



The Einstein Telescope standard siren simulations for $f(Q)$ cosmologies

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Abstract To investigate the model and extra frictional effects in standard siren simulation of $f(Q)$ cosmologies, we simulated three types of standard siren data based on different fiducial models (Λ CDM and $f(Q)$ models). Both effects are important in standard siren simulation. Explicitly, the $f(Q)_P$ and $f(Q)_E$ models need more observational data (e.g. growth factor) to further study. The $f(Q)_{PE}$ model could be ruled out by the EM data. And both the $f(Q)_{HT}$ models will be excluded by the future standard siren data.

1 Introduction

The standard siren (SS) of gravitational wave (GW) provides an absolute measurement of distance without dependence on other sources [1]. This standard siren method is widely used to constrain cosmological models especially for the modified gravity. Presently, the direct detection of gravitational wave has discovered at least 99 standard siren events [2–8], but only one single confirmed standard siren event (GW170817) and one possible standard siren event (GW1905 21) have been detected. These two events are unable to do effective cosmological constraints. In the coming decade, ground-based (e.g. Einstein Telescope (ET) [9–11] and space-based telescopes (e.g. Taiji [12], Tianqin [13], and LISA [14])) experiments are predicted to discover more standard siren sources. Therefore, to forecast fundamental properties of gravity, the mock catalogs of standard sirens should be created.

In the standard siren method, the luminosity distance D_L could be extracted from the GW amplitude h_A . And the standard siren simulation should be based on the background cosmology but it could not be determined at present. The cosmological constant called Λ CDM model is the simplest theoretical explanation for our accelerating universe which is preferred by the majority of observational survey releases

(e.g. Planck data [15, 16]). And, dark energy and modified gravity are the candidates of explaining the accelerating universe as well. All the cosmological models could affect the amplitude of standard siren. Then conversely, the standard siren data could constrain cosmological models. Here, the choice of fiducial model of standard siren simulation is called the model effect. And compared with the Λ CDM and dark energy models of general relativity (GR), the propagation equation of gravitational wave in modified gravity has an extra friction term which affects the standard siren simulation as well. It is called the extra frictional effect. Roughly, based on affine connections [17–22], there are mainly three types of modified gravity: $f(R)$, $f(T)$ and $f(Q)$ cosmologies. Here, we choose the Λ CDM model in general relativity (GR) as baseline and discuss the $f(Q)$ cosmologies which are relatively simple. Among the various $f(Q)$ models [23–66], two Λ CDM-like models (the power-law $f(Q)_P$ [23, 24] and square-root exponential $f(Q)_E$ [25] models) which could come back to Λ CDM model are chosen to discuss. Correspondingly, two non Λ CDM-like models (the power exponential $f(Q)_{PE}$ [26, 27] and hyperbolic tangent $f(Q)_{HT}$ [28–30] models) are chosen to constrain as well.¹

To invest the mode and extra friction effects, we intend to simulate three types of mock standard siren data in this paper: the first one (SSI _{Λ}) is based on the Λ CDM model; the second one (SSII) is based on $f(Q)$ cosmologies but assuming the extra friction term zero; the third one (SSIII) is based on the $f(Q)$ model as well and using its true extra friction term. Then we could compare the SSI _{Λ} and SSII to see the model effect, and compare SSII and SSIII to see the effect of extra friction term. The electromagnetic (EM) data is used as

¹ The description of gravity using Teleparallel Gravity with Weitzenböck connection T which is called torsion scalar is equivalent with the Symmetric Teleparallel gravity using the non-metricity scalar Q in the background level. In the both gravities, the curvature R in GR is replaced by T or Q [67–84].

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baseline in all models, including direct determination of the Hubble parameters derived from cosmic chronometer (CC) method [85, 86], baryon acoustic oscillations (BAO) of Dark Energy Spectroscopic Instrument (DESI) [87] and the type Ia supernovae of PantheonPlus compilation (PantheonPlus) [88, 89].

The rest of this paper is organized as follows. In Sect. 2, we will introduce the standard siren of gravitational wave. In Sect. 3, we will introduce the EM observational data and the mock standard siren data. In Sect. 4, we will briefly describe the $f(Q)$ cosmologies. In Sect. 5, we will summarize the used data. In Sect. 6, we will report the constraint results. Finally, a brief summary will be given in Sect. 7.

2 The standard siren

The gravitational waves from compact systems are viewed as standard sirens to probe the evolution of the universe [1, 90]. From the GW signal, the luminosity distance D_L^{SS} is measured directly, without invoking the cosmic distance ladder, since the standard sirens are self-calibrating. And it could be extracted from the GW amplitude

$$h_A = \frac{4}{D_L^{SS}} \left(\frac{GM_c}{c^2} \right)^{5/3} \left(\frac{\pi f_{GW}}{c} \right)^{2/3}, \quad (1)$$

where h_A is the SS amplitude, “A” could be “+” or “ \times ”, D_L^{SS} is the luminosity distance for gravitational wave standard siren, M_c is the chirp mass, and f_{GW} is the GW frequency. Obviously, the amplitude of gravitational wave standard siren will be affected by the background cosmological model.

Especially, the propagation equation of standard siren in Fourier form is [91, 92]

$$\bar{h}_A'' + 2\mathcal{H}[1 + \delta(\eta)]\bar{h}_A' + k^2\bar{h}_A = 0, \quad (2)$$

where \bar{h}_A denotes the Fourier mode of the standard siren amplitude, the prime “ $'$ ” denotes a derivative with respect to conformal time η , and $\mathcal{H} = a'/a$. And especially, δ is the so-called extra friction term which is zero for the Λ CDM model or dark energy model in General Relativity.

In order to simplify the propagation equation within modified gravity theories, by defining a modified scale factor $\tilde{a}'/\tilde{a} = \mathcal{H}[1 + \delta(z)]$ and $\chi_A = \tilde{a}h_A$, we get [93]

$$\chi_A'' + \left(k^2 - \frac{\tilde{a}''}{\tilde{a}} \right) \chi_A = 0. \quad (3)$$

Then, the relation between the EM luminosity distance and the SS luminosity distance could be expressed as

$$D_L^{SS}(z) = \exp \left(\int_0^z \frac{\delta(z')}{1+z'} dz' \right) D_L^{EM}(z), \quad (4)$$

where $D_L^{EM}(z)$ and $D_L^{SS}(z)$ are EM and GW luminosity distances separately. Obviously, the extra friction term δ characterizes the difference between the GW luminosity distance and the EM luminosity distance. When δ is negative, $D_L^{EM} > D_L^{SS}$, there is a smaller D_L^{SS} which denotes a larger Hubble parameter H , and then a larger Hubble constant H_0 which parameterizes the current expansion rate of our universe.

The H_0 parameter is related with the famous “Hubble tension” problem. Cosmologically, H_0 could be measured from the cosmic microwave background which fit to a cosmological model such as Λ CDM (for instance, from Planck [94]). And locally, H_0 could be measured from the observed redshift–distance relation in the Hubble flow for distant objects (for instance, from Cepheid variables and Type Ia supernovae by the SH0ES Team based on the three-rung distance ladder method [95–97]). Explicitly, the Hubble tension refers to a discrepancy of more than 5σ between H_0 measured using these two measurements (see reviews by Refs. [98, 99]). In cosmology, many models which could return to Λ CDM meet Hubble tension problem as the Λ CDM does [100, 101]. Here, we will discuss Hubble tension for various $f(Q)$ cosmologies in the standard siren simulation. The calculated values of Hubble tension could be denoted as [102]

$$T_1(\theta) = \frac{|\theta(D_1) - \theta(D_2)|}{\sqrt{\sigma_\theta^2(D_1) + \sigma_\theta^2(D_2)}}. \quad (5)$$

where θ is the best fitted values of H_0 from different data sets; the first data set D_1 represents the constraining results of cosmological fitting; the second data set D_2 is the chosen baseline measurement which is $H_0 = 73.17 \pm 0.86$ km/s/Mpc from the latest SH0ES Team [95–97]; and $\sigma_\theta(D_1)$ and $\sigma_\theta(D_2)$ represent the errors from D_1 and D_2 data sets respectively.

3 The method

The Markov Chain Monte Carlo (MCMC) package Cosmomc [103] is employed to infer the posterior probability distributions of parameters, and further to derive the best fitted values and their corresponding errors. And numerical simulation is also used to forecast results of surveys and targeted observations. We choose the Einstein Telescope as the representative of third generation instruments which will detect thousands of Neutron Star Binary (NSB) and Black Hole Binary (BHB) mergers to probe the cosmic expansion at high redshifts [91, 104]. Here, we simulate the standard siren data by using the best fitted parameter values from EM combination and Einstein Telescope design index.

Firstly, we employ current EM observational data sets which are related to the cosmic distance, including the type

Ia supernovae of PantheonPlus compilation (PantheonPlus) [88, 89], the direct measurements of the Hubble parameter derived from cosmic chronometer method (CC) [85, 86], and baryon acoustic oscillations of Dark Energy Spectroscopic Instrument (DESI) [87], to perform the MCMC analysis which give out the fiducial parameter values required by standard siren simulation.

The PantheonPlus compilation is acquired from 18 distinct surveys [88, 89]. It contains 1701 distance modulus data spanning within a redshift range of $0.00122 < z < 2.26137$. The distance modulus, which is the observable quantity in the supernovae (SN) data, is defined as

$$\mu = m - M = 5 \log_{10}[d_L(z)] + 5 \log_{10} \left[\frac{c/H_0}{Mpc} \right] + 25, \quad (6)$$

where the luminosity distance is

$$d_L = (1+z) \int_0^z \frac{d\tilde{z}}{H(\tilde{z})}. \quad (7)$$

In general, the goodness of fit for theoretical model is measured by χ^2 and likelihood functions (L) which is expressed as $\chi^2 = -2\ln L$. To get best fit, the value of χ^2 is needed to minimize. In the context of the PantheonPlus compilation, the χ^2 likelihood function could be computed by

$$\chi^2_{PantheonPlus} = \sum_{i=1}^{1701} \Delta\mu^T \mathbf{C}_{stat+sys}^{-1} \Delta\mu, \quad (8)$$

where the covariance matrix ($\mathbf{C}_{stat+sys}$) includes both the systematic and statistical errors, $\Delta\mu$ is the vector of 1701 SN distance modulus residuals computed as

$$\Delta\mu_i = \mu^{model}(z_i) - \mu_i^{data}. \quad (9)$$

The Hubble parameter data could be obtained through the cosmic chronometer (CC) method which calculate the differential ages of passively evolving galaxies. Here, the used CC compilation contains 32 data points [105–114] which are tabulated in Table 1 of Ref. [86]. And we use the covariance matrix for computations as described in Ref. [85]. Then, the form of χ^2 of Hubble parameter data through the cosmic chronometer (CC) method is

$$\chi^2_{CC} = \sum_{i=1}^{32} \Delta H(z)^T \mathbf{C}_{stat+model+young+met}^{-1} \Delta H(z), \quad (10)$$

where $\Delta H(z) = H(z_i)^{model} - H_i^{data}$, “stat”, “young”, “model” and “met” denote the contributions to the covariance due to statistical errors, young component contamination, dependence on the chosen model, and stellar metallicity respectively.

Furthermore, the properties of BAO are derived from the matter power spectrum which are related with the matter

fluctuation perturbation. In the clustering of matter of late universe, they could serve as a standard ruler to map the expansion history of the universe. Here, we adopt the first-year data released by the DESI collaboration [87], which includes observations from four different classes of extragalactic targets: the bright galaxy sample (BGS) [115], luminous red galaxies (LRG) [116], emission line galaxies (ELG) [117], and quasars (QSO) [118]. The DESI provides robust measurements of the transverse comoving distance (D_M), the Hubble distance (D_H) and the angle-average distance (D_V) relative to the drag-epoch sound horizon (r_d) in seven redshift bins from over 6 million extragalactic objects. The DESI data are summarized in Table 1 of Ref. [87]. Firstly, we calculate the χ^2 related to the BGS and QSO data as below

$$\chi^2_{DESI1} = \sum_{i=1}^2 \frac{((D_V/r_d)^{model} - (D_V/r_d)^{data})^2}{\sigma_{z_i}^2}. \quad (11)$$

And, the data of D_M/r_d and D_H/r_d from tracers LRG1, LRG2, LRG3+ELG1, ELG2, and Ly α QSO, degenerate at the same redshift z . Following Ref. [119, 120], the data vector D could be constructed as

$$D \equiv \begin{pmatrix} D_M/r_d \\ D_H/r_d \end{pmatrix}, \quad (12)$$

with its covariance matrix defined as [119, 120]:

$$\text{Cov}_{DESI2} = \begin{bmatrix} \sigma_1^2 & r \cdot \sigma_1 \cdot \sigma_2 \\ r \cdot \sigma_1 \cdot \sigma_2 & \sigma_2^2 \end{bmatrix}, \quad (13)$$

where σ_1 and σ_2 denote the standard deviations of D_M/r_d and D_H/r_d respectively. The correlation coefficient between D_M/r_d and D_H/r_d , which is denoted as r , is provided in Table 1 of Ref. [87]. Then the second χ^2_{DESI2} is expressed as

$$\chi^2_{DESI2} = \sum_i^{10} \Delta D_i^T \text{Cov}_{DESI2}^{-1} \Delta D_i, \quad (14)$$

where $\Delta D_i = D_i^{model} - D_i^{data}$ is the data vector constructed by Eq. (12).

We combine the PantheonPlus, CC and DESI observational data as EM compilation which has 1745 data points. And the best fitted values of EM compilation, which correspond to the minimum of sum of $\chi^2_{PantheonPlus}$, χ^2_{CC} and χ^2_{DESI} , is used as fiducial parameter values in standard siren simulations.

Meanwhile, the strain $h(t)$ in the gravitational wave interferometers could be written as [104]

$$h(t) = F_+(\theta, \phi, \psi)h_+(t) + F_\times(\theta, \phi, \psi)h_\times(t), \quad (15)$$

where F_+ , F_\times are the antenna pattern functions sensed by the gravitational wave detector. The redshift range is chosen as $0 < z < 5$. And the standard siren sources considered in this work include the merger events from black hole-neutron star systems and binary neutron star systems, both of which are expected to exhibit afterglows in the EM radiation after they emit bursts of gravitational wave. Thus, BNS and BHNS could be observed not only as a transient standard siren event, but also as an EM counterpart, and could be used as standard siren candidates.

In this work, to calculate the errors of the simulated data, we utilize the one-sided noise power spectral density (PSD) which characterizes performance of the gravitational wave detector. The measurement errors of luminosity distance is also related to the weak lensing effects. Following the studies in Refs. [104, 121], this weak lensing error is assumed to be $0.05z$. Thus, the total uncertainty on the measurement of D_L is taken as

$$\sigma_{D_L} = \sqrt{\sigma_{\text{inst}}^2 + \sigma_{\text{lens}}^2} = \sqrt{\left(\frac{2D_L}{\rho}\right)^2 + (0.05zD_L)^2}, \quad (16)$$

where σ_{inst}^2 is the instrumental error calculated by Fisher Matrix and ρ is the ratio of signal to noise which is usually chosen as $\rho > 8$. In this paper, we will simulate 1000 standard siren data points expected to be detected by Einstein Telescope in its 10-year observation. To achieve this, the Fisher matrix approach is utilized [91]. And, we roughly assume that there are 500 BNS events and 500 BHNS events. The χ^2 of SS data could be expressed as below:

$$\chi_{\text{SS}}^2 = \sum_{i=1}^{1000} \frac{(D_L^{\text{model}}(z_i) - (D_L^{\text{data}})_i)^2}{\sigma_{z_i}^2}. \quad (17)$$

Then, the standard sirens method offers a new independent way to probe the cosmic expansion.

4 The $f(Q)$ cosmologies

In the Symmetric Teleparallel gravity, the non-metricity Q , which represents the variation in length of a vector during parallel transport, is used to describe the gravitational interaction. And, its natural extension, the $f(Q)$ modified gravity, has revealed many interesting cosmological phenomena as shown in the literature.

The action of $f(Q)$ cosmology could be given by [31–33],

$$S = \int \sqrt{-g} \left[-\frac{1}{16\pi G} f(Q) + \mathcal{L}_m \right] d^4x, \quad (18)$$

where $f(Q)$ is an arbitrary function of the non-metricity scalar Q ; \mathcal{L}_m is the matter Lagrangian density and g is the determinant of metric $g_{\mu\nu}$.

As the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) spacetime is considered, we obtain the exact value of non-metricity scalar

$$Q = 6H^2. \quad (19)$$

Here, $H = \dot{a}/a$ is the Hubble parameter. Note that in the $f(Q)$ gravity, the non-metricity scalar Q plays the role of Ricci scalar R in GR, which indicates these two categories of modified gravity theories ($f(Q)$ and $f(R)$) are equivalent in the background level. It is convenient to set $f(Q) = Q + F(Q)$ where the $F(Q)$ part represents the cosmic acceleration effect. Then the Friedmann equations for flat spacetime take the following form [31–33]

$$3H^2 = \rho + \frac{F}{2} - QF_Q, \quad (20)$$

$$\dot{H} = \frac{F - Q - 2QF_Q}{4(2QF_{QQ} + F_Q + 1)}, \quad (21)$$

where ρ and p are the energy density and pressure for the matter fluid, and satisfy conservation equation:

$$\dot{\rho} + 3H(1 + w)\rho = 0, \quad (22)$$

where w is the equation-of-state (EoS) parameter.

Correspondingly, the effective energy density ρ_{eff} and effective pressure p_{eff} for the acceleration part that is sourced from $F(Q)$ could be described as

$$\rho_{\text{eff}} = \frac{F}{2} - QF_Q, \quad (23)$$

$$p_{\text{eff}} = 2\dot{H}(2QF_{QQ} + F_Q) - \rho_{\text{eff}}. \quad (24)$$

Thus, the effective EoS parameter could be given by

$$w_{\text{eff}} = -1 + \frac{1 - 1/(F/Q - 2F_Q)}{1 + 1/(2QF_{QQ} + F_Q)}. \quad (25)$$

In the framework of $f(Q)$ gravity, the extra friction term in Eq. (2) takes the below form:

$$\delta(z) = \frac{d \ln f_Q}{2\mathcal{H} d\eta}. \quad (26)$$

And the dimensionless Hubble parameter could be expressed as

$$E(z) = \frac{H}{H_0}, \quad (27)$$

where the subscript “0” denotes the present time.

Especially, the Λ CDM model could be regarded as $F(Q) = \Lambda$ where $w_{\text{eff}} = -1$ and $\delta = 0$. Here, for convenience, the constraining models are divided into the Λ CDM-like one (the power-law $f(Q)_P$ [23, 24] and square-root exponential $f(Q)_E$ [25] models) and non Λ CDM-like one (the power exponential $f(Q)_{PE}$ [26, 27] and hyperbolic tangent $f(Q)_{HT}$ [28–30] models). Next, we will introduce them one by one.

4.1 The power-law form: $f(Q)_P$ model

The power-law $f(Q)$ model [23,24] (hereafter $f(Q)_P$ model) is a simple and notable model,

$$F(Q) = \alpha Q^b, \quad (28)$$

where $\alpha = Q_0^{1-b}(1 - \Omega_{m0})/(1 - 2b)$, and b is the new freedom which quantifies deviation from Λ CDM model. When $b = 0$, this model degenerates to Λ CDM model. When $b = 1/2$, this model reduces to the Dvali-Gabadadze-Porrati (DGP) model [122]. When $b = 1$ the model is the same as the standard cold dark matter model after re-scaling the Newton's constant. Then it is required that $b < 1$ in order to obtain an accelerating cosmic expansion. Ref. [92] gave out that the $f(Q)_P$ model could be distinguished from Λ CDM model by using GW and EM combined data. This phenomenon is worthy checking by the updating data.

4.2 The square-root exponential form: $f(Q)_E$ model

And, we introduce the square-root exponential model [25] (hereafter $f(Q)_E$ model)

$$F(Q) = \alpha Q_0(1 - e^{-p\sqrt{Q/Q_0}}), \quad (29)$$

where $\alpha = (1 - \Omega_{m0})(1 - (1 + p)e^{-p})$ and p is model parameter. Usually, we set $b = 1/p$ for convenience. As the $f(Q)_P$ model, the $f(Q)_E$ model comes back to Λ CDM model when $b = 0$. And $b \neq 0$ in $f(Q)_E$ model indicates an essential deviation from Λ CDM model. However, $b \rightarrow +0$ corresponds to $p \rightarrow +\infty$, while $b \rightarrow -0$ corresponds to $p \rightarrow -\infty$. Then, getting across $b = 0$ means crossing the singularity p . And when $b < 0$, $e^{-p\sqrt{Q/Q_0}}$ grows exponentially. Therefore, to avoid the singularity, we set the prior $b > 0$ for all constraints which is favored by Ref [78]. In the literature [92], the EM constraint with Pantheon included shows the Hubble tensions as large as 3.21σ . So we constrain it with PantheonPlus included data to see whether the Hubble tension could be alleviated.

4.3 The power exponential form: $f(Q)_{PE}$ model

In Refs. [26,27], a power exponential form of $f(Q)$ model has been introduced (hereafter $f(Q)_{PE}$ model), and it could be expressed as:

$$F(Q) = Q \left(e^{\lambda \frac{Q_0}{Q}} - 1 \right), \quad (30)$$

where the derived parameter λ which is determined by Ω_{m0} could be described as

$$\lambda = \frac{1}{2} + W_0 \left(-\frac{\Omega_{m0}}{2e^{\frac{1}{2}}} \right), \quad (31)$$

and W_0 is the Lambert function. Similar to the Λ CDM model, the $f(Q)_{PE}$ model has two free parameters Ω_{m0} and H_0 . Note that, the $f(Q)_{PE}$ model theoretically could not come back to the Λ CDM for any values of λ , thus it is called the non Λ CDM-like model here.

At high-redshift where $Q_0 \ll Q$, $F_Q \simeq -\lambda^2 Q_0^2/Q^2$ and $F_{QQ} \simeq \lambda^2 Q_0^2/Q^3$. The effective EoS can be expanded as

$$w_{\text{eff}} = -1 - \lambda \frac{Q_0}{Q}. \quad (32)$$

If $\lambda > 0$, the w_{eff} would approach to -1 from the phantom side. And the extra friction term could be expressed as

$$\delta = -\frac{3}{2}\lambda^2 \left(\frac{Q_0}{Q} \right)^2, \quad (33)$$

which tends to 0 from negative side.

The power exponential model has the same free parameters with Λ CDM model, but does not degenerate with Λ CDM model. As it has been proved to alleviate the Hubble tension [26,27], it will be interesting to constrain this model.

4.4 The hyperbolic tangent form: $f(Q)_{HT}$ model

For the purpose of realizing the crossing of the phantom divide line [28], the hyperbolic tangent form of the $f(Q)$ model is proposed as [29,30] (hereafter $f(Q)_{HT}$ model):

$$F(Q) = \alpha Q_0 \left(\frac{Q_0}{Q} \right)^{-b} \tanh \frac{Q_0}{Q}, \quad (34)$$

where b is an additional free parameter compared with Λ CDM model or $f(Q)_{PE}$ model. And the dimensionless parameter α could be expressed as

$$\alpha = \frac{1 - \Omega_{m0}}{(1 - 2b) \tanh 1 + 2 \operatorname{sech}^2 1}. \quad (35)$$

The $f(Q)_{HT}$ model could not come back to the Λ CDM for any values of b as well.

When $Q \gg Q_0$, $\tanh(Q_0/Q) \simeq Q_0/Q$ and $\operatorname{sech}(Q_0/Q) \simeq 1$. Then, $F_Q \simeq \alpha(b-1)(Q_0/Q)^{(2-b)}$ and $F_{QQ} \simeq \alpha(1-b)(2-b)(Q_0/Q)^{(2-b)}/Q$ which are small as well at high redshift. And, the EoS and extra friction are approximately to be

$$w_{\text{eff}} = -2 + b, \quad (36)$$

$$\delta = -\alpha(1-b)(2-b) \left(\frac{Q_0}{Q} \right)^{2-b}. \quad (37)$$

As the effective energy density must be larger than 0, we need to give a prior of b for the model. So, we divide the Hyperbolic Tangent model to two branches:

f(Q)HT1: We set a prior $b < 0.5$ for this model. The w_{eff} would approach to the phantom side at high redshifts. The δ tends to 0 from negative side.

Table 1 The best fitted values with 1σ and 2σ standard errors from the constraints of EM and SS related data for the Λ CDM and $f(Q)$ models. The best fitted values of EM is used as baseline parameter values in standard siren simulations. Explicitly, the best fitted EM values of Λ CDM model are used for SSI_{Λ} simulation. The best fitted EM values

of $f(Q)$ models are used for SSII and SSIII simulations. Especially, we list the Hubble tension at the end of the table where the reference Hubble constant value $H_0 = 73.17 \pm 0.86$ km/s/Mpc is from the SH0ES Team [95,96] as Sect. 2 stated. We did not list the Hubble tensions of SSI_{Λ} and SSII data for $f(Q)$ models because they are not physical

Model	Data	Ω_{m0}	H_0 (km/s/Mpc)	b	δ_0	$w_{\text{eff}0}$	Hubble tension
Λ CDM	EM	$0.322^{+0.011+0.023}_{-0.011-0.022}$	$73.23^{+0.17+0.34}_{-0.17-0.34}$	—	0	-1	0.07σ
	SSI_{Λ}	$0.316^{+0.014+0.028}_{-0.014-0.027}$	$73.56^{+0.54+1.11}_{-0.54-1.11}$	—	0	-1	0.38σ
$f(Q)_P$	EM	$0.327^{+0.012+0.024}_{-0.012-0.023}$	$73.35^{+0.19+0.38}_{-0.19-0.38}$	$-0.082^{+0.061+0.111}_{-0.053-0.122}$	$-0.024^{+0.017+0.033}_{-0.017-0.032}$	$-1.025^{+0.017+0.033}_{-0.017-0.035}$	0.20σ
	SSI_{Λ}	$0.281^{+0.059+0.084}_{-0.031-0.101}$	$73.09^{+0.88+1.81}_{-0.88-1.70}$	$0.201^{+0.401+0.601}_{-0.252-0.672}$	$0.153^{+0.073+0.481}_{-0.252-0.332}$	$-0.915^{+0.121+0.241}_{-0.121-0.230}$	—
	$SSII_{P0}$	$0.330^{+0.036+0.054}_{-0.019-0.066}$	$73.74^{+0.91+1.61}_{-0.80-1.71}$	$-0.211^{+0.421+0.642}_{-0.350-0.711}$	$-0.032^{+0.036+0.221}_{-0.131-0.150}$	$-1.048^{+0.072+0.202}_{-0.130-0.181}$	—
	$SSIII_{P}$	$0.383^{+0.028+0.047}_{-0.018-0.053}$	$73.88^{+0.86+1.51}_{-0.73-1.59}$	$-0.420^{+0.281+0.599}_{-0.442-0.580}$	$-0.090^{+0.022+0.160}_{-0.096-0.101}$	$-1.118^{+0.049+0.190}_{-0.131-0.151}$	0.61σ
$f(Q)_E$	EM	$0.322^{+0.011+0.023}_{-0.011-0.022}$	$73.19^{+0.18+0.35}_{-0.18-0.37}$	$0.106^{+0.035+0.103}_{-0.106-0.106}$	$0.005^{+0.001+0.018}_{-0.004-0.005}$	$-0.992^{+0.001+0.030}_{-0.007-0.008}$	0.02σ
	SSI_{Λ}	$0.289^{+0.038+0.054}_{-0.017-0.070}$	$72.55^{+1.11+1.70}_{-0.88-1.91}$	$0.424^{+0.077+0.577}_{-0.424-0.424}$	$0.098^{+0.048+0.171}_{-0.097-0.098}$	$-0.861^{+0.073+0.221}_{-0.139-0.139}$	—
	$SSII_{E0}$	$0.306^{+0.048+0.066}_{-0.025-0.084}$	$71.23^{+1.00+2.01}_{-1.00-1.91}$	$0.572^{+0.209+0.726}_{-0.438-0.572}$	$0.146^{+0.078+0.181}_{-0.143-0.146}$	$-0.791^{+0.141+0.231}_{-0.161-0.209}$	—
	$SSIII_E$	$0.327^{+0.019+0.036}_{-0.015-0.038}$	$72.36^{+0.78+1.31}_{-0.60-1.40}$	$0.220^{+0.056+0.277}_{-0.220-0.220}$	$0.038^{+0.013+0.101}_{-0.036-0.038}$	$-0.941^{+0.024+0.151}_{-0.059-0.059}$	0.74σ
$f(Q)_{PE}$	EM	$0.347^{+0.012+0.024}_{-0.012-0.023}$	$73.78^{+0.18+0.35}_{-0.18-0.36}$	—	$-0.092^{+0.001+0.002}_{-0.001-0.002}$	$-1.132^{+0.004+0.008}_{-0.004-0.008}$	0.69σ
	SSI_{Λ}	$0.336^{+0.013+0.027}_{-0.013-0.025}$	$74.52^{+0.54+1.11}_{-0.54-1.11}$	—	$-0.091^{+0.001+0.002}_{-0.001-0.002}$	$-1.129^{+0.005+0.009}_{-0.005-0.009}$	—
	$SSII_{PE0}$	$0.339^{+0.016+0.032}_{-0.016-0.029}$	$74.37^{+0.67+1.31}_{-0.67-1.31}$	—	$-0.091^{+0.001+0.003}_{-0.001-0.002}$	$-1.130^{+0.005+0.010}_{-0.005-0.011}$	—
	$SSIII_{PE}$	$0.404^{+0.017+0.033}_{-0.017-0.031}$	$73.94^{+0.62+1.21}_{-0.62-1.21}$	—	$-0.094^{+0.001+0.001}_{-0.001-0.001}$	$-1.151^{+0.005+0.010}_{-0.005-0.010}$	0.73σ
$f(Q)_{HT1}$	EM	$0.336^{+0.012+0.023}_{-0.012-0.022}$	$72.65^{+0.24+0.46}_{-0.24-0.47}$	$0.218^{+0.045+0.081}_{-0.038-0.084}$	$0.048^{+0.024+0.056}_{-0.028-0.050}$	$-0.827^{+0.042+0.099}_{-0.050-0.086}$	0.58σ
	SSI_{Λ}	$0.349^{+0.016+0.033}_{-0.016-0.031}$	$72.30^{+1.50+2.41}_{-1.21-2.70}$	$0.171^{+0.291+0.329}_{-0.097-0.431}$	$0.070^{+0.081+0.311}_{-0.171-0.242}$	$-0.777^{+0.131+0.572}_{-0.310-0.421}$	—
	$SSII_{HT10}$	$0.345^{+0.018+0.035}_{-0.018-0.034}$	$71.07^{+1.21+2.71}_{-1.61-2.52}$	$0.319^{+0.181+0.181}_{-0.030-0.318}$	$0.190^{+0.131+0.302}_{-0.201-0.272}$	$-0.558^{+0.231+0.571}_{-0.392-0.491}$	—
	$SSIII_{HT1}$	$0.435^{+0.019+0.037}_{-0.019-0.036}$	$70.96^{+0.96+2.20}_{-1.31-2.01}$	$0.310^{+0.190+0.190}_{-0.035-0.311}$	$0.164^{+0.141+0.261}_{-0.161-0.242}$	$-0.542^{+0.281+0.562}_{-0.362-0.492}$	1.56σ
$f(Q)_{HT2}$	EM	$0.322^{+0.012+0.025}_{-0.012-0.024}$	$73.37^{+0.18+0.35}_{-0.18-0.36}$	$1.624^{+0.016+0.032}_{-0.016-0.032}$	$-0.415^{+0.023+0.041}_{-0.021-0.045}$	$-0.988^{+0.015+0.030}_{-0.015-0.030}$	0.23σ
	SSI_{Λ}	$0.265^{+0.044+0.084}_{-0.054-0.081}$	$73.06^{+0.63+1.41}_{-0.71-1.30}$	$1.778^{+0.161+0.168}_{-0.051-0.193}$	$-0.238^{+0.170+0.201}_{-0.078-0.231}$	$-0.839^{+0.140+0.171}_{-0.073-0.202}$	—
	$SSII_{HT20}$	$0.327^{+0.048+0.064}_{-0.021-0.085}$	$73.86^{+0.49+0.94}_{-0.49-0.99}$	$1.629^{+0.029+0.191}_{-0.129-0.129}$	$-0.415^{+0.069+0.231}_{-0.161-0.190}$	$-0.985^{+0.038+0.190}_{-0.121-0.132}$	—
	$SSIII_{HT2}$	$0.398^{+0.002+0.002}_{-0.001-0.004}$	$84.74^{+0.33+0.65}_{-0.33-0.63}$	$1.944^{+0.002+0.002}_{-0.001-0.004}$	$-0.031^{+0.002+0.003}_{-0.001-0.001}$	$-0.483^{+0.004+0.005}_{-0.001-0.009}$	12.56σ

f(Q)_{HT2} : As Big Bang Nucleosynthesis (BBN) give the constraint $b < 1.946$ [123]² we set a prior $1.500 < b < 1.946$ for this model. At high redshifts, the w_{eff} would approach to the quintessence side, while the δ tends to 0 from positive side.

Meanwhile, under the constraining of existing data, the hyperbolic tangent ($f(Q)_{HT}$) model is “punished” by Akaike information criterion (AIC) and Bayesian information criterion (BIC) [65,66] which worth further study.

5 The simulated standard siren data

And, we summary the used data as below:

² Another BBN constraints for the $f(Q)_{HT2}$ model are satisfied $b \lesssim 1.88$ [29].

EM: The observational data of PantheonPlus, CC and DESI are combined as EM compilation which has 1745 data. Its best fitted values is used as baseline parameter values in standard siren simulations. And we list the constraining results of EM in Table 1.

SSI_Λ: The SSI_{Λ} simulation used in Λ CDM and $f(Q)$ models is based on the Λ CDM model with $\Omega_{m0} = 0.322$ and $H_0 = 73.23$. The “SS” is the abbreviation of standard siren. And the subscript “ Λ ” denotes the Λ CDM fiducial model. This simulation is mainly used for testing model effect in $f(Q)$ models.

SSII: As Table 1 shows, we use $f(Q)_P$ model with $\Omega_{m0} = 0.327$, $H_0 = 73.35$ and $b = -0.082$ for the $SSII_{P0}$ simulation; use $f(Q)_E$ model with $\Omega_{m0} = 0.322$, $H_0 = 73.19$ and $b = 0.106$ for the $SSII_{E0}$ simulation; use $f(Q)_{PE}$ model with $\Omega_{m0} = 0.347$ and $H_0 = 73.78$ for the $SSII_{PE0}$ simulation; use $f(Q)_{HT1}$ model with $\Omega_{m0} = 0.336$, $H_0 = 72.65$ and $b = 0.218$ for the $SSII_{HT10}$ simulation; and use

Table 2 The χ^2 , AIC and BIC values of the PantheonPlus, CC, DESI, EM and SSI_{Λ} data for Λ CDM model and $f(Q)$ cosmologies. Because SSII and SSIII are simulated based on different $f(Q)$ models, we could

EM	Model	$\chi^2_{PantheonPlus}$	χ^2_{CC}	χ^2_{DESI}	χ^2_{EM}	AIC_{EM}	BIC_{EM}	ΔAIC_{EM}	ΔBIC_{EM}
Λ CDM	Λ CDM	1758.9	17.6	22.6	1799.1	1803.1	1814.0	0	0
	$f(Q)_P$	1761.6	17.5	20.3	1799.4	1805.4	1821.8	2.3	7.8
	$f(Q)_E$	1758.3	17.6	24.6	1800.5	1806.5	1822.9	3.4	8.9
	$f(Q)_{PE}$	1774.6	17.9	29.2	1821.7	1825.7	1836.6	22.6	22.6
	$f(Q)_{HT1}$	1750.4	17.3	14.6	1782.3	1788.3	1804.7	-14.8	-9.3
	$f(Q)_{HT2}$	1763.7	17.2	22.5	1803.4	1809.4	1825.8	6.3	11.8
	Model	—	—	—	$\chi^2_{SSI_{\Lambda}}$	$AIC_{SSI_{\Lambda}}$	$BIC_{SSI_{\Lambda}}$	$\Delta AIC_{SSI_{\Lambda}}$	$\Delta BIC_{SSI_{\Lambda}}$
SSI_{Λ}	Λ CDM	—	—	—	983.9	987.9	997.7	0	0
	$f(Q)_P$	—	—	—	984.1	990.1	1004.8	2.2	7.1
	$f(Q)_E$	—	—	—	984.3	990.3	1005.0	2.4	7.3
	$f(Q)_{PE}$	—	—	—	986.9	990.9	1000.7	3.0	3.0
	$f(Q)_{HT1}$	—	—	—	985.1	991.1	1005.8	3.2	8.1
	$f(Q)_{HT2}$	—	—	—	984.2	990.2	1004.9	2.3	7.2

$f(Q)_{HT2}$ model with $\Omega_{m0} = 0.322$, $H_0 = 73.37$ and $b = 1.624$ for the $SSII_{HT20}$ simulation. Especially, $\delta = 0$ is assumed for all the SSII simulations. Physically, the SSII simulations do not correspond to any true data. This simulation is used to denote the model and extra friction effects by comparing with SSI_{Λ} and SSIII. The subscripts “P”, “E”, “PE”, “HT1” and “HT2” denotes the simulated model. And the subscript “0” denotes $\delta = 0$.

SSIII: The model parameter values are the same as SSII except that we use $\delta_0 = -0.024$, $\delta_0 = 0.005$, $\delta_0 = -0.092$, $\delta_0 = 0.048$ and $\delta_0 = -0.415$ which are the EM constraining results listed in Table 1 for the $SSIII_P$, $SSIII_E$, $SSIII_{PE}$, $SSIII_{HT1}$ and $SSIII_{HT2}$ simulations separately.

Here, the EM and SSI_{Λ} data will be applied to all the models. While as SSII and SSIII simulations are based on different $f(Q)$ model, they will be applied to their fiducial model. To compare the real and simulated data we will plot the evolution of luminosity distances. And, we will use the dashed/dotted lines for SSI_{Λ} /SSII in $f(Q)$ cosmologies. While we use the solid lines for all the EM data, for the simulated data SSI_{Λ} in Λ CDM model and for the simulated data SSIII in $f(Q)$ models.

And to determine the most suitable model according to data, we will make a comparison for the EM and SSI_{Λ} results by using the minimum of χ^2 value which stands for the best fit. However, a higher number of parameters can artificially improve the fit, leading to a smaller χ^2 , making it unreliable for model comparison. To address this issue, we employ the Akaike information criterion (AIC) where $AIC = \chi^2 + 2N$ with N as the number of free parameter

not compare their χ^2 s (AICs, BICs). Then, the χ^2 s (AICs, BICs) of SSII and SSIII are not listed here

[124] and the Bayesian information criterion (BIC) where $BIC = \chi^2 + N \ln m$ with m as the number of data points used in the fit [125]. Here, the χ^2_{SSS} s should be around the number of data 1000. And, for Gaussian errors, the difference between two models could be written as $\Delta AIC = \Delta \chi^2 + 2\Delta N$. Similar to the AIC, the difference denoted by BIC has the form $\Delta BIC = \Delta \chi^2 + \Delta N \ln m$. The $\Delta AIC = 5(\Delta BIC \geq 2)$ and $\Delta AIC = 10(\Delta BIC \geq 6)$ are considered to be the positive and strong evidence against the weaker model.

6 Results and discussion

To see the precision values, the constraining best fitted parameter values with 1σ and 2σ standard errors are presented in Table 1. We also list the χ^2 , AIC and BIC results in Table 2 as supplements. The main results of Λ CDM and $f(Q)$ cosmologies are presented in Figs. 1, 2, 3, 4, 5 and 6 respectively. Explicitly, the data comparisons, the parameter probability density functions (pdfs) with its 1σ and 2σ confidence regions, and the evolutions of δ and w_{eff} within 1σ confidence intervals are plotted here.

As the data comparison figures shows, the evolution values of D_L^{SS} in $f(Q)_E$ model are un-distinguished. And the evolution values of $D_L^{SSI_{\Lambda}}$ are un-distinguished with that of D_L^{SSII} in the $f(Q)_P$ and $f(Q)_{HT2}$ models. Meanwhile, the evolution values of $D_L^{SSII_{\Lambda}}$ are slightly smaller than that of D_L^{SSII} in the $f(Q)_{HT1}$ model, but slightly larger than that of D_L^{SSII} in the $f(Q)_{PE}$ model. And the evolution values of D_L related to SSIII simulation are smaller than that of SSI_{Λ} and SSII simulations. Explicitly, for SSII simulations, $D_L^{SSII_{PE}} < D_L^{SSII_P} \simeq D_L^{SSII_E} \simeq D_L^{SSII_{HT2}} \simeq D_L^{SSI_{\Lambda}} < D_L^{SSI_{HT1}}$; for

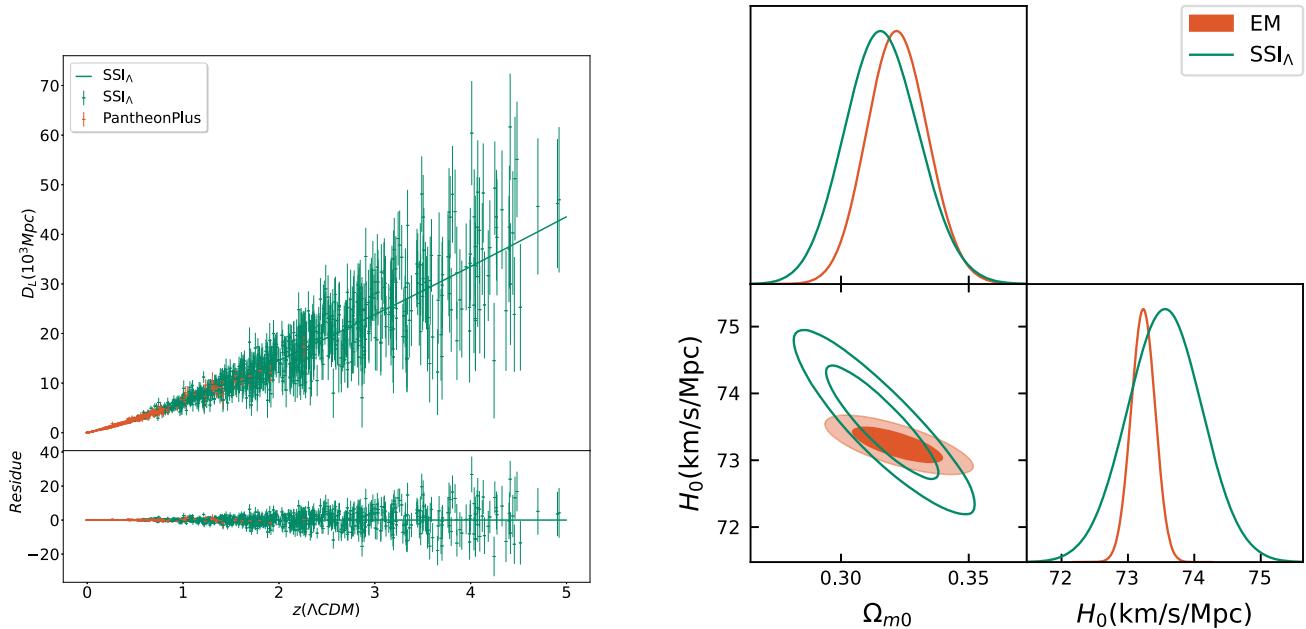


Fig. 1 In the left panel, the D_L s of real and simulated data are compared. The red one with error-bar are from the PantheonPlus combination; the green one with error-bar are from simulated SSI_{Λ} data; and the green line denotes the evolution of D_L following the assumption of

SSI_{Λ} data which are based on Λ CDM model with its best fitted EM values ($\Omega_{m0} = 0.322$ and $H_0 = 73.23$). In the right panel, the probability density functions (pdfs) with its 1σ and 2σ confidence regions for the parameters of Λ CDM model (Ω_{m0} and H_0) are shown

SSIII simulations, $D_L^{SSIII_{HT2}} \ll D_L^{SSIII_{HT1}} \simeq D_L^{SSIII_{PE}} < D_L^{SSIII_P} < D_L^{SSIII_E} \simeq D_L^{SSI_{\Lambda}}$.

And as the triangle plots show, the contours of $f(Q)_E$ model are not closed because of the prior $b > 0$. While in the other models, the contours are smoothly closed and the probability density functions (pdfs) are Gaussian-distributed. Roughly, for all the models, the tightest constraints come from the EM data. Especially, for the Λ CDM, $f(Q)_E$ and $f(Q)_{HT2}$ models, Ω_{m0} is around 0.322 with 0.024 (0.045) as 1σ (2σ) error range. In the other models, the error ranges are similar but the best fitted Ω_{m0} s slightly shift. Furthermore, the error ranges of H_0 in EM constraints are similar. In another saying, all the simulations are based on similar Ω_{m0} and H_0 . The Hubble tension could be alleviated to 0.07σ level under the constraint of EM in Λ CDM model. Then, it is not surprising that the Hubble tension could be alleviated in other constraints except that of $SSIII_{HT2}$. Surprisingly, the simulation of $SSIII_{HT2}$ is problematical where the Ω_{m0} is very closed to its upper limit with 12.56σ Hubble tension.

In this discussion, the Λ CDM model is used as baseline. As Fig. 1 shows, the $\Omega_{m0} - H_0$ contour of EM is much smaller than that of the SSI_{Λ} data in the Λ CDM model. The direction of $\Omega_{m0} - H_0$ contour is changed slightly as well. In the next, we discuss all the $f(Q)$ models one by one.

6.1 Discussion on the $f(Q)_P$ model

As Table 1 shows, by comparing the results from SSI_{Λ} and $SSII_{P0}$, the model effect brings a large shift of the best fitted value of Ω_{m0} which is $\Delta\Omega_{m0} = 0.049$ in $f(Q)_P$ model. And by comparing $SSII_{P0}$ and $SSIII_P$ constraints, the extra friction term brings shift of the best fitted value of Ω_{m0} as $\Delta\Omega_{m0} = 0.053$ which is 221% of the 1σ regime of EM constraint ($\Delta\Omega_{m0}^{1\sigma} = 0.024$). The model effects are comparable with the extra friction term effects in the $f(Q)_P$ model.

As Fig. 2 shows, the constraining tendencies of SS related data are similar. And their contours are overlapped with the EM ones in 2σ ranges. Especially, the Ω_{m0} related to $SSIII_P$ is as large as $0.383^{+0.028+0.047}_{-0.018-0.053}$. The correlation of $\Omega_{m0} - H_0$ contour is negative in the EM constraint while it is positive in the SS related constraints. This phenomenon could help to break the degeneration between parameters.

Furthermore, the $b = 0$, $\delta = 0$ and $w_{\text{eff}} = -1$ are not included in 1σ range of EM and $SSIII_P$ data for $f(Q)_P$ model. In another saying, the $f(Q)_P$ model could be distinguished from Λ CDM in 1σ ranges. And at high z , the deviations from Λ CDM model become evident if $w_{\text{eff}} \neq -1$, while most δ s tends to 0 which correspond to a flat w_{eff} .

As Table 2 shows, the χ^2_{CC} and χ^2_{DESI} of $f(Q)_P$ model are smaller than that of Λ CDM model. While, the $\chi^2_{PantheonPlus}$ of $f(Q)_P$ model is larger than that of Λ CDM model. After EM data combination, $\Delta AIC_{EM} = 2.3$, the $f(Q)_P$

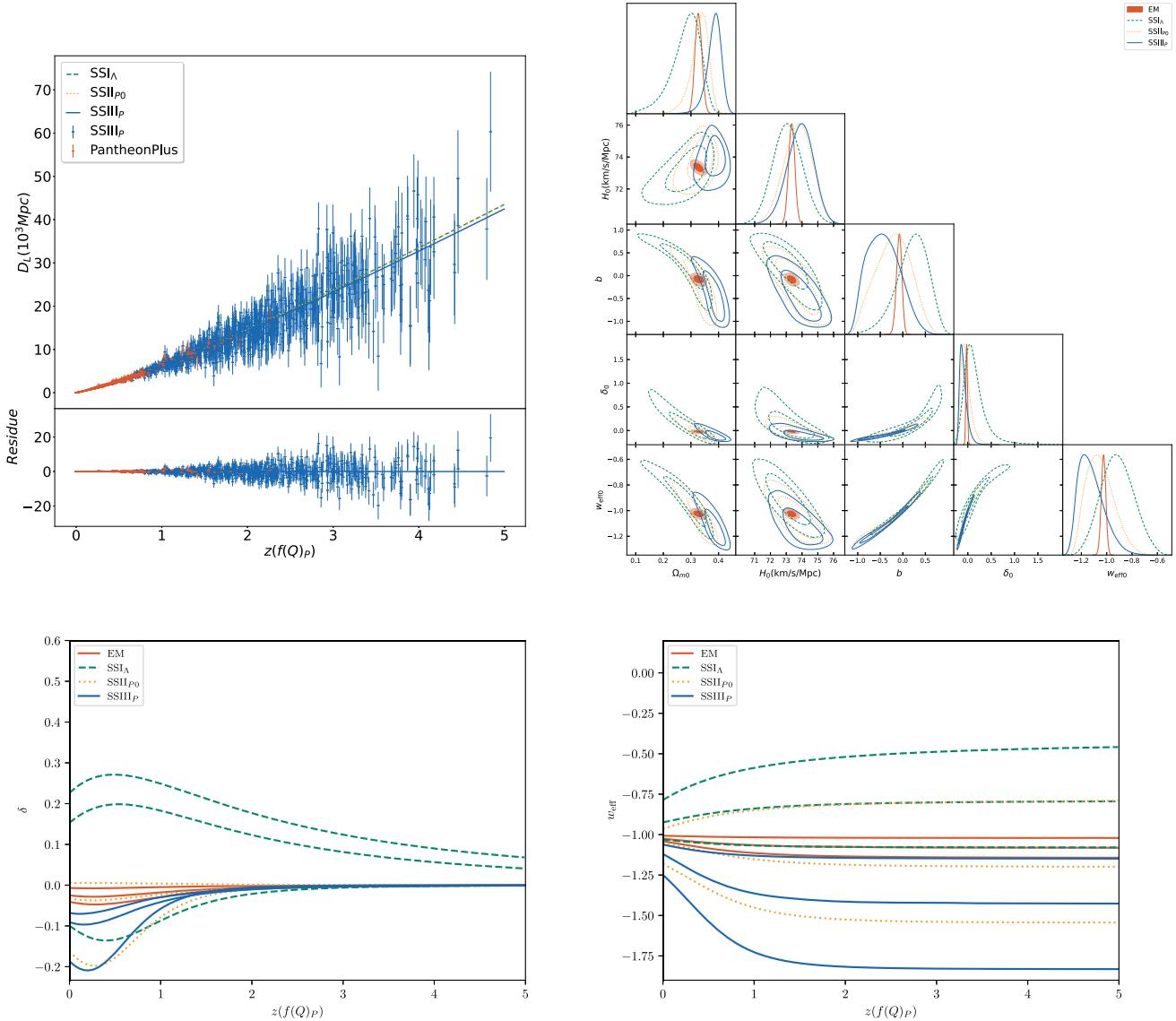


Fig. 2 The two upper panels are similar as Fig. 1. In the upper left panel, the D_L s of real and simulated data are compared. The red one with error-bar denote the PantheonPlus data as Fig. 1. And we change the green solid line for SSI_{Λ} in Fig. 1 to the dashed green one for comparison. Additionally, the orange line denotes the simulated $SSII_{P0}$ data with the best fitted EM values of $f(Q)_P$ model ($\Omega_{m0} = 0.327$, $H_0 = 73.35$, $b = -0.082$ and $\delta_0 = 0$). The blue ones with error-bar

are $SSIII_P$ data which has the same baseline values of Ω_{m0} , H_0 and b as $SSII_{P0}$ simulation, but has a non-zero friction term ($\delta_0 = -0.024$). In the upper right panel, the probability density functions with its 1σ and 2σ confidence regions for the parameters of $f(Q)_P$ model (Ω_{m0} , H_0 , b , δ_0 and $w_{\text{eff}0}$) are shown. And in the two bottom panels, the evolutions of δ and w_{eff} within 1σ confidence intervals for the $f(Q)_P$ model under the constraints of EM and SS related data are shown

model could be regarded as equal as Λ CDM model. But $\Delta BIC_{EM} = 7.8$ denotes the $f(Q)_P$ model is “punished” by the EM data. And in the SSI_{Λ} constraint, $\Delta AIC_{SSI_{\Lambda}} = 2.2$ and $\Delta BIC_{SSI_{\Lambda}} = 7.1$ which are similar to the EM case.

6.2 Discussion on the $f(Q)_E$ model

As Table 1 shows, by comparing the results from SSI_{Λ} and $SSII_{E0}$, the model effect brings a shift of the best fitted value of Ω_{m0} which is $\Delta\Omega_{m0} = 0.017$ in $f(Q)_E$ model. While

the extra friction term brings shift of the best fitted value of Ω_{m0} as $\Delta\Omega_{m0} = 0.021$ by comparing $SSII_{E0}$ and $SSIII_E$. The Ω_{m0} shifts of $f(Q)_E$ model are comparable in the model and extra friction effects.

As the $f(Q)_P$ model, the shapes of SS related data are similar and the smallest contours are still from the $SSIII_E$ simulation. Comparing with the EM and SS related data, the direction of $\Omega_{m0} - H_0$ contour is changed. Specially, the Hubble tension from EM constraint is as small as 0.02σ which is smallest among all constraints.

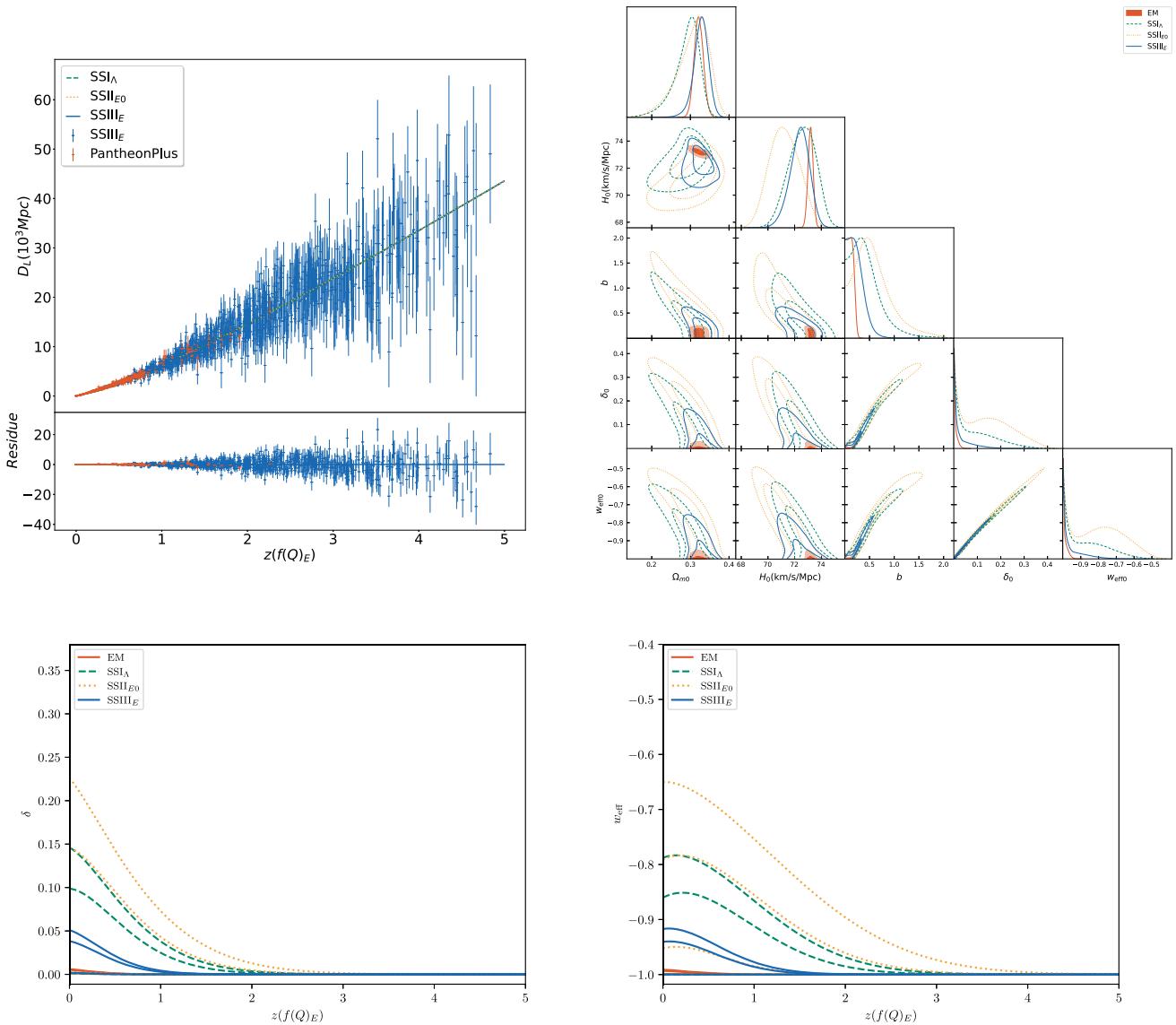


Fig. 3 The fiducial values in the $f(Q)_E$ simulations are $\Omega_{m0} = 0.322$, $H_0 = 73.19$ and $b = 0.106$ with $\delta_0 = 0$ for $SSII_{E0}$ and $\delta_0 = 0.005$ for $SSIII_E$. The others are the same as Fig. 2

And we obtain a positive δ_0 and a quintessence-like w_{eff} in all $f(Q)_E$ constraints. The evolutions of w_{eff} and δ , which are quite flat, almost follow Λ CDM model at $z > 2$. In the other saying, the deviations from Λ CDM model become ignored at high z .

In contrast to the $f(Q)_P$ model, the χ^2_{DESI} of $f(Q)_E$ model is larger than that of Λ CDM model, while the χ^2_{CC} is equal to that of Λ CDM model. And the $\chi^2_{PantheonPlus}$ of $f(Q)_E$ model is smaller than that of Λ CDM model. Finally after combination of EM data, the information criterion gives similar results to $f(Q)_E$ model as $f(Q)_P$ model. Explicitly, $\Delta AIC_{EM} = 3.4$ shows the $f(Q)_E$ model could be regarded as equal as Λ CDM model. And $\Delta BIC_{EM} = 8.9$ which denotes the $f(Q)_E$ model is “punished” by the EM

data. Furthermore in the SSI_A constraint, $\Delta AIC_{SSI_A} = 2.4$ and $\Delta BIC_{SSI_A} = 7.3$ which are similar to the EM case as well.

6.3 Short summary on the Λ CDM-like models

Generally, both the $f(Q)_P$ and $f(Q)_E$ models give out small Hubble tensions as Λ CDM model. And the 1σ (2σ) constraining regions of all the SS related data of $f(Q)_P$ ($f(Q)_E$) model are much larger than that of EM data. Especially, the best fitted Ω_{m0} of SSI_A are much smaller than that of other data while the plots of D_L are un-distinguishable. This phenomenon hints the model effect, which is comparable with

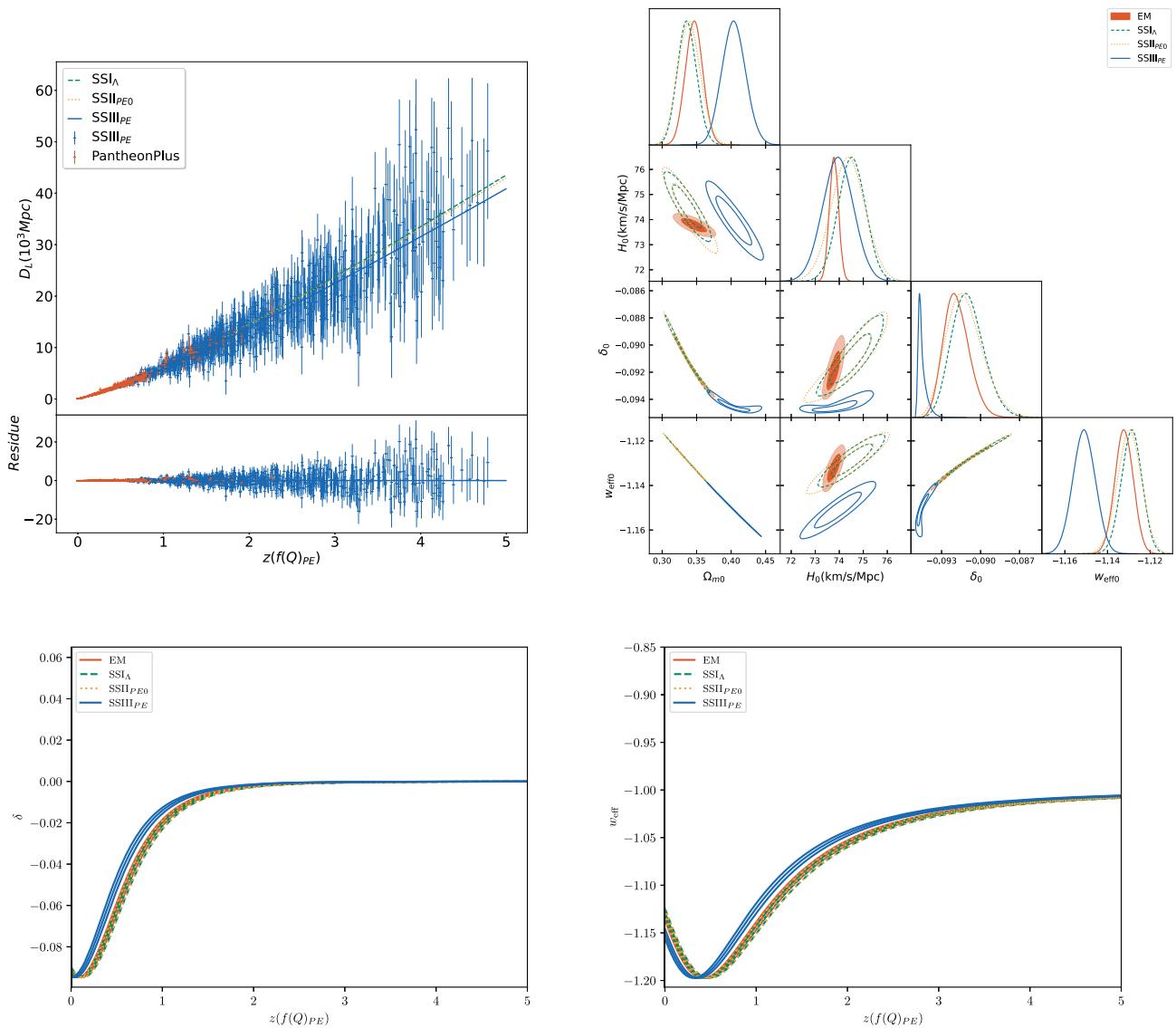


Fig. 4 The fiducial values in the $f(Q)_{PE}$ simulations are $\Omega_{m0} = 0.347$ and $H_0 = 73.78$ with $\delta_0 = 0$ for $SSII_{PE0}$ and $\delta_0 = -0.092$ for $SSIII_{PE}$. The others are the same as Fig. 2

the friction term, could not be ignored in the Λ CDM-like model simulations.

In the EM and SSI_{Λ} data, the $f(Q)_P$ and $f(Q)_E$ models are favored by AIC, but “punished” by BIC. Theoretically, the b parameter may effect cosmic perturbations giving an intriguing division between background and perturbation behavior in terms of model parameters [24, 126, 127]. In the future, the growth factor data which are derived from matter perturbations could be used to give out more information.

6.4 Discussion on the $f(Q)_{PE}$ model

The $f(Q)_{PE}$ model has the same free parameters with Λ CDM model. As Fig. 4 shows, the H_0 related contours derived from the SS related data are parallel. The SSI_{Λ}

and $SSII_{PE0}$ related contours are closed to each other. And they have overlapped with the EM contours whose directions are changed. While the $SSIII_{PE}$ related contours are separated with the other data because of its Ω_{m0} as large as $0.404^{+0.017+0.033}_{-0.017-0.031}$. The shapes of contours $\Omega_{m0} - w_{eff0}$, $\Omega_{m0} - \delta_0$ and $\delta_0 - w_{eff0}$ are narrow. By comparing the results from SSI_{Λ} and $SSII_{PE0}$, the model effect brings a slight shift of the best fitted value of Ω_{m0} which is $\Delta\Omega_{m0} = 0.003$. While the extra friction term brings a much larger shift of the best fitted value of Ω_{m0} which is $\Delta\Omega_{m0} = 0.065$.

In the $f(Q)_{PE}$ model, as Table 1 and Fig. 4 show, the parameter δ_0 is always smaller than 0 in 2σ ranges. The best fitted values of δ and their 2σ regimes are around -0.092 for all the data. Precisely speaking, the $\delta_0 = 0$ is excluded in all the constraints of the $f(Q)_{PE}$ model where the extra

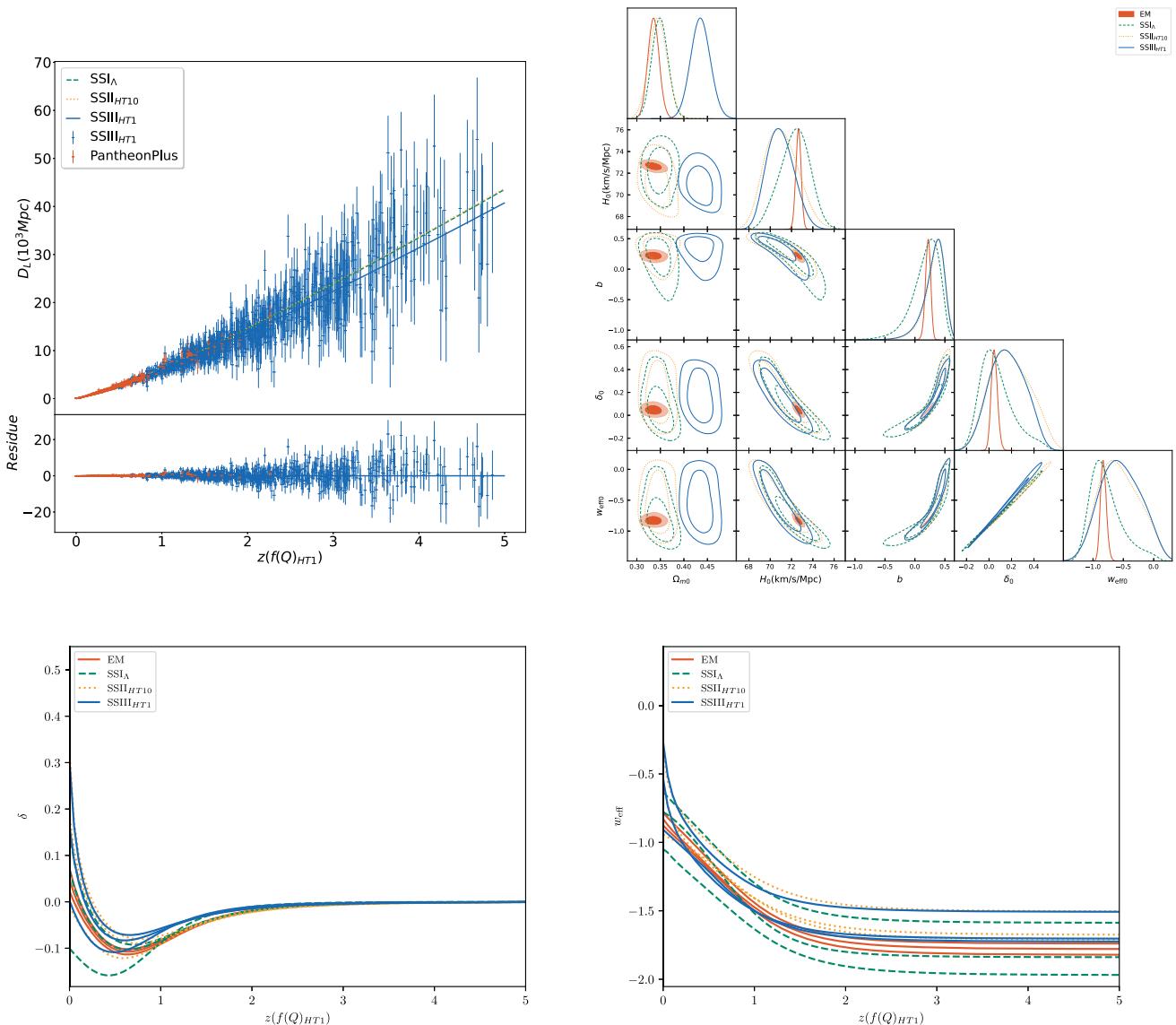


Fig. 5 The fiducial values in the $f(Q)_{HT1}$ simulations are $\Omega_{m0} = 0.336$, $H_0 = 72.65$ and $b = 0.218$ with $\delta_0 = 0$ for $SSII_{HT10}$ and $\delta_0 = 0.048$ for $SSIII_{HT1}$. The others are the same as Fig. 2

friction term plays an important role in the simulations. All the constraint results exclude $w_{\text{eff}0} = -1$ in 2σ regimes as well which means this model could be distinguished from the Λ CDM model. As z increasing, the shapes of evolution of w_{eff} and δ are similar which corresponds to the narrow positive correlation between w_{eff} and δ . The value of w_{eff} gradually approaches -1 , which mimics the standard Λ CDM model while it is still in the phantom range which did not cross $w_{\text{eff}0} = -1$. And, δ gradually tends to 0 which corresponds to Λ CDM model as well. These results are consistent with the analytic calculations in Eqs. (32) and (33).

As Table 2 shows, the χ^2_{CC} and χ^2_{DESI} of $f(Q)_{PE}$ model are larger than that of Λ CDM model. In the contrast, the difference between the $\chi^2_{PantheonPlus}$ of $f(Q)_{PE}$ model and

that of Λ CDM model is as large as 15.7. After combination of EM data, $\Delta AIC_{EM} = 22.6$ and $\Delta BIC_{EM} = 22.6$ which denote the $f(Q)_{PE}$ model could be excluded. And in the SSI_A constraint, $\Delta AIC_{SSI_A} = 3.0$ and $\Delta BIC_{SSI_A} = 3.0$ are opposite to the EM constraint.

6.5 Discussion on the $f(Q)_{HT1}$ model

As Fig. 5 shows, the parallel contours in $f(Q)_{HT1}$ model are the Ω_{m0} related ones which are derived from the SS related data. Both the SSI_A and $SSII_{HT10}$ related contours have overlapped with the EM contours where the directions are changed. The shapes of contours $\delta_0 - w_{\text{eff}0}$ are narrow for all the constraints as well.

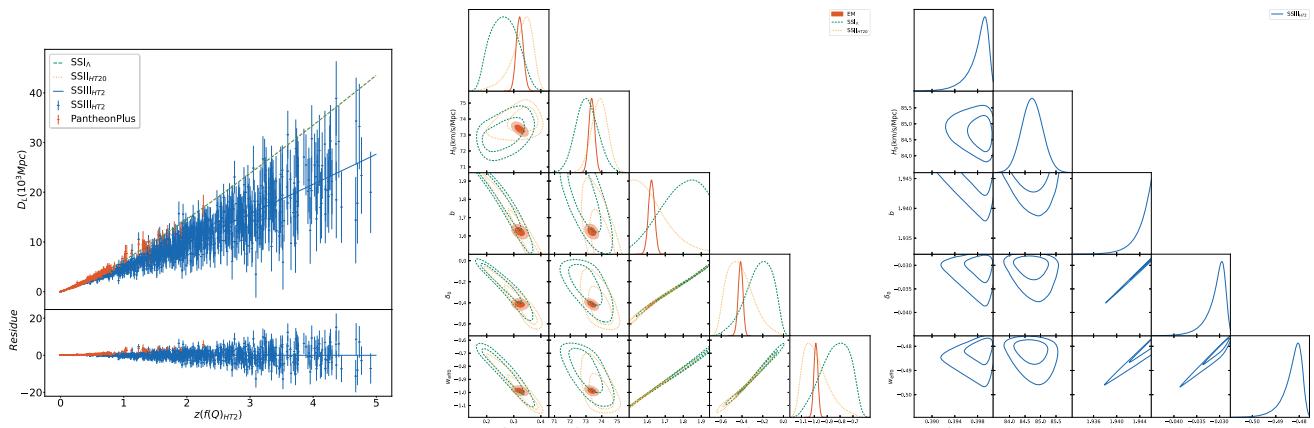


Fig. 6 Here, as the constraining result of Ω_{m0} of SSIII_{HT2} is very close to its upper prior, we assume $\Omega_{m0} < 0.4$ for SSIII_{HT2} and list the results at the right panel. The others are the same as Fig. 2, except that

the fiducial values in the $f(Q)_{HT2}$ SS simulations are $\Omega_{m0} = 0.322$, $H_0 = 73.37$ and $b = 1.624$ with $\delta_0 = 0$ for SSII_{HT20} and $\delta_0 = -0.415$ for SSIII_{HT2}

Surprisingly, in the constraining results of SSIII_{HT1} data, the best fitted result of Ω_{m0} reaches as large as $0.435^{+0.019+0.037}_{-0.019-0.037}$ which are out of all the existing reasonable constraints. There is a tension between the EM and SSIII_{HT1} data. And by comparing the results from SSI_A and SSII_{HT10}, the model effect brings a slight shift of the best fitted value of Ω_{m0} which is $\Delta\Omega_{m0} = 0.004$ in $f(Q)_{HT1}$ model. While the extra friction term brings a rather large shift of the best fitted value of Ω_{m0} which is $\Delta\Omega_{m0} = 0.090$ by comparing SSII_{HT10} and SSIII_{HT1} constraints.

The $\delta = 0$ and $w_{\text{eff}0} = -1$ are excluded in 1σ confidence interval in the EM and SSIII_{HT1} results, while it is included in the SSI_A related results. The evolutions of δ cross 0 and the evolutions of w_{eff} cross -1 in most 1σ intervals. And with increasing of z , δ s gradually approach 0, w_{eff} s gradually deviate from -1 . These results are consistent with the analytic calculations in Eqs. (36) and (37).

As Table 2 shows, the $\chi^2_{\text{PantheonPlus}}$, χ^2_{CC} and χ^2_{DESI} of $f(Q)_{HT1}$ model are smaller than that of Λ CDM model. As a result, $\Delta AIC_{EM} = -14.8$ and $\Delta BIC_{EM} = -9.3$ which denote the $f(Q)_{HT1}$ model is favored by the EM data. And in the SSI_A constraint, $\Delta AIC_{SSI_A} = 3.2$ which denotes the $f(Q)_{HT1}$ model is favored and $\Delta BIC_{SSI_A} = 8.1$ which denotes the $f(Q)_{HT1}$ model is “punished”. Considering the large value of Ω_{m0} in the SSIII_{HT1} constraint, the future standard siren data may rule out the $f(Q)_{HT1}$ model.

6.6 Discussion on the $f(Q)_{HT2}$ model

As Fig. 6 shows, the evolution values of D_L related to SSIII_{HT2} simulation are much smaller than that of SSII_{HT20} and SSI_A simulation. And compared with the PantheonPlus data, the simulated SSIII_{HT2} data, which are based on a large negative $\delta_0 = -0.415$, have much smaller D_L values.

The EM constraint is the tightest one as well. But when the SSIII_{HT2} data is used, it could not give out an effective Ω_{m0} even after set a prior $0 < \Omega_{m0} < 0.4$ as Table 1 and Fig. 6 show. Explicitly, the constraining results of SSIII_{HT2} is $\Omega_{m0} = 0.398^{+0.002+0.002}_{-0.001-0.004}$ under the prior $0 < \Omega_{m0} < 0.4$ with $\chi^2_{SSIII_{HT2}} = 1197.2$. Because of such poor fitting results, we do not plot the evolutions of δ and w_{eff} for this model.

As Table 2 shows, the χ^2_{CC} and χ^2_{DESI} of $f(Q)_{HT2}$ model are smaller than that of Λ CDM model. While, the $\chi^2_{\text{PantheonPlus}}$ of $f(Q)_{HT2}$ model is larger than that of Λ CDM model. Furthermore, $\Delta AIC_{EM} = 6.3$ and $\Delta BIC_{EM} = 11.8$ which denotes the $f(Q)_{HT2}$ model is excluded by the EM data. And, based on SSIII_{HT2} simulation and constraint, we conclude that the $f(Q)_{HT2}$ model will be ruled out by the future standard siren observational data as well.

6.7 Short summary on the non Λ CDM-like models

Here, the model effects are much smaller than the extra friction effects in both models. While the two effects are comparable in the Λ CDM-like models, e.g. the model effect of $f(Q)_{PE}$ model ($\Delta\Omega_{m0} = 0.003$) is 12.5% of the 1σ range of EM constrained Ω_{m0} ($\Delta\Omega_{m0}^{1\sigma} = 0.024$). As for the extra friction effect of $f(Q)_{PE}$ model ($\Delta\Omega_{m0} = 0.065$), it is 270.8% of the 1σ range of EM constrained Ω_{m0} . As the errors caused by model effect could be at the level of 10%, it should not be ignored.

And, the Hubble tensions in the non Λ CDM-like models are slightly larger than that in the Λ CDM-like models. Anyway, compared with the D_L s derived from $f(Q)_P$ and $f(Q)_E$ models, the ones related to the simulated SSIII data of non Λ CDM-like model are smaller. Especially, $D_L^{SSIII_{HT2}}$ s

seem to be smaller than the PantheonPlus data. All the Ω_{m0}^{SSIII} for the non Λ CDM-like models are larger than 0.370 in 2σ ranges which are out of most existing constraints. The $f(Q)_{PE}$ model could be ruled out by the EM data, and both the $f(Q)_{HT}$ models will be excluded by the future standard siren data.

7 Conclusion

To study the model and extra frictional effects in standard siren simulation, we simulated standard siren data based on Λ CDM (SSII $_{\Lambda}$), based on the $f(Q)$ models with $\delta = 0$ (SSII) and based on the true $f(Q)$ models with $\delta \neq 0$ (SSIII) by using the real EM observational data as baseline. And two Λ CDM-like models ($f(Q)_P$ and $f(Q)_E$) and two non Λ CDM-like models ($f(Q)_{PE}$ and $f(Q)_{HT}$) are chosen to constrain. The evolution values of D_L related to SSII and SSIII simulation are $D_L^{SSII_{PE}} < D_L^{SSII_P} \simeq D_L^{SSII_E} \simeq D_L^{SSII_{HT2}} \simeq D_L^{SSII_{\Lambda}} < D_L^{SSII_{HT1}}$ and $D_L^{SSII_{HT2}} \ll D_L^{SSII_{HT1}} \simeq D_L^{SSII_{PE}} < D_L^{SSII_P} < D_L^{SSII_E} \simeq D_L^{SSII_{\Lambda}}$.

And the tightest constraints are from the EM data in all $f(Q)$ cosmologies. The model effects are smaller than the extra friction effects in non Λ CDM-like models. While they are comparable in Λ CDM-like models. Both effects play important roles in standard siren simulation and could not be ignored. By comparing the constraining results, especially the χ^2 , AIC and BIC, the $f(Q)_P$ and $f(Q)_E$ models need more observational data (e.g. growth factor) to further study; the $f(Q)_{PE}$ model could be ruled out by the EM data; and both the $f(Q)_{HT}$ models will be excluded by the future standard siren data.

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Code Availability Statement This manuscript has no associated code/software. [Author's comment: In this theoretical study, all analyses and derivations were performed using established theoretical frameworks and analytical methods.]

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