EXPLORING THE 2^+ ISOSPIN DOUBLET IN ^{8}Be THROUGH MULTIPLE METHODS*

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Received 14 November 2023, accepted 19 February 2024, published online 24 April 2024

The 2^+ doublet in ^{8}Be is a well-known example of nearly equal isospin mixing between two states. However, the degree of mixing has not been experimentally determined as the studies of reaction feeding to the doublet do not distinguish isospin easily, and the β^+/EC feeding rate is very low. The IS633 experiment is the first beta decay study to resolve the 2^+ doublet thanks to the high statistics and, therefore, gives a chance to determine the mixing through the study of the Gamow–Teller and Fermi population of these states. Two approaches have been followed: an R-matrix analysis and the β recoil study. Preliminary results are presented in this contribution.

DOI:10.5506/APhysPolBSupp.17.3-A32

1. Introduction

The existence of a 2^+ isospin doublet in ^{8}Be, with energy levels at 16.6 and 16.9 MeV, has been known since the 60s. A key contribution to its characterization was the study of the population of the excited states of ^{8}B through the ^{10}B$(d,\alpha)^8$Be reaction at different energies [1, 2]. The observed feeding to both states was similar independently of the deuteron energy, giving rise to the hypothesis that both states have some degree of isospin (T) mixing [1]. In addition, the results from the bombardments with 12 MeV deuterons showed that both states exhibit asymmetric Gaussian shape lines, with the tails reducing between 16.6 and 16.9 MeV, while becoming more prominent outside the doublet, a behaviour characteristic of interfering levels of the same spin and parity (2^+) [2]. This interference between the levels was caused by the Coulomb interaction and corroborated the isospin mixture

* Presented at the XXXVII Mazurian Lakes Conference on Physics, Piaski, Poland, 3–9 September, 2023.

hinted in [1]. In these cases, the wave function (w.f.) of each level can be decomposed into a combination of "pure" isospin levels

$$|16.6\rangle = \alpha\,|T=0\rangle + \beta\,|T=1\rangle\ , \tag{1a}$$
$$|16.9\rangle = \beta\,|T=0\rangle - \alpha\,|T=1\rangle\ , \tag{1b}$$

where α and β are the mixing coefficients between the pure isospin levels ($|T=0\rangle$ and $|T=1\rangle$) satisfying $\alpha^2+\beta^2=1$. Using this approximation and assuming that only $T=0$ contributes to the $^{10}\mathrm{B}(d,\alpha)^{8}\mathrm{Be}$ reaction, Barker [3] obtained an expression for the shape of the lines dependent on the energy, width, and degree of isospin mixing of the levels — this formula fits the experimental spectrum shown in figure 1 of [2] if a complete isospin mixture ($\alpha^2/\beta^2=1$) is imposed. This makes the 2^+ isospin doublet in ^{8}Be the best-known case of fully mixed isospin states, however, the proposed total isospin mixture has yet to be directly determined experimentally.

The isospin doublet of ^{8}Be can be studied through the β-feeding of its father nucleus, the 1-proton halo ^{8}B ($Q_{\mathrm{EC}}=17.9798(1)$ MeV). In this case, an expression for the intermixing rate between the two levels is derived assuming that all the Fermi (F) strength goes to the $T=1$ component and all the Gamow–Teller (GT) strength to $T=0$ ($M_{0,\mathrm{GT}} >>> M_{1,\mathrm{GT}}$) — this assumption comes from the shell-model calculation of [4] that predicts that the matrix element $M_{1,\mathrm{GT}}$ is negligible compared with the $M_{0,\mathrm{GT}}$

$$\frac{\alpha^2}{\beta^2} = \frac{B_{\mathrm{GT}(16.6)}}{B_{\mathrm{GT}(16.9)}}\,; \qquad \frac{\alpha^2}{\beta^2} = \frac{B_{\mathrm{F}(16.9)}}{B_{\mathrm{F}(16.6)}}\,, \tag{2a}$$
$$\alpha^2 = \frac{\Gamma_{16.6}}{\Gamma_{16.6}+\Gamma_{16.9}}\,; \qquad \beta^2 = \frac{\Gamma_{16.9}}{\Gamma_{16.6}+\Gamma_{16.9}}\,, \tag{2b}$$

where Γ, B_{GT}, and B_{F} are, respectively, the decay width, the Gamow–Teller and Fermi strength of each level. Since all excited states of ^{8}Be are unbound, all relevant information is extracted from the $\alpha+\alpha$ spectrum.

The main challenge of this approach is that due to the kinematics (f-factor), the feeding to the doublet is not the dominant decay mode of ^{8}B. There are three possible decays within the Q_{EC} window: First, the dominant feeding ($> 88\%$) to a broad state at 3 MeV ($J^\pi=2^+, \Gamma=1513(15)$ MeV). This level is the primary source of high-energy solar neutrinos and has been widely studied due to its astrophysical interest [5]. Second, the allowed EC/β^+ transitions to the 2^+ doublet (16.6 MeV; $\Gamma_{16.6}=108.1(5)$ keV and 16.9 MeV; $\Gamma_{16.9}=74.0(4)$ keV) contributing around 10% of the total feeding (most of the β^+ feeding going to the 16.6 MeV level). Third, a possible EC to a highly excited 1^+ level at 17.640 MeV that decays into ^{7}Li by emitting a low-energy (330 keV) proton (this decay mode has not been observed so far).

Recently *ab initio* calculations using the SA-NCSM framework (NNLO$_{\rm opt}$ chiral potential) predict a 2^+ intruder level at 11 MeV with $\Gamma \sim 10$ MeV [6] — these predictions are in line with those made by Barker in the 80s using simultaneous R-matrix fits to β decay and α scattering data [7], however, none of these extra levels has been previously observed.

Assuming equal matrix elements, the feeding to the 16.9 MeV state is 2.4×10^{-2} times that of 16.6 MeV [8]. This fact, together with the too-low feeding to the doublet, means that a high level of statistics is necessary to resolve the doublet. The lack of statistics has been the main bottleneck in studying the ^{8}Be doublet by β^+/EC decay, in particular the 16.9 MeV level. In the previous benchmark experiment (JYFL08) [8], only 5 events were attributed to the decay of this state.

2. IS633: Experimental set up and data

To populate the 2^+ doublet in ^{8}Be through β^+ decay, the low feeding rate must be compensated with high statistics. This requires a high yield of ^{8}B and a setup that offers a large coverage to detect the α particles emitted from the ^{8}Be decay. The IS633 experiment, performed at the ISOLDE/CERN facility [10], is the latest work of the MAGISOL Collaboration in this research line. A schematic setup is shown in Fig. 1.

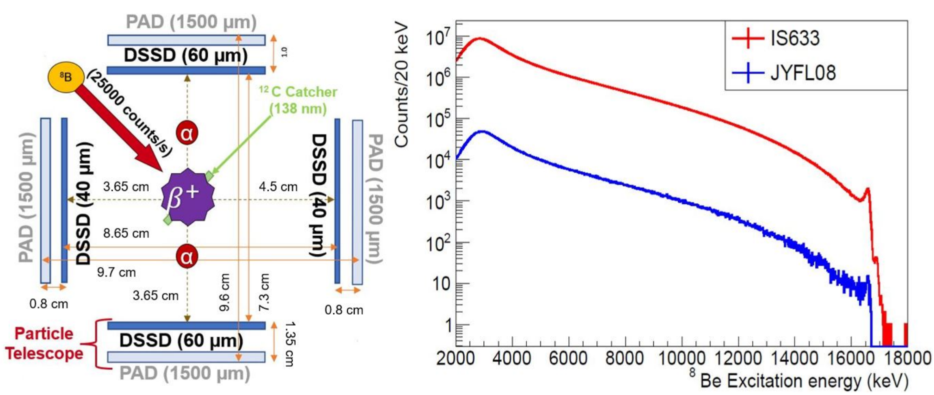

Fig. 1. (Colour on-line) Left: IS633 setup: ^{8}BF$_2$ molecules are implanted in a 31 μg/cm^2 C-foil, ^{8}B decays to ^{8}Be that breaks up into two α particles detected through a system of four particle telescopes. Right: $\alpha + \alpha$ spectrum obtained from IS633 (red/higher curve) compared with the previous benchmark experiment (JYFL08) done in Jyväskylä (Finland).

A 31 μg/cm^2 carbon foil placed at the centre of the setup is used to stop a 50 keV ^{8}BF$_2$ molecular beam, produced online at ISOLDE [9]. The ^{8}B is implanted at a depth of 26 nm with an average rate of 6×10^4 ions/s [10].

The β^+ decay of ^{8}B feeds the 2^+ excited states of ^{8}Be, which in turn break up into an α-particle pair. Four particle telescopes placed perpendicular to each other surrounding the carbon foil catcher are used to detect the α particles. Each telescope is formed by a 60 μm or 40 μm Double-Sided Silicon Strip Detector (DSSD) backed by a 1500 μm thick Si-PAD detector. This geometry gives an angular coverage of $\Omega = 38\%$ of 4π.

The emitted α particles experience an energy loss ($E_{\rm loss}$) directly related to the implantation depth of the ^{8}B. This is because each α particle must traverse a distance $r_{\rm a}$ and $r_{\rm b}$ in the C-foil. $r_{\rm a} = d_{\rm a}/\cos\theta$ and $r_{\rm b} = d_{\rm b}/\cos\theta$, where $d_{\rm a}$ is the implantation depth, $d_{\rm b}$ = (thickness of target) $- d_{\rm a}$, and θ is the angle of the pixel where the alpha-particle is detected with respect to the point where ^{8}B beam is stopped. $d_{\rm a}$ was estimated to be 26 nm on average from the calculated energy losses (SRIM) in the C-foil.

Each valid event must fulfil two conditions: First, two α particles must be detected in coincidence in opposite detectors. Second, the pixels where these α particles are detected must be inside the area formed by the projection of the beam spot over their respective DSSD since only events within this area could come from an α emitted from the foil.

Once the events are selected, corrections for the energy loss in the carbon foil and α breakup energy ($-91.84(4)$ keV) must be applied. The final $\alpha+\alpha$ spectrum is shown in red in Fig. 1. The high statistics obtained enabled to resolve the 16.9 MeV level from the 16.6 MeV one for the first time in a β-decay experiment.

3. R-matrix

To determine the mixing coefficients of the doublet, one needs to extract from the $\alpha+\alpha$ spectrum, the width (Γ), and the F and GT strengths for each level, a challenging task due to the fact that the 3 MeV, 16.6 MeV, and 16.9 MeV resonances are so broad that their contributions are linked, as depicted in Fig. 2. Consequently, fitting with a well-defined function (*e.g.* Gaussian) is not feasible. Instead, a fitting algorithm should distinguish between the three contributions and the R-matrix is the ideal tool for this.

In an R-matrix analysis, a spectrum is decomposed into well-defined resonances [11]. This is achieved by dividing the configuration space into two radial regions: internal (dominated by the nuclear interaction) and external (dominated by the Coulomb interaction), separated by a constant called the channel radius $r_{\rm c} = (A_1^{1/3} + A_2^{1/3})r_o$. Since the wave function and its derivative must be continuous at the border between these regions, the eigenvalues in the internal region must be related to the eigenvalues in the external. This relation is the R-matrix, which can be derived from solving the Schrödinger equation in both regions and imposing continuity at $r_{\rm c}$.

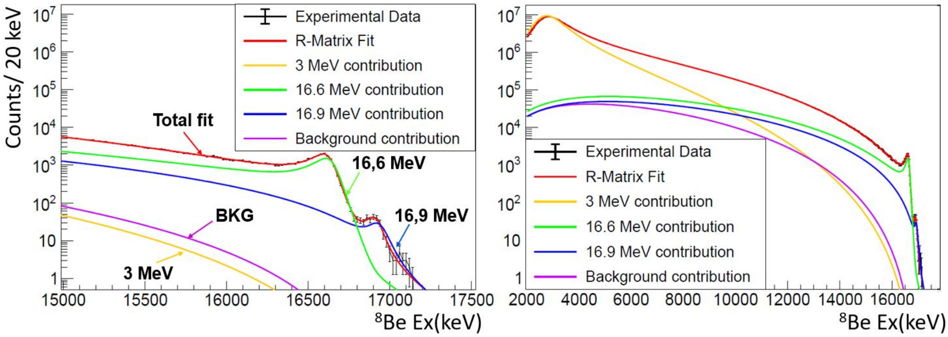

Fig. 2. Left: local R-matrix fit to the doublet with all the individual contributions marked (background level set to 37 MeV). It is important to note that the 3 MeV state does not influence 16.6 MeV. Right: global fit to the whole spectrum.

The R-matrix is expressed as the summation of a series of resonant levels characterized by unique parameters. In Barker's variant of the R-matrix, which adapts it from a reaction framework to the β decay to broad resonances [12], the parameters of each level are the energy (E), level width (Γ), and GT and F strengths, making it an ideal tool for this analysis.

In this study, the R-matrix was implemented into the ORM-FIT program developed at the Aarhus University [8]. Here r_o is fixed at 1.35 fm, while the R-matrix is decomposed in four levels: three representing the 3 MeV, 16.6 MeV, and 16.9 MeV resonances, along with a background (BKG) level. The code employs the MINUIT algorithm to iteratively compare the initial R-matrix curve with experimental data, adjusting the 16 parameters (4 for each level) until the R-matrix adequately fits the data. The values of E, Γ, B_{F}, and B_{GT} are extracted from this fit. All errors for the R-matrix parameters are given by the MINUIT algorithm, in turn, uncertainties for the mixing coefficients shown are derived using standard error propagation.

An iterative procedure has been employed to fit the excitation spectrum. The procedure starts by fitting only the doublet region using a very reduced fitting range. With the parameters of the doublet optimized, the fitting range is expanded to include the 3 MeV and intermediate regions, while keeping the doublet parameters fixed. Once the 3 MeV and intermediate regions are fitted, the doublet parameters are liberated, and the fit is repeated. The process continues until it converges to a set of values for the parameters. The local and full spectrum fits are shown in Fig. 2.

Visually, both fits look good; however, the width of the 3 MeV resonance obtained in the global fit is 1883(13) keV; 350 keV wider than the established literature value from reaction experiments [13]. While a series of consistency checks did not provide any new insight into this problem [14], an interesting observation can be made. If the iteration order is inverted, fitting the ^{8}Be excitation spectrum first from 2 to 4 MeV and then expanding onwards, the local fits to the 3 MeV region do, in fact, produce results in accordance with the literature, while the global fit does not. Following this idea, a series of fits to the 3 MeV resonance with an increasing fitting range were performed. The results, shown in Table 1, indicate that it is the intermediate region which distorts the width of the 3 MeV response.

Table 1. Evolution of the FWHM of the 3 MeV resonance obtained with R-matrix for multiple fitting ranges.

	Literature [13]		IS633	
Fitting range [MeV]	E [keV]	$\Gamma_{\alpha\alpha}^{3\,\mathrm{MeV}}$	E [keV]	$\Gamma_{\alpha\alpha}^{3\,\mathrm{MeV}}$
2–4	3030 (10)	1513 (15)	2997 (36)	1470 (15)
2–5			3006 (32)	1588 (18)
2–6			3020 (38)	1655 (16)
2–7			3030 (37)	1706 (18)

The fits to the 3 MeV level indicate that the intermediate region cannot be modelled as the tail of a very high-energy resonance formed by tail contributions of high-energy 2^+ background states. Instead, we should consider the hypothesis where the intermediate region of the spectrum contains broad intruder resonances; [7] proposes a 2^+ $\Gamma = 10$ MeV intruder resonance near $E = 10$ MeV, while [6] postulates two, a 0^+ at $E \sim 8.5$ MeV and a 2^+ at $E \sim 11$ MeV both with $\Gamma \sim 10$ MeV.

An R-matrix fit was attempted for each 2^+ scenario but in all cases yielded unsatisfactory results — the width of the 3 MeV level moves close to 1600(10) keV, but the fits give $\chi^2 = 2.045 \times 10^6$, five orders of magnitude higher than the χ^2 obtained for Fig. 2.

The possibility of a direct transition to the continuum could be considered. This transition would be linked to the fact that the last proton in ^{8}B is loosely bound, a more technical explanation can be found in [15]. To test if the continuum decay hypothesis is correct, the R-matrix would require an additional number of resonances which the code currently cannot support.

The results of the local fits of the doublet can potentially be used to determine the mixing coefficient but this imposes a considerable uncertainty that could hinder their validity. Instead, an alternative method employing not the $\alpha + \alpha$ spectrum but rather the $\alpha - \alpha$ one might give better results.

4. β-recoil

The recoil method presented here is an attempt to circumvent the problems derived from the R-matrix, *i.e.*, the difficulty of fitting the intermediate region and its consequences for the energies and widths of the 2^+ states. A β^+ decay is a three-body process where the final products are a positron, a neutrino, and a recoiling daughter nucleus. In the ^{8}Be case, the recoiling nucleus is unbound and, while recoiling, breaks up ($T_{1/2} = 67 \times 10^{-16}$ s; $\Gamma = 5.57(25)$ eV) into two α particles which receive a fraction of the recoiling momentum. The recoil transfer is not necessarily symmetric and depends on the direction of the emitted particles and the type (F or GT) of transition, therefore will follow a specific distribution whose shape is given by [16]

$$\begin{aligned} w(t) \;=\; & N_\beta \int\limits_{W_{\min}}^{W_o} F(Z,W)\frac{p_\beta}{2k}\Bigg[(c_1-c_2)\,W\,(W_o-W) \\ & -A\frac{c_2^2-c_1^2}{2}p_\beta\frac{t}{k} - A\frac{c_2^3-c_1^3}{3}p_\beta^2\Bigg]\,\mathrm{d}W + N_{\mathrm{EC}}\frac{f_{\mathrm{BK}}+f_{\mathrm{BL1}}}{2T_{\max}}\,, \end{aligned} \tag{3}$$

where W is the total relativistic energy of the β particle (W_o the maximum energy), p_β its momentum, $F(Z,W)$ is the Fermi function, t the kinematic shift, $k, c_1, c_2, f_{\mathrm{BK}}, f_{\mathrm{BL1}}$ are kinematic constants, N_β and N_{EC} are the β and EC components of the decay, $T_{\max}$ the maximum recoil energy of an α particle. Finally, A is the asymmetry parameter that contains all the information related to the F and GT components of the decay. A more in-depth discussion can be found in [16].

For the current discussion, only the relation between A and the F and GT components is relevant. This relation is expressed as

$$A = \frac{g_{\mathrm{V}}^2 B_{\mathrm{F}} - \left(\frac{1}{3}+\frac{2}{30}\tau\theta\right) g_{\mathrm{A}}^2 B_{\mathrm{GT}}}{g_{\mathrm{V}}^2 B_{\mathrm{F}} + g_{\mathrm{A}}^2 B_{\mathrm{GT}}}\,, \tag{4}$$

where g_{V} and g_{A} are, respectively, the vector and axial-vector weak coupling constants, B_{F} and B_{GT} are the reduced Fermi and Gamow–Teller strengths, and τ and θ are spin-dependent coefficients derived in [16].

For the $^8\mathrm{B}(2^+) \to {}^8\mathrm{Be}(2^+) \to 2\alpha$ case, $\Delta J = 0$ and $\Delta L = 2$, therefore $\tau\theta = 10$, which allows relating B_{F} and B_{GT} with A through the expression

$$\frac{B_{\mathrm{F}}}{B_{\mathrm{GT}}}\left(\frac{g_{\mathrm{V}}^2}{g_{\mathrm{A}}^2}\right) = -\frac{(A+1)}{(A-1)}\,. \tag{5}$$

This implies that if $A = 1$, there is no GT component; if $A = 0$, the F and GT components are equal; and for $A = -1$, the F component is zero.

Equations (3), (4), and (5) give a possible way of obtaining B_F and B_{GT} from a recoil distribution. To obtain this distribution, we employ the ^{8}Be excitation spectrum to select all events where the α particles add up to a specific excitation energy and then subtract their individual energies. The resulting recoil distribution is fitted with equation (3) using A as a fit parameter, and from its value, the F and GT contributions can be deduced.

This method eliminates the need to handle the interfering tails and intermediate region effects. For each fit, Eq. (3) must be convolved with the detector response function. Our initial attempts to model the response use Laplacian or T-Student distributions. Figure 3 shows the outcomes of these initial fits, for a 16,6 MeV excitation energy (16.60 MeV $\pm$ 0.05 MeV). The normalized Laplacian gives a slightly worse result than T-Student ($\chi^2_{\rm Lap} = 0.85$ *vs.* $\chi^2_{\rm Stu} = 1.09$). In both cases the asymmetry parameter is $A = 0.10(8)$ implying a total mixture at 16.6 MeV, a promising result.

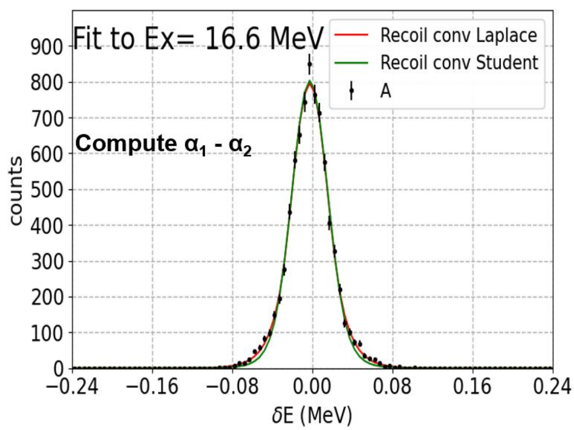

Fig. 3. β recoil distribution obtained from the energy difference of each α-pair in the ^{8}Be Ex = 16.55–16.65 MeV range. The recoil spectrum is fitted with Eq. (3) folded with a response function modelled as Laplacian or as T-Student distribution.

Following the initial fits, the next step is to refine our response function. Presently, we are exploring two distinct methodologies: one analytical and the other numerical. In the analytical approach, we utilize the response function outlined in [8], while in the numerical approach, the response function is derived from G4 simulations conducted for each detector [17]. The result obtained from these new methods is still being evaluated.

5. Summary

IS633 is the first experiment to successfully resolve the β^+-feeding to the 2^+ doublet in ^{8}Be. This doublet has been anticipated to exhibit a complete isospin mixture. Two methods, the R-matrix and the β-recoil, are employed to extract the Fermi and Gamow–Teller strengths, crucial for determining the isospin mixture, from the ^{8}Be complex excitation spectrum. The R-matrix fit to the entire spectrum gives too wide levels due to the influence of tails in the intermediate energy region, an extra intermediate state does not fix this problem. In response to the limitations of the R-matrix, an alternative method based on the β^+-recoil is being applied with promising results.

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