Supermembrane on the PP-wave Background[1]

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We consider superalgebra and BPS conditions in the supermembrane theory on the pp-wave background. The full superalgebra is calculated and we present examples of 1/2 and 1/4 BPS conditions. Also BPS conditions of classical solutions in the matrix model are confirmed by using this formula.

1 Introduction

It is considered that supermembrane theories in eleven dimensions [1, 2, 3] are intimately related to M-theory. For the past years, many works towards the formulation of M-theory have been done. The matrix model approach among these works particularly seems successful at least in a certain region of M-theory [4]. For many years solutions of the eleven-dimensional supergravity have been studied, and the maximally supersymmetric solutions are turned out to be Minkowski space, $AdS_4 \times S^7$, $AdS_7 \times S^4$, and Kowalski-Glikman (KG) pp-wave solution [5]. These are possible candidates for M-theory backgrounds. The matrix model on this pp-wave was proposed by Berenstein-Maldacena-Nastase [6]. The action contains mass terms and Myers term which lead to drastically different features from the flat case. A single supermembrane, which is unstable in the flat case [7], might be stabilized since the flat directions of the potential are completely lifted up. In addition, a fuzzy sphere, rotating ellipsoidal fuzzy sphere, etc. appear as classical solutions due to the presence of the Myers term, and this fact implies close relations to the noncommutative geometry. Thus, supermembrane and matrix model on the pp-wave have many interesting features to be studied.

Here we consider supermembranes on the eleven-dimensional maximally supersymmetric pp-wave background based on our previous results [8, 9] (For our works about other topics, see [10, 11, 12]). First we introduce the effective Lagrangian of a supermembrane and supersymmetries on the pp-wave. The supercharges are conserved but do not commute with the Hamiltonian since these explicitly depend on the world-volume time in the light-cone formulation. Thus the superalgebra and BPS conditions are different from the

[1]This work is based on our papers [8, 9] collaborated with K. Yoshida.

flat case. We calculate the full superalgebra and give the physical interpretation of the additional brane charges as correction terms to those in the flat space. We also comment on the correspondence of brane charges in the supermembrane theory and matrix model. Then the consistency of supercharges and BPS conditions to the time evolution is discussed. Next we present examples of 1/2 and 1/4 BPS conditions and several classical solution. Finally, by employing our BPS formula in terms of matrix model we confirm BPS conditions of classical solutions in the matrix model.

2 Supermembrane Action and Supersymmetries on PP-wave

We shall consider the supermembrane theory [1, 2, 3] on the eleven-dimensional maximally supersymmetric pp-wave background [5]. The metric of this background is given by

$$ds^2 = -2dx^+dx^- + G_{++}(dx^+)^2 + \sum_{\mu=1}^{9}(dx^\mu)^2\,, \tag{1}$$

$$G_{++} \equiv -\left[\left(\frac{\mu}{3}\right)^2(x_1^2+x_2^2+x_3^2) + \left(\frac{\mu}{6}\right)^2(x_4^2+\cdots+x_9^2)\right]\,,$$

where the constant 4-form flux $F_{+123} = \mu\ (\neq 0)$ is equipped.

The effective Lagrangian of a supermembrane on the pp-wave [6, 13] is

$$w^{-1}\mathcal{L} = \frac{1}{2}D_\tau X^r D_\tau X^r - \frac{1}{4}\{X^r,\,X^s\}^2 - \frac{1}{2}\left(\frac{\mu}{3}\right)^2 X_I^2 - \frac{1}{2}\left(\frac{\mu}{6}\right)^2 X_{I'}^2$$
$$-\frac{\mu}{6}\epsilon_{IJK}X^K\{X^I,\,X^J\} + i\psi^T\gamma^r\{X^r,\,\psi\} + i\psi^T D_\tau\psi + i\frac{\mu}{4}\psi^T\gamma_{123}\psi\,, \tag{2}$$

where ψ is an $SO(9)$ spinor and the Lie bracket is defined by $\{A,\,B\} \equiv w^{-1}\epsilon^{ab}\partial_a A\partial_b B$ with an arbitrary function $w(\sigma)$ on the membrane surface. The residual symmetry called area-preserving diffeomorphism (APD) is realized as a gauge symmetry. The covariant derivative for this gauge symmetry is defined by $D_\tau\bullet \equiv \partial_\tau\bullet - \{\omega,\,\bullet\}$ with a gauge connection ω. In the light-cone formulation, symmetries are not manifestly seen but the Lagrangian (2) still has 32 supersymmetries. The 16 linearly-realized supersymmetries are expressed as

$$\delta_\epsilon X^r = 2\psi^T\gamma^r\epsilon(\tau)\,,\quad \delta_\epsilon\omega = 2\psi^T\epsilon(\tau)\,, \tag{3}$$

$$\delta_\epsilon\psi = \left(-iD_\tau X^r\gamma_r + \frac{i}{2}\{X^r,\,X^s\}\gamma_{rs} + \frac{\mu}{3}i\sum_{I=1}^{3}X^I\gamma_I\gamma_{123} - \frac{\mu}{6}i\sum_{I'=4}^{9}X^{I'}\gamma_{I'}\gamma_{123}\right)\epsilon(\tau)\,,$$

$$\epsilon(\tau) = \exp\left(\frac{\mu}{12}\gamma_{123}\tau\right)\epsilon_0 \quad (\epsilon_0 : \text{const. spinor})\,,$$

which are often called dynamical supersymmetries. The other 16 nonlinearly-realized supersymmetries are written as

$$\delta_\eta X^r = 0\,, \quad \delta_\eta \omega = 0\,, \tag{4}$$
$$\delta_\eta \psi = \eta(\tau)\,, \quad \eta(\tau) = \exp\left(-\frac{\mu}{4}\gamma_{123}\tau\right)\eta_0 \quad (\eta_0 : \text{const. spinor})\,,$$

which are often referred to kinematical supersymmetries. Supersymmetry transformations in the flat space are recovered in the limit $\mu \to 0$.

By the use of Noether's theorem, the supercharges Q^+ and Q^- of the linearly and non-linearly realized supersymmetries, respectively, are given by

$$Q^+ = \int d^2\sigma\, w\Bigg[-2e^{-\frac{\mu}{12}\gamma_{123}\tau}\Big(D_\tau X^r\gamma_r\psi + \frac{1}{2}\{X^r, X^s\}\gamma_{rs}\psi + \frac{\mu}{3}\sum_{I=1}^{3} X^I\gamma_I\gamma_{123}\psi + \frac{\mu}{6}\sum_{I'=4}^{9} X^{I'}\gamma_{I'}\gamma_{123}\psi\Big)\Bigg]\,, \tag{5}$$

$$Q^- = \int d^2\sigma\, w\left[-2ie^{\frac{\mu}{4}\gamma_{123}\tau}\psi\right] = -2ie^{\frac{\mu}{4}\gamma_{123}\tau}\psi_0\,, \tag{6}$$

where ψ_0 is the zero-mode of ψ and w is normalized as $\int d^2\sigma\, w(\sigma) = 1$. The Dirac bracket is defined by the standard manner, and the canonical commutation relations are

$$\{X^r(\sigma), P_s(\sigma')\}_{\text{DB}} = \delta^r_s\delta^{(2)}(\sigma - \sigma')\,, \quad \{\psi_\alpha(\sigma), S_\beta(\sigma')\}_{\text{DB}} = \frac{1}{2}\delta_{\alpha\beta}\delta^{(2)}(\sigma-\sigma')\,. \tag{7}$$

Here $P_r \equiv wD_\tau X_r$ and $S_\alpha \equiv iw\psi^T_\alpha$ are canonical momenta of X^r and ψ_α, respectively. The canonical Hamiltonian H does not commute with supercharges

$$\{Q^+_\alpha, H\}_{\text{DB}} = +\frac{\mu}{12}(\gamma_{123}Q^+)_\alpha\,, \quad \{Q^-_\alpha, H\}_{\text{DB}} = -\frac{\mu}{4}(\gamma_{123}Q^-)_\alpha\,, \tag{8}$$

but one can understand these charges are conserved by considering time evolutions

$$\frac{dQ^\pm}{d\tau} = \frac{\partial Q^\pm}{\partial\tau} + \{Q^\pm, H\}_{\text{DB}} = 0\,. \tag{9}$$

By using the Dirac bracket, we obtained the superalgebra[8]. This superalgebra (other than central charges) agrees with that of matrix model on the pp-wave [6][2] while it realizes the superalgebra of the supermembrane theory in the flat space [3] by taking the limit $\mu \to 0$. The superalgebra includes some surfaces terms. In the flat case these are central charges of the supertranslation algebra. These charges indicate brane charges of extended objects.

[2]This type of superalgebra is called super pp-wave algebra. It is closely related with superalgebra of $AdS_4 \times S^7$ or $AdS_7 \times S^4$ [14] via the Penrose limit [15].

Terms S^a_{rs}, S^a_r, and S^a_{rstu} in ref.[8] are the same as in the flat space. The charges S^a_{rs}'s and S^a_r's correspond to transverse M2-branes and longitudinal M2-branes, respectively. The S^a_{rstu}'s correspond to longitudinal M5-branes.

For the pp-wave case, our resulting superalgebra includes additional terms $U^a_{JKI'J'}$ and $U^a_{I'}$. These should be interpreted as correction terms to brane charges in the flat space due to the fact that the pp-wave background is curved. It was suggested in the AdS-string case that topological charges are modified since the AdS background has a non-vanishing curvature while those in the flat case are recovered in the flat limit [16]. This result supports the physical interpretation for our results.

On the other hand, these charges might indicate the existence of extra extended objects only living on the pp-wave. These might be related to fuzzy membrane and giant graviton discussed in [6], or other extended object living only on the pp-wave [17, 10, 18, 19] due to a certain kind of Myers effects [20, 21]. In the next section we discuss BPS conditions to understand non-trivial brane charges.

3 BPS Conditions and Classical Configurations

To study BPS conditions, let us construct the supercharge matrix with 32×32 components[3]

$$
\begin{aligned}
i\{Q_\alpha, Q^T_\beta\}_{\rm DB} &\equiv \begin{pmatrix} i\{Q^-_\alpha, (Q^-)^T_\beta\}_{\rm DB} & i\{Q^-_\alpha, (Q^+)^T_\beta\}_{\rm DB} \\ i\{Q^+_\alpha, (Q^-)^T_\beta\}_{\rm DB} & i\{Q^+_\alpha, (Q^+)^T_\beta\}_{\rm DB} \end{pmatrix} \\
&= \begin{pmatrix} \mathbf{1} & 0 \\ N_2 & \mathbf{1} \end{pmatrix} \begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \tilde{\rm m} \end{pmatrix} \begin{pmatrix} \mathbf{1} & N_1 \\ 0 & \mathbf{1} \end{pmatrix}, \quad \tilde{\rm m} \equiv {\rm e}^{-\frac{\mu}{12}\gamma_{123}\tau} \cdot {\rm m} \cdot {\rm e}^{+\frac{\mu}{12}\gamma_{123}\tau}.
\end{aligned} \tag{10}
$$

The component matrix N_2 is given in ref.[9] with $N_1 = N_2^T$. The matrix "m" is expressed as

$$
\begin{aligned}
{\rm m}_{\gamma\delta} &= 2\left[\tilde{H} - \sum_{r,s=1}^{9} \tilde{z}_{rs}\tilde{z}^{rs}\right]\delta_{\gamma\delta} + 2\sum_{r=1}^{9}\left[\tilde{z}_r - \int d^2\sigma\tilde{\varphi}\tilde{X}^r\right](\gamma^r)_{\gamma\delta} \\
&+ \sum_{r,s,t,u=1}^{9}\left[\tilde{z}_{rs}\tilde{z}_{tu} + 2\tilde{z}_{rstu}\right](\gamma^{rstu})_{\gamma\delta} + 2\mu\sum_{J,K=1}^{3}\sum_{I',J'=4}^{9}\tilde{U}_{JKI'J'}(\gamma_{JKI'J'})_{\gamma\delta} \\
&+ 2\mu\sum_{I'=4}^{9}\tilde{U}_{I'}(\gamma_{I'}\gamma_{123})_{\gamma\delta} + \frac{\mu}{3}\sum_{I,J=1}^{3}\tilde{M}_0^{IJ}(\gamma_{IJ}\gamma_{123})_{\gamma\delta} - \frac{\mu}{6}\sum_{I',J'=4}^{9}\tilde{M}_0^{I'J'}(\gamma_{I'J'}\gamma_{123})_{\gamma\delta},
\end{aligned} \tag{11}
$$

[3]Hereafter, we use expressions of supercharges in which the factor $1/\sqrt{2}$ is absorbed into the normalization of ψ.

where we have shifted variables as $X^r = X_0^r + \tilde{X}^r$ and $P^r = wP_0^r + \tilde{P}^r$. The $\tilde{H}$ is the Hamiltonian and $\tilde{M}_0^{IJ}$, $\tilde{M}_0^{I'J'}$ are generators of the Lorentz symmetry SO(3)×SO(6). Also the $\tilde{\varphi}$ represents the Gauss' law constraint for the area-preserving diffeomorphism of the system. Brane charges $\tilde{z}_r$, $\tilde{z}_{rs}$, $\tilde{z}_{rstu}$, $\tilde{U}_{JKI'J'}$, and $\tilde{U}_{I'}$ are defined by

$$\tilde{z}_r \equiv \sum_{s=1}^{9} \int d^2\sigma\, w\{w^{-1}\tilde{X}^r \tilde{P}_s,\, \tilde{X}^s\} + i \int d^2\sigma\, w\{\tilde{X}_r \psi^T, \psi\} + \frac{3}{8} i \sum_{s=1}^{9} \int d^2\sigma\, w\{\tilde{X}^s, \psi^T \gamma_{rs} \psi\}\,, \tag{12}$$

$$\tilde{z}_{rs} \equiv -\frac{1}{2} \int d^2\sigma\, w\{\tilde{X}^r,\, \tilde{X}^s\} = z_{rs}\,, \tag{13}$$

$$\tilde{z}_{rstu} \equiv \frac{i}{48} \int d^2\sigma\, w\{\tilde{X}_{[r}, \psi^T \gamma_{stu]} \psi\} = z_{rstu}\,, \tag{14}$$

$$\tilde{U}_{JKI'J'} \equiv -\frac{1}{12} \sum_{I=1}^{3} \epsilon_{IJK} \int d^2\sigma\, w\{\tilde{X}^{I'},\, \tilde{X}^I \tilde{X}^{J'}\}\,, \tag{15}$$

$$\frac{1}{2}\tilde{U}_{I'} \equiv -\frac{1}{4} \int d^2\sigma\, w \left\{ \tilde{X}^{I'},\, \frac{1}{3}\sum_{I=1}^{3} (\tilde{X}^I)^2 - \frac{1}{6} \sum_{J'=4}^{9} (\tilde{X}^{J'})^2 \right\}. \tag{16}$$

When we consider BPS states, some supersymmetries are broken. But the remaining unbroken supersymmetries leave the states invariant. Then commutators between the states and the unbroken supercharges vanish. So, by analyzing the rank of the matrix "m", we can discuss BPS conditions.

3.1 1/2 BPS Conditions

The 1/2 BPS conditions are given by the condition m=0. In this case, by rewriting the supercharge matrix as

$$i\{\begin{pmatrix} Q_\alpha^\uparrow \\ Q_\alpha^\downarrow \end{pmatrix}, (Q_\beta^\uparrow, Q_\beta^\downarrow)\}_{\rm DB} = \begin{pmatrix} -\mathbf{1}_{16} & 0 \\ 0 & \tilde{\rm m} \end{pmatrix}, \quad \begin{pmatrix} Q_\alpha^\uparrow \\ Q_\alpha^\downarrow \end{pmatrix} \equiv \begin{pmatrix} \mathbf{1}_{16} & 0 \\ -N_2 & \mathbf{1}_{16} \end{pmatrix} \begin{pmatrix} Q_\alpha^- \\ Q_\alpha^+ \end{pmatrix}, \tag{17}$$

we can read off the unbroken supercharge

$$Q^\downarrow = \int d^2\sigma\, w \Big[-2 e^{-\frac{\mu}{12}\gamma_{123}\tau} \Big(D_\tau \tilde{X}^r \gamma_r \tilde{\psi} + \frac{1}{2}\{\tilde{X}^r,\, \tilde{X}^s\}\gamma_{rs}\tilde{\psi} + \frac{\mu}{3} \sum_{I=1}^{3} \tilde{X}^I \gamma_I \gamma_{123} \tilde{\psi} + \frac{\mu}{6} \sum_{I'=4}^{9} \tilde{X}^{I'} \gamma_{I'} \gamma_{123} \tilde{\psi} \Big) \Big] \equiv \tilde{Q}^+\,. \tag{18}$$

Further, by the use of $\tilde{Q}^+$, the matrix $\tilde{\rm m}$ is represented by

$$\tilde{\rm m} = e^{-\frac{\mu}{12}\gamma_{123}\tau} \cdot {\rm m} \cdot e^{+\frac{\mu}{12}\gamma_{123}\tau} = i\{Q^\downarrow,\, (Q^\downarrow)^T\}_{\rm DB} = i\{\tilde{Q}^+,\, (\tilde{Q}^+)^T\}_{\rm DB}\,. \tag{19}$$

Thus, $\tilde{\mathrm{m}}$ is also conserved and consistent with the time evolution.

Let us now present an infinite planar membrane solution with non-zero brane charge. An example of oscillating planar solutions is described by $X^1 = \sigma^1 \cos\left(\frac{\mu}{3}\tau\right)$, $X^2 = \sigma^2 \cos\left(\frac{\mu}{3}\tau\right)$ where other coordinates $X^3, \ldots, X^9$ are set to zero. This solution has a nonzero brane charge $z_{12} \sim \cos^2\left(\frac{\mu}{3}\tau\right)$ and no angular momentum. Also, this configuration becomes the planar configuration $X^1 = \sigma^1$, $X^2 = \sigma^2$ in the flat limit $\mu \to 0$.

3.2 1/4 BPS Conditions

The 1/4 BPS conditions are that the rank of the matrix "m" is eight. To study the rank of "m", we decompose the $SO(9)$ gamma matrices γ^r $(r = 1, \ldots, 9)$ with 16×16 components into 8×8 gamma matrices as

$$\gamma^{\tilde{r}} = \begin{pmatrix} 0 & \tilde{\gamma}^{\tilde{r}} \\ (\tilde{\gamma}^{\tilde{r}})^T & 0 \end{pmatrix} \quad (\tilde{r} = 1, \ldots, 8), \quad \gamma^9 = \begin{pmatrix} \mathbf{1}_8 & 0 \\ 0 & -\mathbf{1}_8 \end{pmatrix}, \tag{20}$$

where $\tilde{\gamma}^{\tilde{r}}$'s are real matrices with 8×8 components, which satisfy $\tilde{\gamma}^{\tilde{r}}(\tilde{\gamma}^{\tilde{s}})^T + \tilde{\gamma}^{\tilde{s}}(\tilde{\gamma}^{\tilde{r}})^T = 2\delta^{\tilde{r}\tilde{s}}$, $(\tilde{\gamma}^{\tilde{r}})^T\tilde{\gamma}^{\tilde{s}} + (\tilde{\gamma}^{\tilde{s}})^T\tilde{\gamma}^{\tilde{r}} = 2\delta^{\tilde{r}\tilde{s}}$. The general analysis is too complicated and difficult and we shall present some special solution.

We now consider only μ-dependent parts by setting μ-independent parts zero. For simplicity, we impose further conditions $\tilde{M}_0^{IJ} = \tilde{M}_0^{\tilde{I}'\tilde{J}'} = \tilde{U}_{JKI'J'} = \tilde{U}_9 = 0$. Then the matrix "m" can be written as

$$\mathrm{m} = 2\mu \begin{pmatrix} \sum_{\tilde{I}'=4}^{8} \left(\tilde{U}_{\tilde{I}'} - \frac{1}{6}\tilde{M}_0^{9\tilde{I}'}\right) \tilde{\gamma}^{\tilde{I}'}(\tilde{\gamma}^1)^T\tilde{\gamma}^2(\tilde{\gamma}^3)^T & 0 \\ 0 & \sum_{\tilde{I}'=4}^{8} \left(\tilde{U}_{\tilde{I}'} + \frac{1}{6}\tilde{M}_0^{9\tilde{I}'}\right) (\tilde{\gamma}^{\tilde{I}'})^T\tilde{\gamma}^1(\tilde{\gamma}^2)^T\tilde{\gamma}^3 \end{pmatrix}.$$

Thus 1/4 BPS conditions are realized for $\tilde{U}_{\tilde{I}'} = +\frac{1}{6}\tilde{M}_0^{9\tilde{I}'}$ or $\tilde{U}_{\tilde{I}'} = -\frac{1}{6}\tilde{M}_0^{9\tilde{I}'}$ $(I' = 4, \ldots, 8)$, and the unbroken supercharge for this 1/4 BPS condition is given by

$$Q^{\downarrow(\pm)} \equiv \mathrm{e}^{-\frac{\mu}{12}\gamma_{123}\tau} \left(\frac{1 \pm \gamma^9}{2}\right) \mathrm{e}^{+\frac{\mu}{12}\gamma_{123}\tau} Q^{\downarrow}. \tag{21}$$

Here the plus (+) sign corresponds to the first $\tilde{U}_{\tilde{I}'} = +\frac{1}{6}\tilde{M}_0^{9\tilde{I}'}$ and the minus (−) one to the second $\tilde{U}_{\tilde{I}'} = -\frac{1}{6}\tilde{M}_0^{9\tilde{I}'}$, respectively. These BPS conditions denote that the brane charge $\tilde{U}_{\tilde{I}'}$ equals the angular momentum $\tilde{M}_0^{9\tilde{I}'}$, and hence describe certain rotating objects which live only on the pp-wave.

4 BPS Conditions in Matrix Model on the PP-wave

The matrix model action can be obtained from the supermembrane action through the matrix regularization procedure. As the result, the Lie bracket, integral, and variables are replaced by following the rule:

$$\{\, ,\, \} \longrightarrow -i[\, ,\,]\,, \quad \int d^2\sigma\, w(\sigma) \longrightarrow \mathrm{Tr}\,, \\ X(\xi^i) \longrightarrow X(\tau)\,, \quad \psi(\xi^i) \longrightarrow \psi(\tau)\,. \tag{22}$$

We shall consider classical solutions in the matrix model on the pp-wave background.

4.1 1/4 BPS Solutions

Here let us consider the well-known 1/4 BPS rotating solution described by

$$X^4(\tau) + iX^5(\tau) \;=\; \mathrm{e}^{\pm i\frac{\mu}{6}\tau}\left(X^4(0) + iX^5(0)\right)\,, \quad [X^4(0), X^5(0)] \;=\; 0\,, \tag{23}$$

where other X^r's are set to zero. For this solution, the matrix "m" is written as

$$\mathrm{m}_{\alpha\beta} \;=\; \left(\frac{\mu}{6}\right)^2 (1 \mp \gamma_{12345})_{\alpha\beta}\,\mathrm{Tr}\left[(X^4(0))^2 + (X^5(0))^2\right]\,. \tag{24}$$

Thus, the projection operator appears in the expression of "m". This result means that the rank of "m" is 8 and denotes 1/4 BPS state. Hence we have obtained the consistent result as expected. We can replace the indices 4 and 5 with two indices of $4, \ldots, 9$ due to the $SO(6)$ symmetry.

5 Conclusions and Discussions

We have considered superalgebra and BPS conditions in the supermembrane theory on the pp-wave background. We have calculated the full superalgebra and given the physical interpretation of non-trivial brane charges as correction terms to those in the flat space. Such corrections are turned on since the pp-wave background is curved. We have also commented on the correspondence of brane charges in the supermembrane theory and matrix model in the pp-wave case, and transverse M5-branes. Next, we have studied the unbroken supercharges in 1/2 and 1/4 BPS conditions. Also, applying our BPS formula to matrix model, we have confirmed BPS conditions for several classical solutions.

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