SLAC-TN-74-8
Z. D. Farkas
July 1974

Average Power Dissipated in SLED Cavities

In this note, the average power dissipated in the SLED cavities per unit incident average power and the division of dissipated power between cylinder wall and end plates, will be derived.

The energy dissipated by cavities during a single pulse, U, is given by

$U = \int_{\text{over the pulse}} P_d(t)dt$ where $P_d(t)$ is the instantaneous dissipated power, and is given by $P_d(t) = P_e(t)/\beta = E_e^2(t)/\beta$. Using the expression for $E_e(t)$ from Ref. 1,* we obtain

$$P_d(A) = P_{ds}(1 - \epsilon^{-\tau})^2$$

$$P_d(B) = P_{ds}\left[\eta\epsilon^{-(\tau - \tau_1)} - 1\right]^2$$

$$P_d(C) = P_{ds}\left[\eta\epsilon^{-(\tau_2 - \tau_1)} - 1\right]^2 \epsilon^{-2(\tau - \tau_2)}$$

where β is the coupling coefficient, τ is time normalized to the cavity constant, T_c, $\tau = t/T_c$, t_1 is the time when the 180^0 phase shift is applied, t_2 is the time when the pulse ends, P_{ds} is the steady-state dissipated power in the cavity and is equal to $4\beta/(1 + \beta)^2$, $\eta = 2 - \epsilon^{-\tau_1}$.

Integrating $P_d(t)$ in the three regions A, B, C, we obtain

$$U(A) = P_{ds}T_c\left[\tau_1 + 2\epsilon^{-\tau_1} - 1/2\epsilon^{-2\tau_1} - 3/2\right]$$

$$U(B) = P_{ds}T_c\left[(\tau_2 - \tau_1) - 2\eta + \eta^2/2 + 2\eta\epsilon^{-(\tau_2 - \tau_1)} - (\eta^2/2)\epsilon^{-2(\tau_2 - \tau_1)}\right]$$

$$U(C) = P_{ds}T_c\left[\eta\epsilon^{-(\tau_2 - \tau_1)} - 1\right]^2/2$$

---

*1. "SLED: A Method of Doubling SLAC's Energy," Z.D. Farkas et al., SLAC-PUB-1453.

The total energy dissipated per pulse, $U = U(A) + U(B) + U(C)$, can be written as $U = P_{ds}T_cF$, since $P_{ds}T_c$ can be factored out from the expression for U. For a given network, F is a function of t_1 and t_2. The average power dissipated is $P_{dav} = rU = rP_{ds}T_cF$, where r is the pulse repetition rate. A peak incident power of unity was assumed and therefore the above expression for average power has to be multiplied by the actual peak incident power.

The average dissipated power normalized to the average incident power, P'_{dav} is given by

$$P'_{dav} = P_{ds}T_cF/t_2 = P_{ds}F/\tau_2$$

If there were no transients, then F/τ_2, which is a time factor that accounts for the condition that the cavities are generally not completely filled during the pulse, would equal unity, and P'_{dav} would equal P_{ds}.

If the 180^o phase shift is not actuated then we have only two regions, A and C. The above expressions for U(A) and U(C) hold if we substitute τ_2 for τ_1. Thus, with no 180^o phase shift, the expressions for the dissipated energy are

$$U(A) = P_{ds}T_c(\tau_2 - 2\epsilon^{-\tau_2} - 1/2\,\epsilon^{-\tau_2} - 3/2)$$

$$U(C) = P_{ds}T_c(\eta - 1)^2/2$$

For 30 MW incident power, 5 μsec pulse, 180 pulses per second the average incident power is 27 KW. The normalized dissipated power can be obtained by measuring the difference in db of the average power output of the SLED network with the cavities tuned and then with the cavities detuned.

The division of dissipated power between end plates and the cylinder wall of a TE_{01n} cavity is obtained as follows. The power in one end plate, P_e is proportional to the integral over its surface of the square of the magnetic field on its surface, and similarly the power dissipated on the cylinder wall, P_w, is proportional to the integral over its surface of the square of the magnetic field at its surface. The magnetic fields at either end plate, ($z = 0$, or $z = L$), H_{ro}, and at the cylindrical surface ($r = a$), H_{za} are given respectively by

$$H_{ro} = (n\pi/2r_{o1})(D/L)\, J_1(r_{o1}r/a); \quad H_{za} = J_o(r_{o1})\sin(n\pi z/L)$$

Thus, $P_e \sim (n\pi/2r_{o1})^2(D/L)^2 \int_o^a 2\pi 4 J_1^2\,(r_{o1}r/a)dr; \; P_e \sim (n\pi/2r_{o1})^2(D/L)^2(D^2/8)J_2(r_{o1})$

$$P_w \sim \pi D J_o^2(r_{o1}) \int_o^L \sin^2(n\pi z/L)dz \sim (\pi DL/2)\, J_o^2(r_{o1})$$

Thus,

$$P_e/P_w \equiv k = (n^2\pi^2/8r_{o1}^2)(D/L)^3 \left[J_2^2(r_{o1})/J_o^2(r_{o1})\right] = (n^2\pi^2/8r_{o1}^2)(D/L)^3$$

since $\qquad J_2^2(r_{o1}) = J_o^2(r_{o1})$.

If the total power dissipated in the two cavities is P_d, then $P_d = 2(2P_e + P_w)$ and $P_e = P_d/2\,(2 + 1/k)$, $P_w = P_d/2(2k + 1)$. For the SLED cavities $n = 5$, $D/L = .48$, hence $P_e = P_d/8.2$, $P_w = P_d/3.92$.

The table below lists the normalized and actual average power dissipated in the SLED cavities for several combinations of t_1 and t_2 .

TABLE I.

t_1 μsec	t_2 μsec	P'_{dav}	Actual Power KW	Actual Power in one end plate KW	Actual Power in one cylinder KW
4.2	5	.24	6.5	.79	1.66
5	5	.37	10.0	1.22	2.55
1.7	2.5	.08	2.2	.27	.56
2.5	2.5	.26	6.9	.84	1.76
3	4	.16	4.3	.52	1.10
4	4	.33	9.0	.49	2.3

Note : $t_1 = t_2$ implies no 180^o phase shift.