LARGE ANGLE ELASTIC SCATTERING AT HIGH ENERGIES TREATED BY A STATISTICAL MODEL *)

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ABSTRACT

An analysis is made of the contribution of the "not too peripheral" collisions to large angle elastic scattering. This type of collision - which is supposed to give the main contribution to the large angle elastic scattering - is assumed to be well described by the statistical model, which then can predict the weight of the elastic channel as compared to all channels. Therefore, the ratio $\sigma_{el} : \sigma_{inel}$ can be calculated for all angles lying sufficiently outside the diffraction peak. Graphs are given for the ratio $\sigma_{el} : \sigma_{inel}$ for πN and NN scattering up to 25 GeV primary energy together with the corresponding graphs for the transformation of the angle between the centre of momentum and the laboratory system.

The non-diffractive elastic cross - section decreases (according to the statistical model) roughly exponentially with increasing total energy in the CM system.

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*) To be submitted for publication to Nuovo Cimento.

4142/TH.278
19 June 1962

I. INTRODUCTION

Large angle elastic scattering has recently attracted the attention of high-energy physicists. The available field theoretical models of high-energy scattering are mainly based on the assumption of low momentum transfer and the exchange of a few mesons [1]. Therefore they predict essentially the elastic and inelastic scattering for small angles and fail - merely by definition - for large angle elastic scattering.

On the other hand the statistical model is by its very nature mainly, though not exclusively, concerned with more central collisions [2]. In this model the probabilities for the various channels as e.g.,

$$
\begin{aligned}
\pi+N \rightarrow\ & \pi+N \\
& 2\pi+N \\
& \vdots \\
& n\pi+N+K+\bar{K} \\
& n\pi+Y+K \\
& \text{etc.,}
\end{aligned}
$$

are calculated under the assumption that the total energy in the CM system is available for particle production (which is equivalent to assuming a rather central collision). In a pictural language one may say that a compound system is formed, in which all the possible channels are virtually present, after which finally one is selected with a probability proportional to a certain average of the squared transition matrix element times the phase-space volume of this final state. Clearly, there is then a non-vanishing probability that this compound system returns to just the initial channel, although this probability decreases rapidly with increasing energy, since then the number of open channels and their weight becomes large.

It is now this kind of elastic scattering - henceforward called "compound-elastic" scattering - which might possibly give the main contribution to elastic scattering sufficiently outside the diffraction region.

However, the three weak points in this approach are that we do not know:

a) what the angular distribution is; because the statistical model is not fit to answer such detailed questions.

b) what percentage of the collisions should be considered central.

c) whether the contribution considered here is not completely hidden under the tail of the diffraction peak.

As to the angular distribution we remark that clearly in a central collision, as we picture it, the colliding particles have lost their memory as to their initial direction in their way through the compound system and as the contributing angular momenta will be low we might expect a fairly isotropic distribution at larger angles in the CM system. Experiments seem to confirm it.

In fact, a simple consideration shows which angular momenta will contribute to the formation of the compound elastic channel and that this number is independent of the primary energy, unlike for the formation of a compound nucleus in nuclear physics where one expects angular momenta up to $\approx kR$ to contribute. A necessary, though not sufficient, condition for the formation of a compound intermediate state is that the available energy can reach a state of thermodynamic equilibrium during the time of contact of the colliding particles. If b is the impact parameter (see Fig. 1), then the time t of contact between the particles should be greater or of the order of b, so that the interaction, where the maximum propagation velocity is $c = 1$, can spread out over the whole of both particles before they part again from each other.

On the other hand $t \approx d = 2r/\gamma$ if r is the particle radius ($\approx$ interaction radius) and $\gamma = E/2m$ (if both particles have equal mass m; otherwise the γ for the heavier mass is to be taken). Thus we have $b \lesssim \frac{2r}{E}/2m$. Now the angular momentum belonging to the maximum b is $l_{max} = b_{max} \cdot k$, where k is the CM momentum: $k \approx E/2$ for energies of several GeV. This gives then $l_{max} \approx 2rm$. For NN collisions one has $m = 1$ and $r \approx 5$, hence $l_{max} = 10$ independent of the incident energy.

We shall, although we have no good arguments for it, assume isotropy in the CM system. In toto, we thus expect mainly two contributions to the elastic scattering: the forward diffraction peak plus a small isotropic component (see Fig. 2).

In the next paragraph we shall come back to the second point concerning the percentage of central collisions.

About the last difficulty we remark the following: if we consider the simplest scattering model, namely the black sphere, the partial amplitudes $\tau_l(s)$ in the expansion for the scattering amplitude

$$T(s,t) = \sum (2l+1)\, P_l\left(1 + \frac{t}{2k^2}\right) \cdot \tau_l(s)$$

[where $s = E_{CM}^2$ and $t = -(\text{four-momentum transfer})^2$],
will be almost purely imaginary and equal up to $l \approx kr$. Now, if σ_{tot} should be constant (independent of s) then also $\tau_l(s)$ should be independent of s. In this case the elastic cross-section for all partial waves up to $l_{max} \approx 10$ will be proportional to $1/s$. This should then correspond to that part of the elastic diffraction cross-section which is due to the compound

states. However, in the statistical model the elastic cross-section goes down exponentially with the CM energy, that is as $\approx e^{-\alpha\sqrt{s}}$. So it might well be that the thing calculated here is completely hidden under the diffraction scattering even for large angles. There seems, however, to be no experimental indication for large angle scattering to be proportional to $1/s$. In fact, it seems to drop much faster [10], so that the present description might indeed bear some relation to reality. The numbers given here should then be regarded as a lower limit.

II. THE MODEL

1) General description

In the past years many statistical calculations have been carried through at CERN. In all of them the probabilities P_b for all channels (labelled by a number b) have been calculated and, if we give - once and for ever - the compound elastic channel the number $b = 0$, then the quantities

$$R \equiv \frac{P_0}{\sum_{b=0}^{\infty} P_b} \tag{1}$$

are available; some of them have been published [2-8]. These results are partly for pp and partly for πp collisions. Figure 3 shows a graphical representation of these results. The circles and triangles indicate the values of (1) which are taken from actual calculations, whereas the curves are interpolations and extrapolations from these points taking advantage of the fact that we know that the curves must go to 1 for vanishing primary kinetic energy. It is seen that the curve

$$\lg \frac{P_0}{\sum P_l} \quad \text{versus} \quad E_{CM}$$

is almost a straight line in the pp case. As there is little doubt that in the πN case the behaviour should be similar, we may safely extrapolate that curve from the few calculated points to somewhat higher energies.

It is not very well known to what extent the ratio (1) represents the ratio of the (non-diffractive) elastic cross-section to the total minus the diffraction cross-section. Let us denote by $\sigma_{c,el} \equiv \sigma_{el} - \sigma_{diffr}$ the compound elastic cross-section and by $\sigma_{c,tot} = \sigma_{total} - \sigma_{diffr}$ the non-diffractive part of the total cross-section. For vanishing primary kinetic energy we have $\sigma_{c,tot} \to \sigma_{c,el}$, whereas for energies where inelastic processes - in which we are interested - become important, $\sigma_{c,tot} \to \sigma_{inel}$. Therefore, in most cases one may replace $\sigma_{c,tot}$ by σ_{inel}. Then the relation between the experimental (σ) and the theoretical qualities is

$$\sigma_{c,el} = \alpha P_0$$

$$\sigma_{c,tot} = \frac{1}{\beta}\left(\alpha \sum_{l=0}^{\infty} P_l\right)$$

so that

$$\frac{\sigma_{c,el}}{\sigma_{c,tot}} = \beta \frac{P_0}{\sum_{l=0}^{\infty} P_l} \tag{2}$$

The compound elastic cross-section does not contain, by definition, the most peripheral contributions which make up the diffraction peak, whereas $\sigma_{c,tot}$ will contain peripheral as well as central collisions. The $\sum P_b$, however, has been calculated under the assumption of rather central collisions. Therefore $\sigma_{c,tot}$ will contain contributions which partly are neglected in $\sum P_b$. Hence $\beta \leq 1$. In the worst case β should represent the ratio:

$$\frac{\text{Number of central collisions}}{\text{Number of all inelastic collisions}} \text{ (in a given scattering process)}$$

where the "central collision" is defined as a collision where most of the energy is transformed into "heat" available to particle production. Such a ratio has been estimated in Ref. [2] for pp collisions, using the same model as above in the consideration of the maximum angular momentum involved.

In Ref. [2] a table was presented in which the numerical consequences of such a model were shown in two estimates : one of which might be optimistic, and the other one pessimistic. We give here the average of the two estimates:

$\frac{\sigma_{centr}}{\sigma_{geom.}} \approx$	1.0	at	0	GeV	primary energy
	0.8	at	2.75	GeV	primary energy
	0.5	at	6.2	GeV	primary energy
	0.15	at	25	GeV	primary energy

TABLE I

If one compares multiplicities and spectra as calculated by the statistical theory with experimental results at various energies [4),8),9)], one sees that the values in Table I are reasonable if one replaces σ_{geom} by σ_{inel}: J. v. Behr and F. Cerulus *) found, treating $\sigma_{centr}/\sigma_{inel}$ as a parameter to be determined from the experiment, the value ≈ 0.5. At 25 GeV the

*) At 6.2 GeV primary energy (pp collision).

production of antinucleons is smaller than the statistical theory prediction by roughly a factor 10. As the antinucleon production depends critically on the available energy, one may say that in about 10 % of all collisions the full energy is available (these 10 % of all collisions follow the statistical theory) and in the rest there is not enough energy to produce an appreciable number of pairs. This interpretation fits rather well with the value $\sigma_{centr} / \sigma_{inel} \approx 0.15$ which one would take frome Table I.

Even if we would take the values of Table I as granted, we could not say that the value β equals that ratio because it is not known how much is neglected in $\sum P_b$. We shall therefore leave β open and say that it lies between 1 and the values in Table I, but probably more on the side of the latter ones. We have then with (2)

$$\sigma_{c,el} = \sigma_{c,tot} \cdot \beta \cdot \frac{P_o}{\sum P_b} \qquad (3)$$

Taking $\sigma_{c,tot} \approx \sigma_{inel}$ from the experiment and $P_o / \sum P_b$ from the statistical model, we obtain $\sigma_{c,el}$ within the uncertainty of β. As the last factor $P_o / \sum P_b$ varies in the energy region between 0 and 25 GeV by many orders of magnitude [see Fig. (3)], this uncertainty caused by β might be taken not too seriously.

2) <u>Angular distribution</u>

We assume the angular distribution in the CM system to be as shown in Fig. 2. Then the compound elastic differential cross-section will be roughly isotropic, hence

$$\frac{\partial \sigma_{c,el}}{\partial \Omega} \approx \frac{\sigma_{c,el}}{4\pi}$$

and in the CM system the number of particles scattered into the angular interval $\theta^{o}_{CM} \leq \theta_{CM} \leq \theta^{1}_{CM}$ is proportional to

$$\frac{1}{2}\sigma_{c,el} \cdot \int_{\cos\theta^{o}}^{\cos\theta^{1}} d(\cos\theta) = \sigma_{c,el} \cdot \left(\frac{\cos\theta^{o} - \cos\theta^{1}}{2}\right)_{CM} \qquad (4)$$

As the intensity might be very small in view of the smallness of $\sigma_{c,el}$, one will - in many cases - integrate over a large angular interval, thereby reducing also the influence of anisotropy. All particles found in the above CM angular interval will show up in the corresponding lab. angular interval.

For the convenience of the experimentalists we have calculated the angular transformation

$$\cos\theta_{CM} = \left\{\frac{(E+m)(\mu^2+Em)+(E^2-\mu^2)\cos\theta_L\sqrt{\mu^2\cos^2\theta_L+m^2-\mu^2}}{(E+m)^2-(E^2-\mu^2)\cos^2\theta_L} - E\right\} \cdot \frac{m^2+\mu^2+2Em}{m(E^2-\mu^2)} + 1$$

for various total primary lab. energies E, and for the two cases πp and pp (m = proton mass, μ = pion or proton mass respectively). The results are displayed in Figs. 4 and 5 in a self-explanatory way.

One obtains then the compound elastic cross-section for scattering into the angular interval $\theta^{o}_{lab} \leq \theta_{lab} \leq \theta^{1}_{lab}$ by reading off $(\cos\theta^{o} - \cos\theta^{1})_{CM}$ from Fig. 4 or 5, and $P_o/\sum P_b$ from Fig. 3, inserting both into the expression

$$\sigma_{c,el}\Big|_{\theta^{o}_{CM}}^{\theta^{1}_{CM}} = \sigma_{inel} \cdot \beta \cdot \frac{P_o}{\sum P_b} \cdot \left(\frac{\cos\theta^{o} - \cos\theta^{1}}{2}\right) \qquad (5)$$

where σ_{inel} is the experimental inelastic cross-section and β the not well-known correction factor discussed above.

3) Isospin analysis

There are three interesting cases, namely pp, π^+p and π^-p. The first two ones need no further comment since the initial states are isospin eigenstates. The last case is different as it is no pure T state and charge exchange scattering is possible. In this case the curve $P_o/\sum P_b$ versus energy would refer to the sum of compound elastic plus charge exchange scattering. Namely the results of the statistical theory were found by calculating all P_b's for the $T = 1/2$ and $T = 3/2$ channel separately and adding them afterwards with weights 2/3 and 1/3 respectively. This is exact for the elastic channel as there the interference terms drop out [see after Eq. (7)] and it is approximately exact in the production channels, because then the random phase will make the individual interference terms small.

In order to separate $\sigma_{c,el}$ into charge exchange and non-charge exchange parts, we introduce the $T = 1/2$ and $T = 3/2$ elastic amplitudes and the cross-sections, using the following notation

$$\left.\begin{aligned}
\langle p\pi^+ | p\pi^+ \rangle &= a_3 \\
\langle p\pi^- | p\pi^- \rangle &= \tfrac{1}{3}\,(a_3 + 2a_1) \\
\langle p\pi^- | n\pi^o \rangle &= \tfrac{\sqrt{2}}{3}\,(a_3 - a_1) \\
\sigma_{c,el}(p\pi^+ \rightarrow p\pi^+) &\equiv \sigma^+ \\
\sigma_{c,el}(p\pi^- \rightarrow p\pi^-) &\equiv \sigma^- \\
\sigma_{c,el}(p\pi^- \rightarrow n\pi^o) &\equiv \sigma^o
\end{aligned}\right\} \qquad (6)$$

The numerical coefficients are squares of Clebsch - Gordan coefficients pertaining to the πN system. Then, apart from irrelevant factors

$$\left.\begin{aligned} \sigma^+ &= |a_3|^2 \\ \sigma^- &= \frac{1}{9}\left(|a_3|^2 + 4|a_1|^2 + 4\,\mathrm{Re}\,(a_1 \cdot a_3)\right) \\ \sigma^0 &= \frac{2}{9}\left(|a_3|^2 + |a_1|^2 - 2\,\mathrm{Re}\,(a_1 \cdot a_2)\right) \end{aligned}\right\} \qquad (7)$$

[Obviously $\sigma^- + \sigma^0 = \frac{1}{3}|a_3|^2 + \frac{2}{3}|a_1|^2$, which justifies the way in which the statistical results for $\pi^- p \rightarrow$ anything were calculated.] As in the high energy region there is no particular reason why $|a_1|^2$ and $|a_3|^2$ should be very different in the elastic channel, we may put them equal *) and obtain in this case

$$\left.\begin{aligned} &\sigma^- + \sigma^0 = \sigma^+ \\ &\frac{\sigma^-}{\sigma^- + \sigma^0} = \frac{5}{9} + \frac{4}{9}\cos\varphi = \frac{\sigma^-}{\sigma^+} \qquad ; \quad \cos\varphi = \frac{\mathrm{Re}\,(a_1 \cdot a_3)}{|a_1 \cdot a_3|} \\ &\frac{\sigma^0}{\sigma^- + \sigma^0} = \frac{4}{9}\left(1 - \cos\varphi\right) = \frac{\sigma^0}{\sigma^+} = \frac{\sigma^+ - \sigma^-}{\sigma^+} \end{aligned}\right\} \qquad (8)$$

Nothing can be said about $\cos\varphi$ theoretically. But if both, σ^+ and σ^-, have been measured, then Eq. (8) may serve to determine the factors $\sigma^-/(\sigma^- + \sigma^0)$ and $\sigma^0/(\sigma^- + \sigma^0)$ which have to be multiplied into the value $\sigma_{c,el}$ of Eq. (5) in order to find the $\pi^- p \rightarrow \pi^- p$ and $\pi^- p \rightarrow \pi^0 n$ compound elastic cross-sections separately. This factor should be of the order of, but somewhat smaller than one.

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*) In fact, in the statistical theory they are assumed to be equal. But, of course, $P_0 / \sum P_b$ is not equal in the $T = 1/2$ and in the $T = 3/2$ states as, there, all inelastic channels contribute.

III. RESULTS

1) In Fig. 3 are displayed the values of $P_o/\sum P_b$ for pp, and π-nucleon collisions as functions of the total CM energy. [The values of $P_o/\sum P_b$ in the channels T = 1/2 and T = 3/2 are not so different that it is worthwhile to draw two curves. The one shown refers to $\frac{1}{3} \cdot (T = \frac{3}{2}) + \frac{2}{3}(T = \frac{1}{2})$. The values for T = 3/2 are only a few percent lower.]

2) Figures 4 and 5 give for the cases π N and NN the transformation of the angle from the CM to the lab. system with the primary energy (lab) as parameter.

3) The cross-section for compound elastic scattering between given lab. angles is then found from these graphs and Eq. (5) if pp and π^+p collisions are concerned. In the case of π^-p scattering the result has to be multiplied with the factors given by Eq. (8) for the two channels π^-p and π^on respectively.

4) The factor β, taking into account the probability of a central collision, is not well known. A pessimistic estimate is given in Table I.

ACKNOWLEDGEMENTS

We wish to thank Dr. L. Jones for drawing our attention to the fact that measurements had been made and were in qualitative agreement with statistical model results. This initiated the present work, to which Dr. L. Jones contributed valuable discussions. Some critical remarks by Dr. H. Lehmann (Hamburg University) are greatly appreciated.

We thank Mrs. Trentini for drawing the curves and the DD Division at CERN for running our computer programme for the angle transformation.

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FIGURE CAPTIONS

Fig. 1 For the definition of "central collisions".

Fig. 2 The (ad hoc) assumed angular distribution in the centre of momentum frame.

Fig. 3 The branching ratio

$$\frac{P_0}{\sum_{\ell=0}^{\infty} P_\ell}$$

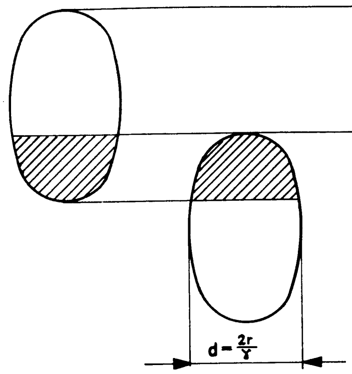
as function of E_{CM} from statistical calculations. References:

- black circles : Šoln [6)]
- open circle : Michel [7)]
- black triangles : Hagedorn [2),5)]
- open triangle : Cerulus and Hagedorn [8)].

Fig. 4 Transformation of the pion scattering angle for πp collisions. The parameter of the curves is the total pion lab. energy in GeV.

Fig. 5 Transformation of the proton scattering angle for pp collisions. The parameter of the curves is the total proton lab. energy in GeV.

FIG. 1



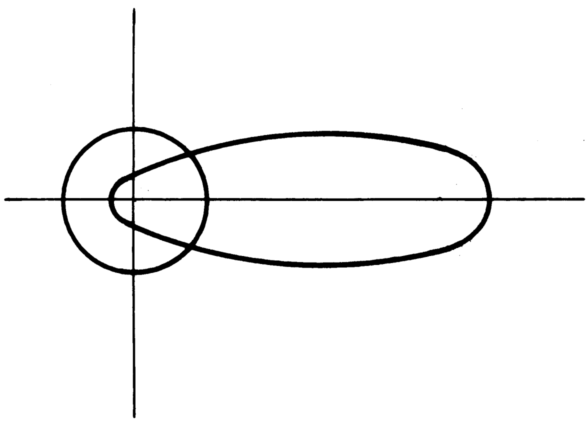


FIG. 2

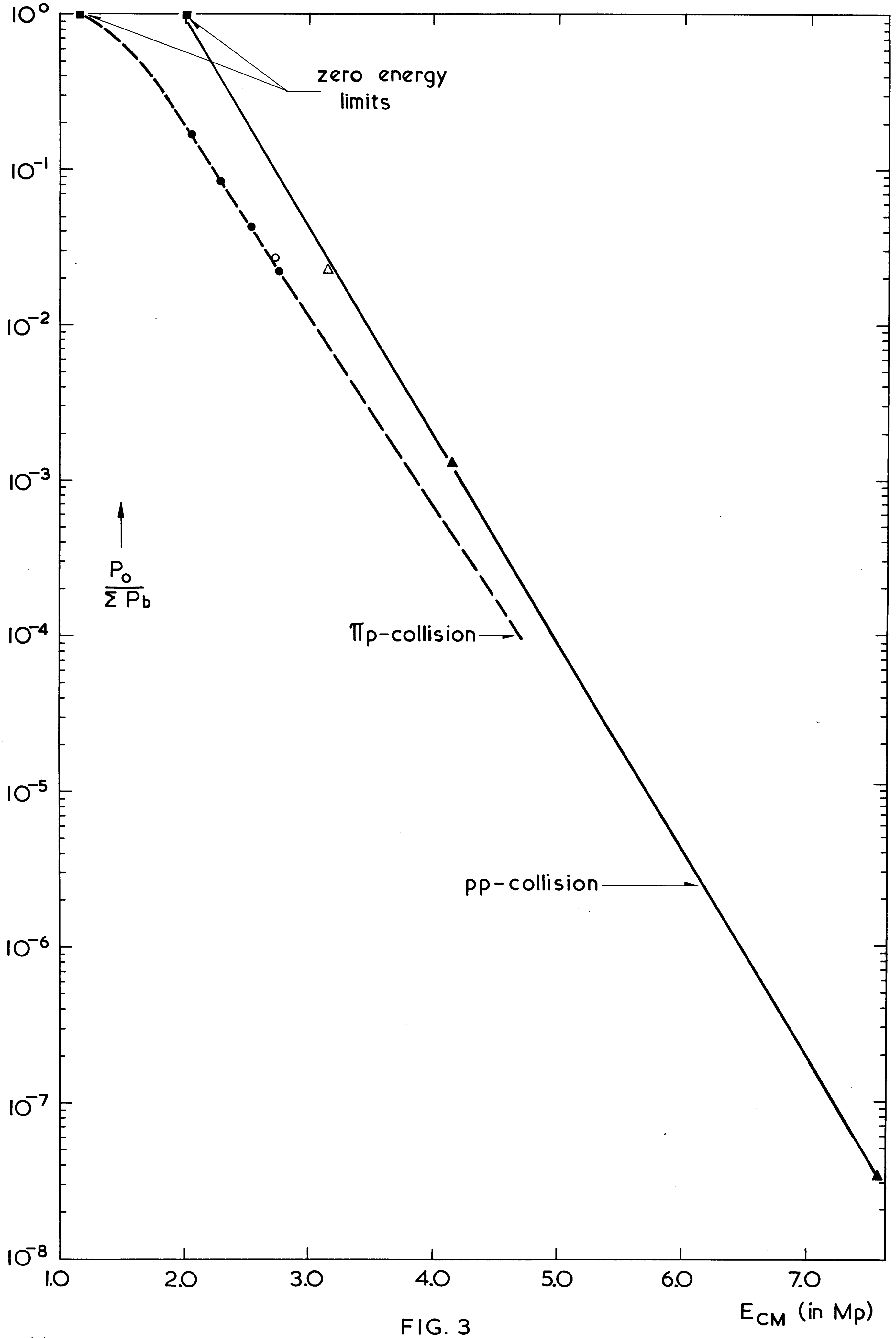


FIG. 3

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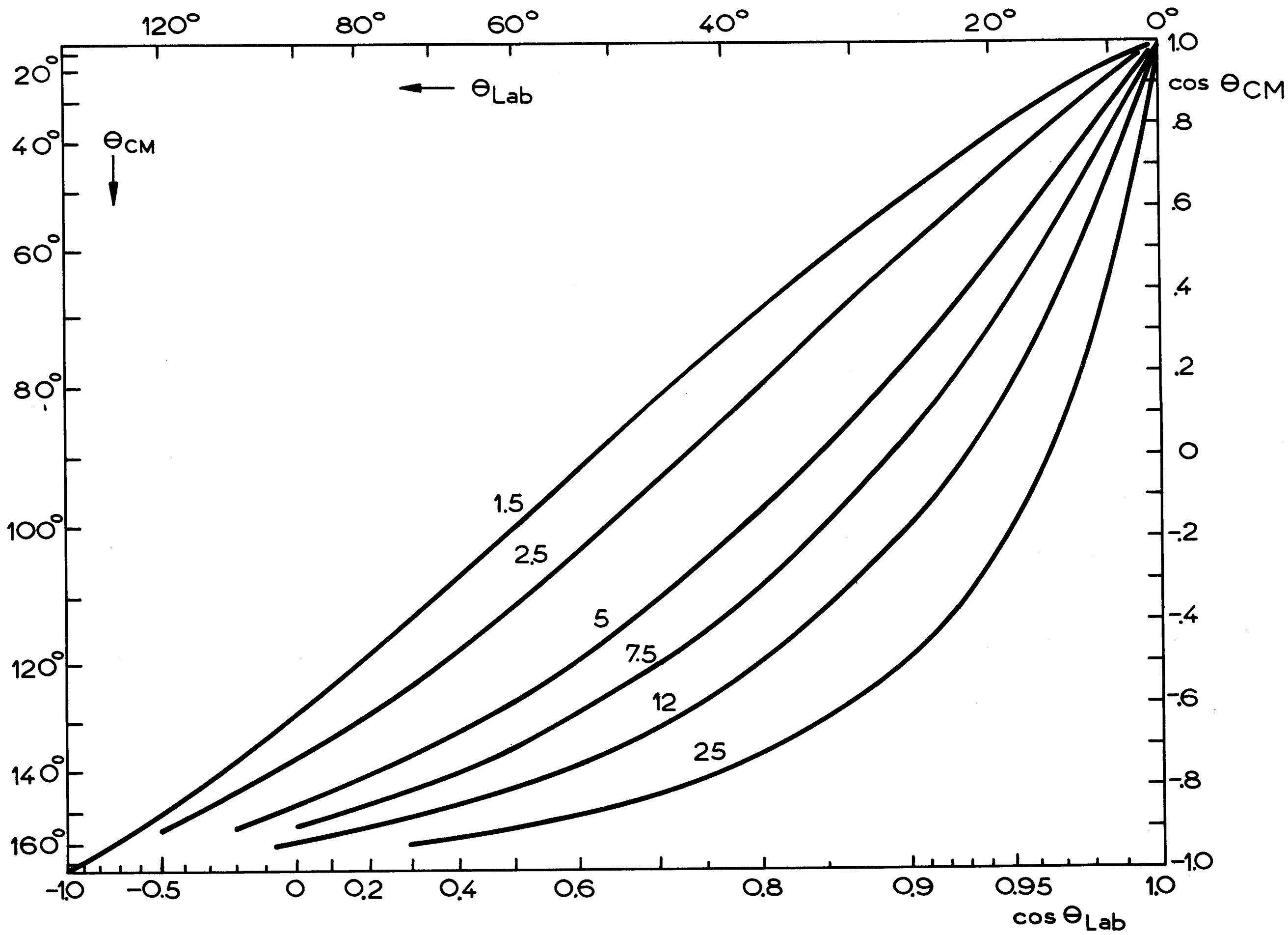
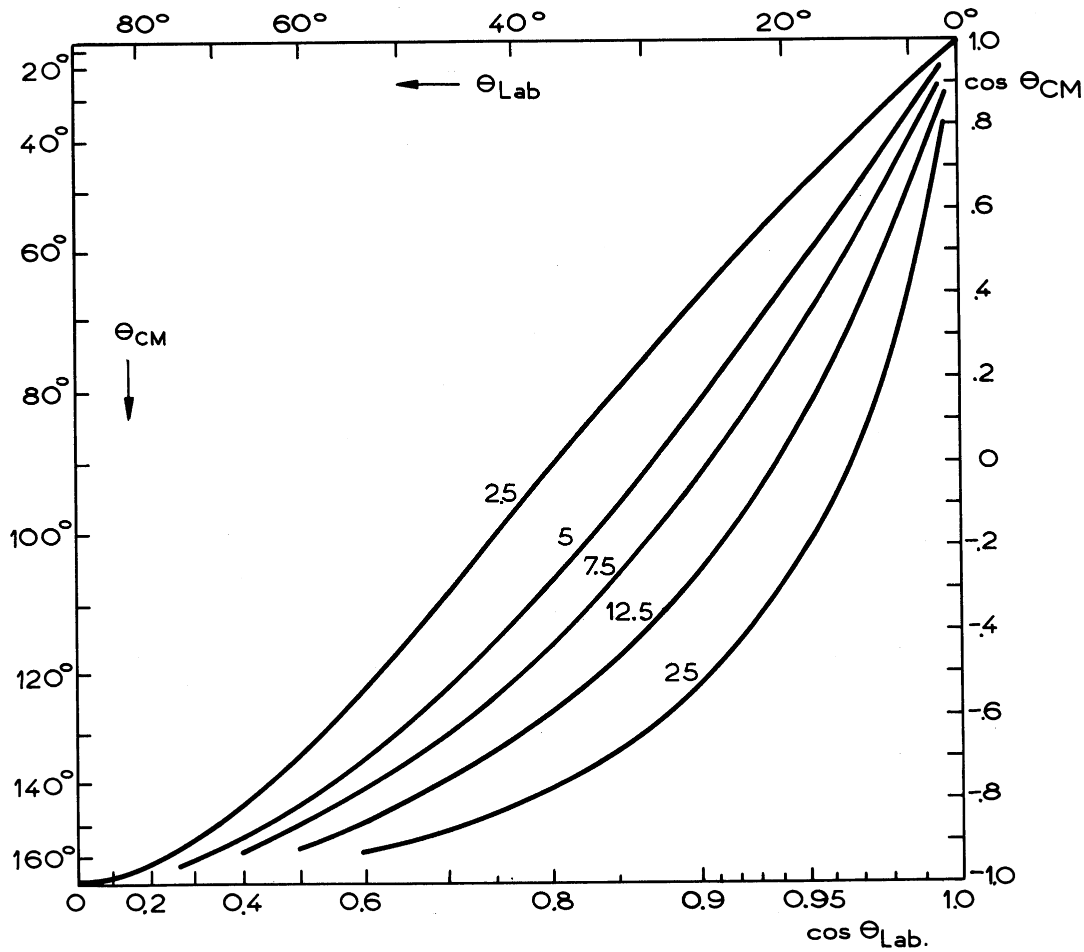


120°
80°
60°
40°
20°
0°
← Θ_{Lab}
Θ_{CM} ↓
20°
40°
60°
80°
100°
120°
140°
160°
1.0
cos Θ_{CM}
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
-1.0
1.5
2.5
5
7.5
12
25
-1.0
-0.5
0
0.2
0.4
0.6
0.8
0.9
0.95
1.0
cos Θ_{Lab}

FIG. 4



80°
60°
40°
20°
0°
← Θ_{Lab}
Θ_{CM} ↓
20°
40°
60°
80°
100°
120°
140°
160°
1.0
cos Θ_{CM}
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
-1.0
2.5
5
7.5
12.5
25
0
0.2
0.4
0.6
0.8
0.9
0.95
1.0
cos $\Theta_{Lab.}$

FIG. 5