

# Cosmic strings, deformed lattices and spontaneous symmetry breaking

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**Abstract.** External conditions have a dramatic impact on the way dynamical symmetry breaking occurs. In a curved background, the natural expectation is that curvature works toward the restoration of the internal symmetry. We show instead that, for topological defects, the competing action of the locally induced curvature and boundary conditions generated by the non-trivial topology allows configurations where symmetries can be spontaneously broken close to the core. Inspired by the effect of geometrical deformations on 2D lattices, we then propose a novel mechanism to induce a superconducting phase by triggering condensation along cosmic strings.

## 1. Introduction

Most of the revolutionary results of modern theoretical physics have been achieved using tools and techniques provided by quantum-mechanical perturbation theory: if the strength of the interaction among particles is weak (namely the coupling constant is small), then it is possible to extract information about how the coupling-induced small corrections affect a system.

Nevertheless, *contemporary* theoretical physics is looking forward. Typically, as energy scales become higher, non-perturbative effects get into the game, and the system is said to have entered the strongly coupled regime. Remarkable examples of strongly coupled systems include QCD and the theory of quarks, high-temperature superconductors and the very primordial plasma filling the universe a few instants after the big bang, but also the theory of the propagation of conducting electrons in graphene, the contemporary high-tech superstar material.

The study of the dynamics of such systems is a formidable task, and clear statements on the physics at strong coupling scales are only possible at the cost of demanding numerical simulations. On the other hand, general phenomenological guidelines for the study of strongly coupled systems can be precisely drafted exploiting mathematical considerations on the underlying symmetries. The physics of symmetry breaking, in fact, notably relies on exact mathematical statements which are intimately non-perturbative, as in the case of the Goldstone theorem.

A seminal paper by Coleman and Weinberg [1] showed how symmetries can be spontaneously broken due to quantum fluctuations modifying the structure of the vacuum. This mechanism of *dynamical* symmetry breaking, initially developed in the context of scalar field self-interactions, can be naturally extended to fermions.

To better understand the picture, let's introduce some concepts for the unfamiliar reader. In a field theory with massless fermions, the interactions between particles preserve helicity: the polarised left- and right-handed sectors evolve separately. However, in D-dimensional four fermion effective theories<sup>1</sup>,

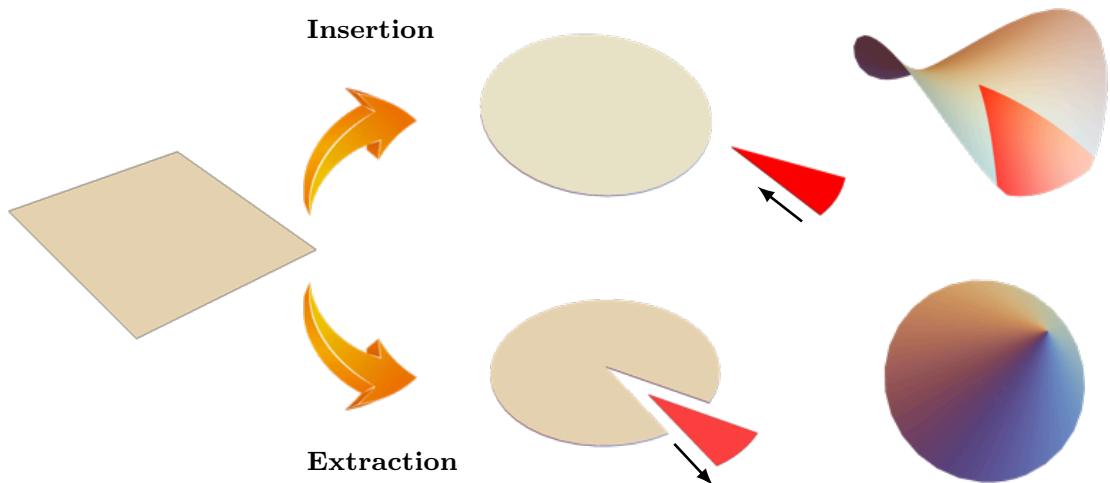
$$S_{4\text{fET}} = \int d^D x \sqrt{g} \left\{ \bar{\psi} i \gamma^\mu \nabla_\mu \psi + \frac{\lambda}{2N} (\bar{\psi} \psi)^2 + \dots \right\}, \quad (1)$$

(like the celebrated Nambu–Jona-Lasinio model [2, 3], and in its progeny of models for strong interactions) such symmetry is spontaneously broken by a non-vanishing order parameter: when the composite operator  $\phi \sim \bar{\psi} \psi$  acquires a non-zero vacuum expectation value, then a dynamical effective mass for the fermions,  $M_{\text{eff}} \sim \langle \bar{\psi} \psi \rangle$ , is generated. This mechanism, analogous to the spontaneous magnetisation, is responsible for most of the observed mass of hadrons.

Transitions between broken symmetry and restored symmetry phases are generally triggered by changes in the external conditions. In particular, a nonzero temperature leads to temperature-dependent mass generation and restoration of broken symmetries once a certain critical value of the temperature is reached. Other factors modifying the phase diagram of self-interacting theories span from finite density effects to the action of a non-vanishing chemical potential or of an external gauge field. Here we instead concentrate on geometrical effects, and how these challenge the vacuum stability of a theory with four fermions interactions [4].

Many are the configurations in which geometry affects the symmetry breaking of strongly interacting systems: in flat spacetime with  $R^3 \times S^1$  topology and periodic boundary conditions, for example, the consequences of the non-trivial topology are very similar to those of nonzero temperature [5]; on the other hand, in curved spacetime the effects of external gravitational fields resemble those of an effective extra mass [6, 7, 8, 9]. The combination of these external factors (in particular topology, nonzero temperature, and curvature) acting on self-interacting theories is likely to have been of considerable importance in the early stages of the evolution of the universe. During those eras a spontaneous breaking of an internal symmetry group results in the production of topological defects – the well-known Kibble–Zurek mechanism [10, 11].

<sup>1</sup> Here,  $\lambda$  is the coupling constant and  $N$  the total number of fermion degrees of freedom.



**Figure 1.** The flat space is modified through the insertion or the extraction of a piece of lattice. Adding a section with a given angle, one finds a saddle; subtracting the same section, and then sewing on the cut, a cone.

## 2. Cosmic defects and crystal lattices

Suppose the dynamical symmetry breaking for some model to be at the origin of the formation of a static straight cosmic string [12, 13, 14, 15] lying along the  $z$ -axis, namely an infinitely long thin tube of false vacuum generated in the sudden temperature-driven transition from a phase to another (here, the transverse size of the cosmic string is neglected while ‘sudden transition’ means a transition with a rate that is fast if compared with the size of the system). Away from the defect, the spacetime associated with the gravitating string is accurately described by the vacuum Einstein equations. It turns out that at large distance from the string, the geometry is locally flat,  $ds_{\text{con}}^2 = dt^2 - dz^2 - dr^2 - r^2 d\theta^2$ , but with an important *caveat*: it is not globally Euclidean, since the angular coordinate does not run on the entire  $2\pi$  circle; instead,  $0 \leq \theta < 2\pi - \Delta$ , with  $\Delta > 0$  ( $\Delta < 0$ ) being the deficit (excess) angle: surfaces at constant  $t$  and  $z$  are cones, not planes.

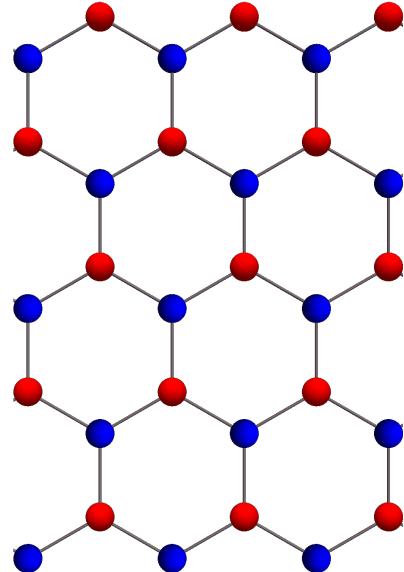
This is a remembrance of defects insertion in crystal lattices: starting from a (locally) flat lattice (see Fig. 1), the subtraction (or addition) of atoms is equivalent to the extraction (or insertion) of sections of the lattice with a given angle. The procedure results in an ice-cream-shaped lattice (or a saddle) which is locally flat everywhere apart for the apex. Note yet that the spacetime surgery does not come for free: the price to pay is the implementation of some non-trivial new boundary conditions along the cut, which will be a reminder of the deficit (excess) angle at the apex. What is the role of the background geometry in modifying the vacuum structure? Surprisingly enough, we can now show that curvature, localised close to a topological defect, enhances, rather than inhibits, the condensation along the defect itself. In order to do so, we will consider the lesson coming from a  $(2+1)$ -dimensional system borrowed from condensed matter [16], a honeycomb lattice characterised by a bipartite symmetry, that is, whose hexagonal structure is obtained by the superposition of two triangular sublattices (see Fig. 2): the breakdown of this discrete symmetry is behind the phenomenon of condensation we describe here.

The dynamics of the delocalized electrons on such a lattice is often described in terms of a generalisation of the Hubbard model, that in the continuum limit is mapped onto a field theory with nine different couplings [17]; however, considering the limit for a large number of fermion flavours  $N$  and after bosonization, the Hubbard model for the honeycomb lattice acquires the form of a bosonized  $(2+1)$  Gross–Neveu model,

$$\mathcal{L}_{\text{GN}} = \bar{\psi}_\sigma i\partial^\mu \psi_\sigma + \sigma \bar{\psi}_\sigma \phi \psi_\sigma + \frac{\phi^2}{2\lambda}, \quad (2)$$

where  $\sigma = \pm$  is a spin index on which one sums. The final goal is to study the behaviour of the order parameter  $\phi$  when moving toward the apex of the cone.

Nature abhors (or discretely hides) singularities: the sympathetic reader will then not complain about a regularization of the space generated by the topological ‘stringy’ defect with a family of smoothed versions of the conical solution (in Euclidean time),  $ds^2 = d\tau^2 + f_\epsilon(r)dr^2 + r^2 d\theta^2$ , where  $f_\epsilon(r)$  is a regularising function: in the limit  $\epsilon \rightarrow 0$  one might recover the singular cone. Using standard technique (Schrödinger–Lichnerowicz–Weitzenböck formula) to square the Dirac operator, the effective action reads



**Figure 2.** The hexagonal honeycomb lattice with the two triangular sublattices (blue and red spheres)

$$\Gamma[\phi] = - \int d^3x \sqrt{g} \frac{\phi^2}{2\lambda} + \frac{1}{2} \sum_{p=\pm} \log \det \left( \square + \frac{R}{4} + \phi_p^2 \right), \quad \phi_{\pm}^2 = \phi^2 \pm \sqrt{g^{rr}} \phi' \quad (3)$$

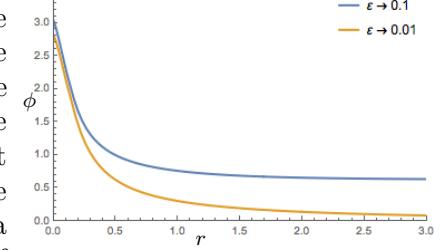
where the metric here employed is the smoothed one,  $ds^2$ , rather than  $ds_{\text{con}}^2$ . As previously mentioned, the change in the topology has a prominent role in altering the boundary conditions on the glued side of the lattice; the response to this change is captured by the employment of a modified covariant derivative,  $D_\mu = \nabla_\mu + \imath \mathcal{A}_\mu$ , acting on spinors and encompassing an effective non-dynamical gauge field,  $\mathcal{A}_\mu$ , that depends on the deficit angle[18]. Expressing the functional determinant of the (squared) Dirac operator in curved space in terms of its heat kernel expansion [19, 20], and using zeta function regularization, it is possible to calculate the effective action (3) [21] and find out the effective equations of motion of the order parameter  $\phi$ , whose solutions can be reconstructed numerically. Although the presence of curvature (which acts, as for the chiral gap effect [22, 23], as an effective mass term) is supposed to enhance a phase of symmetry restoration, the presence of the extra effective gauge field, another reminder of the geometry of the system, catalyse the formation of a bubble of condensed particles close the vertex of the cone, (see Fig. 3, the saddle case is similar). A well known duality between defective crystal configurations and differential geometry [24, 25, 26] raises the question about how generalised non-Riemannian structures can contribute to the mechanism described so far. Similar issues would enter the game in the case of presence of inhomogeneities [27, 28].

### 3. Conclusions

For relativistic cosmic strings, the possibility of condensation in the region surrounding the string core due only to configurational elements is a novel mechanism to provide a superconducting phase around the defect, whose phenomenological potentiality is completely to explore. However, it is worth mentioning that in a more realistic setup, phase transitions in the early universe did not seed the formation of a single straight string, but of a network of cosmic strings, which renders eventually even more striking and intriguing the connection with a crystal lattice, where a distribution of defects is more natural to occur. Another interesting aspect is the following: in order to simplify the discussion we have here considered the spontaneous symmetry breaking for the Gross–Neveu model catalysed by an external string defect, namely originated by the breakdown of some symmetry of a different field. Different would be the situation in which the responsible for the defect formation is the very same field, in which case one might take into account possible backreaction effects.

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**Figure 3.** Profile of the condensate  $\phi$  as a function of the distance from the apex,  $r$ . Different colours refer to different values of the  $\epsilon$  parameter regularising the cone:

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