Physics Letters B 804 (2020) 135390

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

A walking dilaton inflation

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ARTICLE INFO

Article history:
Received 7 January 2020
Received in revised form 12 February 2020
Accepted 18 March 2020
Available online 23 March 2020
Editor: G.F. Giudice

Keywords:
Small field inflation
Scale invariance
Many flavor QCD

ABSTRACT

We propose an inflationary scenario based on a many-flavor hidden QCD with eight flavors, which realizes the almost scale-invariant (walking) gauge dynamics. The theory predicts two types of composite (pseudo) Nambu-Goldstone bosons, the pions and the lightest scalar (dilaton) associated with the spontaneous chiral symmetry breaking and its simultaneous violation of the approximate scale invariance. The dilaton acts as an inflaton, where the inflaton potential is induced by the nonperturbative-scale anomaly linked with the underlying theory. The inflaton potential parameters are highly constrained by the walking nature, which are evaluated by straightforward nonperturbative analyses including lattice simulations. Due to the pseudo Nambu-Goldstone boson's natures and the intrinsic property for the chiral symmetry breaking in the walking gauge dynamics, the inflaton coupled to the pions naturally undergoes the small field inflation consistently with all the cosmological and astrophysical constraints presently placed by Planck 2018 data. When the theory is vector-likely coupled to the standard model in part in a way to realize a dynamical electroweak symmetry breaking, the reheating temperature is determined by the pion decays to electroweak gauge bosons. The proposed inflationary scenario would provide a dynamical origin for the small field inflation as well as the light pions as a smoking-gun to be probed by future experiments.

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1. Introduction

Exponentially expanded cosmological evolution, inflation, is one of the most attractive and plausible period of our Universe to dynamically and simultaneously solve crucial cosmological problems, such as the flatness, the homogeneous and isotropic, and the horizon problems. The dynamics itself would have no doubt that we have experienced such an epoch, however, details of inflation have not been revealed so far both theoretically and experimentally at all.

As one of candidate scenarios for the inflation, models having a log-type potential can be good targets. Since the log-type potential naturally has a plateau, it can be easily applied to the inflation. Such a log-type potential can be given in the context of the Coleman-Weinberg (CW) mechanism [1] where quantum loop contributions modify the shape of the potential to be flatten around at the origin, which is called the CW inflation [2]. However, it is in general quite difficult to satisfy all the cosmological parameters fitted by cosmic microwave background (CMB) observations such as the Planck satellite [3]. Some recent developments have been made where the small field inflation (SFI) of the CW type can be achieved without conflicting with the CMB observations by introducing a linear term originated to a fermion condensation [4] and a dynamical origin of the initial field value [5]. Still, it is however unavoidable to naively assume the quartic coupling of the inflaton to be extremely small, in order to realize the observed amplitude of the scalar perturbation.

At this moment, one should notice that the inflaton quartic coupling λ_χ can be expressed by the ratio of the inflaton mass (m_χ) to the vacuum expectation value of the inflaton (v_χ), $\lambda_\chi \sim (m_\chi/v_\chi)^2$, with the stationary condition taken into account. This implies that the tiny enough λ_χ is directly linked to a large enough scale hierarchy between m_χ and v_χ. Then, an interesting question would be raised: *"Can this large hierarchy be physical in a sense of quantum field theory?"*

It is a walking gauge theory that can naturally supply such a large scale hierarchy, which is characterized by the "journey-distance" during the walking (i.e. almost scale-invariant) behavior, from an ultraviolet (UV) scale ($\Lambda_{\rm UV}$) down to an infrared (IR) scale ($\Lambda_{\rm IR}$).

For instance, in the case of many flavor QCD with the SU(N) group coupled to fermions (F) belonging to the fundamental representation, the walking regime can be established reflecting the

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https://doi.org/10.1016/j.physletb.2020.135390
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perturbative IR fixed point, well known as the Caswell-Banks-Zaks IR fixed point [6,7]. In that case, the fermion dynamical mass scale (m_F) is expected to be generated from the UV scale $\Lambda_{\rm UV}$ having the characteristic scaling relation, called Miransky scaling [8] (sometimes also called Berezinsky-Kosterlitz-Thouless scaling) intrinsic to the conformal phase transition [9]: $m_F \sim \Lambda_{\rm UV} e^{-\pi/\sqrt{\alpha/\alpha_c - 1}}$ (in the chiral broken phase), where α is the fine structure constant of the gauge theory and α_c denotes the critical coupling above which the chiral symmetry breaking takes place. This scaling surely realizes a large scale hierarchy in the chiral broken phase during the walking regime, spanned by the IR m_F and the UV $\Lambda_{\rm UV}$ scales, where the $\Lambda_{\rm UV}$ can be identified as the critical scale at which the m_F is generated.

The lightest composite scalar, so-called walking dilaton, arises as the consequence of the spontaneous breaking of the (approximate) scale invariance, and simultaneously gets massive due to the explicit breaking induced by the scale anomaly [10] arising from the Miransky scaling above: $\beta(\alpha) = \partial\alpha/\partial \ln \Lambda_{\rm UV} \sim -\frac{\alpha_c}{\pi}(\alpha/\alpha_c - 1)^{3/2}$. QCD with eight flavors has been confirmed by lattice simulations to be walking with the chiral broken phase [11–13]. In that case, it has been observed on lattices [14–16] that the walking dilaton formed by a flavor singlet bilinear $\bar{F}F$ can be as light as or less than the chiral symmetry breaking scale m_F, in accordance with the expected particle identity as a pseudo Nambu-Goldstone (pNG) boson for the scale symmetry breaking. Furthermore, it has been found [14,15] that the dilaton decay constant (f_χ) can be much larger than the m_F, in accord with a many-flavor version of the Veniziano limit, dubbed anti-Veneziano limit [17]: $f_\chi \sim \sqrt{NN_F} m_F$ for $N\alpha =$ fixed, $N_F/N =$ fixed $\gg 1$, and $N, N_F \to \infty$. Hence the f_χ, dictated by the scale-chiral breaking, would be on the order of the chiral-critical scale $\Lambda_{\rm UV}$ above. Thus one would have a large scale hierarchy between $m_\chi \lesssim m_F$ and $f_\chi \sim \Lambda_{\rm UV}$, via the Miranksky scaling, $m_\chi \sim f_\chi e^{-\pi/\sqrt{\alpha/\alpha_c - 1}}$, which has indeed been supported by some straightforward Schwinger-Dyson equation-analysis on many-flavor walking gauge theories [18].

Note that the dilaton potential is also generated by the scale anomaly driven by the chiral symmetry breaking, and is fixed by the anomalous Ward-Takahashi identity for the scale symmetry [17,19,20], to generically take the form of CW type,[1] like $V_\chi \sim \lambda_\chi \chi^4 \ln\chi$ with $\lambda_\chi \sim (m_\chi/v_\chi)^2$ and v_χ being identified as the dilaton decay constant f_χ above ($v_\chi \equiv f_\chi$). Thus, one undoubtedly expects that, with the walking dilaton identified as an inflaton, a tiny enough quartic coupling, $\lambda_\chi \sim (m_\chi/v_\chi)^2 \equiv (m_\chi/f_\chi)^2 \sim (m_\chi/\Lambda_{\rm UV})^2 \ll 1$, desired for the consistent inflation, would naturally and dynamically be generated by the walking gauge dynamics, when the IR and UV scales are associated with m_χ and v_χ, respectively.

A possible inflationary history goes like: We start from the chiral-scale broken phase in the vacuum for a many flavor walking gauge theory, where we have the walking dilaton and its potential of a CW type. Allowing some coupling between the walking fermions and some scalar, e.g. the standard model (SM)-like Higgs, a preheating mechanism [21] would work to trap the walking dilaton around the origin of the potential – ("approximate") chiral-scale symmetric point – due to an induced-finite matter-density effect (like a thermal plasma) triggered by parametric resonances, as proposed in [5]. As the temperature cools down, the walking dilaton is going slowly to roll the potential-down hill, i.e. undergoes the SFI due to the underlying walking nature, which governs the evolution of Universe at that time. After ending the inflation (almost in the same manner as in the CW inflation), the dilaton starts to drop down to the vacuum $\chi = v_\chi$, oscillate and will keep reheating until it decays. The reheating would work unless the created temperature gets higher than the critical temperature(s) for the chiral phase transition and/or deconfinement phase transition – expected to be around the temperature $\sim m_F$ – above which the walking dilaton gets disassociated to cease having the potential.

In this paper, we present a dynamical inflationary scenario of the CW-SFI type arising from eight-flavor QCD (a large N_F walking gauge theory). The walking dilaton plays the role of an inflaton, where the inflaton potential parameters are highly constrained by the walking nature. We evaluate the potential parameters using outputs from straightforward nonperturbative analyses including lattice simulations. It is shown that with a tiny dilaton coupling to pions explicitly breaking the chiral-scale symmetry, the walking dilaton inflation resolves a well-known incompatibility intrinsic to the SFI of the CW type for realizing the desired e-folding number and the observed spectral index [2,29]. This makes the proposal in [4] explicitized by a concrete dynamics as the many-flavor walking gauge theory.

To be more realistic, the inflationary scenario is fitted with all the cosmological and astrophysical constraints presently placed by Planck 2018 data [3] and theoretical requirements on the walking dilaton and chiral pion physics. As a reference scenario, the hidden walking dynamics is vector-likely gauged in part by the electroweak (EW) charges and is coupled to a Higgs doublet in a scale-invariant way. Thus the EW symmetry breaking (EWSB) is triggered by the bosonic seesaw mechanism [30–41], as a high-scale dynamical scalegenesis, in which the gauge hierarchy problem is possibly absent. We find that the present inflationary scenario is highly constrained to give a stringent bound on the fermion dynamical mass scale (m_F) to be greater than 10^{11} GeV, and the reheating temperature (T_R) is then fixed by chiral pion decays to EW gauge bosons including photons to be $\gtrsim 10^2$ GeV, where the masses of walking dilaton and pions are also constrained by several theoretical and astrophysical limits to be $\gtrsim 10^8$ GeV and $\gtrsim 10^5$ GeV, respectively. Since $T_R \ll m_F$, this reheating thus works consistently with the deconfinement and or chiral phase transitions in the walking gauge theory, as noted above.

Thus the presently proposed inflationary scenario of the CW-SFI type would provide the dynamical origin and explanation for the tiny inflaton coupling and somewhat light particles (the walking pions) as a smoking-gun to be probed by future experiments.

2. Walking dilaton potential

We begin by writing the dilaton potential induced from a generic many-flavor QCD, which takes the form including the CW type:

$$V(\chi) = -\frac{C}{2N_F}\chi^a \mathrm{tr}[U + U^\dagger] + \frac{\lambda_\chi}{4}\chi^4\left(\ln\frac{\chi}{v_\chi} + A\right) + V_0 \,. \tag{1}$$

V_0 is the vacuum energy, which is determined by taking the normalization of the potential as $V_0(v_\chi) = 0$.

The C term in Eq. (1) has come from the explicit-chiral breaking (flavor universal) mass term for the hidden QCD fermions,

$$\mathcal{L}_m = -m_0 \sum_{i=1}^{N_F} \bar{F}_i F_i \,, \tag{2}$$

(with m_0 being real), by extracting the flavor-singlet component as

$$\bar{F}_{Ri} F_{Lj} \approx \langle \bar{F}_{Ri} F_{Li} \rangle \cdot \left(\frac{\chi}{v_\chi}\right)^a \cdot U_{ij} \,, \tag{3}$$

[1] A similar composite dilaton (or glueball) potential of the CW type was discussed in a context different from the present interest in the SFI, where a large field inflation with non-minimal coupling to general relativity is assumed [22–28].

(and its hermitian conjugate partner). $U = e^{2i\pi/f_\pi}$ is the chiral field parametrized by the pion field $\pi = \pi^\alpha T^\alpha$ with T^α being generators of SU(N_F) ($\alpha = 1, \cdots, N_F^2 - 1$) normalized as $\mathrm{tr}[T^\alpha T^\beta] = \delta^{\alpha\beta}/2$, and the pion decay constant f_π. The exponent a for the χ controls the size of the overlap amplitude between the χ and the composite operator $\bar{F}F$ in the underlying theory, for which we will take $a = 1$, so the χ is allowed to linearly couple to the $\bar{F}F$[2] Hence the parameter C is expressed in terms of the pion mass m_π and the decay constant f_π as

$$C = N_F \frac{m_\pi^2 f_\pi^2}{2 v_\chi}\,, \tag{4}$$

with the canonical form of the pion kinetic term being assumed. Note that the f_π can be related with the fermion dynamical mass m_F as

$$f_\pi \simeq \sqrt{N} \cdot \frac{m_F}{2\pi}\,, \tag{5}$$

which is based on a naive dimensional analysis regarding the definition of the pion decay constant.[3] Then the parameter C in Eq. (4) can be evaluated as

$$C \simeq \frac{N N_F m_\pi^2 m_F^2}{8\pi^2 v_\chi}\,. \tag{6}$$

The quartic coupling λ_χ in Eq. (1) is set by the ratio of the dilaton mass m_χ to the dilaton decay constant v_χ as $\lambda_\chi = (m_\chi/v_\chi)^2$ (by taking into account the stationary condition) in the absence of the explicit chiral-scale breaking by the C term (i.e. chiral limit). To this chiral-limit quantity, some straightforward computation of the scale anomaly in the many-flavor walking gauge theory (called ladder Schwinger-Dyson equation analysis), in combination with the partially-conserved dilatation-current (PCDC) relation, would give a constraint [17][4]:

$$m_\chi^2 v_\chi^2 \simeq \frac{16 N N_F}{\pi^4} m_F^4\,. \tag{7}$$

Thereby the λ_χ may be evaluated as

$$\lambda_\chi \simeq \frac{16 N N_F}{\pi^4} \left(\frac{m_F}{v_\chi}\right)^4\,. \tag{8}$$

The full walking dilaton mass (M_χ) is given (by evaluating $V''(v_\chi) = \partial^2 V(\chi)/\partial\chi^2|_{\chi = v_\chi}$) as the sum of the chiral limit value (m_χ) and the correction from the C-term[5]:

$$M_\chi^2 = m_\chi^2 + \frac{3C}{v_\chi} \left(\simeq m_\chi^2 + 3N_F \frac{m_\pi^2 f_\pi^2}{2v_\chi^2}\right) \simeq m_\chi^2 + 3 N N_F \frac{m_\pi^2 m_F^2}{8\pi^2 v_\chi^2}\,. \tag{9}$$

Through the stationary condition (evaluated at $\chi = v_\chi$), the parameter A in Eq. (1) is given as a function in terms of the C and λ_χ to be $A = -\frac{1}{4} + \frac{C}{\lambda_\chi v_\chi^3}$.[6]

Thus, the walking dilaton potential in Eq. (1) is highly constrained by nontrivial parameter correlations given by Eqs. (6) and (8). Then the potential $V(\chi)$ normalized to $v_\chi^4 (\equiv f_\chi^4)$ is essentially controlled by small underlying-theory parameters, $m_\pi/v_\chi \ll 1$ and $m_F/v_\chi \ll 1$ (with $m_\pi \ll m_F$, to be consistent with the nonlinear realization for the chiral-scale symmetry in which we are currently working, as seen from the potential form in Eq. (1)).

3. Small field inflation

The slow roll parameters (η and ϵ), the e-folding number (N) and the magnitude of the scalar perturbation (Δ_R^2) are respectively defined as

$$\begin{aligned}
\eta &= M_{\rm pl}^2 \left(\frac{V''(\chi)}{V(\chi)}\right), \\
\epsilon &= \frac{M_{\rm pl}^2}{2} \left(\frac{V'(\chi)}{V(\chi)}\right)^2, \\
N &= \frac{1}{M_{\rm pl}^2} \int_{\chi_{\rm end}}^{\chi_{\rm ini}} d\chi \left(\frac{V(\chi)}{V'(\chi)}\right), \\
\Delta_R^2 &= \frac{V(\chi)}{24\pi^2 M_{\rm pl}^4 \epsilon}\,,
\end{aligned} \tag{10}$$

with $M_{\rm pl}$ being the reduced Planck mass $\simeq 2.4 \times 10^{18}$ GeV. Since we work in an extremely tiny explicit-chiral (and scale) symmetry-breaking limit with $m_\pi \ll m_F (\ll v_\chi)$ and the magnitude of the dilaton potential during the inflation (for $\chi \ll v_\chi$) can be approximated by the vacuum energy V_0 as in the CW-SFI case, the slow-roll parameters are well approximately evaluated as

[2] As discussed in the literature [42,20], when the walking dilaton χ purely arises as the $\bar{F}F$-composite scalar, the power parameter a would be identical to $(3 - \gamma_m)$, the dynamical dimension of the $\bar{F}F$ operator with the anomalous mass dimension γ_m, which is fixed by the anomalous Ward-Takahashi identity for the scale symmetry. However, possible mixing with other flavor-singlet scalars, like a glueball or tetraquark states, might be present, so in that sense the parameter a would generically be undermined until fully solving the mixing structure, that is beyond the scope of the current interest. In the present work, thus, we will take a conservative limit with $a = 1$, so that the χ couples to the $\bar{F}F$ as if it were a conventional singlet-scalar component as seen in the linear sigma model: the chiral $\mathrm{SU(N_F)}_L \times \mathrm{SU(N_F)}_R$ linear sigma-model field M ($M^\dagger$) is introduced as the effective local-operator description for the $\bar{F}_R F_L$ ($\bar{F}_L F_R$), and transforms as $M \to g_L \cdot M \cdot g_R^\dagger$ under the chiral symmetry with the transformation matrices $g_{L,R}$. The current (universal) F-fermion mass m_0 is therefore coupled to the M ($M^\dagger$) so as to respect the original form in Eq. (2) in a chiral invariant way, like $\mathrm{tr}[\mathcal{M} M^\dagger + M \mathcal{M}^\dagger]$ with the mass-parameter spurion field $\mathcal{M}$ transforming in the same way as M, with the vacuum value $\langle \mathcal{M} \rangle = m_0 \cdot 1_{\mathrm{N_F \times N_F}}$. In the chiral broken phase, the M can generically be polar-decomposed by hermitian ($\tilde{M}$) and unitary ($\xi_{L,R}$) matrices as $M = \xi_L \cdot \tilde{M} \cdot \xi_R$ where $\xi_L^\dagger \xi_R = U$. Supposing a low-energy limit where only the lightest scalar (χ) survives among the N_F scalars in $\tilde{M}$, one may write $M \approx \chi \cdot U$. Plugging this approximated expression into the above $\mathcal{M}^\dagger M +$h.c. term, to get $\chi \cdot m_0 (U + U^\dagger)$, the coupling form of which coincides with the conservative limit $a = 1$. Thus, the choice $a = 1$ is reasonable if the linear sigma model gives a good low-energy description for the underlying walking gauge theory in terms of the chiral-breaking structure.

[3] The Pagels-Stokar formula [43] applying to the present walking gauge dynamics (with the nonrunning and ladder approximation taken) would yield [17] $f_\pi \simeq \sqrt{N/(2\pi^2)} m_F$, which is larger by about factor of $\sqrt{2}$.

[4] The right hand side corresponds to the vacuum expectation value of the trace of (symmetric part of) energy momentum tensor, $\langle \theta_\mu^\mu \rangle = 4\mathcal{E}_{\rm vac}$, where the $\mathcal{E}_{\rm vac}$ denotes the vacuum energy in the walking gauge theory, which is dominated by the F-fermion loop contribution to the gluon condensate [17], hence it scales solely with the dynamical mass m_F, like $\mathcal{E}_{\rm vac} \propto N N_F m_F^4$. On the other hand, the definition of the dilaton decay constant $f_\chi (= v_\chi)$ gives at the dilaton-soft mass limit $p^2 = m_\chi^2 \to 0$, $\langle 0 | \theta_\mu^\mu(0) | \phi(p) \rangle = -m_\chi^2 f_\chi$ (with $\phi = f_\chi \log(\chi/f_\chi)$), the left hand side of which can be evaluated by the $\mathcal{E}_{\rm vac}$ by using the PCDC: $\theta_\mu^\mu(x) = -m_\chi^2 f_\chi e^{-ixp} \phi(x)$ together with the standard reduction formula, and the Ward-Takahashi identity for the scale symmetry (the low-energy theorem), as $\langle 0 | \theta_\mu^\mu(0) | \chi(p) \rangle = -4 d_{\theta_\mu^\mu} \cdot \mathcal{E}_{\rm vac}/f_\chi$ with the scale dimension of θ_μ^μ, $d_{\theta_\mu^\mu} = 4$. Thus one has $m_\chi^2 f_\chi^2 = -16\mathcal{E}_{\rm vac}$. For more detailed evaluation, see the literature [17] and references therein.

[5] This mass formula is precisely the same as the one (with a factor $(3 - \gamma_m)(1 + \gamma_m)$ taken to be 3) derived in the dilaton-chiral perturbation theory for the many-flavor walking gauge theory at the leading order of the derivative expansion [42,20].

[6] The second term in this stationary condition, which gives a scale invariant χ^4 term proportional to the chiral explicit breaking C parameter, plays the role of stabilization of the dilaton potential in the presence of the fermion current mass, as pointed out it is necessary to have in the literature [42,20].

$$\eta = \frac{M_{\rm pl}^2}{V_0^{\rm LO}} \left(\frac{m_F}{v_\chi}\right)^2 \chi^2 \Bigg[-\frac{72}{\pi^2}\left(1 - 4\pi^2 + 6\pi^2 \ln\frac{\chi^2}{v_\chi^2}\right)\left(\frac{m_\pi}{v_\chi}\right)^2 + \frac{384}{\pi^4}\left(1 + \frac{3}{2}\ln\frac{\chi^2}{v_\chi^2}\right)\left(\frac{m_F}{v_\chi}\right)^2 + \mathcal{O}\left(\frac{m_\pi^4}{v_\chi^2 m_F^2}\right)\Bigg],$$

$$\epsilon = \frac{M_{\rm pl}^2}{2[V_0^{\rm LO}]^2}\left(\frac{m_F}{v_\chi}\right)^4 v_\chi^6 \times \Bigg[\frac{24}{\pi^2}\left(\left\{1 - 12\pi^2 \ln\frac{\chi^2}{v_\chi^2}\right\}\frac{\chi^3}{v_\chi^3} - 1\right)\left(\frac{m_\pi}{v_\chi}\right)^2 + \left\{\frac{192}{\pi^4}\frac{\chi^3}{v_\chi^3}\ln\frac{\chi^2}{v_\chi^2}\right\}\left(\frac{m_F}{v_\chi}\right)^2 + \mathcal{O}\left(\frac{m_\pi^4}{v_\chi^2 m_F^2}\right)\Bigg]^2, \tag{11}$$

with

$$V_0^{\rm LO} = \frac{24}{\pi^4} m_F^4 . \tag{12}$$

The SFI with the extremely small chiral-scale breaking by the m_π will give an overall scaling for ϵ/η with the small expansion factors as $\frac{\epsilon}{\eta} \sim \left(\frac{m_\pi}{m_F}\right)^4 \left(\frac{v_\chi}{\chi}\right)^2$. Hence the inflation would be ended by reaching $\eta = 1$, as long as $\chi/v_\chi > (m_\pi/m_F)^2$, as in the CW-SFI case, which indeed turns out to happen as will be seen later. In that case (with $m_\pi \ll \chi \ll m_F \ll v_\chi$), the η and ϵ as well as the Δ_R^2 and N can further be approximated to be

$$\eta \simeq 24 \frac{M_{\rm pl}^2}{v_\chi^2}\frac{\chi^2}{v_\chi^2}\ln\frac{\chi^2}{v_\chi^2},$$

$$\epsilon \simeq \frac{\pi^4}{2}\left(\frac{M_{\rm pl}}{v_\chi}\right)^2\left(\frac{m_\pi}{m_F}\right)^4,$$

$$\Delta_R^2 \simeq \frac{2}{\pi^{10}}\left(\frac{m_F}{v_\chi}\right)^4 \cdot \left(\frac{v_\chi}{M_{\rm pl}}\right)^6 \left(\frac{m_F}{m_\pi}\right)^4,$$

$$N \simeq \frac{(\chi_{\rm end} - \chi_{\rm ini})}{\sqrt{2\epsilon} M_{\rm pl}} \simeq \frac{(\chi_{\rm end} - \chi_{\rm ini}) v_\chi}{6\pi^2 M_{\rm pl}^2}\left(\frac{m_F}{m_\pi}\right)^2 . \tag{13}$$

Note that the gigantic suppression factor $\frac{2}{\pi^{10}}\left(\frac{m_F}{v_\chi}\right)^4$ for Δ_R^2 in Eq. (13) shows up, corresponding to an extremely tiny quartic coupling λ_χ, realized by the walking nature $(m_\chi/v_\chi)^2 \ll 1$ as seen from the PCDC relation in Eq. (7) (also see Eq. (8)), which gets small enough to cancel the other factors coming from the small ϵ, to easily achieve the right small amount of the observed $\Delta_R^2 \sim 10^{-9}$ at the pivot scale. Given the observed Δ_R^2, the pion mass m_π is actually written as a function of other potential parameters like

$$m_\pi^2 \simeq \sqrt{\frac{2}{\pi^{10}\Delta_R^2}}\left(\frac{m_F}{v_\chi}\right)^2\left(\frac{v_\chi}{M_{\rm pl}}\right)^3 m_F^2 . \tag{14}$$

Note also that the e-folding number N in Eq. (13) is set by the constant ϵ, in contrast to the case of the CW-SFI where it is instead set by η so that one would encounter the incompatibility between the N and the spectral index $n_s \simeq 1 + 2\eta$ in comparison with the observational values [2,29]. As discussed in the literature [4], a small enough tadpole term (corresponding to the C-term at present) helps avoid this catastrophe, which will be more concretely demonstrated by the present walking inflationary model later on.

4. Embedding into a dynamical scalegenesis

To more realistically analyze the present walking dilaton inflationary scenario, we need to consider couplings between the walking gauge sector and SM sector, so that we can access the reheating temperature T_R and the e-folding number detected by the CMB photons at the pivot scale ($k = k_{\rm CMB} = 0.05\,{\rm Mpc}^{-1}$) through the following relation [3]:

$$N_{\rm CMB} \simeq 61 + \frac{2}{3}\ln\left(\frac{V_0^{1/4}}{10^{16}{\rm GeV}}\right) + \frac{1}{3}\ln\left(\frac{T_R}{10^{16}{\rm GeV}}\right). \tag{15}$$

Aa a benchmark model, we shall try to embed the present scenario into a dynamical scalegenesis,[7] in which the EWSB is triggered by what is called the bosonic seesaw mechanism [30–41].[8] In that case, the hidden walking eight-flavor fermion fields F^i ($i = 1, \cdots, 8$) are vector-likely charged in part by the EW gauges. A possible charge assignment goes like

$$\Psi_{L/R} = (F^1, F^2)^T_{L/R} \qquad \sim (3, 1, 2, 1/2)$$
$$\psi_{L/R}^{1,\cdots,6} = F_{L/R}^{3,\cdots 8} \qquad \sim (3, 1, 1, 0), \tag{16}$$

under the SU(N) × SU(3)$_c$× SU(2)$_W$× U(1)$_Y$ symmetry.

The gauge invariance as well as the classical scale-invariance allows us to introduce a Yukawa coupling between the F-fermion fields and a Higgs doublet field H as [35–41]

$$\mathcal{L}_{y_H} = -\sum_{A=1}^{6} y_H^A(\bar\Psi_L H \psi_R^A + \bar\Psi_R H \psi_L^A) + {\rm h.c.}, \tag{17}$$

(with the hidden-fermion parity invariance ensured by the underlying vectorlike gauge theory via Vafa-Witten theorem [78]), where the couplings y_Hs are assumed to be small enough (to be consistent with the fitting later). After the chiral condensate (and confinement) develops, those y_H-Yukawa interactions generate a couple of mixings between the elementary H doublet and composite Higgs doublets $\Theta^A \sim \bar\psi^A \Psi$, to give the negative mass square of the SM-like Higgs field (arising as the lowest mass eigenstate from the mass matrix), that is called the bosonic seesaw mechanism [30–41], as follows:

$$m_H^2 \simeq -\sum_{A=1}^{6}[y_H^A]^2 m_F^2 \equiv -6 y_H^2 m_F^2 , \tag{18}$$

where the y_H^A couplings have been assumed to be flavor universal ($y_H^A \equiv y_H$). Thus, with the quartic coupling for the H at hand, the EWSB is dynamically achieved so that the SM-like Higgs acquires the EW vacuum expectation value ($v_{\rm EW} \simeq 246$ GeV) properly. Then, the m_H scale is fixed by the 125 GeV Higgs mass as $m_H^2 = -m_{h(125)}^2/2 \simeq -(88\,{\rm GeV})^2$, so that the y_H coupling is determined as a function of m_F like

$$y_H^2 \simeq \left(\frac{36\,{\rm GeV}}{m_F}\right)^2 . \tag{19}$$

[7] The explicit-scale breaking m_0 term in Eq. (2) at the quantum level will not generate extra scale anomalies nor quadratic divergent contributions to the Higgs mass parameter (m_H). Thereby one can keep the (almost) quantum scale invariance (up to the tiny m_0) up until the dimensional transmutation triggered in the hidden non-Abelian gauge sector, as long as the theory can be embedded into an asymptotic safety below or at the Planck scale [44–61], and the initial condition $m_H(M_{\rm pl}) = 0$ is realized by some over-Plankian nonperturbative dynamics [62–66].

[8] Embedding inflationary scenarios into dynamical scale generation mechanisms has extensively been addressed recently [67–77] in a context different from the present study.

Another important point arising from the y_H interactions is that Eq. (17) explicitly breaks the F-fermion chiral $U(8)_L \times U(8)_R$ symmetry, even the vectorial part. The breaking effect of this non-vectorial type destabilizes the chiral manifold, yielding the instability for the pions (i.e. making the pions tachyonic) [35–41]. Although some pions charged by EW gauges get sizable enough masses on the order of $\mathcal{O}(\alpha_W m_F^2)$ to safely overcome this instability, other chargeless pions can be unstable. The tachyonic correction to those pion masses are evaluated by using the current algebra, as done in the literature [36,37], to be

$$m_{y_H}^2 \simeq -(y_H v_{\rm EW}) \frac{\langle -\bar{F}F\rangle}{f_\pi^2} \,, \tag{20}$$

where $\langle \bar{F}F\rangle$ denotes the chiral condensate per flavor. This contribution has to be smaller than the pion-flavor universal m_π term arising from the bare m_0 mass term for the F-fermions: $m_{y_H}^2 \ll m_\pi^2$.[9] The chiral condensate in the many-flavor walking gauge theory can be evaluated by a straightforward nonperturbative computation based on the Schwinger-Dyson equation analysis (in the ladder approximation) [17]

$$\langle -\bar{F}F\rangle_{m_F} \simeq \frac{8N}{\pi^4} m_F^3 \,, \tag{21}$$

in which the renormalization scale has been set at the scale m_F.[10] Using Eqs. (19) and (21) together with Eq. (5), the m_{yH} is completely fixed to a constant:

$$m_{y_H}^2 \simeq -(169\,{\rm GeV})^2 \,. \tag{22}$$

Thus the walking pion (particularly for neutral pions) can safely be stabilized when

$$m_\pi \gg 169\,{\rm GeV} \,. \tag{23}$$

In the present inflationary scenario, the reheating temperature is actually determined by the walking pion decays to EW bosons: since the walking dilaton as the inflaton predominantly decays to walking pion pairs with the strong coupling, which will be much faster than the Hubble evolution of the Universe at that time, the rate of the SM particle production is controlled by the pion decays to the EW bosons coupled to the vector-likely charged F-fermion currents.

The interactions between the walking pions and EW bosons, relevant to the pion decay processes, are completely determined by a covariantized Wess-Zumino-Witten term [79,80] (in a way analogously to the three flavor case discussed in [36]):

$$\mathcal{L}_{\pi\mathcal{V}\mathcal{V}} = -\frac{N}{4\pi^2 f_\pi} \epsilon^{\mu\nu\rho\sigma} {\rm tr}[\partial_\mu \mathcal{V}_\nu \partial_\rho \mathcal{V}_\sigma \pi] \,. \tag{24}$$

$\mathcal{V}_\mu$ denotes the external gauge field of 8×8 matrix form, with the $SU(2)_W$ and $U(1)_Y$ gauge fields (W_μ^a, B_μ) embedded following the charge assignment in Eq. (16), which is expressed as

$$\mathcal{V}_\mu = \begin{pmatrix} [\mathcal{V}_\mu^{\rm EW}]_{2\times 2} & 0_{2\times 6} \\ 0_{6\times 2} & 0_{6\times 6} \end{pmatrix} ,$$
$$\mathcal{V}_\mu^{\rm EW} = g_W W_\mu^a \tau^a + \frac{g_Y}{2} B_\mu \cdot 1_{2\times 2} \,, \tag{25}$$

with the $SU(2)_W$ and $U(1)_Y$ gauge couplings, g_W and g_Y, and normalized Pauli matrices τ^a with the normalization ${\rm tr}[\tau^a \tau^b] = \delta^{ab}/2$. The walking pion field π is parametrized in a way similar to the $\mathcal{V}_\mu$ as

$$\pi = \begin{pmatrix} [\pi_{\Psi\Psi}]_{2\times 2} & [\pi_{\Psi\psi}]_{2\times 6} \\ [\pi_{\psi\Psi}]_{6\times 2} & [\pi_{\psi\psi}]_{6\times 6} \end{pmatrix} . \tag{26}$$

One can readily see that only the $\pi_{\Psi\Psi}$ component couples to the EW bosons. The EW charged pions in the $\pi_{\Psi\Psi}$ get large masses of $\mathcal{O}(g_W m_F)$, by the EW interaction, enough to close the decay channel of the walking dilaton into the pion pairs, as will be clarified later. Thereby, only the EW singlet one with the mass m_π, $\pi_{\Psi\Psi} \ni \pi_{\Psi\Psi}^0/2 \cdot 1_{2\times 2}$, contributes to the determination of the reheating temperature.[11]

The $\pi_{\Psi\Psi}^0$ total width is computed to be

$$\Gamma_{\pi_{\Psi\Psi}^0} = \frac{m_\pi^3}{16\pi} \left(\frac{N}{4\pi^2 f_\pi}\right)^2 \left[\left(\frac{g_Y(m_\pi)}{2}\right)^2 + 3\left(\frac{g_W(m_\pi)}{2}\right)^2\right] , \tag{27}$$

where we have taken the EW bosons to be massless because $m_\pi \gg m_{W,Z}$ as evident from Eq. (23). We also specified the renormalization scale for the EW gauge couplings at the $\pi_{\Psi\Psi}^0$ mass scale, which however tuns out to be the same as the ones evaluated at the Z boson pole, $g_W(m_\pi) = g_W(m_Z) \simeq 0.42$ and $g_Y(m_\pi) = g_Y(m_Z) \simeq 0.13$,[12] because possible renormalization corrections from EW-charged F-fermions, with the mass on the scale of m_F, necessarily get decoupled at the scale $m_\pi \ll m_F$. By equating the decay rate in Eq. (27) and the Hubble parameter with the radiation dominance, we determine the reheating temperature T_R as follows:

$$T_R \simeq 0.23 \times \left(\frac{100}{g_*(T_R)}\right)^{1/2} \times \sqrt{\Gamma_{\pi_{\Psi\Psi}^0} M_{\rm pl}} \,, \tag{28}$$

with the effective degrees of freedom at T_R, $g_*(T_R)$, which is set to the SM value $= 106.75$ [81], as long as the walking dilaton and pion masses are larger than T_R, as in the present analysis to be clarified in the next section.

Thus the e-folding number for the CMB photons at the pivot scale ($N_{\rm CMB}$) in Eq. (15) is evaluated as a function of m_π and m_F, which has to be fitted to the theoretically predicted N in Eq. (13), so as for the present inflationary model to surely generate the currently observed CMB photons.

5. Constraints and predictions

Using the observed data for Δ_R^2 and $n_s(= 1 + 2\eta)$, ($\Delta_R^2 \simeq 2.137 \times 10^{-9}$ and $n_s \simeq 0.968$ [3]) with the initial stage of the inflation identified as the pivot scale, together with a couple of formulae derived in the previous sections, we now constrain the present walking-dilaton inflationary-scenarios embedded into a dynamical scalegenesis. Fig. 1 shows the exclusion plot on the parameter space spanned by v_χ and $V_0^{1/4}$ (instead of m_F with Eq. (12) taken into account).

We first see from the figure that the v_χ has to be greater than 10^{11} GeV in order to realize the right amount of the e-folding number for the CMB photons today by the inflation (i.e.

[9] The sign ambiguity for the y_H has been fixed to be positive by the definition of the chiral condensate $\langle \bar{F}F\rangle$ in Eq. (21).

[10] Hence the y_H coupling in Eq. (20) should also be interpreted as the one renormalized at the same scale m_F, so that the corresponding pion mass m_{y_H} is independent of the renormalization scale, as it should be. Note also that the y_H is not evolved by the renormalization below m_F because of decoupling of the F-fermions, so the matching condition in Eq. (19), which should be set at the 125 GeV Higgs mass scale, can be read as $y_H^2(m_F) = y_H^2(m_{h(125)}) \simeq$ right hand side of Eq. (19).

[11] The possible mixing with the η'-like pseudoscalar would be small because of the large enough mass for the many-flavor walking η' generated by the U(1) axial anomaly, $m_{\eta'} \sim \sqrt{N_F/N} m_F (\gg m_F)$, due to the anti-Veneziano limit [17].

[12] Those numbers have been estimated by using the electromagnetic couplings renormalized at the Z-boson mass scale ($m_Z \simeq 91.2$ GeV [81]), $\alpha(m_Z) = g_Y^2(m_Z) c_W^2/(4\pi) \simeq 1/128$ [81] and the (Z-mass shell) Weinberg angle quantity $c_W^2 = m_W^2/m_Z^2 \simeq 0.778$.

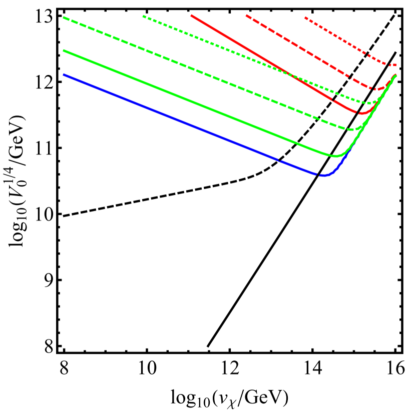

Fig. 1. The exclusion plot on the plane (v_χ, $V_0^{1/4}$). The black solid curve has been created by imposing $N = N_{\rm CMB}$ and the dashed one by $N = N_{30\,{\rm Gpc}}$ [81], below which the causally-connected patch, with the comoving distance (d) generated by the inflation, encompasses the observed Universe today, $d > 30$ Gpc. Hence the region sandwiched by those curves are allowed so that the required e-folding number can be realized. The pion stability is safe above the blue curve (see the main text for the detail). On solid, dashed, and dotted green curves, the reheating temperatures are $T_R = 10^2$, 10^3, and 10^4 GeV, respectively. On solid, dashed, and dotted red curves lines, the pion masses are $m_\pi = 10^3$, 10^4, and 10^5 GeV, respectively.

the bound from N in Eq. (13) equals $N_{\rm CMB}$ in Eq. (15)). Furthermore, the v_χ as well as the $V_0^{1/4}(\simeq m_F)$ are highly constrained by the pion instability set as in Eq. (23) to be $v_\chi \gtrsim 10^{14}$ GeV and $V_0^{1/4}(\simeq m_F) \gtrsim 3 \times 10^{10}$ GeV.

When the EW phase transition is assumed to happen in the thermal history of our Universe, the reheating temperature T_R is necessary to be $\gtrsim 100$ GeV. In that case, the bounds on the v_χ and $V_0^{1/4}$ get shifted further upward, so that we would have

$$\begin{aligned} v_\chi &\gtrsim 1.7 \times 10^{15}\,\mathrm{GeV}\,, \\ V_0^{1/4}(\simeq m_F) &\gtrsim 4.1 \times 10^{11}\,\mathrm{GeV}\,, \\ \text{for} \qquad T_R &\gtrsim 10^2\,\mathrm{GeV}\,. \end{aligned} \tag{29}$$

Then the masses of the walking dilaton (in Eq. (9)) and pion (in Eq. (14)) at those lower bounds are estimated to be

$$\begin{aligned} M_\chi(\simeq m_\chi) &\simeq 3.8 \times 10^8\,\mathrm{GeV}\,, \\ m_\pi &\simeq 6.7 \times 10^4\,\mathrm{GeV}\,. \end{aligned} \tag{30}$$

At this benchmark point, for other model parameters we have $\chi_{\rm ini} \simeq 6.7 \times 10^9$ GeV, $\chi_{\rm end} \simeq 5.8 \times 10^{10}$ GeV, $\epsilon \simeq 5.1 \times 10^{-21}$, $N \simeq 51$, and $y_H \simeq 6.5 \times 10^{-11}$. One can easily see from those reference outputs that the approximated analytic formulae in Eq. (13), derived for $m_\pi \ll \chi_{\rm ini,end} \ll m_F \ll v_\chi$, indeed work well, and surely reflects the large N_F scaling (in the anti-Veneziano limit [17]) intrinsic to the underlying many-flavor walking gauge theory, $v_\chi(\equiv f_\chi) \sim \sqrt{N_F} m_F \gg m_F$, and its associated pNG boson's nature $m_\chi, m_\pi \ll m_F$.

In terms of the bare fermion mass m_0 in Eq. (2), one can also check the size of the explicit-chiral scale breaking regarding the m_0 to be extremely tiny: using the current algebra for the pion mass together with Eqs. (5) and (21), one gets $m_0/m_F = m_\pi^2 f_\pi^2/(2\langle -\bar{F}F\rangle m_F) \simeq \pi^2/64(m_\pi/m_F)^2$, which is $\simeq 4.1 \times 10^{-15}$ at the benchmark point above. This tiny m_0 would be physical to be probed by detecting light walking pions with the mass $\gtrsim 10^3$ GeV.

In addition, we observe that the non-Gausiannity is small enough to be consistent with the latest results by Planck satellite [82]. The non-Gaussianity can be evaluated as $f_{\rm NL} = (5/12)(n_s + f(k) n_t)$ where $n_s(\simeq 0.96)$ and n_t are the spectral indices for scalar and tensor modes, and $f(k)$ is determined by the shape of triangle which has a range of values $0 \le f \le 5/6$ [83]. Since n_t and the scalar to tensor ratio r have a relation: $r + 8n_t = 0$, the second term in the parenthesis gives a negative contribution to the non-Ganssianity. Namely, the maximal value of $f_{\rm NL}$ can be obtained to be roughly 0.4, which is within the current limit: $f_{\rm NL} = -0.9 \pm 5.1$ (at the 68% confidence level) [82]. Therefore, the non-Gaussianity is also consistent with the observation.

6. Summary and discussion

In summary, we have proposed an inflationary scenario of the CW-SFI type, dynamically arising from a large N_F walking gauge theory, what we called the hidden walking gauge theory. The inflation is played by the walking dilaton and the inflaton potential parameters are highly constrained by the walking nature. We have evaluated the potential parameters using outputs from our best knowledge based on straightforward nonperturbative analyses. We showed that due to the intrinsic feature of the large N_F walking dynamics, called the anti-Veneziano-large N_F scaling, the desired tiny inflaton quartic coupling can naturally be realized, and the inflaton coupled to the walking pions can survive the cosmological, astrophysical constraints, from which other models of the CW-SFI generically suffer.

For the inflationary scenario to be realistic, as a reference model, the hidden walking dynamics was vector-likely gauged in part by the EW charges and was coupled to a Higgs doublet in a scale-invariant way, so that the EW symmetry breaking is triggered by the bosonic seesaw mechanism. This benchmark model has been fitted with all the cosmological and astrophysical constraints presently placed by Planck 2018 data and theoretical requirements on the walking dilaton and chiral pion physics. We found that the present inflationary scenario is highly constrained to give a stringent bound on the fermion dynamical mass scale to be $> 10^{11}$ GeV, and the reheating temperature (T_R), which is fixed by chiral pion decays to EW gauge bosons including photons, to be $\gtrsim 10^2$ GeV. It also turned out that the masses of walking dilaton and pions are constrained by several theoretical and astrophysical limits to be $\sim 10^8$ GeV and $\sim 10^5$ GeV, respectively.

In closing, we shall give comments on issues to be left in the future works.

The present walking inflationary scenario works for a so large dilaton decay constant $v_\chi(\equiv f_\chi)$ compared to the pion decay constant f_π, $v_\chi \gtrsim 10^4 f_\pi$. The current lattice simulations [15] have tried to observe those quantities with the current fermion mass $m_0/m_F \simeq 0.18 - 0.45$, to give a preliminary result on the dilaton decay constant $f_\chi(\equiv v_\chi) \simeq 3.7 f_\pi$ based on the lowest-order formula in the dilaton-chiral perturbation theory [20], by simply fitting the dilaton mass formula as in Eq. (9) with the data on the slope with respect to the leading order m_π^2 dependence on the dilaton mass (dM_χ^2/dm_π^2). (This has been consistent with a direct measurement of the pole residue of the scalar current correlator with a simple-minded linear fit assumed in the same range of the current fermion mass.). However, this result cannot simply be compared with our values at present: for our reference-current fermion-mass value $m_0/m_F \sim 10^{-15}$, the next-to leading-order chiral-logarithmic correction would be significant for the estimates on the dilaton mass, and decay constant cannot be measured just by the slope dM_χ^2/dm_π^2, as explicitly demonstrated in [20]. Future upgraded lattice setups, by which the chiral log corrections are visible, could check if our scenario is indeed viable on the ground of the realistic nonperturbative dynamics.

The phenomenological consequence for the present inflationary scenario, distinguishable from other inflation models in the huge ballpark, would be derived from the presence of walking pions with mass $\gtrsim 10^3$ GeV (if an EW phase transition happens by supercooling like in the scenario discussed in the literature [84]). Such a sub TeV-walking pions can have several predictions for terrestrial and satellite experiments. At current or future hadron collider experiments, the walking pions are potentially produced via photon fusion process [36]. The detail analyses for these experimental signatures are to be discussed in future publications.

Moreover, actually we can have rich amount (35 kinds) of dark matter candidates (corresponding to the $\pi_{\psi\psi}$ states in Eq. (26), except the heavy walking η' decaying to EW bosons just like the $\pi^0_{\Psi\Psi}$). Hence the thermal history and testability at underground and satellite experiments for those dark matter particles are deserved to be addressed as well.

As noted in the Introduction, the initial condition for the inflaton has been set to be away (to the left side) enough from the vacuum in the potential, by assuming some trapping mechanism to work there, as discussed in the literature [5] for the CW-SFI scenario. Actually, it would not be so plausible to apply the existing trapping mechanism: first, in our benchmark scenarios the inflaton gets coupled to the SM-like Higgs via a Higgs portal coupling as noted in [41], which would be crucial for the trapping to work [5]. Note the size of the portal coupling ($\lambda_{\rm mix}$) turns out to be extremely smaller than the standard-Higgs quartic coupling of $\mathcal{O}(10^{-1})$: $\lambda_{\rm mix} = m_H^2/v_\chi^2 \lesssim 10^{-26}$ (see Eq. (29)), hence the Higgs parametric resonance may not work well for trapping the χ at around the origin of the potential as argued in [5]. However, in contrast to the literature having arguments based on the potential for elementary scalar fields, another trapping possibility other than coupling to the SM Higgs might be present in the case of the composite inflaton we have employed, which could be closely tied with the underlying nonperturbative gauge dynamics. Though being beyond the current scope, this issue will involve some generic features and expected developments on the particle production mechanism, so it would be worth pursuing in the future.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

We are grateful to Satoshi Iso and Kazunori Kohri for useful comments. We are also grateful to Seishi Enomoto for valuable discussions. H.I. thanks for the hospitality of Center for Theoretical Physics and College of Physics, Jilin University where the present work has been partially done. This work was supported in part by the National Science Foundation of China (NSFC) under Grant No. 11747308 and 11975108, and the Seeds Funding of Jilin University (S.M.). The work of H.I. was partially supported by JSPS KAKENHI Grant Number 18H03708.

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