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EFFECT ON FEL GAIN CURVE USING PHASE SHIFTERS

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Abstract

Phase matching between FEL and electron beam should be precisely controlled for FEL amplification. Phase shifters located between undulators performs the phase matching. An electron beam can be controlled to be in the in- or out-phase by setting the phase shifters from the phase shifter scan. In this article, we show effects of FEL gain curve by setting the in- and out-phase of electron beam. We address reasons of the reduction of FEL intensity in the out-phase condition dividing the linear and saturation FEL amplification regimes. In the linear regime the gain curve is shifted, and in the saturation regime the electron loss occurs during the undulator tapering. Our results show agreements with experiments performed at PAL-XFEL.

INTRODUCTION

Most x-ray free-electron laser (XFEL) facility requires segmented undulators with drift sections. Drift sections include quadrupoles for beam focus, beam path correctors, diagnostics, and beam phase shifters [1,2]. While an electron beam travels drift sections, the electron beam phase can be mismatched to the generated X-ray. This phase mismatch can lead to a reduction of XFEL intensity, therefore the phase matching can be an import optimization process.

Enhancement of FEL efficiency by matching phase can be understood by so-called ‘phase jump’ at the saturation region [3,4]. The key idea is setting a dominant electron beam phase zero so called ‘synchronous phase’ before entering the next undulator. To investigate effects on the FEL gain curve, two cases of the phase-matched condition (the in-phase) and 180^0 off-set condition from the in-phase condition (the out-phase) are considered.

THEORY AND SIMULATION

We show optimization of FEL and FEL gain curve by the phase shifters. To understand the changes of FEL gain curve, we develop a linear theory for the linear region and use KMR analysis [4,5] for the saturation region, those are compared with FEL simulations.

Linear Regime

If we ignore the time dependent terms, 1D FEL equations are given as [6]

$$\frac{d\theta}{d\hat{z}} = \hat{\eta} \tag{1.a}$$

$$\frac{d\hat{\eta}}{d\hat{z}} = ae^{i\theta} + a^{*}e^{-i\theta} \tag{1.b}$$

$$\frac{da}{d\hat{z}} = -\langle e^{-i\theta}\rangle_{slice} \tag{1.c}$$

Here, $\theta \equiv (k+k_u)z - \omega t$ is the electron phase, $\eta \equiv (\gamma - \gamma_0)/\gamma_0$ is the energy deviation ratio to γ_0, and a is the slowly varying electric field envelope. The equations are normalized by $\hat{z} = 2k_u\rho z$, $\hat{\eta} = \eta/\rho$, and $a = eK[JJ]E/4\gamma_0^2 k_u mc^2\rho^2$, where $[JJ]$ is the harmonic factor, and ρ is the FEL parameter. Introducing the collective variables : the bunching factor $b = \langle e^{-i\theta}\rangle_{slice}$ and the collective momentum $P = \langle \hat{\eta} e^{-i\theta}\rangle_{slice}$, Eq. (1) is rewritten by ignoring the non-linear terms : $dP/d\hat{z} = a + a^{*}\langle e^{-i2\theta}\rangle - i\langle \hat{\eta}^2 e^{-i\theta}\rangle \approx a$

$$\frac{da}{d\hat{z}} = -b \tag{2.a}$$

$$\frac{db}{d\hat{z}} = -iP \tag{2.b}$$

$$\frac{dP}{d\hat{z}} = a \tag{2.c}$$

Eq. (2) is the linearized FEL equation, which can be summarized to be an third-order differential equation

$$\frac{d^3a}{d\hat{z}^3} = ia \tag{3}$$

The general solution of Eq.(3) can be obtained as

$$a(\hat{z}) = \frac{1}{3}\sum_{l=1}^{3}\left[a(\hat{z}_0) - i\frac{b(\hat{z}_0)}{\mu_l} - i\mu_l P(\hat{z}_0)\right]e^{-i\mu_l\hat{z}} \tag{4}$$

, where μ_l indicates three radiation modes : a simple propagator $\mu_1 = 1$, a damper $\mu_2 = -(1+\sqrt{3}i)/2$, and a grower $\mu_3 = (-1+\sqrt{3}i)/2$. Assuming the electron beam in the in-phase, the initial conditions at $\hat{z}_0 = 0$ are $a(0) = C_1 + C_2 + C_3$, $b(0) = \mu_1 C_1 + \mu_2 C_2 + \mu_3 C_3$, and $P(0) = i(\mu_1^2 C_1 + \mu_2^2 C_2 + \mu_3^2 C_3)$. Considering a SASE condition of $a(0) = P(0) = 0$ with initial bunching factor of $b(0) = b_0$, a grower mode becomes dominant, then Eq.(4) is simplified to be

$$a_{in}(\hat{z}) \cong \frac{i-\sqrt{3}}{6} b_0 e^{\frac{\sqrt{3}+i}{2}\hat{z}} \tag{5}$$

To calculate the changed gain curve at the out-phase, we simply set the phase replacing θ by $\theta + \pi$. Eq. (1) is then reformulated to be the linearized equations which is reduced to a single third-order differential equation same as Eq. (3). Therefore, the expression of the solution is identical to the in-phase solution of Eq. (4) except of the initial conditions. according to the phase shifter position $\hat{z}_0 \neq 0$. The initial conditions at $\hat{z} = \hat{z}_0$ can be defined obtained using Eq. (5):

$$a(\hat{z}_0) = -\frac{1}{3} b_0 e^{i\left(\frac{\hat{z}_0}{2} - \frac{\pi}{6}\right)} e^{\frac{\sqrt{3}}{2}\hat{z}_0} \tag{6.a}$$

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$$b(\hat{z}_0) = -\frac{da}{d\hat{z}}\bigg|_{\hat{z}_0} = \frac{1}{3} b_0 e^{i\frac{\hat{z}_0}{2}} e^{\frac{\sqrt{3}}{2}\hat{z}_0} \quad (6.b)$$

$$P(\hat{z}_0) = i\frac{db}{d\hat{z}}\bigg|_{\hat{z}_0} = \frac{i}{3} b_0 e^{i\left(\frac{\hat{z}_0}{2}+\frac{\pi}{6}\right)} e^{\frac{\sqrt{3}}{2}\hat{z}_0} \quad (6.c)$$

The solution of the out-phase equation then is derived as

$$a_{out}(z) = \frac{1}{9} b_0 e^{\frac{\sqrt{3}+i}{2}\hat{z}_0}\left[\left(\sqrt{3}+i\right)e^{-i(\hat{z}-\hat{z}_0)} + i e^{\frac{-\sqrt{3}+i}{2}(\hat{z}-\hat{z}_0)} + \frac{\sqrt{3}-i}{2} e^{\frac{\sqrt{3}+i}{2}(\hat{z}-\hat{z}_0)}\right] \quad (7)$$

Note that these expressions are available for $\hat{z} \geq \hat{z}_0$. Eq. (7) agrees well with the 1D static simulation as shown in Fig. 1.

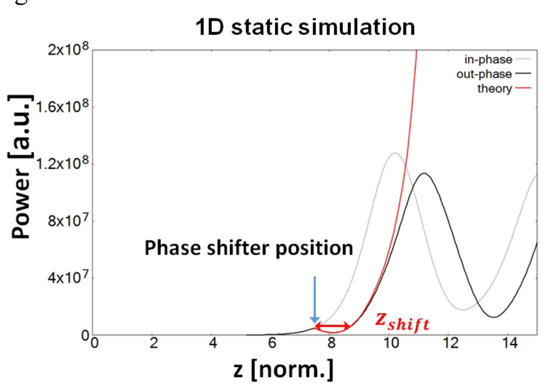

Figure 1: FEL gain curves of the in-phase and the out-phase. The phase is shifted at $\hat{z}_0 = 7.5$. The shifting distance is $\Delta\hat{z} = 1.27$ agreeing with the expectation.

After $\hat{z} = \hat{z}_0$, the FEL gain curve is modulated for a distance where three modes of propagator, damper, and grower are accompanied. However, the grower is dominant being farther from the phase shifter, this resembles the phase shifter shifts the gain curve. To obtain the shifting distance we further modify Eq. (7) by assuming $\hat{z} \gg \hat{z}_0$, which becomes

$$|a_{out}(\hat{z})|^2 \approx \frac{b_0^2}{9} e^{\sqrt{3}(\hat{z}-1.27)} \quad (8)$$

Eq.(8) shows the gain curve shift is shifted about **1.27** from the in-phase gain curve.

Saturation Regime

As the electron beam enters the saturation region, electrons are bunched locally. A dominant FEL amplification or diminishment is determined by the bunched electrons. To express electrons' trajectory in the phase space, the equations of motion of Eq. (1.a) and (1.b) are rewritten as

$$\frac{d\psi}{d\hat{z}} = \hat{\eta},\ \frac{d\hat{\eta}}{d\hat{z}} = -a_0 \sin\psi \quad (9)$$

whose Hamiltonian is

$$H = \frac{1}{2}\hat{\eta}^2 + a_0(1-\cos\psi) \quad (10)$$

, where $a_0 \equiv |a|/2$, $\psi \equiv \theta + \phi + \pi/2$, and $a = |a|e^{i\phi}$. The separatrix can then be defined for the trajectory of $H_{sep} = H(\psi = \pm\pi, \hat{\eta} = 0)$. The separatrix function is expressed as

$$\hat{\eta}(\psi) = \pm 2\sqrt{a_0}\cos\frac{\psi}{2} \quad (11)$$

When electrons locate in the separatrix or 'phase bucket', the electrons participate in FEL process. In the phase space undulator taper normally shifts down the phase bucket and the phase shifter moves the electrons to the right or left.

To study FEL gain curve by a phase shifter depending on a different longitudinal position, we conduct 1D static FEL simulation. 20 segmented undulator modules are prepared and each undulator module is composed of 5 m undulator and 1 m drift section. To prevent confusion, we test only one phase shifter, therefore, the out-phase condition is selected for just one undulator module and all others are the in-phase. Figure 2 summarizes results of two phase shifters at 30 m and 41 m. The out-phase condition reduces the FEL gain curve, which is because of the electron loss from the phase bucket (red lines). FEL amplification is fully supressed at deep saturation region as shown in Fig. 2 (c), that means the phase bucket loses electrons significantly. This addresses FEL optimization using phase shifters is very important at saturation region especially when the electron beam is well bunched.

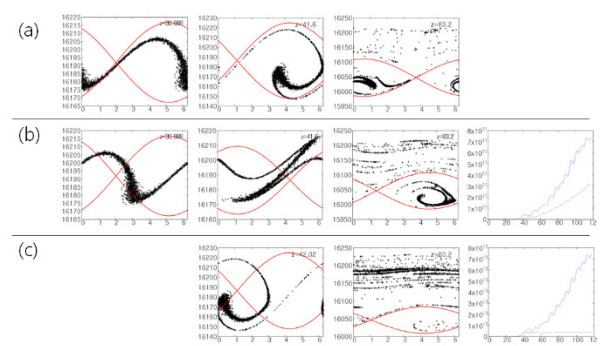

Figure 2: Comparisons of FEL gain curves with the electron phase distribution. (a) The reference without phase shifters. With the phase shifters (b) at 30 m, (c) at 41 m in the out-phase condition. The red line in the phase distribution is the phase bucket following Eq. (11).

EXPERIMENT

Normally the phase shifter is a compact magnetic chicane which elongates the electron beam path. Pohang Accelerator Laboratory X-ray Free Electron Laser (PAL-XFEL) operates the phase shifter made up of permanent magnets [1]. In the operation, the phase scan is performed by changing the gap of magnets (phase shifter gap), which retards the electron beam phase. Phase matching is done by finding the phase shifter gap producing the optimal FEL intensity. In Fig. 3 (a), the installed phase shifter between undulator modules is shown, and Fig. 3 (b) shows an example of the phase shifter scan.

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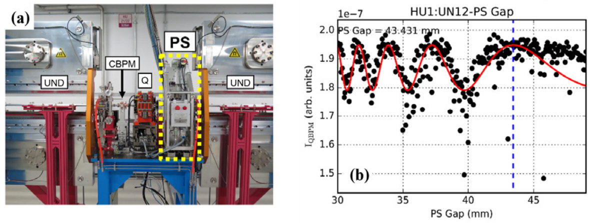

Figure 3: (a) Intersection between undulator (UND) modules. Cavity beam position monitor (CBPM), quadrupole (Q) and phase shifter (PS) has been installed. (b) Measured FEL intensity according to the gap of phase shifter. Fitting by sine function is drawn as the red solid line and the optimal phase is determined by setting PS gap (blue dashed line).

During the FEL optimization all phase shifters are set to be the maximum FEL intensity, so the electron beam can be set in the in-phase condition. To see the gain curve change, one phase shifter is set to be the out-phase condition by selecting the minimum FEL intensity from the phase shifter gap scan. Comparisons of the in- and out-phase conditions are shown in Fig. 4. Targeted experiment conditions are 5 keV FEL energy with the undulator parameter K=1.87, the undulator period 26 mm, and the electron beam charge 200 pC.

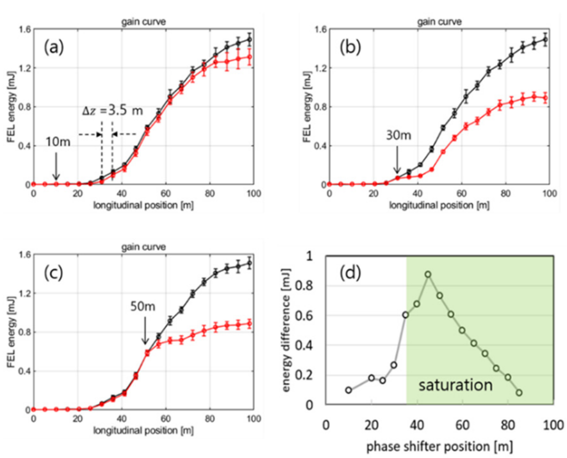

Figure 4: Gain curve comparison between the in- and out-phase conditions. One phase shifter is set for the out-phase at (a) 10 m , (b) 30 m, and (c) 50 m. (d) FEL energy difference between the in- and out-phase from the phase shifter scan.

Fig. 4 lists three phase shifters of the in- and out-phase conditions. From the FEL gain curve (black line), the saturation starts around z=30 m. In Fig. 4 (a) at z=10 m, the phase shifter shifts the gain curve around 3.5 m, which is 1.2 in normalized unit and it is comparable to the theoretical value of 1.27. However after the saturation the gain curve is similar to the reference. The gain curve is reduced significantly around the end of the linear regime as shown in Fig. 4 (b). When the phase shifter is located in deep saturation region as shown in Fig. 4 (c), the FEL amplification is suppressed. These FEL reductions are reproduced by the simulation results of Fig. 2. Therefore the phase shifter scan is efficient and important for the saturation region.

CONCLUSION

We studied the in- and out-phase effect on the FEL gain curve. FEL intensity difference between the in- and out-phase conditions increases as entering the saturation region. FEL reduction by the out-phase condition is separated in the linear and saturation regimes. In the linear regime the gain curve is simply shifted. In the saturation regime electrons are lost from the phase bucket which operates FEL process. From this analysis the phase shifter scan is more efficient for the saturation region and the undulator tapering.

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