national accelerator laboratory

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0430

DAMPING OF COHERENT PHASE ERRORS AT INJECTION INTO THE MAIN RING

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It was proposed by T. L. Collins that phase (or energy) errors in the main ring of an injected booster pulse may be damped out by feedback to the phase of the RF in the cavities. However, the RF phase must be shifted from its normal value to the value with feedback only for the newly injected booster pulse (with phase errors) so as not to affect the previously injected pulses for which all errors have presumably already been damped out. Q. Kerns estimated that within a gap of, say, 10 RF cycles between the previously injected booster pulse and the newly injected pulse the maximum RF phase shift attainable is only about 2°. The essential features to be investigated are (1) the damping rate under the 2° limitation and (2) the effect of nonlinearity in phase oscillation.

During injection the injected beam bunches are held in stationary RF buckets ($\phi_s = 0$) in the main ring. The phase oscillation is given by the equations

$$\begin{cases} \dfrac{d\phi}{dn} = -\Lambda E \\ \dfrac{dE}{dn} = eV \sin \phi \end{cases} \qquad (1)$$

or

$$\frac{d^2\phi}{dn^2} + \Omega^2 \sin\phi = 0, \quad \Omega^2 = eV\Lambda \tag{2}$$

where n is the revolution number, ϕ is the RF phase at the time of passage of the particle through the cavity, E is the deviation of the particle energy from the synchronous energy (8 BeV), V is the peak cavity voltage, and

$$\Lambda = \frac{2\pi h}{mc^2\gamma\beta^2}\left(\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}\right)$$

evaluated at the synchronous energy of 8 BeV. For the main ring h = (harmonic number) = 1113, γ_t = (transition γ) = 19.6, V = 1.5 MV, and we have

$$\Lambda = 0.00666 \text{ MeV}^{-1}, \qquad \Omega = 0.100 \text{ rad/rev.}$$

The bucket boundaries are sine curves with the extensions $\phi = \pm\pi$ and $E = \pm 2\frac{\Omega}{\Lambda} = \pm 30.0$ MeV. The bucket area is

$A = 16\frac{\Omega}{\Lambda} = 240$ MeV-rad.

The feedback scheme is given by the equations

$$\begin{cases} \dfrac{d\phi}{dn} = -\Lambda E \\ \dfrac{dE}{dn} = eV\sin\left(\phi + a\,\dfrac{d\langle\phi\rangle}{dn}\right) \end{cases} \tag{3}$$

where a is the feedback factor which may be a function of n and < > denotes averaging over all particles in the bunch. Thus, $\langle\phi\rangle$ is the phase of the centroid of the bunch and is the signal picked up by the phase sensor. The phase jump $a\,\frac{d\langle\phi\rangle}{dn}$ is to be limited to $\pm 2^\circ$.

For small oscillations we can linearize Equ. (3) by replacing the sine function by its argument. Furthermore, for the linear equations we can drop the averaging symbol and reinterpret ϕ and E as the variables for the centroid of the beam bunch. The second order linear equation for ϕ, then, becomes

$$\frac{d^2\phi}{dn^2} + a\Omega^2 \frac{d\phi}{dn} + \Omega^2\phi = 0 \tag{4}$$

For constant a the damping factor is exp $\left(-\frac{\Omega^2}{2}\, an\right)$ and the number of turns to reduce the amplitude by e^{-1} is

$$n_e = \frac{2}{a\Omega^2} \,. \tag{5}$$

The optimum value of a is the critical damping value

$$a_c = \frac{2}{\Omega} \tag{6}$$

for which n_e has its minimum value $1/\Omega$. For large phase errors, however, the amount of feedback $a_c \frac{d\phi}{dn}$ required for critical damping may easily exceed the hardware limit of $\alpha = 2^o$. The feedback phase jump is, therefore, $\pm\mathrm{Min}\left(a_c \left|\frac{d\phi}{dn}\right|, \alpha\right)$ where the sign is that of $\frac{d\phi}{dn}$. When the phase jump is clipped in this manner, the a $\frac{d\phi}{dn}$ term in Equ.(4) is replaced by $\pm\alpha$ giving

$$\frac{d^2\phi}{dn^2} + \Omega^2\,(\phi \pm \alpha) = 0 \tag{7}$$

which represents just a shift in the origin for ϕ. The origin in the (ϕ,E) plane changes by 2α every time $\frac{d\phi}{dn}$

changes sign. The effect of this is to initiate a new undamped oscillation with initial conditions $E = 0$ and $|\phi| = |\phi - 2\alpha|$ about the new origin. Thus, the damping achieved is 4α per phase oscillation. When the phase error has been reduced to less than 2α this scheme will lead to an overshoot. By this time, however, $a_c \frac{d\phi}{dn}$ is small and the clipping is not in effect. The final damping is that corresponding to critical damping.

One might speculate that a more efficient scheme would be to start with a small feedback coefficient a and increase it in step with the decreasing amplitude of the phase oscillation so that the maximum phase jump is always $\pm\alpha$. An approximate analysis of Equ. (4) with $a = a(n)$ leads to the function

$$a(n) = a_o \left[1 - \log (1 - n/n_e) \right] \tag{8}$$

where n_e is the e-folding revolution number given by Equ. (5) and

$$a_o = \alpha/(\Omega\phi_o). \tag{9}$$

This approximately maintains the condition

$$a \left(\frac{d\phi}{dn}\right)_{max} \approx \pm\alpha$$

where "max" implies the maximum value occurring each half phase oscillation. For $n \geq n_e$ Equ. (8) is nonsense and a is increased linearly up to a_c from $a(n_e - 1)$. The approximate damping factors for small n for the damping schemes considered are:

(a) for "constant feedback" $a = \text{const} = a_o = \frac{\alpha}{\Omega\phi_o}$,

damping factor $= e^{-\frac{1}{2}\frac{\alpha}{\phi_o}\Omega n}$

(b) for "critical feedback" $a = \text{const} = a_c = \frac{2}{\Omega}$ and

no phase jump limit, damping factor $= e^{-\Omega n}$

(c) for "clipped feedback" $a = \text{Min}\left(a_c, \frac{\alpha}{|d\phi/dn|}\right)$,

damping factor $\cong e^{-\frac{2}{\pi}\frac{\alpha}{\phi_o}\Omega n}$

(d) for "logarithmic feedback" $a = a_o\left[1 - \log\left(1 - \frac{n}{n_e}\right)\right]$,

damping factor $= e^{-\frac{\alpha}{\phi_o}\Omega n}$

The feedback programmed according to Equ. (8) seems to have a slight edge over the clipping scheme, but the damping rate decreases for large n in this scheme so that the two are extremely close. Certainly simplicity favors the clipping scheme.

The phase-space area occupied by the beam is estimated to be only about 13 MeV-rad which is small compared to the 240 MeV-rad area of the bucket. Therefore as long as the initial amplitude of the phase error ϕ_o is much smaller than π the nonlinearity in the phase oscillation should not be too severe. The effects of the nonlinearity and of the various feedback schemes are studied by using a computer. Both the clipped feedback and the programmed feedback given by Equ. (8) are examined.

The equations programmed for the computer are the difference equations (now we replace the averaging sign < >)

$$\begin{cases} \phi_{n+1} = \phi_n - \Lambda E_n \\ E_{n+1} = E_n + eV \sin\left[\phi_{n+1} + a\,(\langle\phi_{n+1}\rangle - \langle\phi_n\rangle)\right] \end{cases} \quad (10)$$

which correspond to Equations (3) and describe more closely the actual behavior of the particles in the main ring. Four hundred and forty particles distributed in a matched phase space area of 13 MeV-rad are traced with their average phase (phase of centroid) $\langle\phi_n\rangle$ evaluated every turn. Their phase points are plotted in the (E,ϕ) phase plane at specified number of turns after injection ($n = 0$). Figures 1-5 show bunch shapes after 500 turns around the machine for initial phase errors $\langle\phi_o\rangle = 10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ$. The ellipse represents the path taken by the centroid of the bunch in the absence of feedback. The clipped feedback program is used with a clipping level $\alpha = 2^\circ$. The trajectory of the bunch centroid with feedback is shown for the same cases in Figures 6-10.* Figure 11 shows the bunch after 500 turns for the log feedback function Equ. (8), starting at $\langle\phi_o\rangle = 30^\circ$. The corresponding trajectory of the centroid is shown in Figure 12.* Nonlinearity is measured by the change in the second moment of the bunch in normalized coordinates. Table I compares results for $\langle\phi_o\rangle = 30^\circ$ with the four feedback conditions mentioned previously. Table II gives similar figures for $\langle\phi_o\rangle = 10^\circ, 20^\circ, 30^\circ, 40^\circ$, and 50° using the clipping scheme. The quantity n_{100} is the number of turns for which $\sqrt{\langle\phi\rangle^2 + \langle E\rangle^2} = .01$ in normalized coordinates. The quantity

*Curve is marked every tenth turn.

σ_{500}/σ_o is the ratio of the second moment of the bunch on the 500th turn to that on the zeroth turn.

It appears from these results that something like 30^o phase error or 8 MeV energy error in the injection can be successfully corrected by feedback to the RF phase with the limit of 2^o in phase jump. The damping time required is much less than the $\frac{1}{15}$ sec. between booster pulses. For phase errors larger than 30^o the nonlinearity begins to cause significant distortion in the bunch shape leading to dilution of the phase space density. Because the exponent of the damping factor is proportional to the phase jump limit α, a small increase in this quantity would lead to considerable relaxation in injection tolerances.

ACKNOWLEDGMENT

This investigation was begun by Lloyd Smith. His preliminary result has been helpful to our work. The computer programming for the CALCOMP plots was written by Espie Nelson.

TABLE I

Comparison of Feedback Schemes

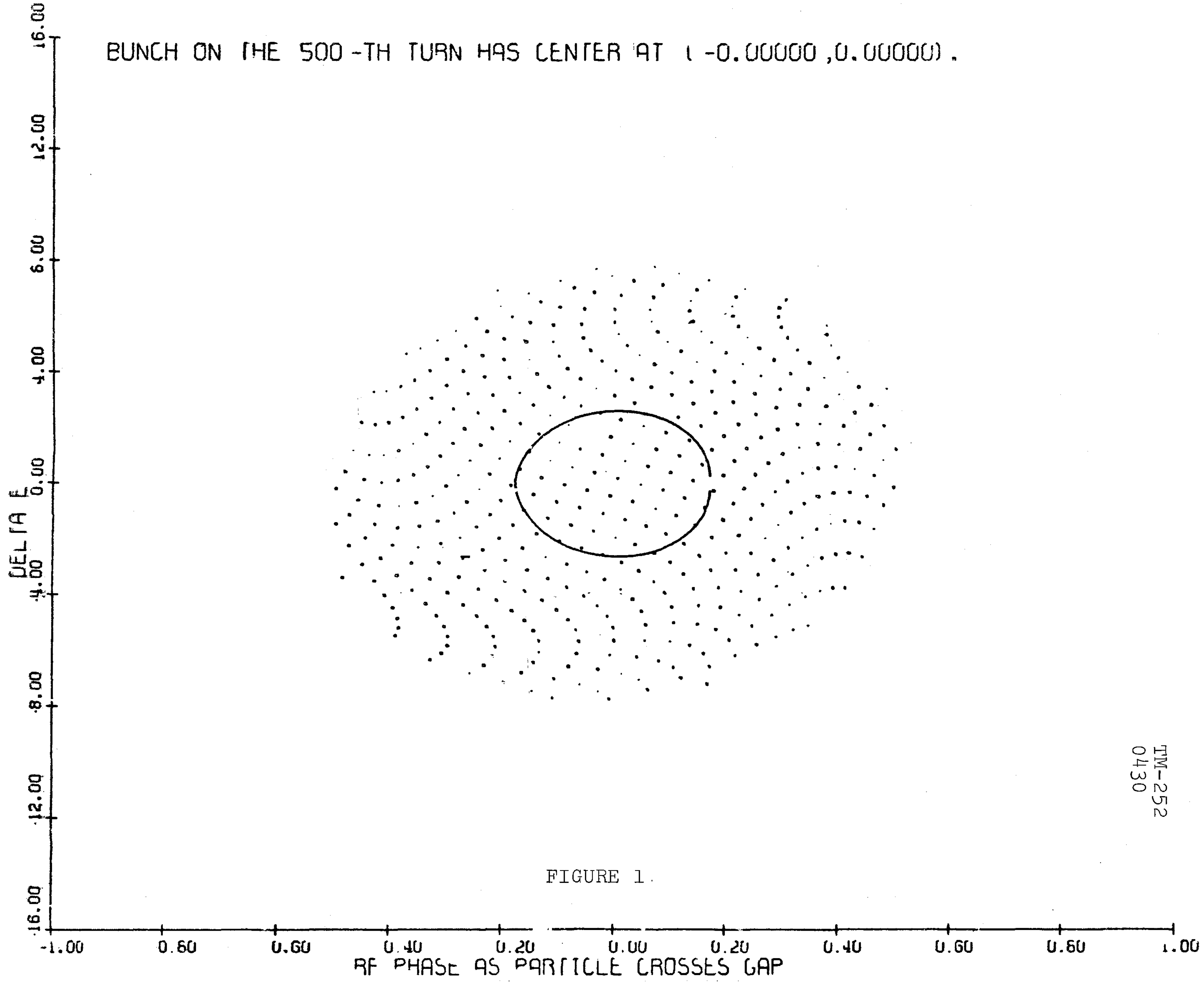
at $\langle\phi_o\rangle = 30^o$, $\langle E_o\rangle = 0$, $\alpha = 2^o$

	Feedback type	n_{100}	σ_{500}/σ_o
(a)	$a = a_o = \frac{\alpha}{\Omega\phi_o}$	∿ 1100	1.087
(b)	$a = a_c = \frac{2}{\Omega}$ (no phase jump limit)	64	1.004
(c)	$a = \mathrm{Min}\left(a_c, \frac{\alpha}{\lvert d\phi/dn\rvert}\right)$	248	1.032
(d)	$a = a_o\left[1-\log(1-n/n_e)\right]$	343	1.052

TABLE II

Clipped Feedback $a = \mathrm{Min}\left(a_c, \frac{\alpha}{\lvert d\phi/dn\rvert}\right)$ Results

$\langle\phi_o\rangle$	n_{100}	σ_{500}/σ_o
10^o	92	1.002
20^o	156	1.007
30^o	248	1.032
40^o	317	1.119
50^o	358	1.326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-12.00
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0.60
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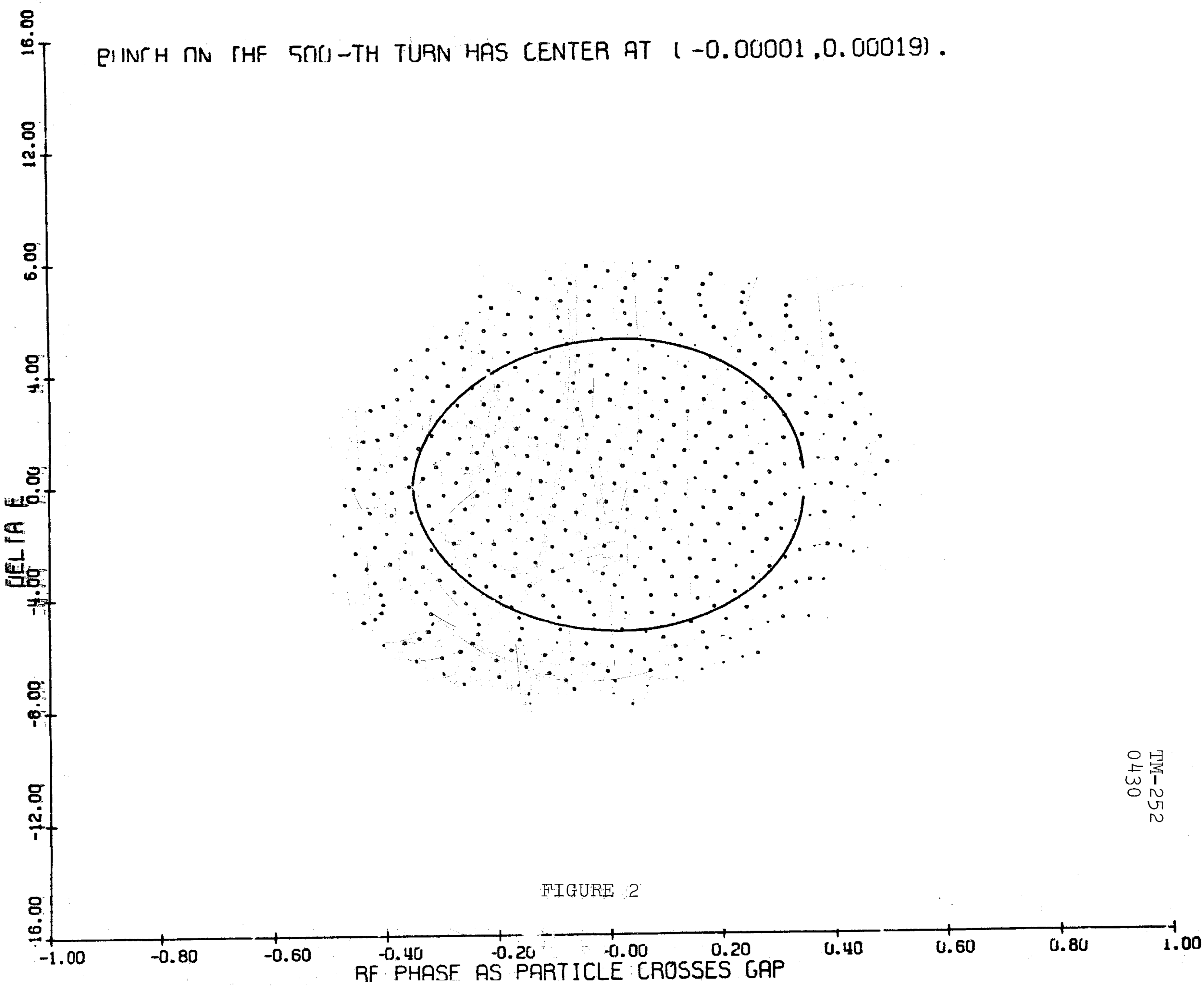
0.00
0.20
0.40
0.60
0.80
1.00
RF PHASE AS PARTICLE CROSSES GAP

FIGURE 1.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-0.60
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0.40
0.60
0.80
1.00
RF PHASE AS PARTICLE CROSSES GAP

FIGURE 2

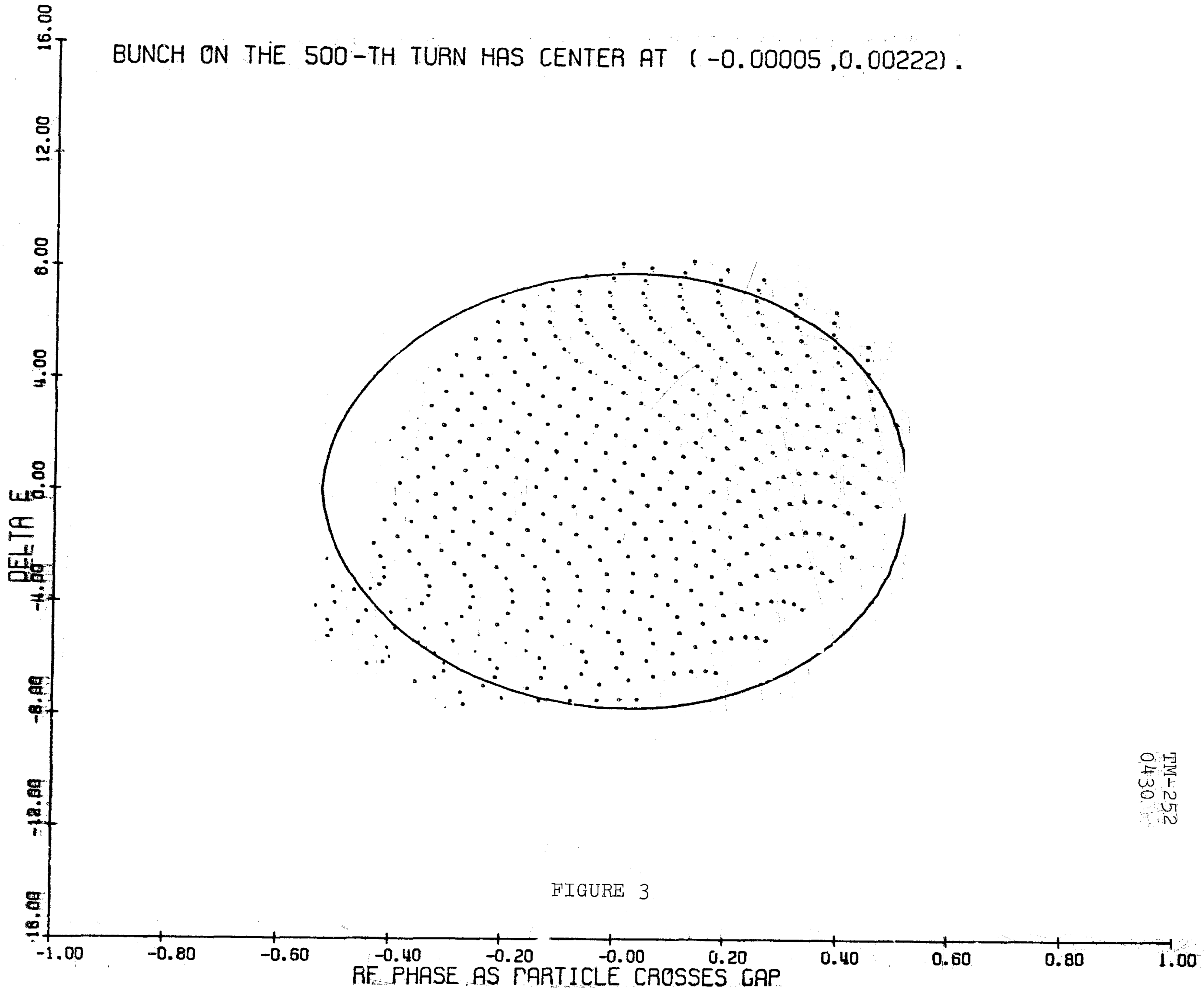

FIGURE 3

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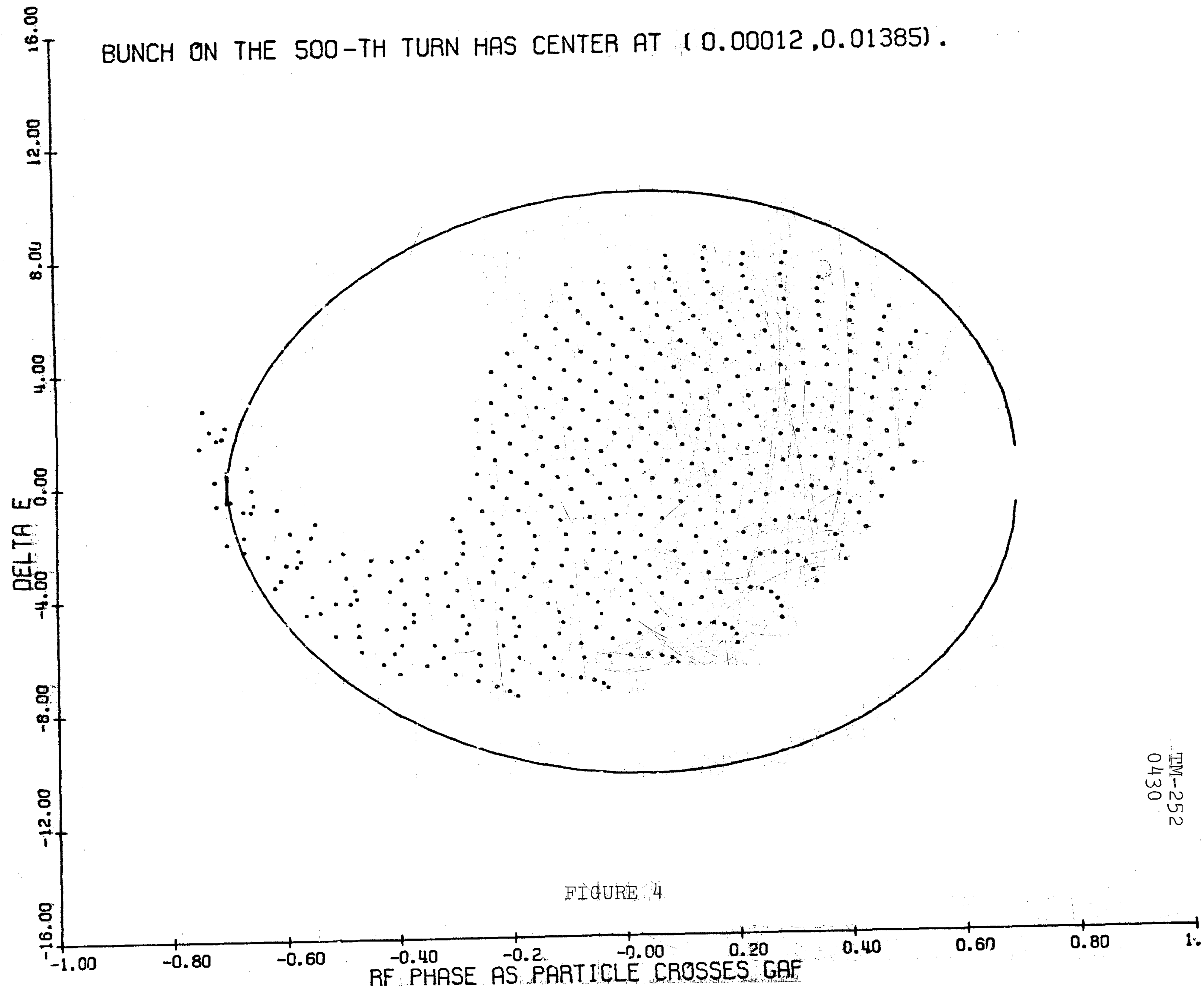

FIGURE 4

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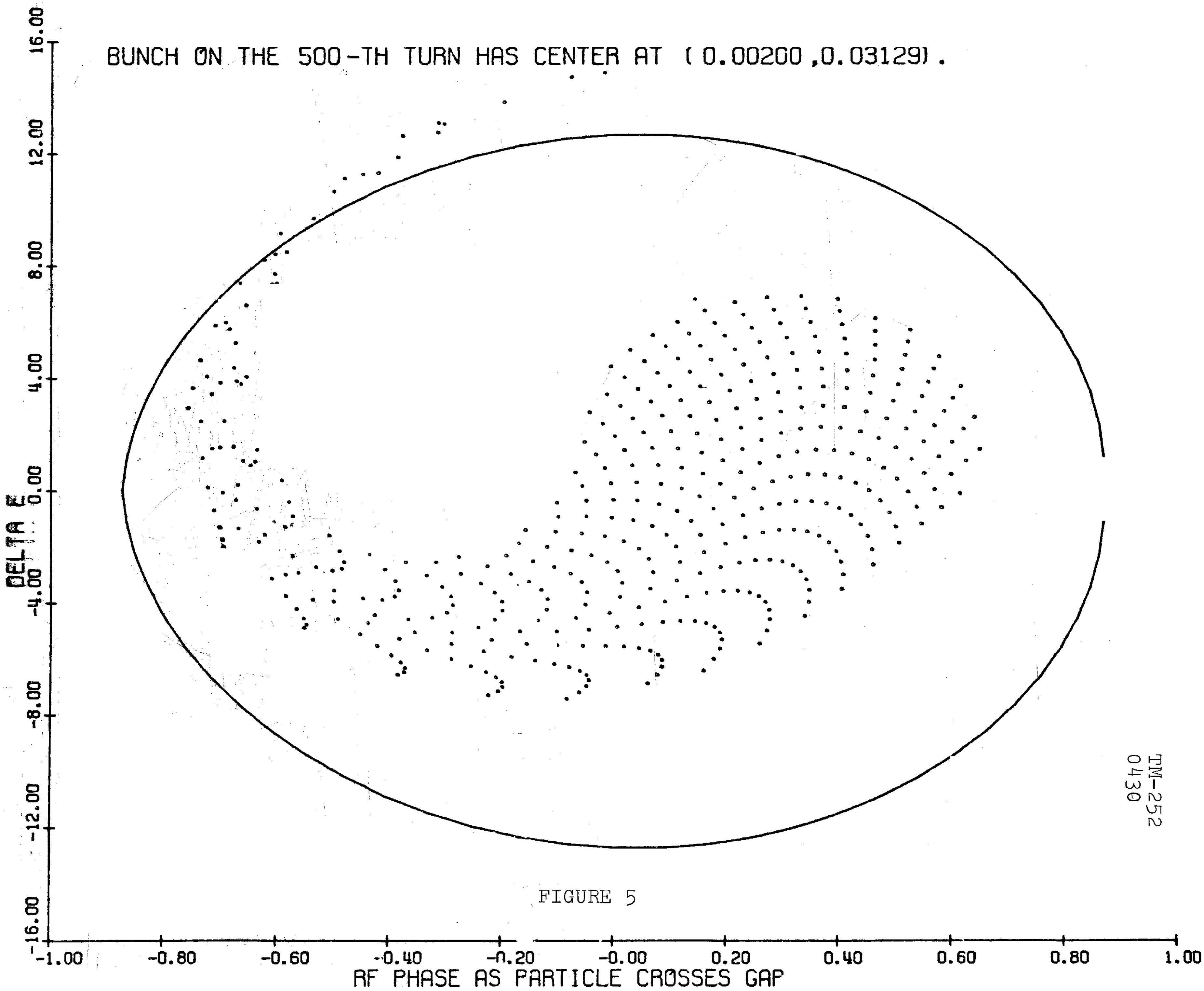

FIGURE 5

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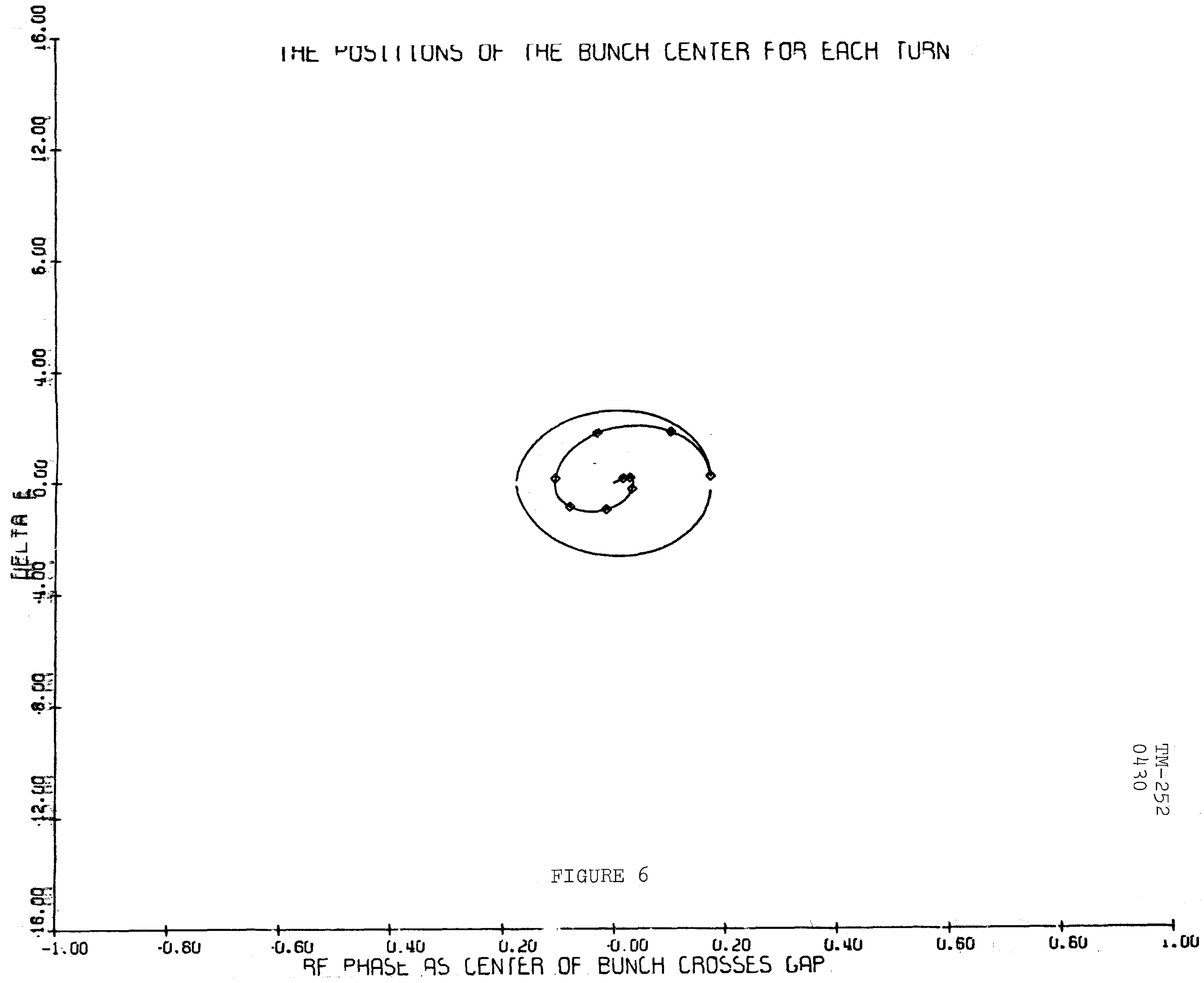
THE POSITIONS OF THE BUNCH CENTER FOR EACH TURN
16.00
12.00
8.00
4.00
0.00
-4.00
-8.00
-12.00
-16.00
DELTA E
-1.00
-0.80
-0.60
0.40
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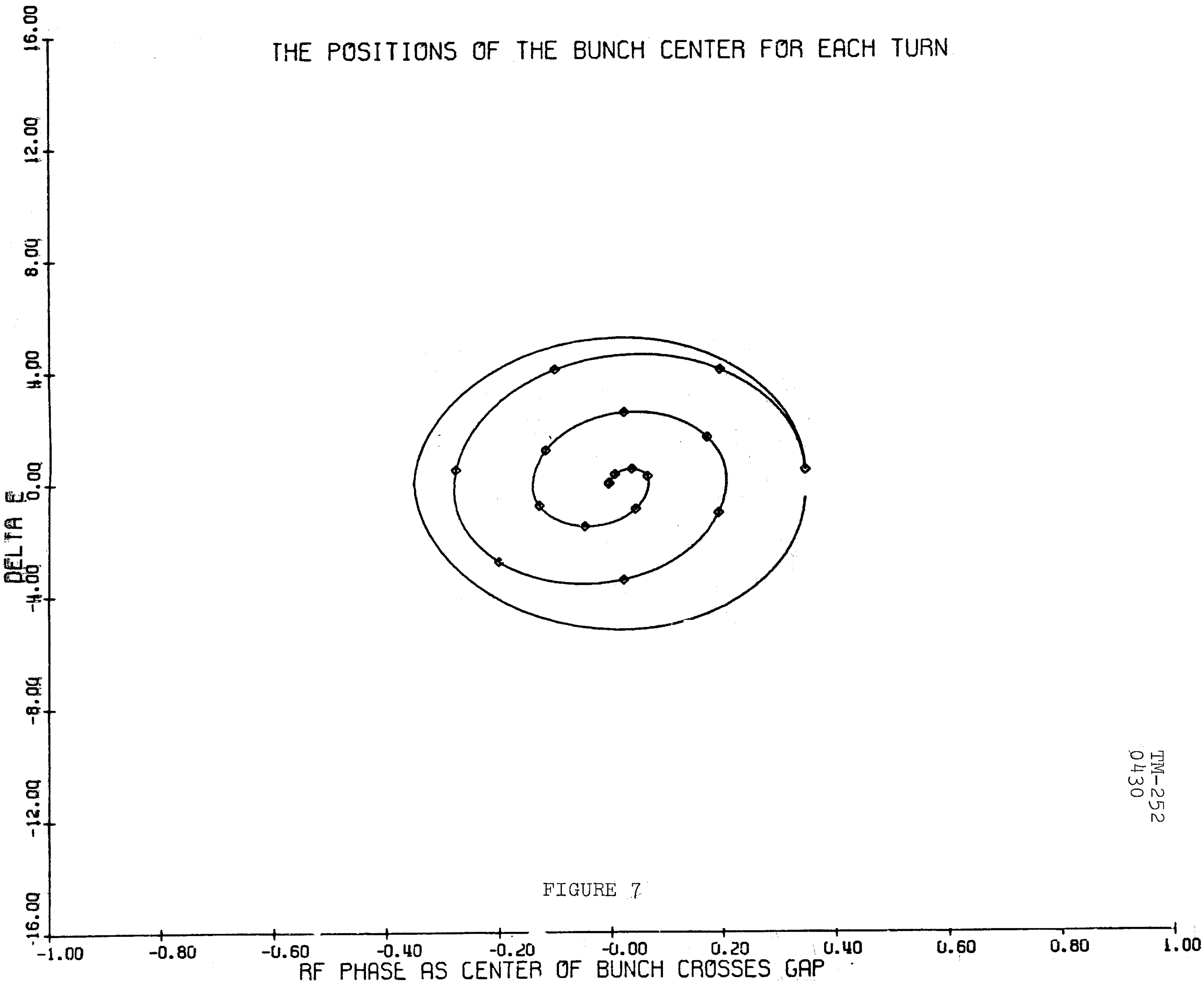
0.20
0.40
0.60
0.80
1.00
RF PHASE AS CENTER OF BUNCH CROSSES GAP

FIGURE 6


THE POSITIONS OF THE BUNCH CENTER FOR EACH TURN
DELTA E
16.00
12.00
8.00
4.00
0.00
-4.00
-8.00
-12.00
-16.00
-1.00
-0.80
-0.60
-0.40
-0.20
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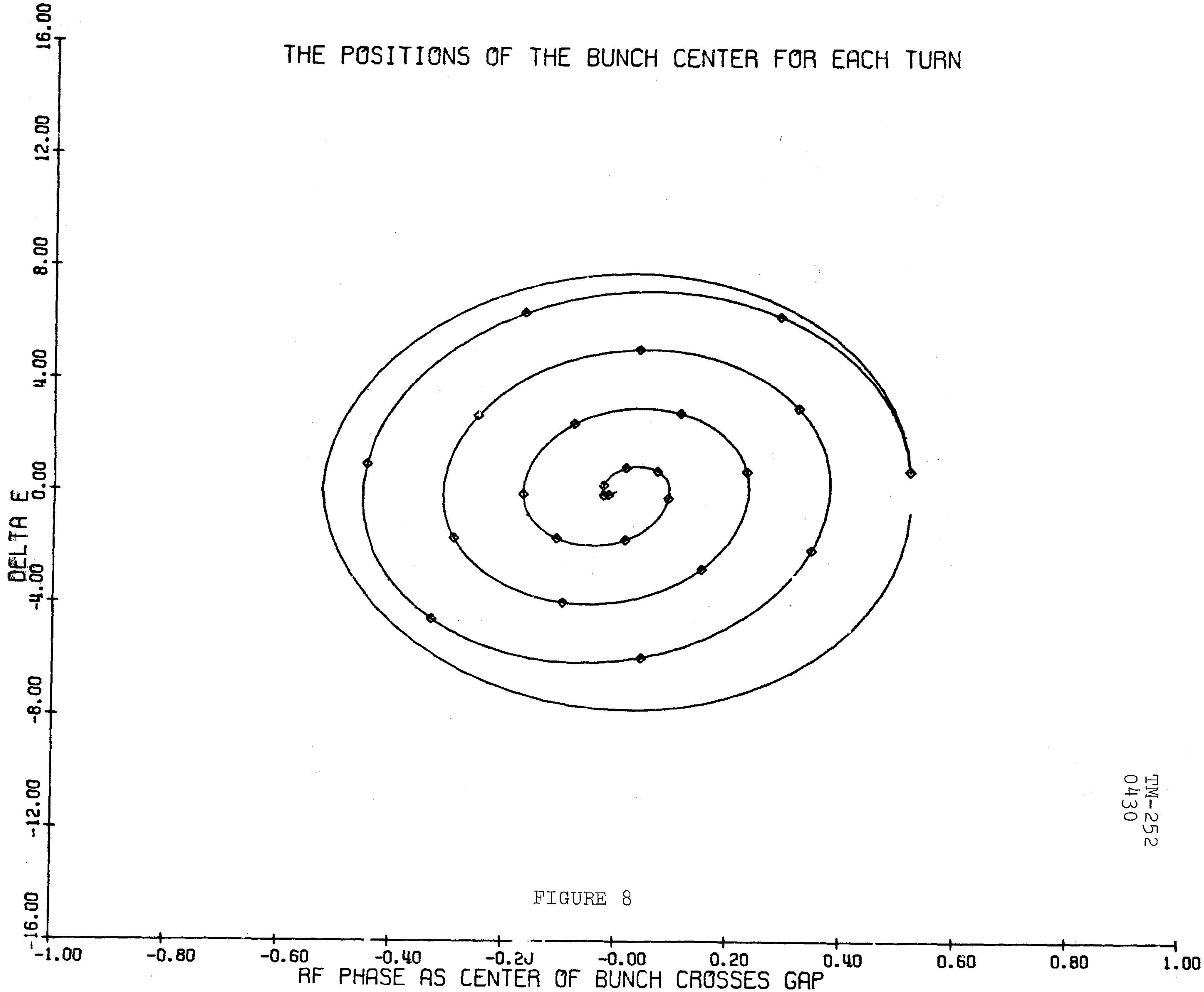
0.20
0.40
0.60
0.80
1.00
RF PHASE AS CENTER OF BUNCH CROSSES GAP

FIGURE 7


THE POSITIONS OF THE BUNCH CENTER FOR EACH TURN
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12.00
8.00
4.00
0.00
-4.00
-8.00
-12.00
-16.00
-1.00
-0.80
-0.60
-0.40
-0.2J
-0.00
0.20
0.40
0.60
0.80
1.00
RF PHASE AS CENTER OF BUNCH CROSSES GAP

FIGURE 8

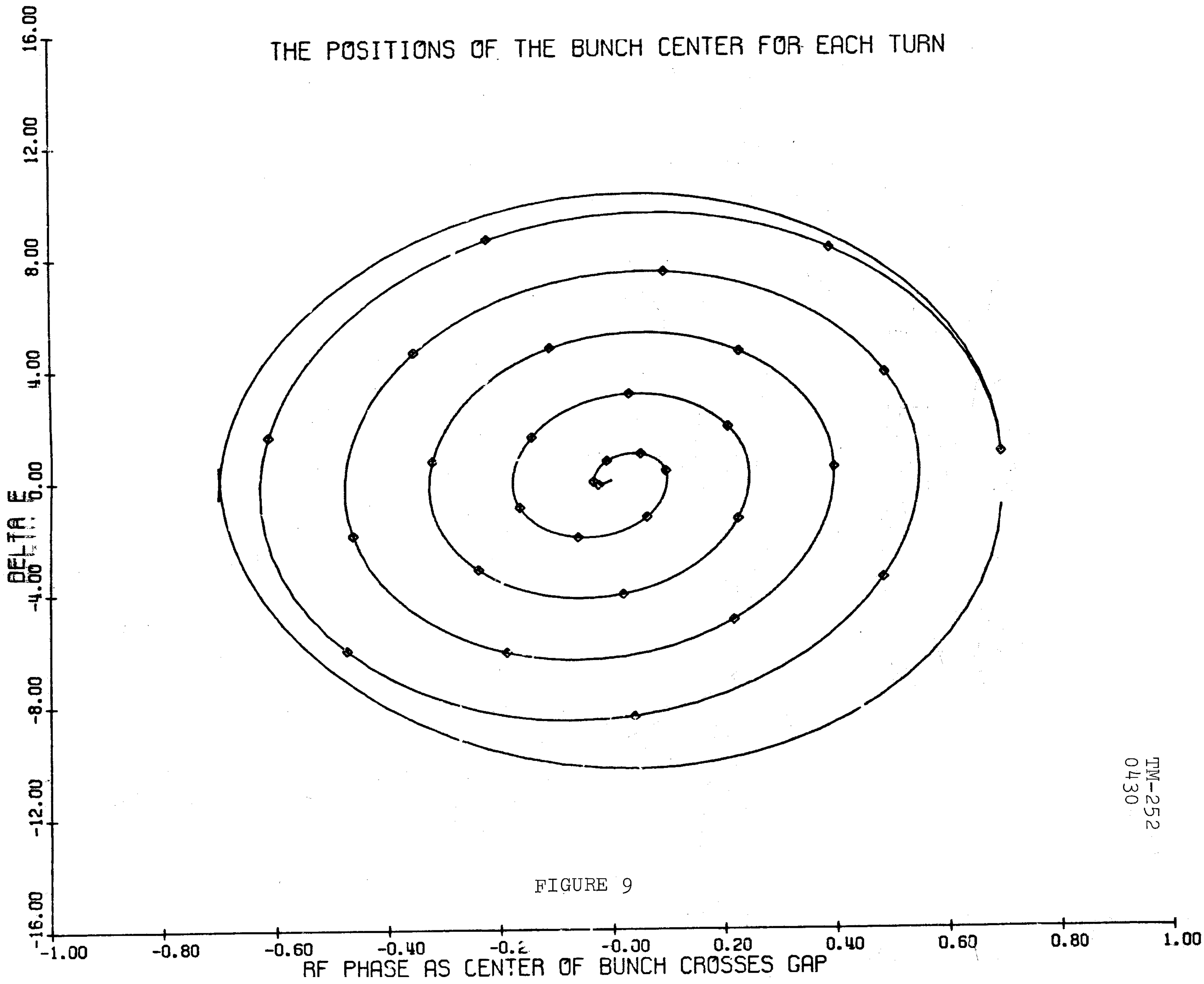

FIGURE 9

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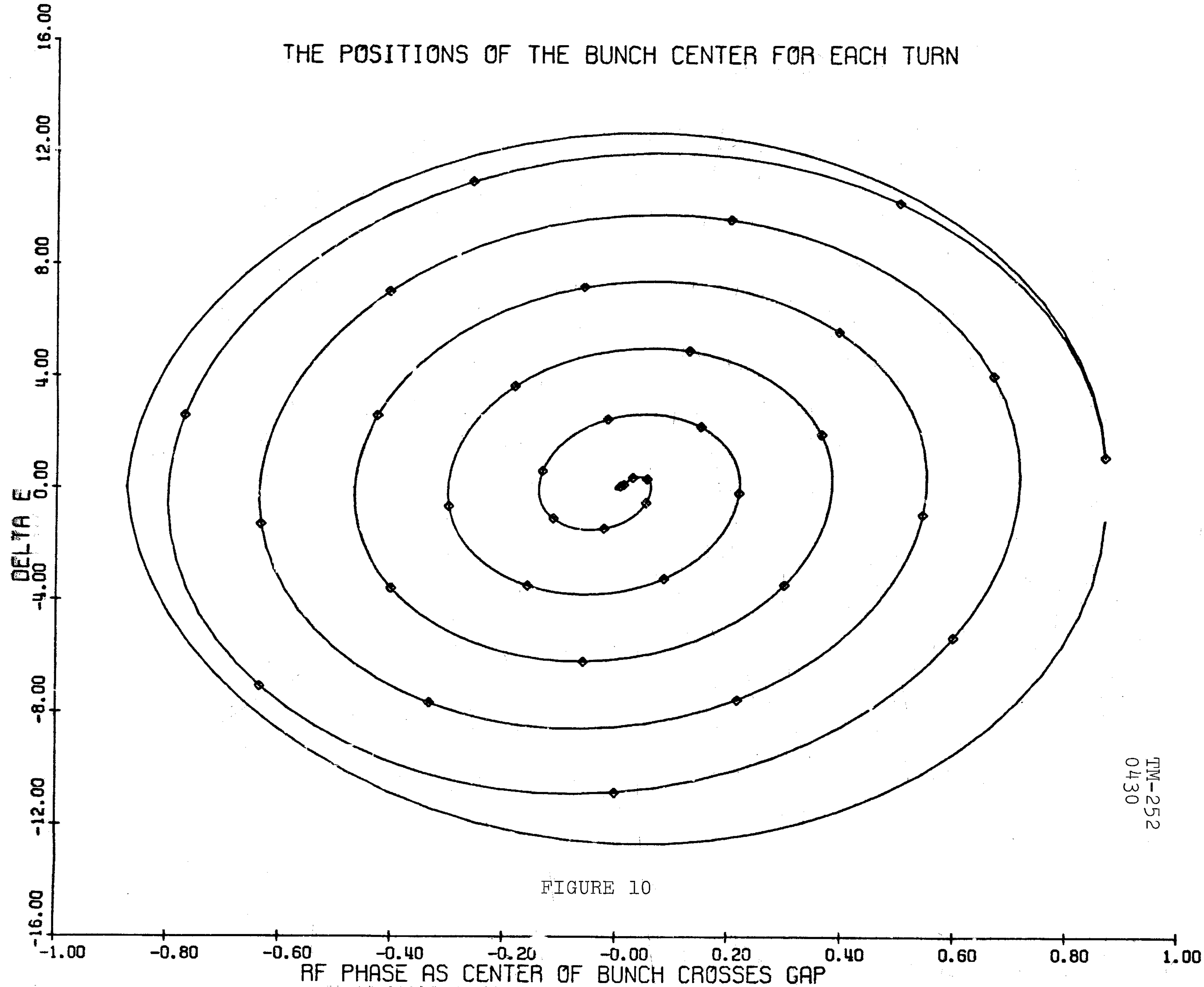

FIGURE 10

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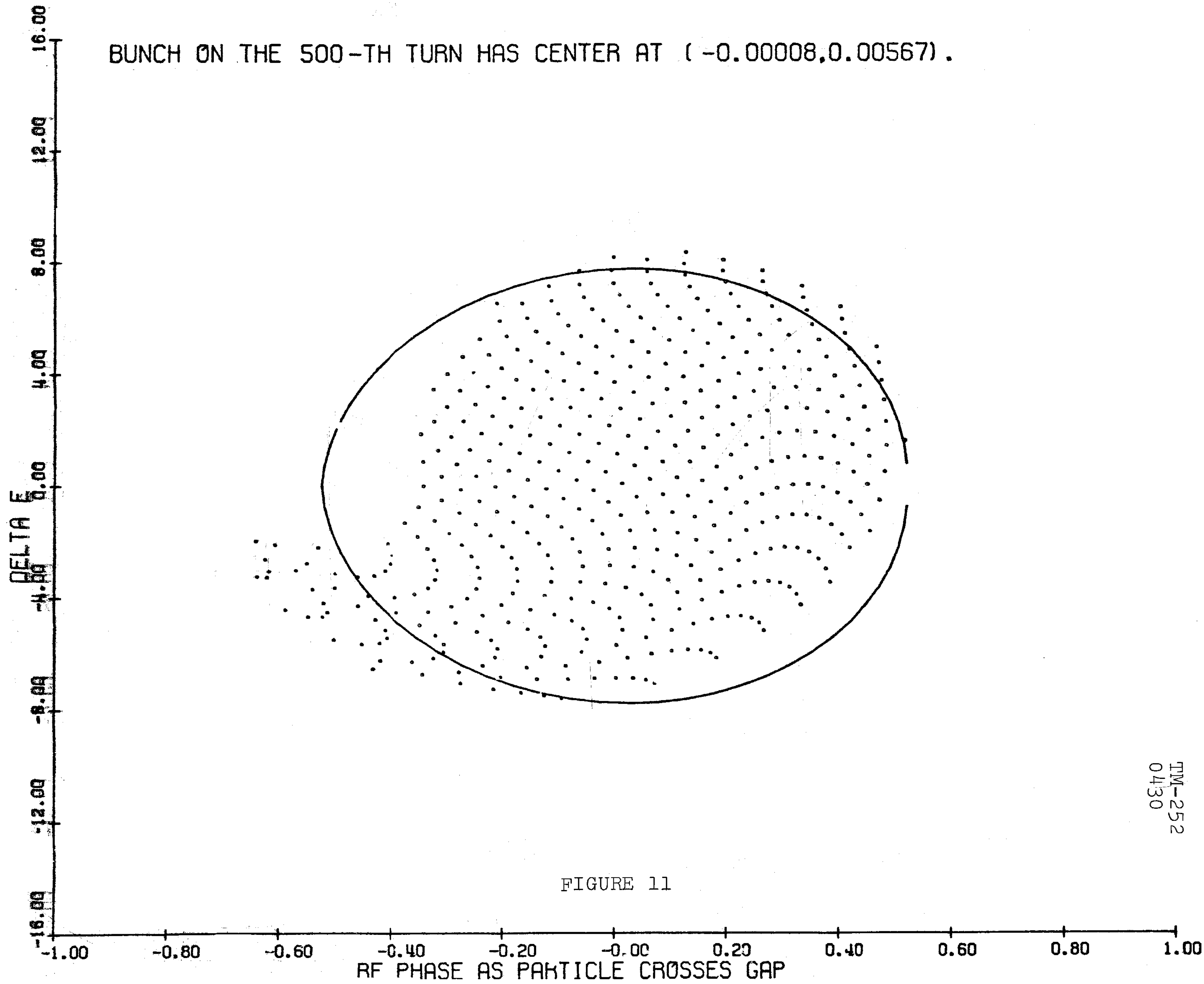
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4.00
0.00
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-8.00
-12.00
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-1.00
-0.80
-0.60
-0.40
-0.20
-0.00
0.20
0.40
0.60
0.80
1.00
RF PHASE AS PARTICLE CROSSES GAP

FIGURE 11

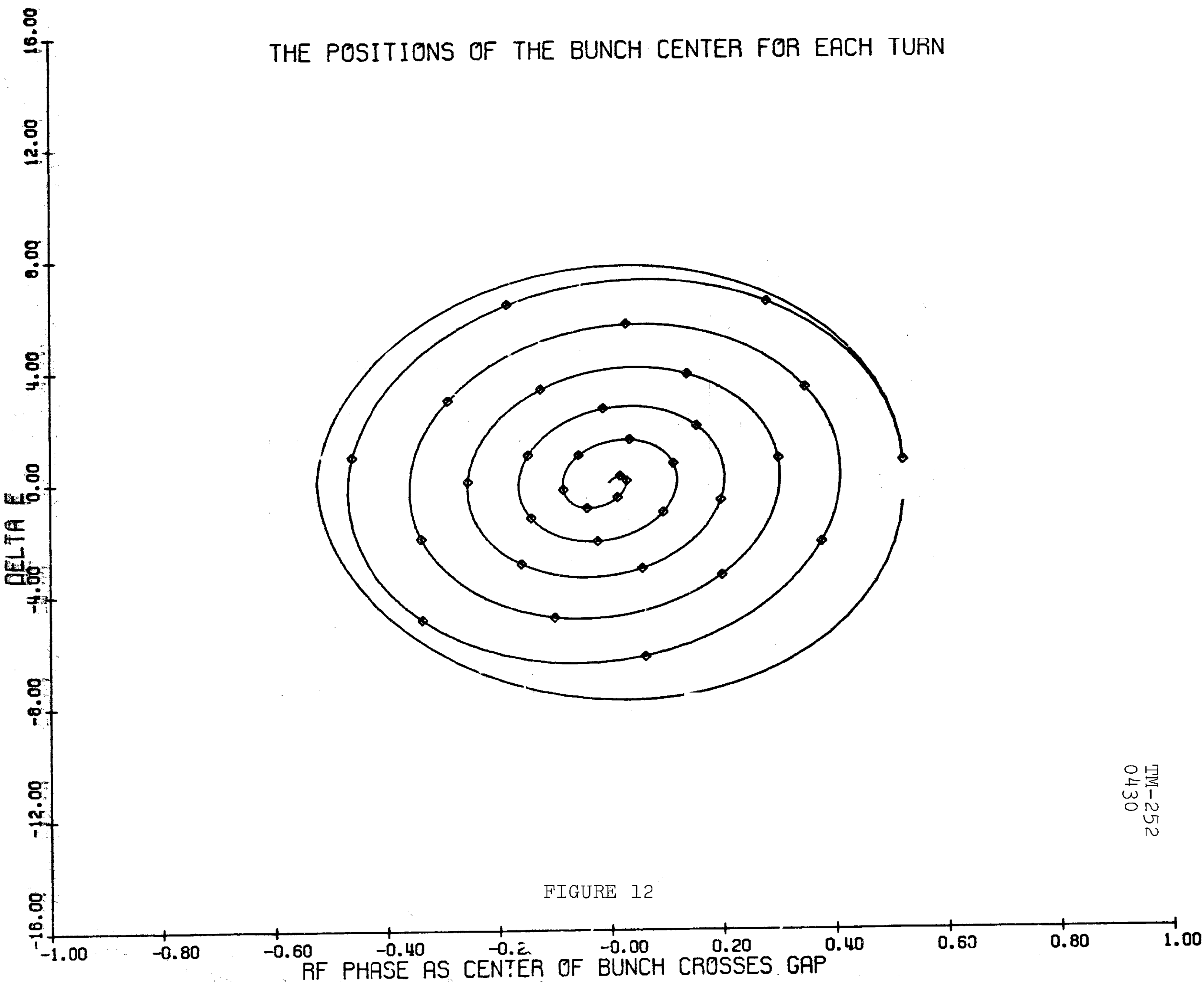
THE POSITIONS OF THE BUNCH CENTER FOR EACH TURN
DELTA E
16.00
12.00
8.00
4.00
0.00
-4.00
-8.00
-12.00
-16.00
-1.00
-0.80
-0.60
-0.40
-0.2
-0.00
0.20
0.40
0.60
0.80
1.00
RF PHASE AS CENTER OF BUNCH CROSSES GAP

FIGURE 12