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ANALYTIC DERIVATIVE OF ORBIT RESPONSE MATRIX AND DISPERSION WITH THICK ERROR SOURCES AND THICK STEERERS IMPLEMENTED IN PYTHON

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Abstract

While large circular colliders rely upon analysis of turn-by-turn beam trajectory data to infer and correct magnetic lattice imperfection and beam optics parameters, historically storage-ring based light sources have been exploiting orbit distortion, via the orbit response matrix. However, even large collider usually benefit of the orbit analysis during the design phase, in order to evaluate and define tolerances, correction layouts and expected performances. The proposed FCC-ee is no different, though its length (about 100 km) and amount of magnets (about 5500) make the standard closed-orbit analysis time consuming. We applied new analytic tools to cope with this issue, showing a significant gain in computational time with practically no loss of accuracy. Examples of applications to the ESRF EBS storage ring and to the CERN FCC-ee are reported with an outlook to an additional challenge provided by the FCC-ee.

INTRODUCTION

Standard optics measurements and corrections in synchrotron light sources are routinely carried out by measuring the orbit response matrix (ORM) and dispersion functions. The error model is inferred from their deviations with respect to the ideal or previous lattice model. Corrections are applied aiming at restoring the ideal ORM and dispersion. This is schematically represented by the following linear systems to be (pseudo-)inverted:

$$\begin{pmatrix} \delta\vec{O}^{(xx)} \\ \delta\vec{O}^{(yy)} \\ \delta\vec{D}_x \end{pmatrix} = \mathbf{N} \begin{pmatrix} \delta\vec{K}_1 \\ \delta\vec{K}_0 \end{pmatrix} , \tag{1}$$

$$\begin{pmatrix} \delta\vec{O}^{(xy)} \\ \delta\vec{O}^{(yx)} \\ \delta\vec{D}_y \end{pmatrix} = \mathbf{S} \begin{pmatrix} \vec{J}_1 \\ \vec{J}_0 \end{pmatrix} . \tag{2}$$

$\delta\vec{K}_1$ and $\delta\vec{K}_0$ are the vectors containing the quadrupole and dipole errors, respectively, whereas $\vec{J}_1$ and $\vec{J}_0$ denote the skew quadrupole fields and the vertical dipole strengths. On the l.h.s. of the above equations we find the blocks of the ORM deviation w.r.t. the initial model ($\delta\vec{O}$) and the deviation between the measured and model dispersion in both planes ($\delta\vec{D}$). The Jacobian matrices **N** (for the focusing part) and **S** (accounting for betatron coupling) are usually computed numerically, by simulating the very same measurement, i.e. by introducing a focusing error or source of coupling and computing the corresponding ORM and dispersion. This is known to be a very time consuming approach, specially for large machines with thousands of magnets, even though its parallelization may reduce its duration. Another weakness of the numerical approach is experienced when very tight optics are simulated, whereby even a small focusing error or source of coupling may render the lattice unstable, thus preventing the numerical calculation from succeeding.

In Ref. [1] analytic formulas for the evaluation of both **N** and **S** are derived, among other observables. This has the great advantage of being much faster and of succeeding even for extreme optics, since it is based on the initial or ideal optical parameters, without further optics calculations required. Even though the basic formulas are derived assuming thin magnets, corrections have been derived in order to account for the variation of the optical parameters across magnets, thus increasing the accuracy of the overall computation and correction. In the following sections examples and figures of applications to different machine lattices are provided. Most of the results reported here have been obtained by an open-source python implementation available to the community [2].

TUNE SHIFT INDUCED BY A QUADRUPOLE ERROR

A first application showing the importance of the including the variation of the optical functions across magnets in already-established analytic formulas is the textbook formula for the tune shift ΔQ induced by a quadrupole error ΔK. This reads $\Delta Q_{x,y}/\Delta K \simeq \pm\beta_{x,y}/4\pi$, where β is the average beta function across the magnet and the sign depends on the plane. If the average beta function is replaced by Eq.(C13) of Ref. [1], which describes its evolution along the quadrupole, the tune shift is computed with a much greater accuracy, as reported in Table 1. For both the ESRF EBS and the CERN FCC-ee lattices the gain in accuracy is of several orders of magnitude.

Table 1: Error in the evaluation of the tune shift induced by a quadrupole error for both the EBS and FCC-ee lattices, as obtained with the average beta function and its more realistic expression.

	$\Delta Q_h/Q_{h,0}^{num}$	$\Delta Q_v/Q_{v,0}^{num}$
EBS QF1J DK/K =0.023 %		
$\langle\beta\rangle/4\pi$	−0.16 %	0.14 %
Eq.(C13) of Ref. [1]	−0.0005 %	0.0024 %
FCC-ee Z QFG2-1 DK/K =0.184 %		
$\langle\beta\rangle/4\pi$	2.2 %	0.38 %
Eq.(C13) of Ref. [1]	0.0003 %	−0.0008 %

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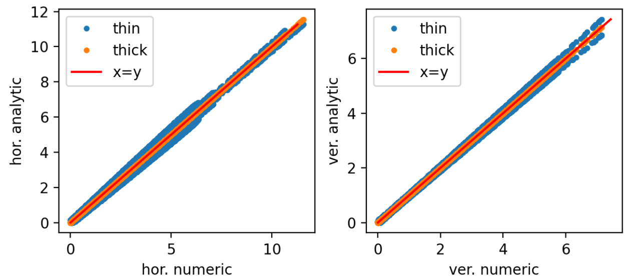

Figure 1: All rows of the diagonal blocks of the ESRF EBS orbit response matrix computed numerically and analytically.

ORBIT RESPONSE TO A DIPOLE KICK

The textbook formula describing the orbit response at a BPM i caused by a dipole kick at a magnet j reads

$$O_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2\sin(\pi Q)} \cos\left(|\phi_j - \phi_i| - \pi Q\right) + L_{ij}\,, \qquad (3)$$

where L_{ij} denotes a second-order dispersive term and ϕ is the betatron phase. The correction of the term $\sqrt{\beta_j}\cos(|\phi_j - \phi_i| - \pi Q)$ in the above formula accounting the variation of both β and ϕ across the orbit corrector can be found in Eq.(C48) of Ref. [1]. A visual comparison between the two formulas is depicted in Fig. 1, where all rows of the ESRF EBS ORM diagonal blocks computed numerically and analytically are shown. The blue dots correspond to Eq. (3), while the result of Eq.(C48) of Ref. [1] is reported in orange, which is by far closer to the correct bisector $y = x$. Normal quadrupole errors are accounted for in the formulas by their impact on beta functions. The contribution of skew quadrupole terms to the analytic ORM is being derived, even though the Jacobian **S** and Eq. (2) can be already used to this end.

APPLICATION TO THE FCC-EE LATTICE

Next we compare the Jacobian **N** of Eq. (1) for the FCC-ee lattice induced by a quadrupole QC1L1-1 and evaluated at 1600 BPMs and 8 steerers. In Fig. 2 a fraction of two columns (one for each plane) of **N** around one interaction point (IP) is displayed. The numerical results are shown by the red curve, whereas the analytic solution of Eq.(14) of Ref. [1] assuming thin magnets corresponds to the blue dashed curve in the top plot. Even though the overall agreement is rather good, a substantial inaccuracy exists, as indicated by the difference between the two vectors (green curve). When the variation of the optical parameters across the magnets are included in the analytic formulas (derived in the Appendix C of Ref. [1]), the agreement is greatly improved (by several orders of magnitude) as demonstrated by the bottom plot of Fig. 2 (thick model).

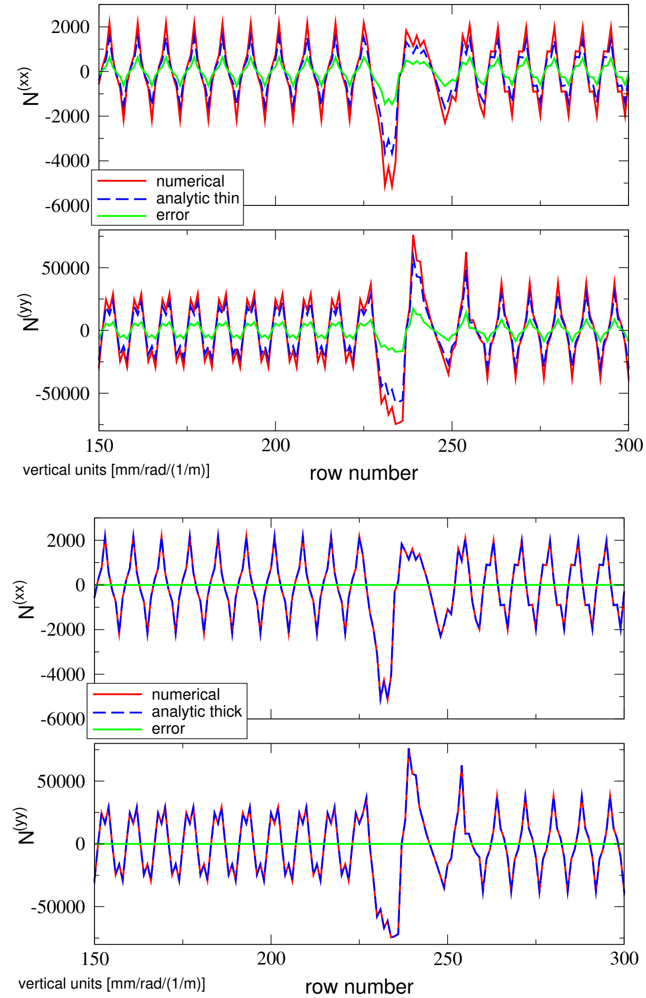

Figure 2: (Color) Example of columns of the FCC-ee ORM Jacobian N computed numerically (red) and analytically (blue). A zoom over only 150 BPMs (among the 1600) is displayed for a better visualization. Top: the analytic N is computed in the thin-lens approximation with constant optical parameters, yielding a sizeable discrepancy (green curve). Bottom: the analytic solution is computed by replacing the optical functions by the corresponding integrals of the previous sections, showing an almost perfect agreement.

In order to provide global and aggregate figures assessing the accuracy of the two models, thin and thick, the rms and peak errors for all rows and columns of the Jacobian **N** have been computed and reported in Fig. 3. The improvement of the accuracy of the thick model accounting for the variation of the optical parameters across magnets is striking and unambiguous.

As far as the computational time is concerned, the analytic evaluation of **N** for the FCC-ee lattice takes about 12% of the CPU time needed to compute its numerical equivalent, and this without any risk of failing the computation of one column if the optics fails when the quadrupole error is included into the model.

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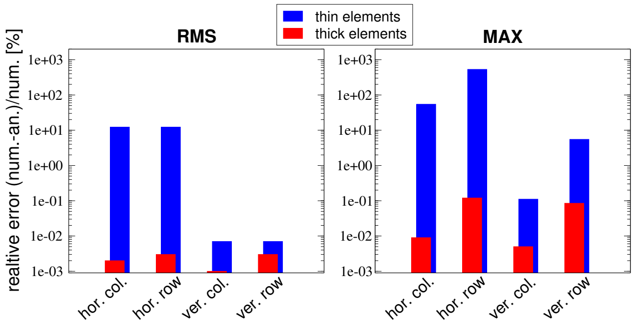

Figure 3: Rms and maximum relative difference from the (true) numerical response and the analytical ones computed along both the matrix columns and rows.

Similar tests on the coupling Jacobian **J** have been carried out (not presented here) with comparable results as far as the accuracy of the analytic formulas is concerned.

FURTHER SIMULATIONS OF THE FCC-EE (TAPERING)

Preliminary simulations have been performed in order to asses the robustness of the analytic formulas in order to compute and correct the linear optics for the FCC-ee lattice, starting with some common sets of errors. Results reported in Table 2 confirm the validity of the analytic ORM on both terms, with no significant difference compared to the numerical option.

Table 2: β-beating, dispersion and emittances after correction of 10 µm random alignment errors on dipole quadrupole and sextupole magnets for the EBS and FCC-ee lattices using analytic or numeric ORM derivative.

$\langle std \rangle_{50}$ units	$\frac{\Delta\beta_h}{\beta_{h,0}}$ %	$\frac{\Delta\beta_v}{\beta_{v,0}}$ %	$\Delta\eta_h$ mm	$\Delta\eta_v$ mm	$\Delta\epsilon_v$ pm rad
		EBS			
err.	19.37	11.08	17.33	6.91	94.17
ana.	0.2	0.2	0.18	0.05	0.003
num.	0.2	0.2	0.18	0.05	0.003
		FCC-ee Z			
err.	3.6	59.4	120.5	82.45	-
ana.	0.81	4.29	26.0	9.57	0.17
num.	0.82	4.30	25.98	9.64	0.18

FCC-ee is known to experience significant energy losses induced by synchrotron radiation along the circumference, making the assumption of constant energy within one revolution unrealistic. The energy losses (and gains at the location of the RF cavities) need therefore to be taken into account and the magnet strength to be varied accordingly in order to ensure the design optics, a scheme known as *tapering* [3, 4]. It is hence of interest to check whether different options in the optics corrections can alter their effectiveness. First results are summarized in Table 3 indicate that the analytic ORM correction provides similar results, regardless the option, be it a pure 4D simulation, or 6D without or with tapering.

Table 3: β-beating, dispersion and emittances after correction of 10 µm random alignment errors on dipole quadrupole and sextupole magnets for the FCC-ee lattice using analytic ORM derivative (1856 BPMs, 18 steerers). The input lattice is tested: without radiation, with radiation and with radiation and tapering. Reference lattice is in all cases without radiation.

$\langle std \rangle_{50}$ units	$\frac{\Delta\beta_h}{\beta_{h,0}}$ %	$\frac{\Delta\beta_v}{\beta_{v,0}}$ %	$\Delta\eta_h$ mm	$\Delta\eta_v$ mm	$\Delta\epsilon_v$ pm rad
4D err	3.63	61.37	118.7	82.36	-
4D cor	0.84	4.24	25.67	9.58	0.71
6D err	3.60	59.45	120.54	82.45	-
6D cor	0.81	4.29	26.0	9.57	0.17
6D err + tapering	3.61	61.33	119.59	82.96	-
6D cor + tapering	0.82	4.22	26.03	9.65	0.18

CONCLUSION

A quick overview with some of the main results obtained in Ref. [1] has been presented with some applications to the ESRF EBS and CERN FCC-ee lattices. The analytic formulas provide a much quicker and numerically robust tool to improve the optimization studies of present and future circular accelerators.

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