

# ANALYSIS OF COUPLED BUNCH INSTABILITIES IN BESSY-VSR\*

M. Ruprecht<sup>†</sup>, P. Goslawski, A. Jankowiak, M. Ries, A. Schällicke, G. Wüstefeld,  
HZB, Berlin, Germany  
T. Weis, TU Dortmund University, Dortmund, Germany

## Abstract

BESSY-VSR, a scheme where 1.5 ps and 15 ps long bunches (rms) can be stored simultaneously in the BESSY II storage ring has recently been proposed [1]. The strong longitudinal bunch focusing is achieved by superconducting high gradient RF cavities. This paper presents investigations of coupled bunch instabilities driven by HOMs of superconducting multi-cell cavities in BESSY-VSR. Analytical calculations and tracking simulations in time domain are performed in the longitudinal and the transverse planes and factors that influence the threshold currents are being discussed. Suitable candidates of cavities which are presently available or in the phase of design are compared with respect to their instability thresholds.

## INTRODUCTION

BESSY-VSR [1] is an upgrade proposal to store short and long bunches simultaneously in the BESSY II storage ring, utilizing two sc 5-cell 1.5 GHz cavities and three sc 4-cell 1.75 GHz cavities to provide alternatingly high and low focusing gradient. Coupled bunch instabilities (CBIs) are an important aspect of beam dynamics, crucial to the stability of operation. This paper presents studies of CBIs, driven by the higher order modes (HOMs)<sup>1</sup> of those cavities.

For this project, highly HOM damped cavities are needed, see [2] for details. In this paper, a recent development from JLab (hereinafter called JLab HC cavity) [3] will be used as a prime example of such a cavity. With a frequency of almost 1.5 GHz and five cells, the cavity is similar to one of the proposed BESSY-VSR cavities. The longitudinal and transverse impedance is depicted in Fig. 1, compared to the present bunch-by-bunch feedback performance of BESSY II with 4 ms<sup>-1</sup> and 1.33 ms<sup>-1</sup> taken for the transverse and longitudinal damping rates respectively [4].

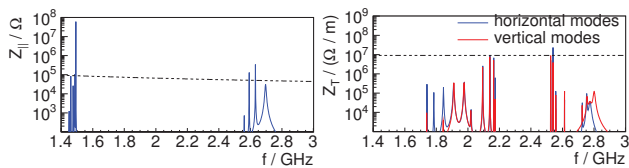


Figure 1: Longitudinal (left) and transverse (right) impedance of the JLab HC cavity compared to present BESSY II feedback performance (dashed line).

\* Work supported by BMBF, partly under contract no. 05K13PEB, and Land Berlin.

<sup>†</sup> martin.ruprecht@helmholtz-berlin.de

<sup>1</sup> In this paper, referring to HOMs includes the same order passband.

A comparison with other designs, namely the Cornell ERL main linac cavity [5] and the bERLinPro main linac cavity [6] is given for the longitudinal case in Fig. 4. The latter two are seven cell designs at a frequency of 1.3 GHz.

## CALCULATION OF CBI

In leading order, HOM driven coupled bunch motion can be described by bunches approximated by point charges that perform dipole oscillations in the longitudinal or transverse planes. In this approximation, the bunches form a system of coupled harmonic oscillators with a driving term given by the wake fields induced by previous bunches in the HOM afflicted cavity. In the frequency domain, wake fields transform into impedances and the solutions can be discussed as small perturbations to the oscillation frequency.

Solutions to the equations of motion are called coupled bunch modes (CBMs). Derivations, both for the case of even fill and uneven fill are presented in [8] and [9] respectively. The growth rate  $\tau^{-1}$  of mode  $\mu$  with its complex angular synchrotron frequency  $\Omega_\mu$  is given by  $\tau^{-1} = \text{Im}(\Omega_\mu - \omega_s)$ , with  $\omega_s$  the unperturbed synchrotron frequency. The case of even fill can readily be solved analytically and the relation of growth rate  $\tau^{-1}$  to the sampled impedance  $Z$  at the frequency  $f$  can be expressed as:

$$\tau^{-1} = \frac{f_{\text{rev}} I}{2E/e} \times \begin{cases} \beta_{x,y} \text{Re}(Z_{x,y}(f)) & \text{transverse} \\ f \alpha \text{Re}(Z_{\parallel}(f)) / f_s & \text{longitudinal} \end{cases} \quad (1)$$

with  $e$  the elementary charge. All other parameters are explained in Table 1, together with the typical values that are used for the calculations in this paper. Throughout this paper, the impedance definitions of [8] are used.

Table 1: BESSY II parameters used for calculation.

Parameter	Value
Energy $E$	1.7 GeV
Momentum compaction $\alpha$	$7.1 \cdot 10^{-4}$
Total beam current $I$	300 mA
Circumference	240 m
Harmonic number $h$	400
RF frequency $f_{\text{rf}}$	500 MHz
Revolution frequency $f_{\text{rev}}$	1.25 MHz
Synchrotron frequency $f_s$	8.0 kHz *
Betatron functions at sc cavities $\beta_{x,y}$	4 m **
Longitudinal radiation damping time $\tau_z$	8 ms
Transverse radiation damping time $\tau_{x,y}$	16 ms

\* Corresponds to the long bunch in BESSY-VSR.

\*\* Conservative number, not minimum of beta function.

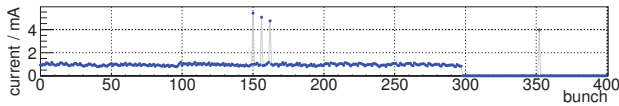


Figure 2: Typical fill pattern at BESSY II with 200 ns gap.

Figure 2 shows the fill pattern that was used for calculations unless otherwise specified. The standard operation of BESSY-VSR is expected to be a setting where at least 75% of the ring is filled with long bunches. In addition, a small number of short bunches will be put in the gap. Thus, the long bunches build up the majority of the total current.

Regarding CBIs, it has to be noted that the synchrotron frequency of the short bunch is about one order of magnitude higher than for the long bunch. As a consequence, the coupling of long and short bunches is suppressed. In addition to the low current in short bunches, the growth rate of longitudinal CBI is reduced by the factor that the synchrotron frequency is higher, making the short bunches less prone to CBI, compare Eq. 1.

In all planes, the growth rates of CBIs depend primarily on the total (average) beam current stored in the machine. The fill pattern is typically a small correction to the expectations from even fill, unless severe deviations from homogeneity are used [7]. In this sense, the fill pattern shown in Fig. 2 is sufficiently realistic to be representative for the desired fill patterns in BESSY-VSR.

## TRACKING CODES

A simple stand-alone tracking code for longitudinal dynamics was written in C++. Each bunch is simulated as a macro particle with a defined charge, allowing for an arbitrary fill pattern. The fundamental interaction of a charged particle with a resonator impedance, such as an HOM or fundamental mode, is calculated by means of phasor addition. Active cavities are implemented as a feedback controlled driven resonator impedance that allows to study transient beam loading in case of uneven fill. Figure 3 shows an example of HOM driven CBI simulated with this tool.

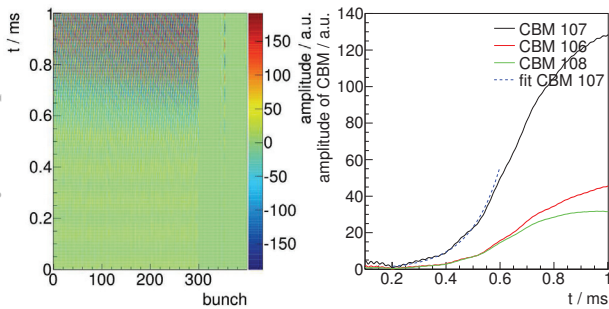


Figure 3: Example of tracking simulation with simple stand-alone code in the longitudinal plane with JLab HC cavity. The strongest HOM is driven almost at its maximum. Left: Longitudinal coordinate of all bunches as a function of time. Right: Longitudinal CBM amplitude vs. time with fit for the strongest mode giving  $\tau^{-1} = 8.9 \text{ ms}^{-1}$ .

The 6D multi particle and multi bunch tracking code mbtrack [10] developed at SOLEIL has recently been extended with the capability of simulating longitudinal long-range resonators [11]. This enables the simulation of HOM driven CBI in a powerful simulation environment.

In this work, a modified version of mbtrack is used. Apart from several smaller changes, the option of transverse long range resonators was added. With this modifications, transverse HOM driven CBI instabilities can be simulated and growth rates extracted. Further studies that combine original functions of mbtrack, such as impedance driven single bunch effects with HOM driven CBI are now possible.

The higher harmonic cavities in BESSY-VSR will offer the possibility of a non-linear longitudinal RF potential for the long bunches, as typically produced with Landau cavities. The central feature is the amplitude dependent synchrotron frequency. Using mbtrack, the dynamics of CBI in such a potential were studied. First findings revealed a counter-productive effect, confirming statements given in [12], namely that the beneficial effect of Landau damping is seen in conjunction with the unfavorable effect of reducing  $f_s$ , compare Eq. 1. Following this thought, increasing  $f_s$  by means of the sc cavities could be helpful too, especially since a higher  $f_s$  also opens up the possibility of faster longitudinal feedback. Hence, it is still an open question as to how the potential for the long bunches should be shaped.

## RESULTS

Table 2 shows solutions of the analytical ansatz described earlier for different fill pattern for the JLab HC cavity, with the frequencies of the HOMs taken as in [3]. This represents only one possible outcome of the fabrication process. The term “hybrid” refers to a proposed fill in BESSY-VSR where 10 consecutive short bunches occupy the center of the gap.

Table 2: Effect of different fill pattern on the growth rates of the JLab HC cavity with a single set of HOM frequencies.

Fill pattern	Horiz.	Vert.	Long.
	Growth rate / $\text{s}^{-1}$		
Uniform	792	945	8806
As in Fig. 2	783	936	8796
Hybrid 100 ns gap	869	946	8802
Hybrid 200 ns gap	865	941	8800

Instability thresholds have been calculated based on Eq. 1 and the parameters in Table 1 for the three cavity designs and different planes in a statistical approach. The result is depicted in Fig. 4, expressed as the fraction of cavities that would result in stable operation, given a maximum growth rate that can be counteracted by damping mechanisms. In other words, for a given feedback damping rate, the probability of stable operation for one cavity can be read out.

As a comparison, the damping rates of the feedback systems at BESSY II are drawn in Fig. 4. With this feedback,

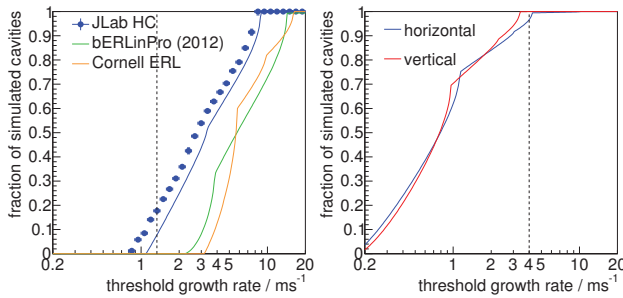


Figure 4: Fraction of simulated cavities that are stable for a given damping rate. Dots and solid lines represent tracking and analytical results respectively. Vertical lines indicate the present feedback performance of BESSY II. Left: Longitudinal plane. Right: Transverse plane.

the probability of stable longitudinal operation would be approximately 10 % (analytical result) for the JLab HC cavity.

The calculation was done in a statistical approach because fabrication uncertainties cause the frequency of each HOM to take a value according to a Gaussian distribution of several MHz spread, centered at the predicted value. This frequency spread is larger than  $f_{\text{rev}}$ , the frequency that defines the spacing at which the impedance is sampled by the beam. Hence, the distribution of HOM frequencies can be considered uniform in an interval of one  $f_{\text{rev}}$ . By calculating the width of each HOM at each threshold growth rate (expressed as  $\text{Re}(Z)$  via Eq. 1) and relating it to the width of the interval defined by  $f_{\text{rev}}$ , the probability of an HOM, and finally a cavity, causing an instability can be obtained.

Additionally, the longitudinal plane for the JLab HC cavity has been studied with the stand-alone tracking code. For this purpose, a statistical sample of 412 cavities has been generated based on the six strongest longitudinal HOMs.  $Q$  and  $R/Q$  have been kept unchanged while the frequency has been randomly shifted according to a Gaussian distribution with a spread of 5 MHz to account for the fabrication uncertainty. Each cavity was tracked for different levels of arbitrarily assumed damping performance of the ring, e.g. provided by a bunch-by-bunch feedback. The crossing of the threshold was determined if the energy deviation of any bunch has surpassed approximately three times the natural energy spread within the first 20k turns. This criterion is arbitrarily chosen, ensuring reasonably fast simulation. However, it tends to underestimate the threshold because the instability may have grown insufficiently large within the limited number of turns. This, and the fact that another fill pattern was used explains some of the deviation between tracking and analytical results in Fig. 4.

## DISCUSSION

Firstly, the results show that likely no restrictions on the fill pattern have to be made. It shall be noted, that the effect of transient beam loading caused by uneven fill, such as a bunch-to-bunch spread of  $f_s$ , is not discussed here.

Secondly, the transverse plane appears less critical than the longitudinal plane, in agreement with experiences at existing machines. Growth rates in the transverse planes are

generally lower and the feedback is significantly faster. Figure 4 right panel suggests a good chance of stability with the present feedback performance of BESSY II.

The longitudinal plane appears to be the critical one. The cavities considered in this paper do not give a good chance for stable operation with the present feedback performance of BESSY II, see Fig. 4 left panel, even if the fact of a reduced  $f_s$  in the measurements of [4] is considered. An improved cavity design in combination with improvements in the feedback systems is therefore recommended.

In addition, HOM detuning could be used to prevent strong HOMs from being driven, hence significantly improving the situation in Fig. 4.

## CONCLUSIONS

Tracking codes for HOM driven CBI have been developed and extended, showing agreement to analytical calculations and opening up possibilities for further studies.

Current designs of highly HOM damped sc multi-cell cavities have been investigated with respect to CBI for the purpose of BESSY-VSR. The expected growth rates of transverse CBI seem to be under control with the present feedback system of BESSY II. Longitudinal CBIs, however, seem to be more troublesome. The present feedback performance is likely not sufficient to guarantee stable operation.

While further improvements in the transverse feedback systems would be straight forward, improvements in the longitudinal feedback system are becoming increasingly challenging. For the latter, a factor of two appears realistic with reasonable efforts, but the question whether and how improvements beyond this are feasible has to be investigated.

Both, an improved cavity design, that is optimized for the purpose of BESSY-VSR, as described in [2] and improvements in the feedback systems are recommended in order to control CBIs in BESSY-VSR.

## REFERENCES

- [1] G. Wüstefeld et al., IPAC 11, San Sebastián, Spain, p. 2936.
- [2] A. Neumann et al., IPAC 14, WEPRI008, these proceedings.
- [3] F. Marhauser et al., “JLab High-Current Cryomodule Development”, ERL 09, Ithaca, New York, USA.
- [4] A. Schlicke et al., IPAC 14, TUPRI072, these proceedings.
- [5] C. Song et al., PAC 07, Albuquerque, USA, p. 1227.
- [6] B. Riemann, “Status and design and HOM calculation for the BERLinPro main linac cavity”, HOMSC12, Daresbury, UK.
- [7] M. H. Wang et al., PAC 01, Chicago, USA, p. 1981.
- [8] A. Wu Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators*, John Wiley & Sons, 1993.
- [9] K. Y. Ng, USPAS Lec. Notes, Los Angeles, USA, Jan. 2002.
- [10] R. Nagaoka et al., PAC 09, Vancouver, BC, Canada, p. 4637.
- [11] M. Klein et al., IPAC 13, Shanghai, China, p. 885.
- [12] A. Mosnier, PAC 99, New York City, USA, p. 628.