

A MEASUREMENT OF  $\alpha_s(Q^2)$  FROM THE GROSS LLEWELLYN SMITH SUM RULE

D. A. Harris<sup>8</sup>, C. G. Arroyo<sup>4</sup>, P. Auchincloss<sup>8</sup>, P. de Barbaro<sup>8</sup>, A. O. Bazarko<sup>4</sup>,  
 R. H. Bernstein<sup>5</sup>, A. Bodek<sup>8</sup>, T. Bolton<sup>6</sup>, H. Budd<sup>8</sup>, J. Conrad<sup>4</sup>, R. B. Drucker<sup>7</sup>,  
 R. A. Johnson<sup>3</sup>, J. H. Kim<sup>4</sup>, B. J. King<sup>4</sup>, T. Kinnel<sup>9</sup>, G. Koizumi<sup>5</sup>, S. Koutsoliotas<sup>4</sup>,  
 M. J. Lamm<sup>5</sup>, W. C. Lefmann<sup>1</sup>, W. Marsh<sup>5</sup>, K. S. McFarland<sup>5</sup>, C. McNulty<sup>4</sup>, S. R. Mishra<sup>4</sup>,  
 D. Naples<sup>5</sup>, P. Nienaber<sup>10</sup>, M. Nussbaum<sup>3</sup>, M. J. Oreglia<sup>2</sup>, L. Perera<sup>3</sup>, P. Z. Quintas<sup>4</sup>,  
 A. Romosan<sup>4</sup>, W. K. Sakumoto<sup>8</sup>, B. A. Schumm<sup>2</sup>, F. J. Sciulli<sup>4</sup>, W. G. Seligman<sup>4</sup>,  
 M. H. Shaevitz<sup>4</sup>, W. H. Smith<sup>9</sup>, P. Spentzouris<sup>4</sup>, R. Steiner<sup>1</sup>, E. G. Stern<sup>4</sup>, M. Vakili<sup>3</sup>,  
 U. K. Yang<sup>8</sup>

<sup>1</sup> Adelphi University, Garden City, NY 11530

<sup>2</sup> University of Chicago, Chicago, IL 60637

<sup>3</sup> University of Cincinnati, Cincinnati, OH 45221

<sup>4</sup> Columbia University, New York, NY 10027

<sup>5</sup> Fermi National Accelerator Laboratory, Batavia, IL 60510

<sup>6</sup> Kansas State University, Manhattan, KS 66506

<sup>7</sup> University of Oregon, Eugene, OR 97403

<sup>8</sup> University of Rochester, Rochester, NY 14627

<sup>9</sup> University of Wisconsin, Madison, WI 53706

<sup>10</sup> Xavier University, Cincinnati, OH 45207

Talk Presented by D.A. Harris

### Abstract

The Gross Llewellyn Smith sum rule has been measured at different values of four-momentum transfer squared ( $Q^2$ ) by combining the precise CCFR neutrino data with data from other deep-inelastic scattering experiments at lower values of  $Q^2$ . A comparison with the  $\mathcal{O}(\alpha_s^3)$  predictions of perturbative QCD yields a determination of  $\alpha_s$  and its dependence on  $Q^2$  in the range  $1 \text{ GeV}^2 < Q^2 < 20 \text{ GeV}^2$ . Low  $Q^2$  tests have greater sensitivity to  $\alpha_s(M_Z^2)$  than high  $Q^2$  tests, since at low  $Q^2$   $\alpha_s$  is large and changing rapidly.

To leading order in perturbative QCD, the structure function  $xF_3$  measured in  $\nu N$  scattering is the difference between the quark and anti-quark momentum distributions. The GLS sum rule predicts that the integral over  $x$  of  $F_3$  is simply 3, the number of valence quarks in a nucleon [1]. There are corrections to the sum rule which introduce a dependence of the GLS integral on  $\alpha_s$ , the strong coupling constant, in the following way [2]:

$$\int_0^1 xF_3(x, Q^2) \frac{dx}{x} = 3\left(1 - \frac{\alpha_s}{\pi} - a(n_f)\left(\frac{\alpha_s}{\pi}\right)^2 - b(n_f)\left(\frac{\alpha_s}{\pi}\right)^3\right) - \Delta HT \quad (1)$$

where  $a$  and  $b$  depend on the number of quark flavors,  $n_f$ , accessible at a given  $x$  and four-momentum transfer squared,  $Q^2$ .  $\Delta HT$  represents a higher twist contribution, which has been estimated using QCD sum rules, a Vector Meson Dominance Model, and a Non-relativistic Quark Model to be  $0.27 \pm 0.14/Q^2(\text{GeV}^2)$  [3]. The  $Q^2$  dependence of  $\alpha_s$  is as follows [4]:

$$\frac{\alpha_s(Q^2)}{4\pi} = \frac{1}{\beta_0(n_f) \ln\left(\frac{Q^2}{\Lambda_{\overline{MS}}^2}\right)} - \frac{\beta_1(n_f)}{\beta_0(n_f)} \frac{\ln\left(\ln\left(\frac{Q^2}{\Lambda_{\overline{MS}}^2}\right)\right)}{\beta_0(n_f)^2 \ln^2\left(\frac{Q^2}{\Lambda_{\overline{MS}}^2}\right)} + \mathcal{O}\left(\frac{1}{\ln^3\left(\frac{Q^2}{\Lambda_{\overline{MS}}^2}\right)}\right). \quad (2)$$

The challenge in evaluating  $\int F_3 dx$  is that for a given  $Q^2$  value, there is a limited  $x$  region that is accessible by any one experiment. The incoming neutrino energy imposes a minimum  $x$  constraint and detector acceptance imposes a maximum  $x$  constraint. CCFR has data at low  $Q^2$  and low  $x$  ( $10^{-2} < x < 10^{-1}$ ), and at high  $Q^2$  and high  $x$  ( $10^{-1} < x < 1$ ). The CCFR detector and the measurement of  $xF_3$  have been described in detail elsewhere [5]. One way to evaluate  $\int F_3 dx$  over all  $x$  is to extrapolate  $xF_3$  from all  $Q^2$  regions to a  $Q_0^2$  value where the data is predominantly at low  $x$ . A previous CCFR analysis found that for  $Q_0^2 = 3 \text{ GeV}^2$ ,  $\int F_3 dx = 2.50 \pm .018(\text{stat}) \pm .078(\text{syst})$  [6]. By using QCD to extrapolate  $xF_3$  to  $Q_0^2$  however, one introduces  $\alpha_s$  *a priori* into the problem. Furthermore, higher twist effects are not included in QCD extrapolations.

The goal of this analysis is to evaluate  $\int F_3 dx$  without introducing any *ad hoc*  $Q^2$  dependence. By combining the CCFR data with that of several other experiments enough data at different energies are obtained to measure  $\int F_3 dx$  without  $Q^2$  extrapolation at values of  $Q^2$  between  $1 \text{ GeV}^2$  and  $20 \text{ GeV}^2$ . The  $xF_3$  measurements from experiments WA59, WA25, SKAT, FNAL-E180 [7], and BEBC-Gargamelle [8] were normalized to the CCFR  $xF_3$  measurements in the  $Q^2$  regions of overlap and then were used along with the CCFR  $xF_3$  data. Furthermore, since at high  $x$  the structure function  $F_2 \approx xF_3$ , one can use  $F_2$  data from  $e^-N$  scattering at SLAC [9] in this region ( $x > 0.5$ ) by normalizing it to the ratio of  $xF_3/F_2$  as measured in the CCFR data. This is particularly important at low  $Q^2$  where there is no  $xF_3$  data at high  $x$ . The published CCFR  $xF_3$  data were modified for new electroweak radiative corrections (Bardin[10]). In addition, the CCFR data were corrected for the contribution from the strange sea [11] of events containing two oppositely charged muons. Finally, by comparing the  $F_2$  values of CCFR to those from SLAC [9], NMC and BCDMS [12], the overall normalization of the CCFR data was determined to be  $1.019 \pm 0.011$ . To integrate over all  $x$ , this analysis sums the binned data for  $x > 0.02$ . For the contribution to the integral at lower  $x$ , the data below  $x = 0.1$  is fit to a power law and then that function is integrated over  $0 < x < 0.02$ . Figure 1a shows the combined  $xF_3$  data and the corrected  $F_2$  data for the four lowest  $Q^2$  bins, as well as the power law fit to the low  $x$  data and the  $\chi^2$  for those fits. To be consistent with theoretical predictions of higher twist effects on the sum rule, the  $\nu$ -nucleon elastic contribution (described in [8]) was added to the integral, and both the elastic and inelastic contributions were corrected for target mass effects [8]. Figure 1b shows  $\int F_3 dx$  as a function of  $Q^2$  and the theoretical prediction (see equations 1 and 2) assuming  $\Lambda_{\overline{MS}}^{(5)} = 150 \text{ MeV}$ .

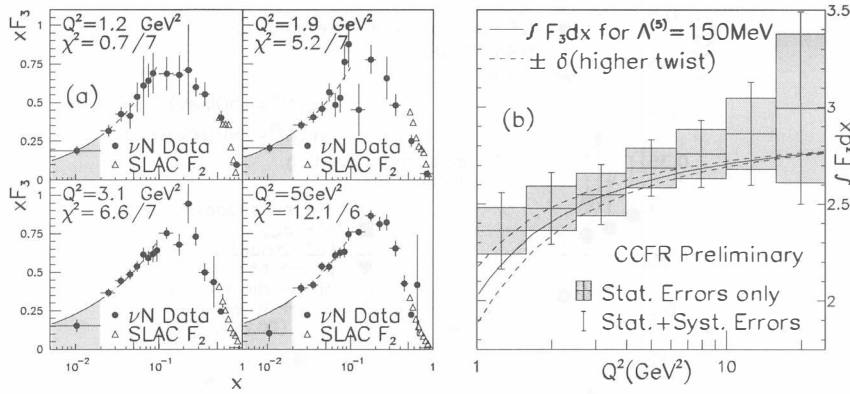


Figure 1: (a)  $xF_3$  vs.  $x$  for four different low  $Q^2$  values. The function shown is a power law fit to data below  $x = 0.1$ . (b)  $\int F_3 dx$  vs.  $Q^2$ , and the theoretical prediction for the integral for  $\Lambda_{\overline{MS}}^{(5)} = 150 \text{ MeV}$ . The dashed lines represent the uncertainty in the higher twist correction.

One can determine  $\alpha_s(Q^2)$  from  $\int F_3 dx$  by using equation 1. The values of  $\alpha_s(Q^2)$  determined by this technique are shown in figure 2. The curves plotted in figure 2 show the evolution of  $\alpha_s$  as a function of  $Q^2$  (see equation 2), for two different values of  $\Lambda_{\overline{MS}}$ . From this plot it is clear that low  $Q^2$  measurements have large potential to constrain  $\alpha_s$  not only because  $\alpha_s$  is large in this kinematic region, but because it is changing rapidly as a function of  $Q^2$ . However, the higher twist uncertainty in  $\int F_3 dx$  is also large in this kinematic region and is the largest single systematic error in this analysis. Evolving the four lowest data points for  $\alpha_s$  to  $M_Z^2$ , we obtain the following value for  $\alpha_s(M_Z^2)$ :

$$\alpha_s(M_Z^2) = 0.108 \pm_{0.005}^{0.003}(\text{stat}) \pm_{0.006}^{0.004}(\text{syst}) \pm_{0.006}^{0.004}(\text{higher twist})$$

For comparison with other low  $Q^2$   $\alpha_s$  measurements, this corresponds to  $\alpha_s(Q^2 = 3.0 \text{ GeV}^2) = 0.26 \pm_{0.03}^{0.02}(\text{stat}) \pm_{0.02}^{0.01}(\text{syst}) \pm_{0.03}^{0.02}(\text{higher twist})$ . Figure 2 puts this result in the context of other measurements by plotting them as a function of  $Q^2$ . In general, the low  $Q^2$  data systematically favor a lower  $\Lambda_{\overline{MS}}$  than do the higher  $Q^2$  data. The result from this analysis is consistent with low energy measurements of  $\alpha_s$ . In particular, it is consistent with the CCFR determination of  $\alpha_s$  from the  $Q^2$  evolution of  $xF_3$  and  $F_2$  for  $Q^2 > 15 \text{ GeV}^2$  ( $\alpha_s(M_Z^2) = 0.111 \pm 0.004$ ), and about  $2\sigma$  lower than that measured from the high  $Q^2$  data [13]. With future experimental improvements (Fermilab NuTeV experiment) and improved theoretical work on higher twist corrections, this fundamental prediction of QCD has promise for being a stringent test of the model.

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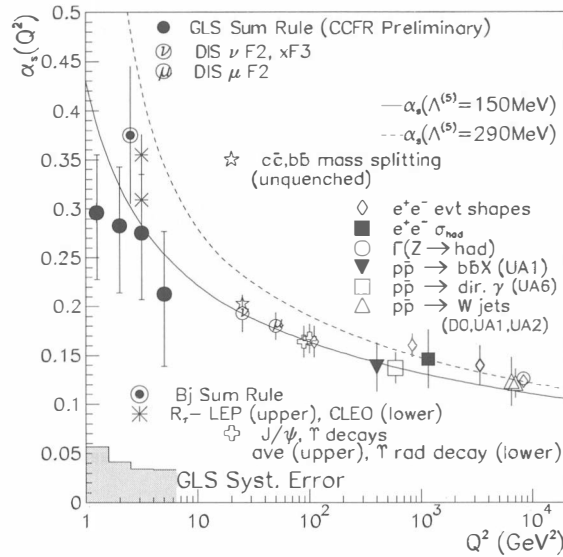


Figure 2:  $\alpha_s$  as measured at several different  $Q^2$  values, and the expected  $Q^2$  dependence of the 3-loop (see equation 2)  $\alpha_s$  for  $\Lambda_{\overline{MS}}^{(5)} = 150 \text{ MeV}$ , and  $\Lambda_{\overline{MS}}^{(5)} = 250 \text{ MeV}$ . Only the statistical errors on the GLS points are plotted at the GLS values, the systematic errors (which are correlated from one  $Q^2$  bin to the next) are shown at the bottom of the plot.

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