SOME CONSEQUENCES OF UNITARY SYMMETRY FOR PROCESSES INVOLVING ω, ϕ AND f^0 MESONS

B.L. Ioffe, I.Yu. Kobzarev and I.Ya. Pomeranchuk

Institute of Theoretical and Experimental Physics,
State Committee of Atomic Energy of the USSR

(Presented by I.Yu. KOBZAREV)

In the scheme of SU_3 unitary symmetry it is usually assumed [1, 2] that the states of ω and ϕ mesons are a mixture of two states: a unitary singlet ψ_1 and a state with $T=0$ and $Y=0$ of a unitary octet ψ_8 i.e.,

$$\left.\begin{aligned}\omega &= \alpha\psi_8 + \sqrt{1-\alpha^2}\,\psi_1,\\ \varphi &= \sqrt{1-\alpha^2}\,\psi_8 - \alpha\psi_1.\end{aligned}\right\} \tag{1}$$

The mixing parameter α, determined by Okubo [1] on the basis of the experimentally observed masses of the ω and ϕ mesons and the theoretical mass of the state $T=0$, $Y=0$ of a unitary octet, obtained from the Gell-Mann–Okubo mass formula, was found to be about 0.64.

Let us consider in the scheme of unitary symmetry decays of some resonant state $A \to C + \phi$ and $A \to C + \omega$. We will assume that these decays are allowed in the SU_3-scheme and that the states A and C refer to representations of different dimensions of the SU_3-group. It is then obvious that only the state of the unitary octet from expression (1) can make a contribution to the matrix element of the transition and, consequently, when taking into account only SU_3-invariant interactions, the ratio of the squares of the matrix elements of the transitions $A \to C + \phi$ and $A \to C + \omega$ will be

$$\frac{|M_{A\to C+\varphi}|^2}{|M_{A\to C+\omega}|^2} = \frac{1-\alpha^2}{\alpha^2}\,. \tag{2}$$

If the energy released in the decays $A \to C + \phi$ and $A \to C + \omega$ are fairly small, it is necessary to allow for the mass difference between the ω and the ϕ mesons. This can be done by writing the matrix element in the form $M_{\phi,\,\omega} = ak^l_{\phi,\,\omega}$, where l is the orbital momentum of the system $C + \omega$, or $C + \phi$, k_ϕ and k_ω are respectively the momenta of the ϕ and ω mesons in the rest system of the particle A. The ratio of the probabilities of the processes $A \to C + \phi$ and $A \to C + \omega$ will then be

$$\frac{w(A\to C+\varphi)}{w(A\to C+\omega)} = \frac{1-\alpha^2}{\alpha^2}\left(\frac{k_\varphi}{k_\omega}\right)^{2l+1}. \tag{3}$$

If A and C refer to representations of a unitary group of the same dimension, it is obvious that no conclusions can be drawn as to the ratio $w(A \to C + \phi)/w(A \to C + \omega)$. Relation (3) makes it possible to investigate the scheme of unitary symmetry in various aspects. If the dimension of the representations to which the particles A and C belong is known, then with the aid of expression (3) it is possible to check the hypothesis that ϕ and ω are described by a mixture of a unitary singlet and a unitary octet. If the dimensions of the representations are not known, then on the basis of the experimentally measured ratio $w(A \to C + \phi)/$ $/w(A \to C + \omega)$ and expression (3) certain information on these dimensions can be obtained.

An example of a reaction of the type under consideration is the decay of a B meson with a mass of 1220 MeV : $B \to \pi + \omega$ [3]. According to experimental data obtained in [3]

$$R_B = w(B \to \pi + \varphi)/w(B \to \pi + \omega) < \\ < 0.2 \pm 0.1^*,$$

whereas from (3) for $l = 0$ it follows that

$$[(1 - \alpha^2)/\alpha^2]\,[k_\varphi/k_\omega] = 0.58.$$

From a comparison of these two figures it follows that, if the B meson has a spin and parity 1^+, it should belong to an octet. This conclusion contradicts the scheme proposed by V.V.Vladimirskii [4], in which a B meson with $I^P = 1^+$ occurs in a 27-supermultiplet. If the spin and parity of the B meson are $I^P = 1^-$ (or 2^-) [5], then $l = 1$ and, according to (3), $R_B \approx 0.1$. Such a value for the ratio R_B does not contradict the experiment for the existing accuracy, and consequently, such a B meson may belong to any of the representations which is a part of the product $\{8\} \times \{8\}$, including a 27-supermultiplet [4].

Data are currently available [6–10] which indicate that the f^0 meson with a mass of 1250 MeV is an isoscalar. If the f^0 meson lies on a vacuum trajectory, as assumed by Chew, Gell-Mann, Frautschi, and Zakhariasen [11, 12] it should be a unitary singlet. Then the amplitudes of the reactions

$$\text{meson } (1,1) + \text{baryon } (1,1) \to \\ \to \text{baryon } (3,0) + f^0 \qquad (4)$$

* The above estimate given for the ratio $w(B \to \pi + \phi)/$ $/w(B \to \pi + \omega)$ is obtained from the experimental data of [3] under the assumption that all the ϕ mesons observed in this experiment were formed in the decay of B mesons.

should be expressed in terms of one unitary amplitude. The relation in this case between the amplitudes of the different charge channels can be easily obtained by means of the U_2-transformations [13] and isotopic invariance, and have the form

$$\left.\begin{aligned} &\pi^- + p \to \Delta^0 + f^0 \quad a, \\ &\pi^+ + p \to \Delta^{++} + f^0 \quad \sqrt{3}a; \\ &K^- + p \to \Sigma_\delta^0 + f^0 \quad \frac{a}{\sqrt{2}}, \\ &\bar{K}^0 + p \to \Sigma_\delta^+ + f^0 \quad a. \end{aligned}\right\} \qquad (5)$$

Verification of the relations between the cross sections of the pairs of the first and third, the second and fourth reactions could yield information on the unitary spin of the f^0 meson. It is profitable to check these relations at energies not close to the formation threshold of f^0 in the fourth reaction and for fairly large transferred momenta. We note that the relations of the first–fourth reactions hold also in the case of formation of nf^0 mesons.

The probabilities of different decay channels of the f^0 meson, if it is a unitary singlet, are given in the following relations:

$$w(f^0 \to \pi^+ + \pi^-) : w(f^0 \to K^+ + K^-) : w \times \\ \times (f^0 \to \eta^0 + \eta^0) : w(f^0 \to \pi^0 + \pi^0) : \\ : w(f^0 \to K^0 + \bar{K}^0) = \\ = K_\pi^{2l+1} : K_K^{2l+1} : \frac{1}{2} K_\eta^{2l+1} : \frac{1}{2} K_\pi^{2l+1} : K_K^{2l+1}, \quad (6)$$

where l is the spin of the f^0 meson, which is equal to 2, as assumed.

The possibility that the f^0 meson does not lie on a vacuum trajectory cannot be excluded at present. In this case the f^0 meson must not be a unitary singlet, but may, for example, belong to a unitary octet. The relations between the cross section of the different processes in the fourth reaction for the meson as member of a unitary octet with $T = 0$ were obtained in [13]. The probabilities of the different decay channels of the f^0 meson, if it belongs to a unitary octet, should be given in the following relations

$$w(f^0 \to \pi^+ + \pi^-) : w(f^0 \to K^+ + K^-) :$$
$$w(f^0 \to \eta^0 + \eta^0) : w(f^0 \to \pi^0 + \pi^0):$$
$$w(f^0 \to K^0 + \bar{K}^0) =$$
$$= K_\pi^{2l+1} : \frac{1}{4} K_K^{2l+1} : \frac{1}{2} K_\eta^{2l+1} : \frac{1}{2} K_\pi^{2l+1} : \frac{1}{4} K_K^{2l+1}. \tag{7}$$

As follows from expressions (6) and (7), when the f^0 meson is a member of a unitary octet, the decay probability of f^0 into K mesons should be four times as small as when the f^0 meson is a unitary singlet. Substituting into (6) and (7) the values for the experimentally measured [6] ratio $R_f =$ $= w(f^0 \to K_1^0 \to K_1^0)/w(f^0 - \pi^+ + \pi^-)$ for an f^0-meson mass of 1260 MeV, we have

$$\left.\begin{array}{l} f^0 - \text{is a unitary singlet} \quad R = 0.048; \\ f^0 - \text{a member of a unitary octet } R = 0.012. \end{array}\right\} \tag{8}$$

Relations (6 – 8) were obtained under the assumption that the interaction range of the particles appearing upon the decay of the f^0 meson is equal to zero. Allowance for the finiteness of the interaction range should increase the value of R_f. The lower limit of the accuracy of the R_f values is determined when the unitary symmetry is violated, and it may be taken, as in other cases [14], not to be worse than 30 – 50%.

According to experimental data of [6], $R_f =$ $= 0.022 \pm 0.01$, which indicates that the f^0 meson is not a unitary singlet. If the f^0 meson belongs to a unitary octet, then the meson X with isotopic spin $T = 1$ should belong to the same octet. The components $X^\pm$ of this meson, besides other decay modes, can decay by the schemes

$$X^\pm \to K^\pm + \bar{K}^0 (K^0), \quad X^\pm \to \pi^\pm + \eta^0.$$

The partial widths of these decays are expressed in terms of the width Γ of the decay of f^0 into $\pi^+ + \pi^-$ and $\pi^0 + \pi^0$ as follows:

$$\Gamma_{X^- \to K^- + K^0} = \frac{q^{2l+1}}{K_\pi^{2l+1}} \Gamma; \; \Gamma_{X^- \to \pi^- + \eta} = \frac{2p^{2l+1}}{3K_\pi^{2l+1}} \Gamma, \tag{9}$$

where q and p are respectively the momenta of the K and η mesons appearing upon the decay of the X meson; K_π is the momentum, of the π meson formed in the decay of f^0. The values of the partial decay widths of X into two K mesons and into π and η can help in finding out whether the observed $\pi\phi$-resonance with a mass 1310 MeV and $I^P = 2^+$ [15] is a member of the same octet with the f^0 meson [6].

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DISCUSSION

Nguyen Van Hieu

In many works different realtions between the cross sections were obtained from the symmetry properties. It is known that these relations are not satisfied experimentally. One should therefore consider symmetry with allowance for the violation which leads to the Gell-Mann–Okubo mass formula. Will there arise any relations between the amplitudes of the processes considered by you when allowance is made for the symmetry violation?

I.Yu. Kobzarev

The relations obtained from SU_3 may be satisfied if the reaction releases a fairly large amount of energy.

Nguyen Van Hieu

I have a remark with regard to the decays of neutral vector mesons. For the unitary singlet ω^0, for example, the decay

$$\omega^0 \longrightarrow \mu^+ + \mu^- \ (e^+ + e^-)$$

is forbidden, if the unitary symmetry is exactly satisfied. Such a unitary singlet is not observed experimentally due to $\omega\phi$-mixing, and the unitary symmetry is violated. In this case, the probabilities of the decays

$$\omega \longrightarrow \mu^+ + \mu^- \ (e^+ + e^-), \quad \varphi \longrightarrow \mu^+ + \mu^- \ (e^+ + e^-)$$

are expressed in terms of the mixing parameters, since ω and ϕ are linear combinations of the unitary singlet ω^0 and of the isotopic singlet ϕ^0 in an octet, and the Feynman diagram for the decay $\omega^0 \to \mu^+ + \mu^- \ (e^+ + e^-)$ (Fig. 1) contains a unit which is related to the Feynman diagram for the transition $\omega^0 \leftrightarrow \phi^0$ (Fig. 2). Similarly, the diagram for the decay $\phi^0 \to \mu^+ + \mu^-$ contains a unit which is related to the proper energy of the ϕ^0 meson.

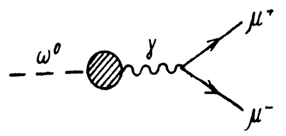

Fig. 1

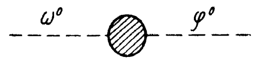

Fig. 2