

# CORRECTION OF NONLINEAR LATTICE WITH CLOSED ORBIT MODULATION \*

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## Abstract

We propose to correct nonlinear lattice optics with the closed-orbit modulation technique. Closed orbit modulation with large amplitude samples the nonlinear optics. Fitting such data measured on the machine to the lattice model with appropriate lattice parameters would reveal the nonlinear errors and provide a means for correction. The method is tested in simulation and is shown to work in principle. Experimental data were also taken. However, more work is needed to understand the other effects on the mode amplitude dependence on modulation amplitude on a real machine.

## INTRODUCTION

The importance of linear optics correction during the commissioning of a storage ring has been universally recognized. There has been many successful methods that have been developed for linear optics correction, using closed orbit response matrix [1–3] or turn-by-turn BPM data [4–7] as input data that sample the optics errors. Correction of the linear optics can often lead to improvement in nonlinear beam dynamics, resulting in higher injection efficiency and longer Touschek lifetime.

Because of the higher order errors in the magnetic fields or alignment errors, the nonlinear beam dynamics of the ring can differ from that of the design lattice, even after the linear optics is corrected. The differences often cause degradation of the nonlinear dynamics performance. Correction of the nonlinear beam dynamics would be needed to restore the machine performance. There have been a number of attempts to correct storage ring nonlinear dynamics experimentally, using parameters such as tune shifts with amplitudes, nonlinear chromaticities, and resonance driving terms (RDTs), to characterize the nonlinear lattice [8–10]. While the correction studies have not definitively led to improvement of nonlinear dynamics performances, and online optimization has been shown to be a reliable approach to improve dynamic aperture (DA) and momentum aperture (MA) [11–13], the correction approach remains a promising approach that could eventually produce a standard method to deterministically find lattice configurations with good DA and MA.

A more recent method for nonlinear lattice correction uses off-momentum orbit response matrix as the input data [14]. The approach of using off-momentum measurements has been adopted in Ref. [15].

In this study, we propose a nonlinear dynamics correction method that uses large amplitude closed orbit modulation

data to characterize the nonlinear lattice behavior. The method is an extension of the linear optics from closed orbit modulation (LOCOM) method [2, 3] to nonlinear dynamics correction. In simulation, we show that nonlinear LOCOM data contain the nonlinear distortion information and the difference between the measured and model calculated data can be fitted to reproduce the nonlinear errors planted in the lattice.

## THE NONLINEAR LOCOM METHOD

The LOCOM uses two correctors to modulate the closed orbit in a storage ring as a way to sample the optics information. The data can be measured on short time scale, especially if resonantly driven modulating signals are used. In Ref. [3], LOCOM mode amplitudes, a concise representation of the data are derived from the modulated orbit measurements. At low orbit oscillation amplitudes, the LOCOM mode amplitudes are determined by the linear optics. As the orbit oscillation amplitude increase, the effects of the lattice nonlinearity will show up. Just like the RDTs are representations of the nonlinear lattice behavior, the nonlinear LOCOM mode amplitudes also represent the nonlinear lattice.

By fitting the nonlinear lattice parameters, such as sextupole strengths, the differences of the mode amplitudes between measurements and the model can be accounted for. The fitting results can be used to change the nonlinear parameters on the machine to restore the lattice behavior toward the design model, as is done for linear optics correction. Since the mode amplitudes are affected by linear optics errors, it is preferred to correct the linear optics first with LOCOM data taken with small corrector modulation amplitude. After that, only nonlinear magnet parameters are fitted for LOCOM data taken with large modulation amplitude. It is also possible to fit the LOCOM data with small and large modulation amplitudes together, with both linear and nonlinear lattice parameters as fitting parameters simultaneously.

## SIMULATION

The SPEAR3 storage ring 10-nm lattice has been used in the simulation study. A pair of correctors in each plane are used to modulate the orbit. Figure 1 shows the mode amplitudes for the in-plane excitation with corrector modulation amplitude of 1 A,  $I_{m1}$ , for the first corrector (corresponding to 0.05 mrad for a horizontal corrector and 0.025 mrad for a vertical corrector).

In the simulation, we also generated data with corrector modulation amplitudes at 0.1 A, and 2 through 10 A with 2 A intervals. To see the effects of nonlinearity, we compare the mode amplitude after they are scaled to the case with

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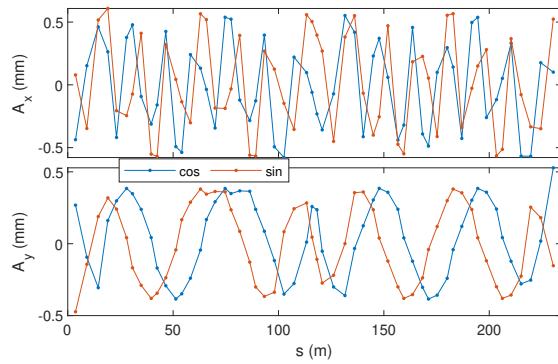


Figure 1: The simulated horizontal (top) and the vertical (bottom) in-plane mode amplitudes with the first corrector modulation amplitude at  $I_{m1} = 1$  A. The cosine and sine modes are both shown.

1 A modulation amplitude. Figure 2 shows the differences of the scaled mode amplitudes (scaled to the 1 A case), for the three cases with modulation amplitude of 2, 6, and 10 A, and that of the 0.1 A case. Figure 3 shows the rms variation of the difference curves for various modulation levels. With a 10 A corrector modulation amplitude, the deviation of the mode amplitude is about 4% in the horizontal plane and 3% in the vertical plane. The rms variations depend on the modulation amplitudes according to a nonlinear scaling rule, roughly  $I_{m1}^{3/2}$ . Clearly, when the corrector modulation amplitude is large, there are significant differences between the scaled LOCOM mode amplitudes and that of the linear lattice. These differences are caused by the lattice nonlinearity and can be seen as features of the nonlinear lattice.

When the distribution of the nonlinear fields in the ring change, the mode amplitudes will also change. In the simulation, we decreased one SF magnet family (including 8 magnets distributed in 4 cells) by 5% and repeated the process to calculate the mode amplitudes. For the 10 A corrector modulation level, the scaled mode amplitudes change by up to  $3 \sim 4 \mu\text{m}$ , roughly 0.5% of the mode amplitudes. It is a small relative change. However, it is on the order of tens of microns (for 10 A modulation amplitude), which can be easily detected with BPMs measuring closed orbits.

SPEAR3 has 10 sextupole families, corresponding to 10 power supplies. The LOCOM data with 10 A modulation amplitude generated with the SF lattice error are fitted with the 10 sextupole strength parameters. The results are shown in Fig. 5, which agree with the expected value of a  $-5\%$  change on the SF family.

## EXPERIMENTS

Experimental data have been taken on the SPEAR3 storage ring to test the nonlinear LOCOM method. Similar to the simulation study discussed in the previous section, the corrector modulation level is varied from low to high with 2 A interval, and at each point, LOCOM data were taken.

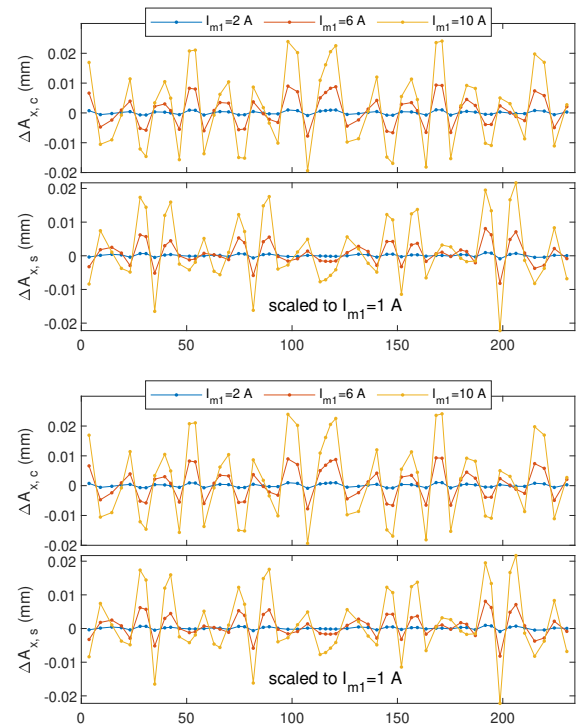


Figure 2: The differences of the cosine and sine mode amplitudes after they are scaled to the modulation amplitude of 1 A, for the horizontal plane (top 2 plots) and the vertical plane (bottom 2 plots), for three modulation levels: 2 A, 6 A, and 10 A. The SPEAR3 10-nm design lattice is used.

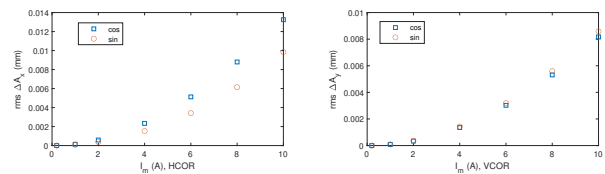


Figure 3: The rms variation of the differences of the scaled mode amplitude vs. the modulation amplitude in simulations.

The data processing is also similar. The mode amplitudes are scaled to the 1 A level and the differences from the very low modulation amplitude case (0.1 A) are calculated. Figure 6 shows the changes of the scaled mode amplitudes for three levels of modulation amplitude, in the same manner as in Fig. 2.

Comparing the curves in Figs. 2 and 6, one can see similarities between the patterns of the measured and simulated data. However, in the measurements, the mode amplitude changes are larger in magnitude for the same corrector modulation level. And, for the low modulation level (e.g. 2 A), the measurement sees a substantial mode amplitude change, while the simulation sees almost nothing. The rms variations of the mode amplitude differences are shown in Fig. 7. Unlike the simulations, the dependence on the modulation amplitude seems to be dominated by a linear term.

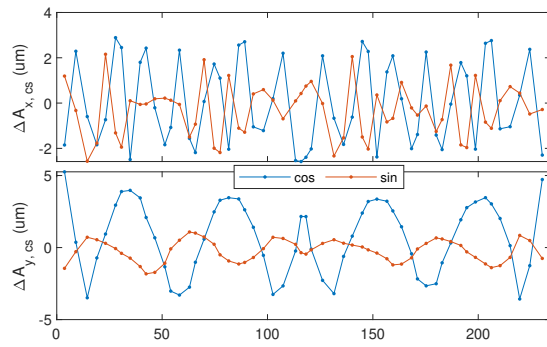


Figure 4: Changes of scaled mode amplitudes (scaled to 1 A modulation amplitude) for the 10 A modulation level when the strength of one SF magnet family decreases by 5%. Top plot: horizontal plane; bottom plot: vertical plane.

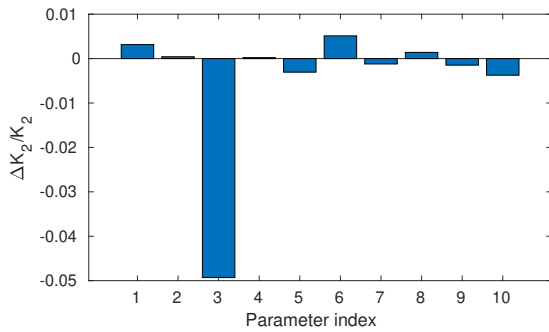


Figure 5: Fitted  $\Delta K_2/K_2$  for the 10 SPEAR3 sextupole families for simulated LOCOM data (with 10 A modulation level). The expected sextupole error of  $-5\%$  is found.

The measurements were repeated after a 5% change to the sextupole strength of the same SF family was made. The differences to the mode amplitudes (linearly scaled to the 1 A case) introduced by the SF magnet change for the 10 A modulation level are as shown in Fig. 8. The patterns do not closely follow the predictions by the simulation (see Fig. 4). It is suspected other effects, such as changes due to the linear optics and coupling caused by “feed-down” have a bigger contribution which shadow the nonlinearity effects.

## SUMMARY

We proposed to use large amplitude closed orbit modulation data to characterize the nonlinear lattice behavior of storage rings and to use it for nonlinear lattice correction by fitting the data to the lattice model. The idea is tested in simulation, which shows that the LOCOM mode amplitudes depend on the orbit modulation amplitude and can be used to find the nonlinear lattice errors. Experimental data also show variations of mode amplitudes that depend on the modulation amplitude. However, there are stronger, unexplained other effects that obscure the contribution of the lattice nonlinearity. Further studies are needed to understand the sources of the dominant effects.

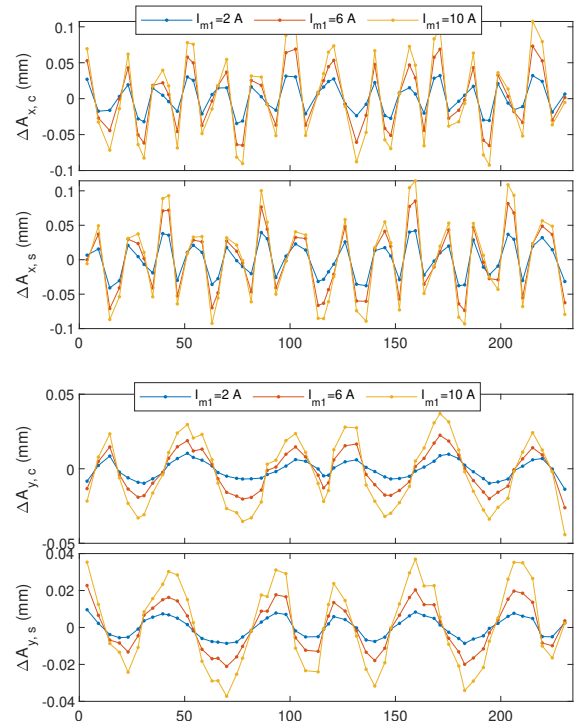


Figure 6: The experimentally measured mode amplitude differences (after scaled to the modulation amplitude of 1 A), for the horizontal plane (top 2 plots and the vertical plane (bottom 2 plots), for three modulation levels: 2 A, 6 A, and 10 A. LOCOM data were taken for the SPEAR3 10-nm nominal lattice configuration.

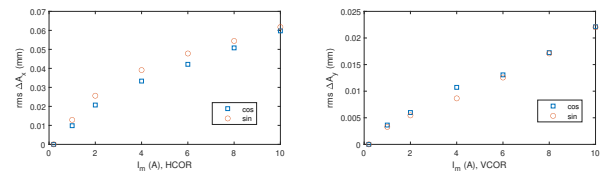


Figure 7: The rms variation of the differences in scaled mode amplitude in measurements.

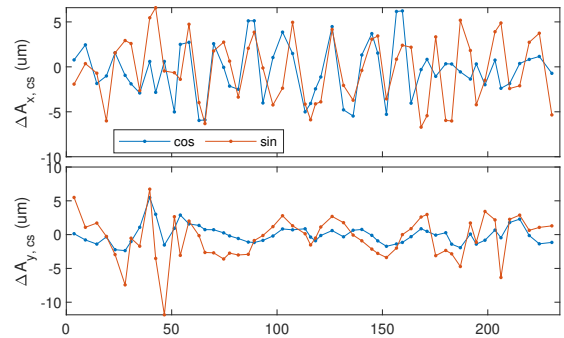


Figure 8: The differences of the scaled mode amplitudes for the 10 A between the case with a 5% change to an SF family and the nominal configuration.

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