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Exceptional super Yang-Mills in $27+3$ and worldvolume M-theory

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ABSTRACT

Bars and Sezgin have proposed a super Yang-Mills theory in $D=s+t=11+3$ space-time dimensions with an electric 3-brane that generalizes the 2-brane of M-theory. More recently, the authors found an infinite family of exceptional super Yang-Mills theories in $D=(8n+3)+3$ via the so-called Magic Star algebras. A particularly interesting case occurs in signature $D=27+3$, where the superalgebra is centrally extended by an electric 11-brane and its 15-brane magnetic dual. The worldvolume symmetry of the 11-brane has signature $D=11+3$ and can reproduce super Yang-Mills theory in $D=11+3$. Upon reduction to $D=26+2$, the 11-brane reduces to a 10-brane with $10+2$ worldvolume signature. A single time projection gives a $10+1$ worldvolume signature and can serve as a model for $D=10+1$ M-theory as a reduction from the $D=26+1$ signature of the bosonic M-theory of Horowitz and Susskind; this is further confirmed by the reduction of chiral $(1,0)$, $D=11+3$ superalgebra to the $\mathcal{N}=1$ superalgebra in $D=10+1$, as found by Rudychev, Sezgin and Sundell some time ago. Extending previous results of Dijkgraaf, Verlinde and Verlinde, we also put forward the realization of spinors as total cohomologies of (the largest spatially extended) branes which centrally extend the $(1,0)$ superalgebra underlying the corresponding exceptional super Yang-Mills theory. Moreover, by making use of an "anomalous" Dynkin embedding, we strengthen Ramond and Sati's argument that M-theory has hidden Cayley plane fibers.

Contents

1. Introduction

After Witten's introduction of M-theory [1] in[1] $D=s+t=10+1$ space-time dimensions, Vafa proposed F-theory in $10+2$ [2]. Bars went further, generalizing F-theory in $10+2$ and $11+1$ [6] by studying S-theory in $11+2$ [7,8] and super Yang-Mills (SYM) theory in $11+3$, with subsequent further investigations by Sezgin et al. [9,10]. Pushing beyond, Nishino defined SYM's in signature $D=s+t=(9+m)+(1+m)$, for arbitrary $m\in\mathbb{N}\cup\{0\}$ [11]. Using the algebraic structure of *Exceptional Periodicity* [12–16], in [17]

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[1] "s" and "t" denote the number of spacelike resp. timelike dimensions throughout.

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the authors defined infinite families of *exceptional* $(1, 0)$ SYM theories; among these, a family in $D = s + t = (8n + 3) + 3$ that directly generalizes Sezgin's SYM in $11 + 3$. It is here worth recalling that the structure of SYM[2] in $11 + 3$, with a **64**-dimensional Majorana-Weyl (MW) semispinor,[3] interestingly arises in a certain 5-grading of "extended Poincaré type" of $\mathfrak{e}_{8(-24)}$ and has found use in unification models (see e.g. [19]).

In this work, we ascend to $D = 27 + 3$ space-time dimensions, in which an electric 11-brane and its 15-brane magnetic dual arise as central extensions of the $(1, 0)$ global supersymmetry algebra. In particular, the 11-brane gives rise to a worldvolume theory with $11 + 3$ signature, thus providing a *worldvolume embedding* for the chiral SYM in $11 + 3$ of Bars and Sezgin [7–9].

Following the 11-brane in the reduction from $27 + 3 \to 26 + 2 \to 26 + 1$ leads to the reduction of the $D = 11 + 3$ worldvolume of the 11-brane to a $D = 10+1$ worldvolume of a 10-brane, suggesting that M-theory may be a worldvolume theory; this is further confirmed by the fact that the reduction of the chiral $(1, 0)$ superalgebra in $D = 11 + 3$ down to $D = 10 + 1$ contains the $\mathcal{N} = 1$ superalgebra pertaining to M-theory [10]. This chain of reductions along the worldvolume of the electric 11-brane yields a natural map of the conjectured "bosonic M-theory" of Horowitz and Susskind [20] in $26 + 1$ down to M-theory in $10 + 1$. Moreover, in $26 + 2$ the electric 10-brane has a 14-brane magnetic dual (both centrally extending the corresponding $(1, 0)$ global superalgebra), and this implies, under reduction to $26 + 1$, that there exists a *"dual"* (worldvolume-realized) M-theory in $D = s + t = 13 + 1$.

2. (1, 0) SYM in 27 + 3 and M-theory

The $(1, 0)$ superalgebra in $D = 27+3$ space-time dimensions (corresponding to the level $n = 3$ of Exceptional Periodicity [12–16]) takes the form [17]

$$27 + 3: \ \left\{Q_\alpha, Q_\beta\right\} = \left(\gamma^{\mu\nu\rho}\right)_{\alpha\beta} Z_{\mu\nu\rho} + \left(\gamma^{\mu_1 \ldots \mu_7}\right)_{\alpha\beta} Z_{\mu_1 \ldots . \mu_7} + \left(\gamma^{\mu_1 \ldots \mu_{11}}\right)_{\alpha\beta} Z_{\mu_1 \ldots . \mu_{11}} + \left(\gamma^{\mu_1 \ldots \mu_{15}}\right)_{\alpha\beta} Z_{\mu_1 \ldots . \mu_{15}}. \tag{2.1}$$

Namely, the central extensions are given by a 3-brane, a 7-brane, an electric 11-brane and its dual, a magnetic 15-brane. Note that the magnetic duals of the 3-brane and 7-brane, *i.e.* the 23-brane resp. 19-brane, do *not* centrally extend the algebra (2.1); however, they can be found as the largest spatially extended central charges at $n = 5$ resp. $n = 4$ levels of Exceptional Periodicity [17].

In $D = 27 + 3$, the electric 11-brane has a multi-time worldvolume, with signature $11 + 3$, which can be used to provide a worldvolume realization for the $11 + 3$ SYM of Bars and Sezgin [7–9]. In other words, the multi-time worldvolume of the electric 11-brane in $27 + 3$ can support a corresponding $(1, 0)$ superalgebra in $11 + 3$:

$$11 + 3: \ \left\{Q_\alpha, Q_\beta\right\} = \left(\gamma^{\mu\nu\rho}\right)_{\alpha\beta} Z_{\mu\nu\rho} + \left(\gamma^{\mu_1 \ldots \mu_7}\right)_{\alpha\beta} Z_{\mu_1 \ldots . \mu_7}, \tag{2.2}$$

whose reduction to lower dimensions contains both the $D = 9 + 1$ type IIB $(2, 0)$ chiral superalgebra and the $D = 10 + 1$ ($\mathcal{N} = 1$) M-theory superalgebra, as discussed by Rudychev, Sezgin and Sundell in [10].

Hence, one can consider a reduction $27 + 3 \longrightarrow 26 + 1$, and focus on the corresponding reduction $11 + 3 \longrightarrow 10 + 1$ of the 11-brane (multi-time) worldvolume down to the (single-time) 10-brane worldvolume (in $26 + 1$); this latter, also in light of the aforementioned reduction of the $(1, 0)$, $D = 27 + 3$ chiral superalgebra (2.2) to the $\mathcal{N} = 1$ superalgebra in $D = 10 + 1$ [10], can be used to provide a worldvolume realization of M-theory.

This simple reasoning yields the following consequences:

- it puts forward the realization of $10 + 1$ M-theory as a worldvolume theory of an electric 10-brane in a higher $26 + 1$ space-time (pertaining to bosonic M-theory of Horowitz and Susskind [20]);
- as such, it provides a map from the bosonic M-theory in $D = 26 + 1$ to M-theory in $D = 10 + 1$;
- we observe that bosonic M-theory can be completed to a two-time theory in $D = 26 + 2$, in which a $(1, 0)$ *exceptional* SYM can be defined, with central extensions given by [17]

  $$26 + 2: \ \left\{Q_\alpha, Q_\beta\right\} = \left(\gamma^{\mu\nu}\right)_{\alpha\beta} Z_{\mu\nu} + \left(\gamma^{\mu_1 \ldots \mu_6}\right)_{\alpha\beta} Z_{\mu_1 \ldots . \mu_6} + \left(\gamma^{\mu_1 \ldots \mu_{10}}\right)_{\alpha\beta} Z_{\mu_1 \ldots . \mu_{10}} + \left(\gamma^{\mu_1 \ldots \mu_{14}}\right)_{\alpha\beta} Z_{\mu_1 \ldots . \mu_{14}}, \tag{2.3}$$

  i.e. by a 2-brane, a 6-brane, and by an electric 10-brane and its dual, a magnetic 14-brane. This implies that there exists a theory which is *dual* to the worldvolume-realized $10 + 1$ M-theory embedded in $26 + 2$, namely the worldvolume-realized $14 + 2$ theory reduced to $13 + 1$ of the magnetic 13-brane embedded in $26 + 1$; we dub such a $13 + 1$ theory the *"dual M-theory"*, and we leave its study for future work.

While the $D = 11 + 3$ superalgebra has been recovered, it is worth clarifying how this sheds light on M-theory, which contains M2 and M5 branes. The gravity dual of the M2 brane is described by the ABJM theory, which describes the near-horizon geometry $AdS_4 \times S^7$ with SO(8) R-symmetry [21]. The gravity dual of the M5 brane is described by the 6D (2,0) SCFT, which describes the near-horizon geometry $AdS_7 \times S^4$ [22]. The near-horizon geometries stem from $SO(3, 2) \times SO(8)$ and $SO(6, 2) \times SO(5)$, respectively, which both can be broken from $SO(11, 2)$, the signature of S-theory. While a generalization of M-theory with $SO(11, 3)$ signature hasn't been discussed, it is generally understood that supergravities as the low-energy limit of string theory come from a double copy of super-Yang-Mills theories.

From the work above, we are also suggesting that S-theory could be recovered as a worldvolume theory from $SO(27, 2)$. This would give rise to $AdS_{12} \times S^{15}$ near-horizon geometry with $SO(16)$ R-symmetry with a 10-brane source, which ultimately stems from an 11-brane in $D = 27 + 3$. Stacking these D-branes gives a $U(N)$ super-Yang-Mills gauge symmetry, while a generalized ABJM theory would give $U(N) \times U(N)$ symmetry. However, something more exotic than super-Yang-Mills theory would be anticipated in the full worldvolume

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[2] It should be recalled that the Bars-Sezgin SYM theories in $D > 10$ are not Lorentz covariant, and so are the generalizations presented here.

[3] It is amusing to observe that the semispinor **64** of $Spin(14)$ recently appeared in the X_1 algebraic structure of the Vogel plane in [18].

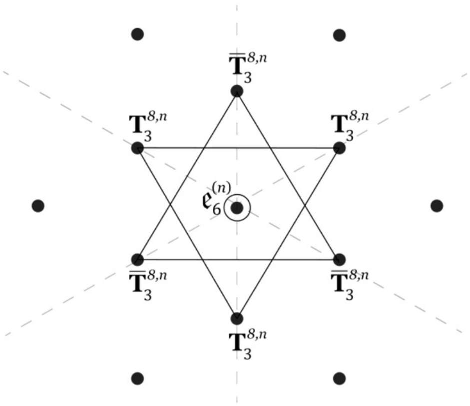

Fig. 1. The *"Magic Star"* of Exceptional Periodicity [12] allows for $\mathfrak{e}_8^{(n)}$ to be found as a finite-dimensional, Jacobi-violating generalization of $\mathfrak{e}_8$ [12–16]. $\mathbf{T}_3^{8,n}$ denotes the Hermitian part of a T-algebra of rank-3 and of *special type* [27], parametrized by the octonions $\mathbb{O}$ with $n \in \mathbb{N}$ [16,27].

theory. Higher-derivative terms would be anticipated to give a non-Abelian Dirac-Born-Infeld theory, similar to how the ABJM model has been extended with higher-derivative terms [23].

Also, it is worth mentioning that the higher Kaluza-Klein states stem from Kac-Moody and Virasoro extensions of Lie algebras [24]. The tower of graviton states comes from a tensor product or double copy of the Virasoro algebra. In particular, M-theory relates to the Kac-Moody algebra $\mathfrak{e}_{11}$ [25]. It can also be shown that $\mathfrak{e}_{11}$ contains $\mathfrak{so}_{20} \oplus \mathfrak{su}_2 \oplus (\mathbf{512}, \mathbf{2})$, which is precisely $\mathfrak{e}_7^{(2)}$. Therefore, it can be anticipated that EP algebras provide finite-dimensional, non-Lie truncations of infinite-dimensional Kac-Moody algebras, up to a rescaling of some roots in order to give a closed algebra. The $D = 27 + 3$ theory that contains the full spectrum of M-theory as a worldvolume most likely would require either $\mathfrak{e}_{16}$ or a Virasoro and Kac-Moody extension of the EP algebra $e_8^{(3)}$. We leave the investigation of this very interesting issues for future work.

3. Exceptional periodicity and spinors as brane cohomologies

Through the algebraic structure of Exceptional Periodicity, let us consider the generalization of the split form $\mathfrak{e}_{8(-24)}$ of the largest finite-dimensional exceptional Lie algebra $\mathfrak{e}_8$ provided by $\mathfrak{e}_{8(-24)}^{(3)}$, the corresponding, so-called *Magic Star algebra* at level $n = 3$ [12–16]:

$$\mathfrak{e}_{8(-24)}^{(3)} := \quad \mathfrak{so}_{28,4} \oplus \mathbf{32768} \tag{3.1}$$

$$= \mathbf{30}_{-2} \oplus (\mathbf{16384})'_{-1} \oplus (\mathfrak{so}_{27,3} \oplus \mathbb{R})_0 \oplus \mathbf{16384}_{+1} \oplus \mathbf{30}_{+2}, \tag{3.2}$$

where $\mathbf{32768} = \mathbf{2^{15}}$ is the MW semispinor in $28 + 4$, while $\mathbf{16384} = \mathbf{2^{14}}$ and $(\mathbf{16384})' = \left(\mathbf{2^{14}}\right)'$ denote the MW spinor and its conjugate in $D = 27 + 3$. $\mathfrak{e}_{8(-24)}^{(3)}$ (3.1) is the $n = 3$ element of the countably, Bott-periodized infinite sequence of generalizations of $\mathfrak{e}_{8(-24)}$. By denoting with $\mathbf{2}^{N-1}$ and $\left(\mathbf{2}^{N-1}\right)'$ the chiral semispinor representations of $\mathfrak{so}_{2N}$, as well as with $\wedge^i\mathbf{N}$ the rank-i antisymmetric (i-form) representation of $\mathfrak{so}_N$, we recall that, as a vector space, the Clifford algebra $Cl(N)$ in N dimensions is isomorphic to the Hodge-de Rahm complex in N dimensions:

$$\dim_{\mathbb{R}} Cl(N) = 2^N = \dim_{\mathbb{R}} \left(\mathbf{2}^{N-1} \oplus \left(\mathbf{2}^{N-1}\right)'\right) = \sum_{i=0}^{N} \binom{N}{i} = \dim_{\mathbb{R}} \left(\bigoplus_{i=0}^{N} \wedge^i \mathbf{N}\right). \tag{3.3}$$

Thus, in the case under consideration, the 2^{15}-dimensional MW semispinor $\mathbf{32768}$ of $\mathfrak{so}_{28,4}$, which branches as $\mathbf{16384} \oplus \mathbf{16384}'$ under $\mathfrak{so}_{28,4} \to \mathfrak{so}_{27,3}$, can be regarded as the total cohomology of a 15-brane, which in turn can be identified with the maximally spatially extended central charge of $\mathcal{N} = (1, 0)$ SYM (2.1) in $27 + 3$ space-time dimensions [17]:

$$\underset{\mathfrak{so}_{27,3}\text{ MW spinor}}{\mathbf{16384}} = \bigoplus_{i=0}^{15} \wedge^{2i+1}\mathbf{15} = \underbrace{\mathbf{15} \oplus \mathbf{455} \oplus \mathbf{3,003} \oplus \mathbf{6,435} \oplus \mathbf{5,005}' \oplus \mathbf{1,365}' \oplus \mathbf{105}' \oplus \mathbf{1}}_{\mathfrak{so}_{15}\text{-cov. odd (co)homology of the 15-brane}}; \tag{3.4}$$

$$\underset{\mathfrak{so}_{27,3}\text{ MW conjug. spinor}}{\mathbf{16384}'} = \bigoplus_{i=0}^{15} \wedge^{2i}\mathbf{15} = \underbrace{\mathbf{1} \oplus \mathbf{105} \oplus \mathbf{1,365} \oplus \mathbf{5,005} \oplus \mathbf{6,435}' \oplus \mathbf{3,003}' \oplus \mathbf{455}' \oplus \mathbf{25}'}_{\mathfrak{so}_{15}\text{-cov. even (co)homology of the 15-brane}}. \tag{3.5}$$

This is nothing but the $n = 3$ case of a general fact, namely that the (chiral) spinor component $\mathbf{2}^{4n+3}$ of the so-called Magic Star algebra [12–16] (see also Fig. 1)

$$\mathfrak{e}_{8(-24)}^{(n)} = \mathfrak{so}_{8n+4,4} \oplus \mathbf{2}^{4n+3} \tag{3.6}$$

$$= (\mathbf{8n+6})_{-2} \oplus \left(\mathbf{2}^{4n+2}\right)'_{-1} \oplus (\mathfrak{so}_{8n+3,3} \oplus \mathbb{R})_0 \oplus \mathbf{2}_{+1}^{4n+2} \oplus (\mathbf{8n+6})_{+2}\,, \tag{3.7}$$

can be realized as the *total cohomology* of a $(4n + 3)$-brane, which in turn can be identified with the largest spatially extended central extension of the $(1, 0)$ supersymmetry algebra in $(8n + 3) + 3$ space-time dimensions [17]

$$(8n+3)+3: \; \left\{Q_\alpha, Q_\beta\right\} = \left(\gamma^{\mu\nu\rho}\right)_{\alpha\beta} Z_{\mu\nu\rho} + \left(\gamma^{\mu_1\cdots\mu_7}\right)_{\alpha\beta} Z_{\mu_1\ldots\mu_7} + \ldots + \left(\gamma^{\mu_1\cdots\mu_{n+3}}\right)_{\alpha\beta} Z_{\mu_1\ldots\mu_{4n+3}}. \tag{3.8}$$

Therefore, the spinor generators of the algebra $\mathfrak{e}^{(n)}_{8(-24)}$ are realized, exploiting the central extensions of the $(1,0)$ supersymmetry algebra in $(8n+3)+3$, in terms of brane cohomology.

We stress that this realization extends the results found by Dijkgraaf, Verlinde and Verlinde [26] which, in the BPS quantization of the 5-brane, realized the **16** components of the central charge as fluxes through the odd homology cycles on the five-brane itself:

$$\underset{\mathfrak{so}_{5,5}\text{ MW spinor}}{\mathbf{16}} = \bigoplus_{i=0}^{5} \wedge^{2i+1}\mathbf{5} = \underset{\mathfrak{so}_{5}\text{-cov. odd (co)homology of the 5-brane}}{\mathbf{5}\oplus\mathbf{10}'\oplus\mathbf{1}} ; \tag{3.9}$$

$$\underset{\mathfrak{so}_{5,5}\text{ MW conjug. spinor}}{\mathbf{16}'} = \bigoplus_{i=0}^{5} \wedge^{2i}\mathbf{5} = \underset{\mathfrak{so}_{5}\text{-cov. even (co)homology of the 5-brane}}{\mathbf{1}\oplus\mathbf{10}\oplus\mathbf{5}'} . \tag{3.10}$$

Since the spinor generators are the very ones responsible for the violation of the Jacobi identity in Magic Star algebras [12–16], this is a further hint that the Lie subalgebras of Magic Star algebras yield purely bosonic sectors. Moreover, it is worth here reminding that the $n=1$ (trivial) level of Exceptional Periodicity boils down to the fact the spinor component **128** of $\mathfrak{e}^{(1)}_{8(-24)} \equiv \mathfrak{e}_{8(-24)}$ can be realized as the total cohomology of the 7-brane which centrally extends the $(1,0)$ superalgebra in $11+3$ [7–10].

4. Brane actions and hidden $\mathbb{OP}^2$ fibers in M-theory

As resulting from the star-shaped algebraic structure of the Magic Star algebras [12–16] (see also Fig. 1), we note that the Hermitian part of the cubic Vinberg's T-algebra[4] [27] at EP level $n=3$ (i.e., $\mathfrak{e}^{(3)}_{8(-24)}$) can be Peirce-decomposed (in a manifestly $\mathfrak{so}_{25,1}$-covariant way) as

$$\mathbf{T}^{8,3}_{3} = \mathbf{26} \oplus \mathbf{4096} \oplus \mathbf{1}, \tag{4.1}$$

where **26** and $\mathbf{4096} = \mathbf{2}^{12}$ respectively are the vector and MW semispinor irreprs. of $\mathfrak{so}_{25,1}$. Adopting Sezgin et al.'s approach [9,10], the larger symmetry $\mathfrak{so}_{28,4}$ can be considered as a multi-particle symmetry, for four particles, where putting all particles but one on-shell yields constant momenta that appear as null vectors [10]; a single particle (described by the algebra $\mathbf{T}^{8,3}_{3}$) enjoys $\mathfrak{so}_{25,1}$ symmetry, while two and three particles acquire enhanced $\mathfrak{so}_{26,2}$ and $\mathfrak{so}_{27,3}$ symmetry, respectively. In such a perspective, bosonic M-theory in $D=26+1$ can be considered as a single time projection of a two-particle system with reduced $\mathfrak{so}_{26,2} \to \mathfrak{so}_{26,1}$ symmetry and Horowitz and Susskind's low energy $D=26+1$ action [20]

$$S = \int d^{27}x\sqrt{-\hat{g}}\left[R(\hat{g}) - \frac{1}{48}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}\right] \tag{4.2}$$

captures a 2-brane in this background where $F = dC$, and this reduces to the bosonic string action in $D=25+1$. As the $D=26+2$ superalgebra admits a 10-brane as well as a 2-brane, one can alternatively consider a $D=26+1$ low energy action in terms of a 12-form field strength

$$S = \int d^{27}x\sqrt{-\hat{g}}\left[R(\hat{g}) - \frac{1}{2(12!)}F_{\mu_1\cdots\mu_{12}}F^{\mu_1\cdots\mu_{12}}\right] \tag{4.3}$$

suggesting a (10,1) signature worldvolume M-theory with sixteen transverse directions. A 2-brane can be immersed in the (10,1) worldvolume as an M2-brane or in the (26,1) signature bulk, reducing to either a type IIA string [5] or $D=25+1$ bosonic string [20] by compactification, respectively. The full worldvolume symmetry in $D=26+2$ for the 10-brane is $SO(10,2)$, the signature of F-theory [2][5]; intriguingly, in such a signature the null reduction of a (2,2) brane in $D=10+2$ yields to type IIB string theory in $D=9+1$ [4].

The $D=27+3$ superalgebra permits an 11-brane with 13-form field strength, which one can use for worldvolume reduction to $D=11+3$, whose $(1,0)$ superalgebra yields the $\mathcal{N}=1$ superalgebra in $D=10+1$ as well as the IIA, IIB and heterotic superalgebras in $D=9+1$ [10], with sixteen dimensions transverse to the 11-brane. Moreover, reducing $11+3$ to $11+1$ allows an identification of the 3-brane with the type IIB D3-brane, as noted by Tseytlin [3].

A non-compact real form of a theorem by Dynkin [28] yields the maximal and non-symmetric "anomalous"[6] embedding [29]

$$\begin{aligned} &\mathfrak{f}_{4(-20)} \subset \mathfrak{so}_{25,1} \\ &\qquad \mathbf{26} = \mathbf{26}, \end{aligned} \tag{4.4}$$

under which the vector representation of $\mathfrak{so}_{25,1}$ stays irreducible, providing the fundamental representation of the minimally non-compact real form $\mathfrak{f}_{4(-20)}$ of $\mathfrak{f}_4$. Since $\mathfrak{f}_{4(-20)}$ is the Lie algebra of the derivations of the Lorentzian version of the exceptional cubic Jordan algebra over the octonions $J^{\mathbb{O}}_{1,2}$ [30,31],

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[4] This has been named *HT-algebra* in [16].

[5] This might seem in contrast with the claim that F-theory has signature $11+1$, made at the start of the paper. However, the signature of the two additional dimensions of F-theory with respect to string theory is somewhat ambiguous due to their infinitesimal character. For example, the supersymmetry of F-theory on a flat background corresponds to type IIB (i.e. $(2,0)$) supersymmetry with 32 real supercharges which may be interpreted as the dimensional reduction of the chiral real 12-dimensional supersymmetry if its signature is $10+2$. On the other hand, the signature $11+1$ is needed for the Euclidean interpretation of the compactification spaces (e.g. the four-folds), and the latter interpretation prevailed in recent years.

[6] This naming goes back to Ramond [29].

$$\mathfrak{f}_{4(-20)} = \mathfrak{der}\left(J_{1,2}^{\mathbb{O}}\right), \tag{4.5}$$

it follows that the vector **26** of $\mathfrak{so}_{25,1}$ may be regarded as the traceless part of $J_{1,2}^{\mathbb{O}}$:

$$\mathbf{26} \simeq \left(J_{1,2}^{\mathbb{O}}\right)_0. \tag{4.6}$$

The threefold Pierce decomposition of $\left(J_{1,2}^{\mathbb{O}}\right)_0$, corresponding to the branching

$$\begin{array}{ccc} \mathfrak{so}_9 & \subset & \mathfrak{f}_{4(-20)} \\ \mathbf{1} \oplus \mathbf{9} \oplus \mathbf{16} & = & \mathbf{26}, \end{array} \tag{4.7}$$

is geometrically realized as three transverse Hopf maps $S^7 \hookrightarrow S^{15} \to S^8$, and it yields to three cosets of type

$$\frac{F_{4(-20)}}{SO(9)}, \tag{4.8}$$

providing the charts of the (non-compact, Riemannian real form of the) Cayley plane $\mathbb{OP}^2$. This, in the worldvolume picture, confirms and strengthens Ramond and Sati's argument that $D = 10+1$ M-theory has hidden Cayley plane fibers [32–35]. Moreover, the $10+1$ worldvolume realization of M-theory proposed in this paper provides a natural realization of M-theory on a manifold with boundary, as it was considered by Horava and Witten in [36], in which the cancellation of anomalies selects the $E_8 \times E_8$ gauge symmetry of heterotic string theory.

Horowitz and Susskind noticed that $D = 26+1$ M-theory predicts the existence of a $2+1$ CFT with $SO(24)$ symmetry [20]. Here, we confine ourselves to noting that the two-particle symmetry $\mathfrak{so}_{26,2}$ is the Lie algebra of the group $SO(26,2)$, whose decomposition $SO(26,2) \to SO(2,2) \times SO(24)$ suggests the existence of an $AdS_3 \times S^{23}$ geometry, which in turn supports the existence of a $2+1$ CFT with $SO(24)$ symmetry. Along this line, the result that the fourth integral cohomology of Conway's group Co_0 [37] is a cyclic group of order 24 [38], as well as the fact that Co_0 is a maximal finite subgroup of $SO(24)$ [39], seem to suggest that such a CFT could be related to Witten's monster CFT construction for 3D gravity [40]; in the multi-particle picture, the twenty-four transverse directions would be discretized as the Leech lattice [41,42].

5. Conclusion

Using the *exceptional* SYM theory in $27+3$ space-time dimensions, whose $(1,0)$ non-standard global superalgebra can be centrally extended by an electric 11-brane and its 15-brane magnetic dual [17], we considered the (multi-time) worldvolume theory of the 11-brane itself as support for the $(1,0)$ SYM theory in $11+3$ space-time dimensions as introduced by Bars and Sezgin some time ago [7–9].

As the $(1,0)$ superalgebra in $11+3$ dimensions reduces to the $10+1$ $\mathcal{N} = 1$ superalgebra, as well as the type IIA, IIB and heterotic superalgebras in $9+1$ [10], we proposed the reduced (single-time) 10-brane worldvolume theory in $10+1$ as a *worldvolume realization* of M-theory (this also entails the existence of a would-be "dual worldvolume M-theory" realized as a worldvolume theory in $13+1$). In this framework, the space-time reduction $27+3 \longrightarrow 26+1$ yields a natural map from the conjectured bosonic M-theory of Horowitz and Susskind [20] in $D = 26+1$ to $D = 10+1$ M-theory. The worldvolume picture is essential in geometrically explaining the origin of the $E_8 \times E_8$ heterotic string; the Horava-Witten domain wall for heterotic M-theory requires a manifold with boundary in eleven dimensions, which occurs naturally if the eleven dimensional manifold is itself a brane worldvolume with boundary.

Moreover, extending the results of [26], we have put forward the intriguing brane-cohomological interpretation of spinors, and in particular of the spinor generators of the recently discovered class of Magic Star algebras [15,16], thus entangling the algebraic structure of Exceptional Periodicity [12–16] with the central extensions of exceptional super Yang Mills theories in higher dimensional space-times.

Last but not least, by recalling an "anomalous" Dynkin embedding [28,29], we identified the vector irrepr. in twenty-six Lorentzian dimensions as the traceless part of (the real, Lorentzian version of) the exceptional cubic Jordan algebra over the octonions, whose Peirce decomposition strengthens Ramond and Sati's argument that $D = 10+1$ M-theory has hidden Cayley plane fibers [32–35].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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