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THE CRAB-CROSSING SCHEME FOR STORAGE-RING COLLIDERS*

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ABSTRACT

It is shown that a scheme of beam-beam collision called crab-crossing is applicable to storage-ring colliders and allows a large crossing angle at the collision point without an excitation of synchrotron-betatron resonances. This scheme will give merits in designing high-luminosity colliders.

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INTRODUCTION

The problem with a crossing angle at the collision point of a storage-ring collider has been studied for years,[1,2] with the realization that the synchrotron-betatron resonance induced by the crossing angle severely limits the luminosity.[3,4] Recently a scheme called 'crab-crossing' was invented by R. B. Palmer to allow a large crossing angle for an electron-positron linear collider without a loss of the luminosity.[5] This scheme is also applicable to any kind of collider. We will show in this paper that it is possible to vanish the synchrotron-betatron coupling-resonances when we apply this scheme to a storage-ring collider with a crossing angle.

1. BEAM-BEAM INTERACTION WITH CRAB-CROSSING

The crab-crossing scheme tilts both bunches before the collision in the crossing plane by an angle φ which is just half the crossing angle, as shown in Fig. 1. It is clear that these bunches collide head-on and there is no loss of the luminosity. Here we assume the crossing is done in the horizontal plane. This tilt of the bunch is made by a transverse RF deflector placed at a point where the horizontal betatron phase advance is $-\pi/2$ from the collision point. We place another deflector after the collision point to restore the tilt to zero.

Now let us calculate the beam-beam effect on a particle of one bunch A from another bunch B. We define the longitudinal coordinate z along the orbit of the bunch A and the horizontal x in the crossing plane, and both are measured from the center of the bunch A. Here we concentrate on the motion in this plane. We use suffixes 0 to 3 to denote each step of this scheme. The first step, the tilt made by the RF deflector, is expressed as

$$\begin{aligned} x_1 &= x_0 + z_0 \tan\varphi \quad , & x_1' &= x_0' \quad , \\ z_1 &= z_0 \quad , & \varepsilon_1 &= \varepsilon_0 - x_0' \tan\varphi \quad , \end{aligned} \tag{1}$$

where x' and ε are the horizontal angle and the relative energy deviation, *i.e.*, the canonical conjugates of x and z, respectively. The change of the energy is inevitable because the motion is symplectic.[6)]

The second step is the kick by the beam-beam force from the other bunch. Here we assume both beams are fully relativistic, so that the electric field $\boldsymbol{E}$ made by the bunch B is perpendicular to the direction of its motion. Thus, we get the electric field which acts on the particle of the beam A at (x, z) as

$$\begin{aligned} E_x &= -B_y \cos 2\varphi \quad , \\ E_z &= B_y \sin 2\varphi \quad , \end{aligned} \tag{2}$$

where B_y is the y-component of the magnetic field. The strength of B_y is a function of the distance from the particle to the tilted axis of the bunch B along the direction of $\boldsymbol{E}$; *i.e.*, $B_y = B_y(x - z\tan\varphi)$. Therefore the force on the particle is written as

$$\begin{aligned} f_x &= e(E_x - B_y) = -2eB_y \cos^2\varphi \quad , \\ f_z &= eE_z = eB_y \sin 2\varphi \quad . \end{aligned} \tag{3}$$

Note that the force is perpendicular to the tilted axis of the bunches. This force gives the transformation of the beam-beam interaction as

$$\begin{aligned} x_2 &= x_1 \quad , & x_2' &= x_1' - F(x_1 - z_1 \tan\varphi)\cos^2\varphi \quad , \\ z_2 &= z_1 \quad , & \varepsilon_2 &= \varepsilon_1 + F(x_1 - z_1 \tan\varphi)\sin\varphi\cos\varphi \quad , \end{aligned} \tag{4}$$

where F is the kick on the particle integrated over the bunch B, and a function of $x_1 - z_1 \tan\varphi$. Here we have neglected the effect from the changes of the beam envelopes along their axes at the collision point. It is easy to see the transformation above is symplectic.

At the last step of the crab-crossing we have a restoring deflector after the collision point which acts as

$$
\begin{aligned}
x_3 &= x_2 - z_2 \tan\varphi \quad , & x_3' &= x_2' \quad , \\
z_3 &= z_2 \quad , & \varepsilon_3 &= \varepsilon_2 + x_2' \tan\varphi \quad .
\end{aligned} \tag{5}
$$

Combining Eqs. (1), (4), and (5) we obtain the entire transformation of the crab-crossing:

$$
\begin{aligned}
x_3 &= x_0 \quad , \\
x_3' &= x_0' - F(x_0)\cos^2\varphi \quad , \\
z_3 &= z_0 \quad , \\
\varepsilon_3 &= \varepsilon_0 \quad .
\end{aligned} \tag{6}
$$

This transformation has the same form as that of a head-on collision,[7] except for the factor $\cos^2\varphi$. All the synchrotron-betatron coupling-terms which appear in a usual crossing-angle scheme[3] have disappeared.

2. SPECIFIC PARAMETERS AND TOLERANCES

As an example, here we calculate several specific parameters of this scheme. Consider a double-ring e^+e^- collider for B-meson physics with asymmetric energies, 12 GeV and 2 GeV.[8,9,10] We choose the crossing angle $2\varphi = 50$ mrad, which gives 100 mm separation of both beams at a point 2 m apart from the collision point. This separation is enough to place different final quadrupoles for both beams. The deflecting RF voltage is given by

$$
V = \frac{cE\tan\varphi}{\omega_{rf}(\beta_x\beta_x^*)^{1/2}} \quad , \tag{7}
$$

where E, ω_{rf}, β_x, and β_x^* are the beam energy, the angular frequency of the RF deflector, the horizontal beta functions at the deflector and the collision point, respectively. For the 12 GeV beam, when we choose these parameters as $\omega_{rf} = 2\pi \times 500$ MHz, $\beta_x = 100$ m, and $\beta_x^* = 0.25$ m, the RF voltage becomes $V = 5.7$ MV.

This scheme requires two kinds of accuracies for the deflectors. One is the relative phase stability $\Delta\theta$ between the deflectors of both beams. This phase error makes a relative horizontal displacement of both beams at the collision point. The tolerance for this error is estimated as

$$\Delta\theta \ll \frac{\omega_{rf}}{c\tan\varphi}\sigma_x^* = \frac{\omega_{rf}}{c\tan\varphi}(\beta_x^*\epsilon_x)^{1/2} \quad , \tag{8}$$

where σ_x^* and ϵ_x are the horizontal beam size at the collision point and the horizontal emittance of the beam. Here we use $\epsilon_x = 3\times10^{-7}$ m as in Ref. (8), then obtain $\Delta\theta \ll 0.11$ rad. The lengths of the wave guides from the common power source to both deflectors must be adjusted and stabilized within the horizontal beam size divided by $\tan\varphi$, which is 1.1 cm in this case.

Another problem is the stability of the amplitude of the deflectors. The amplitude error makes an error on the tilt angle and excites the synchrotron-betatron resonances. We expect that if the tilt-angle error is much smaller than the diagonal angle of the bunch σ_x^*/σ_z, where σ_z is the longitudinal bunch length, this effect is tolerable. Thus we get

$$\frac{\Delta V}{V} \ll \frac{\sigma_x^*}{\sigma_z\tan\varphi} \quad , \tag{9}$$

which is 1.1 in this case if we put $\sigma_z = 1$ cm. An error which randomly varies in every turn accumulates its effect within the damping time. Thus the tolerance for this kind of error becomes (damping time/revolution period)$^{1/2}$ times more severe than those given by Eqs. (8) and (9). This factor is 120 for the radiation damping of the design of Ref. (8).

3. CONCLUSION

The idea of the crab-crossing will make a large crossing angle possible without a loss of the luminosity and the excitation of synchrotron-betatron resonances. A large crossing angle is always useful to design double-ring colliders or ring-linear colliders[11)] because the separation of both beams is easy. It also becomes easier to put final quadrupoles for both beams close to the collision point. This reduces the chromatic effect of the ring and enables us to make the beta functions at the collision point smaller. Another merit of a large crossing angle is the possibility of increasing the frequency of collision without unexpected collisions at other points off the interaction point.

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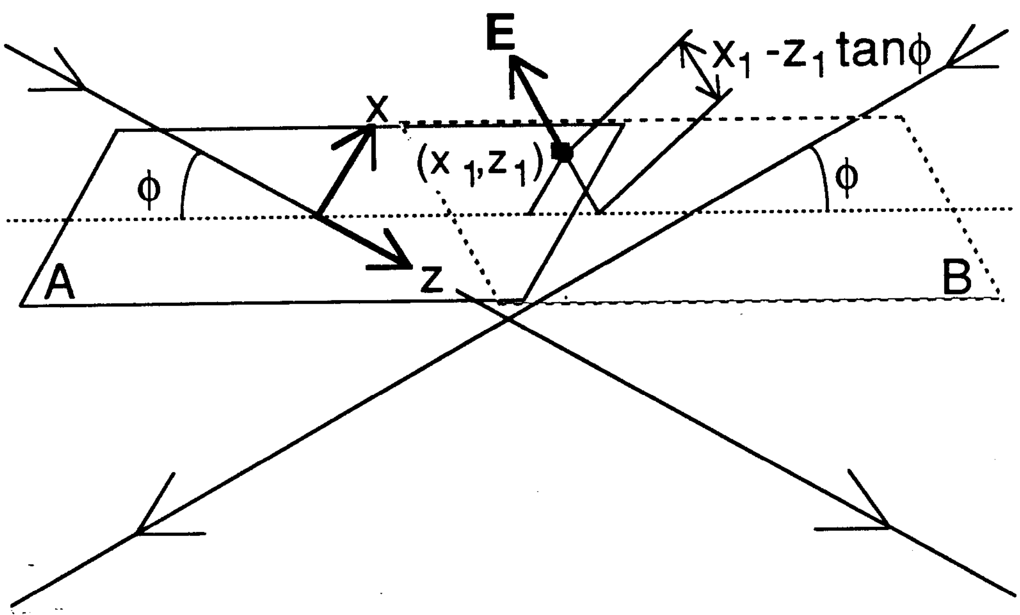

Fig. 1

The crab-crossing scheme. Both bunches A and B are tilted by the angle φ, which is half the crossing angle. The electric field $\boldsymbol{E}$ is perpendicular to the direction of the motion of the bunch B. The strength of the beam-beam force which acts on a particle of the bunch A at (x_1, z_1) is determined by the distance along $\boldsymbol{E}$ to the tilted axis, $x_1 - z_1 \tan\varphi$.