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The $\cos 2\phi_h$ Asymmetry in $K^{\pm}$ Mesons and the Λ-Hyperon-Produced SIDIS Process at Electron Ion Colliders

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Abstract: We investigate the $\cos 2\phi_h$ azimuthal asymmetry contributed by the coupling of the Boer–Mulders function and the Collins function in $K^{\pm}$- and Λ-hyperon-produced SIDIS process. The asymmetry is studied under the transverse-momentum-dependent (TMD) factorization framework at the leading order by considering the TMD evolution effects that utilize the parametrization for non-perturbative Sudakov form factors. The DGLAP evolution effects of the collinear counterpart of the Collins function of the final-state hadrons are considered by introducing the approximated evolution kernels. We utilize the available parametrization for the proton Boer–Mulders function and the Collins function of $K^{\pm}$. For the Collins function of the Λ hyperon, the result of the diquark spectator model is adopted due to the absence of parametrization. The numerical results of the $\cos 2\phi_h$ azimuthal asymmetry are obtained in the kinematic regions of EIC and EicC. It can be shown that the asymmetry is much smaller than the Sivers asymmetry, which means that the convolution of the Boer–Mulders function and the Collins function may not be the main contributor to the $\cos 2\phi_h$ asymmetry. We emphasize the importance of future measurement of the $\cos 2\phi_h$ asymmetry to unravel different contributors.

Keywords: transverse-momentum-dependent factorization; Boer–Mulders function; electron ion colliders

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1. Introduction

The Boer–Mulders function $h_1^{\perp}(x, \boldsymbol{p}_T)$ [1,2] is one of the eight transverse-momentum-dependent (TMD) parton distribution functions (PDFs) at the leading-twist level and gives novel insights into the three-dimensional (3D) partonic structure of hadrons [3–7]. It represents the transverse-polarization asymmetry of partons inside the unpolarized hadron. However, the T-odd Boer–Mulders function was initially thought to be vanished because of the time-reversal invariance of QCD [8]. Several QCD-inspired models [9–13] such as the spectator model [14,15], the light-front constituent quark model [16,17], the MIT bag model [18], the Nambu–Jona–Lasinio model [19,20], etc., have shown that $h_1^{\perp}(x, \boldsymbol{p}_T)$ is actually nonzero. Wilson lines (also known as gauge links), which appear in the full gauge-invariant definition of the TMD distributions, are crucial to this argument. In addition, the Wilson line makes the T-odd distribution functions process-dependent and flips the sign between the semi-inclusive deep inelastic scattering (SIDIS) and the Drell-Yan processes [14].

Since the Boer–Mulders function is a chiral-odd distribution function [2], it must be coupled with another chiral-odd distribution/fragmentation function to survive in the high-energy scattering process. The convolution of the Boer–Mulders function $h_1^{\perp}$ and the Collins function $H_1^{\perp}$ (which describes the fragmentation of transversely polarized quarks into unpolarized hadrons) can give rise to $\cos 2\phi_h$ azimuthal asymmetry in the unpolarized SIDIS process.

In this work, we numerically calculate the $\cos 2\phi_h$ azimuthal asymmetry of the SIDIS process in the kinematic regions of the EIC [21,22] and EicC [23] by utilizing the TMD factorization formalism, which is applicable in the region where the transverse momentum of the produced hadron in the final state is much smaller than the hard-scale Q. TMD factorization has been shown to be valid for processes such as SIDIS [8,24], e^+e^- annihilation [25–27], Drell–Yan [9,28], and W/Z boson production [29,30] processes. We briefly review the key points of the TMD factorization as well as the evolution effect, which has been studied intensively in the literature, for convenience.

According to TMD factorization, the differential cross-section of the SIDIS process can be expressed as the convolution of the hard scattering factor, TMD parton distribution function (PDF), and TMD fragmentation function (FF). One of the most important aspects of the TMD formalism is that it provides a systematic way of dealing with the evolution of TMDs (PDF and FF are collectively referred to as TMDs) and the transverse momentum dependence of TMDs, from which the scale evolution of TMDs is determined by the Collins–Soper equation (CS) [31,32] and the renormalization group equation.

After solving the evolution equations, the TMDs from initial energy μ to another energy Q are encoded in the Sudakov form factor S [33,34] by the exponential form $\exp(-S)$, which can be separated into the perturbatively calculable part $S_{pert}(Q;b_*)$ and the non-perturbative part $S_{NP}(Q;b)$ [33]. $S_{NP}(Q;b)$ cannot be calculated through perturbative theory, while it can be obtained by phenomenological extraction from experimental data. Furthermore, TMDs are expressed in terms of their collinear counterparts with perturbatively calculable coefficients in the perturbative region.

In this work, we will consider the evolution of both the proton Boer–Mulders function and the charged kaon and Λ Collins function to estimate the $\cos 2\phi_h$ asymmetry at the kinematics of EIC and EicC. For the proton Boer–Mulders function and the charged kaon Collins function, we adopt the available parametrization. For the Λ Collins function, we adopt the diquark spectator model result due to the lack of parametrization. The remainder of the paper is organized as follows. In Section 2, we provide the theoretical framework of the $\cos 2\phi_h$ asymmetry in the charged kaon and Λ hyperon production in the SIDIS process within the TMD factorization formalism. In Section 3, we numerically estimate the $\cos 2\phi_h$ asymmetry in the kinematical regions of EIC and EicC. We summarize the work and discuss the results in Section 4.

2. Formalism of the $\cos 2\phi_h$ Asymmetry in the SIDIS Process

In this section, we set up the necessary theoretical framework of the $\cos 2\phi_h$ asymmetry contributed by the Boer–Mulders function of the proton target and Collins function of final-state hadrons in the SIDIS process by applying TMD factorization while considering the evolution effect.

In the studied process, the unpolarized electron beam is scattered off the unpolarized proton target with an unpolarized hadron detected in the final state, which can be expressed as

$$e(\ell) + p(P) \to e(\ell') + h(P_h) + X(P_X), \tag{1}$$

where ℓ and ℓ' stand for the four-momenta of the incoming and outgoing electrons, respectively, and P and P_h denote the four-momenta of the proton target and the final-state hadron (which is a kaon and Λ hyperon in this work), respectively. X denotes the undetected system, which will not be measured in the final state with the four-momentum of P_X.

The following Lorentz invariants are defined to express the differential cross section as well as the physical observables:

$$S = (\ell + P)^2, \quad x = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot \ell}, \quad z = \frac{P\cdot P_h}{P\cdot q}, \quad Q^2 = -q^2, \tag{2}$$

where S is the total center-of-mass energy squared, x is the Bjorken variable, y denotes the lepton (quark) energy–momentum transferring fraction, and z represents the longitudinal momentum fraction of the final fragmented hadron with respect to the parent quark; $q = \ell - \ell'$ denotes the momentum of the virtual photon, with $Q^2 = -q^2$. The reference frame of the studied SIDIS process is shown in Figure 1, in which the z-axis is defined as the momentum direction of the virtual photon according to the Trento convention [35].

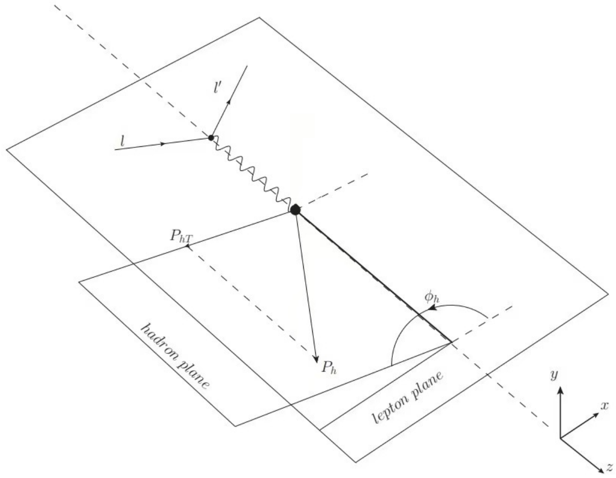

Figure 1. The reference frame in the SIDIS process.

The azimuthal angle ϕ_h between the lepton plane and the hadron plane is defined through

$$\cos\phi_h = -\frac{\ell_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{\ell_T^2 P_{hT}^2}}, \tag{3}$$

with $\ell_T^\mu = g_\perp^{\mu\nu}\ell_\nu$ and $P_{hT}^\mu = g_\perp^{\mu\nu} P_{h\nu}$ being the transverse components of ℓ and P_h with respect to the z-axis, and the tensor $g_\perp^{\mu\nu}$ is

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu P^\nu + P^\mu q^\nu}{P(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right), \tag{4}$$

with $\gamma = \frac{2Mx}{Q}$. Under the one-photon exchange assumption, the cross section of the unpolarized SIDIS process can be written as the general form of [36]

$$\begin{aligned} \frac{d^5\sigma}{dxdydzd^2\boldsymbol{P}_{hT}} &= \sigma_0\left(y, Q^2\right)\Bigg[F_{UU} + \cos\phi_h \frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2} F_{UU}^{\cos\phi_h} + \\ &\qquad \cos 2\phi_h \frac{2(1-y)}{1+(1-y)^2} F_{UU}^{\cos 2\phi_h} + \ldots\Bigg], \\ \sigma_0 &= \frac{2\pi\alpha_{\rm em}^2}{Q^2}\frac{1+(1-y)^2}{y}, \end{aligned} \tag{5}$$

where $\alpha_{\rm em}$ is the fine-structure constant. P_{hT} is the transverse momentum of the final-state hadron with respect to the z-axis. The ellipsis refers to other structure functions that are not considered in this work. The subscripts in the structure function F_{XY} represent the polarization status of the lepton beam (X) and target proton (Y), with U being unpolarized. The structure functions in Equation (5) can be expressed as the convolutions of the corresponding PDFs and FFs [36]:

$$F_{UU} = \mathcal{C}[f_1 D_1], \tag{6}$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[\frac{(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{zM_h}\left(xhH_1^\perp + \frac{M_h}{M}f_1\frac{\tilde{D}^\perp}{z}\right) - \frac{(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{M}\left(xf^\perp D_1 + \frac{M_h}{M}h_1^\perp\frac{\tilde{H}}{z}\right)\right], \tag{7}$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_1^\perp H_1^\perp\right], \tag{8}$$

where the notation $\mathcal{C}$ represents the convolution among the transverse momenta:

$$\mathcal{C}[\omega f D] = x\sum_q e_q^2\int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{hT}/z)\omega(\boldsymbol{p}_T,\boldsymbol{k}_T)f^q(x,p_T^2)\,D^q(z,k_T^2). \tag{9}$$

$F_{UU}^{\cos\phi_h}$ is the structure function related to the Cahn effect [37], which can be written at twist-3 order and can be simplified under Wandzura–Wilczek approximation:

$$F_{UU}^{\cos\phi_h} \approx -\frac{2M}{Q}\mathcal{C}\left[\frac{(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{M}\frac{k_T^2}{M^2}h_1^\perp H_1^\perp + \frac{(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{M}f_1 D_1\right]. \tag{10}$$

Here, $f_1(x,\boldsymbol{p}_T)$ and $h_1^\perp(x,\boldsymbol{p}_T)$ are the unpolarized TMD parton distribution function and the Boer–Mulders function of the target proton, respectively. They depend on the Bjorken variable x and the transverse momentum $\boldsymbol{p}_T$ of the quarks in the proton target. On the other hand, $D_1(z,\boldsymbol{k}_T)$ and $H_1^\perp(z,\boldsymbol{k}_T)$ are the unpolarized fragmentation function and the Collins function, respectively, which depend on the longitudinal momentum fraction z and the transverse momentum $\boldsymbol{k}_T$ of the final-state quark with respect to the z-axis that will fragment to the final-state hadrons; $\hat{h} = \frac{\boldsymbol{P}_{hT}}{|P_{hT}|}$ is the unit vector along $\boldsymbol{P}_{hT}$. One can see from Equation (10) that the Cahn effect can receive a contribution proportional to the convolution of the Boer–Mulders function and the Collins function.

At the leading twist order, the structure function $F_{UU}^{\cos 2\phi_h}$ can be written as the convolution of the Boer–Mulders function $h_1^\perp$ and Collins function $H_1^\perp$ as Equation (8). Meanwhile, the Cahn effect contributes to the $\cos 2\phi_h$ structure function at twist-4 order as well:

$$F_{\mathrm{UU|Cahn}}^{\cos 2\phi_h} = \frac{2M^2}{Q^2}\mathcal{C}\left[2\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)^2 - k_T^2}{M^2}f_1 D_1\right]. \tag{11}$$

Along with the contributions from the convolution of the Boer–Mulders function and the Collins function as well as the Cahn effect, the $\cos 2\phi_h$ structure function may also have a large contribution from the perturbative effect, which has been calculated by the Drell–Yan process [38] as well as the e^+e^- annihilation process [39].

In this work, we only discuss the contribution from the convolution of the Boer–Mulders function and Collins function at the twist-2 level. Therefore, the $\cos 2\phi_h$ asymmetry is defined as the ratio of the $\cos 2\phi$-dependent cross section and the unpolarized differential cross section, which can be written as

$$A_{UU}^{\cos 2\phi_h} = \frac{\sigma_0(y,Q^2)}{\sigma_0(y,Q^2)}\frac{2(1-y)}{1+(1-y)^2}\frac{F_{UU}^{\cos 2\phi_h}}{F_{UU}}, \tag{12}$$

where $\frac{2(1-y)}{1+(1-y)^2}$ is the depolarizing factor. The expressions for the structure functions in Equation (12) are shown in Equation (6) and Equation (8). By integrating other variables among the kinematical regions, we can obtain the x-, y-, and P_{hT}-dependent asymmetries as

$$A_{UU}^{\cos 2\phi_h}(x) = \frac{\int dydzd^2\boldsymbol{P}_{hT}\ \sigma_0(y,Q^2)\frac{2(1-y)}{1+(1-y)^2}F_{UU}^{\cos 2\phi_h}}{\int dydzd^2\boldsymbol{P}_{hT}\ \sigma_0(y,Q^2)F_{UU}}, \tag{13}$$

$$A_{UU}^{\cos 2\phi_h}(z) = \frac{\int dxdyd^2\boldsymbol{P}_{hT}\ \sigma_0(y,Q^2)F_{UU}^{\cos 2\phi_h}}{\int dxdyd^2\boldsymbol{P}_{hT}\ \sigma_0(y,Q^2)F_{UU}}, \tag{14}$$

$$A_{UU}^{\cos 2\phi_h}(P_{hT}) = \frac{\int dxdydz\ P_{hT}\sigma_0(y,Q^2)\frac{2(1-y)}{1+(1-y)^2}F_{UU}^{\cos 2\phi_h}}{\int dxdydz\ P_{hT}\sigma_0(y,Q^2)F_{UU}}. \tag{15}$$

Performing the Fourier transformation of the structure function from the transverse momentum space to b space (which is conjugated to the transverse momentum space [30,40] through the Fourier transformation), the structure function in the b space can be expressed as the simple product instead of the complicated convolution of TMDs in the transverse momentum space. We retrieve the unpolarized structure function from ref. [41] as

$$\begin{aligned} F_{UU}(Q;P_{hT}) &= \mathcal{C}[f_1 D_1] \\ &= x\sum_q e_q^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{hT}/z) f_1^q\left(x, p_T^2\right) D_1^q\left(z, k_T^2\right) \\ &= \frac{x}{z^2}\sum_q e_q^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{K}_T \delta^{(2)}(\boldsymbol{p}_T + \boldsymbol{K}_T/z - \boldsymbol{P}_{hT}/z) f_1^q\left(x, p_T^2\right) D_1^q\left(z, \frac{K_T^2}{z^2}\right) \\ &= \frac{x}{z^2}\sum_q e_q^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{K}_T \int \frac{d^2b}{(2\pi)^2} e^{-i(\boldsymbol{p}_T + \boldsymbol{K}_T/z - \boldsymbol{P}_{hT}/z)\cdot \boldsymbol{b}} f_1^q\left(x, p_T^2\right) D_1^q\left(z, \frac{K_T^2}{z^2}\right) \\ &= \frac{x}{z^2}\sum_q e_q^2 \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{P}_{hT}\cdot \boldsymbol{b}/z} \tilde{f}_1^{q/p}(x,b) \tilde{D}_1^{h/q}(z,b), \end{aligned} \tag{16}$$

where $\boldsymbol{K}_T$ represents the transverse momentum of the final hadrons with respect to the parent quark and has the relation $\boldsymbol{K}_T = -z\boldsymbol{k}_T$ [30]. The TMD distribution function $\tilde{f}_1(x,b)$ and the fragmentation function $\tilde{D}_1(z,b)$ in the b space can be obtained by performing the Fourier transformation from transverse momentum space to b space:

$$\int d^2\boldsymbol{p}_T e^{-i\boldsymbol{p}_T\cdot \boldsymbol{b}} f_1^q(x,p_T^2) = \tilde{f}_1^{q/p}(x,b), \tag{17}$$

$$\int d^2\boldsymbol{K}_T e^{-i\boldsymbol{K}_T/z\cdot \boldsymbol{b}} D_1^q(z,K_T^2) = \tilde{D}_1^{h/q}(z,b). \tag{18}$$

Hereafter, we denote the terms in b space as the terms with a tilde. One should notice that the distribution function and the fragmentation function must depend on the energy scale, which has been neglected in Equation (16) and will be discussed in detail in the following subsection.

Similarly, the structure function $F_{UU}^{\cos 2\phi_h}$ can be rewritten as:

$$\begin{aligned}
F_{UU}^{\cos 2\phi_h}(Q;P_{hT}) &= \mathcal{C}\left[-\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_1^\perp H_1^\perp\right] \\
&= \sum_q e_q^2\int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\delta^{(2)}(\boldsymbol{p}_T-\boldsymbol{k}_T-\boldsymbol{P}_{hT}/z) \\
&\times\left[-\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}\right]h_1^{\perp,q}\left(x,p_T^2\right)H_1^{\perp,q}\left(z,k_T^2\right) \\
&= \frac{x}{z^2}\sum_q e_q^2\int d^2\boldsymbol{p}_T d^2\boldsymbol{K}_T\delta^{(2)}(\boldsymbol{p}_T+\boldsymbol{K}_T/z-\boldsymbol{P}_{hT}/z) \\
&\times\left[\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{K}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T-\boldsymbol{K}_T\cdot\boldsymbol{p}_T}{MM_h\cdot z}\right]h_1^{\perp,q}\left(x,p_T^2\right)H_1^{\perp,q}\left(z,\frac{K_T^2}{z^2}\right) \\
&= \frac{x}{z^2}\sum_q e_q^2\int d^2\boldsymbol{p}_T d^2\boldsymbol{K}_T\int\frac{d^2b}{(2\pi)^2}e^{-i(\boldsymbol{p}_T+\boldsymbol{K}_T/z-\boldsymbol{P}_{hT}/z)\cdot\boldsymbol{b}} \\
&\times\left[\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{K}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T-\boldsymbol{K}_T\cdot\boldsymbol{p}_T}{MM_h\cdot z}\right]h_1^{\perp,q}\left(x,p_T^2\right)H_1^{\perp,q}\left(z,\frac{K_T^2}{z^2}\right) \\
&= \frac{x}{z^3}\sum_q e_q^2\int\frac{d^2b}{(2\pi)^2}e^{i\boldsymbol{P}_{hT}\cdot\boldsymbol{b}/z}(2\hat{h}_\alpha\hat{h}_\beta-g_{\alpha\beta}^\perp)\tilde{h}_1^{\perp,q/p,\alpha}(x,b)\tilde{H}_{1,h/q}^{\perp,\beta}(z,b). \quad (19)
\end{aligned}$$

Here, the Boer–Mulders function and Collins function in b space can be obtained as

$$\tilde{h}_1^{\perp,q/p,\alpha}(x,b;Q) = \int d^2\boldsymbol{p}_T e^{-i\boldsymbol{p}_T\cdot\boldsymbol{b}}\frac{\boldsymbol{p}_T^\alpha}{M}h_1^{\perp,q}\left(x,p_T^2\right), \quad (20)$$

$$\tilde{H}_{1,h/q}^{\perp,\beta}(z,b;Q) = \int d^2\boldsymbol{K}_T e^{-i\boldsymbol{K}_T\cdot\boldsymbol{b}/z}\frac{\boldsymbol{K}_T^\beta}{M_h}H_{1,h/q}^\perp(z,\frac{K_T^2}{z^2}), \quad (21)$$

where α,β are the uncontracted spatial index of the momentum space [41]. Therefore, the transverse-momentum-dependent $\cos 2\phi_h$ azimuthal asymmetry can be rewritten as:

$$A_{UU}^{\cos 2\phi_h} = \frac{\sigma_0(y,Q^2)}{\sigma_0(y,Q^2)}\frac{2(1-y)}{1+(1-y)^2}\frac{\frac{x}{z^3}\sum_q e_q^2\int\frac{d^2b}{(2\pi)^2}e^{i\boldsymbol{P}_{hT}\cdot\boldsymbol{b}/z}(2\hat{h}_\alpha\hat{h}_\beta-g_{\alpha\beta}^\perp)\tilde{h}_1^{\perp,q/p,\alpha}(x,b)\tilde{H}_{1,h/q}^{\perp,\beta}(z,b)}{\frac{x}{z^2}\sum_q e_q^2\int\frac{d^2b}{(2\pi)^2}e^{i\boldsymbol{P}_{hT}\cdot\boldsymbol{b}/z}\tilde{f}_1^{q/p}(x,b)\tilde{D}_1^{h/q}(z,b)}. \quad (22)$$

2.1. TMD Evolution Effects

We establish the framework of the TMD evolution effects that will solve the energy dependence of the TMD PDFs $f_1(x,\boldsymbol{p}_T)$, $h_1^\perp(x,\boldsymbol{p}_T)$ and TMD FFs $D_1(z,\boldsymbol{k}_T)$, $H_1^\perp(z,\boldsymbol{k}_T)$ in this subsection. The evolution effects for the TMDs are performed in b space; the TMD physical observables that can be measured experimentally will be recovered after performing the reverse Fourier transformation from b space to the transverse momentum space. Thus the b behavior of the TMDs is essential for studying the TMD observables.

TMD PDF $\tilde{F}(x,b)$ and TMD FF $\tilde{D}(z,b)$ in b space actually have two energy scale dependencies μ and ζ_F (ζ_D) according to TMD factorization. The scale μ is the renormalization scale related to the corresponding collinear PDFs/FFs, while ζ_F (ζ_D) is the energy scale that serves as a cut-off point to regularize the light-cone singularity in the operator definition of the TMDs. The $\zeta_F(\zeta_D)$ dependence of the TMDs is encoded in the Collins–Soper (CS) equation [30,31]:

$$\frac{\partial\ln\tilde{F}(x,b;\mu,\zeta_F)}{\partial\ln\sqrt{\zeta_F}} = \frac{\partial\ln\tilde{D}(z,b;\mu,\zeta_D)}{\partial\ln\sqrt{\zeta_D}} = \tilde{K}(b;\mu), \quad (23)$$

while the μ dependence is derived from the renormalization group equation:

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(\alpha_s(\mu)), \tag{24}$$

$$\frac{d\ln\tilde{F}(x,b;\mu,\zeta_F)}{d\ln\mu} = \gamma_F\left(\alpha_s(\mu);\frac{\zeta_F^2}{\mu^2}\right), \tag{25}$$

$$\frac{d\ln\tilde{D}(z,b;\mu,\zeta_D)}{d\ln\mu} = \gamma_D\left(\alpha_s(\mu);\frac{\zeta_D^2}{\mu^2}\right), \tag{26}$$

where α_s is the running strong coupling, $\tilde{K}$ is the CS evolution kernel, and γ_K, γ_D and γ_F are anomalous dimensions. Hereafter, we set $\mu = \sqrt{\zeta_F} = \sqrt{\zeta_D} = Q$; then the TMD PDFs and FFs can be written as $\tilde{F}(x,b;Q)$ and $\tilde{D}(z,b;Q)$.

Solving these TMD evolution equations, the general form of the solution for the energy dependence of TMDs can be written as

$$\tilde{F}_{q/p}(x,b;Q) = \mathcal{F}\times e^{-S}\times\tilde{F}_{q/p}(x,b;\mu_B), \tag{27}$$

$$\tilde{D}_{h/q}(z,b;Q) = \mathcal{D}\times e^{-S}\times\tilde{D}_{h/q}(z,b;\mu_B), \tag{28}$$

with $\mathcal{F}$ and $\mathcal{D}$ being the hard scattering factors that depend on the factorization schemes and S being the Sudakov form factor. Equations (27) and (28) show that the energy evolution of TMDs from the initial energy μ to another energy Q can be realized through the Sudakov form factor S by the exponential form $\exp(-S)$.

Studying the b dependence of the TMDs can provide useful information regarding the transverse momentum dependence of the hadronic 3D structure through Fourier transformation, which makes understanding the b dependence quite important. In the small-b region ($b \ll 1/\Lambda_{\rm QCD}$), the b dependence of TMDs is perturbatively calculable, while in the large-b region, the dependence turns non-perturbative and must be obtained from the experimental data due to the lack of non-perturbative calculation. To combine the information about the b dependence from the perturbative calculation valid at the small-b region with the non-perturbative part at the large-b region, a matching procedure must be introduced, with a parameter b_{max} serving as the boundary between the two regions. Then, a b-dependent function b_* is defined with the properties that $b_* \approx b$ at small-b values and b_* is not larger than b_{max} at large-b values:

$$b_* = \frac{b}{\sqrt{1+b^2/b_{max}^2}}\ ,\ b_{max} < 1/\Lambda_{\rm QCD}\,, \tag{29}$$

which was introduced in the original CSS prescription [30]. The typical value of b_{max} is chosen to be around 1.5 GeV^{-1} to ensure that b_* is always in the perturbative region.

In the small-b region ($1/Q \ll b \ll 1/\Lambda_{\rm QCD}$), the TMDs can be expressed as the convolution of the perturbative hard coefficients and the collinear counterpart at fixed energy μ_B, which are the collinear PDFs/FFs or the multi-parton correlation function ($\mu_B = c_0/b_*$, with $c_0 = 2e^{-\gamma_E}$, the Euler constant $\gamma_E \approx 0.577$, and $\otimes$ denotes the convolution in the momentum fraction x):

$$\tilde{F}_{q/p}(x,b;\mu_B) = C_{q\leftarrow i}\otimes F_{i/p}(x,\mu_B) \equiv \sum_i\int_x^1\frac{d\xi}{\xi}C_{q\leftarrow i}\left(\frac{x}{\xi},\mu_B\right)F_{i/p}(\xi,\mu_B), \tag{30}$$

$$\tilde{D}_{h/q}(z,b;\mu_B) = \hat{C}_{j\leftarrow q}\otimes D_{h/j}(z,\mu_B) \equiv \sum_j\int_z^1\frac{d\xi}{\xi}\hat{C}_{j\leftarrow q}\left(\frac{z}{\xi},\mu_B\right)D_{h/j}(\xi,\mu_B), \tag{31}$$

where C represents the hard scattering coefficient and depends on the studied processes, and the summation over i runs over all parton flavors. Combining all of the above information, the expressions of the TMD distribution function and the fragmentation function have the form:

$$\tilde{F}_{q/p}(x,b;Q) = \mathcal{F} \times e^{-S} \times \sum_i C_{q\leftarrow i} \otimes F_{i/p}(x,\mu_B), \tag{32}$$

$$\tilde{D}_{h/q}(z,b;Q) = \mathcal{D} \times e^{-S} \times \sum_j \hat{C}_{j\leftarrow q} \otimes D_{h/j}(z,\mu_B). \tag{33}$$

Introducing the prescription in Equation (29) will separate the Sudakov form factor S into the perturbatively calculable part $S_{\rm pert}(Q;b_*)$ and the non-perturbative part $S_{\rm NP}(Q;b)$:

$$S(Q;b) = S_{\rm pert}(Q;b_*) + S_{\rm NP}(Q;b). \tag{34}$$

According to the intensive studies in refs. [42–45], the perturbative part of the Sudakov form factor $S_{\rm pert}(Q;b_*)$ can be expanded as the series of $(\frac{\alpha_s}{\pi})^n$:

$$S_{\rm pert}(Q;b_*) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A(\alpha_s(\bar{\mu}))\ln(\frac{Q^2}{\bar{\mu}^2}) + B(\alpha_s(\bar{\mu}))\right], \tag{35}$$

with the coefficients A and B as

$$A = \sum_{n=1}^{\infty} A^{(n)}(\frac{\alpha_s}{\pi})^n, \tag{36}$$

$$B = \sum_{n=1}^{\infty} B^{(n)}(\frac{\alpha_s}{\pi})^n. \tag{37}$$

Within the scope of this work, we list $A^{(n)}$ to $A^{(2)}$ and $B^{(n)}$ to $B^{(1)}$ [30,34,42,44,46,47]:

$$A^{(1)} = C_F, \tag{38}$$

$$A^{(2)} = \frac{C_F}{2}\left[C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{10}{9}T_R n_f\right], \tag{39}$$

$$B^{(1)} = -\frac{3}{2}C_F, \tag{40}$$

where $C_F = \frac{4}{3}$, $T_R = \frac{1}{2}$, $C_A = 3$, and $n_f = 5$. The values of the strong coupling α_s are obtained at 2-loop order as an approximation:

$$\alpha_s\left(Q^2\right) = \frac{12\pi}{\left(33-2n_f\right)\ln\left(Q^2/\Lambda^2_{QCD}\right)}\left\{1 - \frac{6\left(153-19n_f\right)}{\left(33-2n_f\right)^2}\frac{\ln\ln\left(Q^2/\Lambda^2_{QCD}\right)}{\ln\left(Q^2/\Lambda^2_{QCD}\right)}\right\}, \tag{41}$$

with fixed $n_f = 5$ and $\Lambda_{QCD} = 0.225$ GeV. Inspired by refs. [46,48], a widely used parametrization of S_{NP} for TMDs at the initial energy $Q_0^2 = 2.4\ \text{GeV}^2$ is proposed [46,49]:

$$S_{\rm NP}^{\rm pdf/ff} = b^2\left(g_1^{\rm pdf/ff} + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right). \tag{42}$$

The 1/2 factor in front of g_2 comes from the fact that only one hadron is involved for the parametrization of $S_{NP}^{\rm pdf/ff}$. The parameter $g_1^{\rm pdf/ff}$ depends on the type of TMDs and can be regarded as the width of the intrinsic transverse momentum for TMDs at the initial energy scale Q_0 [34,47]. Assuming that the dependence of the transverse momentum follows Gaussian form, g_1 can be obtained as

$$g_1^{\mathrm{pdf}} = \frac{\langle p_\perp^2 \rangle_{Q_0}}{4}, \qquad g_1^{\mathrm{ff}} = \frac{\langle k_\perp^2 \rangle_{Q_0}}{4z^2}, \tag{43}$$

where $\langle p_\perp^2 \rangle_{Q_0}$ and $\langle k_\perp^2 \rangle_{Q_0}$ represent the averaged intrinsic transverse momenta squared for TMDs at initial scale Q_0. We adopt the value of g_2 to be $g_2 = 0.184$ [45]. Thus, the non-perturbative Sudakov form factors for PDF and FF have the following form:

$$S_{\mathrm{NP}}^{\mathrm{pdf}}(Q;b) = \frac{g_2}{2} \ln\left(\frac{Q}{Q_0}\right) b^2 + g_1^{\mathrm{pdf}} b^2, \tag{44}$$

$$S_{\mathrm{NP}}^{\mathrm{ff}}(Q;b) = \frac{g_2}{2} \ln\left(\frac{Q}{Q_0}\right) b^2 + g_1^{\mathrm{ff}} b^2. \tag{45}$$

Combining everything mentioned above, the scale-dependent TMD PDFs and FFs in b space can be rewritten as

$$\tilde{F}_{q/p}(x,b;Q) = e^{-\frac{1}{2}S_{\mathrm{Pert}}(Q;b_*) - S_{\mathrm{NP}}^{\mathrm{pdf}}(Q;b)} \mathcal{F}(\alpha_s(Q)) \sum_i C_{q\leftarrow i} \otimes F_{q/p}(x,\mu_B), \tag{46}$$

$$\tilde{D}_{h/q}(z,b;Q) = e^{-\frac{1}{2}S_{\mathrm{Pert}}(Q;b_*) - S_{\mathrm{NP}}^{\mathrm{ff}}(Q;b)} \mathcal{D}(\alpha_s(Q)) \sum_j \hat{C}_{j\leftarrow q} \otimes D_{h/q}(z,\mu_B). \tag{47}$$

The TMD distribution functions and the fragmentation functions can be obtained by Fourier transformation from b space back to $\boldsymbol{k}_T$ space.

2.2. The Structure Functions

In this subsection, we present the detailed formalism of the structure functions F_{UU} and $F_{UU}^{\cos 2\phi}$, which are the denominator and numerator, respectively, of the $\cos 2\phi_h$ asymmetry.

We performed the Fourier transformation to obtain F_{UU} in b space in Equation (16); after solving the evolution equations for the TMDs, we derived the TMDs in b space in Equations (46) and (47). Combing Equations (16), (46) and (47), one can have the specific expression for the unpolarized structure function as:

$$\begin{aligned} F_{UU}(Q;P_{hT}) &= \frac{x}{z^2} \sum_q e_q^2 \int \frac{d^2b}{(2\pi)^2} e^{i\boldsymbol{P}_{hT}\cdot \boldsymbol{b}/z} \tilde{f}_1^{q/p}(x,b) \tilde{D}_1^{h/q}(z,b) \\ &= \frac{x}{z^2} \sum_q e_q^2 \int_0^\infty \frac{bdb}{(2\pi)} J_0\left(\frac{P_{hT}b}{z}\right) e^{-S_{\mathrm{pert}}(Q;b_*) - S_{\mathrm{NP}}^{\mathrm{SIDIS}}(Q;b)} \\ &\quad \times \mathcal{F}(\alpha_s(Q)) \sum_i C_{q\leftarrow i} \otimes f_1^{i/p}(x,\mu_B) \mathcal{D}(\alpha_s(Q)) \sum_j \hat{C}_{j\leftarrow q} \otimes D_1^{h/j}(z,\mu_B) \\ &= \frac{x}{z^2} \sum_q e_q^2 \int_0^\infty \frac{bdb}{(2\pi)} J_0\left(\frac{P_{hT}b}{z}\right) e^{-S_{\mathrm{pert}}(Q;b_*) - S_{\mathrm{NP}}^{\mathrm{SIDIS}}(Q;b)} f_1^{q/p}(x,\mu_B) D_1^{h/q}(z,\mu_B). \end{aligned} \tag{48}$$

Here, we adopt the LO results of the C coefficients, i.e., we take $C_{q\leftarrow i} = \delta_{qi}\delta(1-x)$, $\hat{C}_{j\leftarrow q} = \delta_{jq}\delta(1-x)$ and take the hard factor $\mathcal{F} = 1, \mathcal{D} = 1$. The non-perturbative Sudakov form factor $S_{NP}^{SIDIS}(Q;b)$ is a combination of the unpolarized distribution function part and the unpolarized fragmentation function part:

$$S_{\mathrm{NP}}^{\mathrm{SIDIS}}(Q;b) = S_{\mathrm{NP}}^{\mathrm{f_1}}(Q;b) + S_{\mathrm{NP}}^{\mathrm{D_1}}(Q;b) = g_2 \ln\left(\frac{Q}{Q_0}\right) b^2 + g_1^{\mathrm{f_1}} b^2 + g_1^{\mathrm{D_1}} b^2, \tag{49}$$

where $g_1^{\mathrm{f_1}}$ and $g_1^{\mathrm{D_1}}$ are obtained from Equation (43).

Similarly, we can have the expression for the structure function $F_{UU}^{\cos 2\phi_h}(Q; P_{hT})$. The Boer–Mulders function and the Collins function in the small-b region are also written as the convolution of the correlation function and hard scattering coefficient as

$$\tilde{h}_{1,q/p}^{\perp,\alpha}(x, b; Q) = (\frac{-ib^\alpha}{2})e^{-\frac{1}{2}S_{\rm Pert}(Q;b_*)-S_{\rm NP}^{h_1^\perp}(Q;b)}T_{q/p,F}^{(\sigma)}(x, x; \mu_B), \tag{50}$$

$$\tilde{H}_{1,h/q}^{\perp,\beta}(z, b; Q) = (\frac{ib^\beta}{2})e^{-\frac{1}{2}S_{\rm Pert}(Q;b_*)-S_{\rm NP}^{H_1^\perp}(Q;b)}\hat{H}_{h/j}^{(3)}(z, \mu_B), \tag{51}$$

where the LO result of the C coefficient is used and the hard scattering coefficient $\mathcal{H}_{Collins}(\alpha_s(Q))$ is taken as 1. Here, $T_{q/H,F}^{(\sigma)}(x, x; \mu)$ is the chiral-odd twist-3 quark–gluon–quark correlation function and is related to the first-p_T moment of the Boer–Mulders function $h_{1,q/H}^{\perp(1)}$ by:

$$T_{q/H,F}^{(\sigma)}(x, x; \mu_B) = \int d^2\boldsymbol{p}_T \frac{\boldsymbol{p}_T^2}{M} h_{1,q/H}^{\perp}(x, \boldsymbol{p}_T; \mu_B) = 2M h_{1,q/H}^{\perp(1)}(x). \tag{52}$$

Furthermore, $\hat{H}_{h/q}^{(3)}(z, \mu_B)$ is related to the first-k_T moment of the Collins function as [50]

$$\hat{H}_{h/j}^{(3)}(z) = z^2 \int d^2\boldsymbol{k}_T \frac{|\boldsymbol{k}_T^2|}{M_h} H_{1,h/j}^{\perp} = 2M_h H_{1,h/j}^{\perp(1)}(z)(z, k_T^2). \tag{53}$$

Combining all the ingredients, the structure function $F_{UU}^{\cos 2\phi_h}(Q; P_{hT})$, which is the numerator part of the $cos2\phi_h$ asymmetry, can be rewritten as

$$\begin{aligned} F_{UU}^{\cos 2\phi_h}(Q; P_{hT}) &= \frac{x}{z^3}\sum_q e_q^2 \int \frac{d^2b}{(2\pi)^2} e^{i\boldsymbol{P}_{hT}\cdot\boldsymbol{b}/z}(2\hat{\boldsymbol{h}}_\alpha \hat{\boldsymbol{h}}_\beta - g_{\alpha\beta}^{\perp})\tilde{h}_1^{\perp,q/p}(x, b)\tilde{H}_{1,h/q}^{\perp}(z, b) \\ &= -\frac{x}{z^3}\sum_q e_q^2 \int_0^\infty \frac{b^3 db}{(8\pi)} J_2(\frac{P_{hT}b}{z}) e^{-S_{\rm pert}(Q;b_*)-S_{NP\,Collins}^{\rm SIDIS}(Q;b)} \\ &\times T_{q/H,F}^{(\sigma)}(x, x; \mu_B) H_{h/j}^{(3)}(z, \mu_B), \end{aligned} \tag{54}$$

where the non-perturbative Sudakov form factor $S_{NP}^{\rm SIDIS}(Q; b)$ is the combination of the one for the Boer–Mulders function and the one for the Collins function:

$$\begin{aligned} S_{NP}^{\rm SIDIS}(Q; b) &= S_{NP}^{h_1^\perp}(Q; b) + S_{NP}^{H_1^\perp}(Q; b) \\ &= g_2 \ln\left(\frac{Q}{Q_0}\right) b^2 + g_1^{h_1^\perp} b^2 + g_1^{H_1^\perp} b^2. \end{aligned} \tag{55}$$

3. Numerical Estimate

In this section, based on the formalism established above, we present the numerical estimate for the $\cos 2\phi_h$ asymmetry $A_{UU}^{\cos 2\phi_h}$ in the charged-kaon- and Λ-hyperon-produced SIDIS process at the kinematics regions of EicC and EIC. We need to utilize the collinear unpolarized distribution function $f_1(x)$ and the collinear unpolarized fragmentation function $D_1(z)$ as the inputs of the evolution, for which we adopt the parameterizations of these functions. For the collinear unpolarized distribution function $f_1(x)$ of the proton, we adopt the LO set of CT10 parameterization [51] (central PDF set), while for the fragmentation function $D_1(z)$ of charged kaon, we apply the leading-order DSS parametrization [52]. For the collinear unpolarized fragmentation function $D_1^\Lambda(z)$ of the Λ hyperon, we adopt the model results of the diquark spectator model [53]:

$$D_1^\Lambda(z) = \frac{g_s^2}{4(2\pi)^2} \frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4 L^2} \left\{ z(1-z)\left((m_q + M_\Lambda)^2 - m_D^2 \right) \times \exp\left(\frac{-2zL^2}{(1-z)\Lambda^2} \right) \right.$$
$$\left. + \left((1-z)\Lambda^2 - 2\left((m_q + M_\Lambda)^2 - m_D^2 \right) \right) \times \frac{z^2 L^2}{\Lambda^2} \Gamma\left(0, \frac{2zL^2}{(1-z)\Lambda^2} \right) \right\}. \quad (56)$$

The value of the free parameters g_s, m_q, m_D as well as L^2 and Λ^2 in Equation (56) is taken from ref. [53], and M_Λ is the mass of the Λ hyperon. Since the model result is obtained at an initial energy of 0.23 GeV^2, we use the QCDNUM evolution package [54] to evolve the unpolarized fragmentation function $D_1(z)$ from the initial energy of 0.23 GeV^2 to another energy scale.

For the first p_T-moment of the Boer–Mulders function for the proton target needed in the calculation, we adopt the parametrization of $h_{1,q/p}^{\perp,(1)}$ from ref. [55] as:

$$h_{1,q/p}^{\perp,(1)}(x) = H_q x^{c^q} (1-x)^b f_1^q(x). \quad (57)$$

Here, the parameters H_q, c^q, and b are free parameters that need to be obtained by fitting the experimental data with the parameterized form using the specific fitting results given in ref. [55]. We should note that there is no parametrization for the $s(\bar{s})$ quark Boer–Mulders function; we assume that it follows $h_{1,s(\bar{s})/p}^{\perp q,(1)}(x) = H_{\bar{d}} x^{c^{\bar{d}}} (1-x)^b f_1^{s(\bar{s})}(x)$ due to $s(\bar{s})$ and $\bar{d}$ both being the sea quark of the proton. The collinear correlation function $\hat{H}_{h/q}^{(3)}(z)$ can be obtained from the first k_T-moment of the Collins function for kaon and Λ. The kaon Collins function is extracted from the semi-inclusive hadron pair production in e^+e^- annihilation, which turns out to be in good agreement with the measurements performed by the HERMES [56,57] and COMPASS [58,59] collaborations. $\Delta^N D_{h/q^\uparrow}(z, p_\perp)$ was parameterized in ref. [60], while the Collins function $H_{1,h/j}^\perp(z, p_\perp)$ and $\Delta^N D_{h/q^\uparrow}(z, p_\perp)$ have the following relationship:

$$H_{1,h/j}^\perp(z, p_\perp) = \frac{zM_h}{2|p_\perp|} \Delta^N D_{h/q^\uparrow}(z, p_\perp); \quad (58)$$

one can obtain the expression for $\widehat{H}_{h/j}^{(3)}(z_h)$ as:

$$\hat{H}_{h/j}^{(3)}(z) = \frac{\sqrt{2e}}{M_C} \mathcal{N}_q^C(z) D_{h/q}(z) \left(\frac{M_C^2}{M_C^2 + \langle k_\perp^2 \rangle} \right)^2 \left\langle k_\perp^2 \right\rangle. \quad (59)$$

For kaon meson, the free parameters of the favored and disfavored Collins function are extracted as [60]:

$$\mathcal{N}_{fav}^k = 0.41, \quad \mathcal{N}_{dis}^k = 0.08. \quad (60)$$

In Equations (42) and (43), the free parameter g_1 and the universal parameter g_2 contain information about the evolution of TMDs and are the key parameters that determine the evolution of TMDs from one initial energy μ_B to another Q. We adopt the parametrization as $g_1^{h_1^\perp} = \frac{0.57}{4} GeV^2$ $g_1^{H_1^\perp} = \frac{<p_\perp^2>_c}{4z^2}$, with $< p_\perp^2 >_c = \frac{M_c^2 < p_\perp^2 >}{M_c^2 + < p_\perp^2 >}$ to be consistent with the parametrization from ref. [60].

Since there is no parametrization for the Λ Collins function, we adopt the model calculation performed in ref. [53] using the diquark spectator model as:

$$H_1^{\perp(q)}\left(z,k_T^2\right) = \frac{\alpha_s g_D'^2 C_F}{(2\pi)^4} \frac{e^{\frac{-2k^2}{\lambda^2 z^2 (1-2))^{\beta}}}}{z^2(1-z)} \frac{1}{\left(k^2 - m_q^2\right)} \left(H_{1(a)}^{\perp(q)}\left(z,k_T^2\right) + H_{1(b)}^{\perp(q)}\left(z,k_T^2\right) + H_{1(c)}^{\perp(q)}\left(z,k_T^2\right) + H_{1(d)}^{\perp(q)}\left(z,k_T^2\right)\right). \quad (61)$$

In Equations (42) and (43), the free parameter g_1 and the universal parameter g_2 contain information about the evolution of TMDs and are the key parameters that determine the evolution of TMDs from one initial energy μ_B to another Q. Here, we adopt the results given in ref. [45] for the mean transverse momentum squared:

$$\left\langle p_\perp^2 \right\rangle = 0.38\ \text{GeV}^2, \quad \left\langle k_\perp^2 \right\rangle = 0.19\ \text{GeV}^2. \quad (62)$$

For the universal parameter g_2 in the non-perturbative Sudakov form factor, the specific value $g_2 = 0.184$ is also given in ref. [45]. The kinematical region available at the EIC is chosen as follows [22]:

$$0.001 < x < 0.4, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.8,$$
$$1\ \text{GeV}^2 < Q^2, \quad 5\ \text{GeV} < W, \quad \sqrt{s} = 100\ \text{GeV}, \quad P_{hT} < 0.5\ \text{GeV}. \quad (63)$$

For EicC, the following kinematic region is used:

$$0.005 < x < 0.5, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.7,$$
$$1\ \text{GeV}^2 < Q^2 < 200\ \text{GeV}^2, \quad 2\ \text{GeV} < W, \quad \sqrt{s} = 16.7\ \text{GeV}, \quad P_{hT} < 0.5\ \text{GeV}, \quad (64)$$

where $W^2 = (P+q)^2 \approx \frac{1-x}{x}Q^2$ is the invariant mass of the virtual photon–nucleon system. Since TMD factorization is proven to be valid to describe physical observables in the region $(P_{hT} \ll Q, P_{hT} < 0.5\ \text{GeV})$ is chosen to guarantee the validity of TMD factorization. Combining Equations (12), (48) and (54) and the kinematical regions of EIC and EicC, we can calculate the $\cos 2\phi_h$ asymmetry in the charged-kaon- and Λ-hyperon-produced SIDIS process within the EIC and EicC kinematic range.

The results are shown in Figure 2: the upper, middle, and lower panels of the figure depict the numerical results of $\cos 2\phi_h$ asymmetry times 10^5 in the K^+-, K^--, and Λ-hyperon-produced SIDIS process in the kinematic regions of EIC and EicC, respectively. The left, middle, and right panels denote the $\cos 2\phi_h$ asymmetry as functions of x, z, and P_{hT}, respectively. As can be seen from Figure 2, it can be found that $\cos 2\phi_h$ azimuthal asymmetry in the charged-kaon- and Λ-hyperon-produced SIDIS process is relatively very small compared with other asymmetries, such as Sivers asymmetry [41] and Collins asymmetry [61] in all cases in both the EIC and EicC kinematical regions, which shows that the convolution of the Boer–Mulders function may not be the main contribution of the $\cos 2\phi_h$ asymmetries. The magnitude of the $\cos 2\phi_h$ asymmetry may potentially be recognized as consistent with zero. However, the magnitude of the $\cos 2\phi_h$ asymmetry in the K^+-produced process is larger than that of the K^--produced process. The reason is that the constituent quarks of the K^+ meson are u and $\bar{s}$, and those of K^- are $\bar{u}$ and s. The relatively larger contribution of the valence quark Boer–Mulders function leads to the result $A_{UU}^{\cos 2\phi}(\ell p \to \ell' K^+ X) > A_{UU}^{\cos 2\phi}(\ell p \to \ell' K^- X)$. Another observation is that the asymmetry at the configuration of EicC is larger than that at EIC, which may be due to the fact that the configuration of EicC is more sensitive to the sea quark distributions. In the future, high-energy, high-luminosity electron ion colliders can provide a unique opportunity to extract information about the TMD distribution functions and fragmentation functions, clarify their flavor dependence, and disentangle the different contributions.

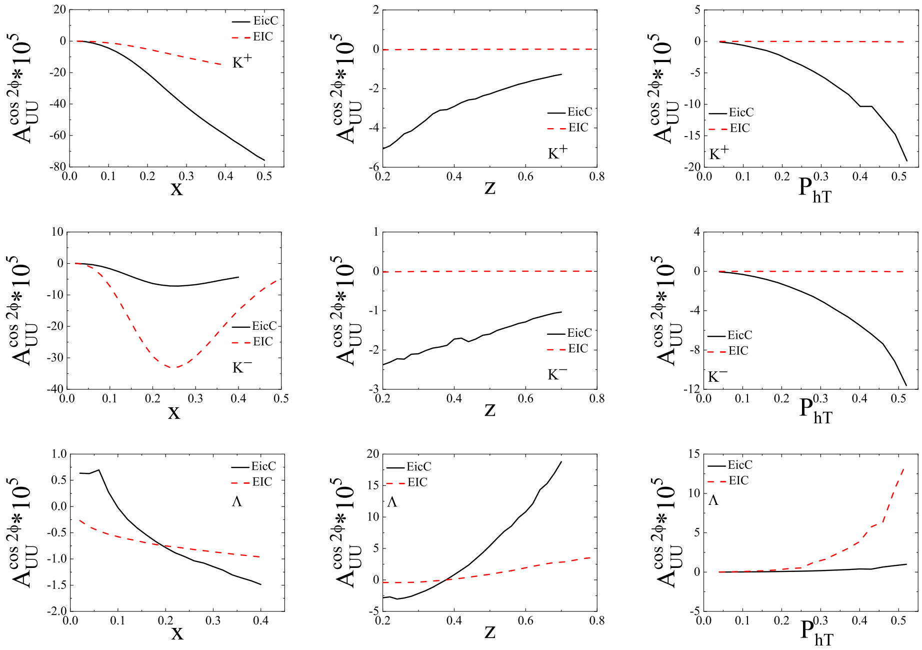

Figure 2. The $\cos 2\phi_h$ asymmetry in the charged-kaon- and Λ-hyperon-produced SIDIS process at the kinematic regions of EIC and EicC as functions of x (left panels), z (middle panels), and P_{hT} (right panels).

4. Conclusions

In this work, we applied the TMD factorization at leading-logarithmic order to study the $\cos 2\phi_h$ asymmetry in the $K^{\pm}$-meson- and Λ-hyperon-produced SIDIS process in the kinematical configurations of EIC and EicC. We present the formalism of SIDIS process $\cos 2\phi_h$ under TMD factorization theorem and study the TMD evolution of the unpolarized proton distribution function and the fragmentation function as well as the Boer–Mulder function and Collins function. We considered the TMD evolution effect of the distribution functions and fragmentation functions that include the Sudakov form factor. The hard scattering coefficient related to the collinear function is taken at leading order. For the Boer–Mulders function of the proton target, the kaon Collins function, we adopted the parametrization for which the TMD evolution effect was considered; for the Λ Collins function, we applied the results from the diquark spectator model. We predict that the $\cos 2\phi_h$ asymmetry in the charged-kaon- and Λ-produced SIDIS process is much smaller than other asymmetries such as the Sivers asymmetry. From this work, we show that the convolution of the Boer–Mulders function and the Collins function in the strange-meson-kaon- and Λ-produced SIDIS process may not be the main contribution of the $\cos 2\phi_h$ asymmetry, which shows the importance of the Cahn effect as well as the perturbative calculation. The measurement of the $\cos 2\phi_h$ asymmetry of semi-inclusive charged kaon and Λ production in future electron ion colliders can provide useful constraints on the different contributors to $\cos 2\phi_h$ asymmetry.

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