Model Building Based on Bootstrap Symmetry Breaking*

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1. At the Nagoya Workshop last year, I presented my ideas concerning the nature of the Higgs boson in the standard model [1]. It was motivated by my long-standing interest in the BCS mechanism as the origin of masses. In essence it boils down to the proposition that the Higgs boson is a top-antitop bound state.

This idea has also been advanced by the Nagoya-Kiev group [2], and some detailed calculations have have been carried out by the Fermilab group [3]. Independtly of the motivation that led me to the model, it looks very natural now that the experimental searches for the top quark are raising the top quark mass higher and higher. (I understand the current lower limit for the top mass from the Fermilab experiment to be in the W-Z mass range.) This makes the Yukawa coupling of the top quark larger and larger, and therefore the picture emerges that the Higgs field is largely made up of the top-antitop component.

I will briefly recapitulate here my ideas about the BCS mechanism, which consist of several ingredients. By the BCS mechanism I mean dynamical generation of the fermion mass due to a short range interaction, together with the Goldstone (pi) and Higgs (sigma) bosons as collective modes. The mass scale is usually small compared to the energy scale at which the original interaction is considered. Remarkably the masses of these low energy modes satisfy a simple relation

$$m_H : m_f : m_G = 2 : 1 : 0, \qquad (1)$$

in the short range, or bubble approximation, limit. Actually this is a special case of a more general sum rule

$$m_1{}^2 + m_2{}^2 = 4\, m_f{}^2, \qquad (2)$$

which is essentially a completeness relation for the composite two-fermion operators. In nonrelativistic examples of superconductivity and ^{3}He superfluidity, these relations are known to be satisfied.

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Once the low energy modes are identified, one can write down an effective Ginsburg-Landau theory for them. Because of the mass relation (1), the Higgs self-coupling λ and the Yukawa coupling f are related by $\lambda = m_f$. The vacuum expectation value v, or the "pion decay constant", represents the high energy scale of the original BCS Hamiltonian, and f is by definition the ratio between the two energy scales.

The simple mass relations suggest a kind of broken supersymmetry inherent in the BCS mechanism. In fact the effective GL Hamiltonian can be factorized as a product of fermionic opeerators as in the real supersymmetry, provided one ingnores the kinetic terms. I will discuss relativistic generalizations of this "quasi-supersymmetry" at the end.

There are also two physical concepts that I have proposed to add to the BCS mechanism. One is tumbling, borrowing from the work of Dimopoulos, Raby and Susskind [4]. The other is what I would call bootstrap. Tumbling means that the Higgs boson associated with the chiral symmetry breaking in turn provides a new attractive interaction that can trigger a second symmetry breaking at a lower energy than the original, and so on. Indeed one such example of tumbling is the formation of nuclei and nucleon pairing in nuclei, due to the existence of the sigma meson which is the "Higgs" boson associated with the chiral symmetry breaking in QCD. One could say that superconductivity is another example:it is induced by the phonons which are the Goldstone bosons associated with the breaking of translational and rotational invariance in crystals.

Instead of a hierarchical chain of symmetry breakings, one can contemplate a theoretical possibility of bootsrtap, namely the possibility that the Higgs boson is the cause of attraction bewteen fermions that is responsible for its very existence. I proposed the hypothesis that this applies to the standard model, which implies that the fermions, gauge bosons, and the Higgs fields that appear in the standard model are dynamically closed among themselves without a need to refer to the possible underlying structure. The condition for bootstrap was expressed by the requirement that the standard theory be free from quadratic divergences on the ground that the theory should be insensitive to the cut-off, which presumbaly represent the energy scale of the underlying structure. Unfortunaely the formulas I used to predict the Higgs and top masses were not correct. Later I will discuss a modified version of the hypothesis.

2. Dyson-Schwinger equation, hard mass, and soft mass

The mass of a fermion can be spontaneouly generated when there is a sufficiently strong attractive interaction, as is known from various examples.

In a renormalizable theory, the interaction may be due to a vector or a scalar boson exchange. There are two types of Feynman diagrams that contribute to the fermion self-energy viewed as the potential energy due to fermions in the vacuum, corresponding to the direct (tadpole) and exchange (usual self-energy) terms which are respectively quadratically and logarithmically divergent in the lowest order. Pursued to all orders, the latter leads to the Dyson-Schwinger equation which may be treated nonperturbativley at various levels of approximation.

For the vector interaction, only the exchange term is nonzero. For the scalar interaction, both terms are present, but the quadratically divergent direct term (tadpole) is dominant, whereas the exchange term has the wrong sign to generate a mass by itself. (In general the sign alternates with the degree of divergence.) The latter fact was the basis for the theory of Sakata and Pais [5], who independently proposed to render the electron self-energy finite by the cancellation between electromagnetic forces and the cohesive force due to a hypothetical scalar field. Their theory is not relevant here because of the neglect of the tadpole, as well as the lack of chiral invariance.

Speaking of old theories, Weisskopf [6] was the first to show the logarithmic nature of electron's self-energy, but he also interpreted it as the result of a cancellation between two quadratically divergent physical effects. In terms of Feynman diagrams, one coresonds to a loop made up of on-shell electron and off-shell photon, and the other made up of off-shell electron and on-shell photon. Actually these quadratic terms belong only to the wave function renormalization, so again this analysis is not of interest here.

A similar but more instructive way of analyzing the self-energy may be the following formal manipulation: Set the external electron momentum to zero (this will not change the divergence properties), and split the two progpagators in the loop as

$$1/[k^2 - m_f{}^2)(k^2 - m_v{}^2)]$$

$$= [1/(k^2 - m_f{}^2) - 1/(k^2 - m_v{}^2)]/(m_f{}^2 - m_v{}^2). \qquad (3)$$

When inserted into the Dyson-Schwinger equation, each term yields a quadratically divergent integral like that for a tadpole or a four-fermion interaction. In fact the two terms look like the fermion loop and boson loop contributions to the tadpole, and their signs are correct for such an interpretation provided that $m_v > m_f$, i. e., a sufficiently short range interaction.

The above excercise was to compare the vector interaction with the scalar and four-fermion interaction cases, and to see how the latter might be interpreted as effective low energy theories of the former.

The Dyson-Schwinger equation for the exchange term gives a running mass m(p) as a nonperturbative solution, whereas the tadpole supplies a bare mass m0 which gets dressed up and becomes a running mass by the exchange term. The bare mass serves as the boundary condition for m(p) at large momentum. If m0 is absent as in the vector interaction, m(p) goes to zero like $\sim 1/p^2$ (up to log factors) as is well known, so the mass is soft. Are the masses (current masses) of quarks and leptons soft or hard? This should be an important question which has a bearing on whether the Higgs field is elementary, or comes from a gauge theory like technicolor.

One way in which the dressing effect manifests itself is in the mass ratio between the fermion and the Higgs. In the bubble approximation of the four-fermion interaction, it was 1:2. In general one has to solve a Bethe-Salpeter type equation in the scalar channel, but even in the context of bubble approximation, one can see the effect of the softness of the fermion mass in the bubble. It is easy to derive the expression

$$m_H{}^2 = 4\langle m_f{}^2\rangle, \qquad (4)$$

where the right-hand side is an average with respect to a weight $dp^4/(p^2 + m_f{}^2)^2 \sim dp/p$. Clearly $m_H \leq 2m_f$ in general. This is in agreement with the more elaborate calculations by Yamawaki et al. [7] and by suwa and So [8]. If one adopts the standard behavior

$$m(p) \sim 1/p^\gamma,\ \gamma = 1 - \surd(1 - \lambda), \qquad (5)$$

where λ characterizes the coupling strength in the Dyson-Shwinger equation, one gets

$$m_H{}^2/m_f{}^2 = 4(1 - \exp(-x))/x,$$

$$x = 4/(1 + \surd(1 - \lambda)). \qquad (6)$$

Even in the limit $\lambda = 0$, this gives $m_H{}^2/m_f{}^2 = 2(1 - \exp(-2))$.

3. Models of bootstrap

First consider nonrelativistic cases like supercondcuctivity. The typical gap equation can be written as

$$1 = (4f^2v^2/m^2)\ 2sh^{-1}(\Lambda/m_f). \qquad (7)$$

Usually the factor in front of $2sh^{-1}$ is <V>N, the product of the average potential <V> and the density of states N of fermions. Here the short range potential is represented by a propagator $1/(m^2 + \nu^2k^2) \sim 1/m^2$ (ν = velocity) and a coupling constant f, whereas N is related to the Higgs condensate v by $N = 4v^2$ in the Ginzburg-Landau translation.

Now apply the bootstrap concept, and say that the propagator is that of the sigma (or Higgs) boson, in which case $m_\sigma = 2m_f = 2fv$. So the factor in Eq.(7) = 1, and the solution is $\Lambda/m_f \sim 1/2$, $\Lambda/m_\sigma \sim 1$. In other words, bootstrap is a self-consistent picture that makes m_σ the only available scale parameter.

Let us next turn to relativistic dynamics. Assume a U(1) * U(1) set of massless fermion fields and massive spin 0 fields (bare mass m_0) with Yukawa coupling f. As was shown above, the fermion mass is generated by the tadpole and exchange diagrams. The equation for m_0 from the tadpole takes the form

$$m_f = m_fF, \text{ or } 1 = F, \text{ where}$$
$$F = (f^2/m_\sigma^2)[\Lambda^2 - m_f^2 \ln(\Lambda^2/m_\sigma 2)]/(4\pi^2). \qquad (8)$$

The exchange diagram dresses m_f, but this can be interpreted as the dressing of the vertex f. The scalar and pseudoscalar masses m_σ and m_π must also include their own self-energies:

$$m_i2 = m_0^2 + S_i(p)\ |p^2{=}m_i^2,$$

$$S_\sigma = -(f^2/4\backslash\pi^2)[\Lambda^2 + (p^2/2 - 3m_f^2)\ln(\Lambda^2/m_f^2)]$$

$$S_\pi = -(f^2/4\pi^2)[\Lambda^2 + (p^2/2 - m_f 2)\ln(\Lambda^2/m_f^2)] \qquad (9)$$

The condition $m_\pi = 0$ serves to relate m_0 to Λ, or to eliminate it between m_π and m_σ:

$$m_\sigma^2 - m_\pi^2 = m_\sigma^2 = S_\sigma(m_\sigma) - S_\pi(0), \text{ or}$$

$$m_\sigma^2\ (1 + (f^2/8\pi^2)\ln(\Lambda^2/m_f^2)) = 2m_f^2\ (f^2/4\pi^2)\ln(\Lambda^2/m_f^2)$$

(10)

This gives the mass ratio m_σ/m_f in terms of f and Λ/m_f, which can be seen to be ≤ 2. When m_σ is substituted in Eq.(9), one gets the gap equation determining m_f. Or one may regard it as an equation for the "vacuum expectation value" v defined by $v = m_f/f$.

The above model is not satisfactory for two reasons: It has a bare mass, and it does not have a quartic coupling, which is not "natural". If the masses are to be generated dynamically in a renormalizable theory, one may allow all dimensionless parameters but no bare mass. The mass scale will then be related only to a scale parameter or a cut-off.

One is thus led back to the conventional Higgs (or Ginzburg-Landau-Gell-Mann-Levy) Lagrangian, except that the vacuum expectation value v must be purely dynamical, i.e., no bare v_0, only the tadpoles. Equating v with the tadpoles, one gets the gap equation

$$1 = (1/m_H{}^2)\, \Sigma\, c_i\, g_i{}^2\, (\Lambda^2 - m_i{}^2 \ln(\Lambda^2/m_i{}^2))/(16\pi^2) \qquad (11)$$

The sum runs over all fermions and bosons that can couple to the Higgs boson. One may conveniently define the coupling constants in such a way that the masses are given by $m_i = g_i\, v$. The numerical coefficeients c_i are then 4 for each Dirac fermion, 3/2 for the Higgs scalar, 1/2 for each pseudoscalar, and 3 for each gauge boson. Eq.(11) is an equation for v, given the coupling constants g_i.

Assuming the Λ^2 terms to dominate the sum, one first gets the inequality

$$\Sigma\, c_i\, g_i{}^2 \gtrsim 0, \text{ or } \Sigma\, c_i\, m_i{}^2 \gtrsim 0. \qquad (12)$$

The presence of quadratic terms means, however, a fine tuning of Λ. It was also argued before that, from the bootstrap point of view, the gap equation should not sensitively depend on Λ since the low energy parameters should be self-consistent among themselves, without a need to refer to an unknown high energy scale. With this ansatz, Eq.(12) becomes an equality

$$\Sigma\, c_i\, m_i{}^2 = 0. \qquad (13)$$

One is then left with

$$1 = -(1/m_H{}^2)\, \Sigma\, c_i\, g_i{}^2\, m_i{}^2 \ln(\Lambda^2/m_i{}^2)/(16\pi^2), \text{ or}$$

$$\Sigma\, c_i\, m_i^4 + 16\pi^2\, m_H^2\, v^2/\ln(\Lambda^2/\mu^2) = 0. \qquad (14)$$

where a common mass μ was inserted in the logarithms for simplicity. One first observes from Eq.(14) that

$$\Sigma\, c_i\, m_i^4 \le 0. \qquad (15)$$

Eqs.(13) and (15) constrain the mass values. For fixed Λ, Eqs.(13) and coupling constants g_i^2 in general behave like $1/\ln(\Lambda/\mu)$, so all the terms are of the same order.

The specific application of these conditions to the standard model lead to the following results.

Eqs.(13) and (15) read

$$m_t^2 = (m_H^2 + 2\, m_W^2 + m_Z^2)/4,$$

$$m_t^4 \le (m_H^4/2 + 2\, m_W^4 + m_Z^4)/4. \qquad (16)$$

Wtih the known values m_W = 80 Gev, M_Z = 91 Gev, there are two regions of compatibility,

$$m_t \le 80 \text{ Gev},\ m_H \le 64 \text{ Gev};$$

$$m_t \ge 150 \text{ Gev},\ m_H \ge 195 \text{ Gev}. \qquad (17)$$

The first region seems excluded by experiment. If Eqs.(13) and (14) are used, one can solve for m_t and m_H as a function of Λ. The mass values turn out to be pushed considerablly higher for standard choices of Λ: m_t = 230 Gev, m_H = 440 Gev for $\Lambda = 10^{19}$ Gev (Planck); m_t = 260 Gev, m_H = 500 Gev for $\Lambda = 10^{15}$ Gev (GUTS); and getting even higher as Λ is further lowered. The general trend is similar to the results of Bardeen et al. [3].

There remain questions of principle and questions of numerical reliability. These have not been addressed yet. The most serious one may be that concerning the quadratic divergence condition. For it to make sense, one must have a prescription for handling higher order terms as well. (See [9] for computations using dimensional regularization, but the physical meaning of such a procedure is not clear.) If it makes sense at all, it may perhaps be understood in terms of a dynamical supersymmetry like quasi-supersymmetry. The quantitative results like those given above of course will change when renormalization correctons are included, but one must also have a definite prescription for them.

4. Quasi-supersymmetry

I will now briefly dicuss a different topic. Quasi-supersymmetry was found in non-relativistic BCS mechanisms. Whatever the orign of the symmetry, can it be made relativistic? This question has led to the following results [10].

The static part of the effective Landau-Ginzburg Hamiltonian satisfying the BCS mass ratios can in general be factorized in terms of fermionic operators

$$Q = \Pi\,\psi + iW\,\psi^{+},$$

$$Q^{+} = \Pi^{+}\psi^{+} - iW\,\psi,$$

$$W = G(\phi^{+}\phi - v\hat{\ }2),$$

$$H = \{\underline{Q}, \underline{Q}^{+}\}/n. \qquad (18)$$

ψ and ψ^{+} are n-component fermion fields, ϕ and ϕ^{+} are n by n complex matrix Higgs fields acting on the former, $\Pi = \partial\phi/\partial t$ and $\Pi^{+} = \partial\phi^{+}/\partial t$ are their canonical cojugates. The underline indicates spatial integral.

The fermionic currents in (extended) supersymmetry, on the other hand, are made up of the following types of pieces in general.

$$Q^{\mu}{}_{R}(x) = \partial_{\nu}\,\phi(x)\,\sigma^{\nu}\,\sigma^{\sim\mu}\,\psi_{R}(x),$$

$$Q^{\mu}{}_{R}(x) = F_{\lambda\rho}(x)\,\sigma^{\lambda}\,\sigma^{\sim\rho}\,\sigma^{\mu}\,\psi_{L}(x),$$

$$Q^{\mu}{}_{R}(x) = W\,\sigma^{\mu}\,\psi_{L}(x), \qquad (19)$$

and similar forms with L and R interchanged. The first and second lines represent kinetic currents for chiral and gauge multiplets, the third one represents the potential for the scalar field. The Q's have internal indices which are suppressed. Anticommutators $\{Q, Q^{+}\}$ generate the Poincare algebra, $\{Q, Q\}$ and $\{Q^{+}, Q^{+}\}$ generate central charges.

Comparing Eqs.(18) and (19), one can see the correpondence: $\psi \to \psi_{R}$, $\psi^{+} \to \psi_{L}$; the first term of Eq.(18a) -> Eq.(19a), the second term -> Eq.(19c).

So the relativistic generalization seems easy. However, the Higgs potential in Eq.(18) is not the Kaehler potential in Eq.(19c), the main reason the masses do not come out equal. Since one does not have exact supersymmetry, one has to decide which part of supersymmery relations to keep and which part to give up. I have proposed to keep one Poincre algebra (N=1 subalgebra), and give up all others. That means, in addition to the energy part of $\Sigma \{Q_i, Q_i^+\}$, the momentum part of it must come out right. It turns out that the follwoing conditions must be met:

a. Matching of fermionic and bosonic degrees of freedom, in order that kinetic energies of the various fields in the Hamiltonian have the same weight;

b. Absence of interaction pieces in the momentum part of the Poincare agebra. For this, one must have fermion, Higgs, and gauge fields all present in such a way that the interactions arising from various cross terms in the anticommutators cancel each other for the momentum algebra, but not for the energy algebra. The Yukawa and gauge couplings must then be related, but the relation does not seem to be unique. It is due to the fact that the currents in our cases carry gauged quantum numbers, and there are ambiguities in the gauge-invariant definition of anticommutators.

The physical meaning of relativistic quasi-supersymmetry thus defined is unclear, but some models satisfying the above criteria can be constructed. They seem to have some resemblance to the hidden symmetry scheme in chiral dynamics. It should be interesting if Higgs and gauge fields both are found to play dynamical roles in the BCS mechanism and lead to quasi-supersymmetry. One would also hope that the quasi-supersymmetry eliminate the quadratic divergences like in real spersymmetry, but this does not seem to be the case in general.

A final comment concerns an observation on quasi-supersymmmetry in SU(5) grand unification. The degree matching beween fermions and bosons works out fine if there are three generations of fermions $(1 + 5 + 10^{*}) * 2 * 3 = 96$ against the SU(5) gauge fields $(24 * 2)$ and a set of complex adjoint Higgs fields $(24 * 2)$. The Higgs fields can break SU(5) down to SU(3)*SU(2)*U(1), but there are no Yukawa couplings, so fermions remain massless at this energy scale. The Higgs fields of the standard model presumbaly will arise later as composites.

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