NEW ISOBARIC SPIN GROUP

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(Presented by J. P. VIGIER)

The usual assumption on the new quantum numbers associated with isobaric spin, strangeness and baryon number, is that, since the particles are δ-functions, they cannot be related to kinematical degrees of freedom within the frame of space and time.

However this assumption does not seem to be corroborated by the new diffraction experiments of Hofstaedter, Wilson and their collaborators. It is therefore reasonable to start a priori from the idea that particles are not δ-functions, but rather (classically) material distributions enclosed within time-like tubes.

This leads to the idea that one can add to the position kinematical variable x_μ some new internal kinematical variables $q_\mu^{(\xi)}$ within the frame of space and time. These variables correspond to possible internal motion of our internal structures and their quantization would lead to internal stable quantized states associated with various elementary particles.

If we now assume the particle structure is spherically summetric and that internal motions can be schematized by internal rotations, we can represent such motions by those of two frames $a_\mu^{(\xi)}$ and $b_\mu^{(\xi)}$ located at the same point ($a_\mu^{(4)}$ and $b_\mu^{(4)}$ being time-like vectors). Now the relative orientations of two such frames can be represented by the parameters ω of a proper Lorentz transformation $\Lambda(\omega)$, and we can add to the usual external Lagrangien, an internal Lagrangien $L_i = \frac{1}{4} I \, \Omega_{\mu\nu} \Omega_{\mu\nu}$ where $\Omega_{\mu\nu}$ is the relativistic rotation velocity and I the internal moment of inertia. The invariance group of such a Lagrangien is composed with Lorentz transformations acting independently on the fixed and the moving tetrads, namely $\Lambda(\omega') = \Lambda(\alpha)\, \Lambda(\omega)\, \Lambda^{-1}(\beta)$. Now it is known [1] that each Lorentz transformation (expressed for instance in terms of generalized Euler angles [2]) is equal to the product of two complex conjugate three dimensional rotations, so that the above invariance group is the direct product of the right and left translations on the three dimensional complex rotation group SO_3^*. In particular the infinitesimal operations of this group multiplied by $\frac{\hbar}{i}$ according to the usual quantization rule are composed from the infinitesimal complex conjugate rotation to the left and to the right hand side, namely :

$$1 + \alpha_k^+ J_k^+ + \alpha_k^- J_k^- + \alpha_k'^+ J_k'^+ + \alpha_k'^- J_k'^-$$

(the α being infinitesimal independent parameters, and the signs + and - denoting complex conjugated quantities) and the operators obey to the commutation relation [3] :

$$[J_i^\pm, J_j^\pm] = -i\hbar\, \varepsilon_{ijk} J_k^\pm, \quad [J_i'^\pm, J_j'^\pm] = i\hbar\, \varepsilon_{ijk} J_k'^\pm, \quad [J_i^+, J_j^-] = [J_i'^+, J_j'^-] = [J_i^\pm, J_j'^\pm] = 0$$

It can be shown the internal quantum states are represented by the common eigenfunctions of the following six commuting operators :

$$J_3^\pm, \; J_3'^\pm, \; (J^\pm)^2 = J_i^\pm J_i^\pm, \quad S_3' = J_3'^+ + J_3'^- \quad S'^2 = (J_i'^+ + J_i'^-)(J_i'^+ + J_i'^-)$$

As a consequence we can now classify the internal states by the eigenvalues of these operators.

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These states are grouped in levels $H_{l^+l^-}$ according to the eigenvalues l^+, l^- of $(J^+)^2$ and $(J^-)^2$, and each of the levels splits into sublevels invariant under the (left) three dimensional complex rotations, $E^{m'}_{l^+,l^-,s'}$ according to the eigenvalues s', m' of $(S')^2$ and S'_3. One also defines the CPT operations and sees that the antiparticles correspond to the sign-inversal values of m^+, m^-, and m'.

It appears immediately that the sum $m^+ + m^-$ of the eigenvalues of the two complex conjugate operators $J_3^+ + J_3^-$ correspond to the gauge transformation $e^{i\alpha}$, α being real, and the same holds for the eigenvalue m' of the operator $S'_3 = J_3^+ + J_3^{!-}$. As it exists two well known gauges, namely the electric and baryonic ones, we are led to identify the latters with some independent linear combinations of m' and $m^+ + m^-$. The suitable identification is :

$$-2\,m' = \text{baryonic number}, \quad m^+ + m^- - m' = \text{charge}.$$

Consequently we are led, according to the Nishijima-Gell-Mann formula to the identification :

$$m^+ = \text{isobaric spin} \qquad 2\,m^- = \text{strangeness}.$$

This yelds the following table, which is equivalent to that of Nishijima and Gell-Mann, except it now contains four fermionic leptons, and new excited states which can be thought to be associated with the recently observed resonance states.

The theory of interactions results without difficulty from the preceding considerations. One starts by building as usual in the representations of our new group the so called "interaction vectors" which are irreducible under the considered transformations. Simple calculations show that these vectors associate the various particles into multiplets which constitute a scheme only slightly different from the usual scheme utilized in the ordinary isotopic spin space.

One then builds all the possible scalars with the preceding vectors and verify we obtain the correct scheme for strong interactions. By introducing the parity operator into the constitution of the scalars we also get the universal four fermion interactions, and consequently all the weak interactions.

As an example we can write the interaction Hamiltonian for the antibaryons and baryons of representation D (1/2, 1) with production of pions :

$$H = \bar{B}_k\,\sigma_j\,B_k\,M_j + \text{h.c.} = (\bar{\Xi}^\circ\,\pi^\circ\,\Xi^\circ - \bar{\Xi}^-\,\pi^\circ\,\Xi^- + \bar{\Xi}^-\pi^-\Xi^\circ + \bar{\Xi}^\circ\pi^+\Xi^- + \bar{P}\pi^\circ P + \bar{P}\,\pi^+ N + \bar{N}\,\pi^\circ P - \bar{N}\,\pi^\circ N) + \text{h.c.}$$

as we are led to put :

$$B_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} X^+ - \Xi^- \\ X^{++} - \Xi^\circ \end{pmatrix}, \qquad B_2 = \frac{1}{i\sqrt{2}}\begin{pmatrix} X^+ + \Xi^- \\ X^{++} + \Xi^\circ \end{pmatrix}, \qquad B_3 = \begin{pmatrix} N \\ P \end{pmatrix}$$

$$M_1 = \frac{\pi^+ + \pi^-}{\sqrt{2}}, \qquad M_2 = \frac{\pi^- - \pi^+}{i\sqrt{2}}, \qquad M_3 = \pi^\circ$$

(one has dropped in the developed result the terms containing the as yet unknown particles X^+ and X^{++}). This yields the usual strong interactions of the experience. One sees also this justifies immediately the absence of strong interactions between the (anti) nucleons and cascade particles which cannot be understood directly in the usual schemes.

Likewise we can give an example of weak Fermi interaction describing baryon decay with lepton pair creations, with intervention of the parity operator P which transforms the spinors of $D\left(0, \frac{1}{2}\right)$ into those of $D\left(\frac{1}{2}, 0\right)$ namely :

$$H = \bar{B}_k\,\sigma_j\,B_k\,\bar{L}\,\sigma_j\,\rho\,L' = (\bar{\Xi}^\circ\,\Xi^- + \bar{P}N)\bar{e}\,\nu_e + \text{h.c.} + 2\,(\bar{\Xi}^-\,\Xi^- - \bar{\Xi}^\circ\,\Xi^\circ + \bar{N}N - \bar{P}P)\,(\bar{e}e - \bar{\nu}_e\,\nu_e)$$

by putting :

$$L = \begin{pmatrix} e \\ \nu_e \end{pmatrix} \qquad L' = \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}$$

This is once more the list of observed (or reasonably assumed) weak interactions. The last product corresponds to scalars built with neutral currents which have been recently been shown to be reconciliable with present experimental evidences.

As a conclusion we want to remark that our treatment differs from many former attempts which utilized the Lorentz group, by the fact that our new isobaric spin group is built, not directly from Lorentz transformations, but from transformations which act on the Lorentz group as left and right translations. This yields for each type of representation D (l^+, l^-) several different subspaces according to the values of the supplementary quantum number m'.

Moreover the splitting of the Lorentz transformations into complex conjugate three dimensional rotations has the consequence, in the case of the weak interactions, which are invariant independently under the two conjugated groups, that it appears an invariance under a real three dimensional rotation group, which is indeed the ordinary isobaric spin group. The usual strong interaction group appears thus to be a particular case of our more general group.

REFERENCES

[1] EINSTEIN and MAYER - Sitz. der. Preuss. Akad. zu. Berlin 1932.

[2] HALBWACHS, HILLION, VIGIER - Ann. Inst. H. Poincaré 1. (1960).

[3] VAN WINTER - Thesis (Groningen) 1957.

Représentation	$m' = B/2$	$m^+ = i_3$	$m^- = S/2$	$Q = m^+ + m^- - m'$	Particle	Levels
$D(1/2,0)m' = \frac{1}{2}$	- 1/2	- 1/2	0	- 1	e^-	$Z^{-1/2,0,1/2}_{1/2,0,1/2}$
	- 1/2	1/2	0	0	ν_e	$Z^{1/2,0,1/2}_{1/2,0,1/2}$
$D(1/2,0)m' = -\frac{1}{2}$	1/2	1/2	0	1	$e^- = e^+$	$Z^{1/2,0,-1/2}_{1/2,0,1/2}$
	1/2	- 1/2	0	0	$\bar{\nu}_e$	$Z^{-1/2,0,-1/2}_{1/2,0,1/2}$
$D(0,1/2)m' = \frac{1}{2}$	- 1/2	0	- 1/2	- 1	μ^-	$Z^{0,-1/2,1/2}_{0,1/2,1/2}$
	- 1/2	0	1/2	0	ν_μ	$Z^{0,1/2,1/2}_{0,1/2,1/2}$
$D(0,1/2)m' = -\frac{1}{2}$	1/2	0	1/2	1	$\mu^- = \mu^+$	$Z^{0,1/2,-1/2}_{0,1/2,1/2}$
	1/2	0	- 1/2	0	$\bar{\nu}_\mu$	$Z^{0,-1/2,-1/2}_{0,1/2,1/2}$
$D(1,0)m' = 0$	0	1	0	1	π^+	$Z^{1,0,0}_{1,0,1}$
	0	0	0	0	π°	$Z^{-1,0,0}_{1,0,1}$
	0	- 1	0	- 1	$\pi^- = \bar{\pi}^+$	$Z^{0,0,0}_{1,0,1}$
$D(1/2,1/2)\ S' = 0$	0	- 1/2	- 1/2	- 1	K^-	$Z^{-1/2,-1/2,0}_{1/2,1/2,0}$
	0	1/2	1/2	1	K^+	$Z^{1/2,1/2,0}_{1/2,1/2,0}$
	0	1/2	- 1/2	0	K°	$Z^{1/2,-1/2,0}_{1/2,1/2,0}$
	0	- 1/2	1/2	0	K°	$Z^{-1/2,1/2,0}_{1/2,1/2,0}$
$D(1/2,1)$ $m' = -1/2$ (particles) $s' = 1/2$	1/2	1/2	1	2	x^{++}	$Z^{1/2,1,-1/2}_{1/2,1,1/2}$
	1/2	- 1/2	1	1	x^+	$Z^{-1/2,1,-1/2}_{1/2,1,1/2}$
	1/2	1/2	0	1	P	$Z^{1/2,0,-1/2}_{1/2,1,1/2}$
	1/2	- 1/2	0	0	N	$Z^{-1/2,0,-1/2}_{1/2,1,1/2}$
	1/2	1/2	- 1	0	Ξ°	$Z^{1/2,-1,-1/2}_{1/2,1,1/2}$
	1/2	- 1/2	- 1	- 1	Ξ^-	$Z^{-1/2,-1,-1/2}_{1/2,1,1/2}$
$D(1,1/2)$ $s' = 1/2$ $m' = -1/2$ (particles)	1/2	1	1/2	2	Y^{++}	$Z^{1,1/2,-1/2}_{1,1/2,1/2}$
	1/2	0	1/2	1	Y^+	$Z^{0,1,-1/2}_{1,1/2,1/2}$
	1/2	- 1	1/2	0	Y°	$Z^{-1,1/2,-1/2}_{1,1/2,1/2}$
	1/2	1	- 1/2	1	Σ^+	$Z^{1,-1/2,-1/2}_{1,1/2,1/2}$
	1/2	0	- 1/2	0	Σ°	$Z^{0,-1/2,-1/2}_{1,1/2,1/2}$
	1/2	- 1	- 1/2	- 1	Σ^-	$Z^{-1,-1/2,-1/2}_{1,1/2,1/2}$