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# SCALING, DUALITY, AND THE BEHAVIOR OF RESONANCES IN INELASTIC ELECTRON-PROTON SCATTERING†

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## ABSTRACT

We propose that a substantial part of the observed behavior of inelastic electron-proton scattering is due to a non-diffractive component of virtual photon-proton scattering. The behavior of resonance electroproduction is shown to be related in a striking way to that of deep inelastic electron-proton scattering. Relations between the elastic and inelastic form factors and the threshold behavior of the inelastic structure functions in the scaling limit are found.

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High energy inelastic electron-nucleon scattering provides a unique way to probe the instantaneous charge distribution of the nucleon and to search for possible substructure.[1] If one observes only the scattered electrons' energy and angle, then the results of such scatterings are summarized in the structure functions $W_1$ and $W_2$, which depend on the virtual photon's laboratory energy, $\nu$, and invariant mass squared, $q^2$. Considered as a collision between the exchanged virtual photon and the proton, one is studying the total cross section for the process "$\gamma$" + p → hadrons, where the hadrons have an invariant mass W which is related to $\nu$ and $q^2$ by $W^2 = s = 2M\nu + M^2 - q^2$.

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Experiments have revealed a very large cross section for inelastic ep scattering — a cross section which when integrated over $\nu$ at fixed $q^2$ is the same order of magnitude as the Mott cross section for scattering from a point proton[1]. This has led to descriptions of the scattering in terms of point-like constituents of the proton (partons), and to the proposal of scaling[2]: as $\nu$ and $q^2 \rightarrow \infty$, $W_1(\nu, q^2)$ and $\nu W_2(\nu, q^2)$ are to become functions of the single variable $\omega = 2M\nu/q^2$. If we restrict ourselves to the region $W \geq 2.0$ GeV (above the prominent resonances) and $q^2 \geq 0.5$ GeV$^2$, then the resulting subset of data is consistent with scaling, i.e., with a single smooth curve for $\nu W_2$ (and $W_1$) as a function of $\omega$. This curve (for $\nu W_2$) starts at zero at $\omega = 1$, the position of the elastic peak, rises to a maximum at $\omega \simeq 5$, and then appears to fall off at large $\omega$.[1,3] Since $\nu W_2$ is proportional to the virtual photon-proton total cross section, such a fall off of $\nu W_2$ at large $\omega$ implies the presence of a non-diffractive (non-Pomeranchukon exchange) component of virtual photon-proton scattering. In hadronic reactions, at least, such a non-diffractive component at high energy is correlated with the presence and behavior of resonances at low energy. For example, the $K^+p$ total cross section, which shows no prominent resonance bumps at low energy, is constant at high energy, while the $K^-p$ total cross section, with many $Y^*$ resonances at low energy, falls at high energy. This correlation between resonances at low energy and non-Pomeranchukon exchanges (falling total cross sections) at high energies is part of the more general concept of duality, and takes quantitative form in terms of finite energy sum rules.

This directs our attention to the behavior of the resonances in electroproduction and the comparison of their behavior to that of $\nu W_2$ in the scaling limit, $\nu$ and $q^2 \rightarrow \infty$. In particular we want to investigate whether the resonances disappear at large $q^2$ relative to a "background" which has the scaling behavior, or whether the resonances and any "background" have the same behavior, which might then be related to scaling and the apparent fall off in $\nu W_2$ at large $\omega$. When $\nu W_2$ is considered as a function of $\omega$, the resonances occur at values of $\omega > 1$, with the position of any given resonance moving towards $\omega = 1$ as $q^2$ increases. On the other hand, the zeroth resonance or nucleon pole, corresponding to elastic scattering, always occurs at a fixed value of $\omega = 1$. There have been recent attempts[4] within the framework of parton models to derive a connection between the $q^2$ dependence of the elastic form factors and the behavior of $\nu W_2$ in the scaling limit near $\omega = 1$. But when $\nu W_2$ is considered as a function of $\omega$ the elastic peak is always at $\omega = 1$ where $\nu W_2$ vanishes in the scaling limit. With the nucleon pole, always at $\omega = 1$, and the resonances, at varying values of $\omega > 1$, on a different footing, the connection of either elastic scattering or resonance electroproduction to the scaling behavior of $\nu W_2$ is difficult to see.

To easily see the behavior of the resonances and of elastic scattering in comparison to $\nu W_2$ in the scaling limit, one should plot the data for $\nu W_2$ versus the variable $\omega' = (2M\nu + M^2)/q^2 = 1 + s/q^2 = \omega + M^2/q^2$ (or more generally, $\omega' = \omega + m^2/q^2$ with $m^2 \simeq 1$ GeV$^2$). This variable originally arose in the analysis[5] of the large angle inelastic ep data near $\omega = 1$. In the scaling limit where $\nu$ and $q^2 \to \infty$, the variables $\omega'$ and $\omega$ are clearly the same. For finite values of $q^2$ there is a difference; in particular, the elastic peak is no longer at $\omega' = 1$, but appears at $\omega' = 1 + m^2/q^2 > 1$, and moves to smaller values of $\omega'$ as $q^2$ increases, just as the other resonances do[6].

The striking results of making such a plot versus $\omega' = 1 + s/q^2 = \omega + M^2/q^2$ are shown in Figure 1. The dashed line, which is the same in all cases, is a smooth curve through the high energy $\theta = 10^{\circ}$ data[7] in the region beyond the prominent resonances (W > 2.0 GeV) and with large $q^2$ ($3 < q^2 < 7$ GeV$^2$). This is a region where the scaling behavior has occurred experimentally, and we call this the "scaling limit curve", $\nu W_2(\omega')$. The solid lines are smooth curves through

$6^{\circ}$ data at incident electron energies of 7, 10, 13.5, and 16 GeV, and typical values of $q^2$ of 0.4, 1.0, 1.7, and 2.4 $GeV^2$, respectively. As $q^2$ increases the resonances move toward $\omega' = 1$, each clearly following in magnitude the smooth scaling limit curve. As similar graphs of the $10^{\circ}$ data in the resonance region also show, the prominent resonances do not disappear at large $q^2$ relative to a "background" under them, but instead fall at roughly the same rate as any "background" and closely, resonance by resonance, follow the scaling limit curve. We emphasize that this behavior of the resonances, which is of central importance in our arguments, can be seen by careful examination of the data when they are plotted with respect to other variables; with respect to $\omega'$ it just becomes obvious at a glance.

Thus the resonances have a behavior which is closely related to that of $\nu W_2$ in the scaling limit. For large values of $\omega'$, the data for $\nu W_2$ with $q^2 > 0.5$ $GeV^2$ are consistently on a single curve which falls with increasing $\omega'$, just as when plotted versus $\omega$. We therefore propose that the resonances are not a separate entity, but are an intrinsic part of the scaling behavior of $\nu W_2$ and that a substantial part of the observed scaling behavior of inelastic electron-proton scattering is non-diffractive in nature. Appropriately averaged, the nucleon and the resonances at low energy build, in the strong interaction duality sense, the relevant non-Pomeranchukon exchanges at high energy, which result in a falling $\nu W_2$ curve.

What is unique to electroproduction is the experimentally observed scaling behavior which allows us to consider points at the same $\omega'$ arising from different values of $q^2$ and $s = W^2$, both within and outside the low energy resonance region. If we choose $\nu_m$ and $q^2$ in the region where $\nu W_2$ scales, i.e., beyond the region of prominent resonances and where $\nu W_2(\nu, q^2) = \nu W_2(\omega')$ = a smooth function of $\nu$ (see Figure 1), then a finite energy sum rule for $\nu W_2$ at fixed $q^2$ tells us that

$$\frac{2M}{q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu,q^2) = \int_1^{(2M\nu_m+m^2)/q^2} d\omega'\ \nu W_2(\omega'), \tag{1}$$

since the integrands are the same for $\nu > \nu_m$ or $\omega' > (2M\nu_m + m^2)/q^2$ (by the assumption that $\nu_m$ and $q^2$ are in the region where $\nu W_2$ scales). Eq. (1) states that for $\nu < \nu_m$, $\nu W_2(\omega')$ acts as a smooth average function for $\nu W_2(\nu, q^2)$ in the sense of finite energy sum rules. Thus, because we can vary the external photon mass in electroproduction and have scaling, we can directly measure a smooth curve which averages the resonances in the finite energy sum rule and duality sense. High energy electroproduction thus becomes a beautiful example of the duality between resonances and non-Pomeranchukon exchanges at high energy.

Looked at the other way, by appropriate averages over the resonances we would build up the curve for $\nu W_2$ in the scaling limit. But how can resonances, which have form factors which fall rapidly with $q^2$, be consistent with a scaling limit curve which is supposed to characterize a very slow $q^2$ variation? Let us fix $s = M_R^2$, the mass squared of a given resonance (possibly this could be the zeroth resonance, the nucleon) and vary $q^2$. Then if $G(q^2)$ is the excitation form factor of the resonance,

$$\nu W_2 = 2M\nu \left[G(q^2)\right]^2 \delta(s-M_R^2) = (M_R^2 - M^2 + q^2)\left[G(q^2)\right]^2 \delta(s-M_R^2) \tag{2}$$

is its contribution to $\nu W_2$ in the narrow resonance approximation. For large $q^2$, the form factor falls off as some power, say

$$G(q^2) \rightarrow (1/q^2)^{n/2} . \tag{3}$$

As $q^2$ increases, the resonance is pushed down toward $\omega' = 1$, where $\nu W_2(\omega')$ can be parametrized by some power behavior,

$$\nu W_2 \xrightarrow[\omega' \to 1]{} c(\omega'-1)^p = c\left(\frac{s-M^2+m^2}{q^2}\right)^p . \tag{4}$$

If Eqs. (2), (3), and (4) are to be consistent and the resonances build up the scaling limit curve locally, then we must have

$$n = p+1 , \tag{5}$$

i. e., all resonances (including the nucleon) which are to follow the scaling limit curve as $q^2 \to \infty$ must have the same power of fall off in $q^2$ for large $q^2$ and this power is related to the power with which $\nu W_2(\omega')$ rises at threshold.

Eq. (5) is just the relation first derived by Drell and Yan[4] in the parton model for the elastic form factor $F_1(q^2)$. Here, by demanding that the resonances must locally build up the scaling-limit curve, we obtain it for the elastic and inelastic form factors. That all the resonance excitation form factors have approximately the same behavior as the elastic form factor at large $q^2$ is a well noted curiosity of nucleon resonance electroproduction[8].

Conversely, this experimental fact means that each resonance individually follows the scaling limit curve in magnitude as $q^2 \to \infty$ (i. e., as it approaches $\omega' = 1$). Indeed, Figure 1 suggests that for $m^2 \simeq M_N^2$, the finite energy sum rule average of Eq. (1) can be made over a quite local region, i. e., the area under the scaling-limit curve $\nu W_2(\omega')$ equals the total area under a given resonance bump integrated over an energy region (in W) of a few hundred MeV below and above the resonance.

It is instructive to take the variable $\omega'$ on a more serious basis and to carry the idea of local averaging to an extreme: we make the very strong assumption that, in the sense of Eq. (1), the area under the elastic peak in $\nu W_2$ for large $q^2$ is also the same as the area under the scaling-limit curve, $\nu W_2(\omega')$, from $\omega' = 1$ to an $\omega'$ corresponding to a hadron mass $W = W_t$ near physical pion threshold, i. e.

$$\int_1^{1+(W_t^2-M^2+m^2)/q^2} d\omega' \, \nu W_2(\omega') = \left(\frac{2M}{q^2}\right) \int d\nu \, \nu W_2^{\text{elastic}}(\nu, q^2)$$

$$= \left[G(q^2)\right]^2 = \left[F_1(q^2)\right]^2 + \frac{q^2 \mu_A^2}{4M^2} \left[F_2(q^2)\right]^2 \qquad (6)$$

$$= \frac{\left[G_E(q^2)\right]^2 + \frac{q^2}{4M^2}\left[G_M(q^2)\right]^2}{1 + q^2/4M^2} \ .$$

Taking the derivative with respect to $q^2$, we obtain

$$\nu W_2 \left(\omega' = 1 + \frac{W_t^2 - M^2 + m^2}{q^2}\right) = \left(\frac{1}{\omega' - 1}\right)\left(-q^2 \frac{d}{dq^2}\left[G(q^2)\right]^2\right) , \qquad (7)$$

which allows us to calculate $\nu W_2(\omega')$ near threshold in terms of the elastic form factors once we have chosen $W_t^2 - M^2 + m^2$. If $G(q^2) \to (1/q^2)^{n/2}$ as $q^2 \to \infty$, then from (7), $\nu W_2 \xrightarrow[\omega' \to 1]{} (\omega'-1)^p$ where $n = p + 1$, so we again recover the relation (5) between the elastic form factor and threshold behavior of $\nu W_2$.

We might expect such radical assumptions to work when the elastic peak is pushed into the threshold region of $\nu W_2(\omega')$, i.e. when $q^2 >> 1$ GeV$^2$ and $\omega' - 1 = (W_t^2 - M^2 + m^2)/q^2 << 1$. A value of $W_t^2 - M^2 + m^2 \simeq 1.5$ GeV$^2$ results in a $\nu W_2(\omega')$ curve calculated from Eq. (7) which approximately averages the $\theta = 10^o$, $E = 17.7$ GeV ($q^2 \simeq 7$ GeV$^2$) data with $W < 1.8$ GeV ($\omega' - 1 \lesssim 0.5$). Presently available data with $W > 2$ GeV does not extend into the region $\omega' - 1 << 1$, but preliminary indications from the large angle data[9] indicate[5,10] a smooth scaling limit curve which also approximately averages the $\theta = 10^o$, $E = 17.7$ GeV data.

We note that similar assumptions applied to $W_1$ yield that $R = \sigma_S/\sigma_T$, the ratio of longitudinal to transverse total cross sections, goes to zero near threshold; and when applied to inelastic electron-neutron scattering, predicts that for large $q^2$, $\nu W_{2n}/\nu W_{2p} = (\frac{d}{dq^2}\left[G_n(q^2)\right]^2)/(\frac{d}{dq^2}\left[G_p(q^2)\right]^2) \xrightarrow[q^2\to\infty]{} (\mu_n/\mu_p)^2$, or approximately one-half near threshold[11]. A difference between the neutron and proton is generally to be expected if a substantial part of the inelastic electron-nucleon scattering is non-diffractive, as we propose. If we take the non-diffractive parts of the $q^2 = 0$ $\gamma p$ and $\gamma n$ total cross sections[12] as a guide, then we expect in general that $\nu W_{2n}$ will be smaller than $\nu W_{2p}$.

Finally, we note that in this paper we have found correlations of other observations with the experimentally observed scaling behavior of inelastic ep scattering but not predicted it. The connection between the behavior of the resonances and scaling which we propose hints again at a common origin for both in terms of a point-like substructure of the nucleon. Translating this "hint" at a common origin into a real quantitative theory remains, as before, an unsolved problem.

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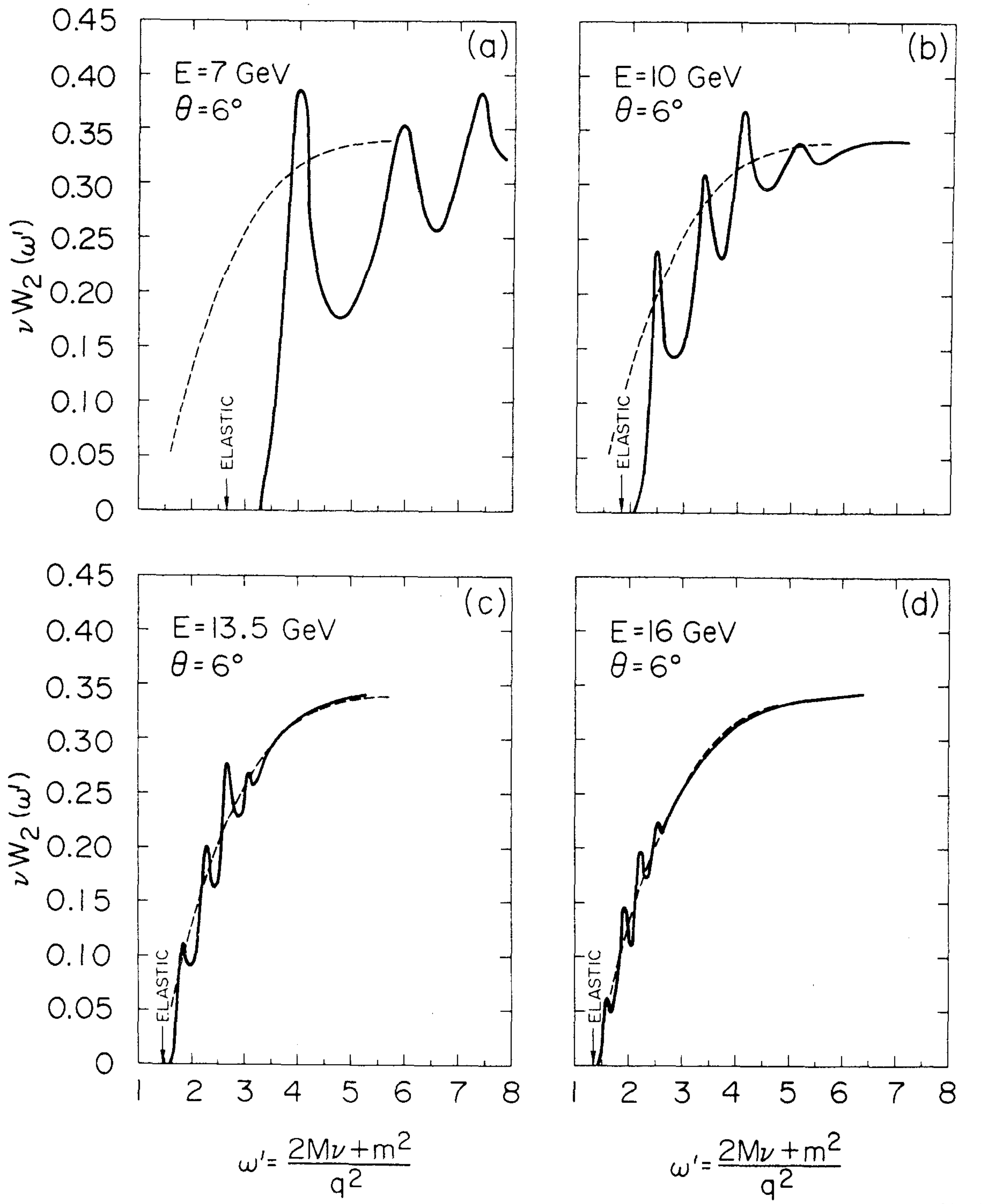


Figure 1 The function $\nu W_2$ plotted versus $\omega' = (2M\nu + m^2)/q^2$, with $m^2 = M_N^2$. The solid lines are smooth curves drawn through the $\theta = 6^o$ data at various incident electron energies. The dashed curve is the same in all cases and is a smooth curve through large $\nu$ and $q^2$ ($3 < q^2 < 7$ GeV$^2$) $W \geq 2$ GeV) $\theta = 10^o$ data. All data is plotted assuming $R = \sigma_S/\sigma_T = 0$ (see Ref. 1). Note that the E = 7 GeV, $\theta = 6^o$ data involves values of $q^2$ all of which are outside the scaling region.