Cosmological Distances Scale: Cosmology at a Crossroads?

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Abstract. A brief review of the results of identification of the cosmological distance scale based on the redshift in the spectra of extragalactic sources based on the data used for detecting the "acceleration of the expansion of the Universe" is given. The data deviation scattering parameter is about 13 ... 20% of the calculated distance value, which is an order of magnitude greater than the accuracy estimates accepted in a number of studies as the average quadratic deviations of arithmetic averages. In addition, there is a dipole anisotropy in this data. It is shown that among the reasons for the so-called "metrological and scientific deadlock" in cosmology may be the lack of error estimates of the inadequacy of the models used and violations of the logic of statistical inference when identifying them by supernovae of type SN Ia.

1. Introduction

In 1923, Arthur Eddington in a textbook on the mathematical theory of relativity [1] published the results of determining the radial velocities of "spiral nebulae" obtained by Vesto Slipher from 1912 to 1922, almost all of which were associated with the beautiful displacement of z in the spectra of their radiation. The shift, including the "violet" one, was interpreted as a Doppler effect, and the establishment of distances to the "spiral fogs", which turned out to be galaxies, served as the basis for the construction of non-stationary cosmological models, although at first specialists did not pay attention to the works [2, 3].

A similar situation exists with the equations by Alexander Friedmann in 1922 [4]

$$\begin{gathered} \frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 + \frac{2}{R^2}\frac{d^2R}{dt^2} + \frac{c^2}{R^2} - \lambda = 0,\ \ \frac{3}{R^2}\left(\frac{dR}{dt}\right)^2 + \frac{3c^2}{R^2} - \lambda = \kappa c^2\rho, \\ \frac{1}{c^2}\left(\frac{dR}{dt}\right)^2 = \frac{1}{R}\left(A - R + \frac{\lambda}{3c^2}R^3\right),\ \ \frac{d\rho}{dt} + \frac{3}{R}\frac{dR}{dt}\cdot\left(\rho + \frac{P}{c^2}\right) = 0, \end{gathered} \tag{1}$$

where $R(t)$ is the radius of curvature, t is the cosmological time, c is the speed of light, $\kappa = 8\pi G/c^2$, λ and A are constants, G is the gravitational constant, ρ is the density, P is the isotropic pressure. One of the solutions of these equations was the non-stationary cosmological model of the "expanding Universe". The equations became known after albert Einstein's recognition of an error with respect to his stationary cosmological model [5] and anticipated the discovery by Edwin Hubble of the "red shift law" for the photometric distance

$$D_L = (c/H_0)\cdot z, \tag{2}$$

where H_0 is the Hubble constant, since the relation (2) can be considered as an approximation of the 1st order of the solution of equations (1). Similar equations were independently arrived at by Georges Lemaitre in 1927 [6], Howard Robertson in 1928 [7], and Arthur Walker in 1933 [8].

In 1942, Otto Heckman obtained a 2nd order approximation [9] to solve the equations (1)

$$D_L \cong (c/H_0) \cdot \left[z + \frac{1}{2}(1 - q_0)z^2 \right], \tag{3}$$

where q_0 is the deceleration parameter and in 1958 Wolfgang Mattig was a strict solution for the "usual" substance [10]:

$$D = \left[c/(H_0 q_0^2) \right] \cdot \left[q_0 z + (q_0 - 1) \cdot \left(\sqrt{2q_0 z + 1} - 1 \right) \right]. \tag{4}$$

In 1960, Fred Hoyle proposed a model to detect the curvature of space [11]

$$cz = H_0 r + K r^2, \tag{5}$$

where K is the nonlinearity parameter. And two years later, Alan Sandage discovered a deviation from Hubble's law:

$$(cz - H_0 r)|_{r \sim 10^9 \mathrm{lightyears}} \sim 10^4 \mathrm{km} \cdot \mathrm{s}^{-1} [12].$$

In 1965, the existence of microwave background radiation, predicted in 1948 by George Gammov in the framework of the Big Bang theory, was experimentally confirmed [13] and became a source of data for verification of cosmological models.

In 1966 it turned out [14–16] that the deviation from the Hubble law [12] gives $K/c = 3.73 \cdot 10^{-46}$, i.e. almost $(H_0/c)^2 = 3.38 \cdot 10^{-46}$ [km^{-2}] for the model (5), from where $z \approx (H_0/c) \cdot r + (H_0/c)^2 \cdot r^2$ and

$$z = (H_0/c) \cdot r/[1 - (H_0/c) \cdot r] \to D_L = (c/H_0) \cdot z/(1 + z), \tag{6}$$

and Hubble radius c/H_0 acquired the physical meaning of the expanding boundary of the "Universe" at $q_0 = 3$, which was consistent with $q_0 = 2.6 \pm 0.8$ [17] and the model (4).

By the mid-1970s, with the accumulation of data, the estimates of H_0 in cosmological models decreased by an order of magnitude, and the estimates of the acceleration parameter were reset to $q_0 = 0.03 \pm 0.4$ [17]. Among cosmologists even became a popular joke, which is attributed to Alan Sandage: "There is nothing more variable than the Hubble constant".

In 1980, General interpolation models with the shape parameter α were found [14, 18]

$$z = [1 + H_0 \cdot r/(\alpha c)]^\alpha - 1 \to D_L = \alpha \cdot (c/H_0) \cdot \left[(1 + z)^{1/\alpha} - 1 \right], \tag{7}$$

for small redshifts, they resulted in a model (5) with a free parameter K.

At the beginning of 1990-ies the model of Friedman–Robertson–Walker received new view [19]

$$D_L = \frac{c}{H_0} \frac{1+z}{\sqrt{|\Omega_k|}} \begin{cases} \sin \varphi(z), \Omega_k < 0 \\ \varphi(z), \Omega_k = 0 \\ \mathrm{sh}\varphi(z), \Omega_k > 0 \end{cases}$$
$$\varphi(z) = \sqrt{|\Omega_k|} \int_0^z \frac{dx}{\sqrt{(1+x)^2(1+\Omega_M x) - x(2+x)\Omega_\Lambda}}, \quad \Omega_k = 1 - \Omega_M - \Omega_\Lambda, \tag{8}$$

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where Ω_M is the density of "dark matter", Ω_Λ is the density of "dark energy", Ω_k is the curvature parameter, and ΛCDM is a model of the microwave background radiation power spectrum as a parameterization of equations (1) in the form of a correlation function of the anisotropy of the rate expressed by the squares of the amplitudes of a_{lm} modes of harmonics for a given number l [20]:

$$C_l = \frac{1}{2l+1}\sum_{m=-l}^{l}|a_{lm}|^2 \equiv C_l(H_0, \Omega_b, \Omega_M, \Omega_\Lambda, \Omega_\nu, n, ...), \tag{9}$$

where Ω_b and Ω_ν are densities of baryon matter and massive neutrinos, respectively, and n is an indicator of the adiabatic perturbation spectrum.

From the point of view of the structure of the model of Friedman–Robertson–Walker takes the simplest form for parameter curvature $\Omega_k = 1 - \Omega_M - \Omega_\Lambda = 0$:

$$D_L = c\frac{1+z}{H_0}\int_0^z \frac{dx}{\sqrt{(1+x)^2(1+\Omega_M x) - x(2+x)\Omega_\Lambda}}. \tag{10}$$

This corresponds to the "flat Universe" model, which in the ΛCDM model is related to the first peak of the microwave background radiation fluctuation spectrum at an angle of $\sim 1°$ [20].

In 1998, "acceleration of Universe expansion" was found by fitting the parameters of the model (9) by "Riess minimum χ^2" method under the current data on the photometric distances of supernovae SN Ia and the redshifts of their mother galaxies [21]. "Best fitting" at $\Omega_k = 0$ gave the following estimates: $H_0 = 65.2 \pm 1.3$ km · s^{-1}·Mpc^{-1} for $\Omega_M = 0.24$ [21] and $H_0 = 63.0 \pm 1.3$ km·s^{-1}·Mpc^{-1} for $\Omega_M = 0.28$ [22], and $\Omega_\Lambda = 0.76$ and $\Omega_M = 0.24$ were accepted.

In 2015, microwave background radiation maps gave $H_0 = 67.8 \pm 1.9$ km·s^{-1}·Mpc^{-1} [20].

In 2016, the authors of the discovery of "accelerating the expansion of the Universe" according to 300 SN Ia at $z < 0.15$ and 19 Cepheids received an estimate of $H_0 = 73.24 \pm 1.74$ km·s^{-1}·Mpc^{-1} [23], which differs by "3.4σ" from the assessment of Plank project – $H_0 = 67.2 \pm 0.7$ km·s^{-1}·Mpc^{-1} [24], using the model when $\Omega_k = 0$ with the push parameter j_0 in the 3rd order approximation [25] :

$$D_L \cong (c/H_0) \cdot \left[z + \frac{1}{2}(1-q_0)z^2 - \frac{1}{6}(1 - q_0 - 3q_0^3 + j_0)z^3 + O(z^4)\right]. \tag{11}$$

At the same time, it was stated that the achieved "accuracy of the model (11)" was 2.4 %!

However, in the report for 7 years of the WMAP experiment [26] it was noted that the introduction of one or two additional parameters of the ΛCDM model increases its accuracy by 90...300 %, while the SKO of H_0 estimates increases by 1.28...6 times! In the same report for the first 9 years of WMAP experiment [27] $H_0 = 69.7 \pm 2.4$ km·s^{-1}·Mpc^{-1}, at the same time, according to the final data more accurate measurements for the space probe Planck $H_0 = 67.8 \pm 0.9$ km·s^{-1}·Mpc^{-1} [28].

By 2017, in cosmology, a situation which the program Manager of the Hubble Space Telescope Wendy Friedman called deadlock [29]: on the background of the General growth trends in the precision of astrophysical measurements, it was noticed a statistically significant discrepancy of estimates of the Hubble constant H_0 obtained from measurements microwaves. model the background radiation in the framework of standard ΛCDM-model, $H_0 =$ for 67.3 ± 1.2 km·s^{-1}·Mpc^{-1} and according to the "ladder distance" to the Cepheids $H_0 = 74 \pm 3$ km·s^{-1}·Mpc^{-1}. "*To break the current impasse, the steps in the extragalactic distance scale will need to be tested at the percent level*" [29].

The similar dynamics of the accuracy of the estimates of the fundamental gravitational constant G, which plays an equally important role in cosmology, including in determining the distances to supernovae according to the Chandrasekhar condition, was noticed at the end of the 20th century. Then the confidence intervals of three of the four best definitions of G were not

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covered at all [30], and in connection with the analysis of experiments to find neutrino oscillations the problem of “wrong confidence intervals” was considered [31]. In 1998, the Committee on Data for Science and Technology recommended that the new value for the constant of gravity as a weighted average of the experimental results and its standard deviation – $G_{1998} = 6.673(10)\cdot 10^{-11}$ $\mathrm{m}^3\cdot\mathrm{s}^{-2}\cdot\mathrm{kg}^{-1}$. This was a “step back” from the 1986 estimate: 6.67259(85) $\mathrm{m}^3\cdot\mathrm{s}^{-2}\cdot\mathrm{kg}^{-1}$. This was followed by:

$$G_{2004} = 6.6742(10); G_{2006} = 6.67428(67); G_{2010} = 6.67384(80)[10^{-11}\mathrm{m}^3 \cdot s^{-2} \cdot \mathrm{kg}^{-1}].$$

And in 2014 by the method of precision atomic interferometry was obtained unexpected accuracy estimate $G = 6.67191(99)10^{-11}$ $\mathrm{m}^3\cdot\mathrm{s}^{-2}\cdot\mathrm{kg}^{-1}$ [32].

The discrepancy in the definitions for the fundamental constant of gravity Terry Quinn from the International Bureau of weights and measures at a special meeting of the British Academy of Sciences called “*metrological and scientific deadlock*”.

Since the dependence of the distance D to extragalactic objects on the red shift z in the spectra of their radiation is a random sequence $D(z)$ with a space-time trend, the results of the identification of this trend by isotropic averaging models were actually used as scales of cosmological States. Their accuracy was characterized by “averaged hypothetical estimates of the scattering parameters of the regression function estimates” or, simply, the mean arithmetic mean. However, a full assessment of the accuracy of the scales gives a scale factor – the dependence of the parameter of the distribution of deviations from the distance.

Although the description of models of scales of distances, as a rule, be limited to consideration of characteristics of the provisions, greater interest in the current situation is the probabilistic assessment of the distribution and the scale factor the model scale as a function of distance.

The article analyzes the situation with respect to the accuracy requirements of the scale of cosmological distances at the level of 1 % and considers the problem of its structural-parametric identification based on the Friedman–Robertson–Walker model approximations according to the data [21, 22] on SN Ia supernovae used to detect the “universe expansion acceleration”.

2. Situation analysis

The determination of the Hubble parameter and the identification of models of scales of cosmological States are associated with a number of problems.

First, the General problem is caused by the so-called “Riess minimum χ^2” method, used to approximate the measurement data by models (10) and (11) without checking the fulfillment of the known conditions for the applicability of regression analysis, and first of all – Gaussian. Essentially it was the best fitting Friedman–Robertson–Walker model to a “flat” Universe.

In fact, this “method” is a weighted least squares method with a known set of advantages and disadvantages. For non-Gaussian data, such averaging leads to the fact that the meaning of the C_l coefficients becomes non-obvious [20], and the accuracy estimates are untenable [33]. If we exclude the condition $\Omega_k = 0$, “Riess minimum χ^2” method would correspond to $\Omega_M = 0,72^{+0.44}/_{-0.56}$ and $\Omega_\Lambda = 1.48 +^{0.56}/_{-0.68}$ [21] and $\Omega_M = 0.73$ and $\Omega_\Lambda = 1.32$ [22] .

Second, not any SN Ia, but supernovae with refined photometric distances and spectral features are suitable as reference points in the calibration of the scale of cosmological distances. Although the supernova SN Ia are stable stars-Noah value in the maximum luminosity, for what they called “standard candles”, the problem is the unpredictability of the moments of flash and the method of recovering of the luminosity curve.

Third, the estimates of the μ and redshift z modules had instrumental uncertainty $\sigma_\mu \sim$ 0.14...0.30 at $\mu < 44.4$ and $\sigma_z \sim 0.001...0.01$ at $z < 1$ [21, 22]. This inequality in the parametric identification of the model (10) led to the method of weighted least squares and estimates of the type of standard deviation of the arithmetic mean based on the “normal” hypothesis. But even then it was known that the use of regression analysis without taking into account the sins of inadequacy of mathematical models is not only a common mistake, but also requires compliance with the logic of statistical inference.

Fourth, the metrological examination [34] showed that the truncated Laplace distribu]]tion is more plausible by the criterion of the maximum probability of agreement for the distribution of errors of data approximation [21, 22] model (10). Statistical non-homogeneity and anisotropy of the data [21, 22] were observed [35] and earlier in the identification of one-number interpreting models. Their clustering on the transparency windows of the Milky Way allowed to justify the transition from equatorial coordinates to galactic (l, b). As a result, a series decomposition with anisotropy parameters $\theta_k(l, b)$ of the form [36]

$$D_L(l,b,z) = \sum_{k=0}^{K} [1 + \theta_k(l,b)] \cdot z^k \ \text{ or } \ D_L(l,b,z) = \theta_{000} + \sum_{i=0}^{I}\sum_{j=0}^{J}\sum_{k=0}^{K} \theta_{ijk} l^i b^j z^k|_{ijk\neq 000}. \quad (12)$$

The model proved to be insensitive to the SN 1997ck anomaly ($z = 0.97$), which confirmed the doubts of the authors of the High-Z SN Search Team and Supernova Cosmology Project in relation to its influence on the final result in the nonlinear Friedman–Robertson–Walker model with zero curvature parameter [21].

Recall data allowed to detect "acceleration of Universe expansion", held 27 SN Ia at $z < 0.1245$ and 10 SN Ia at $z = 0.30...0.97$ [21], and 42 SN Ia at $z = 0.172...0.830$ [22]. Every element demand-ed special training – breeding types, the recovery curve of luminosity, estimation of the brightness at the maximum and an error-abilities "standard luminosity", as well as what was not done – an error estimation-abilities of the inadequacy of the theory of mechanism adopted flash and the test data on statistical homogeneity. And in this case it should be not just about the statistical, but strictly speaking, about the compositional homogeneity [36]. The peculiarity of this check is taking into account the structural elements of the model (12), allowing to determine the parameters q_0 and H_0. The fact is that only the parameters θ_{001} and θ_{002} can be used to determine the corresponding param-eters H_0 and q_0, because the physical meaning of models with anisotropy parameters without rela-tion to the variable z, i.e. at a non-zero point, is not clear.

To the data [21, 22] it is possible to add 33 "pure" (removed from the centers of maternal galax-ies) SN Ia at $z = 0.010...1.390$ due to obtaining the same result on "acceleration" [37].

Table 1 presents the testing data for compositional homogeneity for the maximum complexity mode at $K = 3$ samples of SN Ia [21, 22, 37] by the criterion of minimum of the mean error module of inadequacy (MEMI) by the MCMMLS algorithm of maximum compactness method (MCM) [36].

An analysis of the data in Table 1 showed the following.

1. The verification of the three SN Ia Data sources for compositional homogeneity showed that all three sources do not form a homogeneous set according to the criterion of the MEMI minimum when representing the model (10) by power series (12) on the red shift and galactic coordinates up to the 3rd order inclusive. The best result is given by the data [21] – 11.07 Mpc, but for the data [21, 22, 37] separately due to the imbalance of the supernovae distribution over the distance, this representation gives a nonzero zero point, which is not consistent with the physical meaning of the distance scale. Any combination of sources increases the total MEMI. A full Union describes a zero–point model

$$D_L(l,b,z)|_{N=112} = -\,0.21844190l + (5811.1265 + 7.9642649l – 6.4452453b) \cdot z$$
$$–1135.2145 \cdot l \cdot z^2 \pm 249.81485.$$

It is primarily associated with the unbalanced and random nature of the plan of formation samples because of the unpredictability of the moments of occurrence of supernovae.

Table 1. Verification of compositional homogeneity of data on SN Ia* [21, 22, 37].

No.	Composition	N	Presence of structural elements					q_0	H_0	MEMI,	Total
p/n	samples		θ_{000}	$\theta_{..0}(l,b)$	$\theta_{001}z$	$\theta_{002}z^2$	$\theta_{..3}z^3$		km s^{-1}Mpc^{-1}	Mpc	MEMI,Mpc
1	27	27	-	+	+	-	-	-	61.61158337	**11.068886**	-
2	33	33	-	+	+	-	-	-	62.52214331	78.911354	-
3	10	10	-	+	-	-	-	-	-	89.506042	-
4	42	42	+	+	-	+	+	-	-	257.43274	-
5	27+33	60	-	+	+	-	-	-	57.61546093	70.458221	-
6	{27}+33	27+33	$(27 \cdot 11.068886 + 33 \cdot 78.911354)/60 = 48.3822434$								
7	27+10	37	+	+	+	-	+	-	-	58.518127	-
8	{27}+10	27+10	$(27 \cdot 11.068886 + 10 \cdot 89.506042)/37 = 32.26811735$								
9	27+42	69	+	-	+	-	-	1.000733861	52.66547093	173.63211	-
10	{27}+42	27+42	$(27 \cdot 11.068886 + 42 \cdot 257.43274)/69 = 161.0294928$								
11	33+10	43	-	+	+	+	-	–2.150035384	72.65568241	136.25961	-
12	33+{10}	33+10	$(10 \cdot 89.506042 + 33 \cdot 78.911354)/43 = 81.37523493$								
13	33+42	75	+	+	+	+	-	1.541311597	87.55912376	227.31390	-
14	33+{42}	33+42	$(42 \cdot 257.43274 + 33 \cdot 78.911354)/75 = 178.8833302$								
15	27+10+33	70	-	+	+	-	-	–3.350638341	93.90931934	78.289024	-
16	$\{27+10\}$+33	37+33	$(37 \cdot 58.518127 + 33 \cdot 78.911354)/70 = 68.13207687$								
17	27+10+42	79	-	-	+	+	-	–0.1676786792	**61.43289962**	249.81485	-
18	$\{27+10\}$+42	37+42	$(37 \cdot 58.518127 + 42 \cdot 257.43274)/79 = 164.2701997$								
19	10+42+33	85	-	-	+	+	-	–0.247404035	**63.47458096**	261.63760	-
20	27+42+33	102	+	+	+	+	+	–1.441257709	86.27635027	170.36908	-
21	27+10+42+33	112	-	+	+	+	-	0.6092962354	51.5893877	227.00719	
22	$\{27+10+33\}$+42	70+42	$(70 \cdot 78.289024 + 42 \cdot 257.43274)/112 = 145.4679175$								
23	$\{10+42+33\}$+27	85+27	$(27 \cdot 11.068886 + 85 \cdot 261.63760)/112 = 201.2326422$								
24	$\{27+33+42\}$+10	102+10	$(10 \cdot 89.506042 + 102 \cdot 70.36908)/112 = 163.1491659$								
25	$\{27+10+42\}$+33	79+33	$(79 \cdot 249.81485 + 33 \cdot 78.911354)/112 = 199.4593556$								

MCMMLS – the method of least squares in the cross-monitoring scheme for inadequacy errors of MCM.

2. The characteristics of the dependence position $D_L(l,b,z)$ of the boundary structure of the model of maximal complexity (12) at $K = 3$ did not reach, except for samples No 4, No 7 and No 20.

3. Elimination of zero-point was possible only for samples No 17 and No 19:

$$D_L(l,b,z)|_{N=79} = (4879.9985 + 10.070535 \cdot b) \cdot z + (2854.0151–12.452332 \cdot l) \cdot z^2 \pm 249.81485;$$
$$D_L(l,b,z)|_{N=85} = 4723.0317 \cdot z + (2945.7644–9.6923456\Delta l) \cdot z^2 \pm 261.63760.$$

4. In the angular distribution of the red shift SN Ia, the dipole anisotropy associated with the galactic center was found to be statistically significant by the MEMI minimum criterion, in this regard, the Heckman approximation (3) was reduced to the form

$$D_L = (c/H_0) \cdot \left[(1 + \alpha \cdot b) \cdot z + \frac{1}{2}(1 - q_0) \cdot (1 + \beta \cdot l) \cdot z^2 \right], \qquad (13)$$

where l and b are galactic coordinates, α and β are anisotropy coefficients [36], and by the criterion of MEMI minimum it was preferable to models (3) and (11).

5. For further analysis, sample No 17 of 79 SN Ia [21, 22] is preferred at a lower value of MEMI.

3. Structural-parametric identification of the scale factor

Perhaps there is no "deadlock in cosmology". After all, large values of H_0 are obtained by SN Ia for $z \sim 1$ and less, and smaller values are obtained by microwave background at $z \gg 1$.

Recall that in 2001 the Hubble Space Telescope Key Project recorded the final result:

$$H_0 = 72 \pm 8 \, \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} = \text{const}$$

for 56...467 Mpc [38], and later, according to the criterion of the minimum of MEMI characterizing its functional component, a "matching" was found [36]:

$$H_0(D) = \begin{cases} (72.60 \pm 3.82) \quad \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}, \, D \le 309.5 \, \text{Mpc} \\ (65.95 \pm 2.50) \quad \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}, \, D > 391.5 \, \text{Mpc} \end{cases}$$

and on an interval 391,5...467 Mpc there were only 2 SN Ia from 36.

In addition, data on SN Ia [21, 22] and data on the same SN Ia from [39] for the interpolation model [6] with respect to the "acceleration of the Universe expansion" lead to opposite conclusions [40] for a banal reason – for a mismatch!

The replacement of the model (10) with the model (11) is of interest, since in the theory of measurement problems the structural-parametric identification of mathematical models of objects is accompanied by the effect of the existence of the model of optimal complexity, when an increase in the number of its parameters leads not to a decrease, but to an increase in the errors of inadequacy.

The results of a more complete structural and parametric identification of model (10) the form of crystals of different orders of Taylor on the criterion of minimum MEMI presented in Table 2.

Table 2. Friedman–Robertson–Walker model in the approximation by structured series

Algorithm identification	Code of structure	$D_L(z) = \theta_0 + \theta_1 \cdot z + \theta_2 \cdot z^2 + \theta_3 \cdot z^3 + \theta_4 \cdot z^4 + \theta_5 \cdot z^5$							
		θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	MAD, Mpc	MEMI, Mpc
MCMMLS	1110	43.594540	4264.6030	1529.0123	0	–	–	271.4289	334.03558
MCMLMM	0110	0	4646.8843	5892.5537	0	–	–	266.34543	414.36618
MCMMLS	10111	105.73219	0	64268.574	–70840.438	44468.270	–	243.23727	299.79135
MCMLMM	01001	0	4653.5825	0	0	629.95447	–	270.9549	383.15613
MCMMLS	011111	0	7042.6611	–37816.098	167427.84	–266400.19	138876.73	222.35149	296.18359
MCMLMM	011001	0	3399.3240	1552.0405	0	0	3341.3684	345.07959	369.00238
Algorithm identification	Code of structure	$D_L(l,b,z) = \theta_{000} + \theta_{001} \cdot z + \theta_{002} \cdot z^2 + \theta_{110} \cdot z^3 + \theta_{101} \cdot z^4 + \theta_{102} \cdot z^5$							
		θ_{001}	θ_{002}	θ_{110}	θ_{101}	θ_{011}	θ_{102}	MAD, Mpc	MEMI, Mpc
MCMLMM	0101110	6585.8984	0	6073406.4	–7.4480181	10.567254	0	253.66292	280.88651
MCMMLS	0110011	4930.4692	2819.7024	0	0	9.9955969	–12.664675	226.03539	**247.42842**

MCMLMM – the method of least modules in the cross-monitoring scheme for inadequacy error of MCM.
MAD – mean absolute deviation.

The best for the data [21, 22], as noted in [36], is an anisotropic model

$$D_L = (c/H_0) \cdot \left[(1 + 2.027311498 \cdot 10^{-3} b) \cdot z + \frac{1}{2} \cdot (1 – q_0)(1 – 4.491493499 \cdot 10^{-3} l) \cdot z^2 \right], \quad (14)$$

where $H_0 = 60.80404234$ km·s^{-1}·Mpc^{-1} and $q_0 = –0.14378664$ at MEMI = 247.42842 Mpc (Figure 1a).

Note, the Visser approach (11) to structure code 0111 is not included in the list of models that are optimal according to the criterion of minimum MEMI, in contrast to the Heckman approach (3) to structure code 0110, i.e. the complexity of the model structure did not lead to increased accuracy [36].

Data deviations [21, 22] from models (10), (14) and (15) are given in Table 3.

For data [21] at $H_0 = 65.2$ km·s^{-1}·Mpc^{-1} and $\Omega_M = 0.24$ MAD from model (10) $d = 429.34$ Mpc, for data [22] at $H_0 = 63.0$ km·s^{-1}·Mpc^{-1} and $\Omega_M = 0.28$–$d = 460.38$ Mpc, and for interpolation model (7) at $\alpha = 0.499160639$ and $H_0 = 77,2924661$ km·s^{-1}·Mpc^{-1} $d = 278.32$ Mpc.

In Figure 1b, deviations $\delta_n = D_{Ln} - (c/H_0) \cdot \left[(1 + \alpha \cdot b_n) \cdot z_n + 1/2(1 - q_0) \cdot (1 + \beta \cdot l_n) \cdot z_n^2\right]$ as a function of D_{Ln} distances at the values of the variables l_n, b_n and z_n are represented as a percentage of D_{Ln}, and as the result of structural-parametric identification, the scale factor in the class of Taylor formulas of the 4^{th} order is a constant ($d = 10.550125\%$). This means that the random component of the model error (14) is multiplicative and, at the same time, predominant, since the instrumental errors are less than $\sim 0.7\%$.

Table 4 shows the results of identification only the random component of the relative error of the cosmological distance scale and the nonparametric component of the error of the accepted probability distribution without taking into account the functional component of the error of

Table 3. The distribution of deviations from the characteristics of situation models

SN Ia	D_{Ln}, Mpc	$\delta_{(13)n}$, %	$\delta_{(9)n}$, %	$\delta_{(14)n}$, %	SN Ia	D_{Ln}, Mpc	$\delta_{(13)n}$, %	$\delta_{(9)n}$, %	$\delta_{(14)n}$, %
1992bo	87.90225168	15.576073	4.461711881	19.85797425	1994H	1348.962883	-26.852166	-62.36360044	-27.72100323
1992bc	94.18895965	7.9885584	0.7732109532	16.81414871	1994al	2228.435149	9.1018628	-12.90201243	11.48762516
1992aq	480.8393484	5.8388039	-4.348934592	14.40005399	1994am	1729.816359	1.63902694	-25.81138002	1.015693806
1992ae	363.0780548	8.5434752	-0.6968679529	16.86440754	1994an	2004.472027	6.03513476	-10.65877571	12.98057211
1992P	141.9057522	-3.347137	13.97141905	28.00783448	1995aq	2477.422058	11.2988828	-11.24511358	12.95259106
1990af	202.3019179	-10.03567	-18.23665093	1.731660164	1995ar	2606.153550	15.0211205	-9.148699993	14.64332939
1994M	119.6740531	-11.66121	5.550856653	20.91679287	1995as	3006.076303	20.5973305	-2.843478942	19.68256803
1994S	71.44963261	-30.94071	-5.302982971	11.6172527	1995at	2500.345362	-29.024617	-73.14321263	-35.01689045
1994T	172.9816359	-13.47406	2.237090774	18.42040752	1995aw	1674.942876	-10.092907	-41.67810504	-11.223837
1995D	39.99447498	-11.16161	3.460499742	18.77752565	1995ax	2398.832919	-21.100724	-66.94269355	-30.1384233
1995E	52.48074602	-17.49785	-3.969125071	12.61915435	1995ay	2523.480772	11.3917978	-17.14894797	8.447579464
1995ac	234.4228815	7.1588247	0.3240151331	17.13217869	1995az	2301.441817	7.94616412	-18.79392618	7.031287053
1995ak	107.1519305	9.3141820	3.731641985	19.34370544	1995ba	1949.844600	-3.2398070	-17.35452939	7.788512348
1995bd	67.60829754	-11.32970	-9.739623612	7.887281139	1996cf	2594.179362	-15.953316	-40.57883187	-9.607392767
1996C	157.0362804	-0.097879	16.14373337	29.86715798	1996cg	2322.736796	-8.9246629	-30.50256447	-1.97495736
1996ab	633.8697113	-12.09104	0.79628019	19.05701755	1996ci	2051.162179	-21.219162	-49.61757872	-16.85893844
1992ag	118.5768748	-15.95334	-2.953889711	13.84405756	1996ck	3090.295433	-7.2027149	-40.35439449	-9.449749242
1992al	60.81350013	-4.134940	-7.062949508	10.08104622	1996cl	3801.893963	-12.044932	-52.39964825	-19.58876483
1992bg	178.6487575	4.7011340	4.63593175	20.42805929	1996cm	2546.830253	-11.204997	-7.348068838	15.9888715
1992bh	240.9905429	15.243304	11.01302251	25.93858829	1996cn	2454.708916	10.9400163	-5.435846526	17.39232826
1992bl	178.6487575	-1.968842	-14.52806535	4.627856632	1997F	2844.461107	3.13751481	-30.98061382	-2.1140667
1992bp	338.8441561	-3.414712	-13.98925468	5.986102209	1997G	3854.783577	4.66727670	-35.75078251	-6.181718167
1992br	438.5306978	14.087611	1.240438646	18.73004207	1997H	2570.395783	2.30558639	-28.57534269	-0.3284953505
1992bs	332.6595533	17.226671	8.502964235	24.22274893	1997I	762.0790100	-6.2354308	-17.6876307	4.903365123
1993H	109.6478196	-17.69800	-6.030457937	11.23747208	1997J	3341.950400	4.39074404	-20.79352492	5.835490679
1993O	254.6830253	-5.588166	2.175070759	18.74092215	1997K	4168.693835	27.1583972	8.345213033	28.55007855
1993ag	229.0867653	-10.62503	-4.412409936	13.22120424	1997L	2884.031503	1.49542814	-21.02607472	5.610720949
1996E	2228.435149	0.6531466	-16.14172961	9.00440229	1997N	816.5823714	-13.718278	-15.53809583	6.779790037
1996H	3944.573021	19.176703	-2.543797227	20.06174335	1997O	2937.649652	34.2711087	25.4429573	41.3507691
1996I	3564.511334	16.372452	-2.310434785	22.22995311	1997P	2387.811283	-3.9447040	-21.30595298	5.166361899
1996J	1887.991349	17.585065	10.52107742	29.0894378	1997Q	1887.991349	-20.081710	-37.08448011	-7.403981723
1996K	2118.361135	7.9455213	-5.369414999	17.1534533	1997R	3388.441561	-0.7731796	-28.24645447	-0.009295823116
1996U	3235.936569	27.513718	20.0187312	37.33567268	1997S	3019.95172	-5.6693936	-31.80934087	-2.749933177
1997ce	2454.708916	-6.363111	-8.396561174	15.12213438	1997ac	1412.537545	-17.048983	-28.97384623	-1.979514442
1997cj	3019.95172	1.6502731	-2.87179183	19.66602027	1997af	2924.152378	-2.3940973	-27.14084547	0.8786566892
1997ck	7555.922277	12.332040	6.60529619	25.95459484	1997ai	2108.62815	-13.441220	-29.65648273	-1.469803317
1995K	3090.295433	19.346646	4.338234296	25.23990736	1997aj	2421.029047	-25.315721	-54.21529657	-20.22716571
1992bi	1986.094917	-40.30204	-40.61938407	-10.00453265	1997am	1853.531623	-18.636420	-34.18742619	-5.226970003
1994F	1949.844600	0.3920903	-5.237665617	17.07092385	1997ap	3265.878322	-18.511742	-77.94676543	-39.65171442
1994G	1506.607066	-60.91095	-69.38623739	-32.75264125	–	–	–	–	–

Note: the number in parentheses in the lower index for deviations δ (no)n is the model number.

the inadequacy of the models of the position characteristics and the instrumental component of the error in the red shift and the module of the photometric distances.

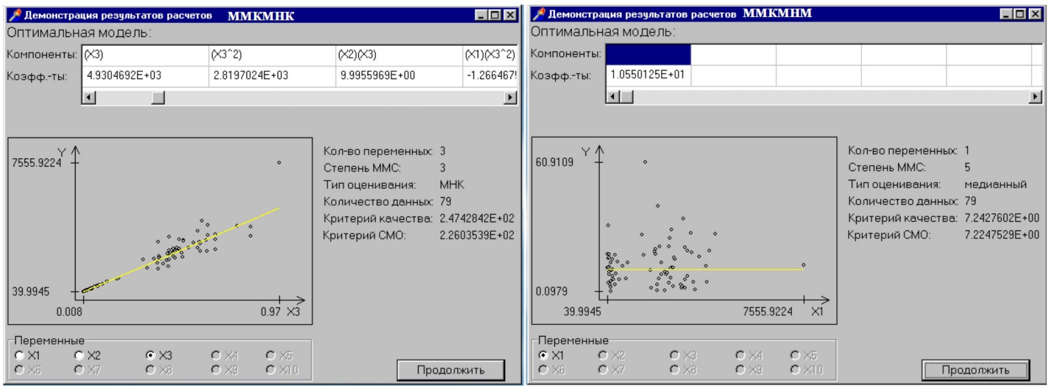

Figure 1. MCM–stat M2 program [35, 33]. Model (14) in coordinates: a) $Y = D_L$, Mpc, and X3= z; MEMI= 247.42842 Mpc; $d = 226.03539$ Mpc. b) $Y = 100 \cdot |\delta_n|/D_L$, %, and X1=$D_L$, Mpc; MEMI=7.2427602 Mpc; $d = 7.2247529$ Mpc.

The truncated probability distributions – uniform, Laplace, Gauss, Cauchy, Trubitsyn and double exponential with the form parameter "4" – were considered as nonparametric hypotheses concerning for random component of error scales. The points of settlement were taken by

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an equivalent uniform distribution. The same distribution was adopted for nonparametric component of inadequacy error.

Table 4. Identification results for the distribution of deviations from the position characteristics of the models.

Model	Distribution of random component	Truncations interval for random component, %	Bias, %	Scale factor, %	Scope for nonparametric component's of inadequacies errors, %	Border of convolutions, %
(9)	Truncated Cauchy	−79.2723; +26.7685	−9.3927	19.8295	13.1461	−89.4459; +29.741
(13)	Truncated Laplace	−62.1312; +35.4914	−0.8485	16.4155	23.3252	−72.1487; +48.7991
(14)	Truncated Cauchy	−40.6902; +42.3893	11.57	13.9494	10.2204	−48.7215; +44.5783

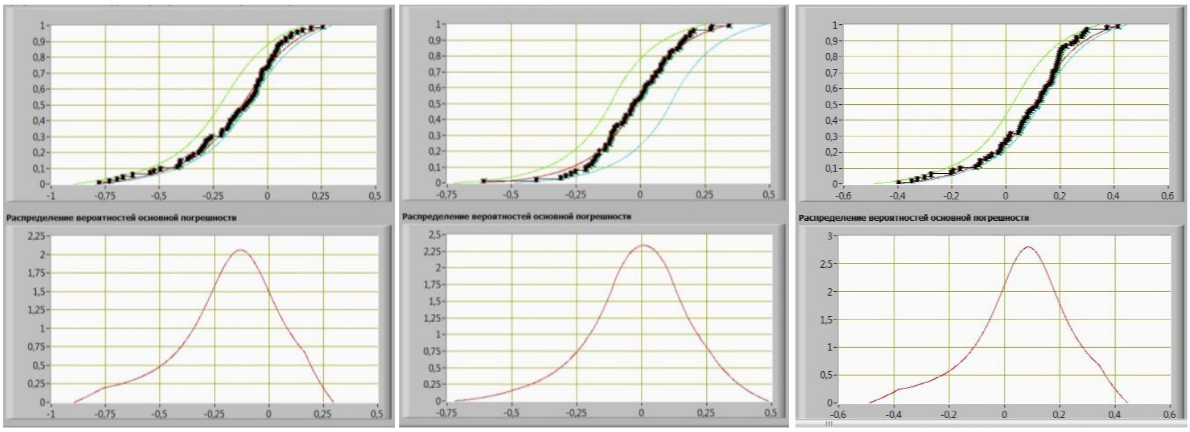

Figure 2. Random component of relative error: a) model (10); b) anisotropic model (14); c) interpolation model (15).

Note, the hypothesis of Gaussian deviations from the model (11) gives the offset of the drop 2.8% and standard deviation $s = 16,3\%$. But the standard deviation of the arithmetic mean of 1.8 % is close to the "estimate" of 2.4 % [23].

The results of identification by the criterion of minimum span of nonparametric error of inade-quacy are presented by convolutions of distributions of components in Figures 2 and 3.

Thus, the anisotropic model (14) based on the Heckman approximation is non-biased and more accurate than the model (11) in the Visser approximation, which illustrates the existence of an optimal complexity model at a given level of instrumental errors in redshift and photometric distance measurements. The interpolation model (15) is more compact, and the model (10) has a displacement of more than 9% and a larger scale factor of Cauchy $\sim 20\%$.

4. Conclusion

The physical meaning of the deadlock in cosmology is related not so much to the divergence of estimates of the Hubble constant, as Wendy Friedman noted, and not so much to the "metrological and scientific deadlock", according to Terry Quinn, but to the fact that the potential possibilities of the Friedman–Robertson–Walker model in the representation of the scale of cosmological races by the function of one argument, the red shift, are exhausted. Therefore, the idea of testing "distances on an extragalactic scale at the percentage level" is not only unrealistic, but is also associated with an inadequate choice of the accuracy indicator in the form of the "average standard deviation".

To a large extent this contributed to the use of untested statistical hypothesis of "normality" and use as the dispersion characteristics of the result of identification of the model, not the

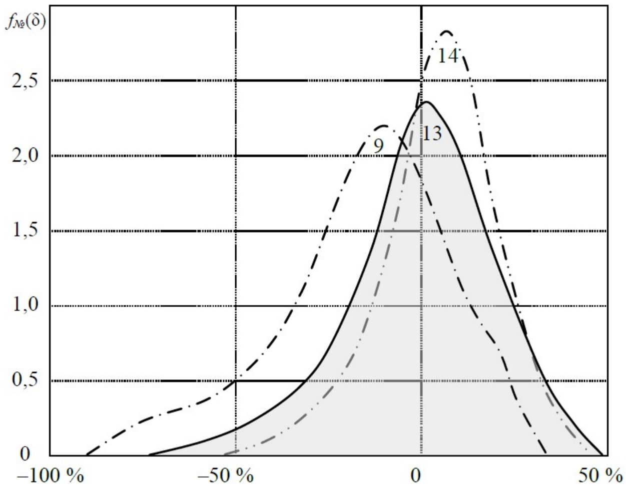

Figure 3. Probabilistic estimates of the multiplicative scale factor for models.

dispersion characteristics of the scale as a whole, but only the dispersion characteristics of the actual character-istics of the situation, which is the "root of N" times less.

Unfortunately, this is a very common error in the application of regression analysis, when the "result of measurement" is the arithmetic mean of the set of "results of observations", and the esti-mate of accuracy is the standard deviation of the arithmetic mean.

The analysis showed that for the models of cosmological distance scales on the red shift axis, us-ing the Friedman–Robertson–Walker model and its representation by Taylor formulas of different orders, the most significant component of the error is the error of inadequacy.

Note that although the statistical analysis of the accuracy of the models of cosmological scales was carried out without taking into account the functional component of the error of the inadequacy of their position characteristics, the fact remains that the scale of cosmological distances on the ba-sis of the red shift of the status of the metric scale does not have any comments.

References

[1] Eddington A S 1923 *The Mathematical Theory of Relativity* (Cambridge: University Press)
[2] Slipher V M 1913 The Radial Velocity of the Andromeda Nebula *Lowell Observatory Bulletin* **2** p 56
[3] Slipher V M 1917 Nebulae *Proceedings of the American Philosophical Society* **56** p 403
[4] Friedmann A A 1922 Über die Krümmung des Raumes *Zeitschrift für Physik* **10** pp 377–86
[5] Einstein A 1923 Notiz zu der Bemerkung zu der Arbeit von A. Friedmann "Über die Krümmung des Raumes" *Zeitschrift für Physik* **16** 326
[6] Lemaître G 1927 Un univers homogene de masse constante et de rayon croissant, redant compte de la vitesse radiale des nébuleuses extra-galactoqies *Annales de la Société scientific de Bruxelles* **A47** p 49
[7] Robertson H P 1928 On relativistic cosmology *Philosophical Magazine* **5** p 835
[8] Walker A G 1933 Distance in an expanding universe *Monthly Notices of the Royal Astronomical Society* **94** p 159
[9] Heckmann O 1942 *Theorien der Kosmologie* (Berlin: Springer)
[10] Mattig W 1958 Über den Zusammenhang zwischen Rotverschiebung und scheinbaren Helligkeit *Astronomischen Nachrichten* **284** pp 109-11
[11] Hoyle F 1961 Verification of cosmology by observations (44 Guthrie Lectures, 21.03.1960) *Proceeding of Physical Society* **A77** 1 pp 1-16
[12] Sandage A R 1962 The distance scale *Problems in extragalactic research* ed G Mc Vittie (New York: Macmillan)
[13] Gamow G 1946 Expanding Universe and the Origin of Elements *Physical review* **70** pp 572–3

[14] Humason M L, Mayall N U and Sandage A R 1956 Redshift and magnitudes of extragalactic nebulae *Astronomical Journal* **61** pp 97–162

[15] Levin S F 2009 On spatial anisotropy of red shift in spectrums of extragalactic sources *Physical Interpretation of Relativity Theory: Proceedings of XV International Meeting: Moscow, 6-9 July 2009* eds M C Duffy et al pp 234–40

[16] Levin S F 2011 Measurement problems in the statistical identification of the scale of cosmological distances *Measurement Techniques* **54** 12 pp 1334–41

[17] Lang K 1974 *Astrophysical formulae* (Berlin, Heidelberg, New York: Springer-Verlag)

[18] Levin S F 2012 Cosmological distances scale based on a red shift interpolation model *Measurement Technique* **55** 6 pp 609-12

[19] Carroll S, Press W and Turner E 1992 The Cosmological Constant *Annual Review of Astronomy & Astrophysics* **30** pp 499-54

[20] Verkhodanov O V, Parijskij Yu N and Starobinsky A A 2005 Determination of Ω_Λ and H_0 from photometric data of radiogalaxies *Bulletin of Special astrophysical observatory Russian Academy of Science* **58** pp 5-16

[21] Riess A G et al 1998 Observational evidence from supernovae for an accelerating universe and a cosmological constant *Astronomical journal* **116** pp 1009–38

[22] Perlmutter S et al 1999 Measurements of Ω and Λ from 42 high-red shift supernovae *Astrophysical Journal* **517** pp 565–86

[23] Riess A et al 2016 A 2.4 % Determination of the Local Value of the Hubble Constant *Preprint* 1604.01424v3 [astro-ph.CO]

[24] Planck Collaboration: Ade P et al 2016 Planck 2015 results: XIII. Cosmological parameters *Astronomy & Astrophysics* **594** A13 (*Preprint* 1502.01589v2 [astro-ph.CO])

[25] Visser M 2004 Jerk, snap and the cosmological equation of state *Preprint* gr-qc/0309109v4

[26] Larson D et al 2010 7 year WMAP observations: power spectra and WMAP-derived parameters *Preprint WMAP* http://lambda.gsfc.nasa.gov/product/map/dr4/pub_papers/sevenyear/powspectra/wmap_7yr_power_spectra.pdf

[27] Hinshaw G et al 2013 Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Cosmological parameter results *Preprint* 1212.5226v3 [astro-ph.CO]

[28] Planck Collaboration 2016 Planck intermediate results. XLVI. Reduction of large-scale systemic effects in HFI polarization maps and estimation of the reionization optical depth *Preprint* 1605.02985v2 [astro-ph.CO]

[29] Freedman W L 2017 Cosmology at a Crossroads: Tension with the Hubble Constant *Preprint* 1706.02739

[30] Feldman G and Cousins R 1998 Unified approach to the classical statistical analysis of small signals *Physical review* D **57** 7 pp 3873–89

[31] Rosi G, Sorrento F, Caccia-puoti L et al 2014 Precision measurement of the Newtonian gravitational constant using cold atoms *Nature* **510** pp 518–21

[32] Quinn T, Parks H, Speake C and Davis R 2013 Improved Determination of G Using Two Methods *Physical Review Letters* **111** 101102

[33] Levin S F 2015 Problems of Applicability of Statistical Methods in Cosmology *Physics of Atomic Nuclei* **77** 13 pp 1-6

[34] Levin S F 2016 Cosmological distances scale. Part 5. Metrological Expert Opinion on Type SN Ia Supernovae *Measurement Techniques* **59** 8 pp 791-802

[35] Levin S F 2011 Measurement problems in the statistical identification of the scale of cosmological distances *Measurement Techniques* **54** 12 pp 1334–41

[36] Levin S F 2018 Cosmological Distance Scale. Part 7. A New Special Case with the Hubble Constant and Anisotropic Models *Measurement Techniques* **61** 11 pp 1057–65

[37] Lipunov V M, Panchenko I E and Pruzhinskaya M V 2011 The mechanism of supernova Ia explosion in elliptical galaxies *New Astronomy* **16** pp 250–2

[38] Freedman W L, Madore B F, Gibson B K et al 2001 Final Results from the Hubble Space Telescope Key Project to Measure the Hubble Constant *Astrophysical Journal* **553** pp 47–72

[39] Tsvetkov D Yu, Pavlyuk N N, Bartunov O S and Pskovsky Yu P 2005 Supernovae Catalogue. (M.: Sternberg State Astronomical Institute) www.astronet.ru/db/ sn/catalog.html

[40] Levin S F 2015 Measurement problem of structural-parametric identification on supernovae type SN Ia for cosmological distances scale of red shift based *Physical Interpretations of Relativity Theory: Proceedings of International Meeting. Bauman Moscow State Technical University, Moscow, 29 June – 2 July 2015* pp 299–310