

Research Article

Extracting Kinetic Freeze-Out Properties in High-Energy Collisions Using a Multisource Thermal Model

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We study the transverse momentum (p_T) spectra of neutral pions and identified charged hadrons produced in proton–proton (pp), deuteron–gold (d–Au), and gold–gold (Au–Au) collisions at the center of mass energy $\sqrt{s_{NN}} = 200$ GeV. The study is made in the framework of a multisource thermal model used in the partonic level. It is assumed that the contribution to the p_T -value of any hadron comes from two or three partons with an isotropic distribution of the azimuthal angle. The contribution of each parton to the p_T -value of a given hadron is assumed to obey any one of the standard (Maxwell-Boltzmann, Fermi-Dirac, and Bose-Einstein) distributions with the kinetic freeze-out temperature and average transverse flow velocity. The p_T spectra of the final-state hadrons can be fitted by the superposition of two or three components. The results obtained from our Monte Carlo method are used to fit the experimental results of the PHENIX and STAR Collaborations. The results of the present work serve as a suitable reference baseline for other experiments and simulation studies.

1. Introduction

The transverse momentum (p_T) distributions of the identified hadrons in the final state of high-energy collisions reflect the excitation degree of the particle emission source as well as the speed of collective motion of the particles. The excitation degree of particle emission source reflects the speed of thermal motion of the particles. Different distribution functions can be potential candidates for the description of the p_T spectra measured in experiments. The distributions include, but are not necessarily limited to, the standard (Maxwell-Boltzmann, Fermi-Dirac, and Bose-Einstein) distributions obtained from the Boltzmann-Gibbs statistics, the Tsallis distribution obtained from the Tsallis statistics [1–6], the Tsallis form of the standard distribution

(or the Tsallis standard distribution), the q -dual distribution obtained from the q -dual statistics [7], the q -dual form of the standard distribution (or the q -dual standard distribution), the Erlang distribution obtained from the multisource thermal model [8–12], and the Hagedorn function [13] (inverse power law [14–18]) obtained from the quantum chromodynamics (QCD) calculus and their superposition.

The flow effect may cause a red shift of the p_T spectra. The influence of flow effect is not excluded in the above-mentioned distributions [1, 7, 8, 13, 14]. The temperature parameters used in these distributions are the effective temperature (T) of particle emission source which is larger than the real temperature one wants to extract from the p_T spectra. Generally, T contains the contributions of thermal motion and collective motion (flow effect) which are described by the kinetic freeze-

out temperature (T_0) and average transverse flow velocity ($\langle\beta_t\rangle$), respectively. The contributions of both the thermal and collective motions to p_T are not exactly separable, though the magnitudes of the two effects can be approximately calculated by some model methods.

One has at least four methods to separate the two types of motions. In method 1, one may use the blast-wave model with the Boltzmann-Gibbs statistics or Tsallis statistics, in which a determined velocity profile is assumed, and T_0 and $\langle\beta_t\rangle$ can be obtained simultaneously [19–22]. The temperature given by the blast-wave model with the Boltzmann-Gibbs statistics is larger than that with the Tsallis statistics. In method 2, one may use the Lorentz-like transformation in a p_T distribution, in which the collective motion is treated as the motion of the reference system and T_0 and $\langle\beta_t\rangle$ can be also obtained simultaneously [23–28]. In method 3, one may use the intercept–slope method in which T_0 is the intercept in the linear relation of T versus the rest mass (m_0) of particle and $\langle\beta_t\rangle$ is the slope in the linear relation of the average p_T ($\langle p_T \rangle$) versus the average energy (i.e., the average mass (\bar{m}) of moving particles in the source rest frame) [29–37]. Here, m_0 refers to a given kind of particle in all components in the data sets. In addition, in the linear relation of T versus m_0 , $\langle\beta_t\rangle$ is related to, but not equal to, the slope [38, 39]. In method 4, one may propose the contribution fractions of thermal and collective motions to $\langle p_T \rangle$ to be determinate values which are from some models. For example, empirically, $T_0 = \langle p_T \rangle / 3.07$ in the hydrodynamic simulations [40], and naturally, one obtains $\langle\beta_t\rangle = (1 - 1/3.07)\langle p_T \rangle / \bar{m} = (2.07/3.07)\langle p_T \rangle / \bar{m}$.

The above four methods were used in our previous work [27, 28, 33–37], though only few distributions were performed. The results from different methods are inconsistent in some cases. These inconsistent results appear not only for the magnitudes but also for the tendencies of T_0 and $\langle\beta_t\rangle$ with increasing the collision energy and centrality. A robust method should be used to obtain a more reasonable result. The result of the blast-wave model is model dependent. Although the third and fourth methods seem to be model independent, they are hard to connect with the basic physics processes. Considering the fact that the standard distribution is the most basic one, which is from the relativistic ideal gas model in the thermodynamics, one prefers to use it with the Lorentz-like transformation to extract T_0 and $\langle\beta_t\rangle$. Here, the particles (or partons) emitted from the hot and dense system are assumed to obey the law of the relativistic ideal gas model, though the matter contained within the intermediate fireball is known to behave like a strongly interacting fluid of partons. To our best knowledge, this direct extraction method, using the standard distribution, is rarely reported in the literature.

In the framework of multisource thermal model at the quark or gluon level [8–12], one may use the standard distribution with T_0 and $\langle\beta_t\rangle$ to describe the behavior of partons. For the production of a given particle of any type, its contributors contain mainly two or three partons with isotropic azimuthal angles. Of course, one does not expect that the single-component nonanalytical description based on the superposition of two or three standard distributions with T_0 and $\langle\beta_t\rangle$ by the Monte Carlo method is enough to fit

the experimental data. Considering different violent degrees of binary nucleon-nucleon process in high-energy collisions, two or three or even more sets of parameters are possible, which results in a non-single-component distribution. Going from the binary process with the lowest intensity to the most violent one, the fractions of the corresponding components are getting smaller and smaller. This means that the fraction of the first component with smallest T_0 and $\langle\beta_t\rangle$ is the largest.

In the case of using the two-component nonanalytical description, the spectrum in high- p_T region is regarded as the result of hard scattering process which implies high- T_0 and $\langle\beta_t\rangle$, and the spectrum in low- p_T region is regarded as the result of soft excitation process which implies low- T_0 and $\langle\beta_t\rangle$. In the case of using the three-component function, one needs one more component, i.e., the intermediate- T_0 and $\langle\beta_t\rangle$ for the spectrum in intermediate- p_T region. The multicomponent function corresponds to the multiregion fine structure of p_T spectra [41–43], which is a natural result of the multisource thermal model [8–12] if the standard distributions with different values of parameter T are used to describe different components.

In this article, the standard distribution with T_0 and $\langle\beta_t\rangle$ will be used to describe the transverse momentum of partons. The transverse momentum p_T of given particle is the sum of contributions of two or three partons. The related calculations are performed in the framework of multisource thermal model [8–12], where the two- or three-component description is available. The calculated results are fitted to the experimental data measured in high-energy collisions by the PHENIX [44] and STAR [45] Collaborations.

The remainder of this article is structured as follows. The picture and formalism of the multisource thermal model are described in Section 2. Results and discussion are given in Section 3. In Section 4, we give our summary and conclusions.

2. Picture of Multisource and Formalism of Multicomponent

According to the multisource thermal model [8–12], one may assume that there are lots of energy sources to form in high-energy collisions. These energy sources can be quarks and/or gluons if one studies the production of particles. For a given particle of any type, its contributors may be generally two (for mesons) or three (for baryons) energy sources of contributor partons [8, 10]. The number of contributor partons is the same as that of constituent quarks of a given hadron. In most of the cases, the contributions of two or three partons are suitable to fit the hadronic spectra. If the two or three partons are not enough in the analysis, one may include the contributions from the fourth or more partons, which corresponds to the hadronic state of multiple quarks [8, 10]. Here, the contributor partons refer to the constituent quarks of identified hadrons. In the case of studying the spectra of leptons, one may consider two contributor partons as the energy sources, in which one is from the projectile and the other is from the target.

In the relativistic ideal gas model, the invariant particle momentum (p) distribution can be given by [1]

$$E \frac{d^3 N}{d^3 p} = \frac{gV}{(2\pi)^3} E \left[\exp \left(\frac{E - \mu}{T} \right) + S \right]^{-1}, \quad (1)$$

where $E = \sqrt{p^2 + m_0^2} = m_T \cosh y$ is the energy, $m_T = \sqrt{p_T^2 + m_0^2}$ is the transverse mass, $y = (1/2) \ln [(E + p_z)/(E - p_z)]$ is the rapidity, p_z is the longitudinal momentum, N is the particle number, g is the degeneracy factor, V is the volume, μ is the chemical potential, and $S = 0, 1,$ and -1 correspond to the Maxwell-Boltzmann, Fermi-Dirac, and Bose-Einstein distributions, respectively.

The density function of momenta is obtained by

$$\frac{dN}{dp} = \frac{2gV}{(2\pi)^2} p^2 \left[\exp \left(\frac{E - \mu}{T} \right) + S \right]^{-1}. \quad (2)$$

The unit-density function of transverse momentum and rapidity is written as [1]

$$\frac{d^2 N}{dp_T dy} = \frac{gV}{(2\pi)^2} p_T E \left[\exp \left(\frac{E - \mu}{T} \right) + S \right]^{-1}. \quad (3)$$

The density function of transverse momentums is

$$\frac{dN}{dp_T} = \frac{gV}{(2\pi)^2} p_T \int_{y_{\min}}^{y_{\max}} E \left[\exp \left(\frac{E - \mu}{T} \right) + S \right]^{-1} dy, \quad (4)$$

where y_{\min} and y_{\max} denote the minimum and maximum rapidities, respectively.

In the near midrapidity region, $E = m_T \cosh y \approx m_T$. In the case of having no collective flow, the transverse momentum p'_{ti} of the i -th parton contributing to the transverse momentum p_T of the given particle is assumed to obey the probability density function of the standard distribution

$$f_{p'_{ti}}(p'_{ti}) = \frac{1}{n} \frac{dn}{n dp'_{ti}} = C p'_{ti} m'_{ti} \left[\exp \left(\frac{m'_{ti} - \mu_i}{T} \right) + S \right]^{-1}, \quad (5)$$

where n is the number of partons, C is the normalization constant, $m'_{ti} = \sqrt{p'^2_{ti} + m_{0i}^2}$ is the transverse mass of the i -th parton, m_{0i} is the constituent mass that is $0.31 \text{ GeV}/c^2$ for up and down quarks [46], and μ_i is the chemical potential of the i -th parton that is nearly 0 at high energy [47–50]. One can obtain easily $f_{p'_{ti}}(p'_{ti}) dp'_{ti}$ from Eq. (5). In addition, if $1/n$ in Eq. (5) is replaced by $1/N_{\text{EV}}$, where N_{EV} denotes the number of events, one may obtain the mean multiplicity of particles in an event.

It should be noted that Eq. (5) is not the united probability density function of transverse momentum and rapidity (or longitudinal momentum), but only the probability density function of transverse momentum at the midrapidity which is the concerned major region in experiments. From a practical point of view, Eq. (5) is an approximate expres-

sion and easy to use. In addition, the constituent mass, but not the current mass, of a given quark is used in Eq. (5) due to the considered quarks being the constituents of the collision system and produced hadrons.

One may introduce the average transverse flow velocity $\langle \beta_t \rangle$ and the Lorentz-like factor $\langle \gamma_t \rangle = 1/\sqrt{1 - \langle \beta_t \rangle^2}$ [23–28] at the parton level. The quantities m'_{ti} and p'_{ti} , as well as the transverse mass m_{ti} and transverse momentum p_{ti} containing the flow effect, can be transformed into each other. One has the Lorentz-like transformation

$$\begin{aligned} m'_{ti} &= \langle \gamma_t \rangle (m_{ti} - p_{ti} \langle \beta_t \rangle), \\ p'_{ti} &= \langle \gamma_t \rangle |p_{ti} - m_{ti} \langle \beta_t \rangle|, \\ dp'_{ti} &= \frac{\langle \gamma_t \rangle}{m_{ti}} (m_{ti} - p_{ti} \langle \beta_t \rangle) dp_{ti}, \end{aligned} \quad (6)$$

where the absolute value $|p_{ti} - m_{ti} \langle \beta_t \rangle|$ is used due to p'_{pt} being positive and $p_{ti} - m_{ti} \langle \beta_t \rangle$ being possibly negative in low- p_{ti} region. The Lorentz-like factor or transformation is called, instead of the Lorentz factor or transformation is called. The reason is that $\langle \beta_t \rangle$ and $\langle \gamma_t \rangle$, but not β_t and γ_t , are used in the analysis.

After the Lorentz-like transformation, the probability density function, $f_{p'_{ti}}(p'_{ti})$, of p'_{ti} is transformed into the probability density function, $f_i(p_{ti})$, of p_{ti} . The relation between the two probability density functions is

$$f_{p'_{ti}}(p'_{ti}) |dp'_{ti}| = f_i(p_{ti}) |dp_{ti}|. \quad (7)$$

From Eqs. (5)–(7), one has

$$\begin{aligned} f_i(p_{ti}) &= f_{p'_{ti}}(p'_{ti}) \frac{|dp'_{ti}|}{|dp_{ti}|} \Big|_{m'_{ti}, p'_{ti}, dp'_{ti} \rightarrow m_{ti}, p_{ti}, dp_{ti}} \\ &= C \frac{\langle \gamma_t \rangle^3}{m_{ti}} |p_{ti} - m_{ti} \langle \beta_t \rangle| (m_{ti} - p_{ti} \langle \beta_t \rangle)^2 \\ &\quad \times \left[\exp \left(\frac{\langle \gamma_t \rangle (m_{ti} - p_{ti} \langle \beta_t \rangle) - \mu}{T_0} \right) + S \right]^{-1}, \end{aligned} \quad (8)$$

in which T is naturally rewritten as T_0 due to the introduction of $\langle \beta_t \rangle$. As the quantities at the parton level, T_0 and $\langle \beta_t \rangle$ may show different tendencies with centrality. The reason is that multiple scattering of secondary particles may happen in the participants and spectators which are centrality dependent.

Equation (8) is obtained from Eq. (5) due to the conversion of probability densities of transverse momentums in which $\langle \beta_t \rangle$ and $\langle \gamma_t \rangle$ are considered. Because of Eq. (5) being only for the case of midrapidity, Eq. (8) is deserved for the same case. Meanwhile, the application of $\langle \beta_t \rangle$ and $\langle \gamma_t \rangle$ at the parton level, but not β_t and γ_t of each parton, can avoid using too many parameters. In fact, the kinetic freeze-out temperature and transverse flow velocity extracted from the data fitting are usually the average quantities.

In the Monte Carlo calculations, let R_i and r_i denote random numbers distributed evenly in $[0,1]$. A concrete p_{ti} satisfies the relation

$$\int_0^{p_{ti}} f_i(p''_{ti}) dp''_{ti} < R_i < \int_0^{p_{ti} + \delta p_{ti}} f_i(p''_{ti}) dp''_{ti}, \quad (9)$$

where p''_{ti} denotes the integral variable to differ from the integral upper limit p_{ti} and δp_{ti} denotes a small amount shift from p_{ti} . The contributor partons are assumed to move isotropically in the transverse plane. To obtain a discrete azimuthal angle ϕ_i that satisfies the isotropic distribution, we have

$$\phi_i = 2\pi r_i. \quad (10)$$

In the transverse plane of the rectangular coordinate system, the x - and y -components of the vector \mathbf{p}_{ti} of the parton transverse momentum are

$$\begin{aligned} p_{xi} &= p_{ti} \cos \phi_i, \\ p_{yi} &= p_{ti} \sin \phi_i, \end{aligned} \quad (11)$$

respectively. If n partons contribute to p_T , the x - and y -components of the vector \mathbf{p}_T of the particle transverse momentum are

$$\begin{aligned} p_x &= \sum_{i=1}^n p_{ti} \cos \phi_i, \\ p_y &= \sum_{i=1}^n p_{ti} \sin \phi_i, \end{aligned} \quad (12)$$

respectively. Then, we have

$$p_T = \sqrt{\left(\sum_{i=1}^n p_{ti} \cos \phi_i \right)^2 + \left(\sum_{i=1}^n p_{ti} \sin \phi_i \right)^2}. \quad (13)$$

In particular, if $n = 2$ meaning that two contributor partons taken part in the formation of a particle, we have

$$\begin{aligned} p_T &= \sqrt{\left(\sum_{i=1}^2 p_{ti} \cos \phi_i \right)^2 + \left(\sum_{i=1}^2 p_{ti} \sin \phi_i \right)^2} \\ &= \sqrt{p_{t1}^2 + p_{t2}^2 + 2p_{t1}p_{t2} \cos |\phi_1 - \phi_2|}. \end{aligned} \quad (14)$$

If $n = 3$ meaning that three contributor partons taken part in the formation of a particle, we have

$$\begin{aligned} p_T &= \sqrt{\left(\sum_{i=1}^3 p_{ti} \cos \phi_i \right)^2 + \left(\sum_{i=1}^3 p_{ti} \sin \phi_i \right)^2} \\ &= \left(p_{t1}^2 + p_{t2}^2 + p_{t3}^2 + 2p_{t1}p_{t2} \cos |\phi_1 - \phi_2| + 2p_{t1}p_{t3} \cos |\phi_1 - \phi_3| + 2p_{t2}p_{t3} \cos |\phi_2 - \phi_3| \right)^{1/2}. \end{aligned} \quad (15)$$

The p_T spectra of particles can be divided into two or three regions. This means that one needs two- or three-component function to fit the p_T spectra. If the first component describes the spectra in low- p_T region, which corresponds to the contribution of soft excitation process, the last component describes the spectra in high- p_T region which corresponds to the contribution of hard scattering process. Naturally, the intermediate component (in three-component function) describes the spectra in intermediate- p_T region. Generally, at low energy or for narrow p_T spectra, one or two components are needed. At high energy or for wide p_T spectra, three or more components are needed.

In the Monte Carlo calculations, one may obtain the digitized probability density function, $f_j(p_T, T_{0j}, \langle \beta_t \rangle_j) = (1/N) dN/dp_T|_j$, for the contribution of the j -th component, where T_{0j} and $\langle \beta_t \rangle_j$ denote the kinetic freeze-out temperature and average transverse flow velocity corresponding to the j -th component. Due to the multicomponent, one has the p_T distribution measured in experiments to be

$$f(p_T) = \frac{1}{N} \frac{dN}{dp_T} = \sum_{j=1}^K k_j f_j(p_T, T_{0j}, \langle \beta_t \rangle_j), \quad (16)$$

where K denote the number of components and k_j denote the contribution fraction of the j -th component. The normalization $\sum_{j=1}^K k_j = 1$ is naturally obeyed.

In the above discussions, from the physics point of view, the origin of multiple sources has two meanings. For a given kind of particle, the multiple sources originate from multiple mechanisms of interactions or different excitation degrees of the system, which results in the multicomponent distribution. If the particles in low-, intermediate-, and high- p_T regions are produced by three different mechanisms or from three excitation degrees, the multiple sources become three sources. For a given particle, the multiple sources refer to multiple energy sources when the particle is formed. Generally, two or three energy sources are considered in the formation of the given particle.

It is noteworthy that the collective motion, which pertains to a common velocity or momentum of the particles, does not lead to the kinetic freeze-out temperature. In fact, the temperature is known to originate only from random thermal motion and reflects the degree of intensity of the thermal motion. In the present work, to obtain the kinetic freeze-out temperature, the transverse flow velocity is introduced to exclude the influence of collective motion. This treatment method is easy to use in the fit of experimental data. If the influence of collective motion is not excluded,

i.e., if the transverse flow velocity is not considered, one will obtain the effective temperature which is larger than the kinetic freeze-out temperature.

In the present work, to fit the experimental invariant yield, $(1/2\pi p_T)d^2N/dydp_T$, one needs to structure the relation, $(1/2\pi p_T)N_0f(p_T) = (1/2\pi p_T) \int [d^2N/dydp_T]dy$, where N_0 is the normalization constant that is generally the area under the data, dN/dp_T . In the fit, N_0 is determined by the data itself and has no relation to the model. Although N_0 does not appear as a parameter in the present work to avoid triviality, one more “1” is subtracted when counting the number of degree of freedom (ndof).

3. Results and Discussion

As an application of the above extraction method based on the description of p_T spectra, the invariant yields, $(1/2\pi p_T N_{EV})d^2N/dydp_T$, of neutral pions (π^0) produced in mid-pseudorapidity ($|\eta| < 0.35$) in gold-gold (Au–Au) collisions with different centrality percentages at the center-of-mass energy per nucleon pair $\sqrt{s_{NN}} = 200$ GeV are presented in Figure 1, where y is the rapidity as defined above, the pseudorapidity $\eta = -\text{Lntan}(\theta/2)$, and θ denotes the emission angle of the particle. The various symbols represent the experimental data measured by the PHENIX Collaboration [44]. The curves are our results fitted by the three-component function. For clarity, the samples for different centrality percentages are rescaled by different factors as mentioned in the legends in the figure. The values of the parameters T_{0j} , $\langle\beta_t\rangle_j$, k_j , χ^2 , and ndof are listed in Table 1, where $j = 1, 2$, and 3. Here, $k_1 (= 1 - k_2 - k_3)$ is not listed due to the fact that it can be obtained from the normalization.

One can see that the PHENIX data are fitted satisfactorily by the three-component function. Although there is no data in the region of $p_T > 12$ GeV/c in peripheral collisions, one may show an extension of the fitted curves based on the parameters extracted from the data in the region $p_T < 12$ GeV/c. The extension of the fitted curves in peripheral collisions can be compared with those in central and semi-central collisions. The wavy structure in each case is caused by the low statistics in high- p_T region.

Based on Table 1, the dependences of parameters on centrality percentage C are also shown in Figure 2. From Figures 2(a)–2(c), the dependences are for T_{0j} , $\langle\beta_t\rangle_j$, and k_j , respectively. It is observed that with the decrease in centrality (or with the increase in centrality percentage) from central to peripheral collisions, the parameters studied here do not have significant change and no obvious fluctuations are observed. If there are any fluctuations, they are insignificant. One can say that these parameters are centrality independent. The reason is that the p_T spectra in Figure 1 are very similar, if not equal, in the shape in different centrality intervals in Au–Au collisions. This implies that the kinetic freeze-out parameters extracted from the spectra of π^0 are nearly independent of the centrality.

It should be noted that most of the parameters in Table 1 have the same uncertainties across the centrality, though one

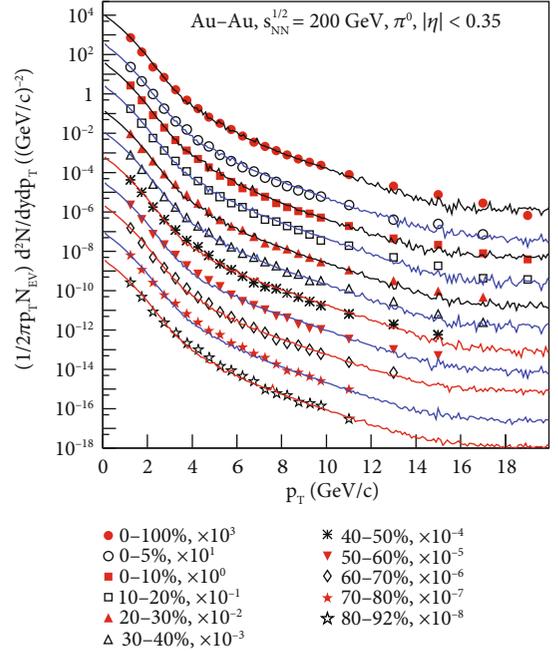


FIGURE 1: The invariant yields, $(1/2\pi p_T N_{EV})d^2N/dydp_T$, of π^0 produced in $|\eta| < 0.35$ in Au–Au collisions with different centrality percentages at $\sqrt{s_{NN}} = 200$ GeV. The symbols represent the experimental data measured by the PHENIX Collaboration [44]. The curves are our results fitted by the three-component function in which each component is regarded as the sum of contributions of two contributor partons. The contribution of each parton of given particle is assumed to obey the standard distribution with the isotropic azimuthal angle.

has seen some differences in the fourth decimal place. Only three decimal places are kept in the table. Because of parameter uncertainties being less than the symbol size, they are not visible in Figure 2.

Table 2 shows the values of correlation coefficients (r_{xy}) between two parameters, where the derived parameter k_1 is also included. It is observed that the correlations are not present for the case of having no k_j ($|r_{xy}| < 0.8$). For example, there is no significant correlation between T_{0j} and $\langle\beta_t\rangle_j$, T_{0j} , and $T_{0j'}$, as well as $\langle\beta_t\rangle_j$ and $\langle\beta_t\rangle_{j'}$, where both j and j' are the sequence numbers of the component. Some correlations involved to k_j are significant ($|r_{xy}| > 0.8$). The reason is that the parameters T_{0j} and $\langle\beta_t\rangle_j$ are restrained to the special p_T regions and affect the local shapes of p_T spectra, while the parameters k_j affect the spectra in whole p_T range due to the normalization. Concretely, only T_{01} and k_1 (or k_3 due to $k_3 = 1 - k_1 - k_2$), as well as k_1 and k_2 (or k_3), are highly correlated. This does not give rise to the problem of multicollinearity.

To see the influence of azimuthal angular difference between the two contributor partons, Figure 3 displays a comparison of the results of the isotropic azimuthal angle (the solid curves), the parallel or identical case ($\phi_1 = \phi_2$, the dashed curves), the vertical case ($|\phi_1 - \phi_2| = \pi/2$, the dotted curves), and the special case of $|\phi_1 - \phi_2| = 3\pi/4$ (the dot-

TABLE 1: Values of T_{0j} , $\langle\beta_t\rangle_j$, k_j , χ^2 , and ndof corresponding to the solid curves in Figure 1 for various centrality percentages C , where $i = 1, 2$, and 3. In particular, $k_1 = 1 - k_2 - k_3$ that is not listed.

C (%)	T_{01} (GeV)	T_{02} (GeV)	T_{03} (GeV)	$\langle\beta_t\rangle_1$ (c)	$\langle\beta_t\rangle_2$ (c)	$\langle\beta_t\rangle_3$ (c)	k_2 (%)	k_3 (%)	χ^2/ndof
0–100	0.178 ± 0.002	0.365 ± 0.004	0.433 ± 0.004	0.239 ± 0.002	0.403 ± 0.004	0.605 ± 0.006	0.160 ± 0.002	0.014 ± 0.001	24/16
0–5	0.175 ± 0.002	0.366 ± 0.004	0.434 ± 0.004	0.239 ± 0.002	0.403 ± 0.004	0.605 ± 0.006	0.156 ± 0.002	0.015 ± 0.001	20/15
0–10	0.176 ± 0.002	0.365 ± 0.004	0.432 ± 0.004	0.238 ± 0.002	0.401 ± 0.004	0.610 ± 0.006	0.145 ± 0.001	0.010 ± 0.001	33/16
10–20	0.175 ± 0.002	0.366 ± 0.004	0.434 ± 0.004	0.239 ± 0.002	0.403 ± 0.004	0.600 ± 0.006	0.145 ± 0.001	0.018 ± 0.001	35/16
20–30	0.177 ± 0.002	0.363 ± 0.004	0.436 ± 0.004	0.239 ± 0.002	0.405 ± 0.004	0.608 ± 0.006	0.145 ± 0.001	0.018 ± 0.001	25/15
30–40	0.178 ± 0.002	0.366 ± 0.004	0.433 ± 0.004	0.239 ± 0.002	0.402 ± 0.004	0.602 ± 0.006	0.145 ± 0.001	0.018 ± 0.001	21/15
40–50	0.178 ± 0.002	0.367 ± 0.004	0.436 ± 0.004	0.239 ± 0.002	0.403 ± 0.004	0.602 ± 0.006	0.152 ± 0.002	0.031 ± 0.001	17/14
50–60	0.180 ± 0.002	0.366 ± 0.004	0.434 ± 0.004	0.239 ± 0.002	0.402 ± 0.004	0.604 ± 0.006	0.152 ± 0.002	0.034 ± 0.001	23/14
60–70	0.180 ± 0.002	0.366 ± 0.004	0.431 ± 0.004	0.239 ± 0.002	0.405 ± 0.004	0.605 ± 0.006	0.152 ± 0.002	0.031 ± 0.001	14/13
70–80	0.181 ± 0.002	0.365 ± 0.004	0.431 ± 0.004	0.240 ± 0.002	0.403 ± 0.004	0.602 ± 0.006	0.165 ± 0.002	0.028 ± 0.001	15/12
80–92	0.180 ± 0.002	0.367 ± 0.004	0.430 ± 0.004	0.238 ± 0.002	0.403 ± 0.004	0.602 ± 0.006	0.165 ± 0.001	0.028 ± 0.002	17/12

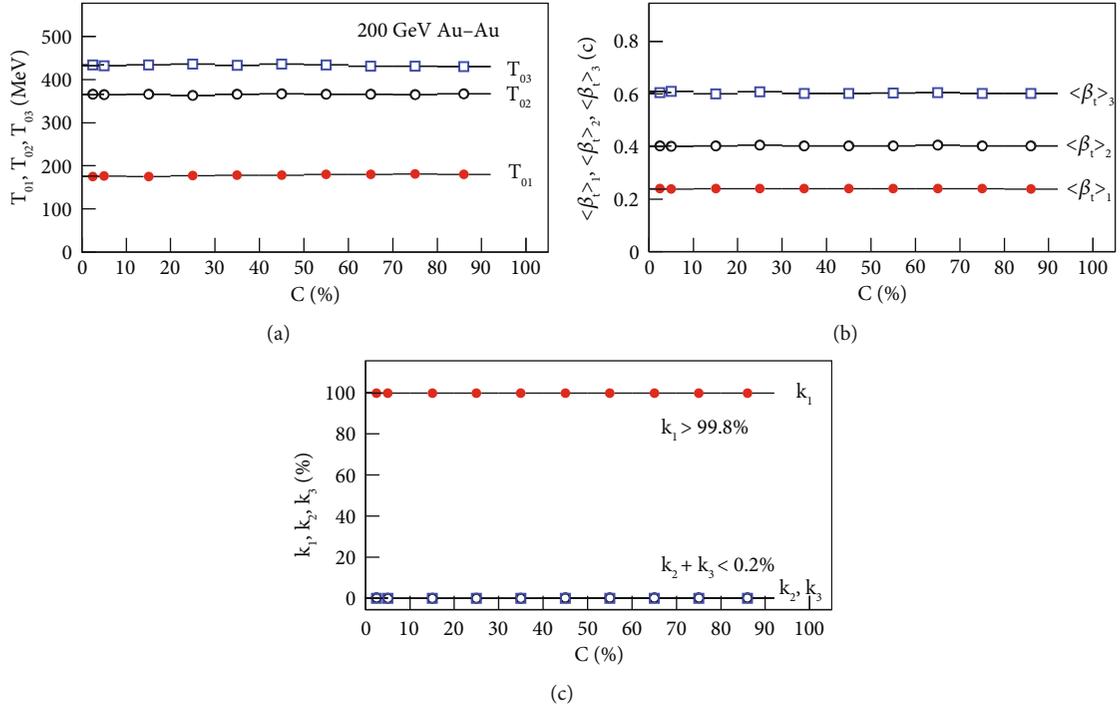


FIGURE 2: Dependences of parameters on centrality percentage C for π^0 production in Au–Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV. (a–c) The dependences are for T_{0j} , $\langle\beta_t\rangle_j$, and k_j , respectively. The contribution ratio of the first component is larger than 99.8%, which means that $T_0 \approx T_{01}$ and $\langle\beta_t\rangle_1 \approx \langle\beta_t\rangle_{01}$.

dashed curves) for three centrality percentages as an example, where the tail parts with several sharp drops in the curves have been cut to avoid confusion. One can see that the shapes of the curves in low- p_T ($p_T < 1$ GeV/c) and high- p_T ($p_T > 14$ GeV/c) regions are significantly different. In the intermediate- p_T region, the dashed curve is more similar to the solid curve if one renormalizes the former. This reflects that in the intermediate- p_T region, one may use the result of $\phi_1 = \phi_2$ to replace approximately that of the isotropic azimuthal angle. The cases of $|\phi_1 - \phi_2| = \pi/2$ and $3\pi/4$

need larger T_{0j} and $\langle\beta_t\rangle_j$ to cater to that of the isotropic azimuthal angle.

To show the contribution of each component with the isotropic azimuthal angle, Figure 4 displays the contributions of the first (the dashed curves), the second (the dotted curves), and the third (the dot-dashed curves) components together with that of the total three components (the solid curves), where the tail parts with several sharp drops in the curves have been cut to avoid confusion. Naturally, the first component contributes mainly in the low- p_T ($p_T < 6$ GeV/c)

TABLE 2: Values of correlation coefficients r_{xy} between two parameters.

	T_{01}	T_{02}	T_{03}	$\langle\beta_t\rangle_1$	$\langle\beta_t\rangle_2$	$\langle\beta_t\rangle_3$	k_1	k_2	k_3
T_{01}	1.000	0.174	-0.507	0.266	0.164	-0.242	-0.822	0.611	0.814
T_{02}	0.174	1.000	-0.263	-0.222	-0.296	-0.614	-0.445	0.350	0.423
T_{03}	-0.507	-0.263	1.000	0.202	0.131	0.086	0.397	-0.562	-0.137
$\langle\beta_t\rangle_1$	0.266	-0.222	0.202	1.000	0.320	-0.376	-0.285	0.180	0.312
$\langle\beta_t\rangle_2$	0.164	-0.296	0.131	0.320	1.000	0.003	-0.207	0.080	0.280
$\langle\beta_t\rangle_3$	-0.242	-0.614	0.086	-0.376	0.003	1.000	0.465	-0.349	-0.455
k_1	-0.822	-0.445	0.397	-0.285	-0.207	0.465	1.000	-0.861	-0.875
k_2	0.611	0.350	-0.562	0.180	0.080	-0.349	-0.861	1.000	0.509
k_3	0.814	0.423	-0.137	0.312	0.280	-0.455	-0.875	0.509	1.000

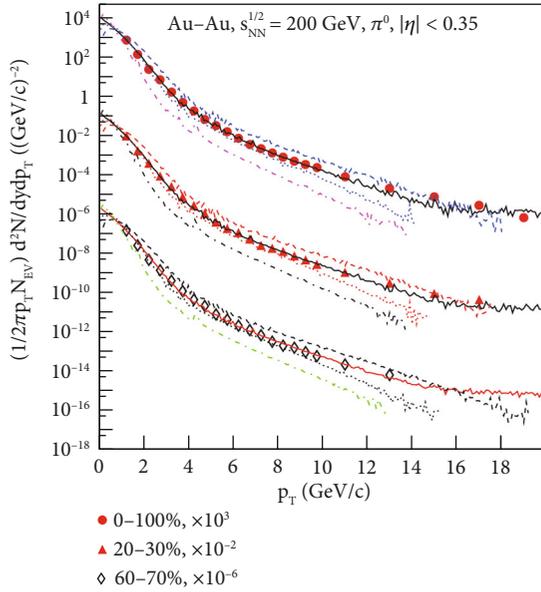


FIGURE 3: Comparison of the results of the isotropic azimuthal angle (the solid curves), the parallel contributions ($\phi_1 = \phi_2$, the dashed curves), the vertical contributions ($|\phi_1 - \phi_2| = \pi/2$, the dotted curves), and the special case of $\phi_1 - \phi_2 = 3\pi/4$ (the dot-dashed curves) for three centrality percentages. The tail parts with several sharp drops in the curves have been cut to avoid confusion. The symbols and the solid curves are the same as Figure 1.

region. The second component contributes mainly in the low- and intermediate- p_T ($p_T < 15$ GeV/c) regions. The third component contributes in the whole p_T region. Although the second and third components contribute in wider p_T region than the first one, the most important contributor is the first component with $k_1 = 1 - k_2 - k_3 > 0.998$ calculated from Table 1.

Due to very small amounts for $k_{2,3}$ (here $k_2 < 0.002$ and $k_3 < 0.0004$), the second and third components do not affect the values of T_0 ($= k_1 T_{01} + k_2 T_{02} + k_3 T_{03}$) and $\langle\beta_t\rangle$ ($= k_1 \langle\beta_t\rangle_1 + k_2 \langle\beta_t\rangle_2 + k_3 \langle\beta_t\rangle_3$) significantly. One has $T_0 \approx T_{01}$ and $\langle\beta_t\rangle \approx \langle\beta_t\rangle_1$. In fact, including k_2 and k_3 causes the increase of T_0 and $\langle\beta_t\rangle$ to be less than 0.5%. One may say that although there are 8 free parameters in the fit for the

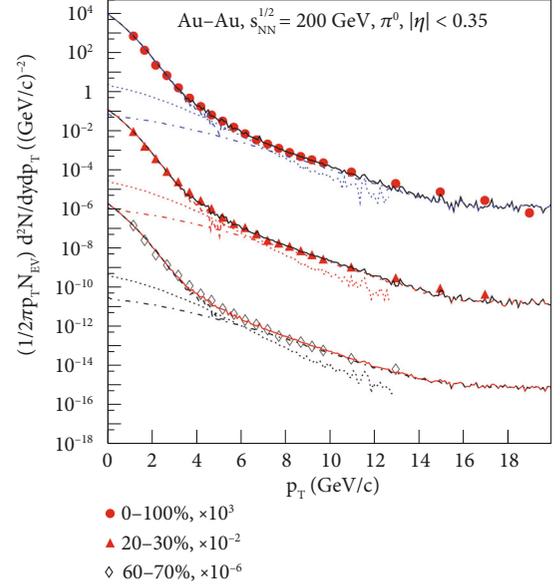
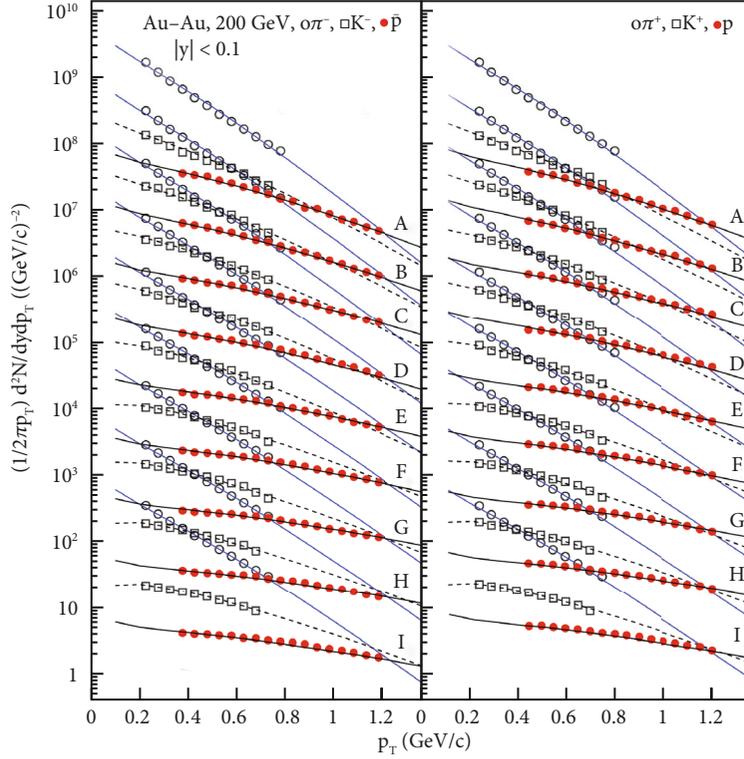


FIGURE 4: Comparison of the results from the three components for three centrality percentages, where the dashed, dotted, and dot-dashed curves reflect the contributions of the first, second, and third components, respectively. The tail parts with several sharp drops in the curves have been cut to avoid confusion. The symbols and the solid curves are the same as Figure 1.

data sets in Figure 1, the main parameters are only T_{01} and $\langle\beta_t\rangle_1$ which represent reasonably T_0 and $\langle\beta_t\rangle$.

In the data analysis, for the purpose of the extraction of kinetic freeze-out parameters, one does not need too wide p_T spectra. Generally, the range of $p_T < 6$ GeV/c (even $p_T < 4$ GeV/c) is enough, though the p_T range is 0~20 GeV/c in some cases in Figures 1, 3, and 4. If the spectrum in low- p_T region is contributed by the soft excitation process and that in high- p_T region is contributed by the hard scattering process, the present work shows that only the contribution of soft process is enough to extract the kinetic freeze-out parameters. The contribution fraction of the hard process is smaller than 0.2%.

The value of T_0 is extracted together with $\langle\beta_t\rangle$ at the parton level. Due to the introduction of $\langle\beta_t\rangle$, T_0 becomes



A 70–80%, $\times 10^8$	F 20–30%, $\times 10^3$
B 60–70%, $\times 10^7$	G 10–20%, $\times 10^2$
C 50–60%, $\times 10^6$	H 5–10%, $\times 10^1$
D 40–50%, $\times 10^5$	I 0–5%, $\times 10^0$
E 30–40%, $\times 10^4$	

FIGURE 5: The invariant yields, $(1/2\pi p_T)d^2N/dydp_T$, of (a) π^- , K^- , and \bar{p} , as well as (b) π^+ , K^+ , and p produced in $y < 0.1$ in Au–Au collisions with different centrality percentages at $\sqrt{s_{NN}} = 200$ GeV. The symbols represent the experimental data measured by the STAR Collaboration [45]. The curves are our results fitted by the single-component function (or the first component in the three-component function), where the sum of contributions of two (for mesons) or three (for baryons) contributor partons is considered.

smaller than the effective temperature T because the flow effect is excluded. The introduction of $\langle\beta_t\rangle$ also affects the tendency of T_0 with changing the centrality and other factors. At both the parton and particle levels, the obtained T_0 or $\langle\beta_t\rangle$ may be inconsistent in the tendency and/or size due to different extraction methods. One should have a determined and uniform method to extract T_0 and $\langle\beta_t\rangle$. As a tentative work, the present work focuses on the parameters at the parton level. Meanwhile, the standard distribution used in the relativistic ideal gas model is applied for the system. In the aspect of extracting the parameters at the kinetic freeze-out, this work has an important significance in methodology.

Many data sets have been analyzed in our previous work [33–37] by the thermal-related models, though only the p_T spectra of π^0 in Au–Au collisions at $\sqrt{s_{NN}} = 200$ GeV are mainly analyzed in this work as an example to check the validity of the model in methodology. In our recent work [51], using the blast-wave model with fluctuations [52–54], the kinetic freeze-out parameters extracted from the p_T spectra of charged pions ($\pi^- + \pi^+$) and kaons ($K^- + K^+$) pro-

duced in lead–lead (Pb–Pb) collisions at $\sqrt{s_{NN}} = 2.76$ TeV [55], proton–lead (p–Pb) collisions at $\sqrt{s_{NN}} = 5.02$ TeV [56], and xenon–xenon (Xe–Xe) collisions at $\sqrt{s_{NN}} = 5.44$ TeV [57] show almost the independence of centrality, and the parameters extracted from the p_T spectra of antiprotons and protons ($\bar{p} + p$) show the dependence of centrality.

The difference between the parameters from the spectra of mesons and $\bar{p} + p$ is caused by different production mechanisms. Generally, mesons are newly produced and some protons already exist in the projectile and target nuclei before the collisions. In our opinion, the extracted parameter values for $\bar{p} + p$ production are reduced by the influence of these pre-existing protons, which are the leading particles with low excitation in the collisions. They increase relatively the yield in low- p_T region. The relative increase is more obvious in peripheral collisions due to lesser multiple scattering.

It is expected that for a narrow spectrum in low- p_T region, a single- or two-component function is suitable; for a not too wide p_T spectrum, the three-component function is suitable; and for a wider p_T spectrum, the fourth

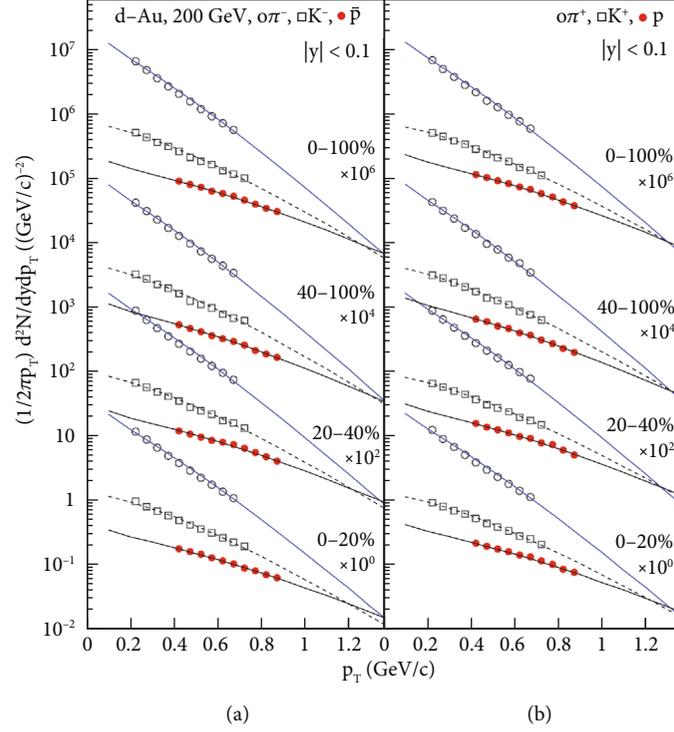


FIGURE 6: The invariant yields, $(1/2\pi p_T) d^2N/dydp_T$, of (a) π^- , K^- , and \bar{p} , as well as (b) π^+ , K^+ , and p produced in $y < 0.1$ in d-Au collisions with different centrality percentages at $\sqrt{s_{NN}} = 200$ GeV. The symbols represent the experimental data measured by the STAR Collaboration [45]. The curves are our results fitted by the single-component function.

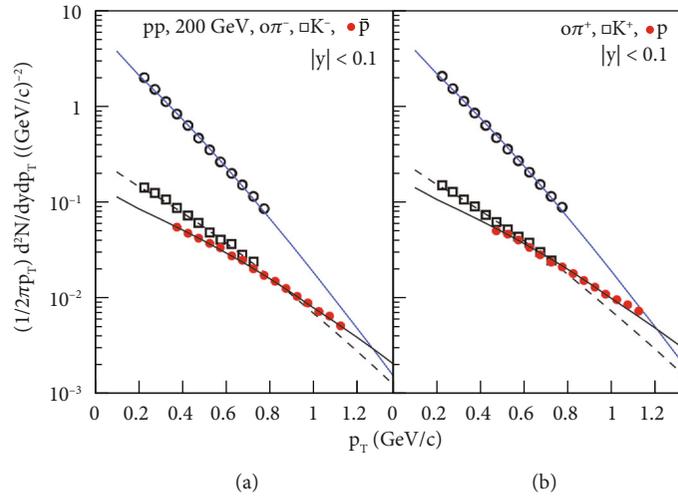


FIGURE 7: The invariant yields, $(1/2\pi p_T) d^2N/dydp_T$, of (a) π^- , K^- , and \bar{p} , as well as (b) π^+ , K^+ , and p produced in $y < 0.1$ in pp collisions at $\sqrt{s} = 200$ GeV. The symbols represent the experimental data measured by the STAR Collaboration [45]. The curves are our results fitted by the single-component function.

component may be included if the three components are not enough. To check the methodology by the spectra in a narrow p_T region, for which only one component is used in the fit, Figures 5(a)–7(a) show the spectra of π^- , K^- , and \bar{p} , and Figures 5(b)–7(b) show the spectra of π^+ , K^+ , and p , produced in Au–Au, deuterium–gold (d–Au), and proton–proton (pp) collisions at $\sqrt{s_{NN}} = 200$ GeV, where $\sqrt{s_{NN}}$ is

simplified to \sqrt{s} for pp collisions. The symbols represent the experimental data measured by the STAR Collaboration [45]. The curves are our fitted results by a single-component function in which only T_{01} and $\langle \beta_t \rangle_1$ are free parameters which are listed in Table 3. For the productions of $\pi^- + \pi^+$ and $K^- + K^+$, two contributor partons contribute to p_T . For the production of $\bar{p} + p$, three contributor partons contribute

TABLE 3: Values of T_{01} , $\langle\beta_t\rangle_1$, and ndof corresponding to the curves in Figure 5 for Au-Au collisions, in Figure 6 for d-Au collisions, and in Figure 7 for pp collisions.

Figure	C (%)	π^\mp			K^\mp			$\bar{p}(p)$		
		T_{01} (GeV)	$\langle\beta_t\rangle_1$ (c)	χ^2/ndof	T_{01} (GeV)	$\langle\beta_t\rangle_1$ (c)	χ^2/ndof	T_{01} (GeV)	$\langle\beta_t\rangle_1$ (c)	χ^2/ndof
Figure 5	0–5	0.132 ± 0.012	0.128 ± 0.012	14/8	0.202 ± 0.019	0.245 ± 0.021	2/8	0.209 ± 0.021	0.423 ± 0.039	0.1/14
	5–10	0.132 ± 0.012	0.128 ± 0.012	11/8	0.197 ± 0.019	0.241 ± 0.021	2/8	0.209 ± 0.021	0.423 ± 0.039	0.2/14
	10–20	0.132 ± 0.012	0.128 ± 0.012	8/8	0.190 ± 0.017	0.233 ± 0.020	2/8	0.202 ± 0.020	0.412 ± 0.037	0.2/14
	20–30	0.131 ± 0.012	0.127 ± 0.011	7/9	0.188 ± 0.017	0.231 ± 0.020	1/8	0.197 ± 0.020	0.362 ± 0.034	0.1/14
	30–40	0.130 ± 0.012	0.127 ± 0.011	5/9	0.170 ± 0.015	0.210 ± 0.018	1/8	0.193 ± 0.019	0.345 ± 0.031	0.2/14
	40–50	0.127 ± 0.011	0.125 ± 0.011	3/9	0.169 ± 0.015	0.185 ± 0.016	2/8	0.186 ± 0.017	0.253 ± 0.022	0.1/14
	50–60	0.124 ± 0.010	0.121 ± 0.010	2/9	0.169 ± 0.015	0.185 ± 0.016	1/8	0.184 ± 0.017	0.251 ± 0.021	0.1/14
	60–70	0.123 ± 0.009	0.120 ± 0.010	1/9	0.165 ± 0.014	0.145 ± 0.013	1/8	0.179 ± 0.016	0.181 ± 0.016	0.1/14
Figure 6	0–100	0.119 ± 0.008	0.120 ± 0.010	4/7	0.149 ± 0.012	0.180 ± 0.016	0.1/8	0.167 ± 0.014	0.162 ± 0.013	0.01/7
	0–20	0.123 ± 0.010	0.122 ± 0.011	6/7	0.152 ± 0.013	0.183 ± 0.017	0.2/8	0.172 ± 0.015	0.167 ± 0.014	0.01/7
	20–40	0.119 ± 0.008	0.120 ± 0.010	5/7	0.149 ± 0.012	0.180 ± 0.016	0.2/8	0.168 ± 0.014	0.163 ± 0.013	0.01/7
	40–100	0.117 ± 0.007	0.118 ± 0.009	2/7	0.146 ± 0.011	0.177 ± 0.014	0.1/8	0.161 ± 0.012	0.155 ± 0.011	0.01/7
Figure 7	—	0.116 ± 0.007	0.118 ± 0.008	1/9	0.155 ± 0.012	0.118 ± 0.008	0.02/8	0.151 ± 0.014	0.118 ± 0.008	1/13

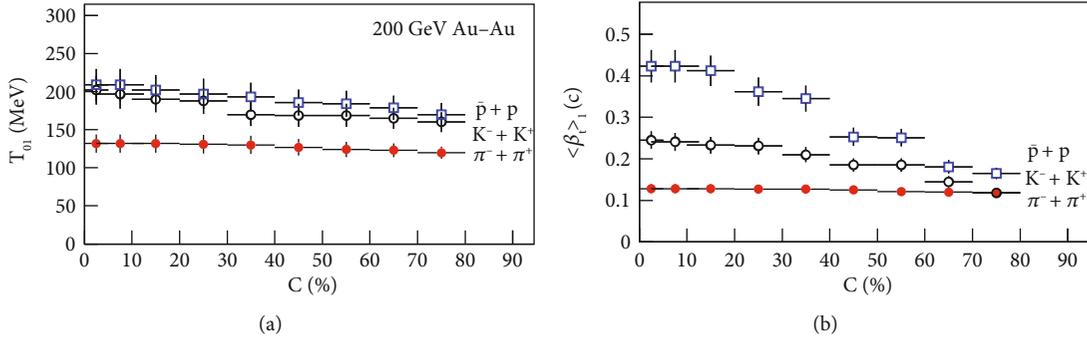


FIGURE 8: Dependences of (a) T_{01} and (b) $\langle\beta_t\rangle_1$ on centrality percentage C and particle kind. In each panel, the closed circles, open circles, and open squares represent the results for $\pi^- + \pi^+$, $K^- + K^+$, and $\bar{p} + p$, respectively.

to p_T . One can see that the single-component function can fit the spectra in the narrow p_T region. Note that some of the χ^2/ndof values in Table 3 are very small. This is because the combined errors in p_T spectra are predominantly determined by large systematic uncertainties, which do not follow a normal distribution. The statistical uncertainties, which have a random nature, are very small and negligible.

The parameter values listed in Table 3 show that pp collisions are similar to peripheral d-Au and Au-Au collisions at the same $\sqrt{s_{\text{NN}}}$. Central, semicentral, and peripheral d-Au collisions are similar to peripheral Au-Au collisions. Larger values of parameters in central Au-Au collisions imply that larger amounts of collision energy are deposited in the system and more multiple scatterings occur in the hot and dense matter.

Table 3 also shows that T_{01} and $\langle\beta_t\rangle_1$ decrease with the decrease in centrality from central to peripheral events and decrease with the decrease in particle mass for the consid-

ered three kinds of particles, in Au-Au and d-Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV. Figure 8 shows the dependence of T_{01} and $\langle\beta_t\rangle_1$ on centrality percentage in Au-Au collisions. One can see clearly the dependence of the parameters on centrality and particle kind. Comparing with that for π^0 , the dependence for charged massive particles ($K^- + K^+$ and $\bar{p} + p$) is significant and the distributions for $\pi^- + \pi^+$ are almost flat, also showing no dependence on centrality. The reason is that charged massive particles have more probability to interact with spectator nucleons, which causes a large red shift of the spectra in peripheral collisions.

Generally, T_0 and $\langle\beta_t\rangle_1$ reflect the excitation and expansion degrees of the system, respectively. The larger these parameters are, the higher the excitation and expansion degrees of the system is. In the case of charged massive particles, central collisions correspond to higher excitation and expansion degrees due to larger amounts of collision energy

being deposited. Meanwhile, charged massive particles have relative large elastic scattering cross-section due to their large sizes, while neutral and charged pions have very small elastic scattering cross-section in the system, which results their parameters showing almost independent of centrality.

The reason why one may apply the same distribution to the energy sources with different degrees of excitation and expansion in high-energy collision systems is because of the similarity, commonality, and universality, especially the universality, in the collisions [58–65]. In particular, in high-energy collisions, the underlying reason is the contributor partons appearing as the energy sources or influence factors. This also explains the consistency of some quantities in high-energy collision systems with different sizes and centralities. Some model independent dependencies, if available, are mainly caused by the effective energies of the contributor partons or energy sources.

In the above discussion, the classical concepts of temperature and equilibrium are tentatively used. However, the collision system is very small. In particular, only two or three contributor partons are considered in the formation of given particle. It seems that the mentioned concepts are not applicable. In fact, although few partons are considered to contribute directly and mainly to particle's p_T , lots of partons exist in the system of high-energy collisions. In addition, the experiments of high-energy collisions involve lots of events. One may use the grand canonical ensemble to study the particle production. Then, the concepts of temperature and equilibrium are applicable. At least, one may regard T_0 and $\langle\beta_t\rangle$ as parameters that describe the average kinetic energies of thermal and collective motions of partons, respectively.

Before summary and conclusions, we would like to emphasize that although many parameters are used in this article, this is only for the wide p_T spectra with multiregion structure. Indeed, mathematically, increasing the number of parameters increases the probability of a good fit as one has more free parameters to play around. Technically, one may not completely discard the usage because there are different particle production mechanisms in different p_T regions. This work uses the standard distribution for a given parton source and the multicomponent distribution for the parton sources with different excitation degrees. In the case of extracting T_0 and $\langle\beta_t\rangle$, only two parameters T_{01} and $\langle\beta_t\rangle_1$ are enough. The distribution with T_{01} and $\langle\beta_t\rangle_1$ ($\neq 0$) describes wider spectra than that with only T_{01} (where $\langle\beta_t\rangle_1 = 0$). Because this work is based on the standard distribution which is widely used in statistical analysis in modern physics, the results are suitable to be the baseline for comparing with other experiments and simulation studies.

In addition, it has been established for almost 20 years that the high- p_T spectra of hadrons in nuclear collisions are explained by jet quenching; i.e., high- p_T partons lose energy and then fragment to hadrons. This work shows an alternative and uniform explanation for the statistical behavior of particle spectra in various p_T regions. There is no contradiction between the two explanations. If the explanation of jet quenching is focused on the production mechanism, the present work is focused on the statistical law obeyed by

the contributor partons and produced particles. Even if in the high- p_T region, the standard distribution is applicable to extract the kinetic freeze-out parameters. Although this results in quite a large T_{02} and T_{03} (~ 0.365 – 0.435 GeV) in the fitting for all centralities, the value of T_0 weighted by T_{0i} is small. Here, the weighted factor is k_i . T_0 is determined by T_{01} that is extracted from the pion spectra to be ~ 0.132 – 0.175 GeV in central Au–Au collisions, ~ 0.123 GeV in central d–Au collisions, and 0.116 GeV in pp collisions at $\sqrt{s_{NN}} = 200$ GeV. It is likely that the unexpected large T_{02} and T_{03} are obtained by the hard process due to violent head on collisions between partons, though this probability is very small.

Different methods or functions used in the extractions of temperature and flow velocity are different “thermometers” and “speedometers.” Although the tendencies of parameters based on different methods are almost the same or approximately the same, there are differences in concrete values. Obviously, before giving a comparison, these thermometers and speedometers should be uniformed according to the selected baseline. In our opinion, the standard distribution is a good candidate to be the baseline. The fact that T_{0j} and $\langle\beta_t\rangle_j$ for pion production are almost independent of centrality is an indicator that there is something more universal than bulk medium flow that governs the physics in different p_T regions. The underlying contributors are partons, but not nucleons, in various collisions at high energy. This also implies the similarity, commonality, and universality, in particular universality, in high-energy collisions [58–65].

4. Summary and Conclusions

In the framework of multisource thermal model used in the parton level, the transverse momentum spectra of the final-state neutral pions and identified charged hadrons produced in mid-(pseudo)rapidity region in Au–Au and d–Au collisions with various centralities and in pp collisions at $\sqrt{s_{NN}} = 200$ GeV have been studied. For a given particle of any type, its contributors may be two or three partons with isotropic azimuthal angle distribution. The contribution of each parton to transverse momentum of the hadron is assumed to obey the standard distribution with given kinetic freeze-out temperature and average transverse flow velocity. The transverse momentum spectra of the final-state hadrons can be fitted by the superposition of two or three components. The number of components is related to the width of transverse momentum spectra.

The results calculated by the Monte Carlo method fit satisfactorily the experimental data measured by the PHENIX and STAR Collaborations. With the decrease in centrality from central to peripheral collisions, the kinetic freeze-out temperature and average transverse flow velocity for each component in pion production do not change significantly. Due to the very small contribution fractions of the second and third components, the main parameters are determined by the first component in the low transverse momentum region. The result corresponding to the isotropic azimuthal

angles is similar to that of the identical azimuthal angles. The kinetic freeze-out parameters decrease with the decrease in centrality for the production of the charged massive hadrons. The work based on the standard distribution is suitable to be the baseline in comparing with other experiments and simulation studies.

Data Availability

The data used to support the findings of this study are included within the article and are cited at relevant places within the text as references.

Ethical Approval

The authors declare that they are in compliance with ethical standards regarding the content of this paper.

Disclosure

The funding agencies have no role in the design of the study; in the collection, analysis, or interpretation of the data; in the writing of the manuscript; or in the decision to publish the results. A preprint has previously been published in arXiv [66]. This paper applies the multisource thermal model, which has also been applied in our recent work [67, 68]. As a result, some similar statements are inevitably applied in this paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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