2, 0) (1, 1) | (+2, +2, 0) |
| | $T_4$ | (2, 0) (0, 2) (0, 2) | (+2, −2, −2) |
| | $T_5$ | (2, 0) (0, 2) (1, 1) | (+2, −2, 0) |
| | $T_6$ | (2, 0) (1, 1) (1, 1) | (+2, 0, 0) |
| | $T_7$ | (0, 2) (0, 2) (0, 2) | (−2, −2, −2) |
| | $T_8$ | (0, 2) (0, 2) (1, 1) | (−2, −2, 0) |
| | $T_9$ | (0, 2) (1, 1) (1, 1) | (−2, 0, 0) |
| | $T_{10}$ | (1, 1) (1, 1) (1, 1) | (0, 0, 0) |
| $\bar{5}_i$ ($i = 1, 2, 3$) | $F_1$ | (−1, 0)(−1, 0)(−1, 0) | (−1, −1, −1) |
| | $F_2$ | (−1, 0)(−1, 0)(0, −1) | (−1, −1, +1) |
| | $F_3$ | (−1, 0)(0, −1)(0, −1) | (−1, +1, +1) |
| | $F_4$ | (0, −1)(0, −1)(0, −1) | (+1, +1, +1) |
| $H_d(\bar{5})$, $h_u(5)$ | $D_1$ | (−1, 0), (+1, 0) | (−1, +1) |
| | $D_2$ | (−1, 0), (0, +1) | (−1, −1) |
| | $D_3$ | (0, −1), (+1, 0) | (+1, +1) |
| | $D_4$ | (0, −1), (0, +1) | (+1, −1) |
| $H_u(5)$, $h_d(\bar{5})$ | $U_1$ | (−4, 0), (+4, 0) | (−4, +4) |
| | $U_2$ | (−4, 0), (0, +4) | (−4, −4) |
| | $U_3$ | (0, −4), (+4, 0) | (+4, +4) |
| | $U_4$ | (0, −4), (0, +4) | (+4, −4) |
| $S(1)$ | | (+1, −1) | +2 |

to carry the same FN charge. Out of the ten different sets of charges for the **10**-plets $T_i$, only one fulfills this requirement, that is $T_5$ of Table IV. Furthermore, the up sector couples to the composite $H_u$. In order that the up masses do not vanish, $H_u$ must carry charge $(-4, 0)$ under $U(1)_L \times U(1)_R$. Thus, of the four sets $U_i$, only $U_1$ and $U_2$ are viable charge assignments.

We next consider the down sector. We require that the three down-type quarks have masses and that these masses are hierarchical. Out of the four different sets of charges for the $\bar{\mathbf{5}}$-plets $F_i$, only one fulfills this requirement, that is $F_1$. Furthermore, the down sector couples to the elementary $H_d$. $H_d$ must be connected to the same node as the $\bar{\mathbf{5}}$. Thus, of the four sets $D_i$, only $D_1$ and $D_2$ are viable choices.

We are therefore left with a unique set of $U(1)_{L-R}$ charges, up to the choice of the charges for $h_d$ and $h_u$. This freedom, however, only affects the $\mu$-terms and the overall scale of the neutrino masses. The flavor structure is unaffected by this choice. Taking the configuration "$T_5F_1D_1U_1$," we obtain the following parametric suppressions for the various entries in the fermion mass matrices:

$$M_u \sim \langle H_u \rangle \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \tag{11}$$

$$M_d \sim \langle H_d \rangle \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix}, \tag{12}$$

$$M_\nu \sim \frac{\langle h_u \rangle^2}{M} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{13}$$

There are other ways to extend the matter content in order to incorporate the $U(1)_{L+R}$ as a the symmetry of the theory. One way to generate the $\mu$-term for the $H_uH_d$ fields, without introducing another pair of Higgs fields, can be achieved in a similar fashion to the next to minimal supersymmetric standard model. One assumes $H_u$ is a condensate $\bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}$ and introduces another field $T$ which is a **55555** condensate of the same strongly coupled gauge group. Then the coupling $TH_uH_d$ is allowed, while the coupling $T^3$ which breaks the $U(1)_{L+R}$, may be generated by nonperturbative effects below the scale at which the $SU(5)_{\rm diag}$ becomes strong, or may not be generated at all, as in [45–47]. We do not discuss this idea further, and just note that, as before, the neutrino sector is predicted to be anarchical.

### C. Breaking $U(1)_{L+R} \rightarrow \mathbb{Z}_5$

Another way to get the up Yukawa terms is by breaking the symmetry through nonperturbative effects. In general, operators which violate an anomalous symmetry are generated nonperturbatively and can be calculated using the holomorphic structure of the superpotential. It is important to note that the Lagrangian of the minimal $SU(5)$ possesses a $\mathbb{Z}_5$ symmetry under which the fields are charged as in Eq. (9). This $\mathbb{Z}_5$ is a subgroup of $U(1)_{L+R}$. In our case, of an $SU(5)_L \times SU(5)_R$ gauge group, it is simple to see that if in one of the nodes, say the left, there is only a single antisymmetric **10** (and another $\bar{\mathbf{5}}$ to cancel gauge anomalies), then the related $\Lambda_{\rm QCD}^b$, where $b$ is the coefficient in the $\beta$ function, is charged 5 under the anomalous symmetry. Therefore, instantons, if they exist, break this $U(1)$ down to $\mathbb{Z}_5$ and can generate masses in the up sector. Unfortunately, at the field theory level, no nonperturbative terms are generated in the superpotential even in this case. The reason is that the number of flavors in this model is $\geq 5$ [48,49].

Nevertheless, it could be that nonperturbative corrections which break the anomalous $U(1)$ arise already at the string level. Such corrections cannot be calculated explicitly. However, they break the $U(1)$ in a way that follows from the anomaly and hence may generate the required up-quark Yukawa couplings. Assuming that these terms are indeed generated in this way, we consider the set of models where there is a single antisymmetric representation on one of the nodes. The list of the 80 possible charge assignments is given in Table V.

Just as for the previous case, most of the possible charge assignments lead to phenomenologically excluded models. We find again that up mass hierarchy requires that the three **10**-plets have three different charges ($T_5$), and down mass hierarchy requires that the $\bar{\mathbf{5}}$ and $H_d$ fields connect to the left node ($F_1D_1$) in which the $U(1)$ is broken down to $\mathbb{Z}_5$. Both possible charge assignments for $H_u$ give nonvanishing hierarchical masses. Note, however, that $U_2$ gives an overall suppression of order $\epsilon$ in the up sector. The neutrino

TABLE V. All possible charge assignments for the model with instanton breaking of the $U(1)_{L+R}$.

| $SU(5)$ | Model | $U(1)_L \times U(1)_R$ | $U(1)_{L-R}$ |
|---|---|---|---|
| $10_i$ | $T_2$ | (2, 0) (2, 0) (0, 2) | (+2, +2, −2) |
| | $T_4$ | (2, 0) (0, 2) (0, 2) | (+2, −2, −2) |
| | $T_5$ | (2, 0) (0, 2) (1, 1) | (+2, −2, 0) |
| | $T_6$ | (2, 0) (1, 1) (1, 1) | (+2, 0, 0) |
| | $T_9$ | (0, 2) (1, 1) (1, 1) | (−2, 0, 0) |
| $\bar{5}_i$ | $F_1$ | (−1, 0)(−1, 0)(−1, 0) | (−1, −1, −1) |
| | $F_2$ | (−1, 0)(−1, 0)(0, −1) | (−1, −1, +1) |
| | $F_3$ | (−1, 0)(0, −1)(0, −1) | (−1, +1, +1) |
| | $F_4$ | (0, −1)(0, −1)(0, −1) | (+1, +1, +1) |
| $H_d(\bar{5})$ | $D_1$ | (−1, 0) | −1 |
| | $D_2$ | (0, −1) | +1 |
| $H_u(5)$ | $U_1$ | (1, 0) | +1 |
| | $U_2$ | (0, 1) | −1 |
| $S(1)$ | | (+1, −1) | +2 |

flavor structure is again anarchical (independent of the choice $U_i$, though the overall scale depends once again on this choice).

The last class of models involves spontaneous breaking of $U(1)_{L+R}$. This can be achieved by adding a condensate of five fields in the fundamental representations of one $SU(5)$ factor. This condensate includes a singlet of the non-Abelian gauge group with a $U(1)_{\mathrm{L+R}}$ charge $+5$, which we denote by $K$. By giving $K$ a VEV, we break $U(1)_{\mathrm{L+R}} \to \mathbb{Z}_5$ and allow up-type mass terms. A second, conjugate, field $\bar{K}$ is needed in order to allow for a mass term. The list of the 320 different charge sets is given in Table VI. The majority of the sets of charges are not viable. Only two models are viable: $T_5F_1D_1U_1K_1$ and $T_5F_1D_1U_2K_1$, where the latter, as in the previous case, has an overall suppression in the up sector. Once again, anarchy is predicted for the neutrino sector.

In all three classes of models, we arrived at an essentially equivalent configuration for the matter content. The differences between the three models are just with respect to new fields that are added to solve the $U(1)_{\mathrm{L+R}}$ problem. This unique configuration, which leads to the flavor structure of Eq. (11), is presented in the quiver of Fig. 9. (The theory described by this quiver diagram suffers from non-Abelian gauge anomalies. A nonanomalous extension is presented in the Appendix and Fig. 10.) This theory produces, at low energy, the minimal $SU(5)$ with the correct hierarchy in the up and down sector and with the predicted neutrino anarchy. We stress that, given our assumptions, this theory is unique and thus the anarchy is predicted.

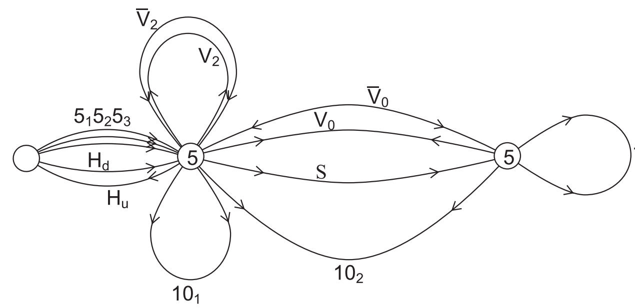


FIG. 9. The unique configuration for the $SU(5)$ GUT fields.

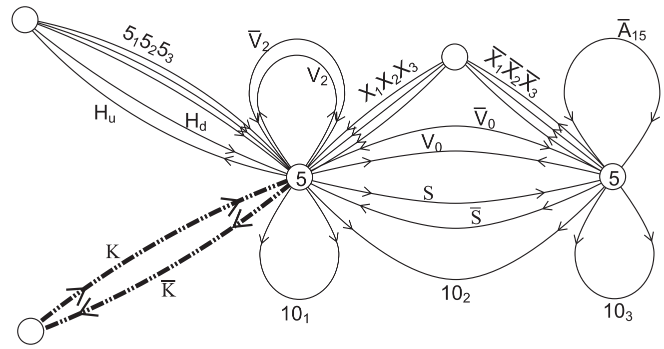


FIG. 10. An $SU(5)$ GUT quiver.

TABLE VI. All possible charge assignments for the model with spontaneous breaking of the $U(1)_{L+R}$ symmetry.

| $SU(5)$ | Model | $U(1)_L \times U(1)_R$ | $U(1)_{L-R}$ |
|---|---|---|---|
| $10_i$ | $T_1$ | (2, 0) (2, 0) (2, 0) | (+2, +2, +2) |
| | $T_2$ | (2, 0) (2, 0) (0, 2) | (+2, +2, −2) |
| | $T_3$ | (2, 0) (2, 0) (1, 1) | (+2, +2, 0) |
| | $T_4$ | (2, 0) (0, 2) (0, 2) | (+2, −2, −2) |
| | $T_5$ | (2, 0) (0, 2) (1,1) | (+2, −2, 0) |
| | $T_6$ | (2, 0) (1, 1) (1, 1) | (+2, 0, 0) |
| | $T_7$ | (0, 2) (0, 2) (0, 2) | (−2, −2, −2) |
| | $T_8$ | (0, 2) (0, 2) (1, 1) | (−2, −2, 0) |
| | $T_9$ | (0, 2) (1, 1) (1, 1) | (−2, 0, 0) |
| | $T_{10}$ | (1, 1) (1, 1) (1, 1) | (0, 0, 0) |
| $\bar{5}_i$ | $F_1$ | (−1, 0)(−1, 0)(−1, 0) | (−1, −1, −1) |
| | $F_2$ | (−1, 0)(−1, 0)(0, −1) | (−1, −1, +1) |
| | $F_3$ | (−1, 0)(0, −1)(0, −1) | (−1, +1, +1) |
| | $F_4$ | (0, −1)(0, −1)(0, −1) | (+1, +1, +1) |
| $H_d(\bar{5})$ | $D_1$ | (−1, 0) | −1 |
| | $D_2$ | (0, −1) | +1 |
| $H_u(5)$ | $U_1$ | (+1, 0) | +1 |
| | $U_2$ | (0, +1) | −1 |
| $K(1)$, $\bar{K}(1)$ | $K_1$ | (+5, 0), (−5, 0) | (+5, −5) |
| | $K_2$ | (0, +5), (0, −5) | (−5, +5) |
| $S(1)$ | | (+1, −1) | +2 |

## V. SUMMARY

The fermion flavor parameters of the standard model have a special, nongeneric structure, that finds no explanation within the standard model. The hierarchy and the smallness of the Yukawa couplings are suggestive of an approximate symmetry. The Froggatt-Nielsen (FN) mechanism is a simple and attractive realization of this idea. The mechanism is, however, limited in its predictive power. In particular, the FN charges are not dictated by the theory, and there is only information on the parametric suppression, but not the order-one coefficients, of the Yukawa couplings. In this work, we investigated whether the embedding of the FN mechanism in string theory improves its predictive power.

Specifically, we examined quiver gauge theories which arise at low energy from type II string theory with D-branes placed on singular manifolds. The quiver gauge theories can be described by (non)directed graphs for (un)oriented strings, in which the nodes represent the gauge groups ($U(N)$, $SO(N)$ or $Sp(N)$), while lines represent matter fields charged under the corresponding gauge groups.

In general, these theories contain several anomalous $U(1)$ symmetries whose anomaly is canceled by a generalized GS mechanism. The unbroken global symmetries can be used to generate the hierarchy of the Yukawa couplings, thus realizing the FN mechanism. Since charges of the matter fields under these $U(1)$ factors are fixed by their representation under the gauge groups, the FN charges are fixed. Consequently, one of the inherent limitations of FN models is removed.

Precisely because it is highly predictive, the above framework does not easily lend itself to the construction of viable models. We have discussed this problem and its possible solutions. Concentrating on a large class of $SU(5)$ GUT models, we have demonstrated the predictive power of quiver gauge theories. Requiring mass hierarchy in the up sector, we showed that the $SU(5)$ must come from an extended product group such as $U(5) \times U(5)$. Furthermore, there must be either quasi- or full-anarchy in the neutrino sector and either one or all three down mass ratios are of the same order as the corresponding mixing angles (e.g. $m_s/m_b \sim |V_{cb}|$). Further requiring mass hierarchy in the down sector, the FN charges of all matter fields are essentially fixed. Consequently, the lepton flavor structure is predicted and, in particular, there is anarchy (that is, no special structure) in the neutrino sector.

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## APPENDIX A: AN *SU*(5) GUT QUIVER

As we showed in Sec. IV, all three classes of models have an equivalent configuration for the matter content, which leads to the flavor structure of Eq. (11). A non-anomalous quiver which realizes this structure is presented in Fig. 10. This figure corresponds specifically to the method of spontaneously breaking the $U(1)_{\rm L+R}$ symmetry. In particular, we explicitly show the condensate field, $K$.

Note that the two $SU(5)$ gauge groups are not asymptotically free, which is a typical problem in realizations of the FN mechanism [2]. In our framework, however, the absence of asymptotic freedom does not pose a problem since the theory is defined at the string scale, where new heavy degrees of freedom are integrated in. The two $SU(5)$ factors are broken into the diagonal $SU(5)$ after giving a VEV to the Froggatt-Nielsen field $S$. The fields $\mathbf{10_i}$, $\bar{\mathbf{5}}_\mathbf{i}$, $H_u$ and $H_d$, together with $\bar{S}$ that becomes an adjoint, are the matter content of $SU(5)$ GUT. The fields denoted by $V_i$ are the vectorlike fields discussed in Sec. III.

All other fields are necessary for non-Abelian gauge anomaly cancellation. The three $X$ fields are connected to a $U(1)$ gauge group. However, to cancel the anomalies, we can alternatively employ a single field connected to a $U(3)$ group. All these additional fields have mass terms in the superpotential and can be integrated out in pairs: $X_i$ with $\bar{X}_i$ and $\bar{A}_{15}$ with the symmetric part of $\mathbf{10}_2$. At the level of the massless spectrum, these fields may obtain masses through couplings to $S$ or to other moduli which obtain VEVs and therefore, by definition, are not shown in our effective quiver.

Reverse geometrically engineering a singular string theory background with this quiver is a complicated task. A generic construction is known for a very limited number of cases [38,50,51]. In fact, we view our model as an effective quiver obtained by Higgsing a larger quiver. One reason for this is that the (symmetric) $\bar{A}_{15}$ field cannot be directly obtained together with the (antisymmetric) $\overline{\mathbf{10}}$ field through the orientifold projection. Nevertheless, the symmetric $\bar{A}_{15}$ can originate from a broken $SU(5) \times SU(5) \rightarrow SU(5)$ if a $\bar{\mathbf{5}} \times \bar{\mathbf{5}}$ field is broken down to a $\bar{A}_{15}$ and a $\overline{\mathbf{10}}$, where the latter is integrated out with another $\mathbf{10}$.

Thus indeed this theory produces, at low energy, the minimal $SU(5)$ with the correct hierarchy in the up and down sector and with the predicted neutrino anarchy.

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## 2.3 Dynamical Supersymmetry Breaking from Simple Quivers

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# Dynamical supersymmetry breaking from simple quivers

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We construct a simple local model of dynamical supersymmetry breaking. The model is a one-generation $SU(5)$ that arises from a IIB $\mathbb{Z}_N$ orientifold. It does not admit a runaway direction and is argued to stabilize the blowup mode related to the corresponding $U(1)$ factor. The theory demonstrates the existence of a new class of ''blowup'' fractional branes

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## I. INTRODUCTION

Dynamical supersymmetry breaking (DSB) [1] is an intriguing solution to the hierarchy problem. Examples of such models were first presented more than 20 years ago [2], and the idea has been extensively studied both from the theoretical and phenomenological points of view (for a review see, e.g., [3]).

For string theory to make contact with reality, some mechanism to break supersymmetry must be employed. In recent years, following the understanding of flux compactifications and moduli stabilization [4,5], the problem of breaking supersymmetry has attracted a vast amount of attention. Many models have been presented, employing various stringy mechanisms, however only very few break supersymmetry dynamically. The reason for the lack of DSB models in string theory is twofold. On the one hand, models with completely stable DSB vacua are nongeneric at the field theory level. On the other hand, compactifying such models and taking care of the stabilization of all moduli and, in particular, Kähler moduli, is very laborious [6].

Constructions of local models were attempted on D-branes [7]. In [8–10], a classification of the gauge dynamics on fractional branes was introduced, where it was argued that the corresponding quiver theories typically break supersymmetry dynamically. However, as was stressed in [11], these brane configurations generically posses a runaway direction which corresponds to a blowup of the singular geometry. This problem can be ameliorated in compact models by stabilizing the runaway directions through some nonperturbative effects [12,13].

Recently it was suggested that metastable vacua that exhibit DSB may be more generic [14]. While this is indeed true at the field theory level, such constructions in string theory still lack a good explanation for the origin of small mass terms which appear in most theories. There have been several attempts to realize such models in string theory; however, most do not address the above issue [15–17] and cannot be compactified in a direct manner.

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It is therefore worthwhile to construct new local models of DSB which are simple enough to allow for a straightforward embedding in a compact background. In this note we report on progress in this direction. Here we concentrate on a simple local construction, while the details of the compact model will be given in [18]. The local construction is a type IIB $\mathbb{Z}_N$ orientifold. Specifically, we construct an $SU(5)$ gauge theory with one generation of $\mathbf{10} + \bar{\mathbf{5}}$ [19]. After imposing the orientifold projection, only one anomalous $U(1)$ is present. We argue that the corresponding closed string Kähler blowup mode that shows up as a Fayet-Iliopoulos (FI) term is stabilized close to the origin. As opposed to the generic quiver, this (bidirectional) quiver does not suffer from a runaway behavior and demonstrates the existence of a new class of fractional branes which we call *blowup fractional branes*.

While this work was being completed, we became aware of [20] which partially overlaps with the local construction of our model.

## II. LOCAL MODELS

As a first step towards writing a complete compact solution, one must specify a local quiver model which exhibits DSB. Here we concentrate on the noncalculable $SU(5)$ gauge theory with one generation of $\mathbf{10}$ and $\bar{\mathbf{5}}$. This model is known to break supersymmetry dynamically [19]. The construction is based on fractional branes located at fixed points of $\mathbb{C}^3/\mathbb{Z}_N$ orientifolds. Quiver models that arise from placing D-branes at such singularities have been extensively studied. The reader is referred to [21,22] and references therein for more details.

*Formalism.* We begin by setting up our notations, closely following [22]. Consider a $\mathbb{C}^3/\mathbb{Z}_N$ singularity. The $\mathbb{Z}_N$ generator $\theta$ acts on the three complex coordinates as $\theta$: $(z^1, z^2, z^3) \rightarrow (\omega^{b_1} z^1, \omega^{b_2} z^2, \omega^{b_3} z^3)$ where $\omega = e^{2\pi i/N}$ is the $N$th root of unity. To preserve $\mathcal{N} = 1$ supersymmetry, the $\mathbb{Z}_N$ must be a subgroup of $SU(3)$ which translates into taking $b_1 + b_2 + b_3 = 0 \pmod N$. The action on the Chan-Paton indices is

$$A^\mu \rightarrow \gamma(\theta) A^\mu \gamma(\theta)^{-1}, \tag{1}$$

$$Z^i \rightarrow \omega^{b_i} \gamma(\theta) Z^i \gamma(\theta)^{-1}, \tag{2}$$

 

where $\gamma(\theta)$ is a representation of $\mathbb{Z}_N$. Since $\mathbb{Z}_N$ is Abelian, all its irreducible representations are one-dimensional, and without loss of generality we may take $\gamma(\theta)$ to be

$$\gamma(\theta) = \mathrm{diag}(\mathbf{1}_{n_0}, \omega\mathbf{1}_{n_1}, \ldots, \omega^{N-1}\mathbf{1}_{n_{N-1}}) \tag{3}$$

where $\sum_a n_a = n$ is the number of fractional branes. The invariant spectrum at the singularity is described by a $U(n_0) \times U(n_1) \times \ldots \times U(n_{N-1})$ theory with matter multiplets transforming as $(\mathbf{n_a}, \bar{\mathbf{n}}_{\mathbf{a}+\mathbf{b_i}})$ for $i = 1, 2, 3$ and $a + b_i$ is taken mod $N$. Such a field theory can be efficiently described by a quiver diagram, where each node denotes a $U(n)$ factor and the bifundamental chiral fields are represented by directed lines connecting two such nodes. A line originating and ending on the same node describes a field in the adjoint representation of the corresponding $U(n)$ factor.

Next we would like to consider the spectrum of D-branes located on top of orientifold planes. As usual, to preserve the same supersymmetry as $D3$ branes, only $O3$- or $O7$-planes may be included, located on the fixed locus of the orientifold action, $\Omega R(-1)^{F_L}$ (where $R$ is the $Z_2$ geometric involution and $(-1)^{F_L}$ is the left-handed world-sheet fermion number). In terms of the open string modes, the effect of the orientifold action is to identify each $U(n_a)$ gauge group with $U(n_{-a})$ while identifying the representation $(\mathbf{n_a}, \bar{\mathbf{n}}_{\mathbf{a}+\mathbf{b_i}})$ with $(\mathbf{n}_{-\mathbf{a}-\mathbf{b_i}}, \bar{\mathbf{n}}_{-\mathbf{a}})$. In particular, the $U(n_0)$ and $U(n_{N/2})$ gauge factors (if they exist) are projected onto themselves, resulting in an $Sp(SO)$ gauge group, depending on the exact orientifold action. Similarly chiral fields transforming in the $(\mathbf{n_a}, \bar{\mathbf{n}}_{-\mathbf{a}})$ are projected into the (anti)symmetric representation of $SU(n_a)$.

Finaly, the quiver diagrams must be extended to accommodate these unoriented theories [23]. Since each end of the string can independently be in either the fundamental or the antifundamental, it must be represented as a bidirected line with an arrow at each of the two ends, indicating the representation of the string under each of the two gauge group factors. In such a bidirected quiver (biquiver for short), a symmetric or an antisymmetric field is represented by a line with both ends coming out of the same set of branes.

*DSB Quivers.* It is now a simple matter to construct the desired $SU(5)$ model. As an example, consider a $\mathbb{Z}_6$ orientifold with the orbifold action $(b_1, b_2, b_3) = (1, 2, -3)$ and orientifold $R = (-1, -1, -1)$. Furthermore, we take the action on the Chan-Panton indices to be

$$\mathrm{diag}\,(\mathbf{1}_1, \omega^2\mathbf{1}_5, \omega^4\mathbf{1}_5), \qquad \omega = e^{i\pi/3}, \tag{4}$$

so altogether we have 11 fractional branes. There is a single orbifold fixed point and an $O3$-plane at the origin. The biquiver is shown in Fig. 1. As required, the theory is $SO(1) \times U(5)$ with one generation of $\mathbf{10} + \bar{\mathbf{5}}$. The $U(1)$ corresponding to the $SU(5)$ is anomalous and becomes massive through the generalized Green-Schwarz mechanism [24,25]. Hence it remains as a global symmetry and has no effect on the low-energy dynamics.

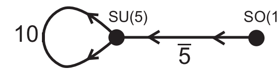


FIG. 1. A one-generation $SU(5)$ biquiver.

TABLE I. The four different orientifold models giving rise to the noncalculable $SU(5)$ DSB theory.

| N | $b_i$ | $\gamma(\theta)$ |
|---|---|---|
| $\mathbb{Z}_6$ | (1, 2, −3) | $\mathrm{diag}(\mathbf{1}_1, \omega^2\mathbf{1}_5, \omega^4\mathbf{1}_5)$ |
| $\mathbb{Z}_6$ | (1, 1, −2) | $\mathrm{diag}(\mathbf{1}_1, \omega^2\mathbf{1}_5, \omega^4\mathbf{1}_5)$ |
| $\mathbb{Z}_9$ | (2, 4, −6) | $\mathrm{diag}(\mathbf{1}_1, \omega^3\mathbf{1}_5, \omega^6\mathbf{1}_5, )$ |
| $\mathbb{Z}_{12}$ | (1, 4, −5) | $\mathrm{diag}(\mathbf{1}_1, \omega^4\mathbf{1}_5, \omega^8\mathbf{1}_5)$ |

It is also possible to construct this biquiver on other singularites, as we (nonexhaustively) list in Table I.

*Blowup Fractional Branes.* Such DSB biquivers exhibit a new class of fractional brane models. In [8] fractional branes were classified as follows:

(1) $N = 2$ *fractional branes*: These exhibit flat directions along which the dynamics are those of $\mathcal{N} = 2$. They typically arise at nonisolated $\mathbb{C}^2/\mathbb{Z}_N$ singularities.

(2) *Deformation fractional branes:* The theory exhibits confining dynamics which translates into (partial) complex-structure deformation of the geometry.

(3) *Runaway (DSB) fractional branes:* This is the generic case. In general, the gauge factors have different ranks and the dynamics lead to a runaway behavior through a nonperturbative superpotential.

It is clear that the DSB biquiver described above does not fit into any of the above classes but instead demonstrates the existence of a new class of fractional branes.[1]

(4) *Blowup fractional branes:* These are fractional branes which do not have flat or runaway directions and are associated with the stabilization of Kähler moduli, corresponding to the possible (partial) blowup of the singularity. In accordance with the classification above we expect blowup fractional branes to be related to unoriented singularities.

In our example the singularity indeed blows up, as we now explain. Ignoring for the moment the nonperturbative dynamics, the $SU(5)$ model has a classical supersymmetric minimum at the origin of field space. Turning on a FI term $\xi^2$ for the corresponding $U(1)$ breaks supersymmetry due to the incompatibility between the $SU(5)$ and $U(1)$ D-terms. At large $\xi^2$ where the classical theory is reliable, a potential

$$V \sim |\xi|^4 \tag{5}$$

[1]We thank Angel Uranga for drawing our attention to this point.

is generated, driving the dynamical FI field to zero. Taking the nonperturbative effects into account, one cannot determine the exact location of the minimum, and on dimensional grounds we expect $\xi^2$ to stabilize near the origin at $\xi^2 \sim \Lambda_5^2$. Such stabilization corresponds to blowing up a 2-cycle in the geometry.

This is in contrast to the case of the runaway class, for which the D-term of a massive anomalous $U(1)$ is necessary in order to stabilize a classical flat direction that becomes unstable quantum mechanically. However, as was already noted in [8] and stressed in [11], such D-term equations should not be imposed, as the massive $U(1)$ is not exhibited at low energy. Imposing the massive D-terms comes at the expense of introducing a new runaway direction of a blowup mode which appears as a FI term. For the model at hand, the field theory does not have a runaway direction and this, in turn, translates into having a stabilized Kähler modulus.

For the specific $SU(5)$ local model, one encounters at the field theory level a single FI blowup mode. In order to embed this quiver in a compact model (away from the decoupling limit), one must worry about other Kähler moduli which must be stabilized without changing the theory at the singularity. There are two mechanisms: First, for the specific $\mathbb{Z}_6$ orbifold, the local geometry consists of four exceptional divisors (arising from one fixed-point and two fixed-curves) out of which only one is compact. Thus, out of the four twisted Kähler moduli, one is stabilized as seen through the gauge dynamics, while the others may be stabilized away from the orbifold fixed-point without affecting the quiver. Second, it is not at all clear which (if any) of the Kähler moduli remain after the orientifold projection. It is possibly misleading to understand the geometry by first resolving the singularity and then orientifolding. Still, the analysis of [26] suggests that at least some of these moduli might be projected out. More details of the Kähler stabilization will appear in [18].

Finally, let us remark that at this stage it is still not clear how generic the blowup class is or whether examples exist where the Kähler moduli are stabilized exactly at the origin, corresponding to the orbifold limit. Furthermore, it would be very interesting to understand whether such quivers exhibit a large-$N$ limit with DSB and Kähler stabilization at the bottom of a duality cascade. We postpone the investigation of this question to future work.

## III. SUMMARY

In this article, a novel realization of the one-generation $SU(5)$ DSB model in string theory was introduced. The model arises in a simple type IIB $\mathbb{Z}_N$ orientifold with fractional branes at the singular locus. The corresponding biquiver model is easily extracted from the geometry. At the field theory level the model has no flat directions, which translates into a stabilization of the Kähler modulus. The latter appears as a dynamical FI term related to the anomalous $U(1)$. The dynamics are therefore in a new class of fractional branes, which (partially) blow up the geometry.

Such models are very simple and are generated from singularities that appear generically on the moduli space of Calabi-Yau manifolds. Therefore it should be easy to construct a compactify version of our construction embedding it in a Calabi-Yau 3-fold. It will be interesting to further study such constructions as they will allow for complex-structure moduli stabilization by turning on fluxes. Such a setup is a step forward in constructing realistic models of particle physics and may allow one to address issues of DSB in the landscape.

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## 2.4 On the Conformal Field Theory Duals of Type IIA $AdS_4$ Flux Compactifications

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# On the Conformal Field Theory Duals of type IIA $AdS_4$ Flux Compactifications

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**Abstract**

We study the conformal field theory dual of the type IIA flux compactification model of DeWolfe, Giryavets, Kachru and Taylor, with all moduli stabilized. We find its central charge and properties of its operator spectrum. We concentrate on the moduli space of the conformal field theory, which we investigate through domain walls in the type IIA string theory. The moduli space turns out to consist of many different branches. We use Bezout's theorem and Bernstein's theorem to enumerate the different branches of the moduli space and estimate their dimension.

# 1 Introduction and Summary of Results

Flux compactifications of string theory (for reviews see [1, 2, 3, 4, 5, 6]) populate large parts of the string landscape, and may describe our universe. However, the theoretical basis for the construction of these compactifications is still far from rigorous (see [7] for criticism), and is based on using low-energy supergravity actions in a regime which is different from the flat-space regime where they are usually derived from string theory. It would be very interesting if a non-perturbative construction of some flux compactifications could be found, providing further support for their consistency, and perhaps leading to new methods for their analysis. A promising arena for such a construction is in flux compactifications involving four dimensional anti-de Sitter (AdS) space. Such compactifications are dual, by the AdS/CFT correspondence [8, 9, 10], to three dimensional conformal field theories. Thus, understanding the three dimensional conformal field theory dual to some $AdS_4$ flux compactification would give a non-perturbative definition for that background. Eventually we would like to study the statistics of conformal field theories that are dual to $AdS_4$ backgrounds, in order to learn about statistics of flux compactifications, and to try to understand how to describe also backgrounds with a positive cosmological constant.

For general flux compactifications, it seems that understanding the dual conformal field theory must be very complicated (see [11, 12] for attempts in this direction). This is because the cosmological constant of the resulting background, which is related to the central charge of the dual conformal field theory, depends in a very complicated way on the fluxes, and it seems that extremely complicated dynamics is needed to reproduce this on the field theory side. The situation seems to be much simpler in the type IIA flux compactifications constructed in [13] (these solutions were further analyzed from the ten dimensional point of view in [14]). These backgrounds have a "large-flux limit" in which some of the fluxes (the three four-form fluxes $f_4^1$, $f_4^2$ and $f_4^3$) are taken to be large, such that in that limit the cosmological constant becomes small, the string coupling becomes weak, and the compact space becomes large. This means that these backgrounds can reliably be studied in the supergravity approximation (except perhaps near the orientifold where the string coupling may be large), and that in the "large-flux limit" the properties of the dual conformal field theories depend in a simple way on the fluxes. One can then hope to reproduce this simple dependence in some field theoretic model. A first attempt at such an analysis, in a different "large-flux limit" which does not lead to a weakly coupled string theory (not all four-form fluxes are taken to be large) appeared in [15]; we will attempt here to describe the field theories appearing in the generic "large-flux limit", which is described by a weakly coupled string theory.

The naive way to construct a field theory dual for flux backgrounds is to imagine constructing the flux gradually from branes carrying that flux, in a manner similar to that in which the $AdS_5 \times S^5$ background of string theory is constructed from D3-branes in flat space. In particular, the 4-form fluxes $f_4^i$ in our background are carried by D4-branes wrapped on 2-cycles in the compact space, and it is natural to imagine building the background from such D4-branes [11, 12, 16]. It is certainly possible to go from a background with a large flux (which is already a weakly coupled weakly curved background) to a background with an even larger flux by adding such branes, and we will use this in our discussion of the moduli space of the conformal field theory. However, it is not clear if one can construct the full theory from such branes, since in the limit of a small flux the background becomes not only strongly curved (this happens also for D3-branes) but also strongly coupled. Nevertheless, it is still natural to guess that the dual conformal field theory arises from some decoupled low-energy theory living on three sets of D4-branes. However, we will find that assuming that the degrees of freedom in this theory are weakly coupled open strings (in adjoint and bi-fundamental representations of the resulting $U(f_4^1) \times U(f_4^2) \times U(f_4^3)$ gauge theory) leads to a contradiction, since the central charge of the dual conformal field theory (which scales as $(f_4^1 f_4^2 f_3^3)^{3/2}$, as we will compute in section 3) grows faster than the number of such degrees of freedom. Thus, the field theory must be more complicated than the naive theory of open strings, perhaps involving a larger gauge group, or [12] fields in multi-fundamental or other higher representations, or perhaps not coming from any gauge theory at all.

In order to find clues about this mysterious field theory we investigate in some detail its moduli space, which can be described using configurations of domain walls in $AdS_4$. Of course, generic flux backgrounds preserve no supersymmetry so they would not be expected to have a moduli space. The flux backgrounds of [13] preserve a four dimensional $\mathcal{N} = 1$ supersymmetry, so they are dual to three dimensional $\mathcal{N} = 1$ superconformal field theories. This amount of supersymmetry is not enough to protect the moduli space from quantum corrections, since generic scalar potentials are consistent with three dimensional $\mathcal{N} = 1$ supersymmetry. Nevertheless, in our study (performed in the weak coupling weak curvature limit) we will find a large moduli space in these backgrounds. We expect this moduli space to be lifted by quantum corrections (perhaps non-perturbative), but these quantum corrections are small in the “large flux limit”, and we expect the existence of a moduli space in this limit to be a useful clue for the construction of the dual field theory. The moduli space turns out to be very complicated, with many different branches that may be interconnected. For each such branch we employ some mathematical theorems that count the number of solutions of polynomial equations, in order to compute its dimension. We will show that for large values of the fluxes, the dimension of the moduli space scales as $\sum_{i<j} f_4^i f_4^j$.

The effective field theory at generic points on the moduli space includes $U(1)$ gauge fields, scalars and fermions; however, in $2+1$ dimensions a $U(1)$ gauge field is equivalent to a compact scalar, so the presence of these gauge fields does not necessarily imply that the full theory is related to a $U(f_4^1) \times U(f_4^2) \times U(f_4^3)$ gauge theory. However, there are special submanifolds of the moduli space in which we can see gauge groups corresponding to all subgroups of $U(f_4^1) \times U(f_4^2) \times U(f_4^3)$, suggesting that the conformal field theory may be described as the low-energy limit of some gauge theory which includes this gauge group. This is further supported by the scaling of the dimension of the moduli space, that is reminiscent of strings in the bi-fundamental representation of each pair of gauge groups (and such bi-fundamental fields indeed appear on the special submanifolds mentioned above).

So far we have not been able to find a simple field theory model that would reproduce all the properties that we find; in particular it seems hard to explain the large number of degrees of freedom, and the complicated form of the moduli space. We hope that these properties will provide useful clues for the construction of such a field theory in the future.

We begin in section 2 with a review of the type IIA backgrounds of [13] that we will be studying and of their supersymmetry equations. In section 3 we compute various basic properties of the dual field theory, like its central charges and the generic features of its operator spectrum. In section 4 we consider branes spanning domain walls in the $AdS_4$ space, and find the condition that they preserve supersymmetry. We then go on in section 5 to study the structure of their moduli space. We compute the moduli space explicitly for a simple example and find some properties, such as the dimension, for the generic case. In the appendices we include some additional calculations, including an explicit calculation of the supersymmetry in the bulk in appendix A. In appendix B we show that the domain walls found in section 4 obey the BPS condition, and in appendix C we consider the possibility of additional domain wall brane configurations.

# 2 The Model

In this section we review the low-energy limit of the background of massive type IIA string theory described by an orientifold of type IIA string theory on $T^6/\mathbb{Z}_3^2$. This model was studied extensively in [13], where it was shown that by turning on generic values for the background fluxes it is possible to stabilize all moduli without the use of non-perturbative effects. We will start by reviewing the geometrical properties of the compact manifold, and then discuss the possible moduli and the way in which they can be stabilized. Finally we will show that the background satisfies the supersymmetry equations in the bulk.

## 2.1 The Geometry

The compact space is an orbifold of $T^6$. We parameterize the torus by the three complex coordinates $z_i = x_i + iy_i$, with $i = 1, 2, 3$. We take the complex structure moduli of the tori to be $\tau_i = \alpha \equiv e^{2\pi i/6}$, so that the $z_i$ coordinates are periodic with the identifications

$$z_i \simeq z_i + 1 \simeq z_i + \alpha. \tag{2.1}$$

At this point in the moduli space of the torus, the $T^6$ has a $\mathbb{Z}_3$ symmetry, under which the coordinates transform as

$$z_i \to \alpha^2 z_i. \tag{2.2}$$

It is then possible to orbifold by this symmetry. This gives rise to a singular space, with 27 singular points corresponding to the fixed points of the $\mathbb{Z}_3$ symmetry [17, 18]. After this identification, there is a second $\mathbb{Z}_3$ symmetry acting freely on the coordinates as

$$(z_1, z_2, z_3) \to (\alpha^2 z_1 + \frac{1+\alpha}{3}, \alpha^4 z_2 + \frac{1+\alpha}{3}, z_3 + \frac{1+\alpha}{3}). \tag{2.3}$$

This symmetry identifies triplets of fixed points, thus leading, after a second orbifold by the second $\mathbb{Z}_3$ symmetry, to a singular Calabi-Yau manifold with only 9 singular points (that can be locally described as a $C^3/\mathbb{Z}_3$ singularity). The cohomology of this manifold is given by $h^{2,1} = 0$ and $h^{1,1} = 12$. There are therefore no complex structure moduli and 12 Kähler moduli. Nine of them are associated to blow-up modes of the singular points, while the other three Kähler moduli describe the volume of the three tori. These volume moduli $\gamma_i$ appear in the metric as

$$ds^2 = \sum_{i=1}^{3} \gamma_i dz^i d\bar{z}^i, \tag{2.4}$$

or in the Kähler form for the manifold as

$$J = ig_{i\bar{j}}\, dz^i \wedge d\bar{z}^j = \sum_{i=1}^{3} i\frac{\gamma_i}{2} dz^i \wedge d\bar{z}^i. \tag{2.5}$$

It will be useful to write an explicit basis for the cohomology of the compact space. There are no one-forms, since the two $\mathbb{Z}_3$ orbifolds project out all of the one-forms of the torus. There are three two-forms that form the basis of the untwisted part of $H^2$. These are the two-forms that remain invariant under the $\mathbb{Z}_3^2$, and they can be chosen as

$$w_i = (\kappa\sqrt{3})^{1/3} i dz^i \wedge d\bar{z}^i, \tag{2.6}$$

in an arbitrary normalization (in which the triple intersection is $\kappa$). Their Poincaré-dual four-forms form the basis for the untwisted part of $H^4$,

$$\tilde{w}^i = \left(\frac{3}{\kappa}\right)^{1/3} (idz^j \wedge d\bar{z}^j) \wedge (idz^k \wedge d\bar{z}^k), \tag{2.7}$$

where $\{i, j, k\}$ are different elements of the set $\{1, 2, 3\}$. We choose the normalizations such that

$$\int_{T^6/\mathbb{Z}_3^2} w_1 \wedge w_2 \wedge w_3 = \kappa, \qquad \int_{T^6/\mathbb{Z}_3^2} w_i \wedge \tilde{w}^j = \delta_i^j. \tag{2.8}$$

There are also two-forms and four-forms associated with the blow-up modes of the orbifold fixed points, which we will not write down explicitly.

Since $h^{2,1} = 0$, the only 3-forms in the compact geometry are the holomorphic 3-form

$$\Omega = \sqrt{\gamma_1\gamma_2\gamma_3} i dz_1 \wedge dz_2 \wedge dz_3 \tag{2.9}$$

and its complex conjugate $\bar{\Omega}$. These are normalized such that

$$\frac{i}{8}\int_{T^6/\mathbb{Z}_3^2} \Omega \wedge \bar{\Omega} = vol(T^6/\mathbb{Z}_3^2) = \frac{1}{8\sqrt{3}}\gamma_1\gamma_2\gamma_3, \tag{2.10}$$

and can be verified to obey the standard relations

$$J \wedge \Omega = 0, \quad \frac{i}{8}\Omega \wedge \bar{\Omega} = \frac{1}{3!}J^3. \tag{2.11}$$

As a last step in defining the geometry we quotient by an orientifold action. We will use the orientifold of $T^6/\mathbb{Z}_3^2$ presented in [19]. The total orientifold action is given by $\Omega(-1)^{F_L}\sigma$, where $\Omega$ is reflection on the worldsheet, $F_L$ is the worldsheet left moving fermion number, and $\sigma$ is the spacetime involution

$$z_i \to -\bar{z}_i. \tag{2.12}$$

Under this action there is a 3 dimensional space left fixed, given by $\mathcal{R}\text{e}[z_i] = 0$. Thus, the theory contains an $O6$-plane wrapping this 3-cycle and filling the non compact directions.

Under the orientifold action the different forms have non trivial transformation properties. The forms defined above transform as

$$w_i \to -w_i, \qquad \tilde{w}^i \to \tilde{w}^i, \qquad \Omega \to \bar{\Omega}. \tag{2.13}$$

One can write the three-forms in a diagonal basis with respect to the orientifold by decomposing $\Omega$ to its real and imaginary parts, $\Omega = \frac{\sqrt{\gamma_1\gamma_2\gamma_3}}{3^{1/4}\sqrt{2}}(\alpha_0 + i\beta_0)$. These transform as

$$\alpha_0 \to \alpha_0, \qquad \beta_0 \to -\beta_0. \tag{2.14}$$

## 2.2 Moduli and Their Stabilization

In order to stabilize all the moduli we will need to turn on a 10-form (or 0-form) RR flux, so that in the low-energy limit we obtain Romans' massive IIA supergravity theory [20] (with a mass parameter proportional to the RR 0-form field strength), compactified to four dimensions on the $T^6/\mathbb{Z}_3^2$ orientifold discussed in the previous subsection. In addition to the background metric and dilaton, the theory includes a NS-NS 2-form $B_2$ (whose field strength is $H_3$), and a RR 1-form and 3-form, $C_1$ and $C_3$ (with field strengths $F_2$ and $F_4$). [1]

Before turning on fluxes, the massless spectrum includes the Kähler parameters from the metric, $\gamma_i$, and the dilaton $\phi$. Since $B_2$ is odd under $\Omega$, its zero modes are related to the forms $\omega_i$ in the $\sigma$-odd cohomology $H^2_-$, and it can be expanded as

$$B_2 = \sum b_i \omega^i. \tag{2.15}$$

The three zero modes $b_i$ combine with $\gamma_i$ to form the bosonic part of a chiral multiplet. Similarly we can expand the RR forms. Since $h^1 = 0$, the one-form has no zero modes. The three-form, being even under $\Omega$, has one zero mode, related to the unique even three-form, $\alpha_0$. Thus we have

$$C_3 = \xi \alpha_0. \tag{2.16}$$

The four dimensional axiodilaton superfield contains the combination of this axion $\xi$ with the dilaton $\phi$.

All of these moduli can be stabilized by turning on fluxes along the compact directions. In order to preserve Poincaré invariance, the fluxes can be written as

$$F_n = \hat{F}_n + vol_4 \wedge \tilde{F}_{n-4}, \tag{2.17}$$

where all the indices in $\hat{F}$ and $\tilde{F}$ are internal, and they are Poincaré dual using the 6 dimensional metric, $\tilde{F}_n = (-1)^{(n-1)(n-2)/2} *_6 \hat{F}_{6-n}$. The background values for the fluxes can then be written by expanding the fields in the relevant cohomology (having the correct parity under the orientifold) :

$$H_3 = -p\beta_0, \quad \hat{F}_0 = -m_0, \quad \hat{F}_2 = -m_i w_i, \quad \hat{F}_4 = e_i \tilde{w}^i, \quad \hat{F}_6 = -e_0 \frac{\alpha_0 \wedge \beta_0}{\text{vol}}. \tag{2.18}$$

They obey the following integrality condition

$$\frac{\sqrt{2}}{(2\pi\sqrt{\alpha'})^{p-1}} \int F_p = f_p \in \mathbb{Z} \,, \qquad \frac{1}{(2\pi)^2\alpha'} \int H_3 = h_3 \in \mathbb{Z}, \tag{2.19}$$

[1] Note that we use the following conventions for the RR fields. We follow the convention of [13, 21] including an additional factor of $\sqrt{2}$ with respect to the standard convention, while working with signs as in [22]. So, we use opposite signs for $F_0$ and $F_6$ compared to [13, 21].

so that the integer fluxes are related to the ones in (2.18) by

$$f_0 = -\sqrt{2}2\pi\sqrt{\alpha'}m_0, \qquad f_2^i = -\frac{\sqrt{2}\kappa^{1/3}}{2\pi\sqrt{\alpha'}}m_i, \qquad f_4^i = \frac{\sqrt{2}}{\kappa^{1/3}(2\pi\sqrt{\alpha'})^3}e_i,$$
$$f_6 = -\frac{\sqrt{2}}{(2\pi\sqrt{\alpha'})^5}e_0, \qquad h_3 = \frac{1}{(2\pi\sqrt{\alpha'})^2}p. \tag{2.20}$$

We will split the field strengths into the background part and an excitations part. They can then be written as

$$\begin{aligned} H_3 &= H_3^{bg} + dB_2, \\ F_2 &= F_2^{bg} + dC_1 + m_0 B_2, \\ F_4 &= F_4^{bg} + dC_3 - C_1 \wedge dB_2 - \frac{m_0}{2} B_2 \wedge B_2. \end{aligned} \tag{2.21}$$

The background values of the fluxes are constrained by the tadpoles of the different fields. These were analyzed in [13], where it was found that there is a unique tadpole for $C_7$ which requires

$$m_0 p = -\sqrt{2}\, 2\pi\sqrt{\alpha'}. \tag{2.22}$$

In terms of the integer fluxes (2.20) this means $f_0 h_3 = 2$, so that there are four different possibilities, $(f_0, h_3) = (1,2), (2,1), (-1,-2), (-2,-1)$. All other fluxes are not constrained by tadpoles.

The scalar potential was analyzed in detail in [13], and it was found that by turning on such fluxes $(e_0, e_i, m_0, m_i, p)$ the moduli are stabilized at values given by

$$\begin{aligned} \gamma_i &= 2(\kappa\sqrt{3})^{1/3}\frac{1}{|\hat{e}_i|}\sqrt{\frac{-5\hat{e}_1\hat{e}_2\hat{e}_3}{3m_0\kappa}}, \\ b_i &= \frac{m_i}{m_0}, \\ e^{-\phi} &= \frac{4}{3}\frac{1}{|p|}\left(-\frac{12}{5}\frac{m_0\hat{e}_1\hat{e}_2\hat{e}_3}{\kappa}\right)^{1/4}, \\ \xi &= \frac{1}{p}\left(e_0 + \frac{e_i m_i}{m_0} + \frac{2\kappa m_1 m_2 m_3}{m_0^2}\right), \end{aligned} \tag{2.23}$$

with $\hat{e}_i \equiv e_i + \kappa m_j m_k / m_0$ (where $\{i,j,k\} = \{1,2,3\}$). From the four dimensional point of view, this solution has a negative cosmological constant

$$\Lambda = -\frac{p^2}{2}\frac{\sqrt{3}}{\gamma_1\gamma_2\gamma_3}, \tag{2.24}$$

and we will consider the maximally symmetric solution of the resulting four dimensional action, which is given by $AdS_4 \times T^6/\mathbb{Z}_3^2$.

There are several things to note here regarding this solution. From supersymmetry we get (see the next subsection) a constraint on the signs of the fluxes

$$\mathrm{sign}(m_0 p) = \mathrm{sign}(m_0 e_i) = -, \tag{2.25}$$

which also guarantees that the $\gamma_i$ and $e^{-\phi}$ are real. When we take large values for the quantized fluxes, $f_4^i \gg 1$ (without making some of them much larger than the others), we get to a regime with large volume and weak coupling where we can trust our computation. Throughout this paper we will work in this regime. We also note that there is a non-singular solution with $e_0 = m_i = 0$, which has no 2-form and 6-form background fluxes.

There are additional moduli localized near the $C^3/\mathbb{Z}_3$ singularities. One can turn on $\hat{F}_2$ and $\hat{F}_4$ fluxes on the corresponding localized cycles, which we denote, respectively, by $n_A$ and $f_A$ ($A = 1, \cdots, 9$ goes over the different singular points). The blow up Kähler modes $t_{BA}$ are then stabilized at

$$t_{BA} = \frac{n_A}{m_0} - i\sqrt{-\frac{10\hat{f}_A}{3\beta m_0}}, \tag{2.26}$$

where we defined $\hat{f}_A \equiv f_A + \beta n_A^2/2m_0$, and the integer $\beta$ is the non-trivial triple intersection of the twisted cycles. The values for $e^\phi$ and $\xi$ are modified by these additional fluxes (the dilaton by a small amount when $f_4^i \gg 1$):

$$\begin{aligned} e^{-\phi} &= \frac{4}{3}\frac{1}{|p|}\left[\sqrt{-\frac{12}{5}\frac{m_0\hat{e}_1\hat{e}_2\hat{e}_3}{\kappa}} + \frac{3}{25}m_0^2\beta\sum_A\left(-\frac{10f_A}{3\beta m_0}\right)^{3/2}\right]^{1/2} \\ \xi &= \frac{1}{p}\left(e_0 + \frac{e_i m_i + \sum_A f_A n_A}{m_0} + \frac{6\kappa m_1 m_2 m_3 + \beta\sum_A n_A^3}{3m_0^2}\right). \end{aligned} \tag{2.27}$$

## 2.3 Supersymmetry

In this subsection we review how the background described above satisfies the supersymmetry equations. We will write the background as a warped product of a four-dimensional Anti de-Sitter space with $T^6/\mathbb{Z}_3^2$, with the metric

$$ds^2 = e^{2A}h_{MN}dx^M dx^N + g_{AB}dy^A dy^B, \tag{2.28}$$

where $A = A(y)$ is the warp factor, $h_{MN}$ is the 4 dimensional AdS metric and $g_{AB}$ is the metric on $T^6/\mathbb{Z}_3^2$. We will use the double spinor convention, which in type IIA amounts to writing the Majorana Killing spinor as two Majorana Weyl spinors with opposite chirality,

$$\epsilon = \epsilon_+ + \epsilon_-, \quad \Gamma_{(10)}\epsilon_\pm = \pm\epsilon_\pm. \tag{2.29}$$

We can decompose the ten dimensional Clifford algebra into the $4d \otimes 6d$ algebras in the following way,

$$\Gamma_{\underline{\mu}} = \gamma_{\underline{\mu}} \otimes \mathbb{I}, \quad \Gamma_{\underline{m}} = \gamma_{(4)} \otimes \hat{\gamma}_{\underline{m}}, \tag{2.30}$$

where the 4d gamma matrices are real and the 6d are purely imaginary and antisymmetric. We denote by underlined indices the tangent space flat indices. The Killing spinors also decompose as

$$\begin{aligned} \epsilon_+(x,y) &= a\,\theta_+(x) \otimes \eta_+(y) + a^*\theta_-(x) \otimes \eta_-(y), \\ \epsilon_-(x,y) &= b^*\theta_+(x) \otimes \eta_-(y) + b\,\theta_-(x) \otimes \eta_+(y), \end{aligned} \tag{2.31}$$

where $\eta_+ = \eta_-^*$ is the unique covariantly constant spinor on the Calabi-Yau, while $\theta_+$, $\theta_-$ (with $\bar{\theta}_+ = \theta_-^T C$) are the Killing spinors on $AdS_4$ satisfying

$$D_\mu\theta_+ = \frac{1}{2}\mu^*\gamma_\mu\theta_-, \quad D_\mu\theta_- = \frac{1}{2}\mu\gamma_\mu\theta_+. \tag{2.32}$$

The complex number $\mu$ is the value of the superpotential, so that the cosmological constant of the $AdS_4$ space is given by $\Lambda = -|\mu|^2$.

The spinor $\eta_+$ on the Calabi-Yau gives rise to an $SU(3)$ structure. Following [22, 23, 24] we can write the two pure spinors as bispinors of $O(6,6)$ in the following way

$$\not{\Psi}^+ = a\eta_+ \otimes b^*\eta_+^\dagger, \qquad \not{\Psi}^- = a\eta_+ \otimes b\eta_-^\dagger. \tag{2.33}$$

Using the Clifford map, there is a one-to-one correspondence between such bispinors and p-forms, given by

$$C \equiv \sum \frac{1}{k!} C_{i_1,\ldots,i_k} dx^{i_1} \wedge \ldots \wedge dx^{i_k} \quad \longleftrightarrow \quad \not{C} \equiv \sum \frac{1}{k!} C_{i_1,\ldots,i_k} \gamma_{\alpha\beta}^{i_1\ldots i_k}. \tag{2.34}$$

Using this map, the pure spinors can also be represented by the almost complex structure 2-form and the holomorphic 3-form,

$$\Psi^+ = \frac{a\bar{b}}{8}e^{-iJ}, \qquad \Psi^- = -\frac{iab}{8}\Omega. \tag{2.35}$$

Following these notations, the equations for preserved supersymmetry are given by [24, 22]

$$\begin{aligned} e^{-2A+\phi}(d + H\wedge)(e^{2A-\phi}\Psi_+) &= 2\mu\,\mathcal{R}\mathrm{e}[\Psi_-], \\ e^{-2A+\phi}(d + H\wedge)(e^{2A-\phi}\Psi_-) &= 3i\,\mathcal{I}\mathrm{m}[\bar{\mu}\Psi_+] + dA \wedge \bar{\Psi}_- \end{aligned} \tag{2.36}$$

$$+\frac{\sqrt{2}}{16}e^{\phi}\left[(|a|^2-|b|^2)\hat{F}+i(|a|^2+|b|^2)\tilde{F}\right], \tag{2.37}$$

where $F = F_0 + F_2 + F_4 + F_6$ are the modified RR fields defined as

$$F = e^{-B}F^{\rm bg} + dC + H \wedge C, \tag{2.38}$$

so that they obey the non-standard Bianchi identity $dF_n = -H \wedge F_{n-2}$.

We solve these equations in Appendix A, finding that for supersymmetry to be preserved the Killing spinors should have $b = -a^*$, and the moduli should obtain values as in (2.23).

# 3 General Properties of the Dual Conformal Field Theory

In the previous section we described a solution of supergravity (and, thus, of string theory) that includes a four dimensional AdS space. According to the AdS/CFT correspondence [8, 9, 10], there is a three dimensional conformal field theory which is the holographic dual of this solution. Many properties of this CFT can be calculated in a simple manner from the supergravity solution. We will discuss these properties in this section, including the central charge, dimensions of operators and the global symmetries of the CFT. We will also discuss D-branes wrapping cycles in the compact space to give particles or strings on $AdS_4$. Throughout this paper we will work only in the limit where all 4-form fluxes are large, so that the string coupling is weak and the supergravity approximation is good.

## 3.1 The Central Charge

We will begin by finding the central charge of the CFT from the curvature of the AdS space. There are various possible definitions of a central charge for three dimensional CFTs, including the coefficient of the two-point function of the stress-energy tensor, and the coefficient multiplying the volume times the temperature squared in the entropy of the theory at finite temperature. In the gravity approximation, all definitions give answers proportional to $R_{AdS}^2/G_4$, where $G_4$ is the four dimensional Newton's constant, since this is the coefficient (in units of $R_{AdS}$) of the four dimensional action, so that all correlation functions are proportional to this. Using our formulas from the previous section, we have (up to constants)

$$\frac{(R_4^{AdS})^2}{G_4} = \frac{Vol(T^6/\mathbb{Z}_3^2)\, e^{-2\phi}\Lambda^{-1}}{\alpha'^4} \propto \frac{(f_4^1 f_4^2 f_4^3)^{3/2}}{f_0^{5/2}h_3^4} \simeq (f_4^1 f_4^2 f_4^3)^{3/2}, \tag{3.1}$$

since the 3-form and 0-form fluxes are numbers of order one. In particular, if we take all the fluxes $f_4^i \sim N$, we find a central charge scaling as $c \propto N^{9/2}$ (this was independently noted in [15]).

Equation (3.1) is reminiscent of the formula for the central charge in the case of $\mathcal{N} = 8$ $SU(N)$ SYM in 2+1 dimensions. In that case the central charge of the theory in the IR (where it is dual to M theory on $AdS_4 \times S^7$) scales like $N^{3/2}$ (which is not understood in terms of any effective field theory degrees of freedom). By analogy, this suggests that in our case there may be some $N_{eff} = f_4^1 f_4^2 f_4^3$, namely that if there is a UV description of any sort it should include an order of $(f_4^1 f_4^2 f_4^3)^2$ degrees of freedom. This is also suggested by the fact that these are the minimal integer powers which are larger than those appearing in the central charge (3.1). This UV description could be for instance an $SU(N_{eff})$ gauge theory, or an $SU(f_4^1) \times SU(f_4^2) \times SU(f_4^3)$ gauge theory with matter in representations whose dimension is of order $N_{eff}^2$ (such representations are consistent with asymptotic freedom in $2+1$ dimensions).

The analysis in [15] give some support to this suggestion. It was argued there that after two T-dualities in the directions of the first 2-torus, and in the limit of $f_4^1 \to \infty,\ f_4^{2,3}\ fixed$, the background should be lifted to M theory, and resembles the near-horizon limit of $f_4^1$ M2-branes (at some singularity). In this case we see that the degrees of freedom are renormalized from $O(1) * (f_4^1)^2$ in the theory on some D2-branes (at the same singularity) to $O(1) * (f_4^1)^{3/2}$ in the theory on the M2-branes.

Below we will use another indicator for the number of branes in the problem which will be the structure (and in particular the dimensionality) of different branches of the moduli space. The moduli space will be made out of holomorphic (in an appropriate sense) D4-branes which wrap different 2-cycles of the torus. Our analysis of the moduli space will be performed in the limit where all fluxes are large, but since it preserves some supersymmetry it is natural to expect that the same results for the form and dimension of the moduli space will hold also in other limits (though we have not verified this directly). Assuming this, we find (using our results derived below) that for the scaling of [15] the dimension of the largest branch of the moduli space will scale like $f_4^1$. Indeed, this branch is described by the motion of D4-branes wrapping the first $T^2$, which become M2-branes (or D2-branes) after 2 T-dualities.

Our more general analysis below will show that the dimension of the maximal branch of the moduli space scales like $\max(f_4^i f_4^j),\ i \neq j$. The previous case is a special case of this. Note that this might suggest that in a scaling limit in which two of the fluxes (say, $f_4^1$ and $f_4^2$) become large while the third remains finite, the theory resembles that of $N_{eff} \simeq f_4^1 f_4^2$ M2-branes. While the dimension of the moduli space and the number of degrees of freedom are consistent with this suggestion, the precise form of the moduli space is very different from what one would obtain from any theory of $N_{eff}$ M2-branes.

## 3.2 Global Symmetries

As described above, the supergravity solution preserves a four dimensional $\mathcal{N} = 1$ supersymmetry. By the AdS/CFT correspondence this maps to a three dimensional $\mathcal{N} = 1$ superconformal symmetry, with two supersymmetry charges and two superconformal charges.

In the AdS/CFT correspondence, the global symmetries of the CFT are related to gauge symmetries of the gravitational theory. Such symmetries arise from reductions of the supergravity fields on the compact space (or from space-filling D-branes). The simplest gauge fields are related to the ten dimensional metric, and are related to the isometry group of the compactification manifold. In our case the compact space is a Calabi-Yau manifold and thus has no isometry group. So, we do not get any gauge fields from the metric. In addition to the metric, the RR 1-form and 3-form can also give rise to gauge symmetries. In our background we have a non-trivial 0-form flux which gives a mass to the 1-form (it is swallowed by the 2-form $B_2$ which becomes massive). Thus, there is no gauge symmetry associated with the 1-form. In order to get a 1-form gauge field from the 3-form we need to integrate it over a 2-cycle. As the compactification manifold contains three such untwisted 2-cycles, we obtain three commuting gauge fields. However, since the 2-cycles are odd under the orientifolding, these gauge fields are projected out by the orientifold. The gauge fields arising from the twisted 2-cycles are similarly projected out.

Thus, the conformal field theory that we are looking for does not have any global symmetry (beyond the $\mathcal{N} = 1$ superconformal algebra, which does not include any continuous R-symmetry group).

## 3.3 Operators and Scalings

Another basic property of a conformal field theory is the spectrum of operators in the theory. The simplest operators are related to the supergravity fields, and their dimensions are related to the masses so we can easily find the spectrum. There are two mass scales for fields in the supergravity. The first is the mass of the moduli, which can be computed from their potential. This was written explicitly in [13] for some of the moduli, and it is easy to see that the others have the same scaling. In units of the four dimensional Planck scale $l_{p4}^2 \simeq G_4$ the moduli masses are

$$m_{moduli}^2 \sim (f_4^1 f_4^2 f_4^3)^{-3/2} l_{p4}^{-2}. \tag{3.2}$$

The other mass scale in supergravity is the mass of the Kaluza-Klein modes, given by the inverse radii of the compact tori,

$$m_{KK}^2 \sim \gamma_i^{-1} \sim (f_4^1 f_4^2 f_4^3)^{-3/2} f_4^i l_{p4}^{-2}. \tag{3.3}$$

The dimensions of the corresponding operators are given using the AdS/CFT correspondence as

$$\Delta_{moduli} \sim m_{moduli} R_{AdS} \sim 1, \qquad \Delta_{KK} \sim m_{KK} R_{AdS} \sim \sqrt{f_4^i}. \tag{3.4}$$

Thus, as in all other conformal field theories dual to theories with a four dimensional supergravity approximation (implying a separation of scales between the moduli and the KK modes), there is a small number of operators with dimensions of order one, and all others have large dimensions. The order one operators correspond to the eight moduli fields, $\phi, \xi, b_i, v_i$.

### 3.4 Wrapped Branes

Another type of operators in the field theory involves D$p$-branes wrapped on $p$-cycles in the compact space, giving particles in the $AdS_4$. Since our background involves massive type IIA string theory, we cannot have any D0-branes (which must have $f_0$ strings ending on them) or D6-branes (which must have $f_0$ NS 5-branes ending on them); this is related to the fact that the RR 1-form is swallowed by the NS-NS 2-form. Naively we can have wrapped D2-branes or D4-branes on our 2-cycles or 4-cycles, but in fact the orientifold maps these to anti-D-branes, so it is unlikely that any stable configurations of this type would exist.

We can also consider a $p$-brane wrapping a $(p-1)$-cycle, leading to a string in $AdS_4$ (mapped to some type of flux tube in the conformal field theory). The only such possible configurations are a D4-brane wrapping a 3-cycle and an NS5-brane wrapped on a 4-cycle. A D4-brane wrapped around the $\alpha_0$ cycle is mapped to an anti-brane by the orientifold, while a D4-brane wrapping the $\beta_0$ cycle is not a consistent configuration, since there is $H_3$-flux on that 3-cycle, implying that such D4-branes must have D2-branes ending on them. The same phenomenon arises for NS5-branes wrapped on the 4-cycles, since these have 4-form flux. Note that the fundamental string is also mapped to a string with opposite orientation by the orientifold. Thus, we do not expect to have any stable extended objects in our theory.

## 4 Supersymmetric Domain Walls

In the next two sections we wish to study the moduli space of the conformal field theory dual to the background described in section 2. To describe the moduli space we need to find Lorentz-invariant configurations with zero energy which have the same asymptotics as the solution described above, but differ in the interior. Usually in the AdS/CFT correspondence such configurations are described by supersymmetric branes sitting at some value of the radial position, giving domain walls in AdS along which the flux which the brane is charged under jumps. Moving along the moduli space of these configurations is described in the field theory side as giving non-trivial vacuum expectation values to operators. Such domain walls break half of the supersymmetry in the bulk; in the conformal field theory they break the superconformal generators and preserve the standard supersymmetry generators.

We will consider here D-brane domain walls, given by D$p$-branes wrapping $(p-2)$-cycles in the compact space, and sitting at fixed radial position in $AdS_4$. For the configuration to be supersymmetric (which is the same as having zero energy in the field theory) these must obey some calibration condition. We will find the supersymmetric cycles over which D-branes can be wrapped by considering the $\kappa$-symmetry equation. In Appendix B we will also verify directly that these configurations are BPS states by considering the DBI+CS action for the D-branes and checking that there is no force acting on them. All of these equations are valid in the probe approximation, in which the back-reaction of the D-brane on the background is small. This approximation will be good in the limit of large four-form fluxes that we are working in. Since in three dimensional $\mathcal{N}=1$ theories the moduli space is generally not protected, we expect some potential along the moduli space to be generated by corrections to our leading order approximation; however, this potential is very small in the limit we are working in, so that there will still be an approximate moduli space in the conformal field theory.

The general supersymmetry condition for a D$p$-brane filling time plus $q$ dimensions and wrapping a $(p-q)$-cycle in the compact directions is the $\kappa$-symmetry equation [25], which in the double spinor notation can be written as in [22]:

$$\hat{\Gamma}_{Dp}\epsilon_- = \epsilon_+, \tag{4.1}$$

where

$$\hat{\Gamma}_{Dp} = \gamma_{\underline{0\ldots q}}\gamma_{(4)}^{p-q} \otimes \hat{\gamma}'_{(p-q)}, \tag{4.2}$$

$$\hat{\gamma}'_{(r)} = \frac{1}{\sqrt{\det(P[g]+\mathcal{F})}} \sum_{2l+s=r} \frac{\epsilon^{\alpha_1\ldots\alpha_{2l}\beta_1\ldots\beta_s}}{l!s!2^l}\mathcal{F}_{\alpha_1\alpha_2}\ldots\mathcal{F}_{\alpha_{2l-1}\alpha_{2l}}\hat{\gamma}_{\beta_1\ldots\beta_s}. \tag{4.3}$$

Here, $P[\cdot]$ indicates the pullback of a bulk field onto the worldvolume of the D-brane, and $\mathcal{F} \equiv f + P[B]$ where $f$ is the field strength of the gauge field on the worldvolume of the D-brane, and we set $2\pi\alpha' = 1$.

We can split the $\kappa$-symmetry equation into an equation in the AdS space,

$$\gamma_{\underline{0\ldots q}}\theta_+ = \alpha^{-1}\theta_{(-)^{q+1}} \tag{4.4}$$

for some constant $\alpha$, and an equation in the compact space

$$b^{(*)^{p+1}}\hat{\gamma}'_{(p-q)}\eta_{(-)^{p+1}} = a^{(*)^{q+1}}\alpha\eta_{(-)^{q+1}}, \tag{4.5}$$

where $x^{(*)^n}$ is defined to be $x$ ($x^*$) for even (odd) values of $n$ (and $a$ and $b$ were defined in (2.31)). From these we can see (using the unitarity of the $\gamma$ matrices) that $\alpha$ must be a pure phase, and that the D-brane can be supersymmetric only if $|a| = |b|$, which is indeed the case for our background. For type IIA (even $p$) the internal equation can be brought to the form

$$b\hat{\gamma}'_{(p-q)}\eta_+ = (-)^{p-q}a^{(*)^{p-q}}\alpha^*\eta_{(-)^{p-q}}, \tag{4.6}$$

from which one gets, as in [22], the following calibration condition on the cycle which the D-brane wraps:

$$\left\{b^* P[e^{-iJ}] \wedge e^{\mathcal{F}}\right\}\Big|_{2k} = -a^*\alpha\sqrt{\det(P[g]+\mathcal{F})}\; d\sigma^1 \wedge \ldots \wedge d\sigma^{2k} \tag{4.7}$$

for D-branes wrapping even $2k$-cycles, and

$$\left\{bP[-i\Omega] \wedge e^{\mathcal{F}}\right\}\Big|_{2k+1} = a^*\alpha^*\sqrt{\det(P[g]+\mathcal{F})}\; d\sigma^1 \wedge \ldots \wedge d\sigma^{2k+1} \tag{4.8}$$

for D-branes wrapping odd $(2k+1)$-cycles. We denote the $n$-form part of an expression by $\{\cdot\}|_n$.

We will next use this formalism to describe different configurations of D-branes in this background and study their supersymmetry properties. We begin by verifying that a D6-brane parallel to the orientifold plane obeys the above equations. We then continue to study the equation for D4-branes spanning domain walls in space-time. After finding the general supersymmetric solution we will study the special case of linear D-branes. In Appendix C we show that there are no other types of D-branes that lead to supersymmetric domain walls.

## 4.1 A Space-Time Filling D6-Brane

We start by considering a probe D6-brane filling the whole non-compact $AdS_4$ space-time and wrapping a three-cycle in the compact space. This is not a domain wall, but we use it to test our equations, since we know that such a configuration carrying the same charges as the O6-plane must be supersymmetric. The $AdS_4$ part of the $\kappa$-symmetry equation (4.4) gives

$$\alpha^{-1}\theta_+ = \gamma_{\underline{0123}}\theta_+ = i\gamma_{(4)}\theta_+ = i\theta_+, \tag{4.9}$$

so it fixes $\alpha = -i$.

Since the orientifold action is

$$z_i \to -\bar{z}_i, \tag{4.10}$$

the orientifold plane is located on $z_i = -\bar{z}_i$, and we wish to put the D6-branes in the same position, so we can parameterize the three compact coordinates of the D6-brane using the embedding

$$\sigma_1 = y_1, \quad \sigma_2 = y_2, \quad \sigma_3 = y_3. \tag{4.11}$$

The induced metric on the worldvolume is

$$ds^2 = \sum_i \gamma_i (d\sigma^i)^2 \tag{4.12}$$

and the induced 3-form is

$$P[\Omega] = \sqrt{\gamma_1\gamma_2\gamma_3} d\sigma_1 \wedge d\sigma_2 \wedge d\sigma_3. \tag{4.13}$$

The right hand side of (4.8) is

$$a^*\alpha^*\sqrt{\det(P[g]+\mathcal{F})}\; d\sigma_1 \wedge d\sigma_2 \wedge d\sigma_3 = a^* i\sqrt{\gamma_1\gamma_2\gamma_3}\; d\sigma_1 \wedge d\sigma_2 \wedge d\sigma_3, \tag{4.14}$$

and the left hand side of the equation is

$$\left\{bP[-i\Omega] \wedge e^{\mathcal{F}}\right\}\Big|_3 = -ib\sqrt{\gamma_1\gamma_2\gamma_3}\; d\sigma_1 \wedge d\sigma_2 \wedge d\sigma_3, \tag{4.15}$$

so in order for the configuration to be supersymmetric we must have $b = -a^*$, precisely as we found from the bulk supersymmetry in section 2.3.

## 4.2 D4-Brane as a Supersymmetric Domain Wall

Next, consider a D4-brane extended as a domain wall in the AdS space and wrapping a generic untwisted 2-cycle[2], in the cohomology class of $\sum N_i w_i$. On such a domain wall, the fluxes jump by $f_4^i \to f_4^i \pm N_i$. In order to find the supersymmetric configuration we will solve the $\kappa$-symmetry equation, starting as before with the $AdS_4$ part,

$$\alpha^{-1}\theta_- = \gamma_{\underline{012}}\theta_+ = \gamma_{\underline{012r}}\gamma_{\underline{r}}\theta_+ = -\gamma_{\underline{r}}\gamma_{\underline{012r}}\theta_+ = -i\gamma_{\underline{r}}\gamma_{(4)}\theta_+ = -i\gamma_{\underline{r}}\theta_+. \tag{4.16}$$

[2] One could also consider D4-branes wrapped around twisted 2-cycles, but it seems that these are never supersymmetric in the presence of the 2-form fluxes stabilizing the twisted sector moduli.

We choose the $AdS_4$ metric

$$ds^2 = \frac{1}{|\mu|}(dr^2 + e^{2r}\eta_{\alpha\beta}dx^\alpha dx^\beta), \tag{4.17}$$

where $\eta$ is a flat Minkowski metric. This is just the standard AdS metric in the Poincaré patch, with the redefinition $r = -\ln(z)$. The covariant derivative can be written as in [26, 27],

$$D_\alpha = \partial_\alpha + \frac{1}{2}|\mu|e^r\gamma_{\underline{\alpha}}\gamma_{\underline{r}}. \tag{4.18}$$

We are interested in the Poincaré supercharges, obeying $\partial_\alpha\theta_\pm = 0$, so using (2.32) we get

$$\frac{1}{2}|\mu|e^r\gamma_{\underline{\alpha}}\gamma_{\underline{r}}\theta_+ = \frac{1}{2}\mu^*\gamma_\alpha\theta_- = \frac{1}{2}\mu^*e^r\gamma_{\underline{\alpha}}\theta_- \tag{4.19}$$

$$\gamma_{\underline{r}}\theta_+ = \frac{\mu^*}{|\mu|}\theta_- = -\text{sign}(p)\frac{b}{\bar{b}}\theta_-. \tag{4.20}$$

Plugging this into (4.16) we find $\alpha = -\text{sign}(p)i\frac{\bar{b}}{b}$.

To solve the internal part of the $\kappa$-symmetry equation we need to choose how to wrap the D4-brane. We start with the simplest case where the D4-brane wraps the torus $z_1$. We can choose the embedding

$$\sigma^1 = x^1, \quad \sigma^2 = y^1, \tag{4.21}$$

with the induced metric being

$$\gamma_1(d\sigma^1)^2 + \gamma_1(d\sigma^2)^2, \tag{4.22}$$

and the pullback of $J$ given by

$$P[J] = \gamma_1 d\sigma^1 \wedge d\sigma^2. \tag{4.23}$$

Plugging into the supersymmetry condition (4.7) we have on the right-hand side

$$-a^*\alpha\sqrt{\det(P[g]+\mathcal{F})}d\sigma^1\wedge d\sigma^2 = i\text{sign}(p)\frac{a^*b^*}{b}\gamma_1 d\sigma^1\wedge d\sigma^2 = -\text{sign}(p)ib^*\gamma_1 d\sigma^1\wedge d\sigma^2, \tag{4.24}$$

while on the other side we have

$$\left\{b^*P[e^{-iJ}]\wedge e^{\mathcal{F}}\right\}\Big|_2 = -ib^*P[J] = -ib^*\gamma_1 d\sigma^1 \wedge d\sigma^2. \tag{4.25}$$

We see that when the background value of $p$ is positive the configuration is supersymmetric.[3] When $p$ is negative one can take the same embedding and flip its

[3] Recall that the signs of $p$ and $e_i$ are the same (A.22).

orientation such that $\sigma^1 = y^1$ and $\sigma^2 = x^1$, to get a supersymmetric configuration. This is just an anti-D4-brane instead of a D4-brane. We see that depending on the sign of the background fluxes, the supersymmetric brane is either a D4-brane or an anti-D4-brane.

Note that in the above we assumed $\mathcal{F} = P[B] + f = 0$. As the $\kappa$-symmetry equation only depends on $\mathcal{F}$, the result won't change if we have a non-trivial background $F_2$ (which generates also a non-trivial background $B$) as long as we turn on fluxes on the worldvolume $f = -P[B]$. If the worldvolume flux is different than this value there will be an additional contribution to both sides of the equation. In the right hand side $\mathcal{F}$ appear only inside the square root so it will change only the absolute value while keeping the phase unchanged. In contrast, the left hand side is proportional to $\mathcal{F} - iJ$ and so will change its phase. We thus conclude that the configuration is supersymmetric only for $\mathcal{F} = 0$.

A different type of cycle the D4-branes can wrap is a twisted cycle at a fixed point. When we go away from the singular limit by turning on 2-form flux on these cycles, the background fluxes and values of the moduli change, see equation (2.27). However the $\kappa$-symmetry equations are only sensitive to changes in the bulk supercharges, that is to the relation between $a$ and $b$ which remains unchanged. Thus, by turning on the appropriate worldvolume flux on D4 branes wrapping the twisted cycles such that $\mathcal{F} = 0$ as before we get additional supersymmetric configurations. We will not consider these configurations in detail, since their contribution to the dimension of the moduli space is finite in the large flux limit.

## 4.3 Generic D4-Brane Configuration

Since the linear embedding described in the previous subsection cannot be realized for generic values of the $N_i$, we will now analyze the most general supersymmetric case of a D4-brane wrapping a generic (untwisted) surface. We will use the complex coordinates $z_a = x_a + iy_a$ in space-time as in (2.1) and define the worldvolume complex coordinate to be $\sigma = \sigma_1 + i\sigma_2$ with the same complex structure. The position of the D4-brane can be written as

$$z^a = z^a(\sigma, \bar{\sigma}). \tag{4.26}$$

The induced metric is given by

$$\begin{aligned} g_{\sigma\sigma} &= \sum_{a=1,2,3} \gamma_a \partial z_a \partial \bar{z}_a, \\ g_{\sigma\bar{\sigma}} &= g_{\bar{\sigma}\sigma} = \sum_{a=1,2,3} \frac{1}{2}\gamma_a (\partial z_a \bar{\partial} \bar{z}_a + \bar{\partial} z_a \partial \bar{z}_a), \\ g_{\bar{\sigma}\bar{\sigma}} &= \sum_{a=1,2,3} \gamma_a \bar{\partial} z_a \bar{\partial} \bar{z}_a, \end{aligned} \tag{4.27}$$

so the right-hand side of the $\kappa$ equation is proportional to

$$\sqrt{\left(\sum_a \frac{1}{2}\gamma_a(\partial z_a\bar{\partial}\bar{z}_a + \bar{\partial} z_a \partial \bar{z}_a)\right)^2 - \sum_a \gamma_a \partial z_a \partial \bar{z}_a \sum_b \gamma_b \bar{\partial} z_b \bar{\partial}\bar{z}_b}. \tag{4.28}$$

This should be equal to the pullback of the almost complex structure, which gives

$$\sum_a \frac{1}{2}\gamma_a(\partial z_a\bar{\partial}\bar{z}_a - \bar{\partial} z_a \partial \bar{z}_a). \tag{4.29}$$

Taking the squares of both sides and equating we get

$$0 \;=\; \frac{1}{2}\sum_{ab}\gamma_i\gamma_j|\partial z_a\bar{\partial} z_b - \partial z_b \bar{\partial} z_a|^2, \tag{4.30}$$

which vanishes if and only if

$$\partial z_a \bar{\partial} z_b = \partial z_b \bar{\partial} z_a. \tag{4.31}$$

In order to understand the meaning of this result, let's consider $z_1$ and $z_2$. We start by defining a new variable $\omega = z_1(\sigma, \bar{\sigma})$. We have

$$\begin{aligned} d\omega &= \partial z_1 d\sigma + \bar{\partial} z_1 d\bar{\sigma}, \\ d\bar{\omega} &= \partial \bar{z}_1 d\sigma + \bar{\partial}\bar{z}_1 d\bar{\sigma}, \end{aligned} \tag{4.32}$$

and

$$\begin{aligned} d\sigma &= \frac{\bar{\partial}\bar{z}_1 d\omega - \bar{\partial} z_1 d\bar{\omega}}{\bar{\partial}\bar{z}_1\partial z_1 - \bar{\partial} z_1 \partial \bar{z}_1}, \\ d\bar{\sigma} &= \frac{\partial z_1 d\omega + \partial \bar{z}_1 d\bar{\omega}}{\bar{\partial}\bar{z}_1\partial z_1 - \bar{\partial} z_1 \partial \bar{z}_1}. \end{aligned} \tag{4.33}$$

We now can write

$$\frac{\partial z_2}{\partial \bar{w}} = \frac{\partial \sigma}{\partial \bar{\omega}}\partial z_2 + \frac{\partial \bar{\sigma}}{\partial \bar{\omega}}\bar{\partial} z_2 = \frac{1}{\bar{\partial}\bar{z}_1\partial z_1 - \bar{\partial} z_1 \partial \bar{z}_1}(-\partial z_2 \bar{\partial} z_1 + \partial z_1 \bar{\partial} z_2) \tag{4.34}$$

which vanishes according to (4.31). We see that the supersymmetry condition can be understood as the statement that the three coordinates $z_a$ can be written as holomorphic functions of each other. In other words, supersymmetry is equivalent to the requirement that the worldvolume wraps a cycle that can be written as the zero locus of two holomorphic functions of the coordinates.

### 4.3.1 Linear D4-Brane

We will study now a simple class of configurations, in which the embedding of the D-brane can be chosen to be a linear map. We can write the embedding as

$$x^i = a^i\sigma^1 + b^i\sigma^2 + \alpha^i, \qquad y^i = c^i\sigma^1 + d^i\sigma^2 + \beta^i. \tag{4.35}$$

Two of the six parameters $\alpha^i,\ \beta^i$ can be absorbed into a shift in $\sigma_1,\ \sigma_2$, while the others parameterize the moduli of the position of the D4-brane. We also need to check that this embedding keeps the periodicity of the tori. The identifications on $\sigma_1, \sigma_2$ are

$$(\sigma_1, \sigma_2) \simeq (\sigma_1 + 1, \sigma_2) \simeq (\sigma_1 + \frac{1}{2}, \sigma_2 + \frac{\sqrt{3}}{2}) \tag{4.36}$$

and similarly for the $(x_i, y_i)$ pairs. Under the first transformation, we get

$$(x_i, y_i) \to (x_i + a_i, y_i + c_i). \tag{4.37}$$

For these two points to be identified we must have $c_i = \frac{\sqrt{3}}{2}m_i$ and $a_i = \frac{m_i}{2} + n_i$ for some integers $m_i, n_i$. The second transformation acts as

$$(x_i, y_i) \to (x_i + \frac{a_i}{2} + \frac{\sqrt{3}}{2}b_i, y_i + \frac{c_i}{2} + \frac{\sqrt{3}}{2}d_i), \tag{4.38}$$

which gives us the restrictions $d_i = \tilde{m}_i - \frac{m_i}{2}$ and $b_i = \frac{1}{\sqrt{3}}(2\tilde{n}_i + \tilde{m}_i - \frac{m_i}{2} - n_i)$ (with integers $\tilde{m}_i, \tilde{n}_i$). We are now able to express $a, b, c, d$ in terms of four integers $m, n, \tilde{m}, \tilde{n}$. The wrapping numbers $N_i$ are given by

$$N_i = \frac{\int_{\sigma^1,\sigma^2} dx^i dy^i}{\int_{x^i,y^i} dx^i dy^i} = \det\left(\begin{bmatrix} a^i & b^i \\ c^i & d^i \end{bmatrix}\right) \frac{\int_{\sigma^1,\sigma^2} d\sigma^1 d\sigma^2}{\int_{x^i,y^i} dx^i dy^i} = a^i d^i - b^i c^i = n^i\tilde{m}^i - \tilde{n}^i m^i. \tag{4.39}$$

Plugging the embedding into the supersymmetry equations (4.31) we get

$$m_j\tilde{m}_i - m_i\tilde{m}_j + n_i\tilde{n}_j - n_j\tilde{n}_i = 0, \tag{4.40}$$

$$m_i\tilde{m}_j - m_j\tilde{m}_i + n_i\tilde{m}_j - m_j\tilde{n}_i - n_j\tilde{m}_i + m_i\tilde{n}_j = 0, \tag{4.41}$$

$$n_i\tilde{m}_i - m_i\tilde{n}_i = N_i. \tag{4.42}$$

We can solve the first two equations for $\tilde{n},\ \tilde{m}$, and plugging into the third we get

$$r_{ij} \equiv \frac{N_i}{N_j} = \frac{m_i^2 + m_i n_i + n_i^2}{m_j^2 + m_j n_j + n_j^2}. \tag{4.43}$$

It turns out that not all charges $N_i$ may be realized by a single linear D4-brane of the type described above, since there is not always an integer solution to (4.43). To see this, we will now prove some things about this ratio. First, note that

$$4(m^2 + mn + n^2) = 3(n + m)^2 + (n - m)^2 \equiv 3x^2 + y^2 \tag{4.44}$$

so we can write the equation as

$$r_{ij} = \frac{3x_i^2 + y_i^2}{3x_j^2 + y_j^2} \tag{4.45}$$

with integer $x_i, y_i$. Next, we will show that a number that can be written as $N = 3x^2 + y^2$ has an even power for the factor of 2 in its prime decomposition. Then, the ratio of two such numbers must also have an even power for the 2 in its prime decomposition (if the ratio is a fraction its prime decomposition is the one coming from the prime decompositions of the numerator and denominator).

We will prove this by induction, showing that if $N$ is divisible by $2^{2n+1}$ then it is also divisible by $2^{2n+2}$. For $n = 0$, if both $x, y$ are even, $N$ is obviously divisible by 4. Else for $N$ to be even both $x, y$ have to be odd, i.e. of the form $x = 2a+1,\ y = 2b+1$. We then get $N = 3x^2+y^2 = 3(4a^2+4a+1)+4b^2+4b+1 = 4(3a^2 + b^2 + 3a + b) + 4$ which is divisible by 4.

We next consider general $n$. Again, since $N$ is even, $x, y$ are both even or both odd. In the first case we can divide the entire equation by 4 and reduce it to the case with $n - 1$. For the latter case, we can write again $N = 3x^2 + y^2 = 3(4a^2 + 4a + 1) + 4b^2 + 4b + 1 = 4(3a(a + 1) + b(b + 1) + 1)$, which after division by 4 is an odd number, specifically it is not a multiple of $2^{2n+1}$, so this case cannot arise.

# 5 The Geometry of the Moduli Space

We have seen that the background of section 2 allows for supersymmetric domain walls, described by D4-branes wrapped on 2-cycles. Over each domain wall the 4-form fluxes jump according to the number of times the domain wall is wrapped over each cycle. When we go far away from the domain walls, we arrive at a background with specific values for the 4 form fluxes. However there are many different configurations of domain walls which result in the same background in the interior of AdS space (beyond all the domain walls). For example, we can take one D-brane wrapped $N_i$ times over the $i$'th cycle, or several branes whose total wrapping number is $N_i$. From this we see that the moduli space may be composed of many different branches. The parameterization of each branch includes the radial position of the domain walls, so each branch is a cone, and all the branches are connected at the origin (when all the domain walls go to the horizon of AdS space). Naively, the full moduli space is made out of all configurations of D4-branes carrying total wrapping numbers equal to the total fluxes $f_4^i$ (some of the D4-branes can of course sit at the origin). However, it is not completely clear that this is true, since our approximations break down when the 4-form fluxes $f_4^i$ become small (and it is certainly not clear if there is an $AdS_4$ solution when one of the fluxes vanishes). Nevertheless, we expect that this naive approach will be a useful tool for counting the dimension of the moduli space at large $f_4^i$.

Note that often configurations made out of different sets of D4-branes (with the same total wrapping number) can be connected without the need to send some of the branes to the origin of the moduli space. When two D4-branes intersect, new light degrees of freedom arise at their intersection point which may deform the configuration and smooth it into a configuration of a single D4-brane with the same total flux.

We will begin by considering a simple branch of the moduli space where there is only one D4-brane wrapping a simple cycle. We will study it in detail and describe its global structure. We will then go on to describe some properties of the general moduli space. Specifically, we will parameterize the different branches, and estimate the dimension of a generic branch.

## 5.1 The Moduli Space of a Single D4-Brane

We start with the simplest branch of the moduli space, which includes branes with wrapping numbers $(N_1, N_2, N_3) = (1, 0, 0)$. For these values we can have only one possible configuration of domain walls, which consists of a single D4-brane wrapping the first $T^2$ inside the compact space. The geometry of its moduli space can be simply read from its low-energy effective action. We consider the D4-brane to be located at specific values of $r, u^2, v^2, u^3, v^3$ and embedded as

$$t = \xi^0, \quad x^1 = \xi^1, \quad x^2 = \xi^2, \quad v^1 = \xi^3, \quad u^1 = \xi^4. \tag{5.1}$$

We begin by assuming that the D4-brane is away from all fixed points of the orbifold and orientifold. The DBI action is given (up to quadratic order in the fields) by

$$\begin{aligned}\mathcal{L}_{DBI} =& -\mu_4 \int d^5\xi e^{-\phi}\sqrt{-g_{ik}} \\ \approx& -\mu_4 \int d^5\xi e^{-\phi}\frac{r^3}{R^3}\gamma_1 \left[1 + \frac{1}{2}G_{\iota\kappa}\partial_i X^\iota \partial_k X^\kappa g^{ik} + \frac{1}{4}\mathcal{F}_{ik}\mathcal{F}_{i'k'}g^{ii'}g^{kk'}\right]\end{aligned} \tag{5.2}$$

where $g_{ik} = \frac{\partial X^I}{\partial \xi^i}\frac{\partial X^K}{\partial \xi^k}G_{IK}$ is the induced metric on the D-brane, and $G_{IK}$ is the ten dimensional metric which we now write in the form (with $R = R_{AdS}$)

$$ds^2 = \frac{r^2}{R^2}\left(dt^2 + (dx^1)^2 + (dx^2)^2\right) + \frac{R^2}{r^2}dr^2 + \sum_{i=1}^{3}\gamma_i\left((du^i)^2 + (dv^i)^2\right). \tag{5.3}$$

We use $i, k$ to denote the worldvolume indices, $\iota, \kappa$ are transverse coordinates and $I, K$ denote full ten dimensional indices. Reducing the action on the torus we take the fields $r, u^2, v^2, u^3, v^3$ to depend only on $t, x_1, x_2$. The 5 dimensional gauge field $A$ can be expanded as [4]

$$A = \hat{A} + a_1 du^1 + a_2 dv^1, \tag{5.4}$$

giving rise to two Wilson lines $a_i$, and a three dimensional gauge field, $\hat{A}$, which can be dualized to another scalar $*_3 d\hat{A} = d\phi$. Using $\int du^1 dv^1 = \frac{\sqrt{3}}{2}$ we get the three dimensional action

$$\begin{aligned}\mathcal{L}_{DBI} = &- \mu_4 \frac{\sqrt{3}}{2} \int d^3\xi e^{-\phi} \frac{r}{R} \gamma_1 \bigg[ \frac{r^2}{R^2} + \frac{1}{2} \partial_i \phi \partial^i \phi + \frac{1}{2} \frac{1}{\gamma_1} \partial_i a_1 \partial^i a_1 + \frac{1}{2} \frac{1}{\gamma_1} \partial_i a_2 \partial^i a_2 \\ &+ \frac{1}{2} \frac{R^2}{r^2} \partial_i r \partial^i r + \frac{1}{2} \gamma_2 \partial_i u^2 \partial^i u^2 + \frac{1}{2} \gamma_2 \partial_i v^2 \partial^i v^2 + \frac{1}{2} \gamma_3 \partial_i u^3 \partial^i u^3 \\ &+ \frac{1}{2} \gamma_3 \partial_i v^3 \partial^i v^3 \bigg],\end{aligned} \tag{5.5}$$

with indices raised and lowered using the flat metric.

The Chern-Simons term is

$$\sqrt{2} \mu_4 \int \mathcal{C}_5, \tag{5.6}$$

where $\mathcal{C}_5 = C_5 + C_3 \wedge \mathcal{F}_2 + \frac{1}{2} C_1 \wedge \mathcal{F}_2 \wedge \mathcal{F}_2 + \frac{1}{6} m_0 \omega_5$, with $d\omega_5 = \mathcal{F}_2 \wedge \mathcal{F}_2 \wedge \mathcal{F}_2$. This can be written as an integral of a 6-form, $\mathcal{F}_6 = d\mathcal{C}_5$, over the volume bounded by the D4-brane. In our background only $F_6$ contributes, and using the calculation in Appendix B, we have

$$\sqrt{2} \mu_4 \int F_6 = \mu_4 \int dt dx_1 dx_2 e^{-\phi} \frac{r^3}{R^3} \gamma_1 \left[ du^1 \wedge dv^1 + \frac{\gamma_2}{\gamma_1} du^2 \wedge dv^2 + \frac{\gamma_3}{\gamma_1} du^3 \wedge dv^3 \right], \tag{5.7}$$

which becomes

$$\sqrt{2} \mu_4 \int F_6 = \mu_4 \int d^5\xi e^{-\phi} \frac{r^3}{R^3} \left[ \gamma_1 + \gamma_2 \left( \frac{\partial u^2}{\partial \xi^3} \frac{\partial v^2}{\partial \xi^4} - \frac{\partial u^2}{\partial \xi^4} \frac{\partial v^2}{\partial \xi^3} \right) + \gamma_3 \left( \frac{\partial u^3}{\partial \xi^3} \frac{\partial v^3}{\partial \xi^4} - \frac{\partial u^3}{\partial \xi^4} \frac{\partial v^3}{\partial \xi^3} \right) \right] \tag{5.8}$$

when we use our specific embedding of the D4-brane. When we compactify, we assume that no fields depend on the compact coordinates, so the only term that contributes to the low-energy effective action is the constant, which is canceled with the constant term in the DBI part.

We can also redefine the radial coordinate to be $\rho = \sqrt{2\sqrt{3} \frac{\mu_4}{g_s} \gamma_1 R \sqrt{r}}$ so that

[4]For $F_2 \neq 0$ we need to take the gauge field to have non vanishing background flux so that $\mathcal{F} = 0$. This doesn't change the rest of the analysis.

the action is given by

$$\mathcal{L}_{DBI} = -\int d^3\xi \Big[\frac{1}{2}\partial_i\rho\partial^i\rho + \frac{1}{2}\frac{\rho^2}{4R^2}\big(\partial_i\phi\partial^i\phi + \gamma_1^{-1}\partial_i a_1\partial^i a_1 + \gamma_1^{-1}\partial_i a_2\partial^i a_2 + \gamma_2\partial_i u^2\partial^i u^2 + \gamma_2\partial_i v^2\partial^i v^2 + \gamma_3\partial_i u^3\partial^i u^3 + \gamma_3\partial_i v^3\partial^i v^3\big)\Big]. \qquad (5.9)$$

This describes an 8 dimensional moduli space which is a cone (with radial coordinate $\rho$) over a 7 dimensional space parameterized by $\phi, a_1, a_2, u^2, v^2, u^3, v^3$.

To study the global structure of the moduli space we will consider now each of the scalar fields. Starting with the dual scalar, $\phi$, one can see that it is actually periodic. In a 3d YM theory, whose action is given by

$$\int d^3x\sqrt{g}\frac{1}{4g_{YM}^2}f_{\mu\nu}f^{\mu\nu} = \frac{1}{g_{YM}^2}\int f\wedge *f, \qquad (5.10)$$

the electric charge inside an $S^1$ is given by

$$Q_e = \frac{1}{g_{YM}^2}\int_{S^1} *f = \frac{1}{g_{YM}^2}\int_{S^1} d\phi = \frac{1}{g_{YM}^2}(\phi(2\pi) - \phi(0)). \qquad (5.11)$$

Since the field values $\phi(0)$ and $\phi(2\pi)$ are the same, and $Q_e$ are integers we have

$$\phi \simeq \phi + g_{YM}^2. \qquad (5.12)$$

In our case $g_{YM}^2 = \frac{g_s}{\mu_4}\frac{2}{\sqrt{3}\gamma_1}$.

The Wilson lines are also periodic fields. Performing a gauge transformation $A \to A + d\Lambda$ with $\Lambda = c_1 u^1 + c_2 v^1$, on a torus of complex structure $\tau$, shifts the Wilson lines by $a_i \to a_i + c_i$. Since $e^{i\Lambda}$ must be periodic under the identifications of the coordinates given by $(u^1, v^1) \sim (u^1 + 1, v^1) \sim (u^1 + \mathcal{R}\mathrm{e}[\tau], v^1 + \mathcal{I}\mathrm{m}[\tau])$, we need

$$c_1 = 2\pi n_1, \qquad c_1\,\mathcal{R}\mathrm{e}[\tau] + c_2\,\mathcal{I}\mathrm{m}[\tau] = 2\pi n_2, \qquad (5.13)$$

for integers $n_1$ and $n_2$. These are solved for integral linear combinations of

$$\{c_1 = 2\pi, \qquad c_2 = 2\pi\frac{1 - \mathcal{R}\mathrm{e}[\tau]}{\mathcal{I}\mathrm{m}[\tau]}\}$$
$$\{c_1 = 0, \qquad c_2 = 2\pi\frac{1}{\mathcal{I}\mathrm{m}[\tau]}\}. \qquad (5.14)$$

Under the corresponding gauge transformation the fields do not change so we must identify these points on the moduli space of the Wilson lines. Therefore we get (using $\tau = e^{i\pi/3}$)

$$(a_1, a_2) \sim (a_1 + 2\pi, a_2 + \frac{2\pi}{\sqrt{3}}) \sim (a_1, a_2 + \frac{4\pi}{\sqrt{3}}). \qquad (5.15)$$

This is a torus with complex structure $\tau = e^{i\pi/3}$ in the coordinate $\tilde{z} = \frac{\sqrt{3}}{4\pi}(a_2 + ia_1)$.

Thus, we see that the moduli space has the structure of a cone with a seven dimensional base $S^1 \times (T^2)^3$ (before imposing the orbifold and orientifold identifications), where the circumference of the $S^1$ is

$$2\pi R_\phi = \frac{2}{\sqrt{3}} \frac{g_s}{\mu_4} \frac{1}{\gamma_1}, \tag{5.16}$$

and the complex structure of the three tori are all $\tau = e^{i\pi/3}$, while their volumes are

$$\left(\frac{4\pi}{\sqrt{3}}\right)^2 \frac{1}{\gamma_1}, \qquad \gamma_2, \qquad \gamma_3. \tag{5.17}$$

The above analysis was for a D4-brane located at a generic point, where it is separated from its images. However, we can consider also a D4-brane located at the fixed points of the $T^6/\mathbb{Z}_3^2$. We start with the non fractional brane (and obtain the fractional ones from it via higgsing). The D-brane wraps $u^1, v^1$, and it can sit at a fixed point on the other two tori. There are three such points, distinct after all identifications. In the covering space of the orbifold action, $T^6$, the D4-brane has nine copies, which are divided into three separate groups of three coincident branes. To study the moduli space we need to consider the transformation of the Chan-Paton indices. Under the first $\mathbb{Z}_3$ The fields transform as

$$\begin{array}{rcl} \phi_{ij} & \to & \alpha^{2(i-j)}\phi_{ij}, \\ r_{ij} & \to & \alpha^{2(i-j)} r_{ij}, \\ a_{ij} = (a_1 + ia_2)_{ij} & \to & \alpha^{2(1+i-j)} a_{ij}, \\ z^2_{ij} = (u^2 + iv^2)_{ij} & \to & \alpha^{2(1+i-j)} z^2_{ij}, \\ z^3_{ij} = (u^3 + iv^3)_{ij} & \to & \alpha^{2(1+i-j)} z^3_{ij}, \end{array} \tag{5.18}$$

where $\alpha = e^{i\pi/3}$. The invariant fields are then

$$\begin{aligned} \phi &= \begin{pmatrix} \phi_{00} & 0 & 0 \\ 0 & \phi_{11} & 0 \\ 0 & 0 & \phi_{22} \end{pmatrix}, \\ r &= \begin{pmatrix} r_{00} & 0 & 0 \\ 0 & r_{11} & 0 \\ 0 & 0 & r_{22} \end{pmatrix}, \\ a &= \begin{pmatrix} 0 & a_{01} & 0 \\ 0 & 0 & a_{12} \\ a_{20} & 0 & 0 \end{pmatrix}, \\ z^i &= \begin{pmatrix} 0 & z^i_{01} & 0 \\ 0 & 0 & z^i_{12} \\ z^i_{20} & 0 & 0 \end{pmatrix}. \end{aligned} \tag{5.19}$$

The moduli space is determined by considering commuting matrices, since the scalar potential contains terms with commutators. We then find two branches. On one branch the fields $a, z^2, z^3$ vanish, and $\phi, r$ can have any value, giving rise to 3 scalars each. This describes the D4-brane and its images at the fixed point as fractional branes with the corresponding gauge group of $U(1)^3$, each at a different radial position. The second is when $a, z^2, z^3$ are generic and $\phi$ and $r$ are proportional to the identity matrix, in which case the D-brane is away from the fixed points and has some non-trivial Wilson line. Here the gauge group is broken back to a single $U(1)$. The position and Wilson lines are given by $\sqrt[3]{z^i_{01} z^i_{12} z^i_{20}}$ and $\sqrt[3]{a_{01} a_{12} a_{20}}$, respectively. This just spans locally a $\mathbb{Z}_3$ singularity. The global identifications are just as in the case away from the fixed points.

We also need to consider the effect of the orientifold action, $\Omega(-1)^{F_L}\sigma$, where $\sigma$ is the spacetime involution $z_i \to -\bar{z}_i$, $F_L$ is the worldsheet left moving fermion number and $\Omega$ is the worldsheet parity reversal. The action on our fields is

$$\begin{array}{rcl} \phi_{i,j} & \to & -\phi_{-j,-i} \\ r_{i,j} & \to & r_{-j,-i} \\ a_{i,j} & \to & \bar{a}_{-j,-i} \\ z^2_{i,j} & \to & -\bar{z}^2_{-j,-i} \\ z^3_{i,j} & \to & -\bar{z}^3_{-j,-i}. \end{array} \tag{5.20}$$

We then get the following degrees of freedom:

$$\begin{array}{rr} \phi_{22} = -\phi_{11}, & \phi_{00} = 0, \\ r_{22} = r_{11}, & r_{00}, \\ a_{01} = \bar{a}_{20}, & a_{12} = \bar{a}_{12}, \\ z^i_{01} = -\bar{z}^i_{20}, & z^i_{12} = -\bar{z}^i_{12}. \end{array} \tag{5.21}$$

The D-brane position is now $i|\sqrt[3]{z^i_{01} z^i_{12} z^i_{20}}|$ so it can move only along the O-plane. To move out of this plane the D-brane must meet its image and so we need a pair of such D-branes. Similarly, the Wilson line is $|\sqrt[3]{a_{01} a_{12} a_{20}}|$.

## 5.2 Generic Properties of the Moduli Space

In the previous section we found that supersymmetric domain wall configurations are described by a holomorphic curve. Here we will provide a more detailed description of a generic branch of this type, and explain how to count its dimension (in the limit of large charges). The main tools that we will use are the Bezout and Bernstein theorems, which we will review, which will be used to calculate the wrapping numbers of a generic branch. We will be interested primarily in the branch of largest dimension, and examine how this maximal dimension scales with the wrapping numbers.

### 5.2.1 Mathematical Preliminaries

We will now introduce some mathematical theorems that will help us count the number of solutions for a system of generic polynomial equations. More details can be found in [29]. The basic theorem that answers this question is Bezout's Theorem:

> If the equations $f_1 = \ldots = f_n = 0$ have degree $d_1, \ldots, d_n$ and finitely many solutions in $\mathbb{CP}^n$, then the number of solutions (counted with multiplicity) is $d_1 \cdots d_n$.

This theorem holds for any polynomials $f_i$ in the complex projective space.

We will be interested, however, in polynomials in $\mathbb{C}^n$. Given such polynomials $f_i \in \mathbb{C}[x_1, \ldots, x_n]$ with terms of total degree up to $d_i$, we can always add an additional variable, $z$, making all terms of total degree $d_i$. We can then view them as equations in $\mathbb{CP}^n$, and then we can apply Bezout's theorem and find the number of solutions. We will assume generic polynomials, so that one can assume no solutions at $z = 0$. By gauging $z = 1$ we can reduce each solution in the projective space to a solution in $\mathbb{C}^n$. We thus have this version of Bezout's theorem

> Given $n$ generic polynomials $f_1, \ldots, f_n$, if the equations $f_1 = \ldots = f_n = 0$ have maximal total degree $d_1, \ldots, d_n$ and finitely many solutions in $\mathbb{C}^n$, then the number of solutions (counted with multiplicity) is $d_1 \cdots d_n$.

Here we assume that the polynomials are generic in the sense that all terms with degree up to $d_i$ appear with a non vanishing coefficient in the polynomial $f_i$.

In our case we will have polynomials that are generic in a different sense than what was used in the previous case. The polynomial $f_i$ will contain all terms that are up to order $d_i^a$ in each variable $x_a$.[5] This is obviously less generic than needed for Bezout's theorem so we will need to use Bernstein's theorem, a generalization of Bezout's theorem. We will start by introducing some concepts used in Bernstein's theorem.

Let $f \in \mathbb{C}[x_1, \ldots, x_n]$ be a polynomial in $n$ variables. We can describe it by a set of points in the positive integer lattice $\mathbb{Z}^n_{\geq 0}$, each point corresponding to a monomial. We can write

$$f = \sum_{\alpha \in \mathbb{Z}^n_{\geq 0}} c_\alpha x^\alpha, \tag{5.22}$$

and the set of points is given by

$$\mathcal{A} = \{\alpha \in \mathbb{Z}^n_{\geq 0} : c_\alpha \neq 0\}. \tag{5.23}$$

[5] Actually, one can relax this condition, but this will not change the scaling behavior of the dimensionality of the moduli space as we will discuss in the next subsection.

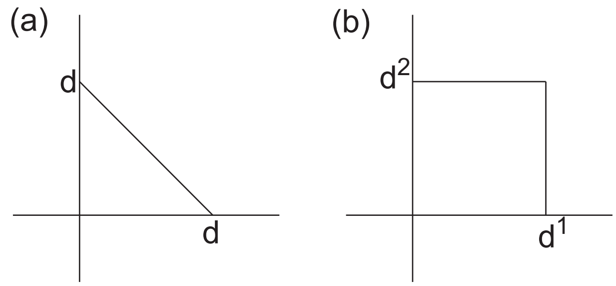


Figure 1: The Newton polytopes of (a) a polynomial with highest total degree $d$. (b) a polynomial with each $x_a$ having highest degree $d_a$.

This set of points can be used to define the Newton polytope of $f$, given by the convex hull of $\mathcal{A}$

$$\mathrm{NP}(f) = \mathrm{Conv}(\mathcal{A}) = \left\{ \sum_{\alpha\in\mathcal{A}} \lambda_\alpha \alpha : \lambda_\alpha \geq 0, \sum_{\alpha\in\mathcal{A}} \lambda_\alpha = 1 \right\}. \tag{5.24}$$

A polynomial is said to be generic if $c_\alpha \neq 0$ for any lattice point $\alpha$ inside its Newton polytope. As an example, for $n = 2$, the Newton polytope for a polynomial with all terms of order up to $d$ is given by the triangle in figure 1(a). The Newton polytope of a polynomial with terms up to $d^a$ in the variable $x_a$ is given by the square in figure 1(b).

There are two operations that can be carried out on polytopes in $\mathbb{R}^n$ in order to generate new ones. Let $P, Q$ be polytopes in $\mathbb{R}^n$ and let $\lambda \geq 0$ be a real number.

1. The Minkowski sum of $P$ and $Q$ denoted $P + Q$, is

$$P + Q = \{p + q : p \in P \text{ and } q \in Q\}, \tag{5.25}$$

where $p + q$ denotes the usual vector sum in $\mathbb{R}^n$

2. The polytope $\lambda P$ is defined by

$$\lambda P = \{\lambda p : p \in P\}, \tag{5.26}$$

where $\lambda p$ is the usual scalar multiplication on $\mathbb{R}^n$.

We will also define the mixed volume of a collection of polytopes $P_1, \cdots, P_n$, denoted

$$MV_n(P_1, \cdots, P_n) \tag{5.27}$$

to be the coefficient of the monomial $\lambda_1\lambda_2\cdots\lambda_n$ in the volume of the polytope $P = \lambda_1 P_1 + \cdots + \lambda_n P_n$.

Using the notions introduced above, we can now write Bernstein's theorem as follows [29, 30]

> Given polynomials $f_1, \ldots, f_n$ over $\mathbb{C}$ with finitely many common zeroes in $(\mathbb{C}^*)^n$, let $P_i = NP(f_i)$ be the Newton polytope of $f_i$ in $\mathbb{R}^n$. Then the number of common zeros of the $f_i$ in $(\mathbb{C}^*)^n$ is bounded above by the mixed volume $MV_n(P_1, \ldots, P_n)$. Moreover, for generic choices of the coefficients in the $f_i$, the number of common solutions is exactly $MV_n(P_1, \ldots, P_n)$.

For the two cases in $\mathbb{R}^2$ described in figure 1 it is simple to calculate the mixed volume. Polynomials $f_1, f_2$ with all terms up to order $d_1, d_2$ have triangular Newton polytopes, as in figure 1(a), and their mixed volume is given by

$$MV_n(P_1, P_2) = d_1 d_2, \tag{5.28}$$

while polynomials with terms up to order $d_i^1$ in $x_1$ and $d_i^2$ in $x_2$ have square Newton polytopes as in figure 1(b), for which

$$MV_n(P_1, P_2) = d_1^1 d_2^2 + d_1^2 d_2^1. \tag{5.29}$$

### 5.2.2 The Branches of Moduli Space

Next we will use Bernstein's theorem to calculate the properties of the moduli space for a generic D4-brane (or several D4-branes) wrapping a 2-cycle on the compact space. We have seen that the supersymmetry condition requires the embedding of the D4-brane to be holomorphic, so the 2-cycle is given by a set of two holomorphic equations in the $z_i$. Since the $z_i$ are doubly periodic the holomorphic equations should be periodic as well. The most general elliptic function over a torus with complex structure $\tau$ can be written in terms of the periodic Weierstrass functions $w_i \equiv \wp(z_i|\tau)$ and their derivatives $w_i' \equiv \wp'(z_i|\tau)$. For the purposes of Berenstein's theorem we will treat these variables as independent and add to the set of polynomials $f_i$ the relations

$$w_i'^2 - (4w_i^3 + g_2(\tau)w_i + g_3(\tau)) = 0, \quad i = 1, 2, 3. \tag{5.30}$$

A general supersymmetric D-brane will thus be located at the zeros of (5.30) and of two holomorphic polynomials of the form

$$P(w_i, w_i') = Q(w_i, w_i') = 0. \tag{5.31}$$

We can restrict the polynomials to have terms only up to first order in $w_i'$, since higher powers can be removed using the relations (5.30). We will take the highest degree of the variable $w_i$ in $P$ and $Q$ to be $p_i, q_i$, respectively[6].

[6] The D-brane configuration is described by the vanishing locus of a set of polynomials where the highest degree of each parameter is constrained separately. Perhaps one can relax this

Given a set of such polynomials, which describe a D4-brane, we will use Bernstein's theorem, applied to subsets of this set, to count the wrapping number of the D-brane on the different cycles. Consider for example $N_1$ – the number of times the brane wraps the $z_1$ cycle. We will evaluate $N_1$ by fixing a value of $z_1$ and then counting how many solutions there are to the equations for this $z_1$ (for a generic $z_1$). Fixing $z_1$ means that we fix both $w_1$ and $w_1'$ which satisfy the constraint (5.30) for $i = 1$. This leaves us with 4 polynomials in the variables $w_2, w_3, w'_2, w'_3$, on which we apply Bernstein's theorem. The number of solutions to these equations is

$$N_1 \sim p_2 q_3 + p_3 q_2, \tag{5.32}$$

and similarly for permutations of $\{1, 2, 3\}$.

Recall that we fix $N_{1,2,3}$ and count the dimensionality of the moduli space for this set of $N$'s. We are interested in finding the values of $p_i$ and $q_i$ for which we obtain the largest dimensionality. The dimension of the moduli space for a given set of $p_i$ and $q_i$ can be estimated by the number of different monomials in the two polynomials, which is $8p_1p_2p_3 + 8q_1q_2q_3$. However, the actual dimension of the moduli space is smaller, since different pairs of polynomials might have the same zero locus. If we assume $q_i < p_i$, then any multiple of $Q$ with degree smaller than $p_i$ can be removed from $P$. We are thus left with a moduli space of dimension

$$D \sim 8(p_1p_2p_3 + q_1q_2q_3 - (p_1 - q_1)(p_2 - q_2)(p_3 - q_3)). \tag{5.33}$$

To summarise, we have found that the moduli space describing a domain wall across which the flux jump by $(N_1, N_2, N_3)$ units of flux, consists of different branches each parametrized by a set of 2 polynomials with degrees satisfying (5.32). The dimension of such a branch is given by (5.33).

### 5.2.3 The Maximal Moduli Space

It is interesting from the point of view of the dual field theory to understand how the dimension of the moduli space scales as we take large wrapping numbers, $N_i >> 1$ (which should still be much smaller than the fluxes since we are using the probe approximation). For this we will find the dimension of the maximal branch with given wrapping numbers. We can use (5.32) to solve for $q_i$ in terms of the $p_i$ for given values of the fluxes,

$$q_1 = \frac{-N_1p_1 + N_2p_2 + N_3p_3}{2p_2p_3} \tag{5.34}$$

condition by considering zeros of this set of equations that enter from infinity or from zeros of $w_i$. On the torus side, in the computations below, we can always avoid such points.

and similarly for $q_2, q_3$. Since the $q_i$'s are positive, this give some non-trivial condition on the $p_i$'s. The requirement $q_i < p_i$ then leads to the inequalities

$$\begin{aligned} -N_1p_1 + N_2p_2 + N_3p_3 &< 2p_1p_2p_3, \\ N_1p_1 - N_2p_2 + N_3p_3 &< 2p_1p_2p_3, \\ N_1p_1 + N_2p_2 - N_3p_3 &< 2p_1p_2p_3, \end{aligned} \tag{5.35}$$

which can be brought to the form

$$N_1 < 2p_2p_3 \tag{5.36}$$

and its permutations. In the same way we can get $N_1 > 2q_2q_3$ and its permutations.

We can now use our results in the equation (5.33) for $D$. The term with three $p_i$'s cancels. For the terms of the form $ppq$ we can use (5.34) to see that they scale like $N_ip_i < N_iN_j$, since $p_i$ can be just as large as $N_j$ $(j \neq i)$. Next, we have $pqq < pN$ so these terms are also smaller than $N_iN_j$. Finally, the term with three $q_i$'s is $qqq < Nq$ and scales as the other terms. Terms with less than three $p$'s or $q$'s are smaller for the same reasons. We thus conclude that for large fluxes the dimension of moduli space behaves as

$$D \leq \sum_{i\neq j} N_iN_j. \tag{5.37}$$

We can actually find a configuration which saturates this bound on the dimensionality of moduli space. For instance, if all $N_i$ are of the same order, then by choosing all $q_i \sim 1$ we get $p_i \sim \sum N_j$, in which case we get $D \sim \sum_{i\neq j} N_iN_j$.

The previous analysis was done under the assumption that each $q_i$ is smaller than $p_i$ so that we can eliminate terms in the polynomial $P$ using $Q$ thus reducing the dimension of moduli space. However it is possible that this is not the case. If one of the $q_i$'s is larger we need to take all monomials, and the dimension of the moduli space is $D \sim p_1p_2p_3 + q_1q_2q_3$. We will assume that $q_1$ is the large $q$ so that $q_1 > p_1, q_{2,3} < p_{2,3}$. We find

$$\begin{aligned} q_1 > p_1 \quad &\to \quad -N_1p_1 + N_2p_2 + N_3p_3 > 2p_1p_2p_3, \\ q_2 < p_2 \quad &\to \quad N_1q_1 - N_2q_2 + N_3q_3 > 2q_1q_2q_3. \end{aligned} \tag{5.38}$$

From the first inequality we get that $p_1p_2p_3 < N_iN_j$ and from the second one we get that also $q_1q_2q_3 < N_iN_j$ so that we arrive again to the same conclusion (5.37) as before.

In addition to the directions in the moduli space that change the two polynomials and control the embedding of the D-brane, there are additional dimensions of the moduli space related to Wilson lines. The number of Wilson lines is related to the 1 dimensional homology group of the Riemann surface the D-brane is wrapping. We can try to estimate this number as follows. We can think of the polynomials $P(w_i, w'_i), Q(w_i, w'_i)$ as polynomials in a projective space by adding a new variable $\lambda$ and making all terms have the same weight of $p = \sum p_i$ and $q = \sum q_i$. The relations (5.30) are then of weight 3. It is then possible using algebraic geometry methods to calculate the Euler characteristic of the complete intersection of these 5 polynomials to be

$$\chi = -27pq(2 - p - q). \qquad (5.39)$$

As before, we have $ppq \sim N_i p_j$ and $qqp \sim N_i q_j$ so that we have $\chi \sim N_i N_j$. Since the number of Wilson lines is just the genus, it scales as the Euler characteristic, and we get that

$$D_{Wilson} \sim \sum_{i \neq j} N_i N_j, \qquad (5.40)$$

as before. We thus conclude that the total dimension of the moduli space with given wrapping numbers scales in the same fashion,

$$D_{total} \sim \sum_{i \neq j} N_i N_j. \qquad (5.41)$$

We note that this behavior may point us towards an $SU(f_4^1) \times SU(f_4^2) \times SU(f_4^3)$ gauge theory, as the dimension of the moduli space can than be viewed as coming from the degrees of freedom of strings sitting in the bifundamental representation of any two $SU(N)$ factors.

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# A Supersymmetry Equations in the Bulk

In this appendix we solve the equations for supersymmetry in the bulk for the background discussed in section 2. We find the unbroken spinors and the values for the stabilized moduli.

The equations for preserved supersymmetry are given by [24, 22]

$$
\begin{aligned}
e^{-2A+\phi}(d+H\wedge)(e^{2A-\phi}\Psi_+) &= 2\mu\,\mathcal{R}\mathrm{e}[\Psi_-]\,, && \text{(A.1)}\\
e^{-2A+\phi}(d+H\wedge)(e^{2A-\phi}\Psi_-) &= 3i\,\mathcal{I}\mathrm{m}[\bar{\mu}\Psi_+] + dA\wedge\bar{\Psi}_- \\
&\quad + \frac{\sqrt{2}}{16}e^{\phi}\left[(|a|^2-|b|^2)\hat{F} + i(|a|^2+|b|^2)\tilde{F}\right], && \text{(A.2)}
\end{aligned}
$$

where $F = F_0 + F_2 + F_4 + F_6$ are the modified RR fields defined as

$$
F = e^{-B}F^{\mathrm{bg}} + dC + H\wedge C, \tag{A.3}
$$

so that they obey the non-standard Bianchi identity $dF_n = -H\wedge F_{n-2}$.

Plugging our background into the first equation we get

$$
\begin{aligned}
& H\wedge\Psi_+ = 2\mu\,\mathcal{R}\mathrm{e}[\Psi_-] \\
& \Rightarrow -p\beta_0\wedge\frac{a\bar{b}}{8}e^{-iJ} = 2\mu\,\mathcal{R}\mathrm{e}\left[-\frac{iab}{8}\Omega\right] \\
& \Rightarrow -pa\bar{b}\beta_0 = 2\mu\,\mathcal{I}\mathrm{m}[ab\Omega] = \sqrt{2}\mu\frac{\sqrt{\gamma_1\gamma_2\gamma_3}}{3^{1/4}}(\,\mathcal{R}\mathrm{e}[ab]\,\beta_0 + \mathcal{I}\mathrm{m}[ab]\,\alpha_0),
\end{aligned} \tag{A.4}
$$

where $\frac{\sqrt{\gamma_1\gamma_2\gamma_3}}{3^{1/4}\sqrt{2}}\beta_0 = \mathcal{I}\mathrm{m}[\Omega]$ and $\frac{\sqrt{\gamma_1\gamma_2\gamma_3}}{3^{1/4}\sqrt{2}}\alpha_0 = \mathcal{R}\mathrm{e}[\Omega]$. The solution is $\mathcal{I}\mathrm{m}[ab] = 0$ and

$$
\begin{aligned}
-pa\bar{b} &= \sqrt{2}\mu\frac{\sqrt{\gamma_1\gamma_2\gamma_3}}{3^{1/4}}ab \Rightarrow \\
\mu &= -\frac{p}{\sqrt{2}}\frac{3^{1/4}}{\sqrt{\gamma_1\gamma_2\gamma_3}}\frac{\bar{b}}{b} \Rightarrow \Lambda = -|\mu|^2 = \frac{p^2}{2}\frac{\sqrt{3}}{\gamma_1\gamma_2\gamma_3}.
\end{aligned} \tag{A.5}
$$

In the second equation we use for the left hand side

$$
H\wedge\Psi_- = -p\beta_0\wedge\frac{-iab}{8}\frac{\sqrt{\gamma_1\gamma_2\gamma_3}}{3^{1/4}}\frac{1}{\sqrt{2}}(\alpha_0 + i\beta_0) = -\frac{ipab}{8\sqrt{2}}\frac{\sqrt{\gamma_1\gamma_2\gamma_3}}{3^{1/4}}\alpha_0\wedge\beta_0. \tag{A.6}
$$

This equation can be split according to the rank of the forms that appear in it. The zero-form part of the equation is

$$
0 = 3i\,\mathcal{I}\mathrm{m}\left[\frac{\bar{\mu}a\bar{b}}{8}\right] + \frac{\sqrt{2}}{16}e^{\phi}\left[(|a|^2-|b|^2)\hat{F}_0 - i(|a|^2+|b|^2)*_6\hat{F}_6\right]. \tag{A.7}
$$

The first term is proportional to $\mathcal{I}\mathrm{m}[ab]$ so it vanishes. The real part implies

$$
|a| = |b|, \tag{A.8}
$$

assuming a non-vanishing value for $\hat{F}_0$, while the imaginary part of this equation requires $\hat{F}_6 = 0$ which gives, using (A.3),

$$\begin{aligned} 0 &= \int \hat{F}_6 = \int H \wedge C_3 + \hat{F}_6^{\rm bg} - \hat{F}_4^{\rm bg} \wedge B + \frac{1}{2}\hat{F}_2^{\rm bg} \wedge B \wedge B - \frac{1}{6} F_0^{\rm bg} \wedge B \wedge B \wedge B \\ &= p\xi - e_0 - e_i b_i - \frac{1}{2}\kappa b_i b_j m_k + \kappa m_0 b_1 b_2 b_3, \end{aligned} \tag{A.9}$$

with $\{i, j, k\}$ being summed over all permutations of $\{1, 2, 3\}$.

The two-form part is

$$0 = 3i\,\mathcal{I}\mathrm{m}\left[\frac{\bar{\mu} a \bar{b}}{8}(-iJ)\right] + \frac{\sqrt{2}}{16} e^{\phi} \left[(|a|^2 - |b|^2)\hat{F}_2 + i(|a|^2 + |b|^2)\tilde{F}_2\right], \tag{A.10}$$

with

$$\begin{aligned} &J = \frac{\gamma_i}{2}(\kappa\sqrt{3})^{-1/3}\omega_i, \\ &\hat{F}_2 = -m_i w_i + m_0 b_i w_i, \\ &\tilde{F}_2 = *_6 \hat{F}_4 = -\hat{e}_i * \tilde{\omega}^i = \frac{2\hat{e}_i\gamma_i^2}{\gamma_1\gamma_2\gamma_3}\left(\frac{\sqrt{3}}{\kappa^2}\right)^{1/3} \omega_i, \end{aligned} \tag{A.11}$$

where we used

$$\begin{aligned} \hat{F}_4 &= \int H \wedge C_1 + \hat{F}_4^{\rm bg} - \hat{F}_2^{\rm bg} \wedge B + \frac{1}{2}\hat{F}_0^{\rm bg} \wedge B \wedge B \\ &= (e_i + \kappa(m_j b_k + m_k b_j) - \kappa m_0 b_j b_k)\tilde{w}^i = \hat{e}_i \tilde{w}^i. \end{aligned} \tag{A.12}$$

Again, (A.10) splits into real and imaginary parts. The real part vanishes since $|a| = |b|$, while the imaginary part reduces to

$$\begin{aligned} 0 &= 3i\frac{-\bar{\mu} a \bar{b}}{8}(J) + i\frac{\sqrt{2}}{16} e^{\phi}(|a|^2 + |b|^2)\tilde{F}_2 \\ &= i\frac{3}{8}\frac{\gamma_i}{2}(\kappa\sqrt{3})^{-1/3}\omega_i \frac{p}{\sqrt{2}} \frac{3^{1/4}}{\sqrt{\gamma_1\gamma_2\gamma_3}} ab + i e^{\phi}|a|^2 \frac{2\sqrt{2}\hat{e}_i\gamma_i^2}{8\gamma_1\gamma_2\gamma_3}\left(\frac{\sqrt{3}}{\kappa^2}\right)^{1/3} \omega_i \end{aligned} \tag{A.13}$$

which gives

$$\frac{e^{-2\phi}\hat{e}_i\gamma_i}{\sqrt{\prod_i e^{-2\phi}\hat{e}_i\gamma_i}} = -\frac{3^{11/12}}{8} p\kappa^{1/3} \frac{ab}{|a|^2} \tag{A.14}$$

(with no summation over $i$). This can be solved to give

$$e^{-2\phi}\gamma_i = \frac{64}{3^{11/6}} \frac{\hat{e}_1\hat{e}_2\hat{e}_3}{\hat{e}_i p^2 \kappa^{2/3}}, \tag{A.15}$$

where we used $\mathcal{I}\mathrm{m}[ab] = 0$ and $|a| = |b|$ to get $b = \pm a^*$ or $ab = \pm|a|^2$. We will later see that we must take the minus sign for the background to be supersymmetric.

The 4-form part of the equation is

$$0 = 3i\,\mathcal{I}\mathrm{m}\left[\frac{\bar{\mu}a\bar{b}}{8}\frac{1}{2}(-iJ)^2\right] + \frac{\sqrt{2}}{16}e^{\phi}\left[(|a|^2 - |b|^2)\hat{F}_4 + i(|a|^2 + |b|^2)\tilde{F}_4\right]. \quad \text{(A.16)}$$

Just as before, the first term vanishes, and since we have $\tilde{F}_4 = *\hat{F}_2$ we get

$$0 = \int_{[w_i]} F_2 = -m_i + m_0 b_i \;\Rightarrow\; b_i = \frac{m_i}{m_0}. \quad \text{(A.17)}$$

Plugging this back into (A.12) we get $\hat{e}_i = e_i + \kappa\frac{m_i m_j}{m_0}$.

Finally, the 6-form part is

$$H\wedge\Psi_- = 3i\,\mathcal{I}\mathrm{m}\left[\frac{\bar{\mu}a\bar{b}}{8}\frac{1}{6}(-iJ)^3\right] + \frac{\sqrt{2}}{16}e^{\phi}\left[(|a|^2 - |b|^2)\hat{F}_6 + i(|a|^2 + |b|^2)\tilde{F}_6\right]. \quad \text{(A.18)}$$

We use (A.6) and

$$\begin{aligned} &\tilde{F}_6 = *_6\hat{F}_0 = -m_0 *_6 1 \\ &\frac{1}{6}J^3 = \frac{i}{8}\Omega\wedge\bar{\Omega} = \frac{1}{8}\frac{\gamma_1\gamma_2\gamma_3}{\sqrt{3}}\alpha_0\wedge\beta_0 = *1 \end{aligned} \quad \text{(A.19)}$$

to get

$$\begin{aligned} 0 &= -H\wedge\Psi_- + 3i\,\mathcal{I}\mathrm{m}\left[\frac{\bar{\mu}a\bar{b}}{8}\frac{1}{6}(-iJ)^3\right] + \frac{\sqrt{2}}{16}e^{\phi}\left[(|a|^2 - |b|^2)\hat{F}_6 + i(|a|^2 + |b|^2)\tilde{F}_6\right] \\ &= \frac{ipab}{8\sqrt{2}}\frac{\sqrt{\gamma_1\gamma_2\gamma_3}}{3^{1/4}}\alpha_0\wedge\beta_0 + 3i\frac{\bar{\mu}a\bar{b}}{8}\frac{1}{8}\frac{\gamma_1\gamma_2\gamma_3}{\sqrt{3}}\alpha_0\wedge\beta_0 - \frac{1}{16}e^{\phi}i2|a|^2\sqrt{2}m_0\frac{1}{8}\frac{\gamma_1\gamma_2\gamma_3}{\sqrt{3}}\alpha_0\wedge\beta_0 \\ &= \frac{ipab}{8\sqrt{2}}\frac{\sqrt{\gamma_1\gamma_2\gamma_3}}{3^{1/4}}\alpha_0\wedge\beta_0 - 3i\frac{p}{\sqrt{2}}\frac{3^{1/4}}{\sqrt{\gamma_1\gamma_2\gamma_3}}\frac{ab}{8}\frac{1}{8}\frac{\gamma_1\gamma_2\gamma_3}{\sqrt{3}}\alpha_0\wedge\beta_0 \\ &\quad -\frac{1}{16}e^{\phi}i2|a|^2\sqrt{2}m_0\frac{1}{8}\frac{\gamma_1\gamma_2\gamma_3}{\sqrt{3}}\alpha_0\wedge\beta_0 \\ &= \frac{5}{8}\frac{ipab}{8\sqrt{2}}\frac{\sqrt{\gamma_1\gamma_2\gamma_3}}{3^{1/4}}\alpha_0\wedge\beta_0 - \frac{1}{16}e^{\phi}i2|a|^2\sqrt{2}m_0\frac{1}{8}\frac{\gamma_1\gamma_2\gamma_3}{\sqrt{3}}\alpha_0\wedge\beta_0, \end{aligned} \quad \text{(A.20)}$$

which gives us

$$e^{4\phi}\sqrt{\prod_i e^{-2\phi}\gamma_i} = \frac{5\cdot 3^{1/4}}{2}\frac{p}{m_0}\frac{ab}{|a|^2}. \quad \text{(A.21)}$$

Using (A.15) we can solve for $e^{\phi}$ and $\gamma_i$. We know that the O6-plane generates a tadpole that is canceled by the fluxes $m_0$ and $p$ according to (2.22), so that we find sign$(m_0 p) = -$. We thus must have, for supersymmetry to hold, $b = -a^*$ as stated in the main text. The results we got for the moduli agree with (2.23).

We note that equations (A.14), (A.21) give us the following conditions on the signs of the background fluxes,

$$\text{sign}(pe_i) = +, \quad \text{sign}(pm_0) = -. \tag{A.22}$$

# B BPS Condition

In this appendix we will consider the supersymmetric domain wall solutions found in section 4. Such a supersymmetric configuration should be a BPS state and therefore feel no radial force. We will verify this fact directly by considering the D-brane effective action. In the supersymmetric configuration the gravitational force coming from the DBI term will be canceled against the RR force coming from the WZ term, related to the charge of the D-brane.

The D-branes extend along a $2+1$ dimensional surface in $AdS_4$ parallel to the boundary at constant $r$, and wrap a $(p-2)$-cycle in the compact space. Their world-sheet action in the string frame is given by

$$I_{brane} = I_{DBI} + I_{WZ} \ , \tag{B.1}$$

where

$$I_{DBI} = -\mu_p \int d^p x e^{-\phi} \sqrt{-\det\left(G + \mathcal{F}\right)} \tag{B.2}$$

is a Dirac-Born-Infeld type action in the string frame, $\mu_p$ is the D-brane tension and

$$\mathcal{F} = f + P[B] \ . \tag{B.3}$$

$I_{WZ}$ is the following Wess-Zumino (WZ) type action

$$I_{WZ} = \sqrt{2}\mu_p \int \left(\mathcal{C} \wedge e^{\mathcal{F}} + m_0 \omega\right) \ , \tag{B.4}$$

where

$$\mathcal{C} = \sum_{i=0}^{9} \mathcal{C}_i \ , \qquad d\omega = e^{\mathcal{F}} \tag{B.5}$$

and $m_0$ is the massive type IIA mass parameter. The $\sqrt{2}$ is due to the different normalization of the RR fields we use (following [13]).

As in [28], the brane action has two contributions which depend on the radial location of the brane in $AdS_4$. One contribution is proportional to the brane area $A$ and comes from the DBI action, the other is proportional to the volume enclosed by the brane $V$ and comes from the WZ action. Next, we are going to evaluate the different terms for wrapped D4-branes. We will assume $\mathcal{F}=0$.

The $D4$-brane domain walls wrap a two-cycle in the compact space. In our background there is a 3-dimensional basis for the untwisted 2-cycles given by $[\omega_i],\ i=1,2,3$. For simplicity we consider wrapping the two-cycle $[\omega_1]$ in $T^6/Z_3^2$.

**DBI**

In this case we have

$$\sqrt{-\det\ G}=\gamma_1 du^1 dv^1 \frac{r^3}{R_{\rm AdS}^3} dt dx^1 dx^2, \tag{B.6}$$

so we have

$$I_{\rm DBI}=\mu_4\int d^5x e^{-\phi}\sqrt{-\det\ G}=\mu_4 a\gamma_1 e^{-\phi}\frac{r^3}{R_{\rm AdS}^3}\int d^3x=\mu_4 3^{-\frac{13}{12}}2^{\frac{7}{2}}5^{\frac{1}{4}}\frac{\kappa^{\frac{1}{3}}E^{\frac{3}{4}}}{e_1 p m_0^{\frac{1}{4}}}\frac{r^3}{R_{\rm AdS}^3}a\int d^3x\ , \tag{B.7}$$

where we define

$$a\equiv\int_{[\omega_1]}du^1 dv^1\ . \tag{B.8}$$

**WZ**

The only non-zero contribution to the WZ term is given by

$$\mu_4\sqrt{2}\int_{W_5}\mathcal{C}_5=\mu_4\sqrt{2}\int_{{\rm Vol}(W_5)}F_6, \tag{B.9}$$

where $W_5$ is the D4-brane worldvolume wrapping a 2-cycle in the compact space and spanning a surface of constant $r$ in the AdS space. $\mathrm{Vol}(W_5)$ is the two cycle times the volume in AdS bounded by the surface of constant $r$. The other boundary of the volume, at $r\to\infty$, gives a contribution $\sim r^{-3}\to 0$ so it does not contribute.

The supergravity fields obey

$$\tilde{F}_6\equiv *\tilde{F}_4=F_6-C_3\wedge H_3+\frac{m_0}{6}B_2\wedge B_2\wedge B_2. \tag{B.10}$$

Integrating over $\mathrm{Vol}(W_5)$ we get that the last two terms vanish, since there are no such background fields with indices in the non-compact space. We can then write

$$\int_{{\rm Vol}(W_5)}*\tilde{F}_4=\int_{{\rm Vol}(W_5)}F_6. \tag{B.11}$$

The right-hand side is what we want to calculate, while the left-hand side is proportional to the integration of $\tilde{F}_4$ over the dual cycle which we can calculate.

In massive type IIA supergravity we have

$$\tilde{F}_4 \;= dC_3 + F_4^{bg} - C_1 \wedge H_3 - \tfrac{m_0}{2} B_2 \wedge B_2. \tag{B.12}$$

since the $B_2$ and $C_1$ are only the fluctuations and vanish in the background, we can replace $\tilde{F}_4$ in the integral by $F_4$.

In addition to a boundary term we are left with

$$\int_{w_2 \times w_3} \tilde{F}_4 = \int_{w_2 \times w_3} F_4^{\text{bg}}, \tag{B.13}$$

and the WZ term can now be written as

$$\mu_4 \sqrt{2} \int_{\text{Vol}(W_5)} *F_4. \tag{B.14}$$

Now

$$\begin{aligned} F_4 &= e_i \tilde{\omega}^i = -\left(\frac{3}{\kappa}\right)^{\frac{1}{3}} e_1 (dz^2 \wedge d\bar{z}^2) \wedge (dz^3 \wedge d\bar{z}^3) + \ldots, \\ *F_4 &= -4\left(\frac{3}{\kappa}\right)^{\frac{1}{3}} e_1 \frac{\gamma_1}{\gamma_2\gamma_3} \Omega_{AdS_4} \wedge (du^1 \wedge dv^1) + \ldots \\ &= -2 e_i \gamma_i \frac{\gamma_i}{\gamma_1\gamma_2\gamma_3} \left(\frac{\sqrt{3}}{\kappa^2}\right)^{1/3} \omega_i, \end{aligned} \tag{B.15}$$

where $\Omega_{AdS_4}$ is the volume form in $AdS$. We find

$$\begin{aligned} I_{\text{WZ}} &= \mu_4 \sqrt{2} \int_W \mathcal{C}_5 = -\mu_4 4\sqrt{2} \left(\frac{3}{\kappa}\right)^{\frac{1}{3}} \frac{a}{2\kappa^{\frac{1}{3}} 3^{\frac{1}{6}}} \frac{e_1 v_1}{v_2 v_3} \frac{r^3}{R_{AdS}^4} \int d^3x \\ &= -\mu_4 3^{-\frac{13}{12}} 2^{\frac{7}{2}} 5^{\frac{1}{4}} \frac{\kappa^{\frac{1}{3}} E^{\frac{3}{4}}}{e_1 p m_0^{\frac{1}{4}}} \frac{r^3}{R_{\text{AdS}}^3} a \int d^3x = -I_{DBI}. \end{aligned} \tag{B.16}$$

Thus, the gravitational force due to the DBI term is canceled exactly by the force from the WZ term, as must be the case for a BPS configuration.

The analysis is very similar for a more general cycle, and we will not write it down explicitly here.

# C Other Domain Walls

Here we consider D2 and D6-branes in domain wall configurations and study their supersymmetry equations. We show that a D2-brane can never be supersymmetric.

A D6-brane can classically be supersymmetric, however due to flux quantization there are generically no such solutions with integer values for the flux.

## C.1 D2-Brane as a Supersymmetric Domain Wall

The $\kappa$-symmetry equation (4.1) takes a simple form when we consider D2-branes, since they are not extended along any compact dimension. In order for these domain walls to be supersymmetric we need

$$\gamma_{\underline{012}}\epsilon_- = \epsilon_+. \tag{C.1}$$

This can be brought to the form

$$\gamma_{\underline{012}}\theta_+ \otimes b^*\eta_- = \theta_- \otimes a^*\eta_-. \tag{C.2}$$

The $AdS_4$ part of the equation is the same as for D4-branes, which results in the equation

$$b^* = \alpha a^* = -\text{sign}(p)i\frac{b^*}{b}a^* \quad \rightarrow \quad b = -\text{sign}(p)ia^*, \tag{C.3}$$

which contradicts our supersymmetric condition in the bulk, $b = -a^*$. Therefore we conclude that D2-branes cannot be supersymmetric domain walls in this background.

## C.2 D6-Brane as a Supersymmetric Domain Wall

Consider now a D6-brane which extends as a domain wall in the AdS and wraps (for instance) the 4-torus spanned by $z_1, z_2$. Its embedding may be chosen as

$$\sigma^1 = x^1, \quad \sigma^2 = y^1, \qquad \sigma^3 = x^2, \quad \sigma^4 = y^2, \tag{C.4}$$

with the induced metric being

$$\gamma_1(d\sigma^1)^2 + \gamma_1(d\sigma^2)^2 + \gamma_2(d\sigma^3)^2 + \gamma_2(d\sigma^4)^2, \tag{C.5}$$

and the pullback of $J$ given by

$$P[J] = \gamma_1 d\sigma^1 \wedge d\sigma^2 + \gamma_2 d\sigma^3 \wedge d\sigma^4. \tag{C.6}$$

Plugging into the supersymmetry condition (4.7) and taking $\mathcal{F} = 0$ as for the D4-branes, we have on the right-hand side

$$-a^*\alpha\sqrt{\det(P[g]+\mathcal{F})}d\sigma^1 \wedge d\sigma^2 \wedge d\sigma^3 \wedge d\sigma^4 = -\text{sign}(p)ib^*\gamma_1\gamma_2 d\sigma^1 \wedge d\sigma^2 \wedge d\sigma^3 \wedge d\sigma^4 \tag{C.7}$$

while on the left-hand side we have

$$\{b^* P[e^{-iJ}]) \wedge e^{\mathcal{F}}\}_4 = b^*\frac{1}{2}(-iP[J]) \wedge (-iP[J]) = b^*\frac{1}{2}(P[J]) \wedge (P[J]))$$
$$= b^*\gamma_1\gamma_2 d\sigma^1 \wedge d\sigma^2 \wedge d\sigma^3 \wedge d\sigma^4. \quad \text{(C.8)}$$

Since the first is purely imaginary and the second is real they cannot be equal, and the domain walls are not supersymmetric. A more generic embedding will not be able to compensate for the factor of $i$, and so even the general case is not supersymmetric. However as we have seen for the D4-branes, adding non trivial $\mathcal{F}$ can add a relative phase between the two sides. With $\mathcal{F} = f_1 d\sigma^1 \wedge d\sigma^2 + f_2 d\sigma^3 \wedge d\sigma^4$ the $\kappa$-symmetry equation becomes

$$b^*(f_1 - i\gamma_1)(f_2 - i\gamma_2) = -\text{sign}(p) i b^* \sqrt{(\gamma_1^2 + f_1^2)(\gamma_2^2 + f_2^2)}. \quad \text{(C.9)}$$

Writing $f - i\gamma = \sqrt{f^2 + \gamma^2} e^{-\tan^{-1}(\gamma/f)}$ we get that for positive $p$

$$\tan^{-1}(\gamma_1/f_1) + \tan^{-1}(\gamma_2/f_2) = \pi/2 \quad \text{(C.10)}$$

which has a solution for $f_i > 0$. For negative $p$ the right hand side should be $3\pi/4$ so there are solutions for $f_i > -\gamma_i$. Classically such supersymmetric configurations exist, however generically there are no such configurations consistent with flux quantization.

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