RESONANT AMPLIFICATION OF NEUTRINO TRANSITIONS IN THE SUN: EXACT ANALYTICAL RESULTS

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Abstract:

We investigate in detail the Mikheyev-Smirnov-Wolfenstein explanation of the solar neutrino puzzle using analytical expressions for the neutrino transition probabilities in matter with exponentially varying electron number density.

The discrepancy between calculation (7.9*(1±.33)SNU)[1] and observations (2.1±0.3SNU)[2] in the ^{37}Cl solar neutrino experiment is a long-standing puzzle. Many different explanations have been proposed so far. Acording to the explanation, suggested by Mikheyev and Smirnov[3], large part of the electron neutrinos born in the solar interior transform on their way to the surface into some other neutrinos (for example into ν_μ or ν_τ).

Two different approaches have been used in order to investigate in detail this possibility: (*i*) numerical solution of the neutrino evolution equation in the sun, and (*ii*) analytical solution of the same equation. In the begining, the analytical results have been obtained assuming that the electron number density varies linearly in the vicinity of the resonant layer[4-6]. Acording to the standard solar model, however, the electron number density distribution in the sun is well discribed by an exponential function except for small regions in the center and near the surface.

Therefore an exact analytical solution of the neutrino evolution equation in matter with exponentially varying electron number density turn out to be very interesting from the solar neutrino puzzle point of view. This solution have been obtained in Ref.7 together with very simple approximate solution[8].

We use these analytical solutions to discribe the nutrino transitions in the sun and to investigate in detail the Mikheyev-Smirnov-Wolfenstein explanation of the solar neutrino puzzle.

The system of two coupled evolution equations[9] for the amplitudes $A_l(L)$ (l=e,σ ;σ = μ or τ) of the probabilities to find neutrino ν_l at distance L in matter can be written in the form:

$$i\frac{d}{dL}\begin{bmatrix} A_e(L) \\ A_\sigma(L) \end{bmatrix} = \frac{1}{4p}\begin{bmatrix} \Delta m^2\cos 2\theta[\exp(-L/r)-1] & \Delta m^2\sin 2\theta \\ \Delta m^2\sin 2\theta & -\Delta m^2\cos 2\theta[\exp(-L/r)-1] \end{bmatrix}\begin{bmatrix} A_e(L) \\ A_\sigma(L) \end{bmatrix} \quad (1)$$

Eq.(1) has been derived assuming that:

(*i*) flavour neutrinos ν_l are superpositions of the neutrinos m_j (j=1,2) having definite masses m_j ($m_1 < m_2$) in vacuum:

$$\nu_e = \cos\theta\nu_1 + \sin\theta\nu_2 \quad , \qquad \nu_\sigma = -\sin\theta\nu_1 + \cos\theta\nu_2 \qquad (2)$$

(ii) the main component of the electron neutrino is the lighter neutrino ν_1, *i.e.*, that

$$\Delta m^2\cos 2\theta > 0 \qquad (\Delta m^2 = m_2^2 - m_1^2) \qquad (3)$$

(iii) neutrinos are relativistic with momentum p:

$$E_j = \sqrt{p^2 + m_j^2} = p + m_j^2/2p \qquad (4)$$

so that, in the convention $c=\hbar=1$, the time variable t has been replaced with the distance L. We note also that G_F is the Fermi coupling constant, $N_e(L)=N_e^R\exp(-L/r)$ is the electron number density and N_e^R, called resonance density, is equal to:

$$N_e^R = \Delta m^2\cos2\theta/2\sqrt{2}G_F p \tag{5}$$

The substitution[7] of the amplitude $A_e(L)$ in the first equation of (1) with the expression

$$A_e(L)=(\Delta m^2\sin2\theta)^{-1}\{i4p[d/dL]+(\Delta m^2\sin2\theta)[\exp(-L/r)-1]\}A_\sigma(L) \tag{6}$$

obtained from the second equation together with the transformation

$$z=i[\Delta m^2 r\cos2\theta/(2p)]\exp(-L/r) \quad , \quad A_\sigma(L)=z^{-1/2}U_\sigma(z) \tag{7}$$

lead to the Whittaker's equation[10]:

$$[d^2/dz^2-1/4+k/z+(1/4-m^2)/z^2]U_\sigma(z)=0 \tag{8}$$

where

$$k=1/2+i\Delta m^2 r\cos2\theta/(4p) \quad , \quad m=i\Delta m^2 r/(4p) \tag{9}$$

If we assume that electron neutrino has been born initially in matter with electron number density $N_e^i(L_i)$ than we need a solution which satisfies the following initial conditions:

$$A_e(L_i)=1 \quad , \quad A_\sigma(L_i)=0 \tag{10}$$

The solution is given by[7]:

$$A_\sigma(L)=i(tg2\theta/2)\exp[(L+L_i)/(2r)]\exp[\pi\Delta m^2 r\cos2\theta/(4p)]$$
$$\times[W_{-k,m}(-z_i)W_{k,m}(z)-W_{k,m}(z_i)W_{-k,m}(-z)] \tag{11}$$

It follows from eq.(6) that

$$A_e(L)=(tg2\theta/2)[\Delta m^2 r\sin2\theta/(4p)]^{-1}\exp[(L+L_i)/(2r)]\exp[\pi\Delta m^2 r\cos2\theta/4p]$$
$$\times[W_{k,m}(z_i)W_{-k+1,m}(-z)-(m-k+1/2)(m+k-1/2)W_{k-1,m}(z)W_{-k,m}(-z_i)] \tag{12}$$

$W_{\pm k,m}(\pm z)$ are the Whittaker's functions.

For the probabilities to detect ν_e or ν_σ in matter with density $N_e^f(L_f)$ if an electron neutrino has been born in matter with density $N_e^i(L_i)$ we have:

$$P_{\nu_e,\nu_e}(N_e^i,N_e^f)=|A_e(L_f)|^2 \quad , \quad P_{\nu_e,\nu_\sigma}(N_e^i,N_e^f)=|A_\sigma(L_f)|^2 \tag{13}$$

In case of very high initial density N_e^i ($N_e^i \gg N_e^R$) and final density equal to zero simple approximate expressions for the average neutrino transition probabilities exist[7,8]:

$$\langle P^{exp}_{\nu_e,\nu_e}(N_e^i,0)\rangle \cong 1/2 + (1/2-P_x^{exp})\cos2\theta_m^i\cos2\theta \tag{14}$$

where P_x^{exp} is the (so-called) probability for transitions between the adiabatic neutrino states which, in case of exponentially varying electron number density, is given by:

$$P_x^{exp}=\{\exp[-2\pi\Delta m^2 r\sin^2\theta/(2p)]-\exp[-2\pi\Delta m^2 r/(2p)]\}/\{1-\exp[-2\pi\Delta m^2 r/(2p)]\} \qquad (15)$$

and θ_m^i is the mixing angle in matter defined by:

$$\cos 2\theta_m^i=(1-N_e^i/N_e^R)/[(1-N_e^i/N_e^R)^2+tg^2 2\theta]^{1/2} \qquad (16)$$

It is easy to verify that eq.(14) is valid also in the adiabatic case, *i.e.* when the probability P_x^{exp} can be neglected, irrespectively of the value of the density N_e^i. Therefore the solution (14) is a good approximation to the exact one (13) when either $N_e^i \gg N_e^R$ or P_x^{exp} is negligible.

In case of linearly varying electron number density P_x^{lin} is given by the well known Landau-Zener factor[11,4-6]:

$$P_x^{lin}=P_{LZ}= \exp[-(\pi/2)(\Delta m^2/2p)r(\sin^2 2\theta/\cos 2\theta)] \qquad (17)$$

Till now the Mikheyev-Smirnov-Wolfenstein effect in the sun has been investigated analytically under the assumption that the electron number density in the solar interior varies linearly in the vicinity of the resonant layer. According to the standard solar model[1], however, the density distribution is described by exponential, not by linear, function of the distance to the center except for small regions near the center and close to the surface.

Let us compare the results for the electron neutrino survival probability $\langle P_{\nu_e,\nu_e}^{exp}\rangle$ obtained from the exponential formula (14) with the results obtaned from the corresponding linear formula (where P_x^{exp} is substituted with P_x^{lin}). We have presented in Fig.1 the case with an electron neutrino born at distance $L=.044194R_\odot$ from the center, which corresponds to the maximum of the spatial distribution of the main (with respect to the ^{37}Cl experiment) ^8B neutrino source, for two different values of the vacuum mixing angle ($\sin^2\theta=.3$,Fig.1a, and $\sin^2\theta=.02$, Fig.1b). For the electron number density distribution in the sun we have used a cubic spline fit of the data given by Bahcall and Ulrich[1].

The differences between the exponential results (full line) and the linear results (dashed line) are substantial when the vacuum mixing angle θ is large[8] (see Fig.1a). We note that for values of the neutrino parameters $p[MeV]/\Delta m^2[eV^2]$ belonging to the interval defined by the two vertical lines the resonant density N_e^R in the sun is in the region where, according to the standard model, the electron number density is well described by an exponential function. In case of small vacuum mixing angle

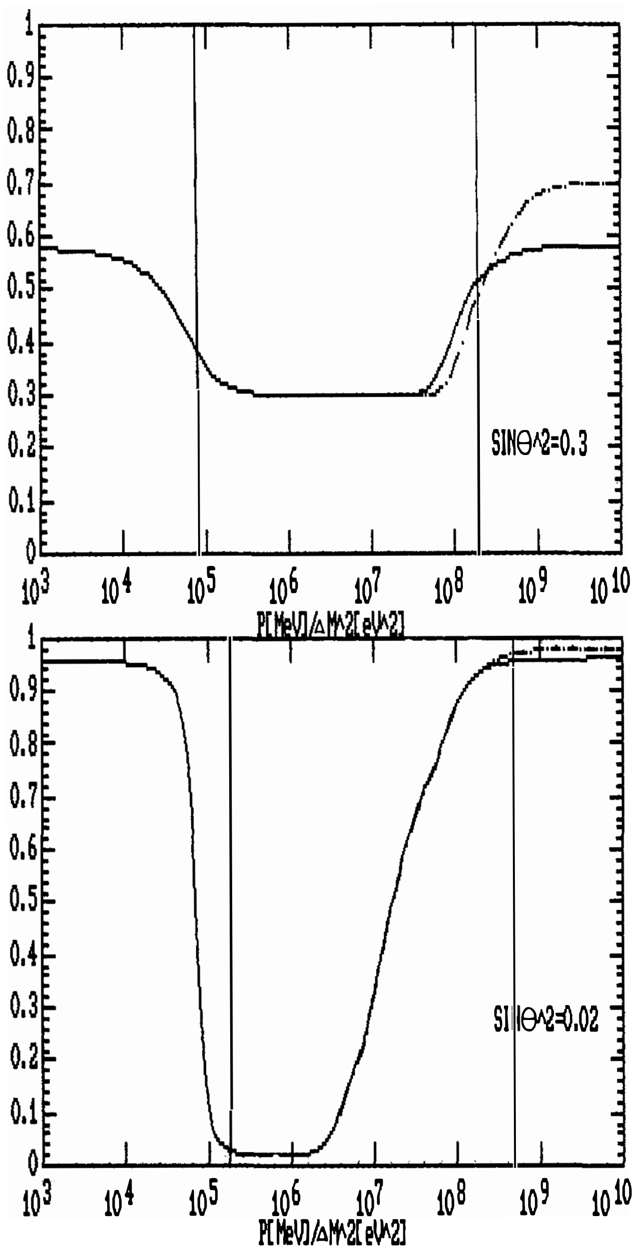

Fig.1a

Fig.1b

Fig.1 Electron neutrino survival probabilities$\langle P^{exp}_{\nu_e,\nu_e}\rangle$(full line) and $\langle P^{lin}_{\nu_e,\nu_e}\rangle$(dashed line) for: a) $\sin^2\theta=0.3$; b) $\sin^2\theta=0.02$.

(Fig.1b) the two curves almost coincide.

In addition the exponential formula (14) and the corresponding linear formula have substantially different asymptotic behaviour in the instantaneous limit, *i.e.* in case of very large $p/\Delta m^2$:

$$\langle P^{exp}_{\nu_e,\nu_e} \rangle \xrightarrow[p/\Delta m^2 \longrightarrow \infty]{} 1-(1/2)\sin^2 2\theta \qquad (18)$$

$$\langle P^{lin}_{\nu_e,\nu_e} \rangle \xrightarrow[p/\Delta m^2 \longrightarrow \infty]{} 1-\sin^2\theta \qquad (19)$$

Simple arguments convince us that the exponential result is the right one because in the instantaneous case, as an opposite to the adiabatic case, the density decreases so fast that the electron neutrino state just born has no time to change. Therefore out of the sun, in vacuum, we have essentially the same electron neutrino state which, later on, oscillates around the average value (18) in perfect agreement with the theory of neutrino oscillations in vacuum.

We conclude that the exponential formulae describe the effect of resonant amplification of neutrino transitions in the sun better than the corresponding linear expressions.

Therefore we have used eq.(14) to obtain detailed information about possible implications of the MSW effect on the detected neutrino flux in the ^{37}Cl solar neutrino experiment.

The data concerning the standard solar model have been taken from Ref.1. The electron number density profile is a cubic spline fit (with continuous first and second derivatives) of Bahcall and Ulrich's results. We have taken into account the contributions of all neutrino sources which are detectable in the ^{37}Cl experiment (^{8}B, ^{7}Be, pep, hep, ^{13}N, ^{15}O, ^{17}F). The neutrino fluxes, the spatial distributions of the neutrino sources in the sun as well as the spectra of ^{8}B, ^{7}Be, pep and hep neutrinos which we used are given in Ref.1, third paper. We have calculated the spectra of the ^{13}N, ^{15}O and ^{17}F neutrinos produced in an allowed β^+-dacay transition using the Fermi function of Konopinski[12] as well as the corrections suggested by Bahcall[1]. The energy dependent $^{37}Cl+\nu_e \rightarrow {}^{37}Ar+e^-$ cross-section has been derived following the procedure outlined in Ref.1

The dependence of the neutrino capture rate in the ^{37}Cl experiment (in fractions of the standard result[1]) on the neutrino parameters Δm^2 and $\sin^2 2\theta$ is presented in Fig.2a. For the contour plot we have used a grid with 2000 points.

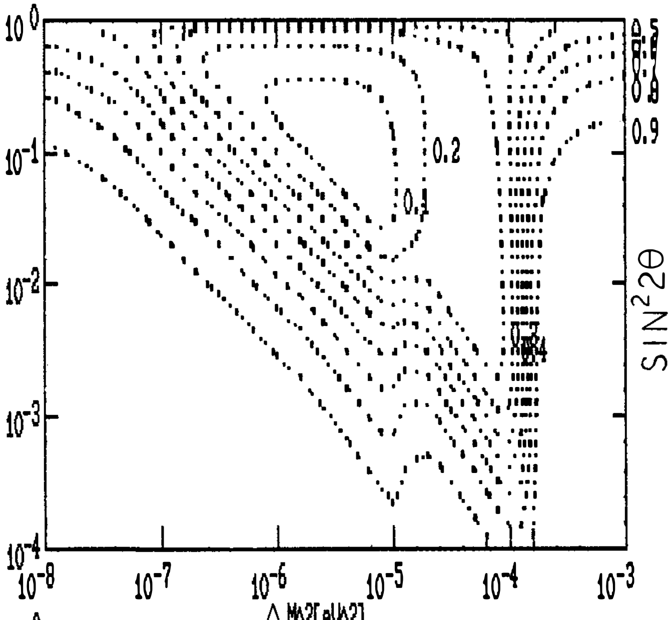

Fig.2a

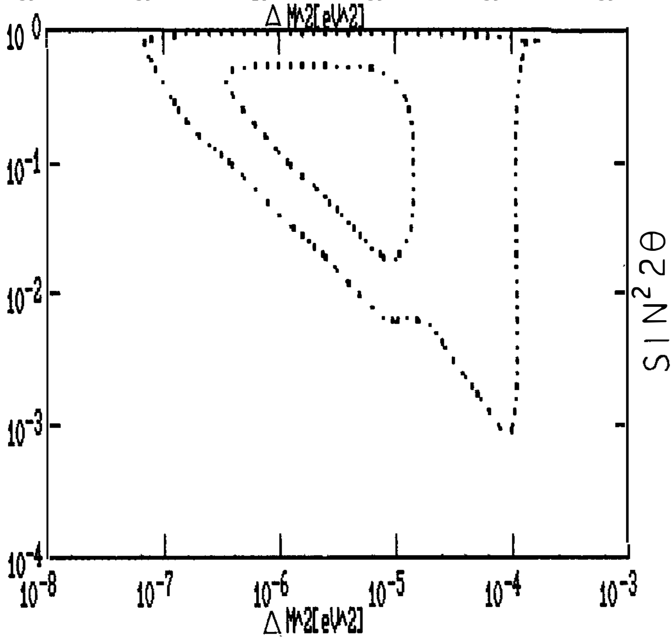

Fig.2b

Fig.2: a)Contour plot of the dependence of the capture rate in ^{37}Cl experiment on Δm^2 and $\sin^2 2\Theta$; b)Region in Δm^2-$\sin^2 2\Theta$ plane corresponding to capture rate in agreement with Davis's result.

In Fig. 2b we present the region in the Δm^2-$\sin^2 2\theta$ plane corresponding to neutrino capture rate which is in agreement with the result of Davis *et. al.*[2]. The uncertainties of the experimental[2] as well as of the theoretical[1] results are taken into account.

In summary we have shown that the analytical expressions for the neutrino transition probabilities in matter with exponentially varying electron number density describe the Mikheyev-Smirnov-Wolfenstein effect in the sun beter than the corresponding linear formulae. Afterwards, we have used these analytical results to investigate in detail the dependence of the neutrino capture rate in the ^{37}Cl solar neutrino experiment on the neutrino parameters (Δm^2 and $\sin^2 2\theta$).

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