Fermi National Accelerator Laboratory

FERMILAB-Pub-79/64-THY
August 1979

A Systematic Investigation of Effects of Heavy Particles via Factorized Local Operators and Renormalization Group II: Explicit Calculations in Quantum Electrodynamics

YOICHI KAZAMA
Fermi National Accelerator Laboratory*
P.O. Box 500, Batavia, Illinois 60510
and
Randall Laboratory of Physics
University of Michigan, Ann Arbor, Michigan 48109
and
YORK-PENG YAO
Randall Laboratory of Physics
University of Michigan, Ann Arbor, Michigan 48109

ABSTRACT

We continue our systematic investigation of effects of heavy particles in low energy physics initiated in a companion paper. It was shown therein that in renormalizable theories without spontaneous symmetry breaking, where there are heavy (M) and light (m) particles, all the $\mathcal{O}(1/M^2)$ heavy particle effects in any proper amputated light Green's function can be written in a factorized form

$$\underset{\text{full theory}}{\Gamma(M,m)} = \underset{\text{light theory}}{\Gamma(m)} + (1/M^2)\sum_N C_N(M,m)\,\underset{\text{light theory}}{\Gamma(O_N;m)}.$$

The functions $C_N(M, m)$, which are the universal coefficients associated with certain local operators O_N, were shown to satisfy a set of Callan-Symanzik like equations. In the present article, these equations are explicitly solved in QED with the relevant anomalous dimensions evaluated to one loop order. As an application within QED, assessment of the effects of muon loops on the electron anomalous magnetic moment is made and absence of leading logarithms in this quantity to all orders in the coupling constant e is proved.

*Present address.

PACS Category Nos.: 12.20.Ds, 11.10.Jj

I. INTRODUCTION

In a previous paper[1] (hereafter referred to as I), we have developed a formalism by which one can systematically calculate physical effects of heavy particles at energies below their production threshold in renormalizable theories without spontaneous symmetry breaking. Specifically, by taking QED (quantum electrodynamics) with photons, electrons (mass m) and muons (mass M) as an example, we have shown that (i) the leading effects due to the heavy particles (muons) on the Green's functions with only light external lines (i.e. those of photons and electrons) are of order $1/M^2$ (times logarithms of M/m) and can be factorized into a sum of products of universal coefficient functions containing all the heavy mass dependence and Green's functions of the light theory with gauge invariant local operators inserted, and that (ii) these coefficient functions obey a set of homogeneous Callan-Symanzik type equations.[2]

In this companion paper, we shall conclude our study in QED by giving the details of the actual calculations involved and discussing some physical applications of our results. In a future communication, we shall report on the more interesting case of quantum chromodynamics.

The organization of the rest of the paper is as follows: in Section II, we recall some of the relevant formulas obtained in I to make this paper more or less self-contained. Section III deals with the detailed form of the Callan-Symanzik type equations. After exhibiting some of the one-loop calculations for the relevant anomalous dimension matrix elements in Section IV, these scaling equations will be explicitly solved in Section V. As an application, we shall assess in Section VI the effects of muon loops on the electron anomalous magnetic moment. A short summary of this paper will be found in Section VII.

II. A BRIEF REVIEW OF SOME RESULTS OBTAINED IN PAPER I

Let $\tilde{\Gamma}^{B,F}$ and $\Gamma^{B,F}$ respectively denote an amputated proper (i.e. one-particle-irreducible, or 1PI) Green's function with B external photons and F external electrons in QED with and without muons. As an efficient bookkeeping device for the mass derivatives, an extra mass λ is introduced[3] uniformly in the free electron propagator so that it takes the form $i/(\not{p} - m - \lambda)$. Renormalization is performed according to the BPHZ scheme,[4,5] where λ may be regarded as an extra external momentum for any subgraph. We have chosen the normalization conditions

$$\left\{\begin{aligned} \tilde{\Gamma}^{0,2}(\not{p}, m, M, \lambda)\Big|_{\substack{\not{p}=0\\ \lambda=0}} &= -m \\ \frac{\partial}{\partial\not{p}}\tilde{\Gamma}^{0,2}(\not{p}, m, M, \lambda)\Big|_{\substack{\not{p}=0\\ \lambda=0}} &= 1 \\ \frac{\partial}{\partial\lambda}\tilde{\Gamma}^{0,2}(\not{p}, m, M, \lambda)\Big|_{\substack{\not{p}=M\\ \lambda=0}} &= -1 \end{aligned}\right. \tag{2.1}$$

$$\left\{\begin{aligned} \Gamma^{0,2}(\not{p}, m, \lambda)\Big|_{\substack{\not{p}=0\\ \lambda=0}} &= -m \\ \frac{\partial}{\partial\not{p}}\Gamma^{0,2}(\not{p}, m, \lambda)\Big|_{\substack{\not{p}=0\\ \lambda=0}} &= 1 \\ \frac{\partial}{\partial\lambda}\Gamma^{0,2}(\not{p}, m, \lambda)\Big|_{\substack{\not{p}=0\\ \lambda=0}} &= -1 \end{aligned}\right. \tag{2.2}$$

$$\left\{\begin{aligned} \tilde{\Gamma}^{2,0}_{\mu\nu}(q^2, m, M, \lambda) &= (-g_{\mu\nu}q^2 + q_\mu q_\nu)(1 + \tilde{\pi}(q^2, m, M, \lambda)) \\ \tilde{\pi}(q^2, m, M, \lambda)\Big|_{\substack{q^2=0\\ \lambda=0}} &= 0 \end{aligned}\right. \tag{2.3}$$

$$\begin{cases} \Gamma^{2,0}_{\mu\nu}(q^2, m, \lambda) = (-g_{\mu\nu}q^2 + q_\mu q_\nu)(1 + \pi(q^2, m, \lambda)) \\ \pi(q^2, m, \lambda)|_{\substack{q^2=0 \\ \lambda=0}} = 0 \end{cases} \qquad . \tag{2.4}$$

$\tilde{\Gamma}^{1,2}_\mu$ and $\Gamma^{1,2}_\mu$ are normalized so that the Ward identity is satisfied.

The factorization theorem proved in the previous paper is then, to $\mathcal{O}(1/M^2)$

$$\begin{aligned} \tilde{\Gamma}^{B,F}(p_i, m, M, \lambda) &= \Gamma^{B,F}(p_i, m, \lambda) \\ &+ \frac{1}{M^2}\sum_{N,b} C_{Nb}\Gamma^{B,F}(O_{Nb}) \end{aligned} \tag{2.5}$$

where O_{Nb} denote gauge invariant local operators whose densities are of mass dimension N, b labels different operators within the same dimension, $\Gamma^{B,F}(O_{Nb})$ is the Green's function of the light theory with O_{Nb} once inserted, and C_{Nb} are the universal (i.e. independent of B and F) coefficient functions which, except for the overall $1/M^2$, contain all the M dependence in the form of the logarithms $\ln (M/m)^n$. C_{Nb} was analyzed to be of the form

$$C_{Nb} = m\lambda^{5-N}\xi^{(5-N)}_{Nb} + \lambda^{6-N}\xi^{(6-N)}_{Nb} \qquad , \tag{2.6}$$

where ξ_{Nb}'s are dimensionless and λ^k is defined to be zero if k is negative. N runs from 3 to 6.

The set of gauge invariant operators that appear may be chosen to be:

$$O_{31} = i\int d^4x(-\bar{\psi}\psi)$$

$$O_{41} = i\int d^4x\bar{\psi}i\not{D}\psi$$

$$O_{42} = i\int d^4x(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu})$$

$$O_{51} = i\int d^4x\bar{\psi}(iD)^2\psi$$

$$O_{52} = i\int d^4x\,\frac{e}{2}\bar{\psi}F_{\mu\nu}\sigma^{\mu\nu}\psi$$

$$O_{61} = i\int d^4x\bar{\psi}(iD)^2 i\not{D}\psi$$

$$O_{62} = i\int d^4x e\bar{\psi}F_{\mu\nu}\gamma^{\nu}D^{\mu}\psi$$

$$O_{63} = i\int d^4x e\bar{\psi}\gamma^{\mu}(\partial^{\nu}F_{\mu\nu})\psi$$

$$O_{64} = i\int d^4x\,\frac{e}{2}\bar{\psi}F_{\mu\nu}\frac{\sigma^{\mu\nu}}{i}\not{D}\psi$$

$$O_{65} = i\int d^4x\tfrac{1}{4}F_{\mu\nu}\partial^2F^{\mu\nu}$$

$$O_{66} = i\int d^4x\,\frac{e^2}{2}\bar{\psi}\psi\bar{\psi}\psi$$

$$O_{67} = i\int d^4x\,\frac{e^2}{2}\bar{\psi}\gamma_{\mu}\psi\bar{\psi}\gamma^{\mu}\psi$$

$$O_{68} = i\int d^4x\,\frac{e^2}{4}\bar{\psi}\sigma_{\mu\nu}\psi\bar{\psi}\sigma^{\mu\nu}\psi \quad . \tag{2.7}$$

These operators are to be understood in the sense of normal products of minimal degree as defined by Zimmerman.[5] Through the BPHZ renormalization scheme, the Green's functions with these operators inserted automatically satisfy the following normalization conditions: Let $(\partial/\partial p)^n$ symbolically denote the differentiation with respect to the external momenta. Then

$$\left(\frac{\partial}{\partial\lambda}\right)^{\ell}\left(\frac{\partial}{\partial p}\right)^{n}\Gamma^{B,F}(O_{Nb})\Big|_{\lambda=p=0} = \left(\frac{\partial}{\partial\lambda}\right)^{\ell}\left(\frac{\partial}{\partial p}\right)^{n}\Gamma^{B,F}(O_{Nb})^{f}\Big|_{p=\lambda=0} \qquad (2.8)$$

if $\ell + n \leq N - B - \frac{3}{2}F$, where the symbol f denotes the free vertex factor corresponding to O_{Nb}. These are listed in the Appendix A.

$\Gamma^{B,F}$, $\tilde{\Gamma}^{B,F}$ and $\Gamma^{B,F}(O_{Nb})$'s were found to satisfy the following scaling equations in the Landau gauge, to be used throughout: $\Gamma^{B,F}$ satisfies

$$\left(m\frac{\partial}{\partial m} + \beta_e\frac{\partial}{\partial e} - m(1-\beta_\lambda)\frac{\partial}{\partial\lambda} - B\gamma_A - F\gamma_e\right)\Gamma^{B,F} = 0 \qquad (2.9)$$

where

$$\left\{\begin{aligned} \beta_e &= m\frac{\partial e}{\partial m} = e\gamma_A \\ -(1-\beta_\lambda) &= m\frac{\partial\lambda}{\partial m} \equiv -1 + \beta_\lambda^{(0)} + \frac{\lambda}{m}\beta_\lambda^{(1)} \\ \gamma_A &= \frac{1}{2}m\frac{d}{dm}\ln Z_3 \\ \gamma_e &= \frac{1}{2}m\frac{d}{dm}\ln Z_2 \end{aligned}\right. \qquad . \qquad (2.10)$$

All of these quantities are defined with the bare charge e_o, the bare mass m_o, and the ultraviolet cutoff Λ held fixed. From Eq. (2.9) and the normalization conditions for appropriate Green's functions, we can express them in terms of the renormalized Green's functions:

$$\left\{\begin{aligned} \beta_\lambda^{(0)} &= 2\gamma_e \\ \frac{2\gamma_e}{1-2\gamma_e} &= -m\frac{\partial}{\partial\lambda}\frac{\partial}{\partial \not p}\Gamma^{0,2}\Big|_{\not p=\lambda=0} \\ \beta_\lambda^{(1)} &= 2\gamma_e - m(1-2\gamma_e)\left(\frac{\partial}{\partial\lambda}\right)^2\Gamma^{0,2}\Big|_{\not p=\lambda=0} \\ 2\gamma_A &= -m(1-2\gamma_e)\frac{\partial}{\partial\lambda}\pi\Big|_{k^2=\lambda=0} \end{aligned}\right. \qquad (2.11)$$

$\tilde{\Gamma}^{B,F}$ satisfies

$$\left(m\frac{\partial}{\partial m} + \tilde{\beta}_e\frac{\partial}{\partial e} + \tilde{\beta}_M m\frac{\partial}{\partial M} - m(1-\tilde{\beta}_\lambda)\frac{\partial}{\partial\lambda} - B\tilde{\gamma}_A - F\tilde{\gamma}_e\right)\Gamma^{B,F} = 0 \qquad (2.12)$$

where

$$\begin{aligned} \tilde{\beta}_e &\equiv m\frac{\partial e}{\partial m} = e\gamma_A \\ -(1-\tilde{\beta}_\lambda) &= m\frac{\partial\lambda}{\partial m} \\ \tilde{\gamma}_A &= \frac{1}{2}m\frac{d}{dm}\ln\tilde{Z}_3 \\ \tilde{\gamma}_e &= \frac{1}{2}m\frac{d}{dm}\ln\tilde{Z}_2 \\ \tilde{\beta}_M &= \frac{\partial M}{\partial m} = \tilde{\beta}_M^{(0)} + \frac{\lambda}{m}\tilde{\beta}_M^{(1)} + \ldots \end{aligned} \qquad (2.13)$$

Again the derivatives are taken with bare parameters held fixed. It was shown that

$$\tilde{\beta}_M^{(0)} = 0 \quad , \quad \tilde{\beta}_M^{(1)} = \mathcal{O}\left(\frac{1}{M}\right) \tag{2.14}$$

and

$$\begin{cases} \tilde{\gamma}_A = \gamma_A + \frac{m^2}{M^2} \Delta\gamma_A \\ \tilde{\gamma}_e = \gamma_e + \frac{m^2}{M^2} \Delta\gamma_e \\ \tilde{\beta}_\lambda = \beta_\lambda + \frac{m^2}{M^2} \Delta\beta_\lambda \end{cases} \tag{2.15}$$

where $\Delta\gamma_A$, $\Delta\gamma_e$, $\Delta\beta_\lambda$ do not have positive powers of M.

$\Gamma^{B,F}(O_{Nb})$'s satisfy

$$\left\{ \left(m\frac{\partial}{\partial m} + \beta_e \frac{\partial}{\partial e} - m(1-\beta_\lambda)\frac{\partial}{\partial\lambda} - B\gamma_A - F\gamma_e \right) \delta_{Ma,Nb} + \gamma_{Ma,Nb} \right\} \Gamma^{B,F}(O_{Nb}) = 0 \tag{2.16}$$

(repeated indices summed)

where the anomalous dimension matrix $\gamma_{Ma,Nb}$ is defined by

$$\gamma_{Ma,Nb} = \left(Zm\frac{d}{dm}Z^{-1}\right)_{Ma,Nb}\Big|_{m_o, e_o \text{ fixed}} \tag{2.17}$$

with Z = matrix relating the bare and the renormalized operators. $\gamma_{Ma,Nb}$ defined above is of dimension M-N and depends on λ. Expanding in powers of λ in the form

$$\gamma_{Ma,Nb} = \sum_{\ell=0} \left(\frac{\lambda}{m}\right)^{\ell} m^{M-N} \gamma^{(\ell)}_{Ma,Nb} \tag{2.18}$$

we found that

(a) $\gamma^{(0)}_{Ma,Nb}$ is block diagonal i.e. $\gamma^{(0)}_{Ma,Nb} \neq 0$ only if M-N,

(b) $\gamma^{(1)}_{Ma,Nb}$ has non-vanishing entries only if $M-N \geq 1$.

From Eqs. (2.9), (2.12) and (2.16) one can derive the scaling equations for the coefficient functions C_{Nb}, viz.

$$\Gamma^{B,F}(O_{Ma}) \left\{ \left(m\frac{\partial}{\partial m} + \beta_e \frac{\partial}{\partial e} - m(1-\beta_\lambda)\frac{\partial}{\partial \lambda} \right) \delta_{Ma,Nb} - \gamma^T_{Ma,Nb} \right\} C_{Nb}$$

$$+ m^2 \left[\Delta\gamma_A \left(e\frac{\partial}{\partial e} - B \right) + m\Delta\beta_\lambda \frac{\partial}{\partial \lambda} - F\Delta\gamma_e \right] \Gamma^{B,F} = 0 \qquad , \qquad (2.19)$$

where T denotes transposition and repeated indices Ma and Nb are summed over. It is important to note that $\Delta\gamma_A$, $\Delta\beta_\lambda$ and $\Delta\gamma_e$ contain large logarithms of the form $(\ln M/m)^n$ and Eq. (2.19), as it is, is a useless set of inhomogeneous equations for the coefficient functions C_{Nb}. However, as we briefly indicated in I (section VI), the scaling equations above may be put into a form in which those governing the physically important coefficients are neatly decoupled from the rest to all orders in the coupling. Moreover these equations are <u>homogeneous</u> and are easily solved. In the next section, we shall demonstrate this in detail.

III. HOMOGENEOUS SCALING EQUATIONS FOR THE COEFFICIENT FUNCTIONS

Let us begin by reminding ourselves that not all the coefficient functions ξ_{Nb}'s are of physical interest. Those of interest are the ones which remain in the limit that the fictitious parameter λ goes to zero, namely $\xi^{(0)}_{5b}$'s and $\xi^{(0)}_{6b}$'s. What we shall demonstrate in this section is that by expanding Eq. (2.19) up to order λ and using some identities to be developed, we shall obtain a closed set of homogeneous equations involving $\xi^{(0)}_{5b}$, $\xi^{(1)}_{5b}$ and $\xi^{(0)}_{6b}$, which we can easily solve. ξ_{3b}'s, ξ_{4b}'s, $\Delta\gamma_A$, $\Delta\gamma_e$ and $\Delta\beta_\lambda$ will not appear in these equations.

We shall expand the quantities in Eq. (2.19) in powers of λ as follows:

$$\Gamma^{B,F}(O_{Ma}) = \sum_{n=0} \lambda^n \Gamma^{B,F}_{(n)}(O_{Ma}) \tag{3.1}$$

$$\Gamma^{B,F} = \sum_{n=0} \lambda^n \Gamma^{B,F}_{(n)} \tag{3.2}$$

$$\Delta\gamma_A = \sum_{n=0} \left(\frac{\lambda}{m}\right)^n \Delta\gamma_A^{(n)} \quad . \tag{3.3}$$

(Similarly for $\Delta\gamma_e$ and $\Delta\beta_\lambda$). We substitute these and Eq. (2.18) into Eq. (2.19) and set the coefficient of each power of λ to zero. The results are as follows: Let us first look at the coefficient of λ^o. The equation we obtain may be written in the form

$$A + B = 0 \tag{3.4}$$

where

$$A = m^2 \left\{ \left[\Delta\gamma_A^{(0)}\left(e\frac{\partial}{\partial e} - B\right) - F\Delta\gamma_e^{(0)} \right] \Gamma_{(0)}^{B,F} \right.$$

$$\left. + \Delta\beta_\lambda^{(0)} m \frac{\partial}{\partial\lambda} \Gamma_{(0)}^{B,F} - (1 - \beta_\lambda^{(0)}) \Gamma_{(0)}^{B,F}(O_{4a}) \xi_{4a}^{(1)} \right\} \tag{3.5}$$

and

$$B = m\Gamma_{(0)}^{B,F}(O_{5a}) \left\{ \left(m\frac{\partial}{\partial m} + \beta_e \frac{\partial}{\partial e} + 1 \right) \delta_{5a,5b} - \gamma_{5a,5b}^{T(0)} \right\} \xi_{5b}^{(0)}$$

$$- m(1 - \beta_\lambda^{(0)}) \Gamma_{(0)}^{B,F}(O_{5a}) \xi_{5a}^{(1)}$$

$$+ \Gamma_{(0)}^{B,F}(O_{6a}) \left\{ \left(m\frac{\partial}{\partial m} + \beta_e \frac{\partial}{\partial e} \right) \delta_{6a,6b} - \gamma_{6a,6b}^{T(0)} \right\} \xi_{6b}^{(0)} \quad . \tag{3.6}$$

In the Appendix B, we have derived useful identities (true to all orders) which express $\Gamma^{B,F}(O_{31})$, $\Gamma^{B,F}(O_{41})$ and $\Gamma^{B,F}(O_{42})$ in terms of $\Gamma^{B,F}$ and its λ derivatives and vice versa. It is most convenient to write them in the form

$$\left(e\frac{\partial}{\partial e} - B\right) \Gamma^{B,F} = -2\,\Gamma^{B,F}(O_{42}) \tag{3.7}$$

$$\frac{\partial}{\partial\lambda} \Gamma^{B,F} = \Gamma^{B,F}(O_{31}) \tag{3.8}$$

$$\frac{F}{2} \Gamma^{B,F} = \left(m + \lambda \frac{1 - \beta_\lambda^{(1)}}{1 - 2\gamma_e} \right) \Gamma^{B,F}(O_{31})$$

$$+ \frac{1}{1 - 2\gamma_e} \Gamma^{B,F}(O_{41}) + \frac{2\gamma_A}{1 - 2\gamma_e} \Gamma^{B,F}(O_{42}) \quad . \tag{3.9}$$

Setting $\lambda = 0$ in Eqs. (3.7)-(3.9) and substituting them into Eq. (3.5), it becomes

$$\frac{A}{m^2} = a\Gamma_{(0)}^{B,F}(O_{31}) + b\Gamma_{(0)}^{B,F}(O_{41}) + c\Gamma_{(0)}^{B,F}(O_{42}) \tag{3.10}$$

where

$$a = m(\Delta\beta_\lambda^{(0)} - 2\gamma_e^{(0)}) \tag{3.11a}$$

$$b = -\left(\frac{2\Delta\gamma_e^{(0)}}{1 - 2\gamma_e} + (1 - 2\gamma_e)\xi_{41}^{(1)} \right) \tag{3.11b}$$

$$c = -\left(\frac{2\gamma_A}{1 - 2\gamma_e}\, 2\Delta\gamma_e^{(0)} + 2\Delta\gamma_A^{(0)} + (1 - 2\gamma_e)\xi_{42}^{(1)} \right) \quad . \tag{3.11c}$$

We shall now utilize the normalization conditions Eq. (2.8) for the operator inserted Green's functions. First choose $B = 0$, $F = 2$ and set the external momentum to zero. The only non-vanishing Green's function is

$$\Gamma_{(0)}^{0,2}(O_{31})|_{\not p=0} = -1 \quad . \tag{3.12}$$

Thus we immediately get $a = 0$. Next consider the part linear in $\not p$ (with still $B = 0$, $F = 2$). Among the remaining Green's functions, only $\frac{\partial}{\partial \not p}\Gamma_{(0)}^{0,2}(O_{41})$ is non-vanishing, which leads to $b = 0$. Similarly, by looking at the normalization condition for the photon two-point function ($B = 2$, $F = 0$), we may easily conclude $c = 0$. Therefore Eq. (3.4) reduces to $B = 0$. This equation may be further decoupled by examining the appropriate normalization conditions, each of which involves only one non-vanishing operator inserted Green's function. In this way one obtains the following set of scaling equations for the coefficient functions ξ_{5b}'s and ξ_{6b}'s:

$$\left(m\frac{\partial}{\partial m}+\beta_e\frac{\partial}{\partial e}+1\right)\xi^{(0)}_{51}-\gamma^{T(0)}_{51,5b}\xi^{(0)}_{5b}-(1-2\gamma_e)\xi^{(1)}_{51} = 0 \tag{3.13}$$

$$\left(m\frac{\partial}{\partial m}+\beta_e\frac{\partial}{\partial e}+1\right)\xi^{(0)}_{52}-\gamma^{T(0)}_{52,5b}\xi^{(0)}_{5b}-(1-2\gamma_e)\xi^{(1)}_{52} = 0 \tag{3.14}$$

$$\left(m\frac{\partial}{\partial m}+\beta_e\frac{\partial}{\partial e}\right)\xi^{(0)}_{6a}-\gamma^{T(0)}_{6a,6b}\xi^{(0)}_{6b} = 0 \quad . \tag{3.15}$$

It should be emphasized that these equations are obtained without any approximations. Notice that the system of equations for $\xi^{(0)}_{6b}$'s is already closed at this stage, while due to the presence of $\xi^{(1)}_{5b}$'s two more equations are needed to close the system for ξ_{5b}'s. These equations will be obtained from the order λ equation, which we shall presently discuss.

The equation that one obtains by equating the coefficient of λ to zero in the expansion appears at first sight to have a much more complicated structure. It is of the form

$$A + B + C = 0 \tag{3.16}$$

where $A = A_1 + A_2$,

$$\begin{aligned} A_1 &= -m^2\Gamma^{B,F}_{(0)}(O_{31})2(1-2\gamma_e)\xi^{(2)}_{31} \\ &\quad + m\Gamma^{B,F}_{(0)}(O_{4a})\left\{\left(m\frac{\partial}{\partial m}+\beta_e\frac{\partial}{\partial e}+1+\beta^{(1)}_{\lambda}\right)\xi^{(1)}_{4a}\right. \\ &\quad \left. -2(1-2\gamma_e)\xi^{(2)}_{4a}-\gamma^{T(0)}_{4a,4b}\xi^{(1)}_{4b}-\gamma^{T(1)}_{4a,5b}\xi^{(0)}_{5b}\right\} \quad , \end{aligned} \tag{3.17}$$

$$A_2 = -m^2\Gamma^{B,F}_{(1)}(O_{4a})(1-2\gamma_e)\xi^{(1)}_{4a}$$

$$+ m^2\left[\Delta\gamma^{(0)}_A\left(e\frac{\partial}{\partial e}-B\right) - F\Delta\gamma^{(0)}_e + \Delta\beta^{(1)}_\lambda\right]\Gamma^{B,F}_{(1)}$$

$$+ m\left[\Delta\gamma^{(1)}_A\left(e\frac{\partial}{\partial e}-B\right) - F\Delta\gamma^{(1)}_e\right]\Gamma^{B,F}_{(0)} + m^3 2\Delta\beta^{(0)}_\lambda\,\Gamma^{B,F}_{(2)} \qquad , \tag{3.18}$$

$$B = m\Gamma^{B,F}_{(1)}(O_{5a})\left\{\left(m\frac{\partial}{\partial m}+\beta_e\frac{\partial}{\partial e}+1\right)\xi^{(0)}_{5a} - (1-2\gamma_e)\xi^{(1)}_{5a} - \gamma^{T(0)}_{5a,5b}\xi^{(0)}_{5b}\right\}$$

$$+ \Gamma^{B,F}_{(1)}(O_{6a})\left\{\left(m\frac{\partial}{\partial m}+\beta_e\frac{\partial}{\partial e}\right)\delta_{6a,6b} - \gamma^{T(0)}_{6a,6b}\right\}\xi^{(0)}_{6b} \qquad , \tag{3.19}$$

and

$$C = \Gamma^{B,F}_{(0)}(O_{5a})\left\{\left[\left(m\frac{\partial}{\partial m}+\beta_e\frac{\partial}{\partial e}+\beta^{(1)}_\lambda\right)\delta_{5a,5b} - \gamma^{T(0)}_{5a,5b}\right]\xi^{(1)}_{5b} - \gamma^{T(1)}_{5a,6b}\xi^{(0)}_{6b}\right\} \ . \tag{3.20}$$

First, due to Eqs. (3.13)-(3.15) previously obtained, B above vanishes. For the remainder, it is clear from the procedure described for decoupling λ^o equations that A will vanish if it is brought into the form $a'\Gamma^{B,F}_{(0)}(O_{31}) + b'\Gamma^{B,F}_{(0)}(O_{41}) + c'\Gamma^{B,F}_{(0)}(O_{42})$ where a', b', c' are independent of B and F. Since A_1 is already in this form, we only need to examine A_2. Using the identities (3.7)-(3.9) it is easy to obtain

$$A_2 = m\Gamma^{B,F}_{(0)}(O_{31})\left\{\frac{F}{2}a + m\Delta\beta^{(1)}_\lambda - 2m\Delta\gamma^{(1)}_e - m\Delta\beta^{(0)}_\lambda\frac{1-\beta^{(1)}}{1-2\gamma_e}\right\} - m\Gamma^{B,F}_{(0)}(O_{41})\frac{2\Delta\gamma^{(1)}_e}{1-2\gamma_e}$$

$$- m\Gamma^{B,F}_{(0)}(O_{42})\left(2\Delta\gamma^{(1)}_A + \frac{2\gamma_A}{1-2\gamma_e}2\Delta\gamma^{(1)}_e\right)$$

$$+ m^2\Gamma^{B,F}_{(1)}(O_{41})b + m^2\Gamma^{B,F}_{(1)}(O_{42})c \tag{3.21}$$

where a, b, c are as defined in Eq. (3.11a)-Eq. (3.11c) and hence vanishing. Thus we find that A_2 is also of the desired form and Eq. (3.15) now reduces to C = 0. Further decoupling of this equation by using appropriate normalization conditions finally brings us the two equations

$$\left(m\frac{\partial}{\partial m}+\beta_e\frac{\partial}{\partial e}+\beta_\lambda^{(1)}\right)\xi_{51}^{(1)}-\gamma^{T(0)}_{51,5b}\xi^{(1)}_{5b}-\gamma^{T(1)}_{51,6b}\xi^{(0)}_{6b}=0 \tag{3.22}$$

$$\left(m\frac{\partial}{\partial m}+\beta_e\frac{\partial}{\partial e}+\beta_\lambda^{(1)}\right)\xi_{52}^{(1)}-\gamma^{T(0)}_{52,5b}\xi^{(1)}_{5b}-\gamma^{T(1)}_{52,6b}\xi^{(0)}_{6b}=0 \quad . \tag{3.23}$$

Eqs. (3.13), (3.14), (3.15), (3.22) and (3.23) constitute twelve homogeneous scaling equations for twelve unknown functions. After describing in the next section the procedure for computing the anomalous dimension matrix elements to one loop order, these equations will be explicitly solved in Section V.

IV. CALCULATION OF THE ANOMALOUS DIMENSION MATRIX ELEMENTS TO ONE LOOP ORDER

The anomalous dimension matrix elements $\gamma_{Ma,Nb}$ were originally defined in Eq. (2.17) in terms of the (cut-off dependent) mixing matrix Z. For calculational purpose, however, it is more transparent to express them in terms of the renormalized Green's functions with operators inserted. This way of evaluation also makes manifest the finiteness of these objects. To obtain these expressions, one simply uses the scaling equation Eq. (2.16) and solve for $\gamma_{Ma,Nb}$.

Let us discuss this in more detail. We first expand $\gamma_{Ma,Nb}$ and $\Gamma^{B,F}(O_{Nb})$ in powers of λ as in Eqs. (2.18) and (3.1) and collect the coefficients of each different power. Since we need $\gamma^{(0)}_{Ma,Nb}$ and $\gamma^{(1)}_{Ma,Nb}$ only, it is sufficient to expand up to order λ. We then obtain the following two sets of equations:

$$\left(m\frac{\partial}{\partial m} + \gamma_A\left(e\frac{\partial}{\partial e} - B\right) - F\gamma_e\right)\Gamma^{B,F}_{(0)}(O_{Ma})$$

$$- m(1 - 2\gamma_e)\Gamma^{B,F}_{(1)}(O_{Ma}) + \gamma^{(0)}_{Ma,Nb}\Gamma^{B,F}_{(0)}(O_{mb}) = 0 \tag{4.1}$$

and

$$\left(m\frac{\partial}{\partial m} + \gamma_A\left(e\frac{\partial}{\partial e} - B\right) - F\gamma_e + \beta^{(1)}_\lambda\right)\Gamma^{B,F}_{(1)}(O_{Ma}) - 2m(1 - 2\gamma_e)\Gamma^{B,F}_{(2)}(O_{Ma})$$

$$+ \gamma^{(0)}_{Ma,Mb}\Gamma^{B,F}_{(1)}(O_{Ma}) + m^{M-N-1}\gamma^{(1)}_{Ma,Nb}\Gamma^{B,F}_{(0)}(O_{Nb}) = 0 \quad . \tag{4.2}$$

Here we used the fact that $\gamma^{(0)}_{Ma,Nb}$ is block-diagonal. Since the treatment of Eq. (4.2) is entirely similar to that of Eq. (4.1), we shall only discuss the calculation of $\gamma^{(0)}_{Ma,Nb}$ to one-loop order through Eq. (4.1). To this order γ_e vanishes in the Landau gauge and it may be dropped in the subsequent equations.

Let us begin with $\gamma^{(0)}_{5a,5b}$'s. First consider the vertex function (i.e. B = 1, F = 2) and look at the linear part in the external momenta. Hereafter we shall use the symbols L.P. for the linear part and Q.P. for the quadratic part in the external momenta. $\Gamma^{1,2}_{\mu}(O_{51})$ and $\Gamma^{1,2}_{\mu}(O_{52})$ are then normalized to its free vertex values, which are respectively $e(2p_\mu + k_\mu)$ and $e(\gamma_\mu \not{k} - k_\mu)$. (See Appendix A.) Then from Eq. (4.1), we find

$$\text{L.P. } m\frac{\partial}{\partial\lambda}\Gamma^{1,2}(O_{51})\,|_{\lambda=0} = \gamma^{(0)}_{51,51}e(2p_\mu + k_\mu) + \gamma^{(0)}_{51,52}e(\gamma_\mu \not{k} - k_\mu) \tag{4.3}$$

and

$$\text{L.P. } m\frac{\partial}{\partial\lambda}\Gamma^{1,2}_{\mu}(O_{52})\,|_{\lambda=0} = \gamma^{(0)}_{52,51}e(2p_\mu + k_\mu) + \gamma^{(0)}_{52,52}e(\gamma_\mu \not{k} - k_\mu) \quad . \tag{4.4}$$

The relvant one-loop diagrams for computing the left-hand side of Eq. (4.3) and (4.4) are listed in Fig. 1. After a straightforward calculation one obtains

$$\text{L.P. } m\frac{\partial}{\partial\lambda}\Gamma^{1,2}_{\mu}(O_{51})\,|_{\lambda=0} = \frac{\alpha}{4\pi}\left\{6e(2p_\mu + k_\mu) + 4e(\gamma_\mu \not{k} - k_\mu)\right\} \tag{4.5}$$

$$\text{L.P. } m\frac{\partial}{\partial\lambda}\Gamma^{1,2}_{\mu}(O_{51})\,|_{\lambda=0} = \frac{\alpha}{4\pi}\left\{6e(2p_\mu + k_\mu) + 4e(\gamma_\mu \not{k} - k_\mu)\right\} \tag{4.6}$$

from which $\gamma^{(0)}_{5a,5b}$ can be simply read off.

By entirely parallel procedures, we may obtain $\gamma^{(0)}_{6a,6b}$. For example, to obtain $\gamma^{(0)}_{6a,6b}$ (b = 1 ∽ 4) to one-loop order, use

$$\text{Q.P. } m\frac{\partial}{\partial\lambda}\,\Gamma_\mu^{1,2}(O_{6a}) = \gamma^{(0)}_{6a,61}e\left\{\gamma_\mu(p+k)^2 + \not p(2p+k)_\mu\right\}$$

$$+\ \gamma^{(0)}_{6a,62}e(p\cdot k\gamma_\mu - p_\mu \not k)$$

$$+\ \gamma^{(0)}_{6a,63}e(k_\mu \not k - k^2\gamma_\mu)$$

$$+\ \gamma^{(0)}_{6a,64}ei\sigma_{\mu\nu}k^\nu \not p \qquad (4.7)$$

which is obtained again from Eq. (4.1)

The complete results for the relevant one-loop anomalous dimension matrix elements are listed in Table 1, together with the values of the other anomalous dimensions defined in Eq. (2.11).

V. SOLUTION OF THE SCALING EQUATIONS

Having computed the relevant anomalous dimension matrix elements to one-loop order, we are ready to perform the leading logarithm sum by solving the set of scaling equations derived in section III.

Let us first clarify the meaning of the leading logarithm in our problem. The lowest order diagram that contains a muon loop is the vacuum polarization diagram in the second order. This however has obviously no $\ln M^2/m^2$ since the electron field is not involved. It is in the 4th order where we first encounter a logarithm, namely the electron self-energy diagram of Fig. 2 . Therefore for our problem the leading logarithm is defined by a term of the form $\alpha(\alpha \ln M^2/m^2)^n$ $(n = 0, 1, 2\ldots)$.

We shall now solve the scaling equations. 1. Solution for $\xi^{(0)}_{6b}$'s: $\xi^{(0)}_{6b}$'s obey Eq. (3.14), which may be written as

$$\left(\frac{\partial}{\partial\kappa} - \beta_e \frac{\partial}{\partial e} + \gamma_6^T\right)\xi_6^{(0)} = 0 \tag{5.1}$$

where $\kappa \equiv \ln M/m$, $\xi_6^{(0)}$ is the column vector composed of $\xi_{6b}^{(0)}$'s, and γ_6 is the matrix connecting them. The eigenvalues of the matrix γ_6^T are easily found to be

$$\{\rho_a\}_{a=1,2,\ldots,8} = \frac{\alpha}{4\pi}\{0, 0, 4, 4, 16/3, 12, 12, -12\} \quad . \tag{5.2}$$

Let U be a matrix which diagonalizes γ_6^T, i.e.

$$(U^{-1}\gamma_6^T U)_{ab} = \rho_a \delta_{ab} \quad . \tag{5.3}$$

U may be chosen to be

$$U = \left(\begin{array}{c|c} U_1 & U_2 \\ \hline 0 & U_3 \end{array}\right) \tag{5.4}$$

where

$$U_1 = \begin{bmatrix} 1 & 0 & -3 & 0 & 3 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 1 & 4 \\ 1 & 0 & 1 & 0 & 3 \\ 0 & -2 & 0 & 0 & 8 \end{bmatrix}$$

$$U_2 = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 0 & 0 \\ 14 & 0 & -22 \\ 3 & 0 & 3 \\ 8 & 0 & 8 \end{bmatrix} , \quad U_3 = \begin{bmatrix} -90 & 0 & 468 \\ -45 & 0 & 0 \\ 0 & 78 & 78 \end{bmatrix} . \tag{5.5}$$

Its inverse then is of the form

$$U^{-1} = \left(\begin{array}{c|c} U_1^{-1} & -U_1^{-1} U_2 U_3^{-1} \\ \hline 0 & U_3^{-1} \end{array} \right) \tag{5.6}$$

with

$$U_1^{-1} = \frac{1}{16} \begin{bmatrix} 4 & -3 & -6 & 12 & -3 \\ 0 & 4 & 8 & 0 & -4 \\ -4 & 0 & 0 & 4 & 0 \\ 0 & -8 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 \end{bmatrix}$$

$$U_3^{-1} = \frac{1}{2340} \begin{bmatrix} 0 & -52 & 0 \\ -5 & 10 & 30 \\ 5 & -10 & 0 \end{bmatrix} . \tag{5.7}$$

After diagonalization, Eq. (5.1) becomes

$$\left(\frac{\partial}{\partial \kappa} - \beta_e \frac{\partial}{\partial e} + \rho_a \right) (U^{-1} \xi_6^{(0)})_a = 0 \qquad \text{(no sum over a)} , \tag{5.8}$$

the solution of which is

$$\left(U^{-1}\xi_6^{(0)}(\kappa, e)\right)_a = \left(U^{-1}\xi_6^{(0)}(0, \bar{e}(\kappa))\right)_a$$

$$\times \exp\left(-\int_e^{\bar{e}(\kappa)} \frac{\rho_a(e')}{\beta_e(e')} de'\right) \quad . \tag{5.9}$$

Here $\bar{e}(\kappa)$ is the running coupling constant defined, in the usual manner, by

$$\frac{\partial \bar{e}(\kappa)}{\partial \kappa} = \beta_e(\bar{e}(\kappa)) \quad , \quad \bar{e}(\kappa = 0) = e \tag{5.10}$$

and has the explicit form

$$\bar{e}^2(\kappa) = \frac{e^2}{1 - \frac{8}{3}\frac{\alpha}{4\pi}\kappa} \underset{\text{def}}{\equiv} e^2 y \quad , \tag{5.11}$$

where we have defined $y = \bar{e}^2/e^2$. Thus from Eq. (5.9), we obtain

$$\xi_{6a}^{(0)}(\kappa, e) = \sum_{b,c} U_{ab}\left[\exp - \int_e^{\bar{e}(\kappa)} \frac{\rho_b(e')de'}{\beta_e(e')}\right] U_{bc}^{-1}\xi_{6c}^{(0)}(0, \bar{e}(\kappa)) \quad . \tag{5.12}$$

For the leading log sum, we only need to compute $\xi_{6c}^{(0)}(0, \bar{e}(\kappa))$ to the lowest order, i.e. to $\mathcal{O}(\bar{e}^2(\kappa))$. Only $\xi_{65}^{(0)}$ is non-vanishing in that order and one has

$$\xi_{65}^{(0)}(0, \bar{e}(\kappa)) = \frac{\bar{\alpha}(\kappa)}{15\pi} = \frac{\alpha}{15\pi} y \quad . \tag{5.13}$$

A straightforward evaluation of Eq. (5.12) then yields the following result:

$$\xi_{61}^{(0)} = -\frac{3}{16}(y - y^{-1})\frac{\alpha}{15\pi}$$

$$\xi_{63}^{(0)} = -\frac{1}{4}(y - y^{-1})\frac{\alpha}{15\pi}$$

$$\xi_{64}^{(0)} = \xi_{61}^{(0)}$$

$$\xi_{65}^{(0)} = \frac{1}{2}(y + y^{-1})\frac{\alpha}{15\pi}$$

$$\text{all other } \xi_{6b}^{(0)}\text{'s} = 0 \text{ (i.e. they contain no leading logs)} \quad . \tag{5.14}$$

We have checked that, when expanded in powers of α, these results agree with the explicit calculation to $\mathcal{O}(\alpha^2)$.

2. Solutions for ξ_{5b}'s: Eqs. (3.13), (3.14), (3.22) and (3.23) may be written in the matrix form

$$\left(m\frac{\partial}{\partial m} + \beta_e\frac{\partial}{\partial e} - \gamma_5\right)\left(m\,\xi_5^{(0)}\right) = \left(m\,\xi_5^{(1)}\right) \tag{5.15}$$

$$\left(m\frac{\partial}{\partial m} + \beta_e\frac{\partial}{\partial e} - \gamma_5'\right)\xi_5^{(1)} = \frac{\alpha}{4\pi}12\begin{bmatrix}\xi_{61}^{(0)} \\ -\xi_{64}^{(0)}\end{bmatrix} \tag{5.16}$$

where

$$\xi_5^{(0)} = \begin{bmatrix}\xi_{51}^{(0)} \\ \xi_{52}^{(0)}\end{bmatrix} , \qquad \xi_5^{(1)} = \begin{bmatrix}\xi_{51}^{(1)} \\ \xi_{52}^{(1)}\end{bmatrix}$$

$$\gamma_5 = \frac{\alpha}{4\pi}\begin{bmatrix}6 & 6 \\ 4 & 4\end{bmatrix} , \qquad \gamma_5' = \frac{\alpha}{4\pi}\begin{bmatrix}12 & 6 \\ 4 & 10\end{bmatrix} \quad . \tag{5.17}$$

These equations are simultaneously diagonalized by a matrix V, i.e.,

$$V^{-1}\gamma_5 V = \frac{\alpha}{4\pi}\begin{pmatrix} 0 & 0 \\ 0 & 10 \end{pmatrix}$$

$$V^{-1}\gamma_5' V = \frac{\alpha}{4\pi}\begin{pmatrix} 6 & 0 \\ 0 & 16 \end{pmatrix}$$

with

$$V = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix} \quad , \quad V^{-1} = \frac{1}{5}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} \quad . \tag{5.18}$$

The resultant equations are

$$\left(m\frac{\partial}{\partial m} + \beta_e\frac{\partial}{\partial e} + 1\right)\left(2\xi_{51}^{(0)} - 3\xi_{52}^{(0)}\right) = 2\xi_{51}^{(1)} - 3\xi_{52}^{(1)} \tag{5.19a}$$

$$\left(m\frac{\partial}{\partial m} + \beta_e\frac{\partial}{\partial e} + 1 - 10\frac{\alpha}{4\pi}\right)\left(\xi_{51}^{(0)} + \xi_{52}^{(0)}\right) = \xi_{51}^{(1)} + \xi_{52}^{(1)} \tag{5.19b}$$

$$\left(m\frac{\partial}{\partial m} + \beta_e\frac{\partial}{\partial e} - 6\frac{\alpha}{4\pi}\right)\left(2\xi_{51}^{(1)} - 3\xi_{52}^{(1)}\right) = 12\frac{\alpha}{4\pi}\left(2\xi_{61} + 3\xi_{64}\right) \tag{5.20a}$$

$$\left(m\frac{\partial}{\partial m} + \beta_e\frac{\partial}{\partial e} - 16\frac{\alpha}{4\pi}\right)\left(\xi_{51}^{(1)} + \xi_{52}^{(1)}\right) = 12\frac{\alpha}{4\pi}\left(\xi_{61} - \xi_{64}\right) \quad . \tag{5.20b}$$

We shall first solve Eqs. (5.20). Consider the equation of the type above

$$\left(m\frac{\partial}{\partial m} + \beta_e\frac{\partial}{\partial e} - \gamma\right)A\left(\frac{m}{M}, e\right) = B\left(\frac{m}{M}, e\right) \quad . \tag{5.21}$$

Using the variable κ, it becomes

$$\left(\frac{\partial}{\partial\kappa} - \beta_e\frac{\partial}{\partial e} + \gamma\right)A(\kappa, e) = -B(\kappa, e) \quad . \tag{5.22}$$

This is easily integrated to yield

$$A(\kappa, e) = A(0, \bar{e}(\kappa)) \exp - \int_0^{\kappa} (\bar{e}(\kappa'))d\kappa'$$

$$- \int_0^{\kappa} d\kappa' B(\kappa - \kappa', \bar{e}(\kappa'))\exp - \int_0^{\kappa'} \gamma(\bar{e}(\kappa''))d\kappa'' \quad . \tag{5.23}$$

This formula, together with the expressions for $\xi_{61}^{(0)}$ and $\xi_{64}^{(0)}$ already obtained in (5.14), then gives

$$\xi_{51}^{(1)} = -\xi_{52}^{(1)} = \frac{\alpha}{15\pi}\left(\frac{3}{8}y - \frac{27}{8}y^{-1} + 3y^{-5/4}\right) \quad . \tag{5.24}$$

The leading log solutions to the remaining equations (5.19) are immediate if one notes that the differential operator $m\frac{\partial}{\partial m} + \beta_e \frac{\partial}{\partial e}$ turns a leading logarithm into a non-leading one. Thus consistency requires

$$2\xi_{51}^{(0)} - 3\xi_{52}^{(0)} = 2\xi_{51}^{(1)} - 3\xi_{52}^{(1)}$$

$$\xi_{51}^{(0)} + \xi_{52}^{(0)} = \xi_{51}^{(1)} + \xi_{52}^{(1)} \quad , \tag{5.25}$$

which in turn gives

$$\xi_{51}^{(0)} = -\xi_{52}^{(0)} = \xi_{51}^{(1)} \tag{5.26}$$

at the leading log level.

We have thus obtained complete leading logarithm sums for the physical coefficients $\xi_{5b}^{(0)}$ and $\xi_{6b}^{(0)}$. It is to be emphasized that, apart from the trivial calculation for $\xi_{65}^{(0)}$ in the second order, all the necessary information is provided by the light theory alone, which is a remarkable result.

VI. AN APPLICATION: EFFECTS OF MUON LOOPS ON THE ELECTRON ANOMALOUS MAGNETIC MOMENT

As an application of our formalism within QED, we shall now discuss the leading logarithm effect of muon loops upon the electron anomalous magnetic moment (g - 2)/2. Recall that for convenience we have defined the electron mass m and normalized Green's functions having external electron lines off the mass shell at zero momenta. Thus to compute the physical (g - 2)/2, we must account for on-shell corrections. These are of the following two types: (a) explicit mass correction and (b) finite multiplicative wave function correction for the relevant Green's functions. Let us denote by * the quantities defined on the mass shell. Then (g - 2)/2 is defined by (suppressing M-dependence)

$$\bar{u}(p')\Gamma^*_\mu(p, p', k, m^*)u(p) = e\bar{u}(p')\left[\gamma_\mu F_1(k^2) + \frac{i\sigma_{\mu\nu}k^\nu}{2m^*}\,\frac{g-2}{2}F_2(k^2)\right]u(p) \quad , \tag{6.1}$$

where, in the vertex function $\Gamma^*_\mu(p, p', k, m^*)$, p and p' are respectively the incoming and the outgoing electron momentum and k = p' - p is the incoming photon momentum. F_1 and F_2 are normalized by $F_1(0) = F_2(0) = 1$. $\Gamma^*_\mu(p, p', k, m^*)$ satisfies

$$\Gamma^*_\mu(p, p', k, m^*)\,\big|_{\not{p}=\not{p}'=m^*,k=0} = e\gamma_\mu \quad , \tag{6.2}$$

whereas the previously defined $\Gamma_\mu(p, p', k, m)$ is normalized by

$$\Gamma_\mu(p, p', k, m)\big|_{\not{p}=\not{p}'=k=0} = e\gamma_\mu \quad . \tag{6.3}$$

Similarly the inverse electron propagators $\Gamma^{*2,0}(\not{p}, m^*)$ and $\Gamma^{2,0}(\not{p}, m)$ satisfy

$$\Gamma^{*2,0}(\not{p}, m^*)\,|_{\not{p}=m^*} \equiv (\not{p} - m^* - \Sigma^*(\not{p}, m^*))\,|_{\not{p}=m^*} = 0$$

$$\frac{\partial}{\partial \not{p}}\Gamma^{*2,0}(\not{p}, m^*)\,|_{\not{p}=m^*} = 1$$

$$\Gamma^{2,0}(\not{p}, m)\,|_{\not{p}=0} = (\not{p} - m - \Sigma(\not{p}, m))\,|_{\not{p}=0} = -m$$

$$\frac{\partial}{\partial \not{p}}\Gamma^{2,0}(\not{p}, m)\,|_{\not{p}=0} = 1 \qquad . \tag{6.4}$$

We shall now discuss one by one the corrections (a) and (b).

(a) Mass correction: $\Gamma^{*2,0}$ and $\Gamma^{2,0}$ are related to each other by a finite multiplicative constant. From the normalization conditions (6.4) one then easily obtains the mass relations

$$m = (m^* + \Sigma^*(\not{p} = 0, m^*))/(1 - (\partial/\partial\not{p})\Sigma^*(\not{p} = 0, m^*))$$

$$\simeq m^*(1 + \mathcal{O}(\alpha) + \mathcal{O}(\alpha^2(m^*/M)^2)) \tag{6.5}$$

where the term $\mathcal{O}(\alpha)$ is due to light theory. When this is substituted into leading logarithm expressions, it only generates corrections which are non-leading. Thus to this level of accuracy, the explicit mass correction may be neglected entirely.

(b) Finite multiplicative correction: By virtue of multiplicative renormalization, we have

$$\Gamma^*_\mu(p, p', k, m^*) = z\,\Gamma_\mu(p, p', k, m) \qquad , \tag{6.6}$$

where z is a finite constant. Applying to the right-hand side the factorization formula and setting $\not{p} = \not{p}' = m^*$, $k = 0$, it becomes

$$e\gamma_\mu = z\Bigg\{ \Gamma_\mu^{\text{light}}(p, p', k, m) + \frac{m}{M^2}\sum_{5a} \xi_{5a}^{(0)}\left(\frac{M}{m}\right)\Gamma_\mu(O_{5a}) + \frac{1}{M^2}\sum_{6a}\xi_{6a}^{(0)}\left(\frac{M}{m}\right)\Gamma_\mu(O_{6a})\Bigg\}\Bigg|_{\not p=\not p'=m^*,k=0} \quad . \tag{6.7}$$

Since we are interested in the leading log result, z should also be computed to the same level. This allows us to use for $\Gamma_\mu(O_{5a})$ and $\Gamma_\mu(O_{6a})$ their free vertex values. Equation (6.7) then reads

$$\begin{aligned} e\gamma_\mu = z\Bigg\{ & \Gamma_\mu^{\text{light}}(p, p', k, m) \\ & + \frac{em}{M^2}\left[\xi_{51}^{(0)}(2p_\mu + k_\mu) + \xi_{52}^{(0)}\frac{\sigma_{\mu\nu}k^\nu}{i}\right] \\ & + \frac{e}{M^2}\Bigg[\xi_{61}^{(0)}(\gamma_\mu(p+k)^2 + \not p(2p_\mu + k_\mu)) \\ & + \xi_{62}^{(0)}(p\cdot k\gamma_\mu - p_\mu \not k) + \xi_{63}^{(0)}(k_\mu \not k - k^2\gamma_\mu) \\ & + \xi_{64}^{(0)}\left(-\frac{\sigma_{\mu\nu}k^\nu}{i}\not p\right)\Bigg]\Bigg\}\Bigg|_{\not p=\not p'=m^*,k=0} \quad . \end{aligned} \tag{6.8}$$

To solve for z, we apply the Gordon decomposition

$$\gamma_\mu = \frac{1}{2m^*}\{(p_\mu + p_\mu') + i\sigma_{\mu\nu}k^\nu\} \quad , \tag{6.9}$$

which is valid when sandwiched between $\bar{u}(p')$ and $u(p)$ as in Eq. (6.1). In the present case with $k = 0$, this simplifies to

$$\gamma_\mu = p_\mu/m^* \qquad . \tag{6.10}$$

Noting that

$$\Gamma_\mu^{\text{light}}(p, p', k, m)\,|_{\not p=\not p=m^*,k=0} = e\gamma_\mu + \mathcal{O}(\alpha) \qquad , \tag{6.11}$$

substitution of Eq. (6.10) into Eq. (6.8), after dropping the common factor ep_μ/m^*, yields

$$z = 1 - (m^*/M)^2\left(2\xi_{51}^{(0)} + 3\xi_{61}^{(0)}\right) + \mathcal{O}(\alpha) \qquad . \tag{6.12}$$

Therefore from Eq. (6.6) we obtain

$$\begin{aligned}
\Gamma_\mu^*(p, p', k, m^*)\,|_{\not p=\not p'=m^*,k^2=0} \simeq & \left[1 - \left(\frac{m^*}{M}\right)^2\left(2\xi_{51}^{(0)} + 3\xi_{61}^{(0)}\right)\right] \\
& \times \Big\{\Gamma_\mu^{\text{light}}(p, p', k, m) + \frac{em}{M^2}\left[\xi_{51}^{(0)}(2p_\mu + k_\mu) - \xi_{52}^{(0)} i\sigma_{\mu\nu}k^\nu\right] \\
& + \frac{e}{M^2}\Big[\xi_{61}^{(0)}(\gamma_\mu(p+k)^2 + \not p(2p_\mu + k_\mu)) \\
& + \xi_{62}^{(0)}(p\cdot k\,\gamma_\mu - p_\mu \not k) + \xi_{63}^{(0)}(k_\mu \not k - k^2\gamma_\mu) \\
& + \xi_{64}^{(0)} i\sigma_{\mu\nu}k^\nu \not p\Big]\Big\}\,|_{\not p=\not p'=m^*,k^2=0} \qquad ,
\end{aligned} \tag{6.13}$$

where we have dropped terms which do not contribute to the leading log. We may now bring this into the form of Eq. (6.1) (with $k^2 = 0$) by using the identities valid on the mass shell, viz.

$$\left\{\begin{aligned}
\not{k} &= 0 \\
2p_\mu + k_\mu &= p_\mu + p_\mu' = 2m^*\gamma_\mu - i\sigma_{\mu\nu}k^\nu \\
p\cdot k\gamma_\mu - p_\mu\not{k} &= \tfrac{1}{2}\left\{(p+k)^2 - p^2 - k^2\right\}\gamma_\mu = 0 \\
k_\mu\not{k} - k^2\gamma_\mu &= 0 \qquad .
\end{aligned}\right. \tag{6.14}$$

Projection of the magnetic part then yields

$$\begin{aligned}
&\text{magnetic part of } \Gamma_\mu^*(p, p', k, m^*)\big|_{\not{p}=\not{p}'=m^*, k^2=0} \\
&= e\,\frac{i\sigma_{\mu\nu}k^\nu}{2m^*}\left[1 - (m^*/M)^2\left(2\xi_{51}^{(0)} + 3\xi_{61}^{(0)}\right)\right] \\
&\times \left\{\mathcal{O}(\alpha) - 2(m^*/M)^2\left(\xi_{51}^{(0)} + \xi_{52}^{(0)} + \xi_{61}^{(0)} - \xi_{64}^{(0)}\right)\right\} \qquad .
\end{aligned} \tag{6.15}$$

Upon multiplying out the terms above and keeping only those containing the leading logs, the right-hand side becomes

$$-\frac{e}{2m^*}\, i\sigma_{\mu\nu}k^\nu (m^*/M)^2 2\left\{\left(\xi_{51}^{(0)} + \xi_{52}^{(0)}\right) + \left(\xi_{61}^{(0)} - \xi_{64}^{(0)}\right)\right\} \qquad . \tag{6.16}$$

But Eqs. (5.14) and (5.26) tell us that the leading logarithms precisely cancel in the above combinations of the coefficient functions. In other words, <u>there is no leading logarithms for (g - 2)/2 to all orders in e.</u>[6] The effect then is of the form $\alpha^2(\alpha\ln(M/m)^2)^n$, the assessment of which requires two loop calculations for the anomalous dimensions.

VII. SUMMARY AND CONCLUSION

In this article, taking QED as an example, we have carried out explicit calculations of effects of heavy particles (muons) in low energy physics using the general formalism developed in I. First the Callan-Symanzik like equations governing the heavy mass dependence of the universal coefficient functions ξ_{Na}'s were brought into a closed set of homogeneous equations, with the help of some all order identities and the appropriate use of the normalization conditions obeyed by the operator inserted Green's functions $\Gamma(O_{Na})$'s (section III). We then expressed the relevant anomalous dimensions appearing in these equations in terms of $\Gamma(O_{Na})$'s and evaluated them to one loop order (section IV). After diagonalizing the anomalous dimension matrices so obtained, the scaling equations were solved to produce leading logarithm expressions for the physical coefficient functions via "improved" perturbation expansion in the running coupling constant $\bar{e}(M/m)$ (section V). It should be emphasized that, aside from the trivial calculation of the second order vacuum polarization due to a muon loop, all the necessary information was furnished by the light theory alone, which contains no knowledge of the heavy sector. This feature is as remarkable as it is pleasing. As an application of the results obtained, we examined the leading effects of muon loops upon the electron anomalous magnetic moment $(g-2)/2$ (section VI). On-mass-shell corrections required due to our definitions of m and Green's functions at zero momenta were taken into account and it was found that the leading effects precisely cancel to all orders in e. Although somewhat unfortunate as it was, it clearly demonstrates the practical capability of our formalism developed and elaborated in this and a previous paper of ours.[1]

As was mentioned in the concluding section of I, application of our method, suitably modified, to asymptotically free theories such as QCD should prove even more useful, since the running coupling constant $\bar{g}^2(M/m)$ decreases as M/m increases. Such extension is currently under investigation.

ACKNOWLEDGMENTS

One of us (Y.K.) would like to thank Professor T.L. Trueman and the members of the theory group at Brookhaven National Laboratory for hospitality, where a part of this work was carried out. This work has been partially supported by the U.S. Dept. of Energy.

APPENDIX A. FREE VERTEX FACTORS FOR O_{Na} AND NORMALIZATION CONVENTION FOR OPERATOR INSERTED GREEN'S FUNCTIONS

In this appendix, we shall list the free vertex factors for O_{Na}'s (Feynman rules) and specify the normalization convention for amputated proper Green's functions with O_{Na}'s inserted, $\Gamma^{B,F}(O_{Na})$.

We only need to consider up to 5-point functions. Free vertex factors are obtained by Fourier transforming the following free field matrix elements after amputating free propagator legs $<\psi_\alpha\overline{\psi}_\beta>$'s and $<A_\mu A_\nu>$'s:

$$
\begin{array}{lll}
B = 0 \;, & F = 2 & <\psi_\alpha\overline{\psi}_\beta O_{Na}> \\[2ex]
B = 2 \;, & F = 0 & <A_\mu A_\nu O_{Na}> \\[2ex]
B = 1 \;, & F = 2 & <\psi_\alpha\overline{\psi}_\beta A_\mu O_{Na}> \\[2ex]
B = 2 \;, & F = 2 & <\psi_\alpha\overline{\psi}_\beta A_\mu A_\nu O_{Na}> \\[2ex]
B = 3 \;, & F = 2 & <\psi_\alpha\overline{\psi}_\beta A_\mu A_\nu A_\lambda O_{Na}> \\[2ex]
B = 0 \;, & F = 4 & <\overline{\psi}_{\alpha_1}\overline{\psi}_{\alpha_2}\psi_{\alpha_3}\psi_{\alpha_4} O_{Na}> \quad .
\end{array}
\tag{A.1}
$$

These are listed in Fig. 3.

To define $\Gamma^{B,F}(O_{Na})$, we must amputate <u>full</u> propagator legs. Thereupon the sign of the 2-point functions are changed relative to the free vertices given in Fig. 3. Specifically, we have

$$\Gamma^{0,2}(O_{Na}) = \frac{1}{i}G^{0,2}(O_{Na})_A \tag{A.2}$$

$$\Gamma^{2,0}_{\mu\nu}(O_{Na}) = \frac{1}{i} G^{2,0}_{\mu\nu}(O_{Na})_A \tag{A.3}$$

where $G^{B,F}(O_{Na})_A$ means the proper Green's function obtained with the vertices of Fig. 3 with free light external lines amputated. In the same notation, all the other Green's functions are defined similarly with 1/i replaced by i.

Several examples should be helpful

$$\Gamma^{0,2}(O_{31}) = -1 + \text{higher order}$$

$$\Gamma^{0,2}(O_{41}) = \not{p} + \text{higher order}$$

$$\Gamma^{2,0}_{\mu\nu}(O_{42}) = (-g_{\mu\nu}k^2 + k_\mu k_\nu)(1 + \text{higher order})$$

$$\Gamma^{1,2}_{\mu}(O_{41}) = e\gamma_\mu + \text{higher order}$$

$$\Gamma^{1,2}_{\mu}(O_{51}) = e(2p_\mu + k_\mu) + \text{higher order}$$

etc., where "higher order" means higher order in momenta and λ.

APPENDIX B. PROOF OF THE IDENTITIES EQ. (3.7)-EQ. (3.9)

In this appendix we shall prove the useful identities Eq. (3.7)-Eq. (3.9) which were crucial in decoupling the renormalization group equations.

(i) First consider $\Gamma^{B,F}(O_{42})$. Given a diagram contributing to $\Gamma^{B,F}$, an insertion of O_{42} may be made to every internal photon line, with the sole effect of changing the sign of the whole diagram. Let V be the number of vertices. Then the number of internal photon lines b is given by $b = (V - B)/2$. Since the operator $e\frac{\partial}{\partial e}$ precisely counts the number of vertices, we can immediately write

$$\Gamma^{B,F}(O_{42}) = -\frac{1}{2}\left(e\frac{\partial}{\partial e} - B\right)\Gamma^{B,F} \tag{B.1}$$

O_{42} being an operator of dimension 4, renormalization is performed in identical fashion for both sides of Eq. (B.1). Thus it is valid for renormalized Green's functions. For two-point photon function, the trivial 0th order diagram has no internal photon line. However it is easily checked that Eq. (B.1) holds for this case as well.

(ii) Next, we shall examine $\Gamma^{B,F}(O_{31})$. Before the subtractions are made, clearly

$$\Gamma^{B,F}_{un}(O_{31}) = \frac{\partial}{\partial\lambda}\Gamma^{B,F}_{un} \tag{B.2}$$

holds. We have to check that after renormalization, this relation remains intact. By power counting, only the electron 2-point function $\overline{\Gamma}^{0,2}(O_{31})$ is (logarithmically) divergent, where bar means that the internal subtractions are already performed. Therefore the renormalized Green's function $\Gamma^{0,2}(O_{31})$ is

$$\Gamma^{0,2}(O_{31}) = \overline{\Gamma}^{0,2}(O_{31}) - \overline{\Gamma}^{0,2}(O_{31})\big|_{\not{p}=\lambda=0} \quad . \tag{B.3}$$

We now prove Eq. (B.2) for renormalized quantities by recursive reasoning. Let us assume $\bar{\Gamma}^{0,2}(O_{31}) = \frac{\partial}{\partial\lambda}\bar{\Gamma}^{0,2}$ holds, which is easily checked for second order. Then

$$\Gamma^{0,2}(O_{31}) = \frac{\partial}{\partial\lambda}\bar{\Gamma}^{0,2} - \frac{\partial}{\partial\lambda}\bar{\Gamma}^{0,2}\Big|_{\not p=\lambda=0} \quad . \tag{B.4}$$

But since

$$\Gamma^{0,2} = \bar{\Gamma}^{0,2} - \bar{\Gamma}^{0,2}\Big|_{\not p=\lambda=0} - \not p\frac{\partial}{\partial\not p}\bar{\Gamma}^{0,2}\Big|_{\not p=\lambda=0} - \lambda\frac{\partial}{\partial\lambda}\bar{\Gamma}^{0,2}\Big|_{\not p=\lambda=0} \tag{B.5}$$

we get

$$\frac{\partial}{\partial\lambda}\Gamma^{0,2} = \frac{\partial}{\partial\lambda}\bar{\Gamma}^{0,2} - \frac{\partial}{\partial\lambda}\bar{\Gamma}^{0,2}\Big|_{\not p=\lambda=0} \quad . \tag{B.6}$$

From Eq. (B.4) and Eq. (B.6), we obtain

$$\Gamma^{0,2}(O_{31}) = \frac{\partial}{\partial\lambda}\Gamma^{0,2} \quad . \tag{B.7}$$

Upon substituting this into the relevant part of a general diagram, one may easily conclude

$$\Gamma^{B,F}(O_{31}) = \frac{\partial}{\partial\lambda}\Gamma^{B,F} \tag{B.8}$$

for any B and F.

(iii) Finally we consider $\Gamma^{B,F}(O_{41})$. It is convenient to deal with the combination

$$O \equiv O_{41} + (m+\lambda)\,O_{31} \quad . \tag{B.9}$$

The free vertices of the operator O are $i(\not{p} - m - \lambda)$ for two fermion vertex and $-ie\gamma_\mu$ for fermion-fermion-photon vertex. Thus, given a diagram contributing to $\Gamma^{B,F}$, we may obtain diagrams contributing to $\Gamma^{B,F}(O)$ in the same order in e by either (a) inserting the vertex $i(\not{p} - m - \lambda)$ into an internal fermion line, or (b) regarding a vertex $-ie\gamma_\mu$ as an insertion of O. Both procedures give back the same diagram except with a minus sign for (a). Thus we obtain V - f original diagrams, where f is the number of internal fermion lines. Using the topological relation $2V = 2f + F$, this number is equal to F/2. So for the unrenormalized functions we have

$$\Gamma^{B,F}_{un}(O) = \Gamma^{B,F}_{un}(O_{41}) + \Gamma^{B,F}_{un}((m+\lambda)O_{31}) = (F/2)\Gamma^{B,F}_{un} \quad . \tag{B.10}$$

(Again it is easily checked that the relation above holds for the trivial free diagrams as well, which do not have any internal lines.) Renormalization, however, alters this simple relation in a non-trivial way. $\Gamma^{B,F}((m+\lambda)O_{31})$ is renormalized in the same manner as $\Gamma^{B,F}(O_{41})$. This means, after renormalization,

$$\Gamma^{B,F}(O_{41}) + \lambda\Gamma^{B,F}(O_{31}) + m\Gamma^{B,F}(O^{(1)}_{31}) = (F/2)\Gamma^{B,F} \tag{B.11}$$

where $O^{(1)}_{31}$ is made finite by <u>once over subtraction</u>. Thus it is necessary to re-express $O^{(1)}_{31}$ in terms of minimally subtracted operators as we described in the appendix of I. (See section 4-3 in particular.) Applying the procedure detailed there, one easily obtains the result

$$m\Gamma^{B,F}(O_{31}^{(1)}) = \frac{\partial}{\partial\lambda}\Gamma^{B,F}\left(m + m\lambda\left(\frac{\partial}{\partial\lambda}\right)^2 \Gamma^{0,2}\Big|_{\not{p}=\lambda=0}\right)$$

$$- \Gamma^{B,F}(O_{41}) m \frac{\partial}{\partial\not{p}}\frac{\partial}{\partial\lambda}\Gamma^{0,2}\Big|_{\not{p}=\lambda=0}$$

$$- \Gamma^{B,F}(O_{42}) m \frac{\partial}{\partial\lambda}\pi\Big|_{k^2=\lambda=0} \quad . \tag{B.12}$$

The derivatives of the Green's functions appearing in this equation are nothing but certain combinations of the anomalous dimensions defined in Eq. (2.11). Substitution of these expressions then yields

$$m\Gamma^{B,F}(O_{31}^{(1)}) = \left(m - \lambda\,\frac{\beta_\lambda^{(1)} - 2\gamma_e}{1 - 2\gamma_e}\right)\frac{\partial}{\partial\lambda}\Gamma^{B,F}$$

$$+ \frac{2\gamma_e}{1 - 2\gamma_e}\Gamma^{B,F}(O_{41}) + \frac{2\gamma_A}{1 - 2\gamma_e}\Gamma^{B,F}(O_{42}) \quad . \tag{B.13}$$

From Eqs. (B.1), (B.8), (B.11) and (B.13), we may solve for $\Gamma^{B,F}(O_{41})$, which gives Eq. (3.8) quoted in the text.

REFERENCES

[1] Yoichi Kazama and York-Peng Yao, University of Michigan report UMHE79-16, to be published. See also UMHE79-24 for a summary of I and II (present paper).

[2] C. Callan, Phys. Rev. D2, 1541 (1970); K. Symanzik, Comm. Math. Phys. 18, 227 (1970).

[3] C.K. Lee, Stanford Ph.D. Thesis (1975), unpublished. See also S.L. Adler and W.A. Bardeen, Phys. Rev. D4, 3045 (1971).

[4] N.N. Bogoliubov and D.V. Shirkov, Introduction to the Theory of Quantized Fields (Interscience Publisher, N.Y., 1959) and references cited in p. 330 thereof. K. Hepp, Comm. Math. Phys. 1, 95 (1965); Théorie de la Renormalisation (Springer, Berlin, 1969).

[5] W. Zimmerman in Lectures on Elementary Particles and Quantum Field Theory, edited by S. Deser et al. (MIT Press, Cambridge, Mass. 1971), Vol. I, p. 397.

[6] We have explicitly checked this cancellation to fourth order in e. What we have shown here is that it is not accidental to this order but rather a general result to all orders.

γ_A	4/3	$\beta_\lambda^{(0)}$	0
γ_e	0	$\beta_\lambda^{(1)}$	-6

$\gamma_{5a,5b}$

5a \ 5b	51	52
51	6	4
52	6	4

$\gamma^{(0)}_{6a,6b}$

6a \ 6b	61	62	63	64	65	66	67	68
61	3	0	0	-1	0	0	0	0
62	1	4	-2/3	1	8/3	0	0	0
63	2	0	8/3	2	16/3	0	0	0
64	-3	0	0	1	0	0	0	0
65	1	0	4/3	1	8/3	0	0	0
66	0	0	2/3	0	0	-12	0	-4
67	0	0	-4	0	0	48	12	8
68	0	0	0	0	0	0	0	12

$\gamma^{(1)}_{6a,5b}$

6a \ 5b	51	52
61	12	0
62	0	0
63	0	0
64	0	-12
65	0	0
66	0	0
67	0	0
68	0	0

Table 1. Relevant anomalous dimensions computed to one loop order in units of $\alpha/4\pi$.

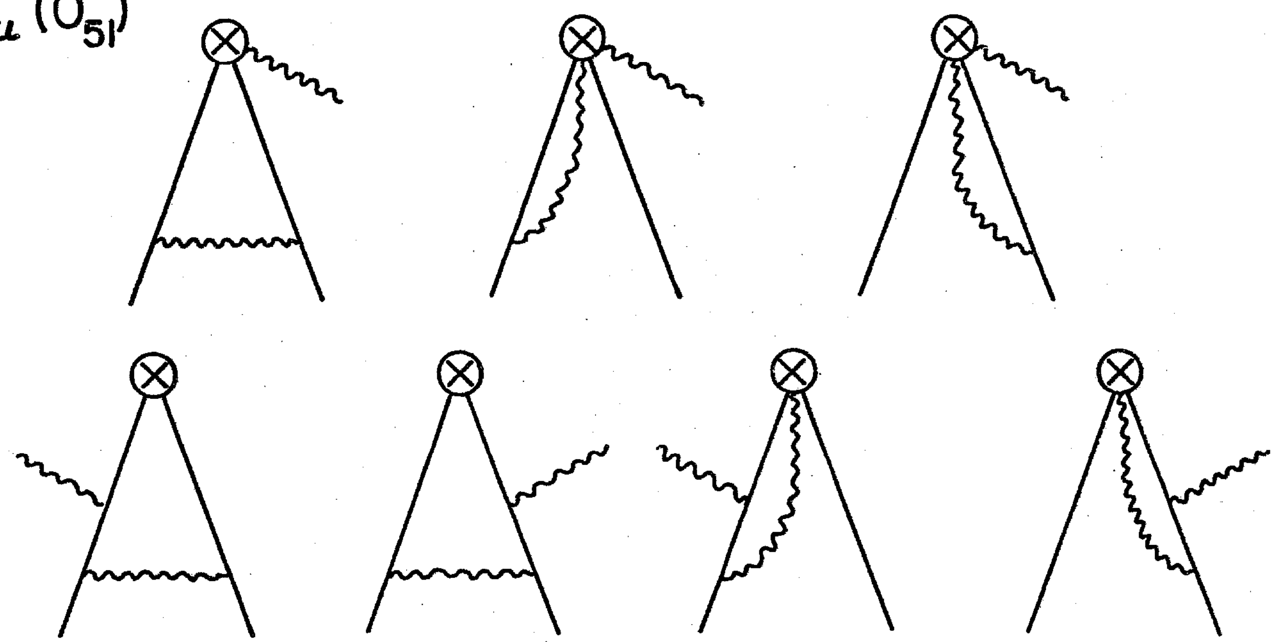
FIGURE CAPTIONS

Fig. 1: Diagrams contributing to the one-loop anomalous dimensions $\gamma^{(0)}_{5a,5b}$. Similar diagrams needed to compute $\gamma^{(0)}_{6a,6b}$ and $\gamma^{(1)}_{6a,5b}$ are easily obtained with the help of the list of free vertices given in Fig. 2.

Fig. 2: Fourth order electron self-energy diagram with a muon loop.

Fig. 3: List of the free vertices of the gauge invariant operators O_{Na}.

$\Gamma_{\mu}^{1,2}(O_{51})$



$\Gamma_{\mu}^{1,2}(O_{52})$

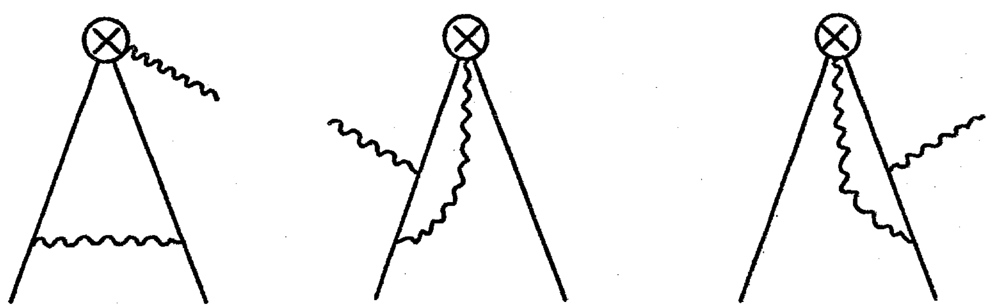


Fig. 1

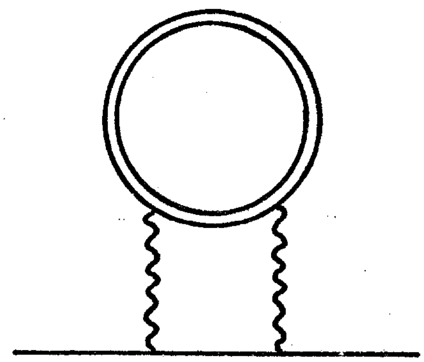
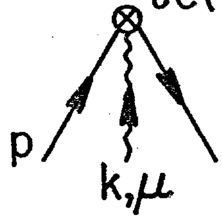


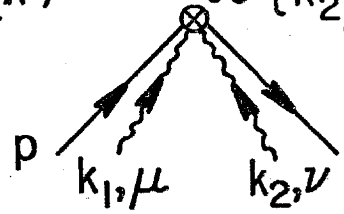
Fig. 2

$O_{62} : ie\,\bar{\psi}\,F_{\mu\nu}\gamma^{\nu}D^{\mu}\psi$

$$ie(-\gamma_\mu\, k\cdot p + p_\mu \not{k})$$

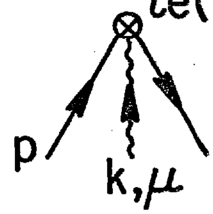


$$ie^2\{k_{2\mu}\gamma_\nu + k_{1\nu}\gamma_\mu - g_{\mu\nu}(\not{k}_1 + \not{k}_2)\}$$



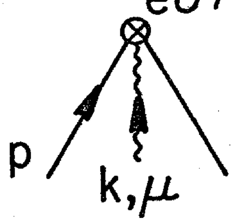
$O_{63} : ie\,\bar{\psi}\gamma^{\mu}(\partial^{\nu}F_{\mu\nu})\psi$

$$ie(k^2\gamma_\mu - \not{k}\,k_\mu)$$

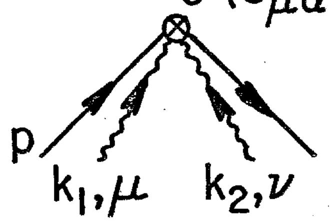


$O_{64} : \dfrac{e}{2}\,\bar{\psi}\,F_{\mu\nu}\sigma^{\mu\nu}\not{D}\psi$

$$e\sigma^{\mu\nu}k_\nu \not{p}$$

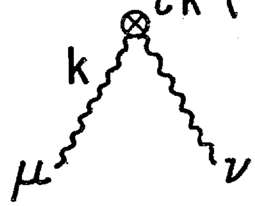


$$-e^2(\sigma_{\mu\alpha}k_1^{\alpha}\gamma_\nu + \sigma_{\nu\alpha}k_2^{\alpha}\gamma_\mu)$$



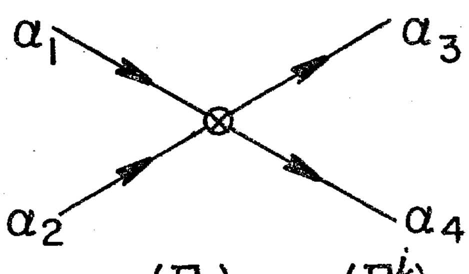
$O_{65} : \dfrac{i}{4}\,F_{\mu\nu}\,\partial^2 F^{\mu\nu}$

$$ik^2(-g_{\mu\nu}k^2 + k_\mu k_\nu)$$



$O_{66} \sim O_{68} : i\dfrac{e^2}{2}\bar{\psi}\,\Gamma_i\,\psi\bar{\psi}\,\Gamma^i\psi$

$\Gamma_i = 1, \gamma_\mu, \dfrac{\sigma_{\mu\nu}}{\sqrt{2}}$



$$e^2\{(\Gamma_i)_{\alpha_4\alpha_1}(\Gamma^i)_{\alpha_3\alpha_2} - (\Gamma_i)_{\alpha_4\alpha_2}(\Gamma^i)_{\alpha_3\alpha_1}\}$$

Fig. 3 cont'd

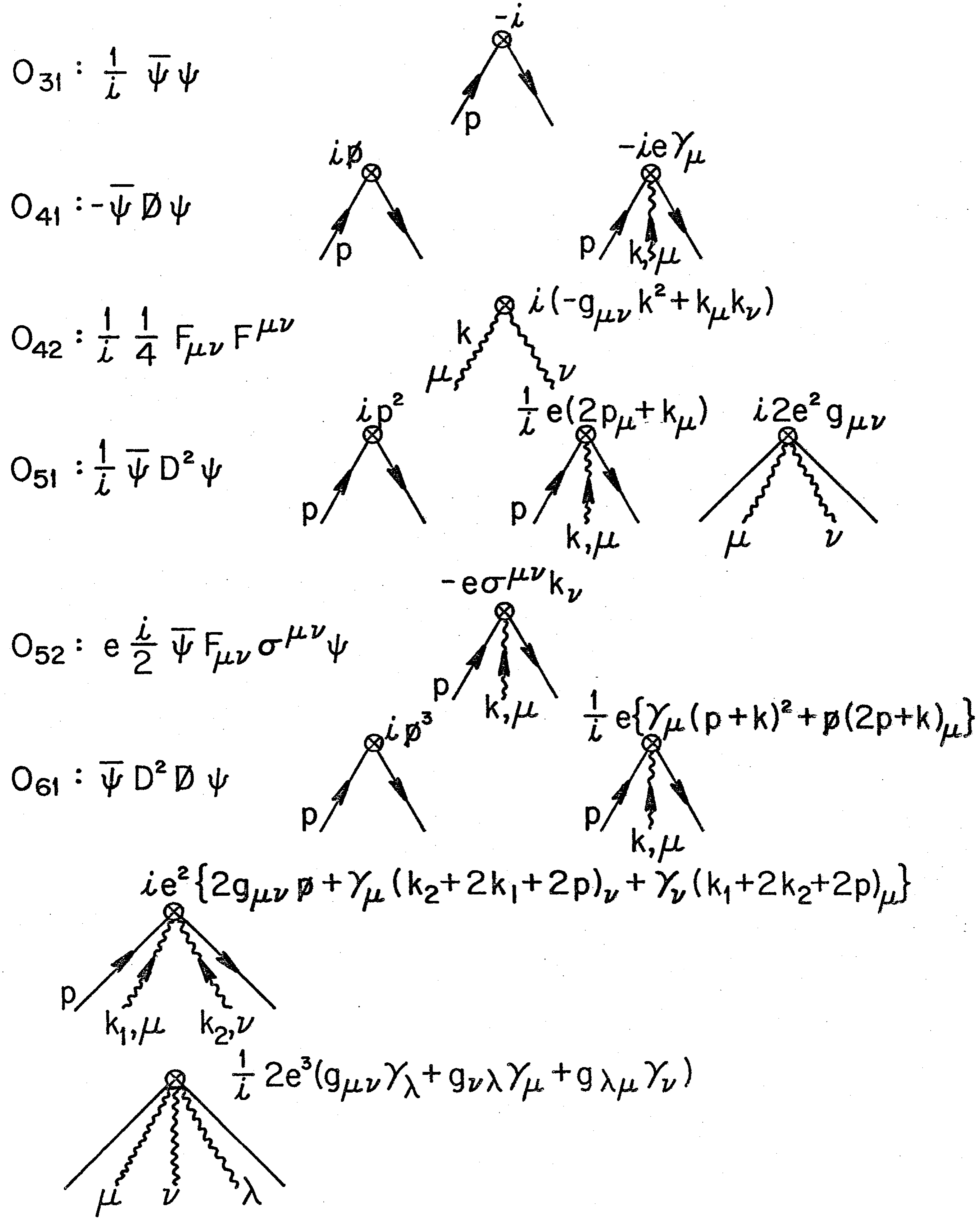

$O_{31}: \frac{1}{i}\,\bar{\psi}\psi$
$-i$
p
$O_{41}: -\bar{\psi}\not{D}\psi$
$i\not{p}$
p
$-ie\gamma_\mu$
p
k, μ
$O_{42}: \frac{1}{i}\,\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu}$
$i(-g_{\mu\nu}k^2 + k_\mu k_\nu)$
k
μ
ν
$O_{51}: \frac{1}{i}\,\bar{\psi}D^2\psi$
ip^2
p
$\frac{1}{i}\,e(2p_\mu + k_\mu)$
p
k, μ
$i2e^2 g_{\mu\nu}$
μ
ν
$O_{52}: e\,\frac{i}{2}\,\bar{\psi}F_{\mu\nu}\sigma^{\mu\nu}\psi$
$-e\sigma^{\mu\nu}k_\nu$
p
k, μ
$O_{61}: \bar{\psi}D^2\not{D}\psi$
$i\not{p}^3$
p
$\frac{1}{i}\,e\{\gamma_\mu(p+k)^2 + \not{p}(2p+k)_\mu\}$
p
k, μ
$ie^2\{2g_{\mu\nu}\not{p} + \gamma_\mu(k_2 + 2k_1 + 2p)_\nu + \gamma_\nu(k_1 + 2k_2 + 2p)_\mu\}$
p
k_1, μ
k_2, ν
$\frac{1}{i}\,2e^3(g_{\mu\nu}\gamma_\lambda + g_{\nu\lambda}\gamma_\mu + g_{\lambda\mu}\gamma_\nu)$
μ
ν
λ

Fig. 3