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D0 Note 1976

Mini-Workshop on Top Mass Finding

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Bloomington, IN

Copies of Transparencies

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Determination of the top mass in dilepton events

Overview and Method I

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Indiana Workshop

October 12 1993

- Dalitz & Goldstein 1st paper + CDF event
- Introduction of $\not{E}_T$ ~~inp~~ into problem.
  Neutrino search Method
  Lessons
- DØ Extension of ellipse solutions involving $\not{E}_T$
- Unweighted solutions
- Weighting Schemes — Method I
- Smearing & Jacobians
- K_T^2 weight
- Results for DØ event 417

1st Dalitz Goldstein paper in PRD
worked on CDF dilepton event from their 1989 run.
No $\not{E}_T$ published for event.

For a given top mass D-G found solutions for top quark 3vector $\vec{t}$ that were consistent with the observed momenta $\vec{b}$, $\vec{l}$. $\vec{t}$ lies on Ellipse

Similarly for $\vec{t}_{bar}$
Chose those $\vec{t}$ + $\vec{t}_{bar}$ that had values of

$$p_T(\vec{t} + \vec{t}_{bar}) < \text{Some amount!}$$

This was essentially a 4 Dimensional search.

The solutions found were then weighted by the term

$$W_T = \underbrace{F(x_1) . F(x_2)}_{\text{D.G scheme} \Rightarrow \underline{\overline{\text{III}}}} \text{ Decay-wt.}$$

Weighting schemes.

1) Likelihood scheme must not change if one has 1 event or 1000 events.

. Mapping of phase space to Measurement space accomplished by number of events and their distribution.

. Detector acceptances likewise

$\Rightarrow$ we use σ_{total} not $\sigma_{visible}$.

$\Rightarrow$

2) Since we need an absolute likelihood, we use $\frac{1}{\sigma}\frac{d\sigma}{d\,\text{Lips}}$

not $\frac{1}{\sigma}\frac{d\sigma}{d\,\underbrace{\text{measured variables}}}$.

What do we actually measure. Do we measure Energy or Energy^2 ... If we change the set of variables, we need an invariant likelihood. Use $\underline{\underline{d\,\text{Lips}}}$.

3) m_t is a parameter we vary. We are at liberty to vary it in any fashion. Equal steps in m_t or not. Likelihood is a function of m_t

$$L \equiv L(m_t)$$

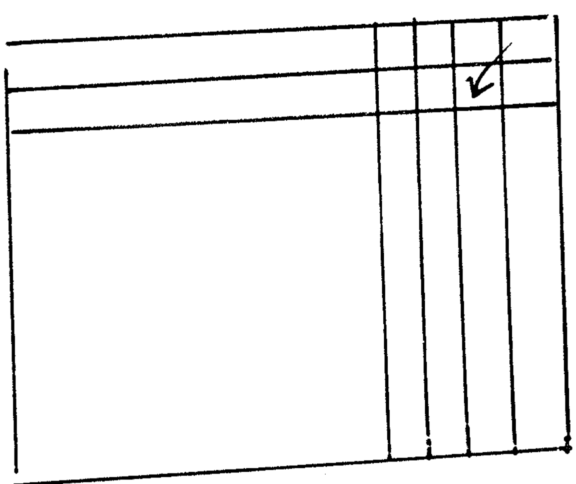
It is not $\frac{dL}{dm_t}$ or $\frac{dL}{d\log m_t}$

We Use FISHER rather than BAYES

EACH m_t is a different Hypothesis.

For each m_t, $\exists$ $\sigma(m_t)$

Event.



14 dimensional phase space

Likelihood ΔL = Probability of getting event in phase space box.

N = Number of events = Luminosity $\times$ $\sigma(m_t)$

ΔN = Number in box = Luminosity $\times$ $\Delta\sigma$

Invariant size of box = Δ Lips

Δ Probability = $\frac{\Delta N}{N}$ = $\frac{\Delta\sigma}{\sigma}$

Probability density = $\boxed{\frac{1}{\sigma}\frac{d\sigma}{d\,\text{Lips}}}$

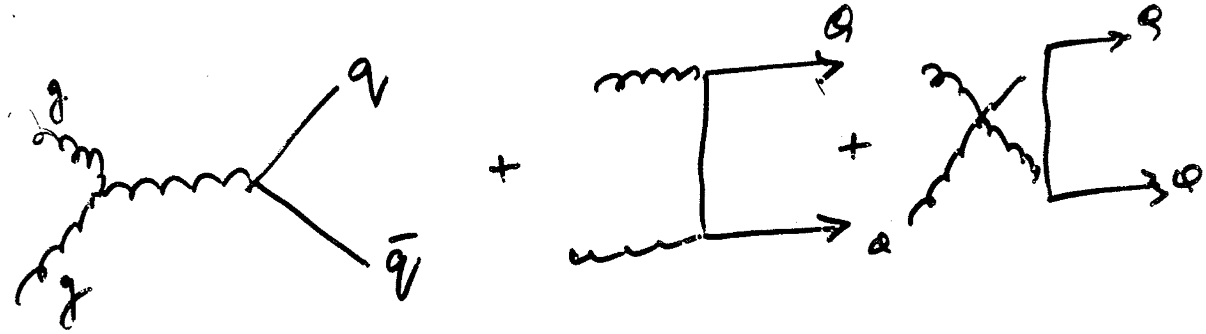
$\frac{d\sigma}{d\,\text{Lips}} \equiv \frac{d^5\sigma}{dk_T^2\, d\phi\, dx_1\, dx_2\, d\hat{t}}$ Decay WA a la D-G

$\equiv F(k_T^2)\ \cancel{d}\ F(x_1)\ F(x_2)\ \frac{d\sigma}{d\hat{t}}$

Calculation of Cross sections

$$\frac{d\sigma}{d\,\mathrm{Lips}} = \frac{d\sigma}{dx_1\,dx_2\,d\hat{t}} = \left[F(x_1)\,F(x_2)\,\frac{d\sigma}{d\hat{t}}\right]_{gg} + q\bar{q}$$

Born term



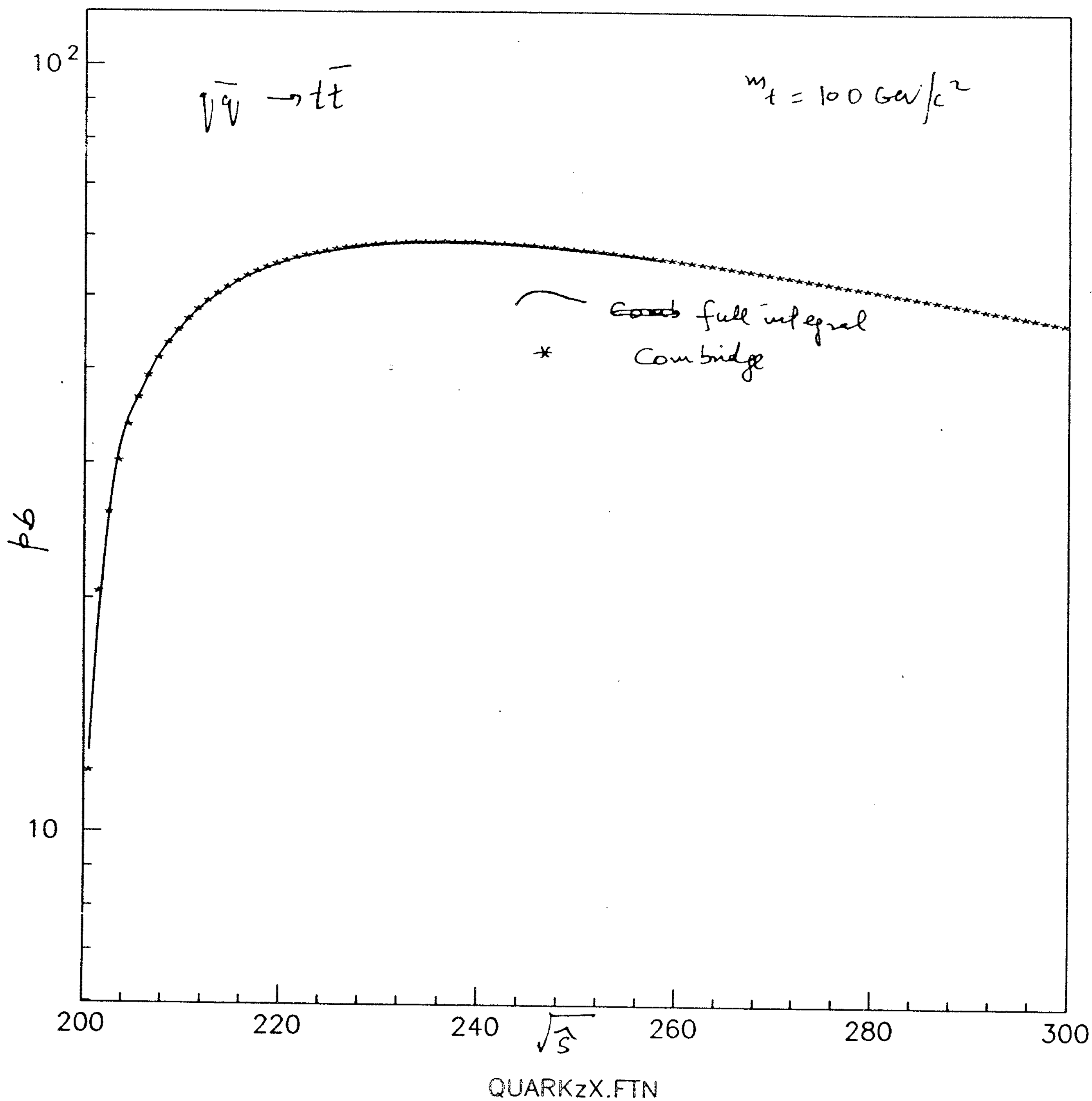
$$\frac{d\sigma}{d\hat{t}}\left(gg \to Q\bar{Q}\right) = \frac{\pi\alpha_s^2}{\hat{s}^2}\,|A|^2$$

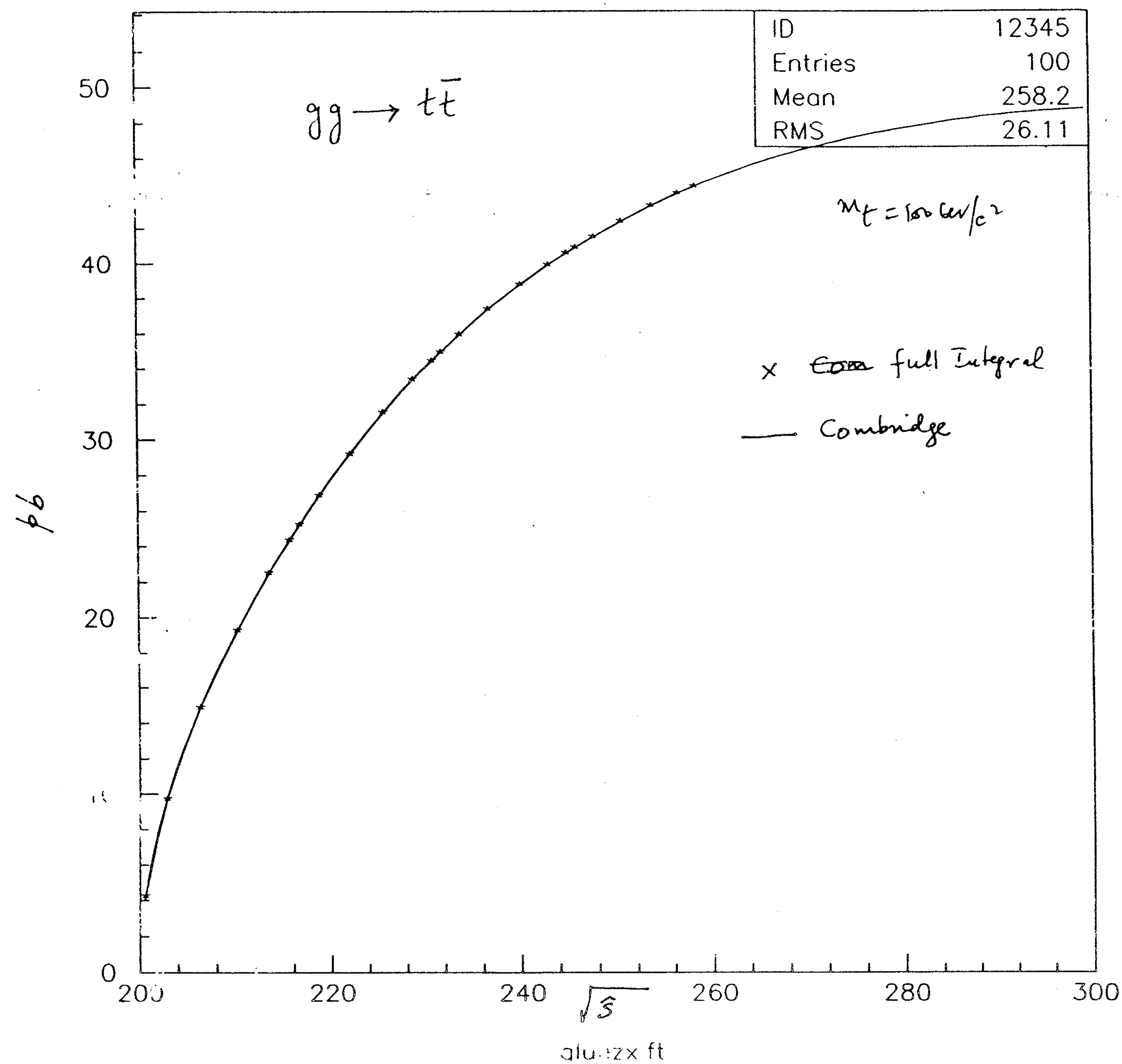
$$|A|^2 = \frac{1}{8}\Bigg\{ \frac{6}{\hat{s}^2}(\hat{t}-m^2)(\hat{u}-m^2) + \left[\frac{4}{3}\frac{\hat{u}-m^2}{\hat{t}-m^2} - \frac{8}{3}\frac{m^2(\hat{t}+m^2)}{(\hat{t}-m^2)^2} + \frac{3(\hat{t}-m^2)(\hat{u}-m^2)+m^2(\hat{u}-\hat{t})}{\hat{s}(\hat{t}-m^2)}\right] + \left[\hat{t} \leftrightarrow \hat{u}\right] - \frac{m^2(\hat{s}-4m^2)}{3(\hat{t}-m^2)(\hat{u}-m^2)} \Bigg\}$$

For $q\bar{q} \to Q\bar{Q}$

$$|A|^2 = \frac{4}{9}\left[\frac{(\hat{t}-m^2)^2 + (\hat{u}-m^2)^2 + 2m^2\hat{s}}{\hat{s}^2}\right]$$

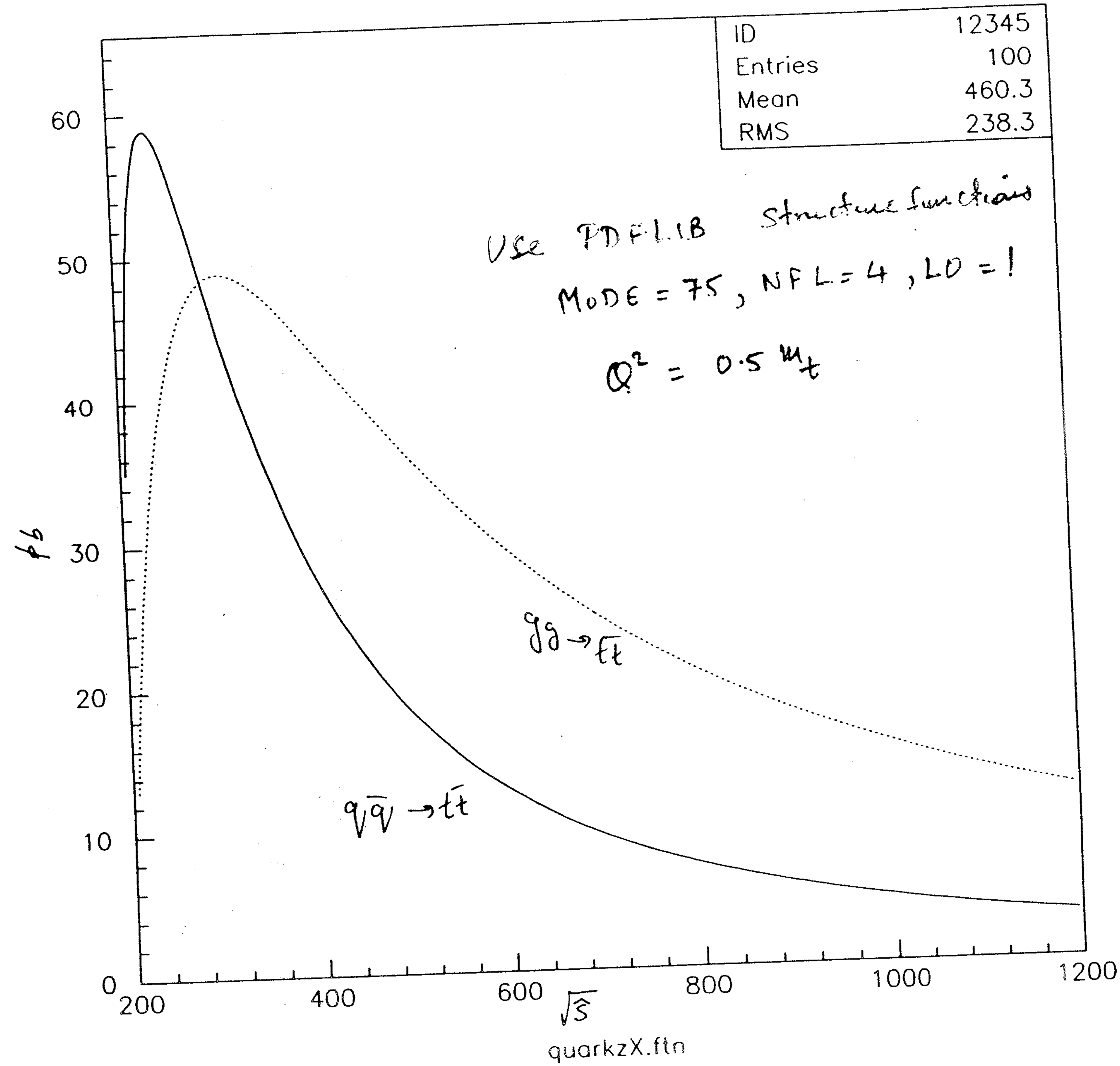
Combined Analysis-- D0 Preliminary





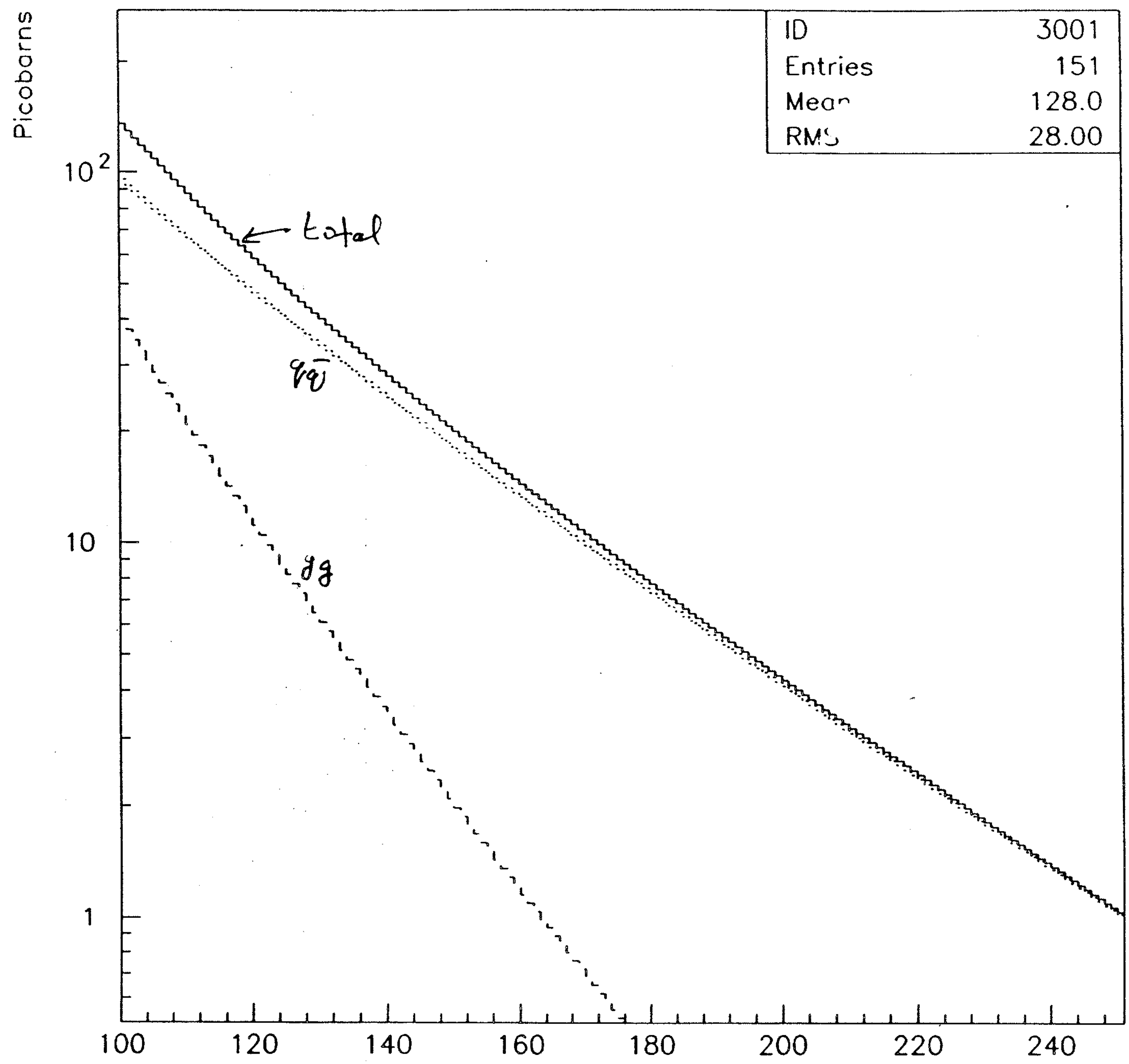
ID 12345
Entries 100
Mean 258.2
RMS 26.11
$gg \to t\bar{t}$
$m_t = 100$ GeV/c^2
× ~~Com~~ full Integral
— Combridge
σ
50
40
30
20
0
200
220
240
260
280
300
$\sqrt{\hat{s}}$
glu:zx ft

8



ID 12345
Entries 100
Mean 460.3
RMS 238.3
Use PDFLIB Structure functions
MODE = 75, NFL = 4, LO = 1
$Q^2 = 0.5\, m_t$
$gg \to t\bar{t}$
$q\bar{q} \to t\bar{t}$
fb
0
10
20
30
40
50
60
200
400
600
800
1000
1200
$\sqrt{s}$
quarkzX.ftn

9



TOP CROSS SECTION VS MASS

FLUCTUATE EVENT

e E Resolution $= \frac{15\%}{\sqrt{E}} = \frac{\sigma^2}{E^2}$

Jet E_T resolution $= \frac{80\%}{\sqrt{E_T}} = \frac{\sigma^2_{E_T}}{E_T^2}$

Muon Resolution $= (0.2)^2 + (0.01\, p_T)^2 = \frac{(\delta p_T)^2}{p_T^2}$

Correct formula [RR DØNOTE 17~~45~~(?)]

$$\frac{1}{\sigma}\frac{d\sigma^{(\text{smeared})}}{d\text{Lips}} = \left|\frac{\partial Q_i}{\partial L_i}\right|_e \frac{1}{N} \sum_{\text{Configurations}} \frac{1}{\sigma}\frac{d\sigma}{d\text{Lips}} \left|\frac{\partial L_i}{\partial Q_i}\right|$$

Where L_i are the variables used to describe LIPS
Q_i are the Variables in Smearing.

After event 417 was found, some of us in Dϕ realized that by adding the $\not{E}_T$ vector, one can find solutions using a search in 2 dimensions.

NEUTRINO SEARCH METHOD

$\Rightarrow$

SEARCH IN EQUAL STEPS OF ϕ

$\Longrightarrow$ Peak in m_t !! Purely Jacobian Induced.

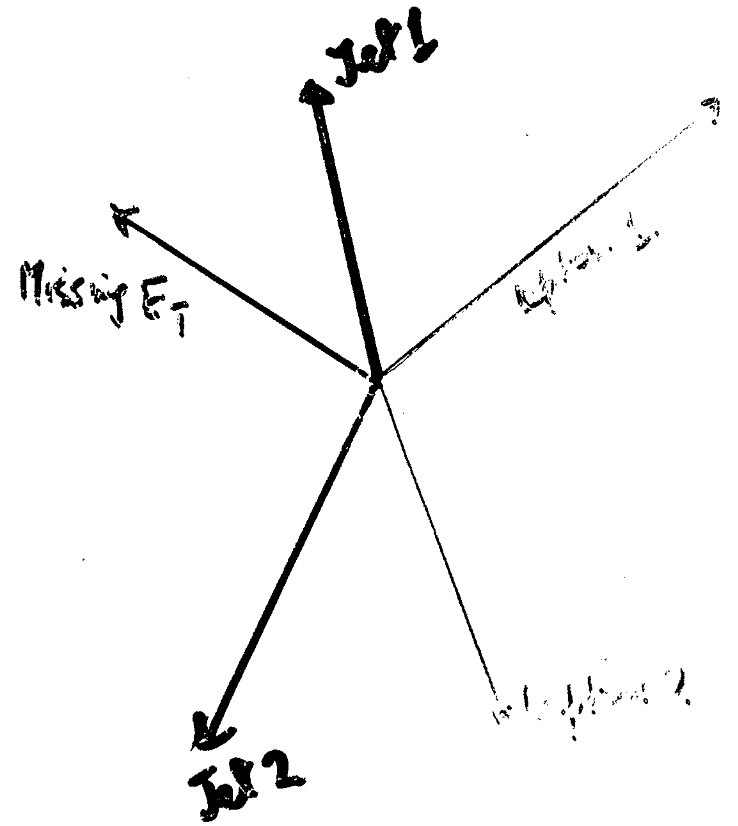
What one needs is solutions for any given m_t.

This leads to a quartic equation in ν_x & ν_y

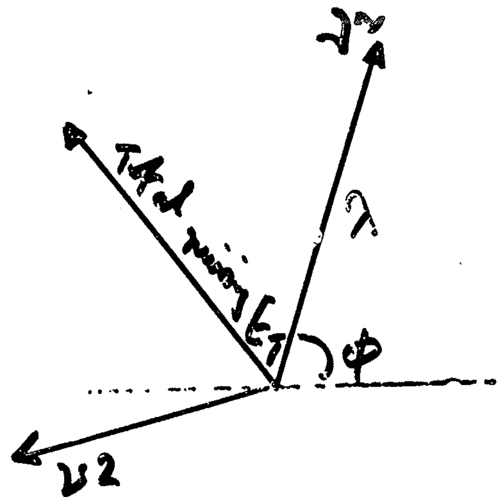
leading to 0, 2 or 4 solutions.

Weighting schemes.

New Method!



Dalitz
Goldstein
does not make use of missing E_T vector explicitly.



$$\nu_1 = \lambda e^{i\phi}$$

ν_2 is given by vector subtraction.

Impose condition that

$$W_1 \longrightarrow \ell_1 + \nu_1$$

Get 2 solutions for longitudinal momentum of 1st neutrino.

$A = \frac{M_W^2}{2}$

$B = E_e$ = Energy of electron

$x = E_z^\nu$ = longitudinal momentum of neutrino

$y = E_e^\nu$ = longitudinal momentum of electron

$\lambda = E_T^\nu$ = transverse momentum of ν

$\mu = E_T^e$ = transverse momentum of electron

θ = angle between E_T^ν and E_T^e

$$C = \frac{A}{B} + \frac{\lambda \mu \cos\theta}{B}$$

$$D = \frac{y}{B}$$

$$x = - \frac{CD \pm \sqrt{C^2 D^2 - (1-D^2)(\lambda^2 - C^2)}}{1-D^2}$$

These are the two solutions

Range of λ for which solutions exist.

$$\lambda_1 = \frac{A}{B\left(\sqrt{1-D^2} - \frac{\mu}{B}\cos\theta\right)}$$

$$\lambda_2 = \frac{A}{B\left(-\sqrt{1-D^2} - \frac{\mu}{B}\cos\theta\right)}$$

when $\theta = \theta + \pi$

$\lambda_1 \longrightarrow -\lambda_2$, $\lambda_2 = -\lambda_1$

So θ should have range $0 \to \pi$.

Similarly for ν_2; Get 2 solutions

Order solutions in increasing P_Z^{ν}.

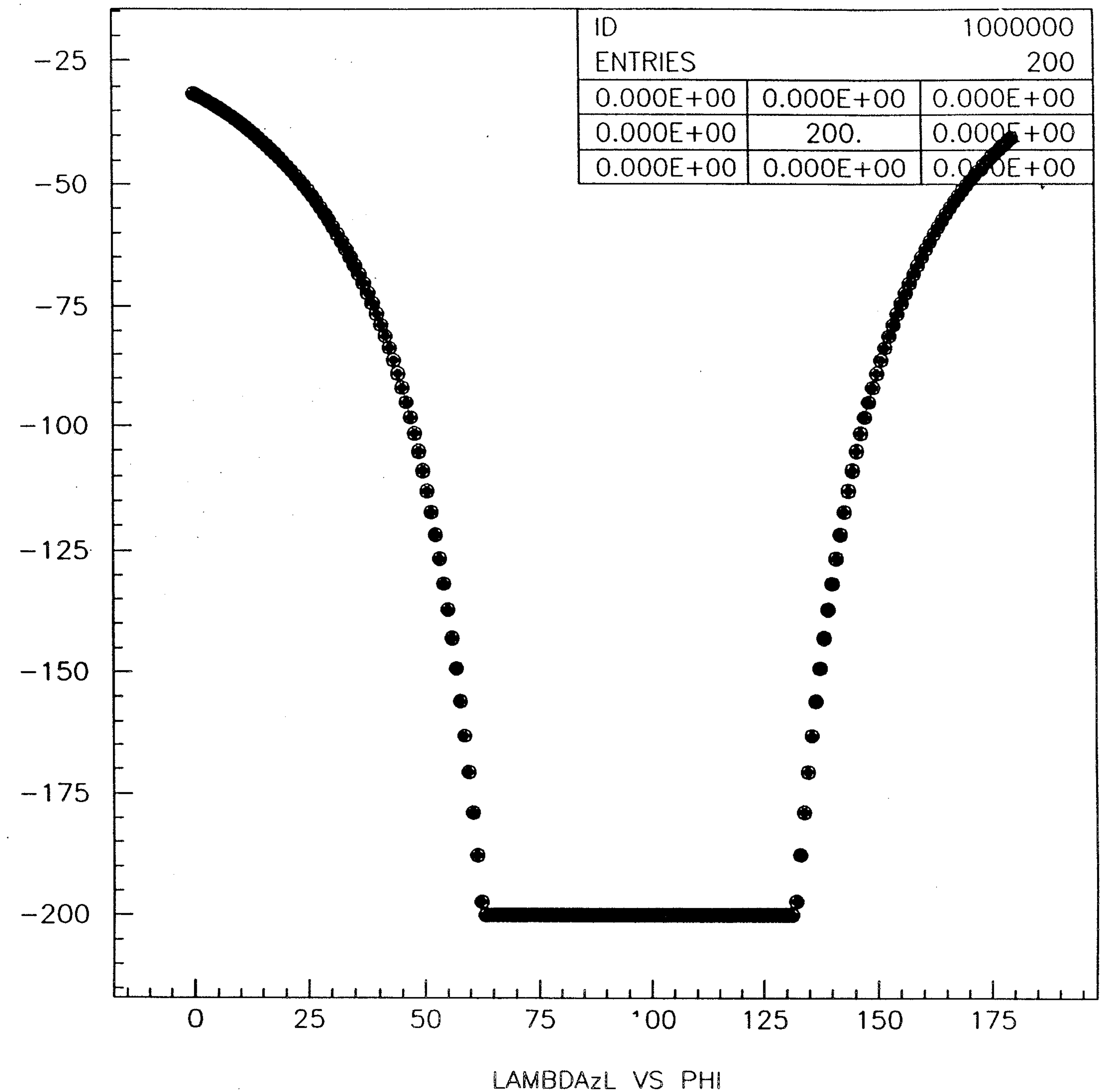
For a given ϕ, move in steps of λ.

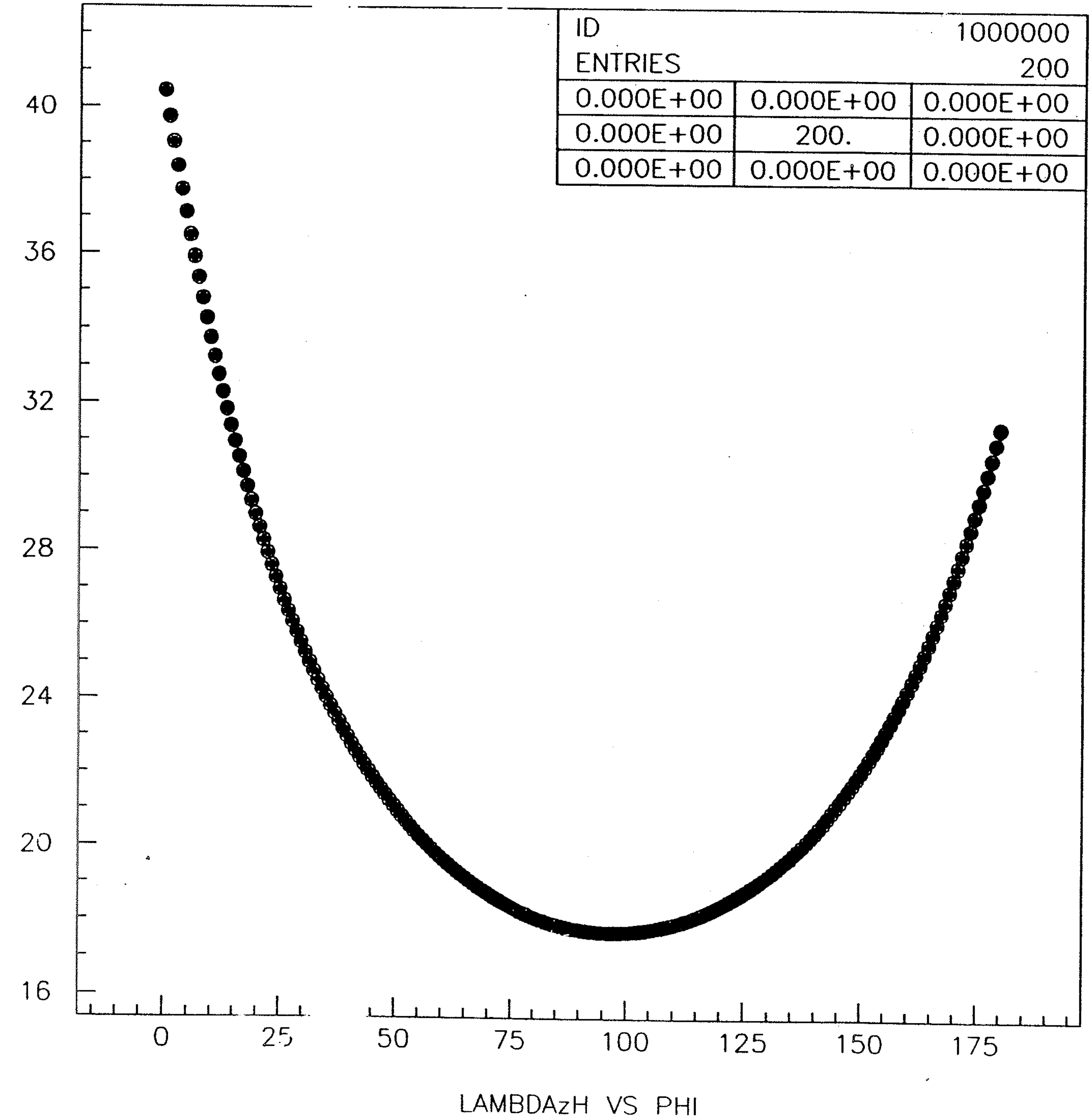
Pick those values of λ for which the two top masses equalize.

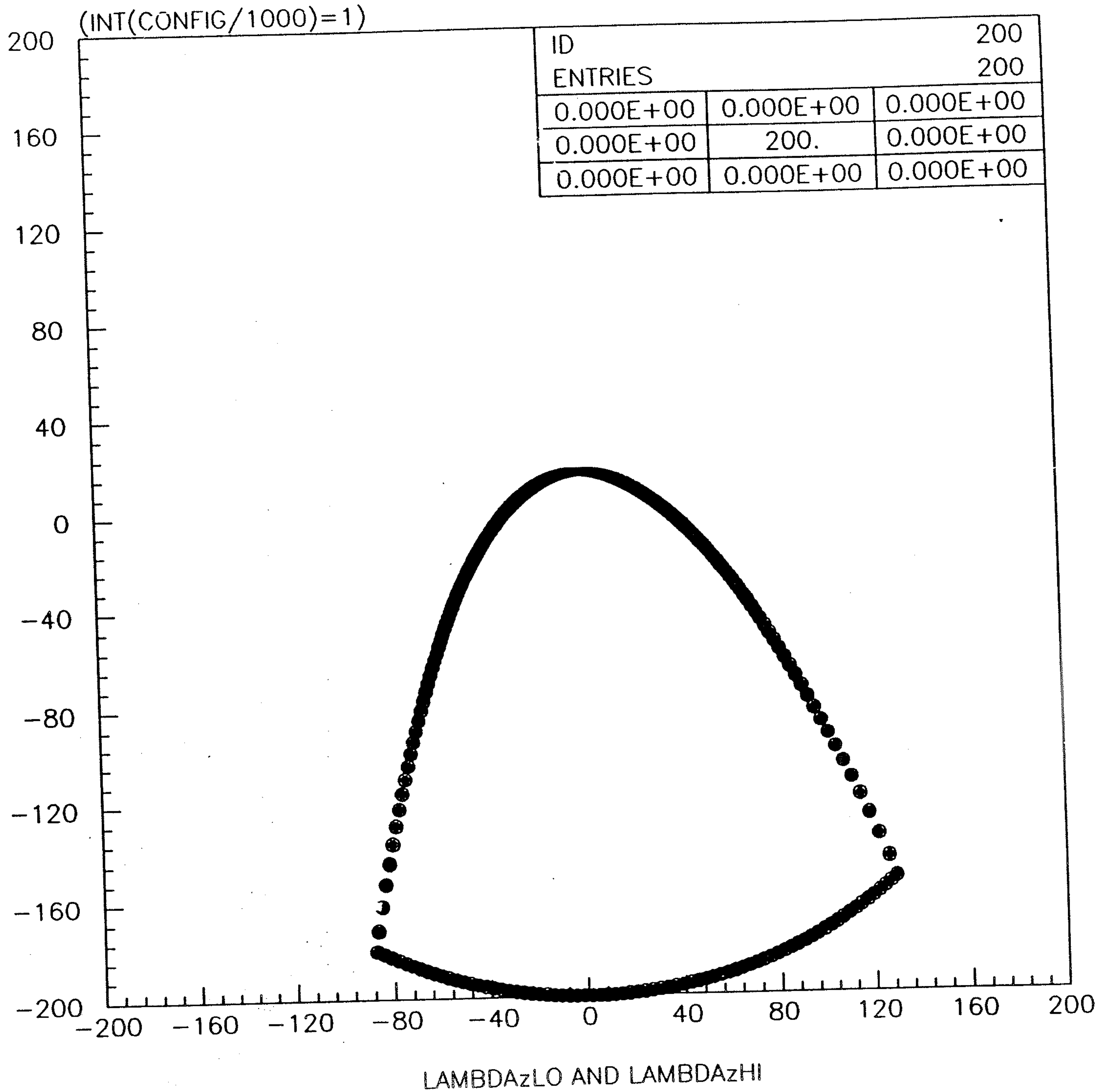
There will in general be 4 values of λ for any given ϕ.

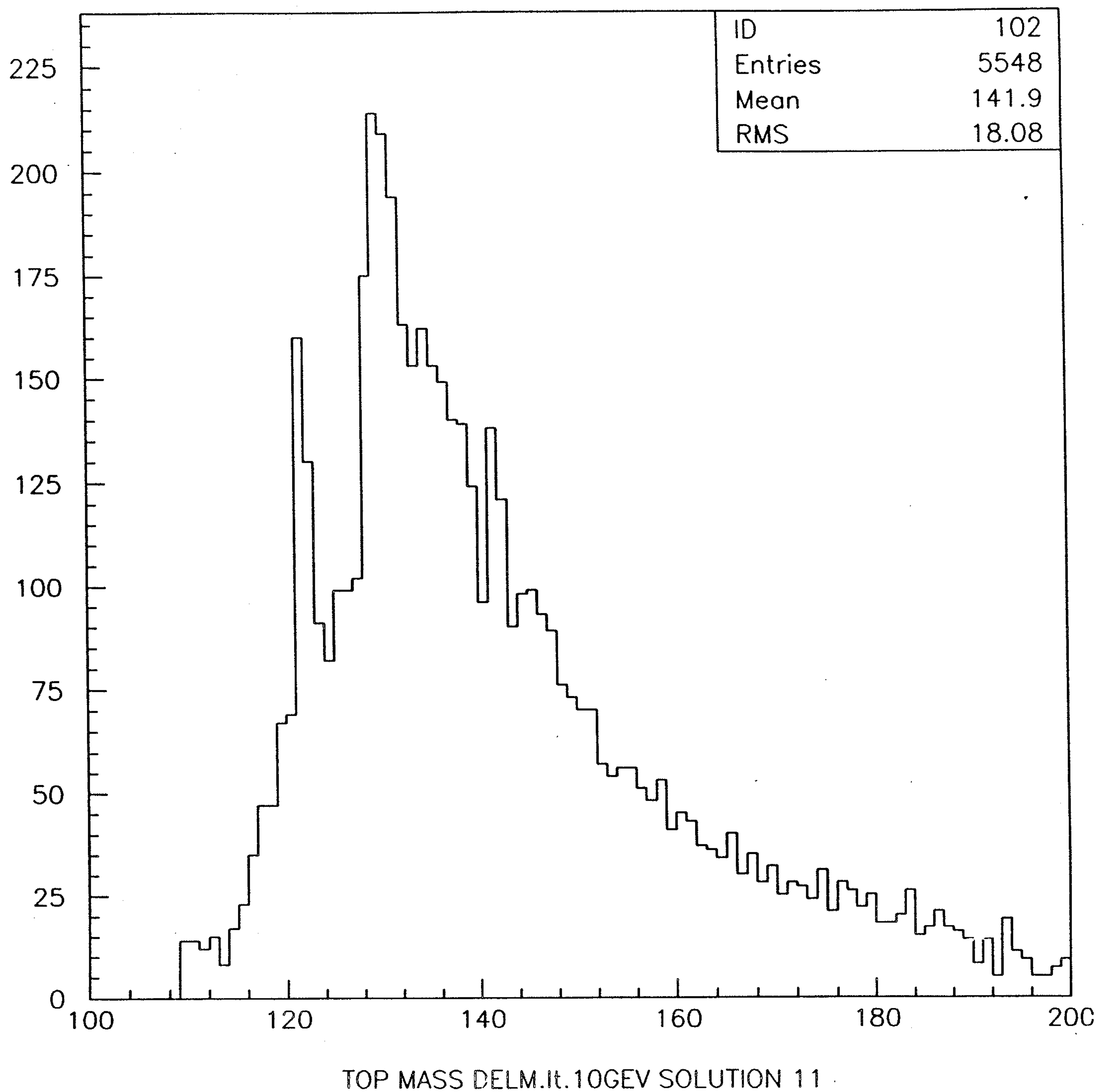
W_1	W_2
1, 2	1, 2

Solution λ_{11} λ_{12} λ_{21} λ_{22}


ID 1000000
ENTRIES 200
0.000E+00 0.000E+00 0.000E+00
0.000E+00 200. 0.000E+00
0.000E+00 0.000E+00 0.000E+00
-25
-50
-75
-100
-125
-150
-175
-200
0
25
50
75
100
125
150
175
LAMBDAzL VS PHI

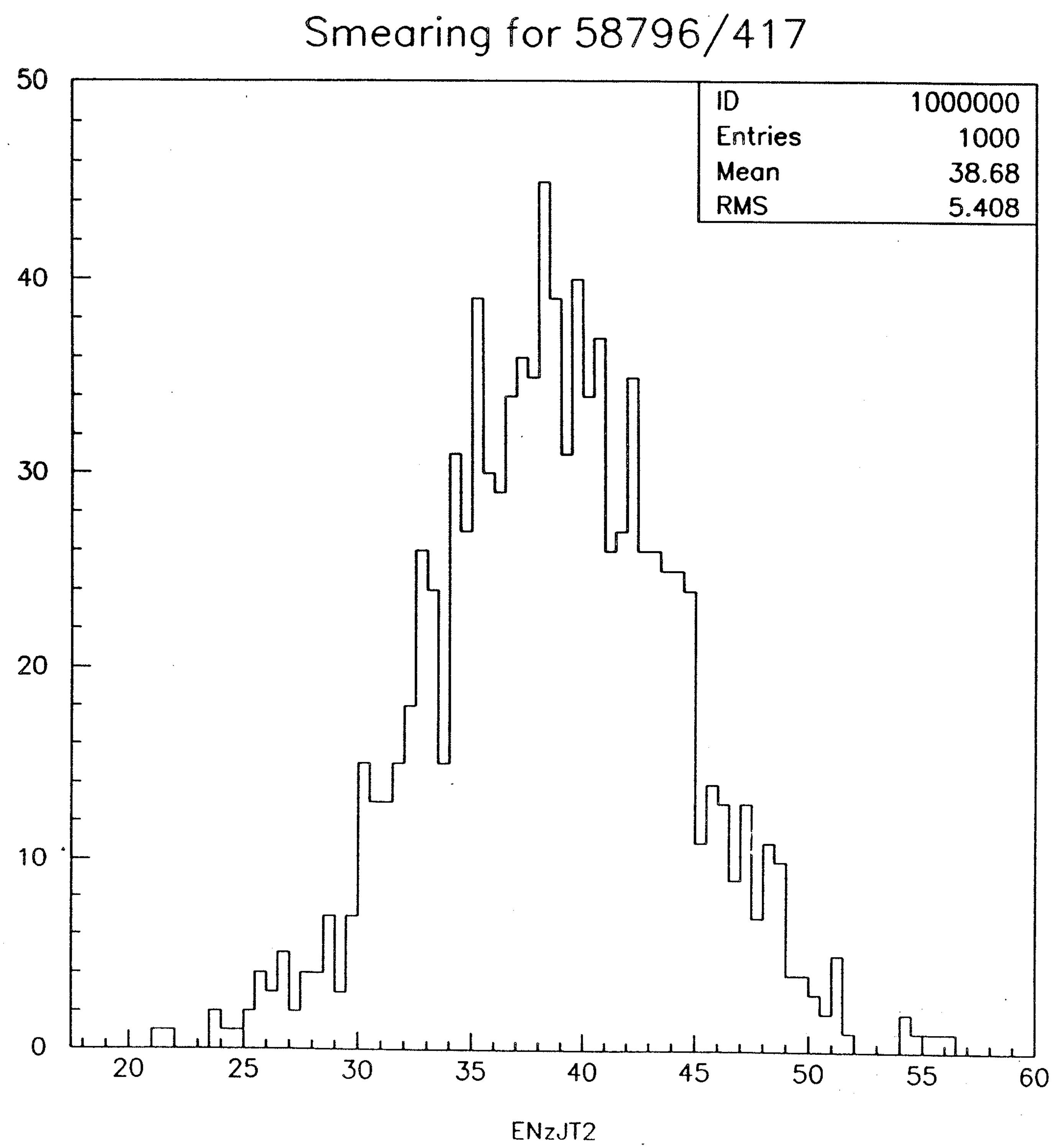

ID 1000000
ENTRIES 200
0.000E+00 0.000E+00 0.000E+00
0.000E+00 200. 0.000E+00
0.000E+00 0.000E+00 0.000E+00
40
36
32
28
24
20
16
0
25
50
75
100
125
150
175
LAMBDAzH VS PHI


(INT(CONFIG/1000)=1)
ID 200
ENTRIES 200
0.000E+00 0.000E+00 0.000E+00
0.000E+00 200. 0.000E+00
0.000E+00 0.000E+00 0.000E+00
200
160
120
80
40
0
−40
−80
−120
−160
−200
−200 −160 −120 −80 −40 0 40 80 120 160 200
LAMBDAzLO AND LAMBDAzHI

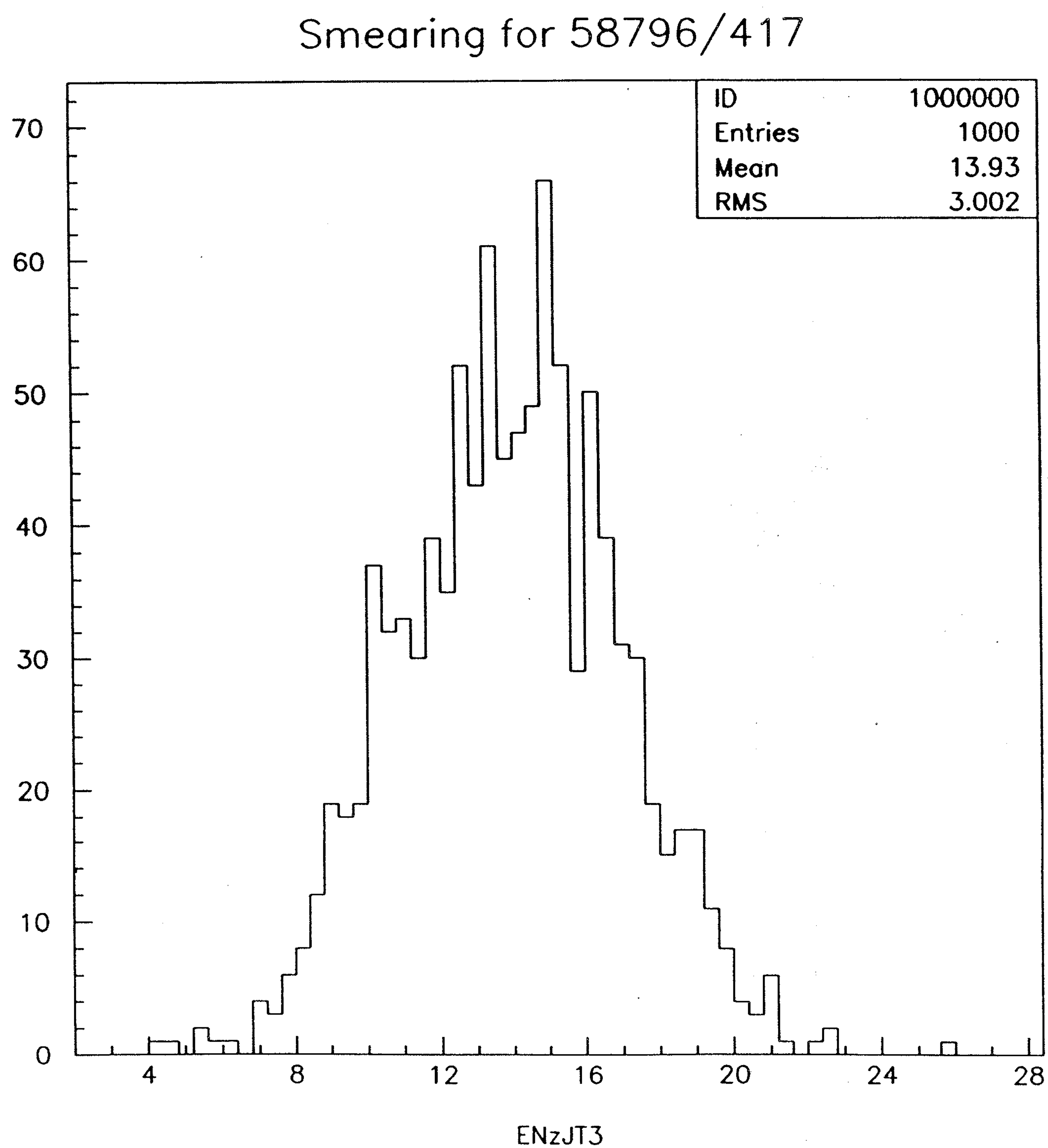


ID 102
Entries 5548
Mean 141.9
RMS 18.08
225
200
175
150
125
100
75
50
25
0
100
120
140
160
180
200
TOP MASS DELM.lt.10GEV SOLUTION 11

21


Smearing for 58796/417
ID 1000000
Entries 1000
Mean 38.68
RMS 5.408
0
10
20
30
40
50
20
25
30
35
40
45
50
55
60
ENzJT2

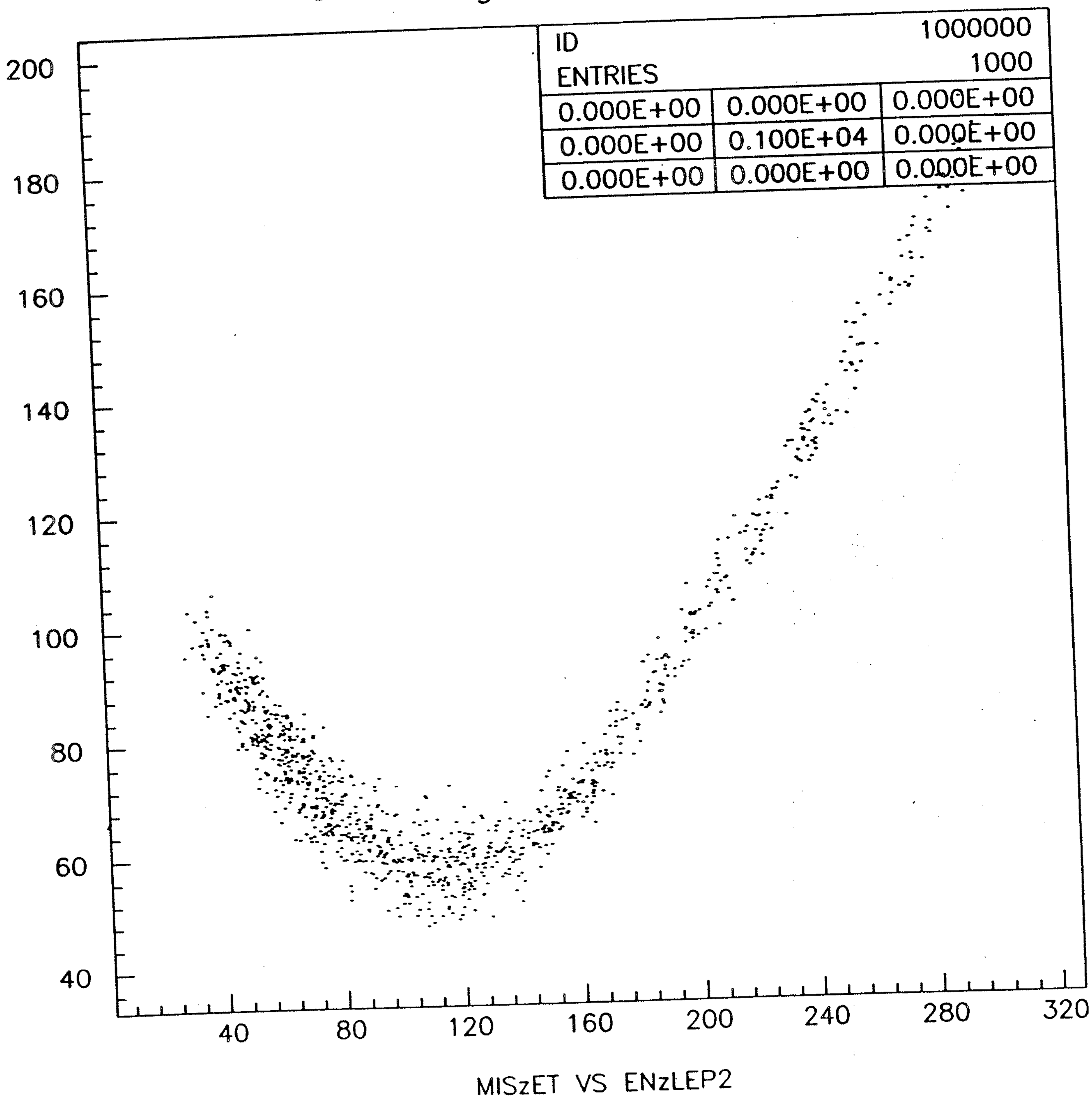
22



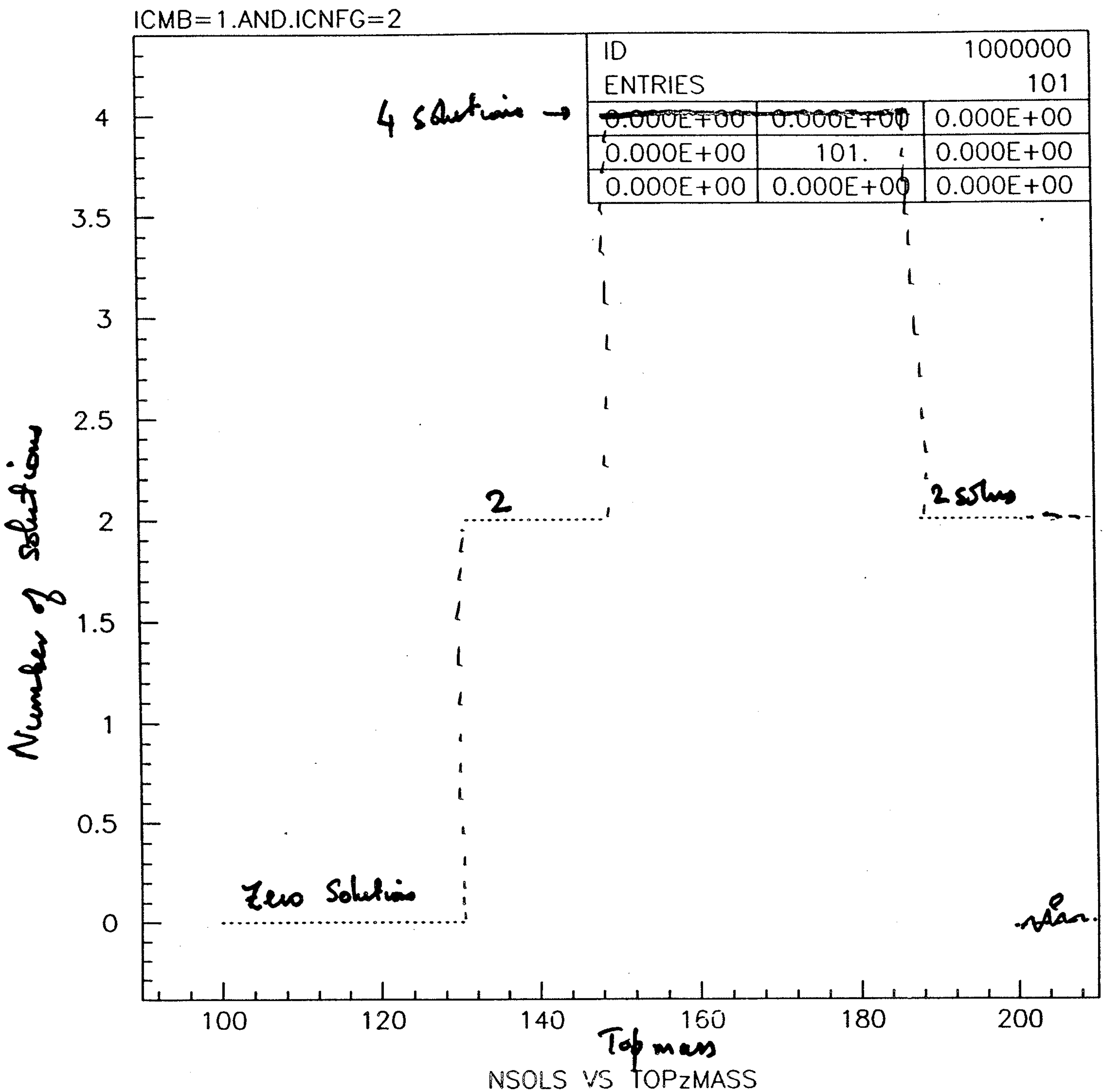
Smearing for 58796/417
ID 1000000
Entries 1000
Mean 13.93
RMS 3.002
0
10
20
30
40
50
60
70
4
8
12
16
20
24
28
ENzJT3

23

Smearing for 58796/417

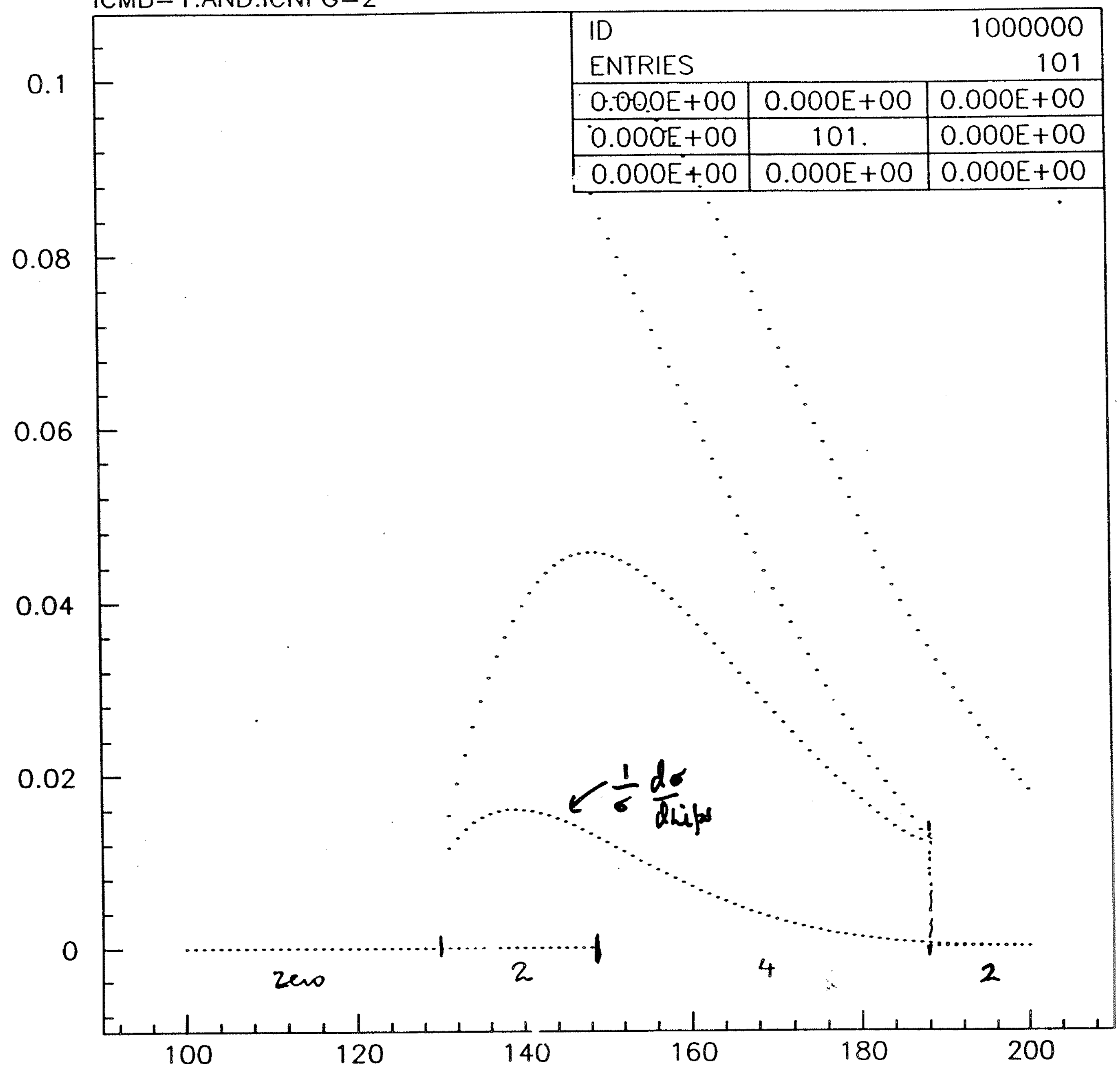


EVENT 417 1st Smeared Configuration



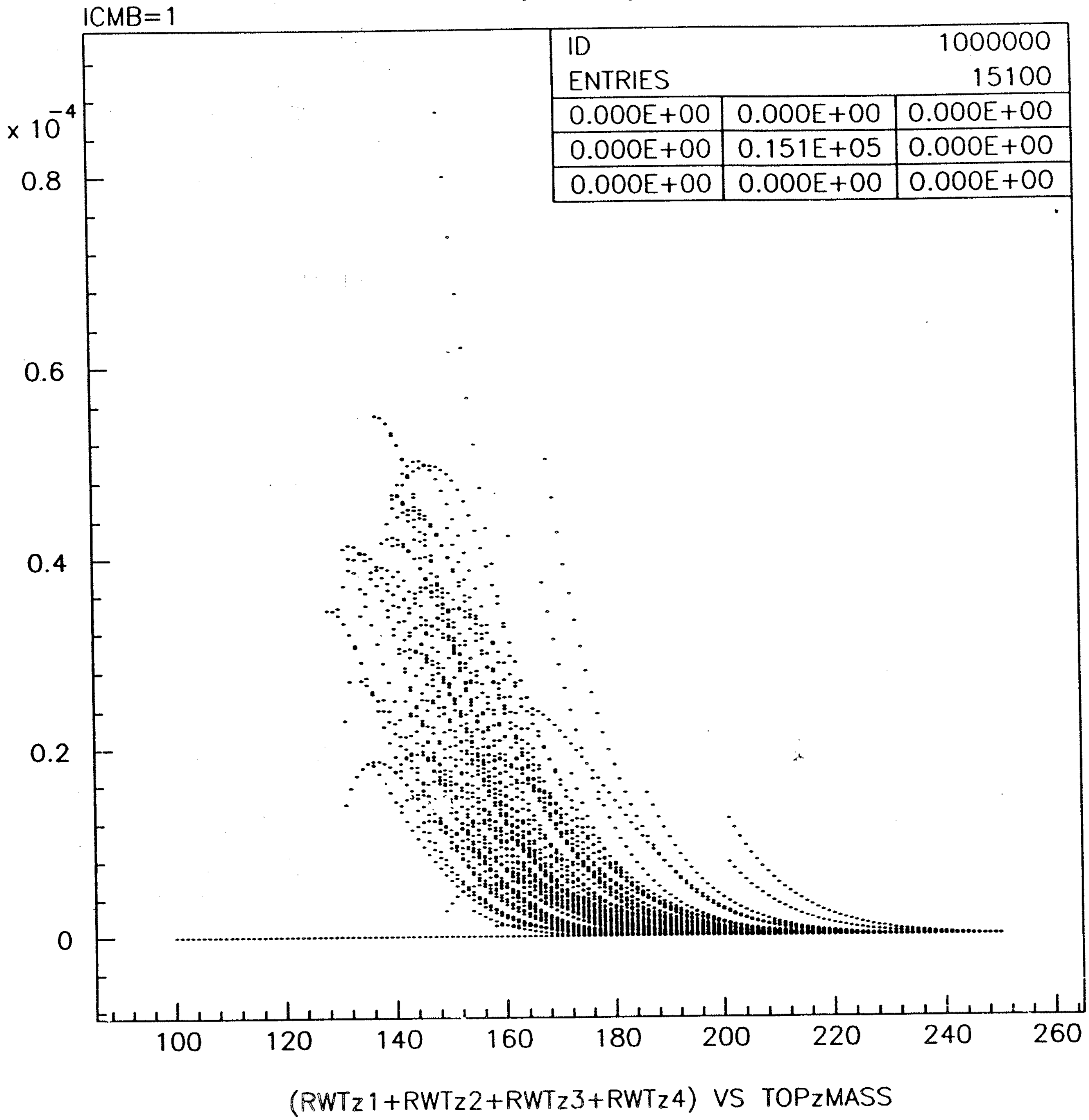
Event 417 1st Smear

ICMB=1.AND.ICNFG=2



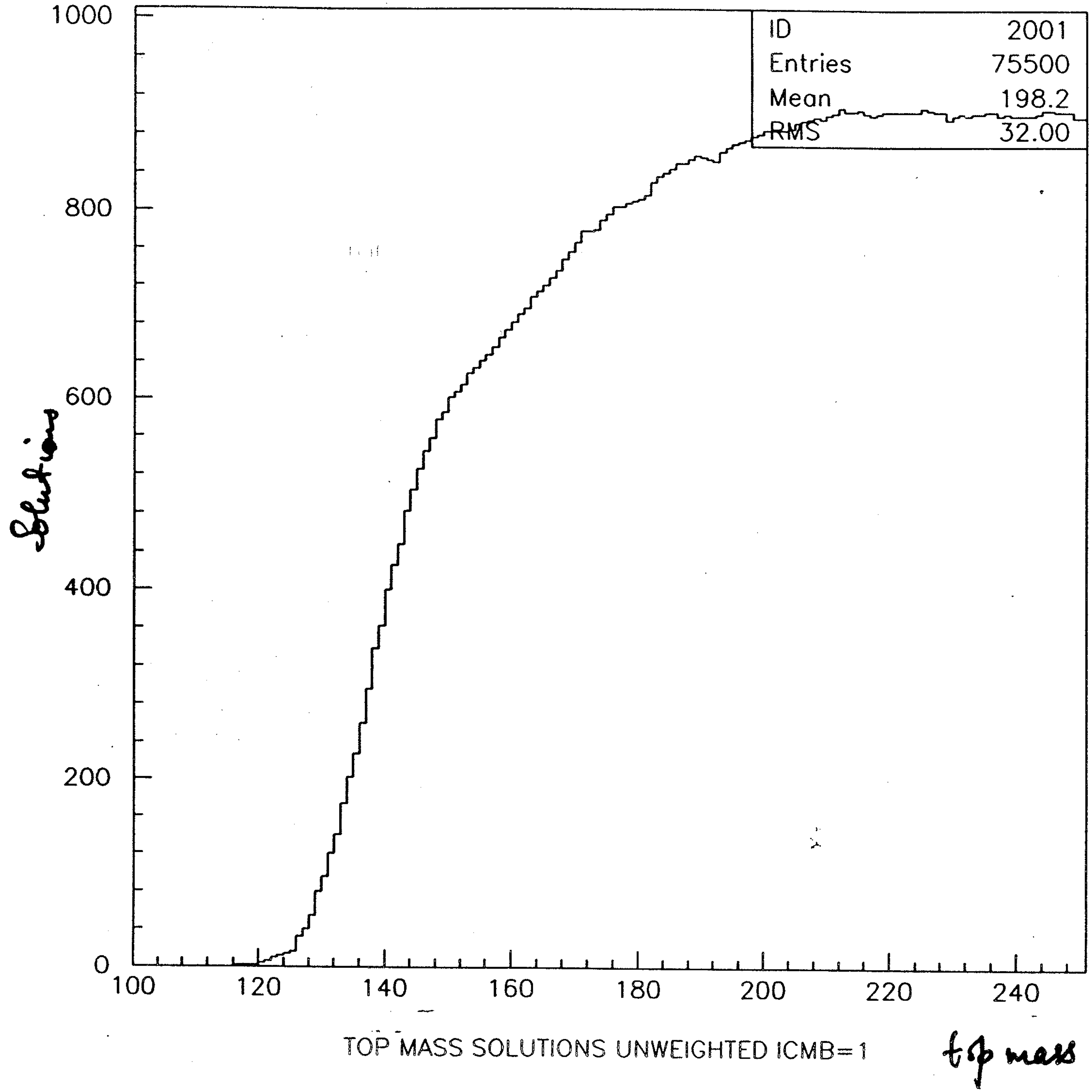
(RWTz1) VS TOPzMASS

Event 417



ICMB=1
ID 1000000
ENTRIES 15100
0.000E+00 0.000E+00 0.000E+00
0.000E+00 0.151E+05 0.000E+00
0.000E+00 0.000E+00 0.000E+00
x 10^{-4}
0.8
0.6
0.4
0.2
0
100
120
140
160
180
200
220
240
260
(RWTz1+RWTz2+RWTz3+RWTz4) VS TOPzMASS

Event 417 100 Smears



Outline of Method

$$t\bar{t} \rightarrow W^+ b\, W^- \bar{b} \rightarrow \underbrace{\ell^+ b\, \ell^- \bar{b}}_{4 \times 3}\ \underbrace{\nu_T}$$

total missing $E_T = 2$

Total number of Observables

$= 4 \times 3 + 2 = 14$ quantities

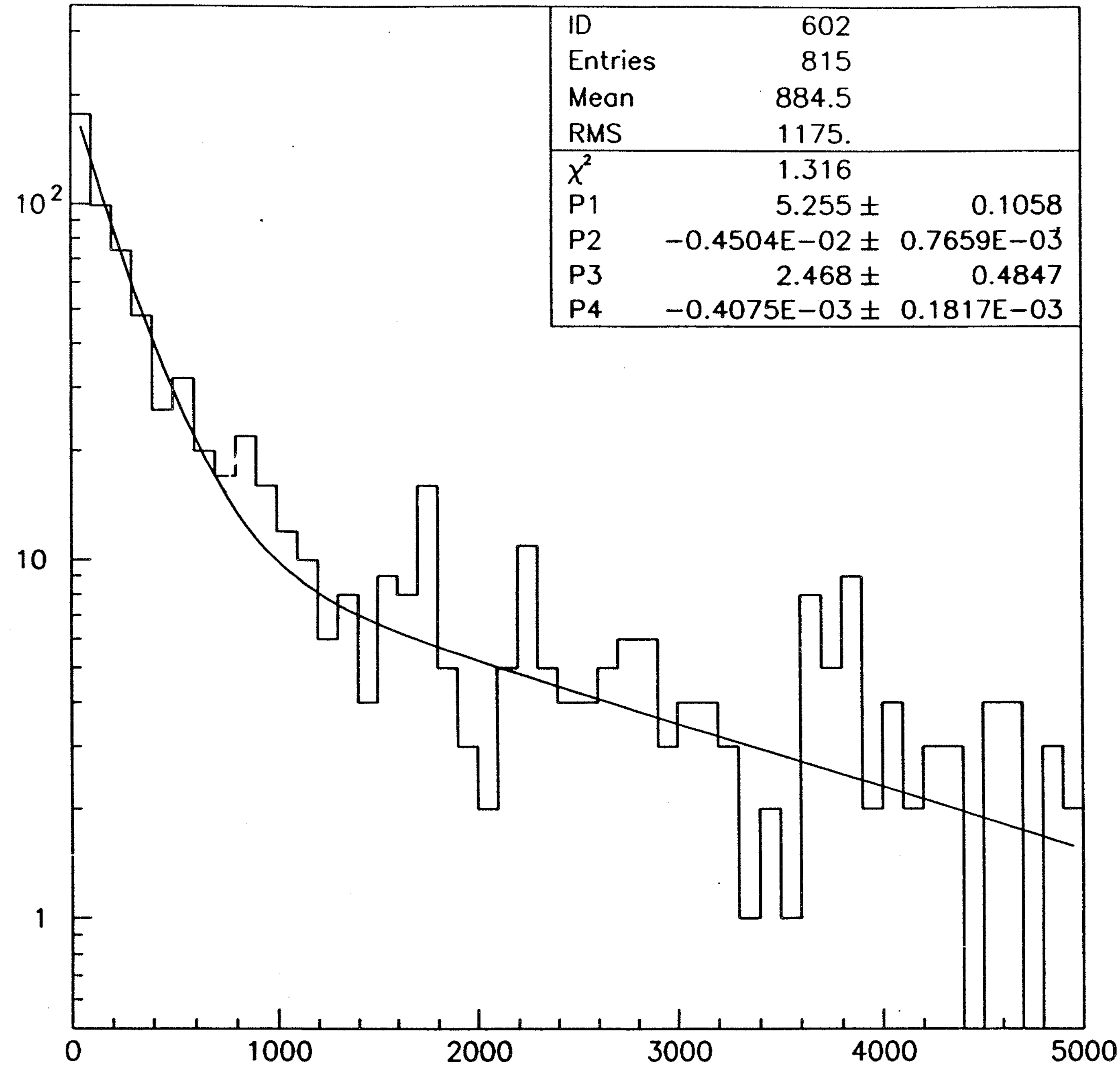
Total number of variables $= \underbrace{\ell^+ b\, \ell^- \bar{b}\, \nu \bar{\nu}}_{6 \times 3} = 18$ variables

Total number of Constraints $= 4$

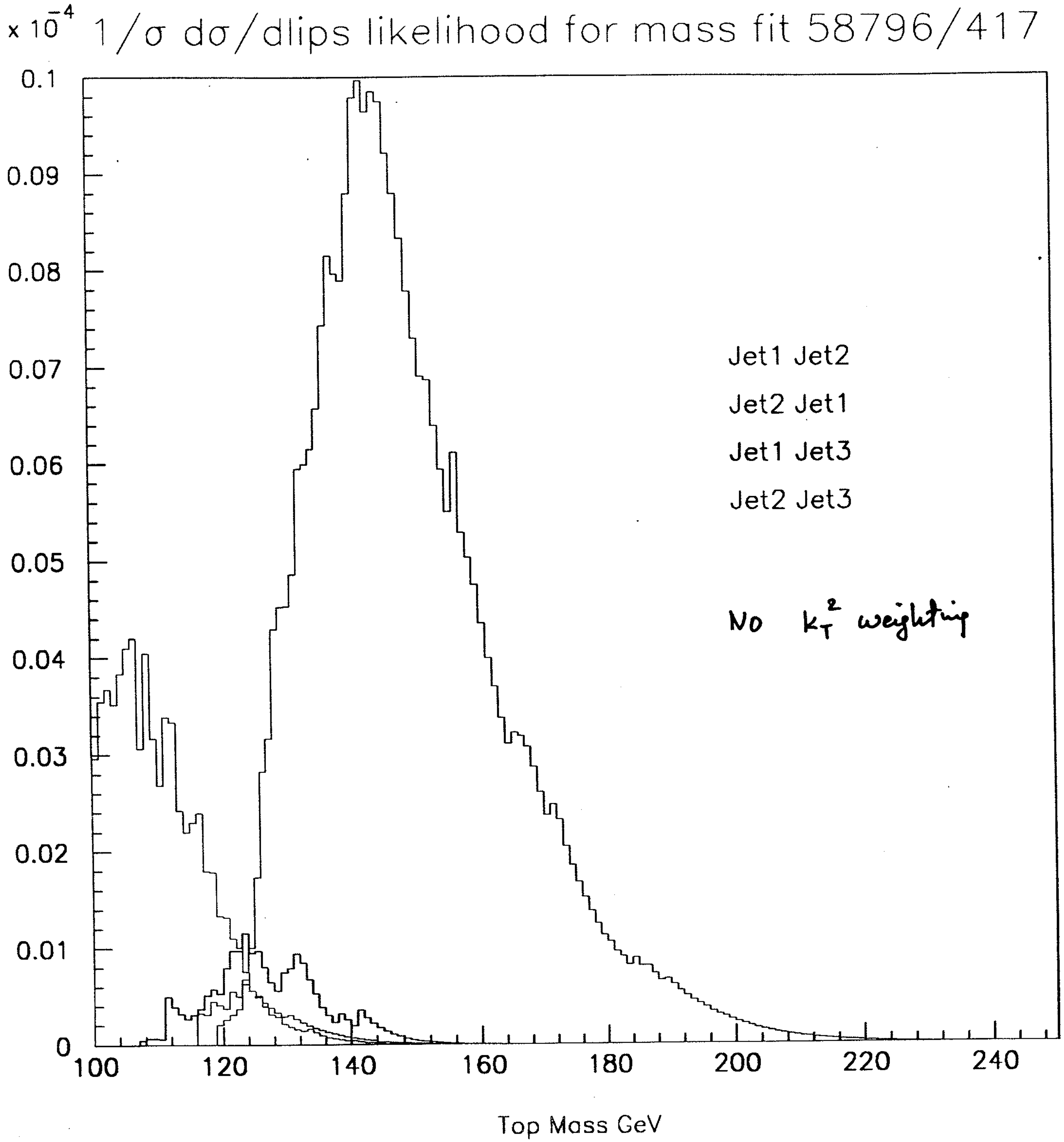
$$W^+ \rightarrow \ell \nu$$
$$W^- \rightarrow \bar{\ell} \bar{\nu}$$
$$t \rightarrow W b$$
$$\bar{t} \rightarrow W^- \bar{b}$$

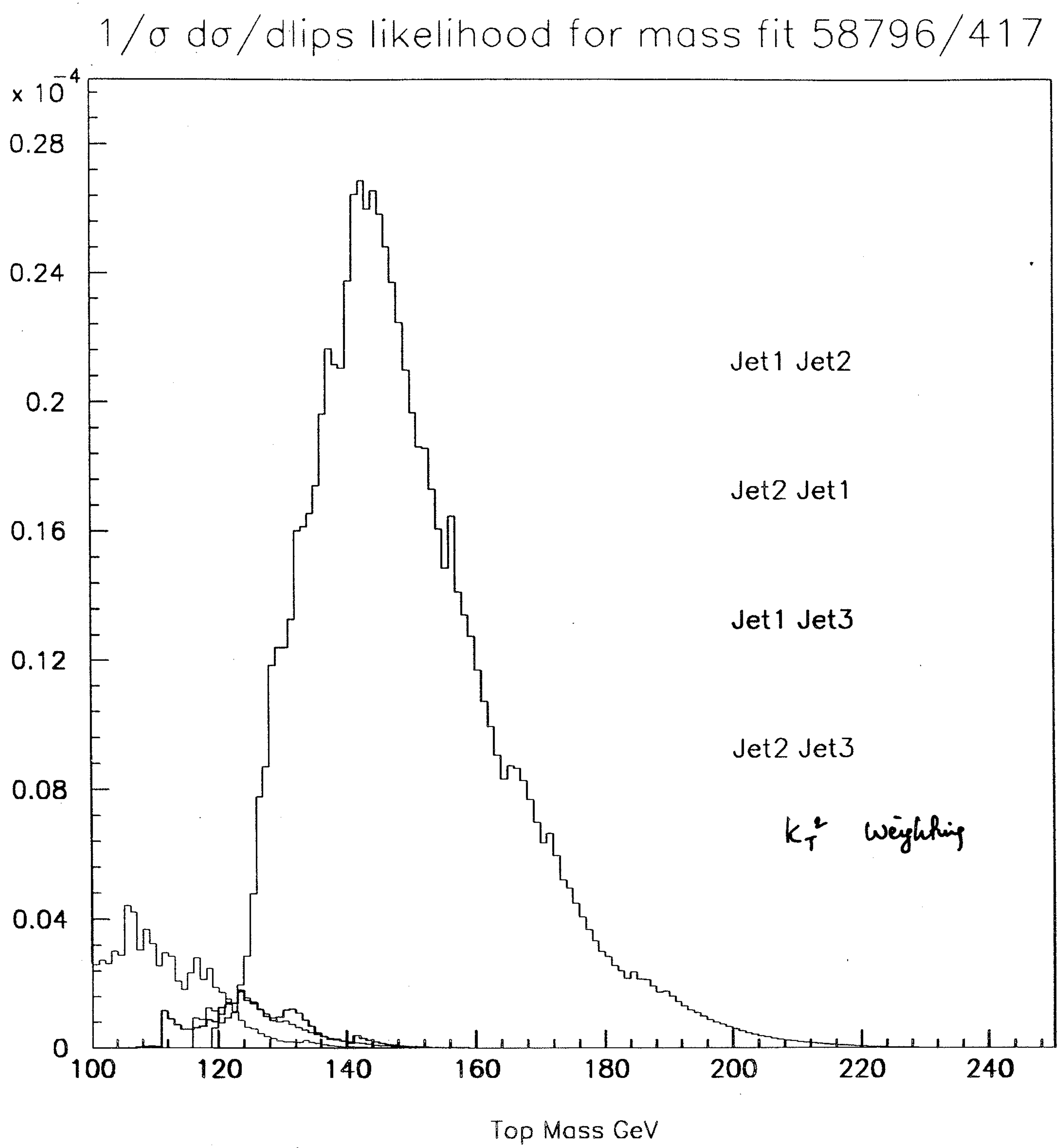
Total number of variables $= 18 - 4 = \underline{14}$

So given the top mass, one can solve the problem exactly. The ν components consistent with the observed event occurs at the intersection of two ellipses. So get either 0, 2 or 4 Solutions. DR/MS solve this <u>exactly</u> using a quartic solver.

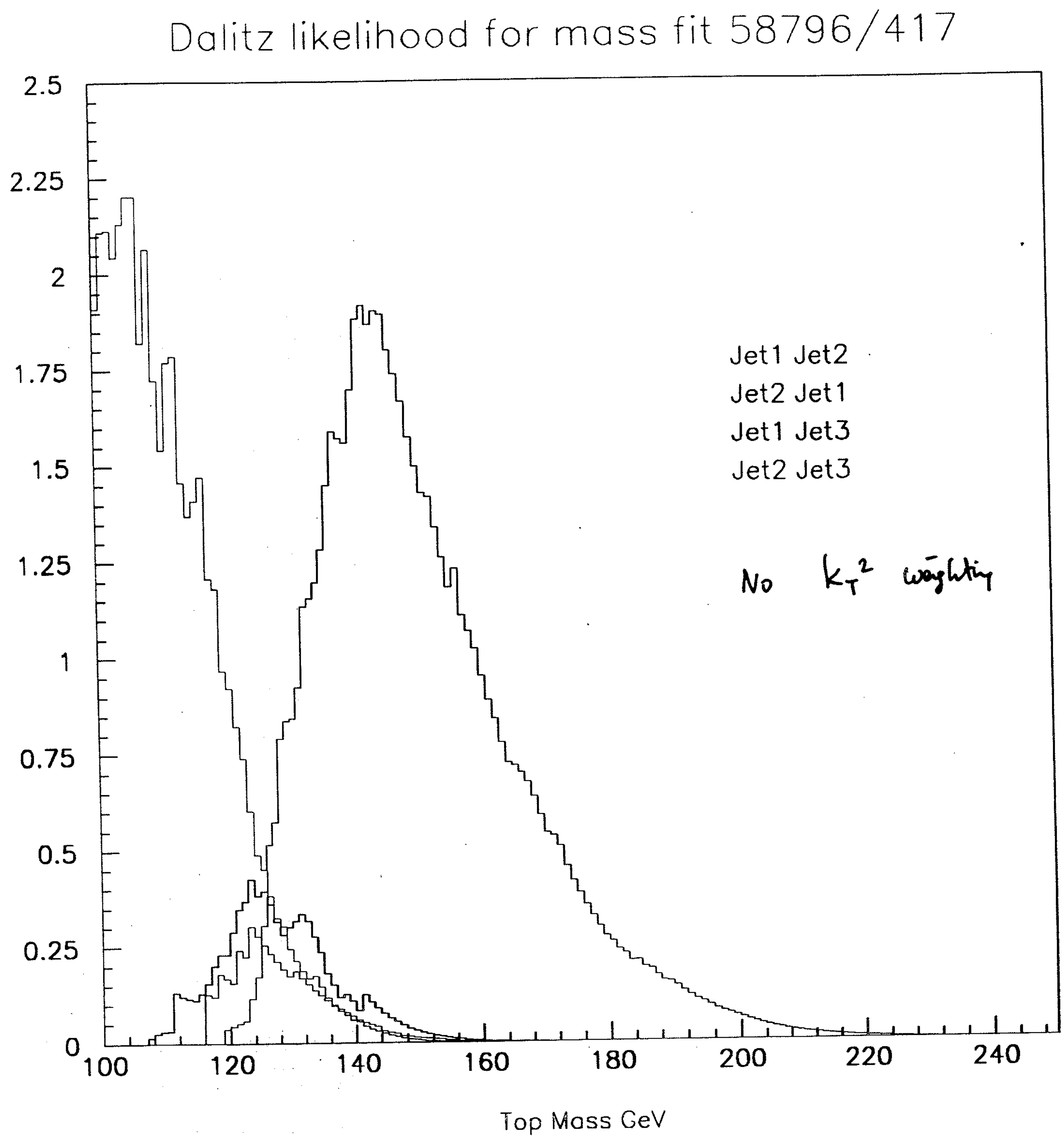


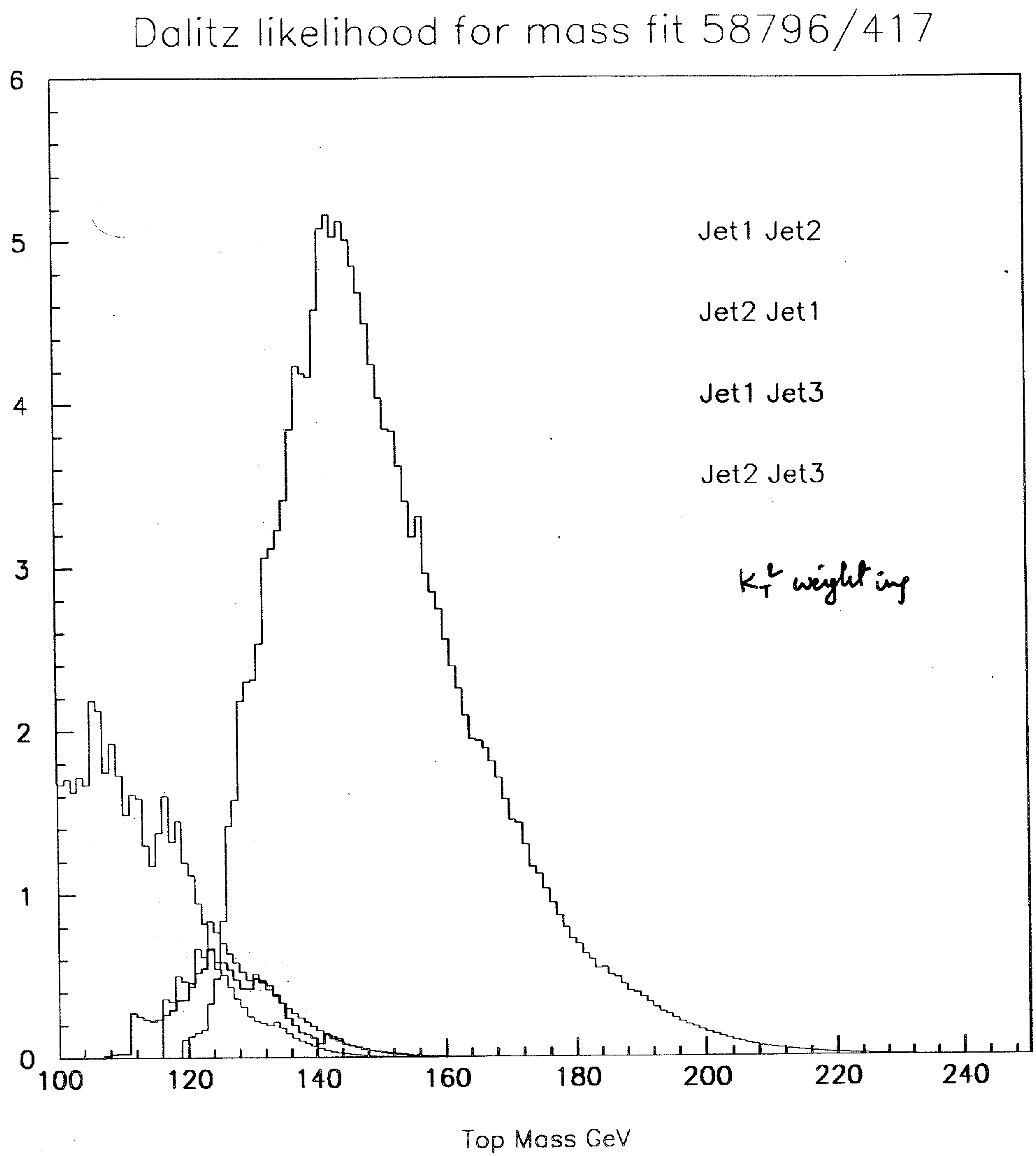
KT**2 SPECTRUM OF TTBAR


x 10^{-4} 1/σ dσ/dlips likelihood for mass fit 58796/417
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
100
120
140
160
180
200
220
240
Top Mass GeV
Jet1 Jet2
Jet2 Jet1
Jet1 Jet3
Jet2 Jet3
No k_T^2 weighting



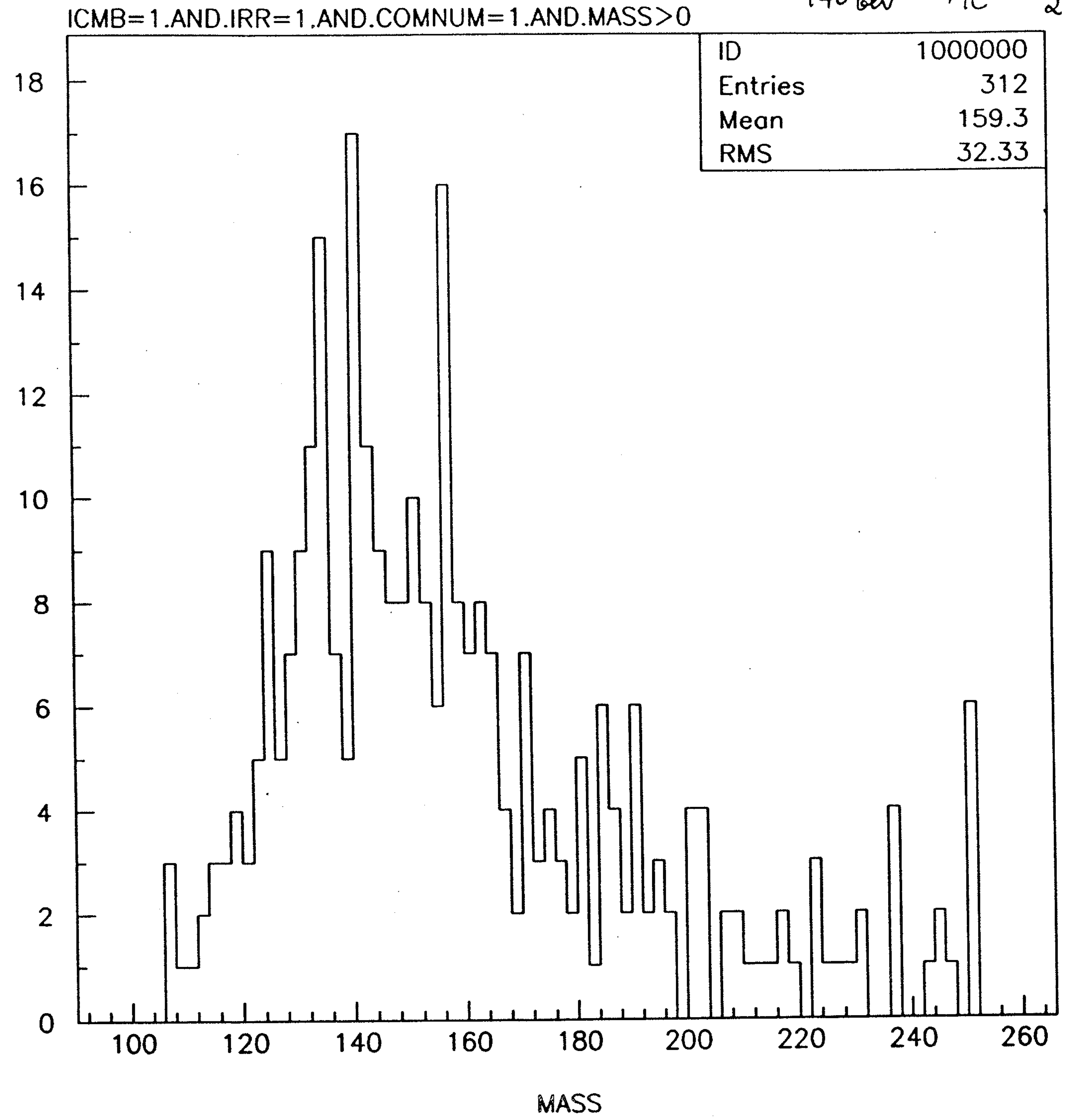
1/σ dσ/dlips likelihood for mass fit 58796/417
x 10^-4
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
100
120
140
160
180
200
220
240
Top Mass GeV
Jet1 Jet2
Jet2 Jet1
Jet1 Jet3
Jet2 Jet3
k_T^2 weighting

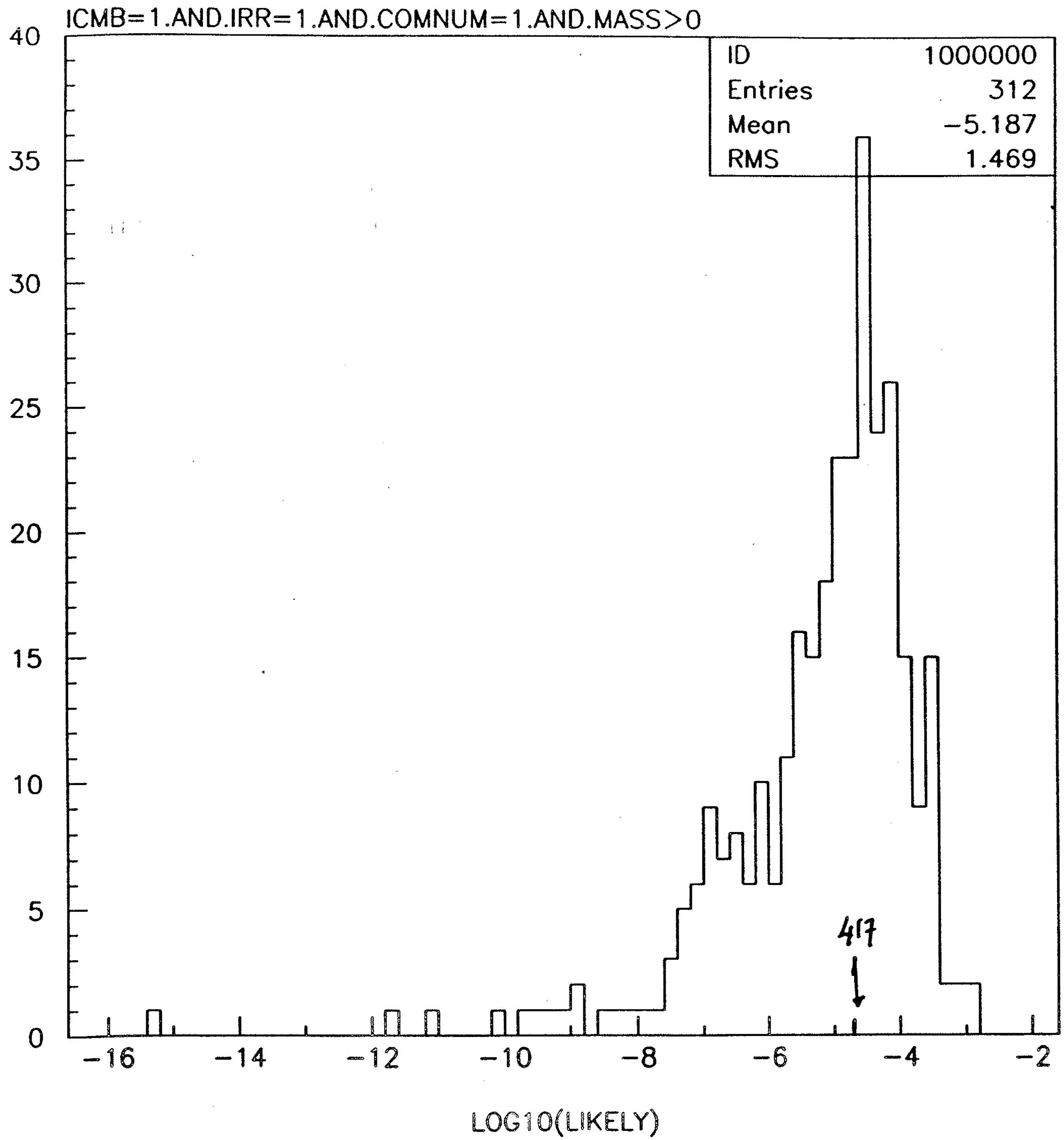

Dalitz likelihood for mass fit 58796/417
2.5
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
100
120
140
160
180
200
220
240
Top Mass GeV
Jet1 Jet2
Jet2 Jet1
Jet1 Jet3
Jet2 Jet3
No k_T^2 weighting

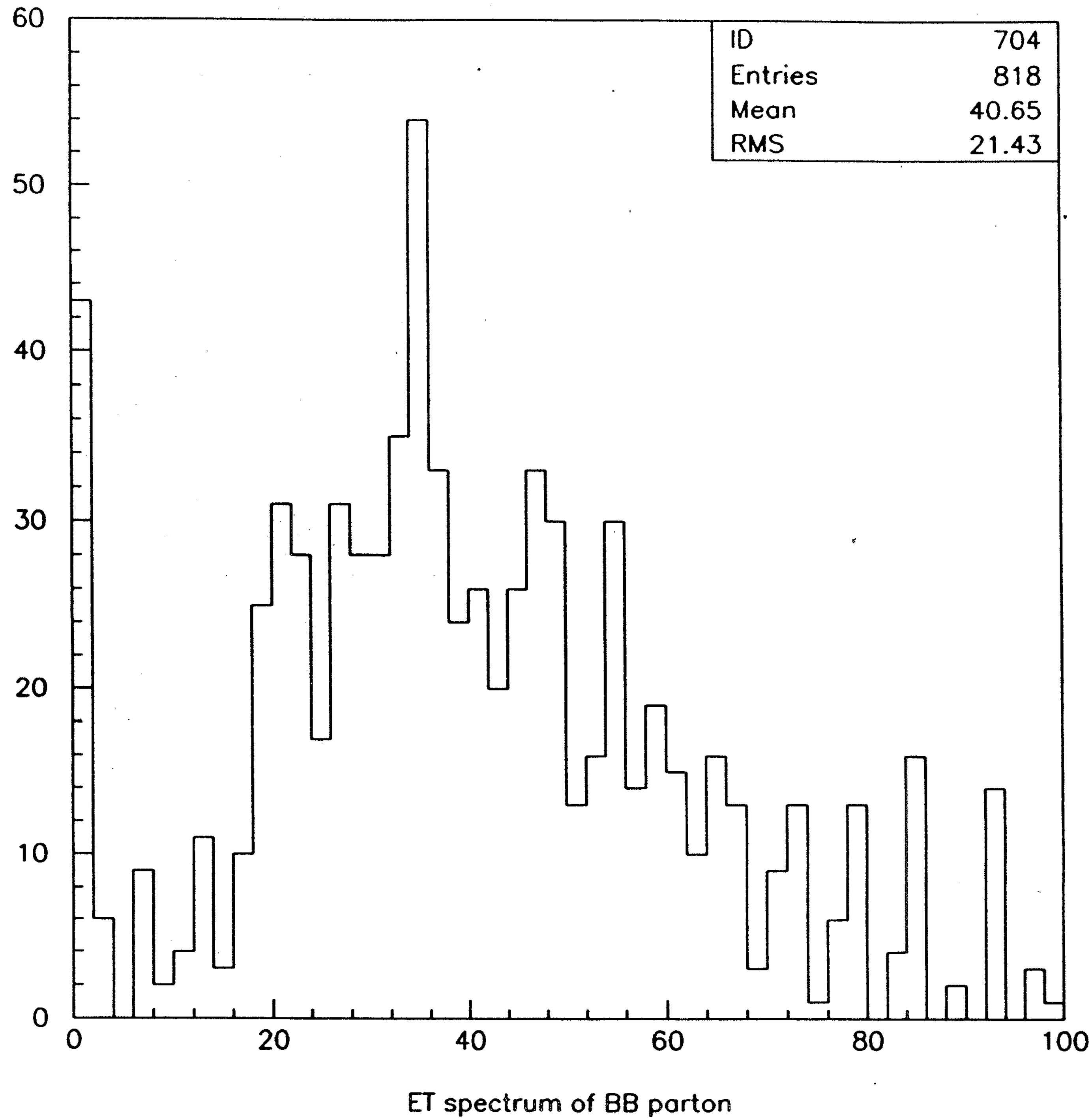


Dalitz likelihood for mass fit 58796/417
Jet1 Jet2
Jet2 Jet1
Jet1 Jet3
Jet2 Jet3
K_T^2 weighting
Top Mass GeV
0
1
2
3
4
5
6
100
120
140
160
180
200
220
240

140 GeV MC 2 highest jets




ICMB=1.AND.IRR=1.AND.COMNUM=1.AND.MASS>0
ID 1000000
Entries 312
Mean −5.187
RMS 1.469
417
0
5
10
15
20
25
30
35
40
−16
−14
−12
−10
−8
−6
−4
−2
LOG10(LIKELY)



ID 704
Entries 818
Mean 40.65
RMS 21.43
60
50
40
30
20
10
0
20
40
60
80
100
ET spectrum of BB parton

Top Mass Analysis Method for Dilepton Channel

M. Strovink
Univ. of Calif., Berkeley
October 13, 1993 at Indiana U.

* * • Motivation
* * • Implementation
* * • Monte Carlo verification
* • Jet combinatorics $\begin{cases} \text{DØ} - 1886 \\ \text{DØ} - 1906 \end{cases}$
* • Result for DØ Event 417

Mass analysis of dilepton top candidates

Given a top mass m_t, one needs 14 additional variables to specify an event of the form

$$\begin{aligned} p+\bar{p} &\rightarrow t+\bar{t}+X \\ t &\rightarrow W^+ + b_{\text{jet1}} \\ \bar{t} &\rightarrow W^- + \bar{b}_{\text{jet2}} \\ W^+ &\rightarrow e^+ + \nu_e \\ W^- &\rightarrow \mu^- + \bar{\nu}_\mu \end{aligned}$$

These variables $(v_1 \cdots v_{14})$ may be chosen to be:

$$\begin{aligned} &\mathbf{p}_\perp(t\bar{t}) \\ &x, \bar{x} \\ &\Omega_t^*, \Omega_{W^+}^*, \Omega_{W^-}^*, \Omega_e^*, \Omega_\mu^*, \end{aligned}$$

where the Ω^* are evaluated in the parent rest frames. x and $\bar{x}$ are the momentum fractions of the partons in the p and $\bar{p}$ which participate in producing the $t\bar{t}$ pair; they specify both the mass and the longitudinal momentum of that pair.

For events of this type it is straightforward to write the LO cross section differential in $dv_1 \cdots dv_{14}$.

There are also 14 experimental observables $(o_1 \cdots o_{14})$, neglecting jet masses. These may be chosen to be (rapidity) y, ϕ, and $\ln p_\perp$ for each of the 4 leptons or jets, in addition to $\mathbf{p}_\perp(t\bar{t})$. (The latter is deduced from transverse momentum balance in the calorimeter.)

Given m_t, a 0C fit is possible.

0C fits

Consider the two neutrino momenta as unknowns.

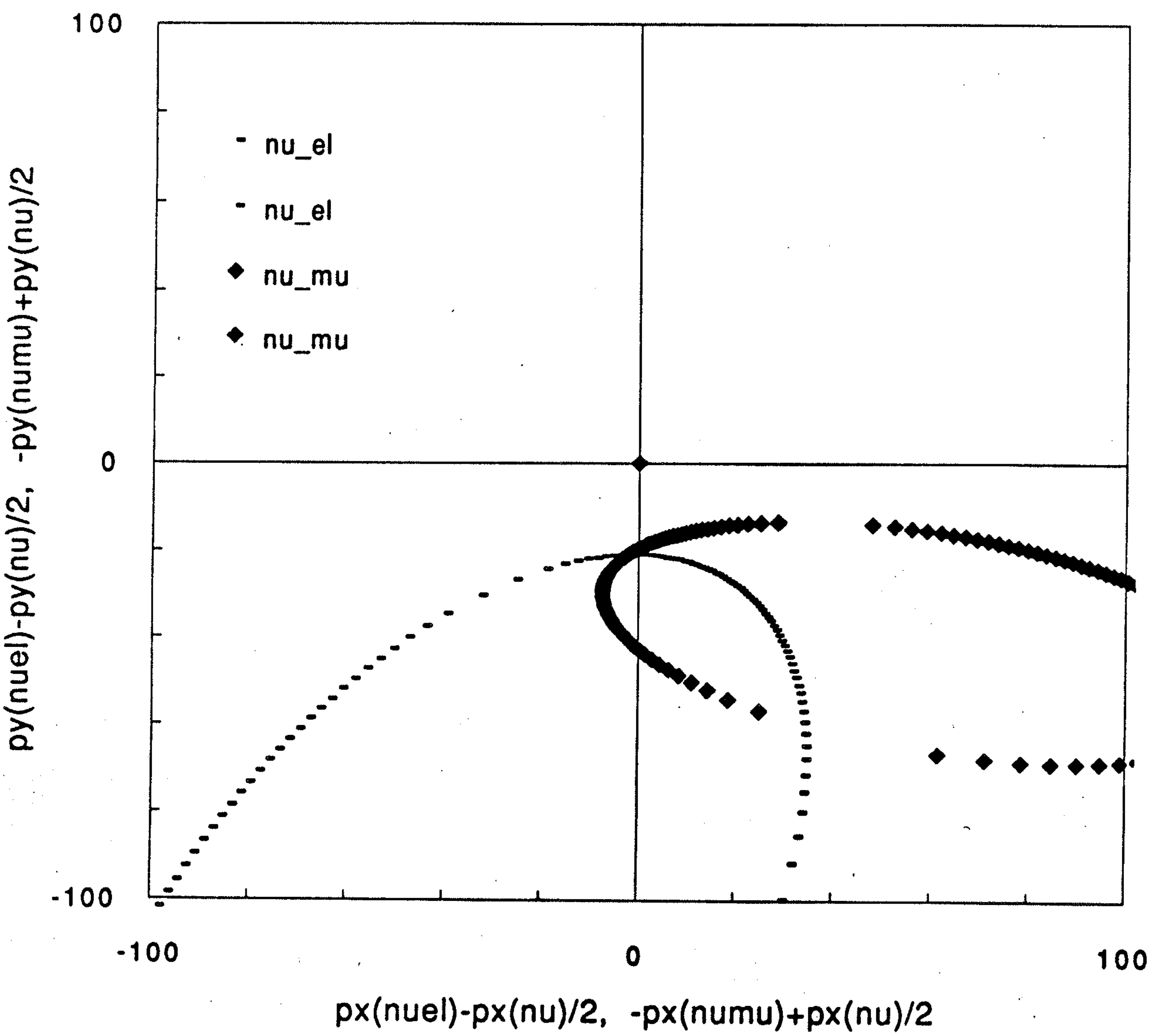
Following Dalitz and Goldstein (PL **B**287, 225 (1992)), requiring ν_e to combine with the e to form a W produces a paraboloid in neutrino momentum space. Requiring the appropriate jet and e to combine with the ν_e to form a t produces an ellipsoid.

The two intersect in an ellipse, which is projected onto the transverse plane. A similar ellipse is obtained for the muon sector.

In DØ we extend the Dalitz-Goldstein technique by using the *measured* missing $\mathbf{p}_\perp$ to impose transverse momentum conservation. We rotate one ellipse by 180°, displace the other by the missing $\mathbf{p}_\perp$, and find the intersections (up to 4 solutions are possible).

We include the effects of measurement error by smearing the events and repeating the 0C fits. To gain speed and accuracy, we bypass the analytic geometry and solve directly the quartic equations which yield these intersections.

Neutrino transverse momenta for M(top)=156



```
 nsoln =              2 ierr =             0

 solution #            1

lepton #            1
 ptop =    277.7685319002619       3.670244195243855      -164.3439313760620
           133.1651562588139
 tmc =     180.0000000000000     Wmc =    80.22000000000006

 lepton #            2
 ptop =    293.5449553706527       11.42975580475614       157.0439313760620
           170.2210478282661
 tmc =     180.0000000000000     Wmc =    80.22000000000005


 solution #            2

 lepton #            1
 ptop =    275.1617582294765       77.58016162983438      -184.4308298227737
           57.27635397185081
 tmc =     180.0000000000000     Wmc =    80.22000000000002
 lepton #            2

 ptop =    348.9537088808154      -62.48016162983438       177.1308298227737
           232.5716867348264
 tmc =     180.0000000000000     Wmc =    80.22000000000001
```

Example of likelihood analysis

Consider a *gedanken* measurement. It is known that a parallel beam of K^0's all decay to $\pi\pi$. The K^0 momenta are known to be uniformly distributed between zero and some high value. There is no background.

Our apparatus is able to measure only the angle $\cos\theta$, relative to the beam, of only one of the two π's.

Our experiment consists of only one such measurement.

What is the likelihood that the K^0 which decayed had momentum K?

Kinematics: if $\beta(K^0) > \beta^*(\pi \text{ in } K^0 \text{ rest frame})$, we expect a Jacobian peak in $\cos\theta$:

$$\begin{aligned}
\cot\theta &= \gamma(\cot\theta^* + (\beta/\beta^*)\csc\theta^*) \\
d\cot\theta &= -\gamma\left(\frac{1}{\sin^2\theta^*} + \frac{\beta}{\beta^*}\frac{\cos\theta^*}{\sin^2\theta^*}\right)d\theta^* \\
&= 0 \text{ when } \beta = \beta^*/(-\cos\theta^*)
\end{aligned}$$

At that peak (in $d\cos\theta^*(K)/d\cos\theta$), $\cos\theta$ is a unique function of K, and *vice versa.*

Method of analysis: Use Bayes' th^{m}:

$$P(K_m|\cos\theta) = \frac{P(K_m)\,P(\cos\theta|K_m)}{\sum_{n=0}^{N} P(K_n)\,P(\cos\theta|K_n)}$$

where K_m is a particular beam momentum (increasing linearly with m), and $\cos\theta$ denotes observation of the pion angle θ.

The denominator is obviously independent of m. Since we are told that the *a priori* probability of any K_m is independent of m, so is the first term in the numerator. Then

$$P(K_m|\cos\theta) \propto P(\cos\theta|K_m)\ .$$

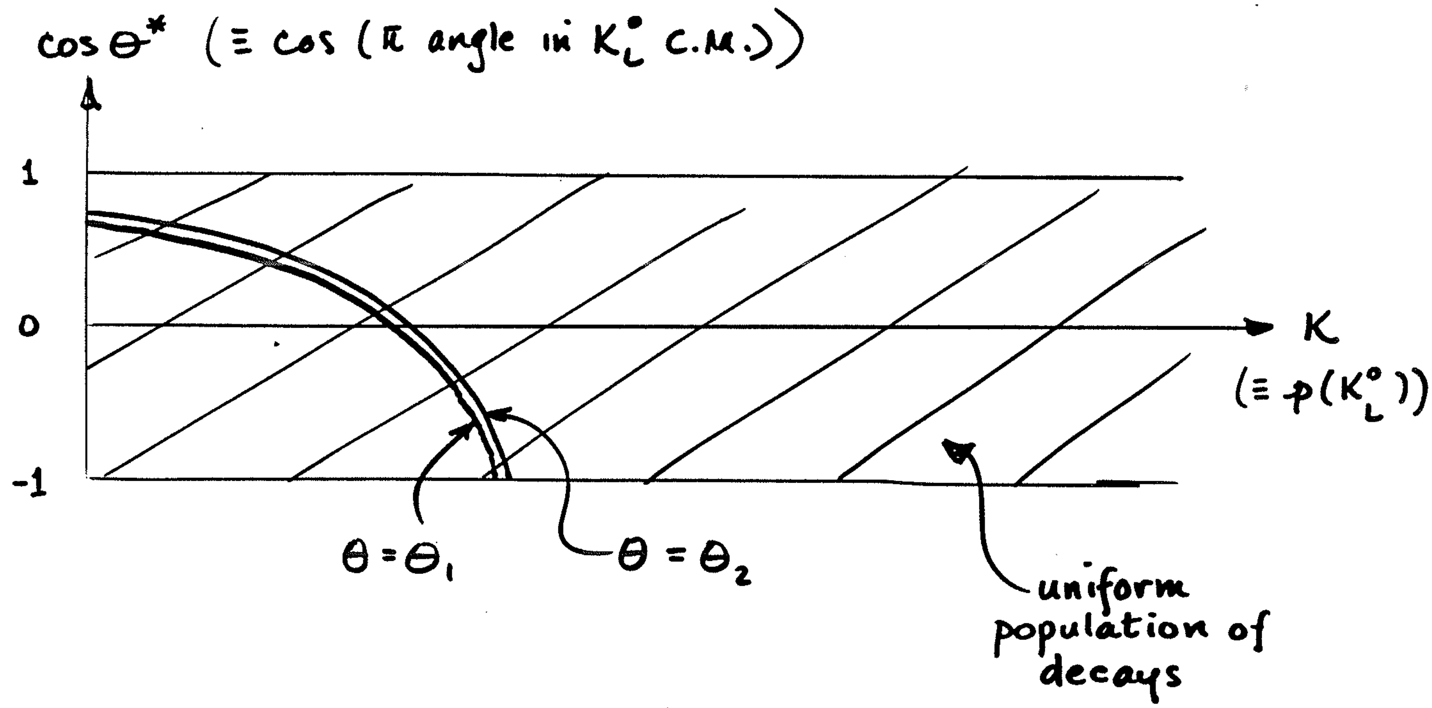
For each K_m in an array of possible values, we calculate the "density of states" factor

$$\left|\frac{\partial(\cos\theta^*(K_m))}{\partial(\cos\theta)}\right|$$

The resulting histogram is our likelihood distribution for K.

If $\cos\theta$ is measured with error δ, we smear it and average the histograms that are obtained.

Note that $\left|\frac{\partial\cos\theta^*}{\partial f(\theta)}\right|$ produces the <u>identical</u> result $\forall\ f(\theta)$, because $\frac{\partial\cos\theta}{\partial f(\theta)}$ is indep. of K_m.



Apparatus detects π's with $\theta_1 < \theta < \theta_2$ (yellow band)
What is the K distribution of these π's?

$$\frac{d^2N}{d\cos\theta^* dK} = \text{const} \equiv \rho \qquad \# \text{ detected events} \equiv n$$

$$n = \int_0^\infty dK\, \rho \left(\cos\theta^*(\theta_2, K) - \cos\theta^*(\theta_1, K)\right)$$

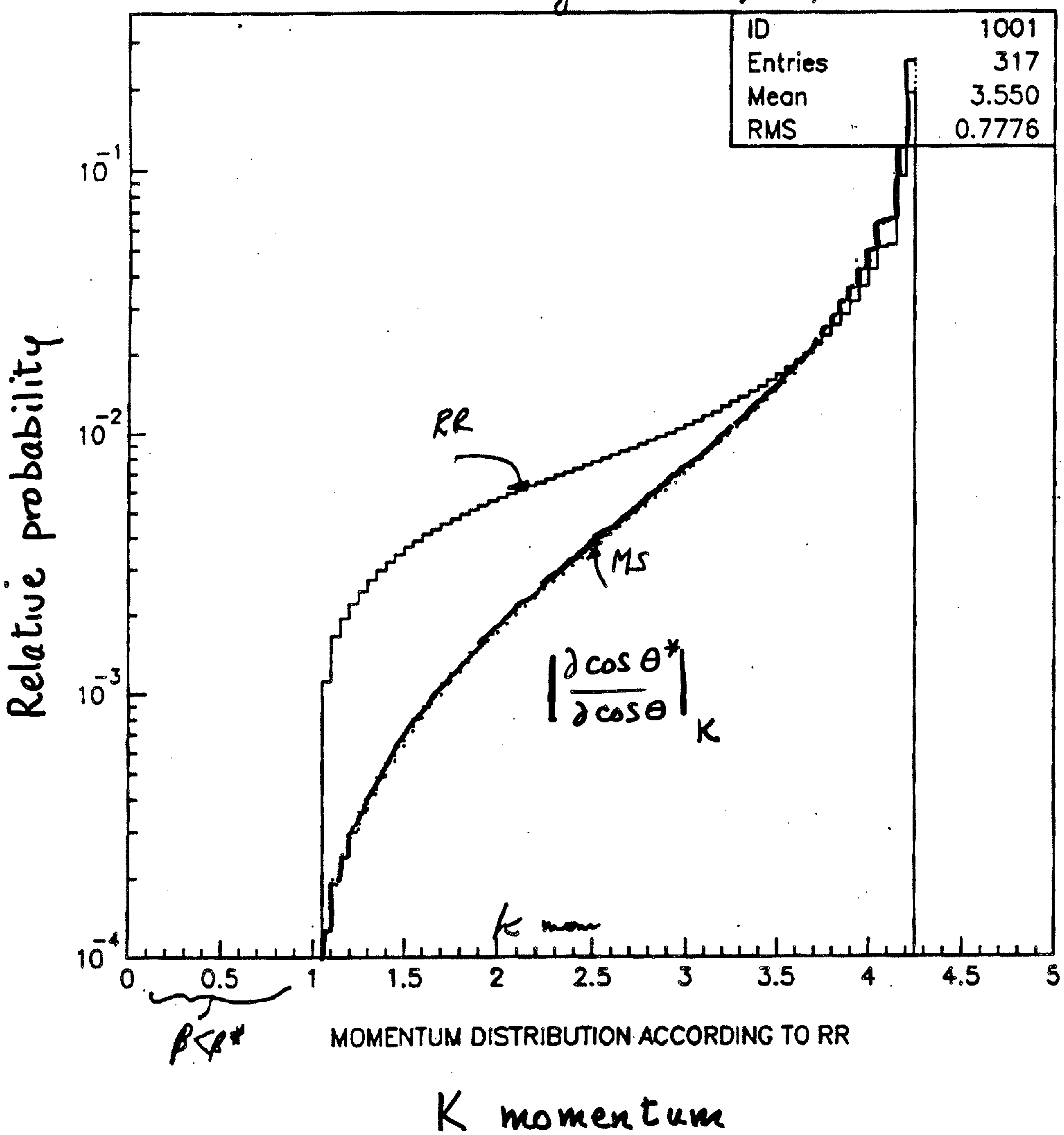
take limit $\Delta\theta \to 0$

$$n = \int_0^\infty dK\, \rho \left.\frac{\partial \cos\theta^*}{\partial\theta}\right|_K \Delta\theta$$

$$\frac{dn}{dK} = \underbrace{\rho\Delta\theta}_{\text{independent of } K} \left.\frac{\partial \cos\theta^*}{\partial\cos\theta}\right|_K \frac{\partial\cos\theta}{\partial\theta}$$

a number independent of K

10° angle in lab for 1 pion



Ignoring production rate in dilepton top mass analysis

When one has a number $N \pm \delta N$ of candidates above background, comparison of N to $\sigma_{\text{vis}} \int \mathcal{L}\, dt$ usefully constrains the top mass. How best to apply this constraint?

In DØ we choose to make the dilepton top mass likelihood analyses independent of the production rate, because

- Information from several channels can be combined to sharpen the mass constraint obtained from the combined production rate.
- Production rate information may be left for inclusion late in the analysis chain when final efficiency corrections, cross sections *etc.* are available.
- Mass information from the production rate may be used as a cross-check on that from the dilepton top mass analysis.

Therefore, the DØ top mass likelihood analysis is designed to be completely independent of any production rate information.

What remains as a basis for dilepton top mass analysis?

As a direct corollary of the preceding, DØ cannot use a differential cross section to weight the solutions found in dilepton top mass analysis, unless that weight is divided by an appropriate total cross section:

- *A priori* we do not consider top masses produced with high cross section to be more probable than those produced with low cross section. This bias would imply that we know the relation between the number of events expected and the number observed.

- Instead we merely ask "given a particular top mass, and given that one event is detected, what is the probability density that the event would have appeared in our apparatus possessing the observed kinematic configuration?" This question is appropriately independent of production rate.

What is "probability density that the event would have appeared in our apparatus possessing the observed kinematic configuration"?

Assume <u>decays isotropic</u>, <u>unit acceptance</u>

$$(1) \quad \frac{d^{14}\mathcal{L}}{do_1 \cdots o_{14}} = \frac{\int |M|^2 \prod_{i=1}^{14} \frac{e^{-\frac{(o_i-\bar{o}_i)^2}{2\sigma_i^2}}}{\sqrt{2\pi\sigma_i^2}} d^{18}LIPS\, \delta^2(m-m_t)\delta^2(m-m_W)}{\int |M|^2 d^{18}LIPS\, \delta^2(\quad)\delta^2(\quad)}$$

$\uparrow \sigma_i$ on RHS has dimension of o_i

Assume <u>meas. errors infinitesimal</u>: $\frac{e^{-\frac{(o_i-\bar{o}_i)^2}{2\sigma_i^2}}}{\sqrt{2\pi\sigma_i^2}} \longrightarrow \delta(o_i-\bar{o}_i)$

Rewrite

$$d^{18}LIPS \propto \left[dx\,d\bar{x}\,d^2\hat{\Omega}\,d^2p_\perp^{t\bar{t}} \frac{d^6 LIPS}{dx\,d\bar{x}\,d^2\hat{\Omega}^*\,d^2p_\perp^{t\bar{t}}}\right] \times$$

$$\times \{E_{j_1}dE_{j_1}d\Omega_{j_1}\, E_{j_2}dE_{j_2}d\Omega_{j_2}\, E_e dE_e d\Omega_e\, E_\mu dE_\mu d\Omega_\mu\}$$

Choose to evaluate $\{\quad\}$ in respective parent CMS's

$$\{\quad\} = f(m_{t_1})dm_{t_1}\, f(m_{t_2})dm_{t_2}\, m_{W^+}dm_{W^+}\, m_{W^-}dm_{W^-}\, d\Omega^*_{j_1}\, d\Omega^*_{j_2}\, d\Omega^*_{j_3}\, d\Omega^*_{j_4}$$

Plugging into (1) and cancelling common factors in num. & denom.

$$(2) \quad \frac{d^{14}\mathcal{L}}{do_1 \cdots do_{14}} = \frac{\int |M|^2 \delta^{14}(o_i-\bar{o}_i)[\quad]d^4\Omega^*}{\int |M|^2[\quad]d^4\Omega^*}$$

Now $|M|^2 \frac{d^6 LIPS}{dx\,d\bar{x}\,d^2\hat{\Omega}^*\,d^2p_\perp^{t\bar{t}}} \equiv \frac{d^6\sigma^{t\bar{t}}}{dx\,d\bar{x}\,d^2\hat{\Omega}^*\,d^2p_\perp^{t\bar{t}}}$

Integrating, the denominator in (2) is just σ_T within a constant factor

$$(3) \quad \frac{d^{14}\mathcal{L}}{do_1 \cdots do_{14}} = \frac{1}{\sigma_T}\int \frac{d^6\sigma^{t\bar{t}}}{dx\,d\bar{x}\,d^2\hat{\Omega}^*\,d^2p_\perp^{t\bar{t}}}\, \delta^{14}(o_i-\bar{o}_i)\, dx\,d\bar{x}\,d^2\hat{\Omega}\,d^2p_\perp^{t\bar{t}}\,d^4\Omega^*$$

00040

constant factors are suppressed.

(Strovink's Jacobian)

Introduce $J \equiv \dfrac{dx\, d\bar{x}\, d^2\hat{\Omega}^* d^2 p_L^{t\bar{t}}\, d^4\Omega^*}{do_1 \cdots do_{14}}$. Then, evaluating $\delta^{14}(o_i - \bar{o}_i)$,

$$\frac{d^{14}\mathcal{L}}{do_1 \cdots do_{14}} = \frac{J}{\sigma_T} \frac{d^6\sigma^{t\bar{t}}}{dx\, d\bar{x}\, d^2\hat{\Omega}^* d^2 p_L^{t\bar{t}}}$$

(4) $$\boxed{\frac{d^{14}\mathcal{L}}{do_1 \cdots do_{14}} = \frac{d^{14}\sigma^{t\bar{t}}}{\sigma_T^{t\bar{t}}\, do_1 \cdots do_{14}}}$$

Isotropic decays; perfect measurements of $\{o_i\}$.

(σ_T becomes σ_{vis} when acceptance considered)

Weights for solutions must be acceptance-dependent

Consider the limiting case of zero detector acceptance, in which only a single kinematic configuration can be observed. Then the only experimental information available is the number of events collected: all events which possibly could be detected appear to be identical.

But DØ dilepton top mass analyses cannot use production rate information. So in this limiting case their only possible basis for weighting the solutions at various dilepton top masses is the multiplicity of solutions (0, 2, or 4). The dilepton top mass likelihood reduces to a sum of θ functions.

The *ad hoc* weights in current DØ use fail this condition. Even in the limit of zero acceptance,

"C" $$\frac{1}{\sigma_T}\frac{d\sigma}{d\text{LIPS}}P(E_e^*)P(E_\mu^*)$$ (method "I")

and

"A" $$f(x)\bar{f}(\bar{x})P(E_e^*)P(E_\mu^*)$$ (method "III")

yield the same top mass likelihood as they do for finite acceptance.

However, the weight used in the top mass analysis described here <u>does</u> satisfy this boundary condition. This weight is

"B" $$\frac{1}{\sigma_{\text{vis}}}\frac{d^{14}\sigma}{do_1\cdots do_{14}}$$ "Method II"

where the $\{o_i\}$ are the experimentally observed variables. In the limit of zero acceptance

$$\Delta o_1 \cdots \Delta o_{14} \rightarrow 0,$$

σ_{vis} reduces to

$$\sigma_{\text{vis}} \equiv \frac{d^{14}\sigma}{do_1\cdots do_14}\Delta o_1 \cdots \Delta o_14,$$

and this weight is equal to the same constant

$$(\Delta o_1 \cdots \Delta o_{14})^{-1}$$

for every solution, as is required.

"Paradoxes" continued ...

3. Using $\dfrac{d^{14}\sigma}{\sigma_{vis}\, do_1 \cdots do_{14}}$ as a weight makes analysis "dependent on an arbitrary choice of observables $\{o_i\}$".

Let $\{o_i'\}$ be an alternate choice of observables.

$$\frac{d^{14}\sigma}{\sigma_{vis}\, do_1' \cdots do_{14}'} = \frac{d^{14}\sigma}{\sigma_{vis}\, do_1 \cdots do_{14}} \cdot \underbrace{\left| \frac{\partial \{o_i\}}{\partial \{o_i'\}} \right|}$$

this factor is independent of top mass ;
<u>no effect</u> on top mass likelihood.

"Paradoxes" continued...

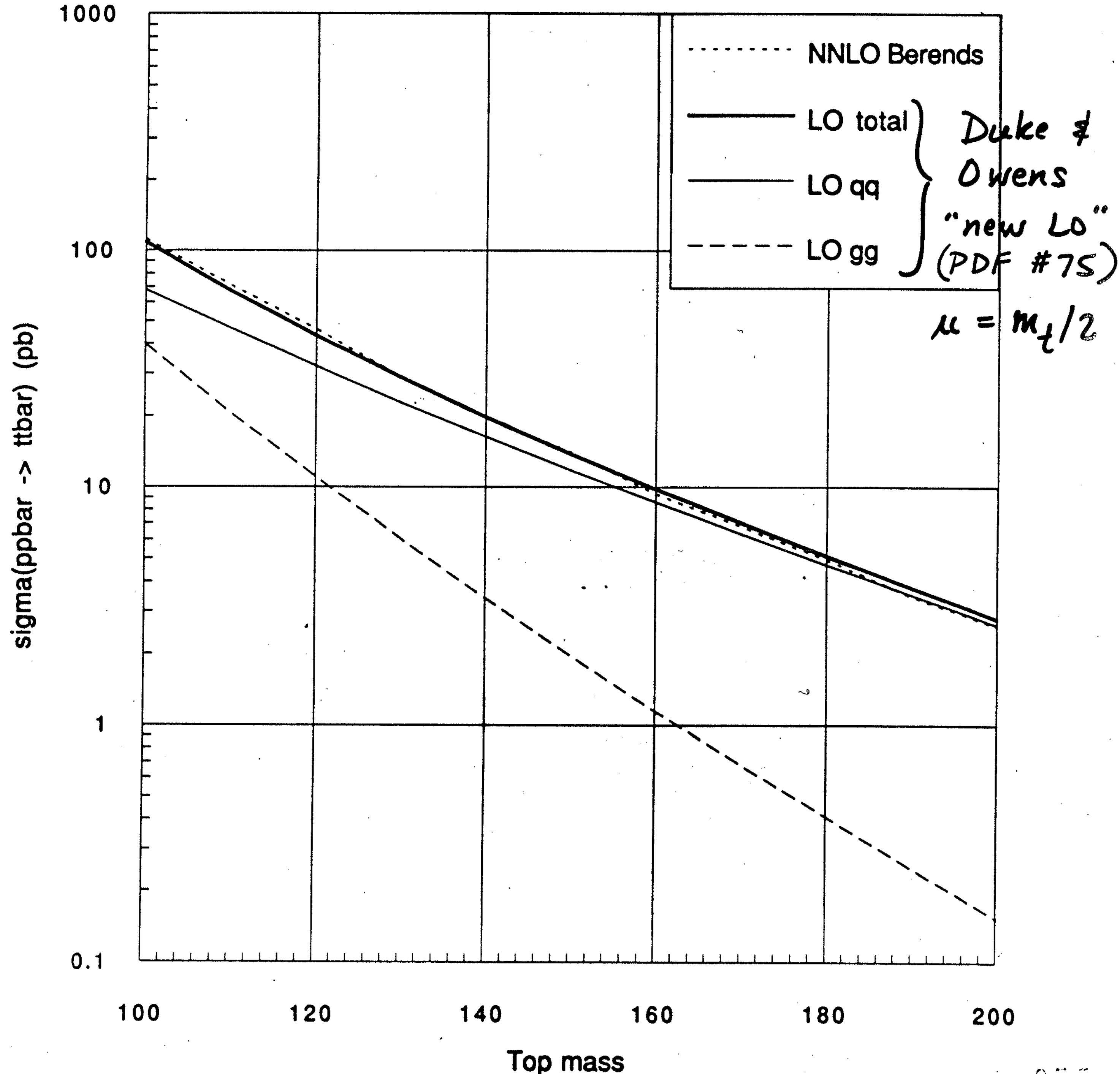
4. ("Theriot paradox"). CDF has 2 top candidates, identical except $p_e^A = 2p_e^B$. Physicist 1 uses $O_e \equiv p_e^{-1}$, physicist 2 uses $O_e = E_e$. Physicist 1 assigns

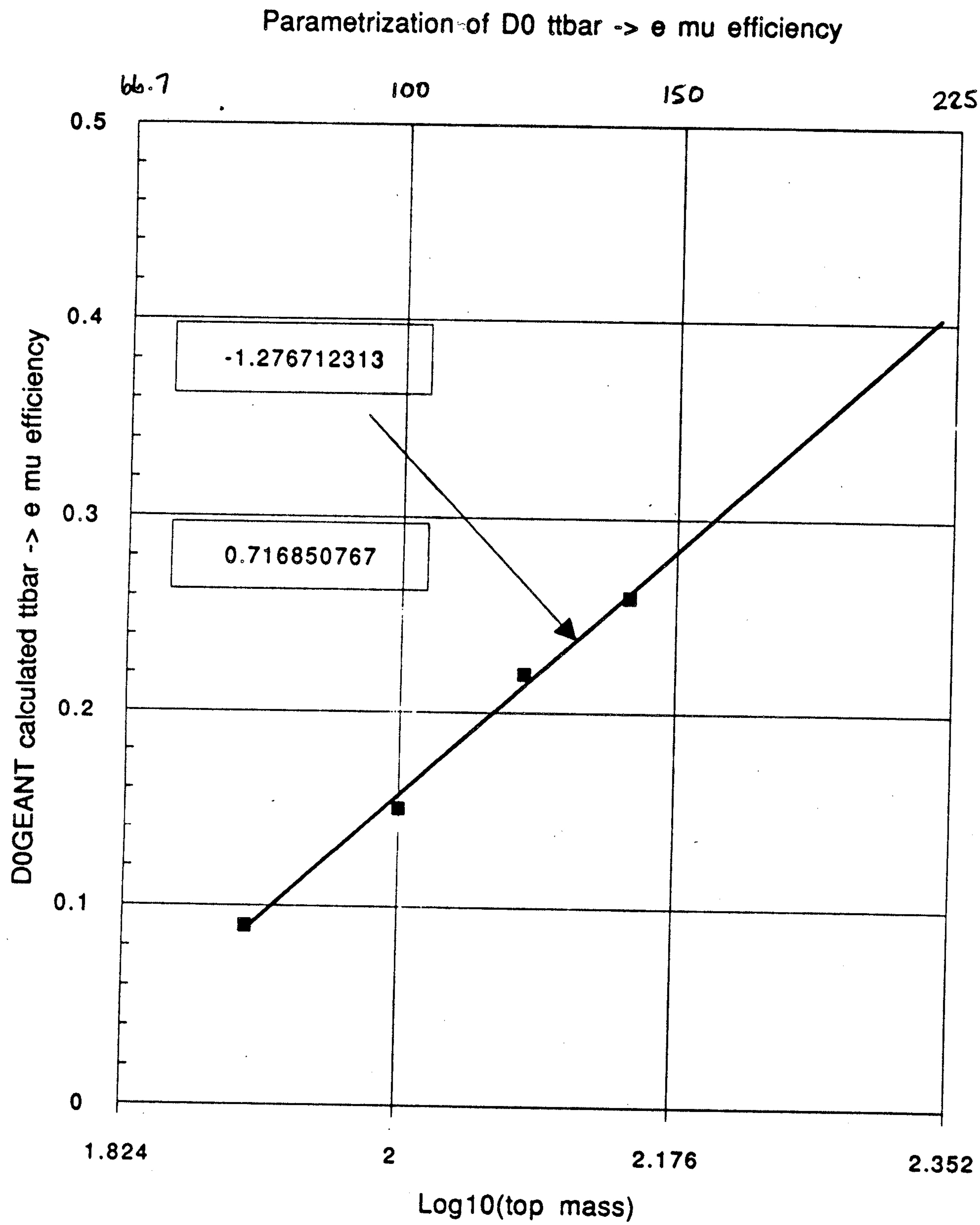
$$\frac{P(A)}{P(B)} = \frac{dO_e^B}{dO_e^A} = \frac{p_e^A}{p_e^B} = 2$$

while physicist 2 gets $\frac{1}{2}$. CDF's tracker and calorimeter are so good that it can't matter which is used.

The weight $\dfrac{d^{14}\sigma}{\sigma_{vis}\, do_1 \cdots do_{14}}$ should be used to assign probabilities to different top masses given the <u>same</u> event, not to assign relative probabilities for top interpretation of <u>different</u> events.

Top cross section vs. top mass





Parametrization of D0 ttbar -> e mu efficiency
66.7
100
150
225
0.5
0.4
0.3
0.2
0.1
0
-1.276712313
0.716850767
D0GEANT calculated ttbar -> e mu efficiency
1.824
2
2.176
2.352
Log10(top mass)

Session Name: csa6 1

```
Csa6>@sig
Csa6>run sig
 dsig_qq =  5.9361205472111768E-02
 dsig_gg =  2.3927438566936087E-02
  sig_qq =  0.1179757291143857
  sig_gg =  4.6937654035596109E-02
  Obtained by integrating dsig..
  sig_qq =  0.1179757559557379
  sig_gg =  4.6937655372402114E-02
```

using GQUAD

Session Name: csa6 1

```
 dsig_qq =  5.9361205472111768E-02
 dsig_gg =  2.3927438566936087E-02
  sig_qq =  0.1179757291143857
  sig_gg =  4.6937654035596109E-02
  Obtained by integrating dsig..
  sig_qq =  0.1179757309420628
  sig_gg =  4.6937654797375261E-02
```

using GQUAD - MS

errors smaller > ×10

Compare $\hat{\sigma}$ (Cambridge) to numerically integrated $\frac{d\hat{\sigma}}{d\hat{t}}$

Transforming $\{o_i\} \rightarrow \{v_i\} \rightarrow \{o_i\}$

```
Running EXTH for solution #          1
pl( ,1) =   105.1905413998806       12.2000000000000
-96.1000000000000         41.0000000000000
pl( ,2) =   107.0071960197070      -24.3000000000000
 98.3000000000000         34.6000000000000
pj( ,1) =   37.58230966824684      -27.3000000000000
-10.3000000000000        -22.4000000000000
pj( ,2) =   46.09696302360927      -16.0000000000000
-22.1000000000000         36.6000000000000
ptop( ,1, ) =   195.5009220115914       20.12306678027462
-109.4733807035237        57.71699583381226
ptop( ,2, ) =   196.8937851334532      -3.323066780274619
 95.67338070352373        84.27766059941914
Returning from EXTH with ierr =          0
oml =  0.3856800697725314     -0.9664082794707615       0.4950909016223313
 1.472362296408078
omj = -0.7302356009572952     -0.9109457571723081      -0.5377500198469648
 2.759473932153241
omt = -1.700769773125329E-02 -0.7616049274923951
x, xbar =  0.2965481105096151       0.1387762700282468
Calling THEX with
rootS, tm, Wm =   1800.000000000000       150.0000000000000
 80.2200000000000
jmex, ptt(2), ptt(3) =   7.700000000000000       6.400000000000000
 16.8000000000000        -13.8000000000000
Returning from THEX with ierr =          0
pl( ,1) =   105.1905413998806       12.2000000000000
-96.1000000000000         41.0000000000000
pl( ,2) =   107.0071960197070      -24.3000000000000
 98.3000000000000         34.6000000000000
pj( ,1) =   37.58230966824684      -27.3000000000000
-10.3000000000000        -22.4000000000000
pj( ,2) =   46.09696302360926      -16.0000000000000
-22.0999999999999         36.6000000000000
ptop( ,1, ) =   195.5009220115914       20.12306678027462
-109.4733807035237        57.71699583381226
ptop( ,2, ) =   196.8937851334532      -3.323066780274615
 95.67338070352373        84.27766059941914
```

(highlighted digits are approximate)

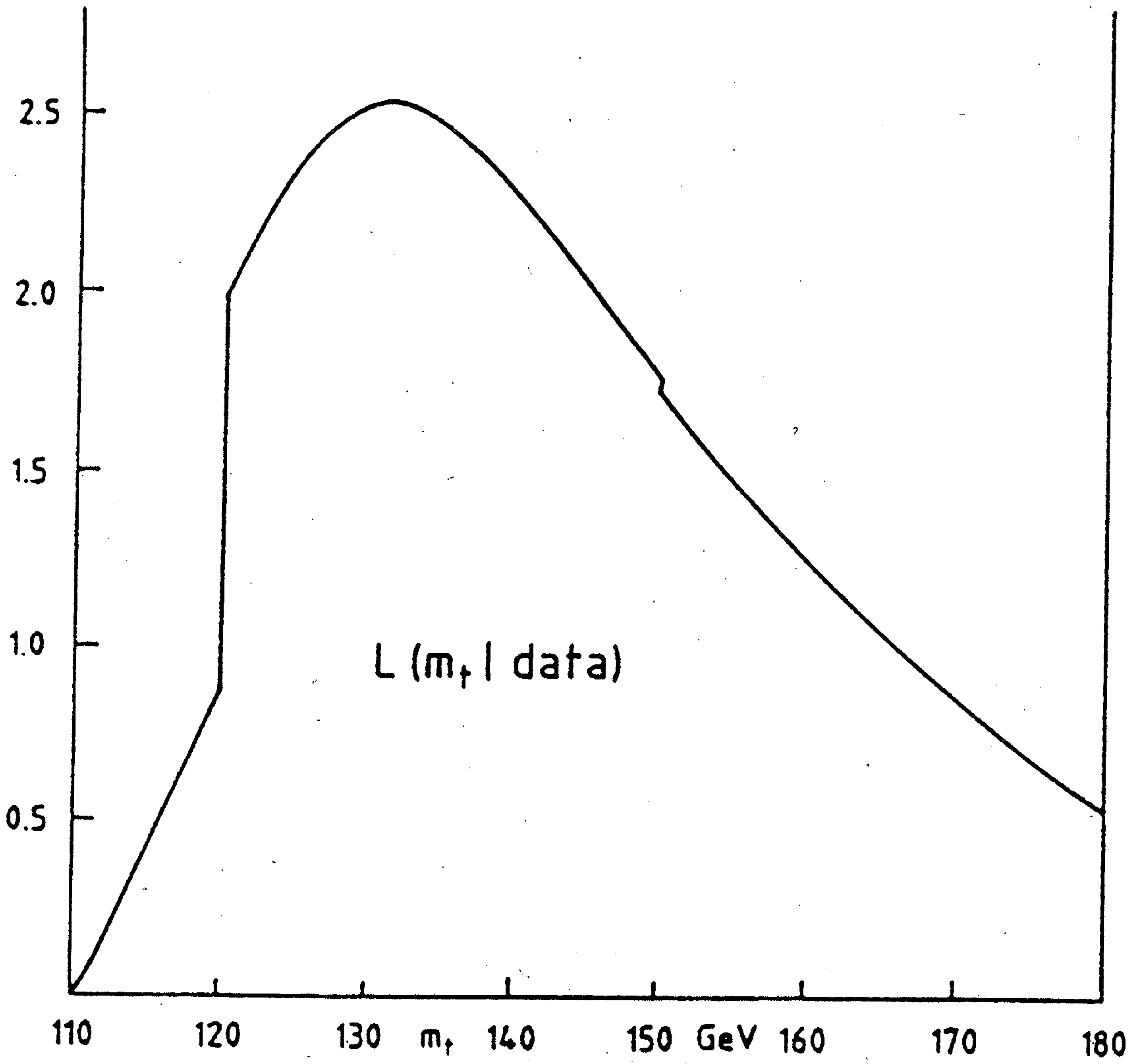
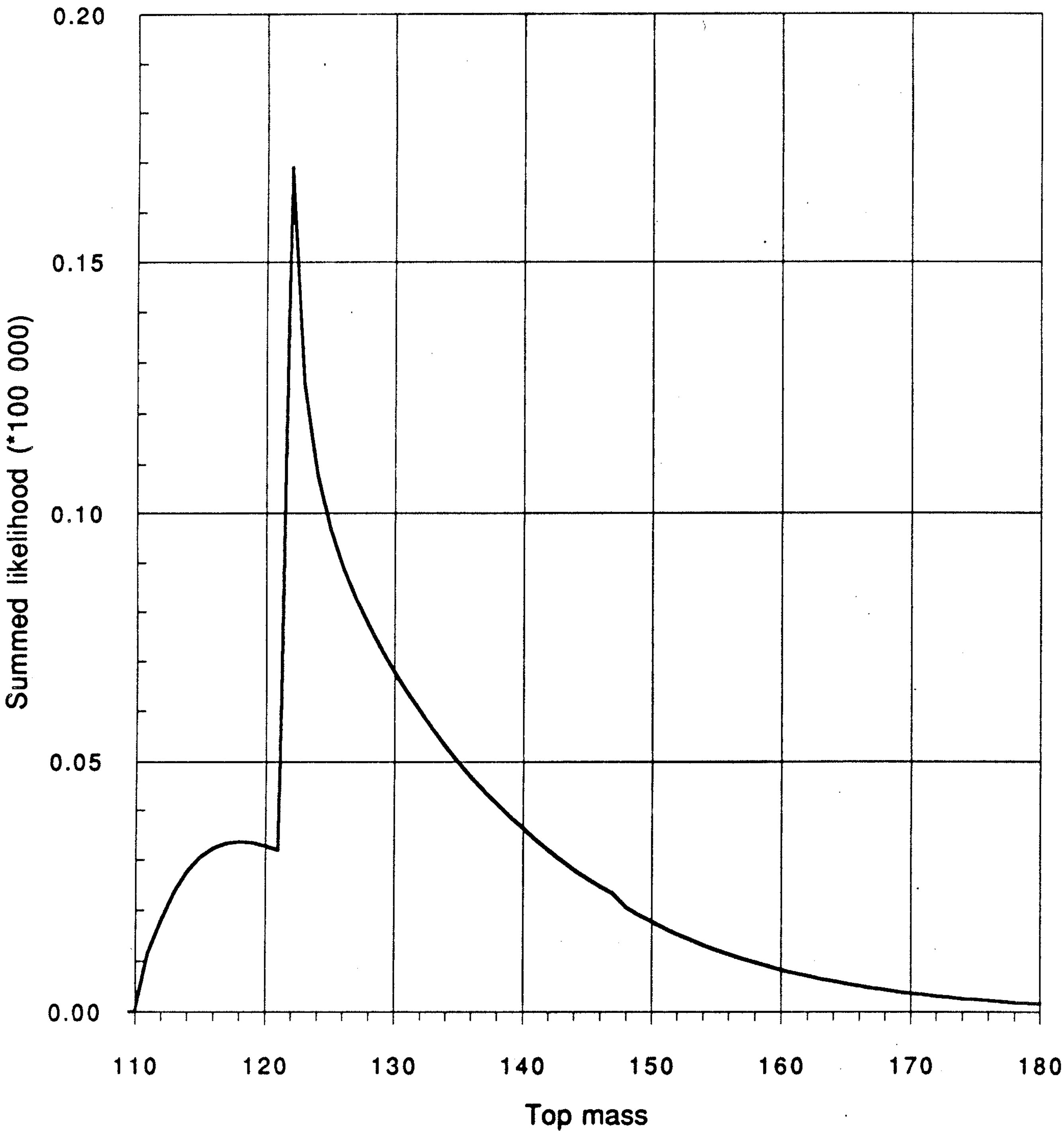


Fig. 4. The likelihood function for the top-quark mass m_t, based on the interpretation of the CDF candidate event as due to $t-\bar{t}$ pair production, followed by W^+ and W^- leptonic decays.

Summed likelihood vs. top mass: CDF 1989 candidate



~~Fine-tuning since 22 May 93~~

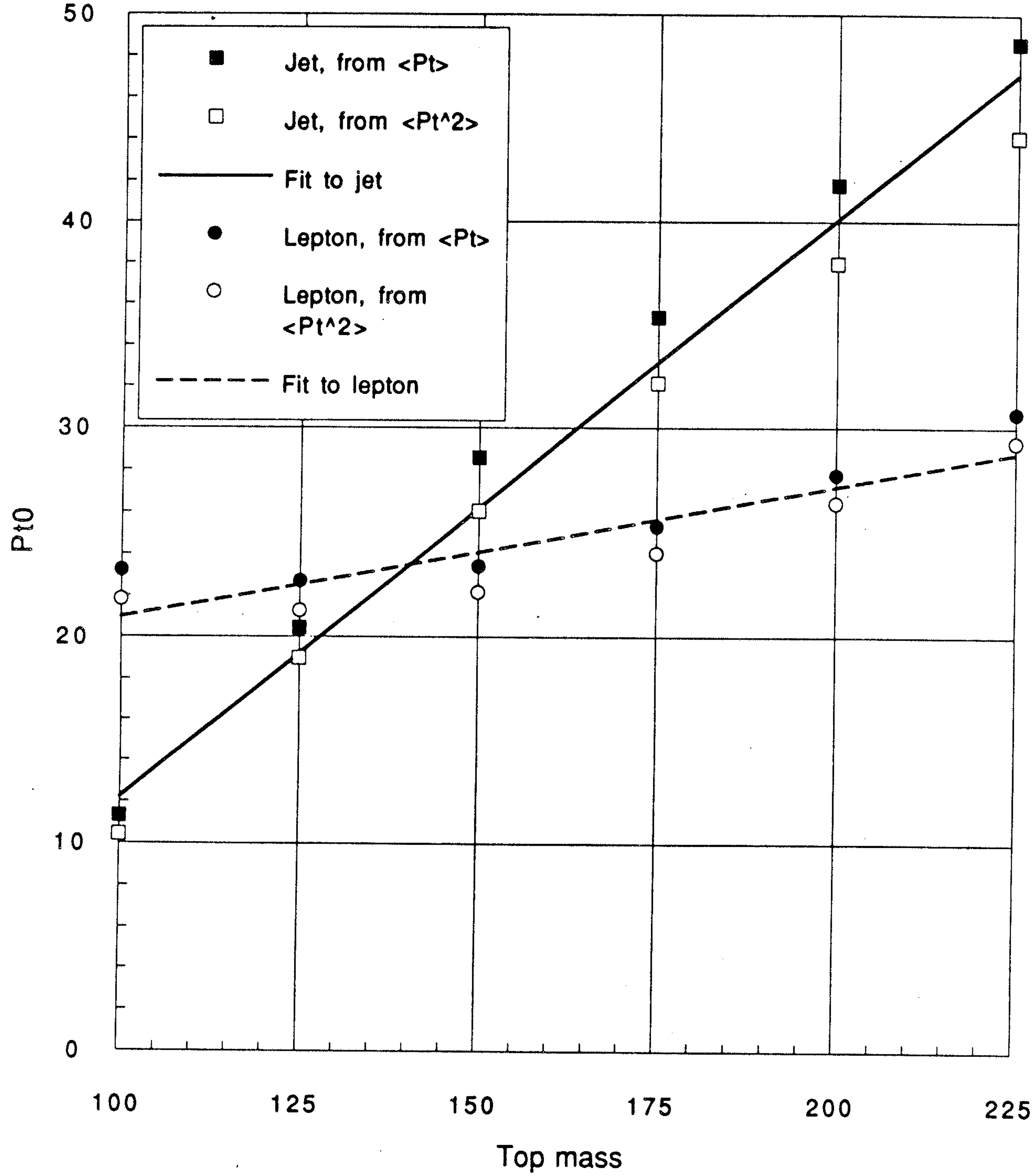
~~The above introduction is intended to apply to two DØ top mass analyses presently yielding results. The fine points below are not necessarily in common between the two. They have a small effect on the fit masses.~~

1. We varied the choice of function $f(m_t)$ which *a priori* is taken to have uniform probability. We considered *(a)* $f(m_t) = m_t$ and *(b)* $f(m_t) = \ln m_t$. [These correspond to *(a)* $P(100 < m_t < 101) \equiv P(200 < m_t < 201)$ and *(b)* $P(100 < m_t < 101) \equiv P(200 < m_t < 202)$]. For Monte Carlo events, function *(b)* yields the more symmetric probability distribution.

2. ~~Following U. Heintz we modified σ_T^{-1} to σ_{vis}^{-1} in the weight. The DØ acceptance is $\approx$ logarithmic in m_t.~~

3. When observed energies are smeared, we adopted a more refined method in which the standard deviation used corresponds to the smeared (parent) rather than the observed (daughter) energy, and the probability for smearing reflects the distribution of parent energies.

Pt0 in distribution Pt exp(-Pt/Pt0) for ttbar jets and leptons



M. Strovink 6/1/93.

Monte Carlo verification of mass fitting method

We use an $\alpha = \alpha_s = 0$ Monte Carlo that is aware of the same physics known to the mass analysis. $t\bar{t}$ events are generated at fixed top mass. The plots are of three types:

1. $d(\text{Probability})/d \ln m_t$, average for many events. Expect a symmetric distribution peaking at the generated m_t. Gives a good overview of the mass analyzer's performance.

2. No. of events *vs.* median $\ln m_t$ from the probability distribution for each event. Expect a symmetric distribution peaking at the generated m_t. Checks reliability of best fit m_t.

3. No. of events *vs.* "confidence" (fraction of the probability distribution lying above the generated m_t). Expect a uniform distribution. Checks reliability of *error* on best fit m_t.

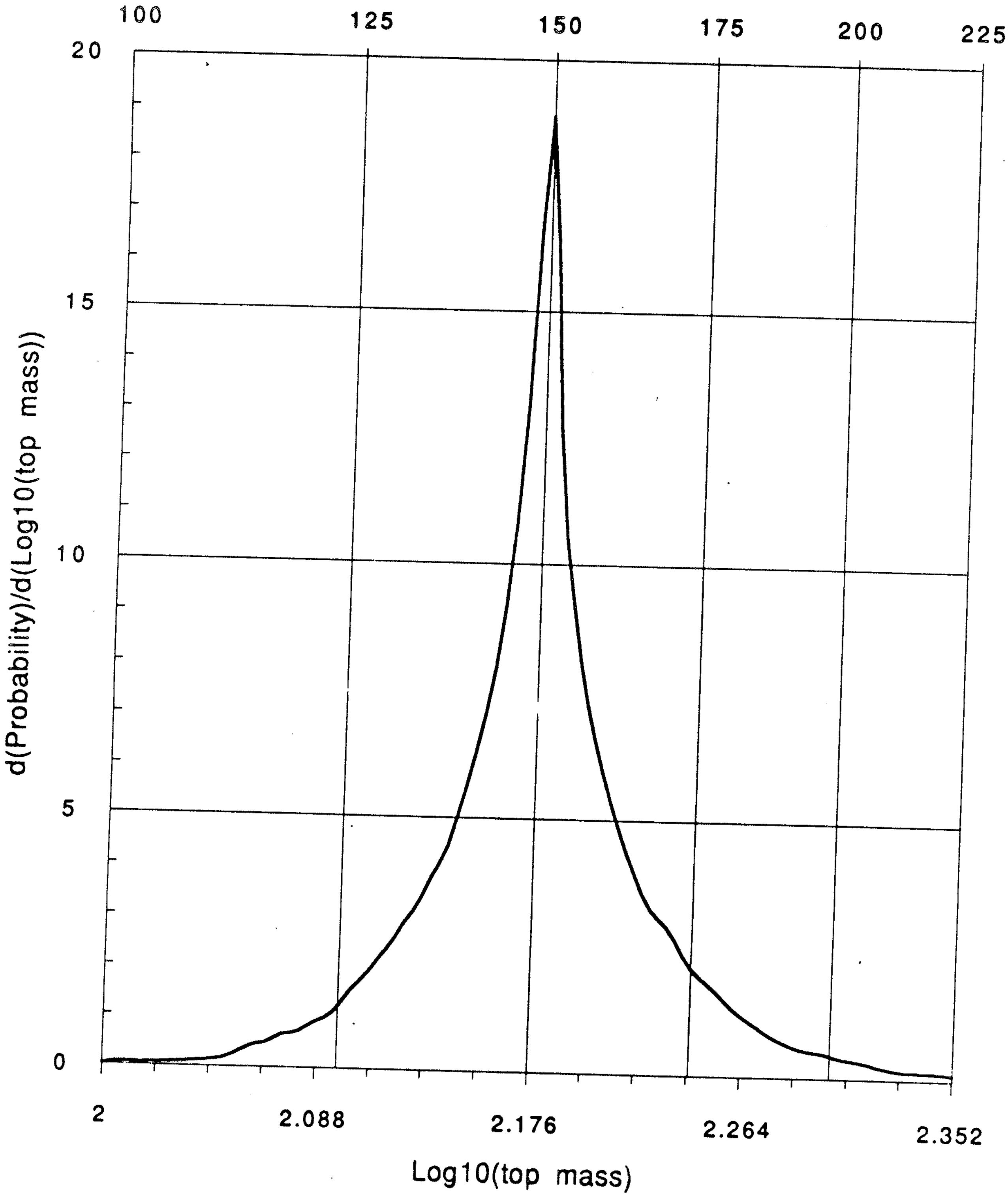
Results are available for the following types of Monte Carlo sample:

- Unsmeared, lepton-jet pairing forced to be correct.
- Unsmeared, all lepton-jet pairings allowed.
- $e\mu$ smeared, all pairings.
- ee smeared, all pairings.

The unsmeared studies, although not realistic, provide an incisive test of the extent to which the mass analysis method is optimized.

Correct pairs. 1/sigT(vis). Ideal measurements.

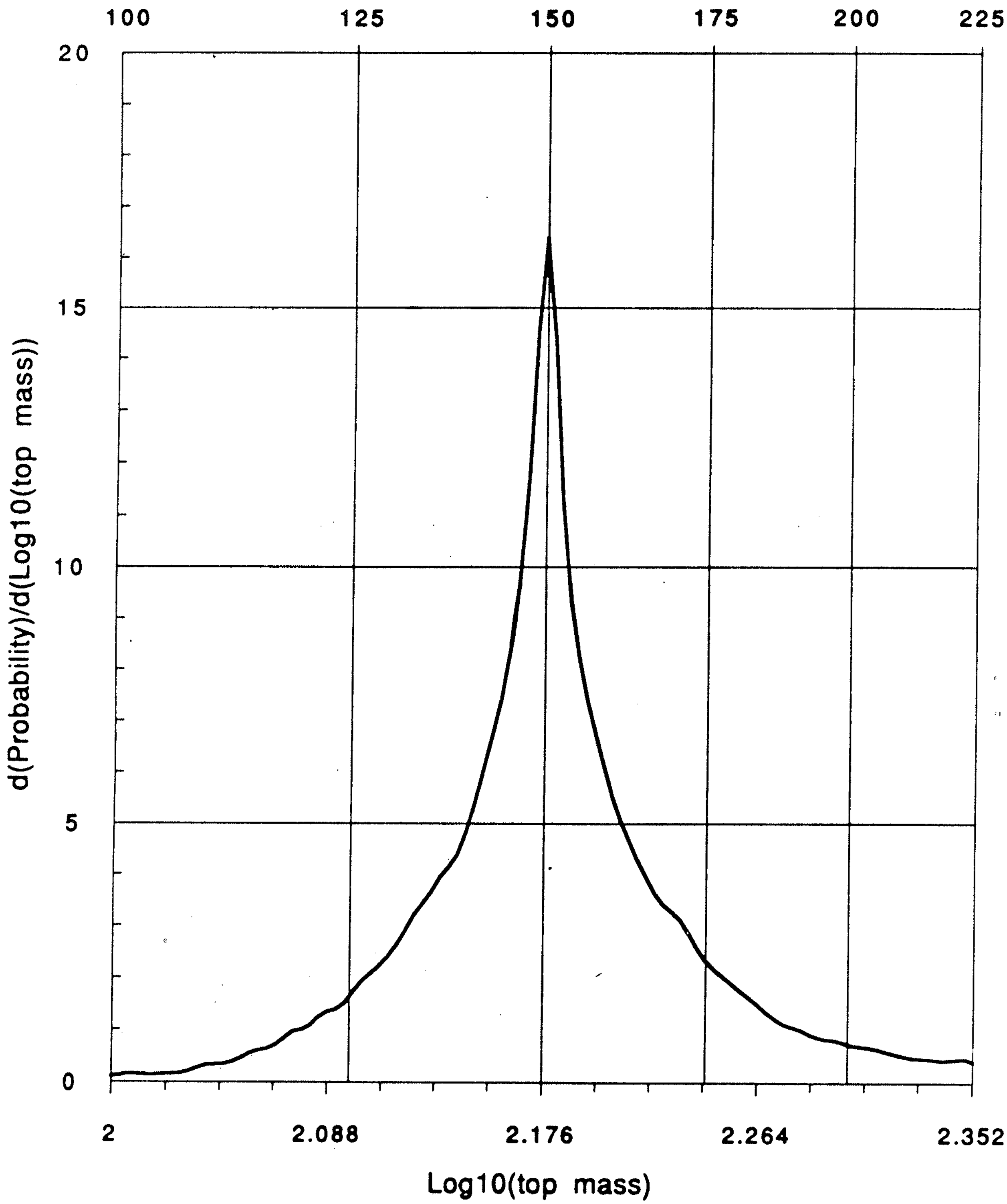
150 GeV Monte Carlo, events averaged with equal weight. Median 150.3.



Variables o_smearing. Smoothed, 3X peak

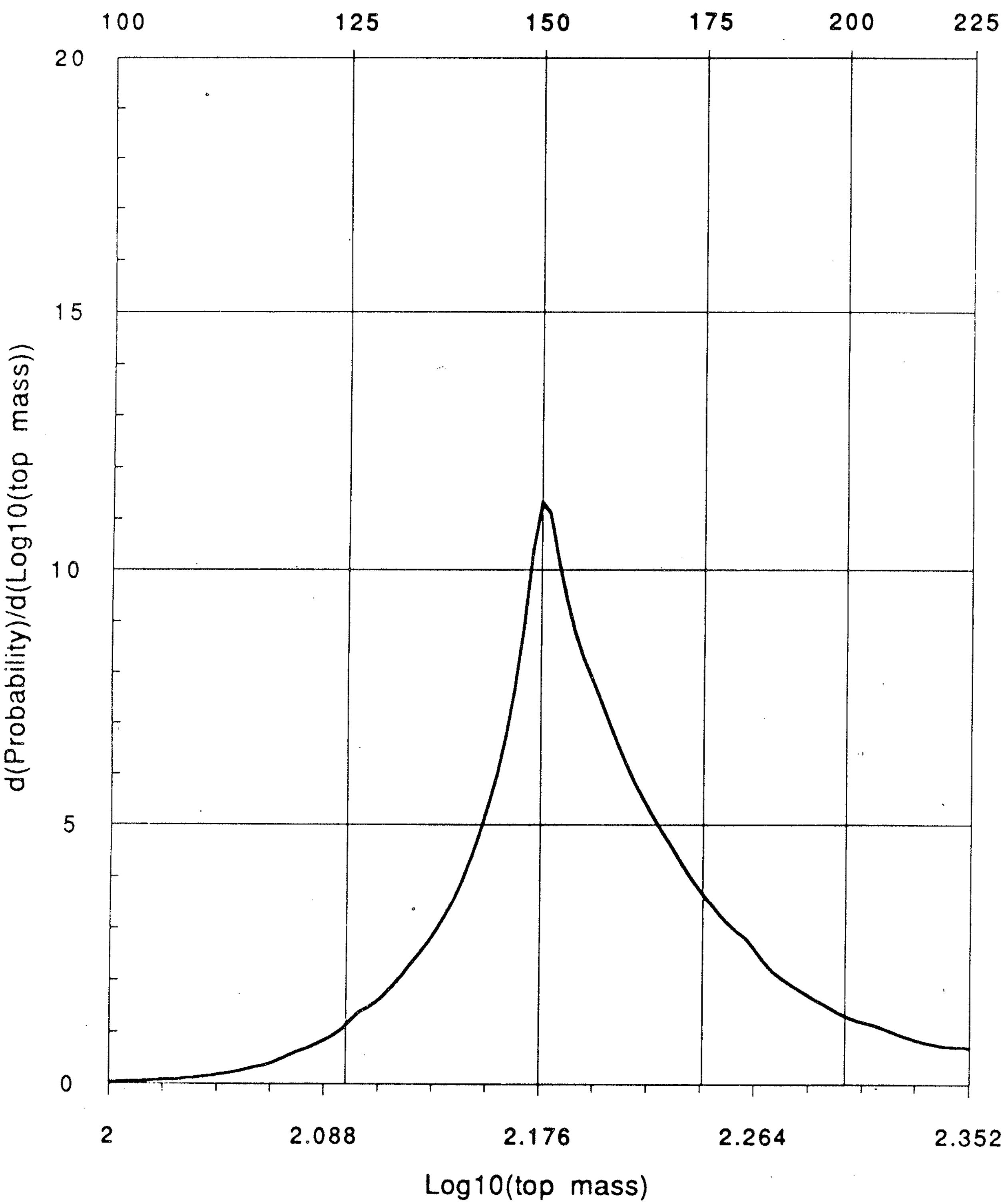
All pairs. 1/sigT(vis). Ideal measurements.

150 GeV Monte Carlo, events averaged with equal weight. Median 150.3.



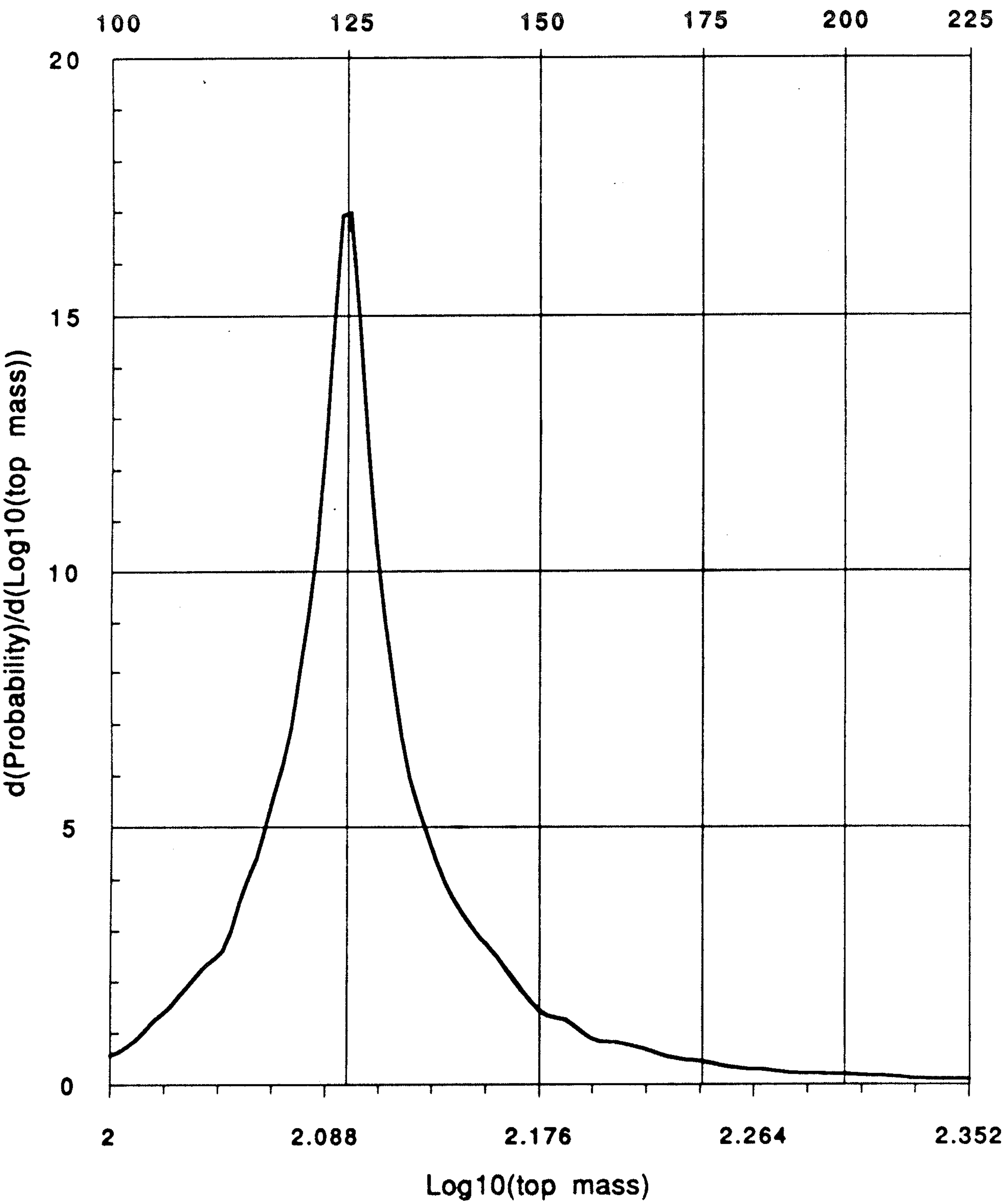
Variables o smearing Smoothed 3X peak M. Strovink 6/1/92

150 GeV Monte Carlo, events averaged with equal weight. Median 154.1.



All pairs. 1/sigT(vis). Ideal measurements.

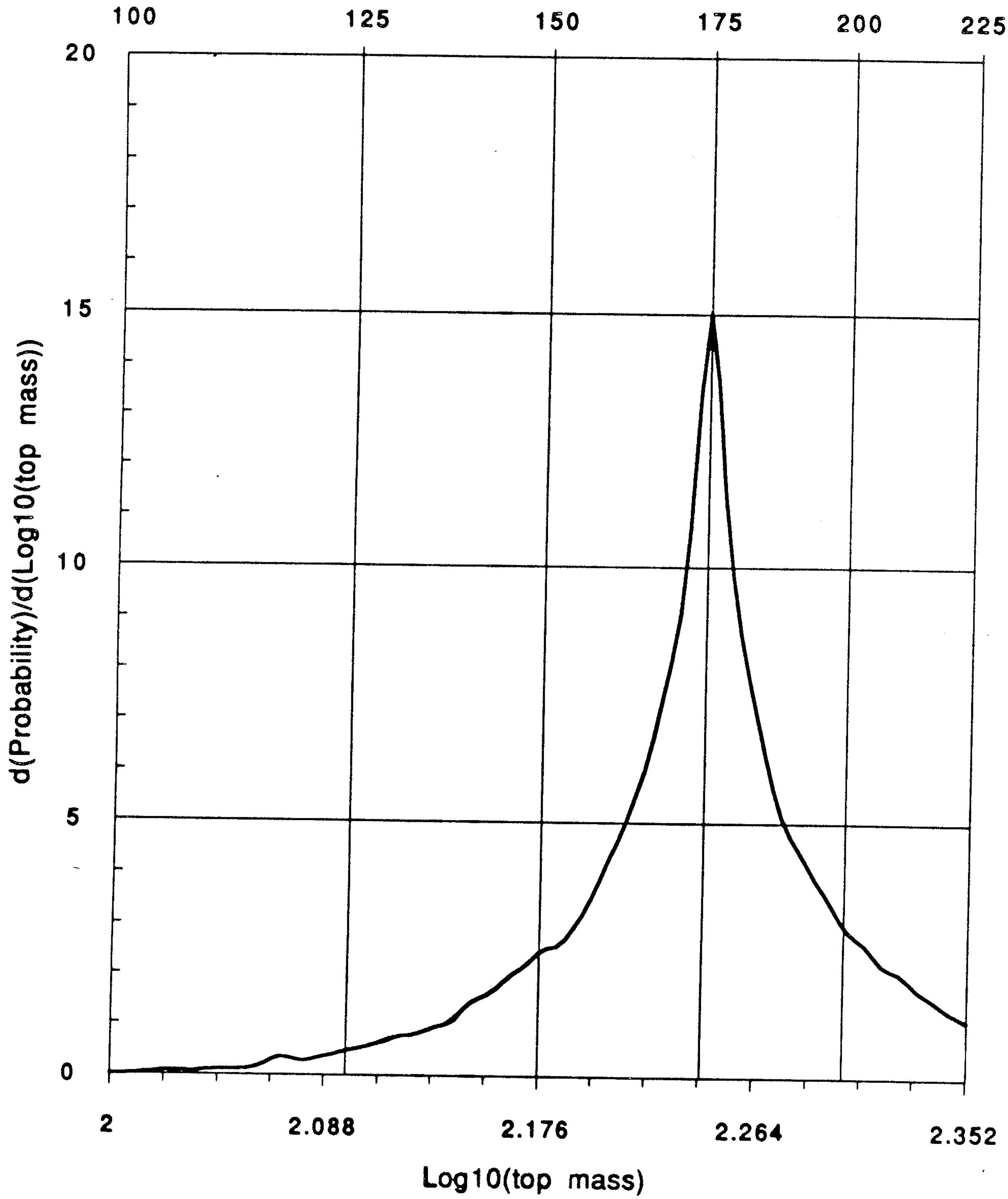
125 GeV Monte Carlo, events averaged with equal weight. Median 125.6.



Variables o_smearing. Smoothed, 3X peak. M. Strovink 6/1/93.

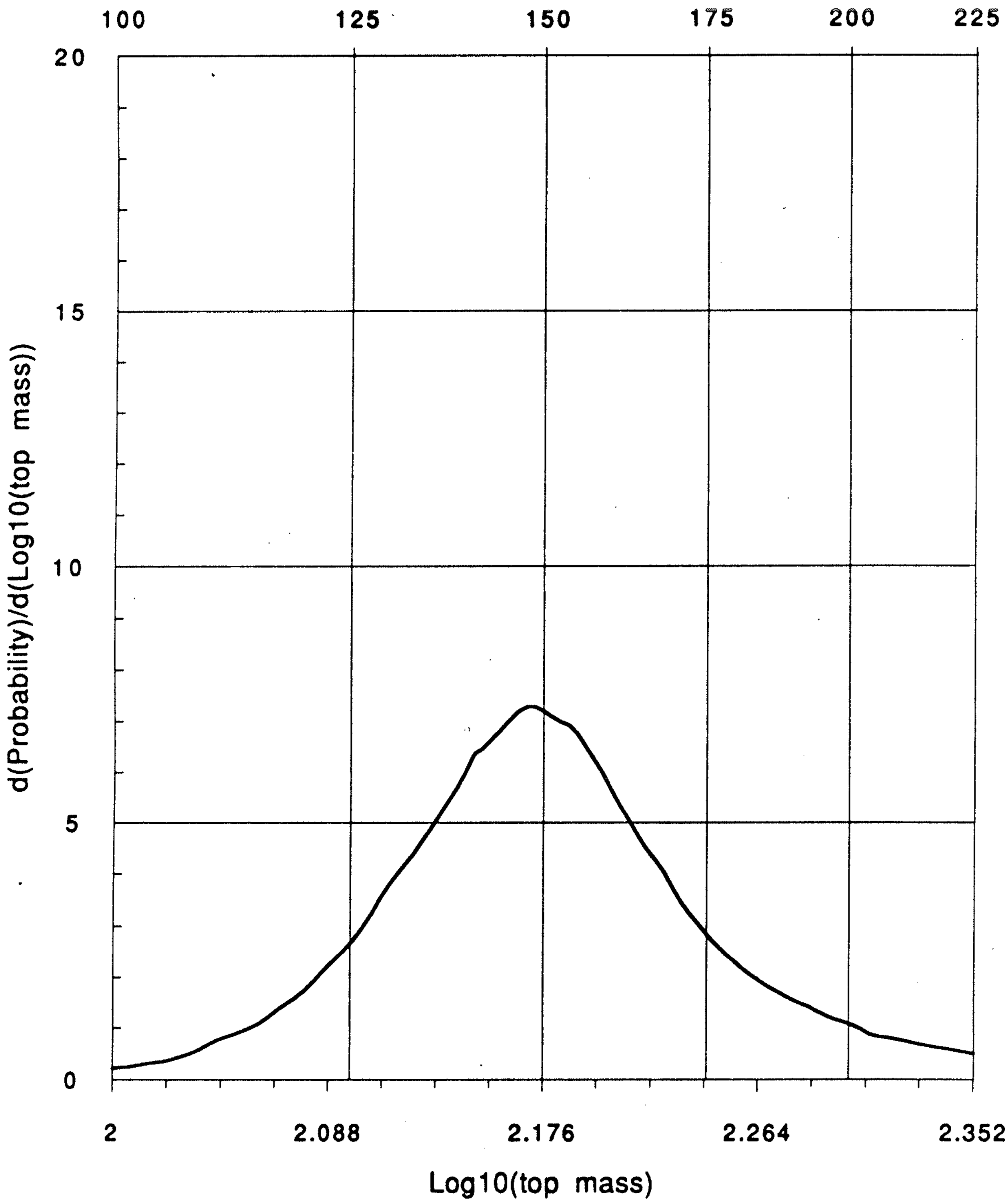
All pairs. 1/sigT(vis). Ideal measurements.

175 GeV Monte Carlo, events averaged with equal weight. Median 174.9.



Variables o smearing. Smoothed 2X peak

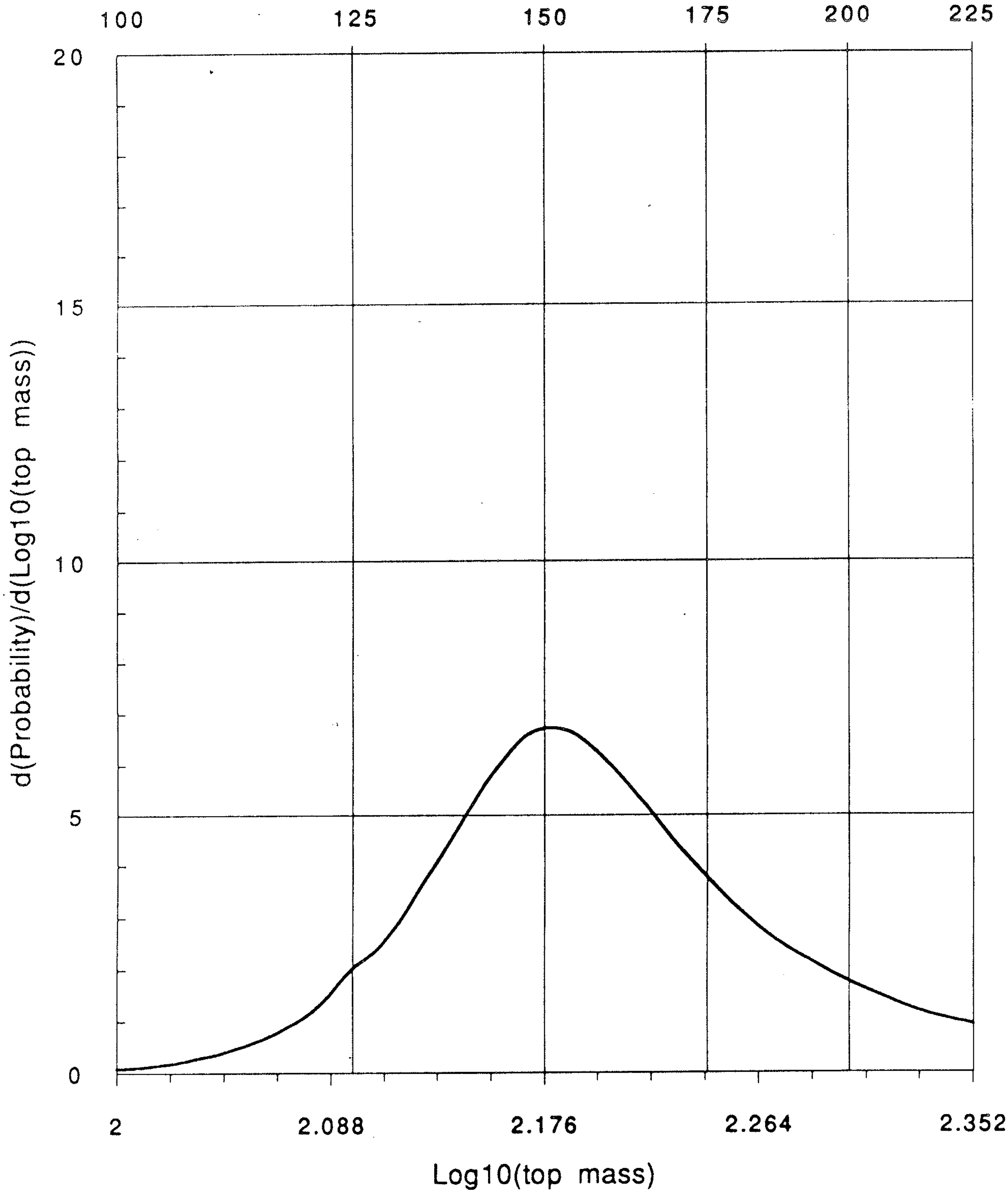
150 GeV Monte Carlo, events averaged with equal weight. Median 149.3.



All pairs.

Bayesian smearing (ee+emu).

150 GeV Monte Carlo, events averaged with equal weight. Median 154.7.

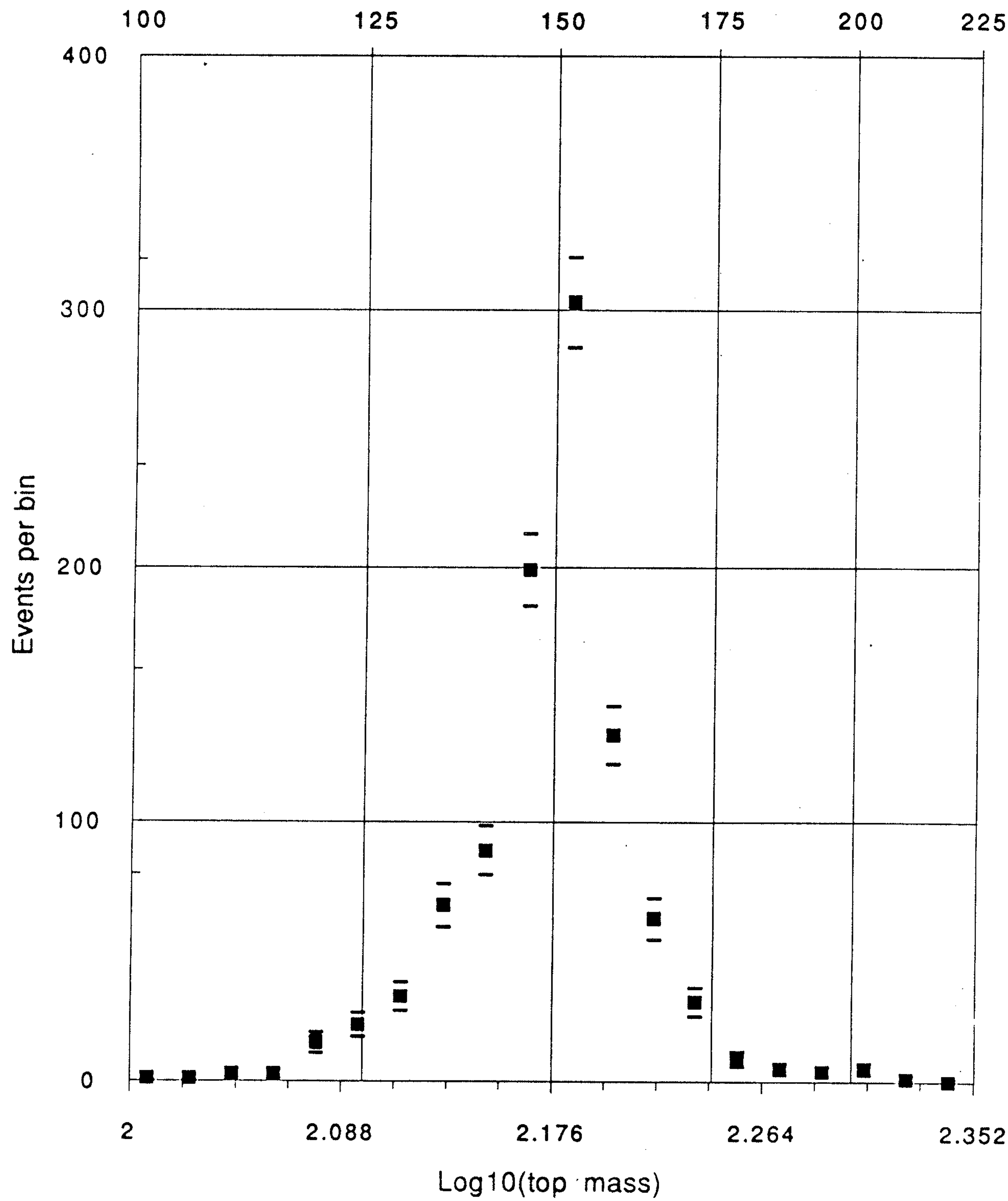


Dalitz-Gildstein weight.

M. Strovink 6/11/93.

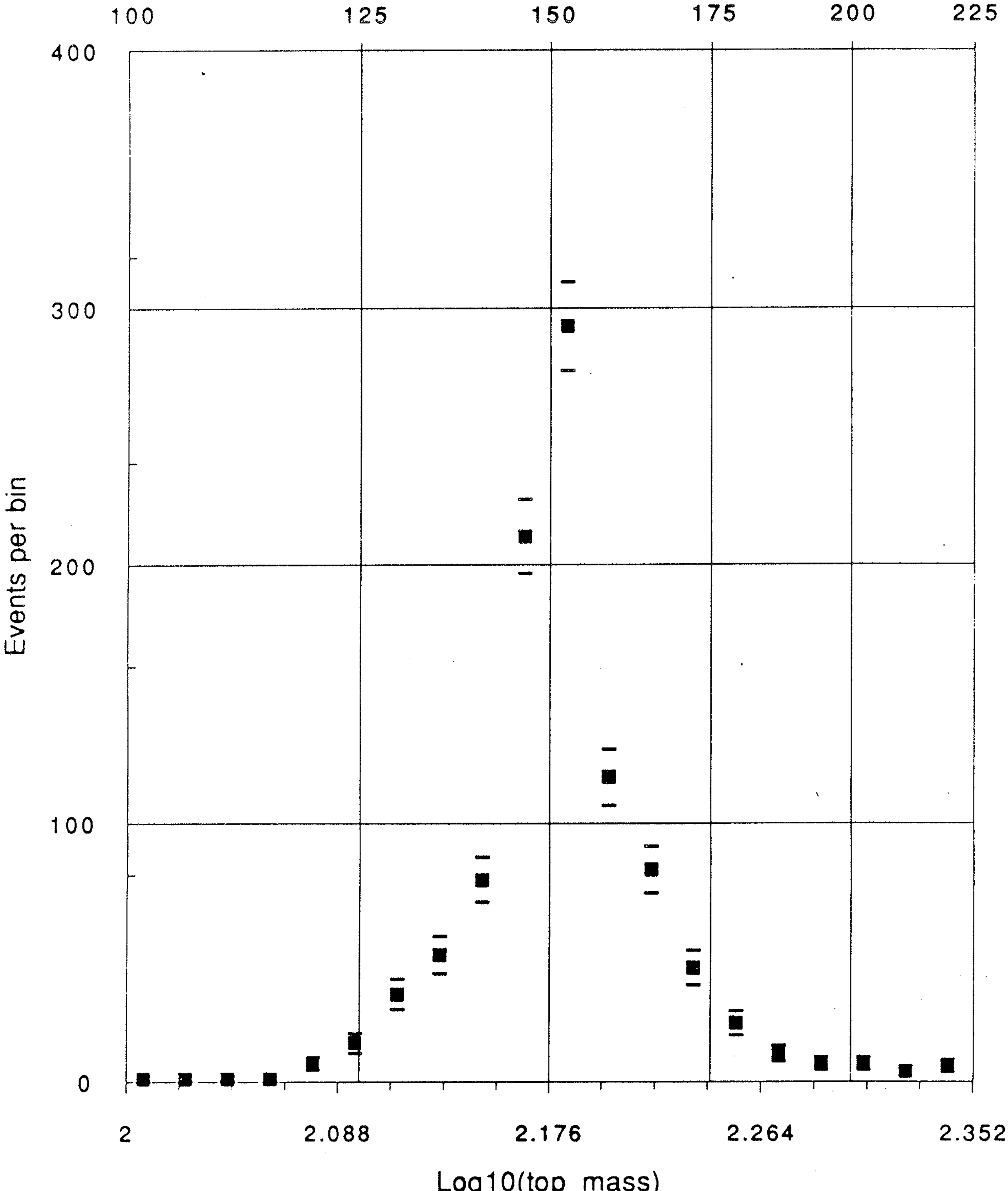
All pairs. 1/sigT(vis). Ideal measurements.

150 GeV Monte Carlo, distribution of medians. Median 150.4.

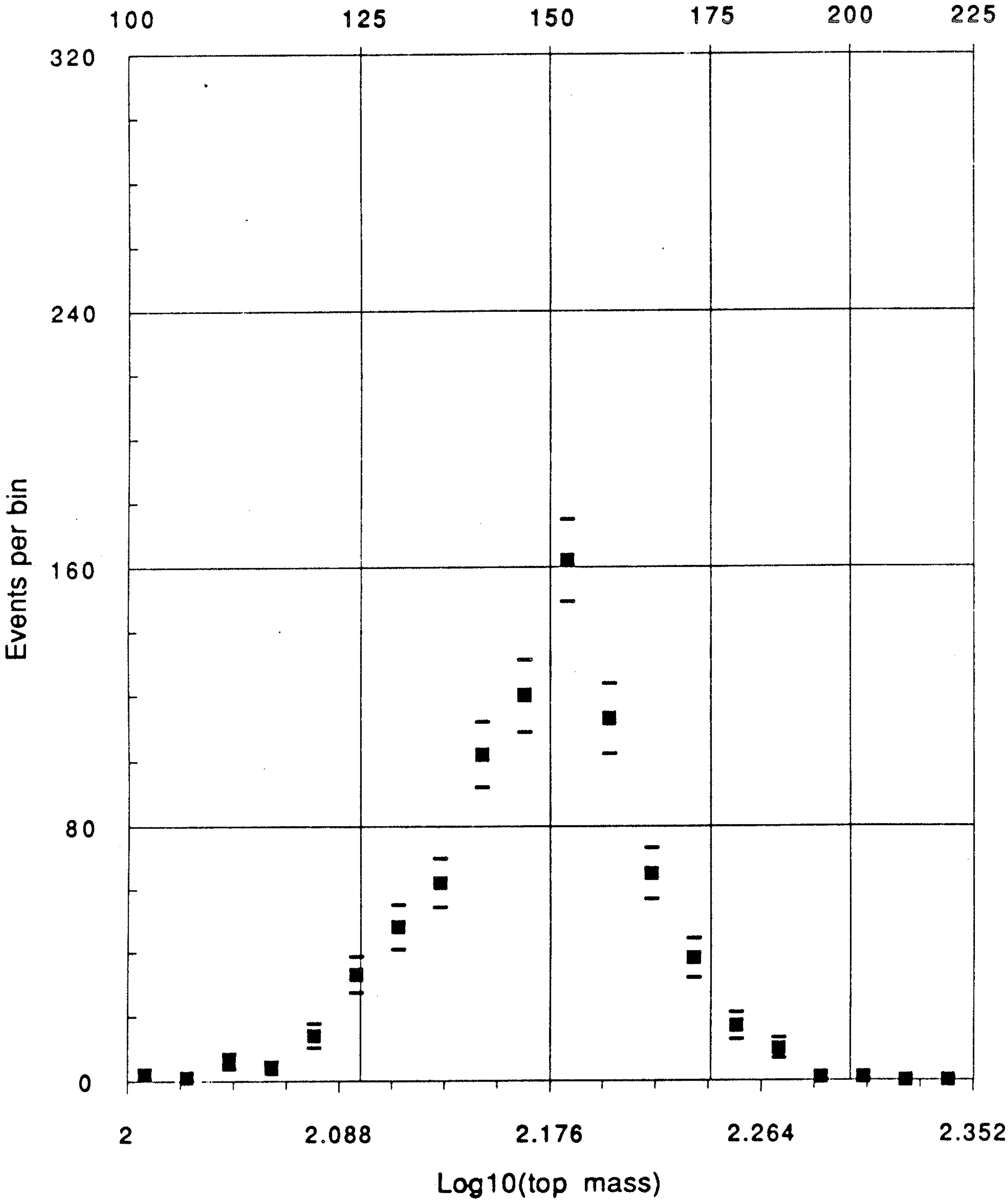


Variables o smearing. Smoothed. 3X peak. M. Strovink 6/1/93

150 GeV Monte Carlo, distribution of modes. Median 151.0.



150 GeV Monte Carlo, distribution of medians. Median 149.7.

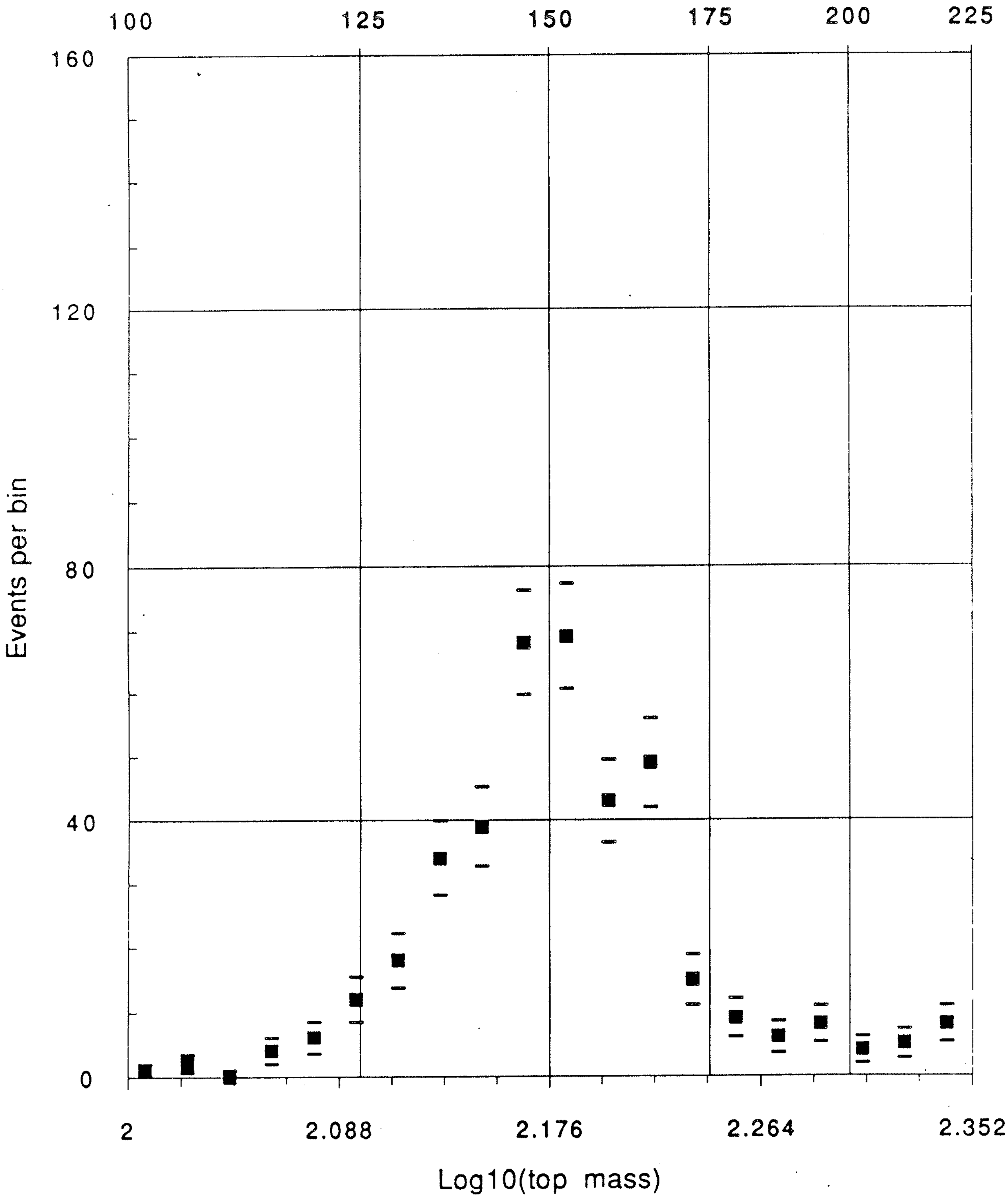


00073

All pairs.

Bayesian smearing (ee+emu).

150 GeV Monte Carlo, distribution of modes. Median 151.0.

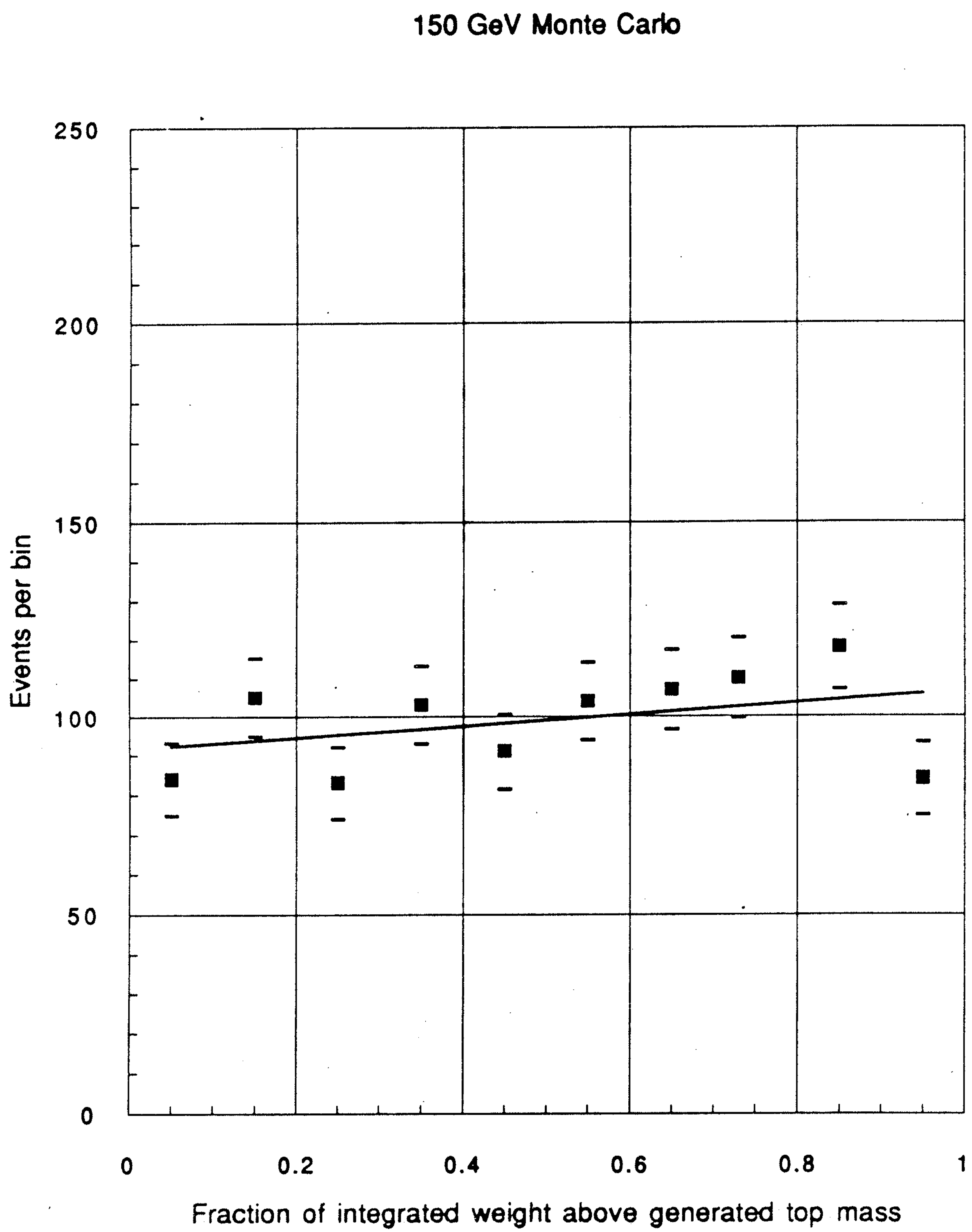


00074

Dalitz-Goldstein weight.

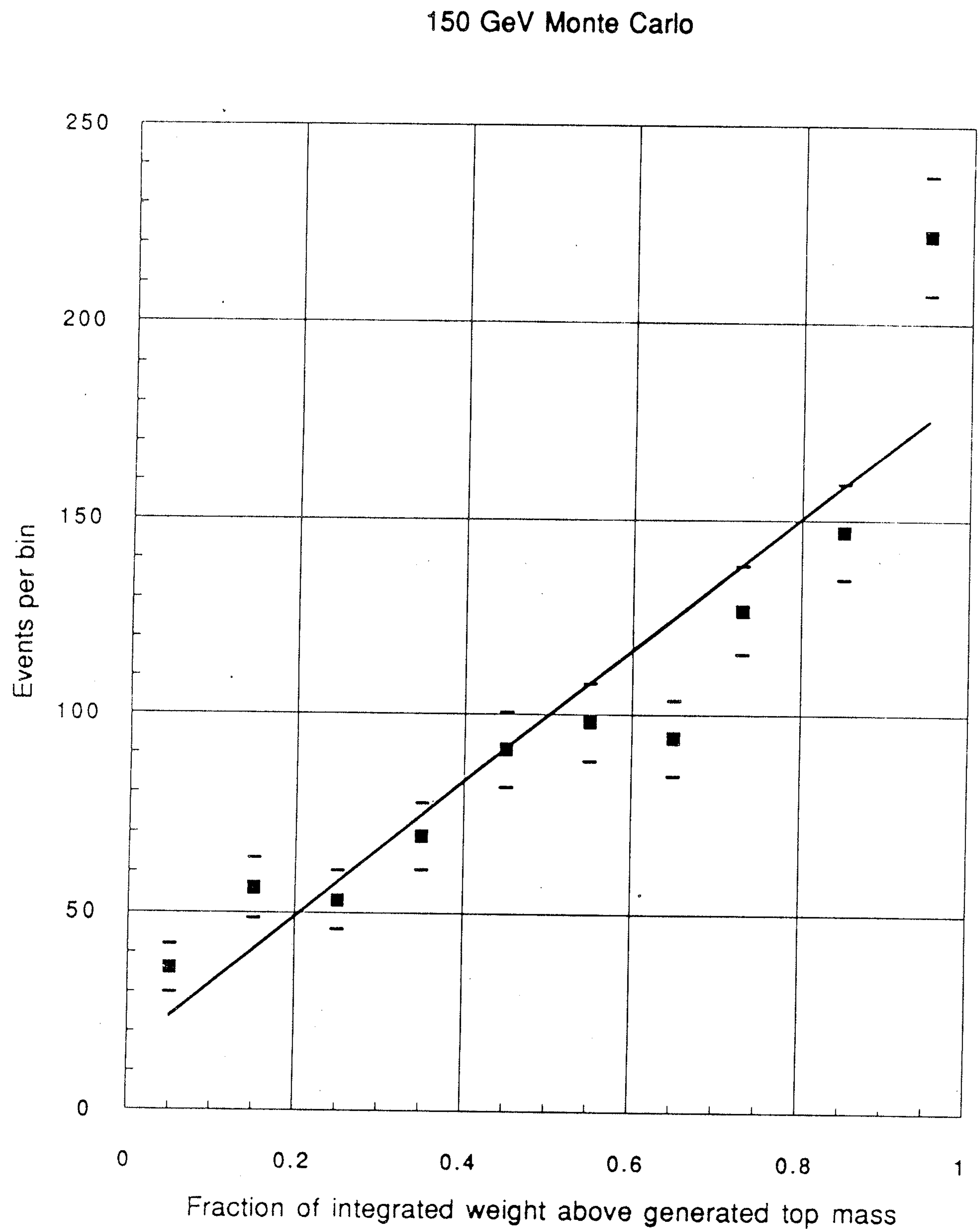
M. Strovink 6/11/93.

All pairs. Log(Mt), 1/sigT(vis). Ideal measurements.



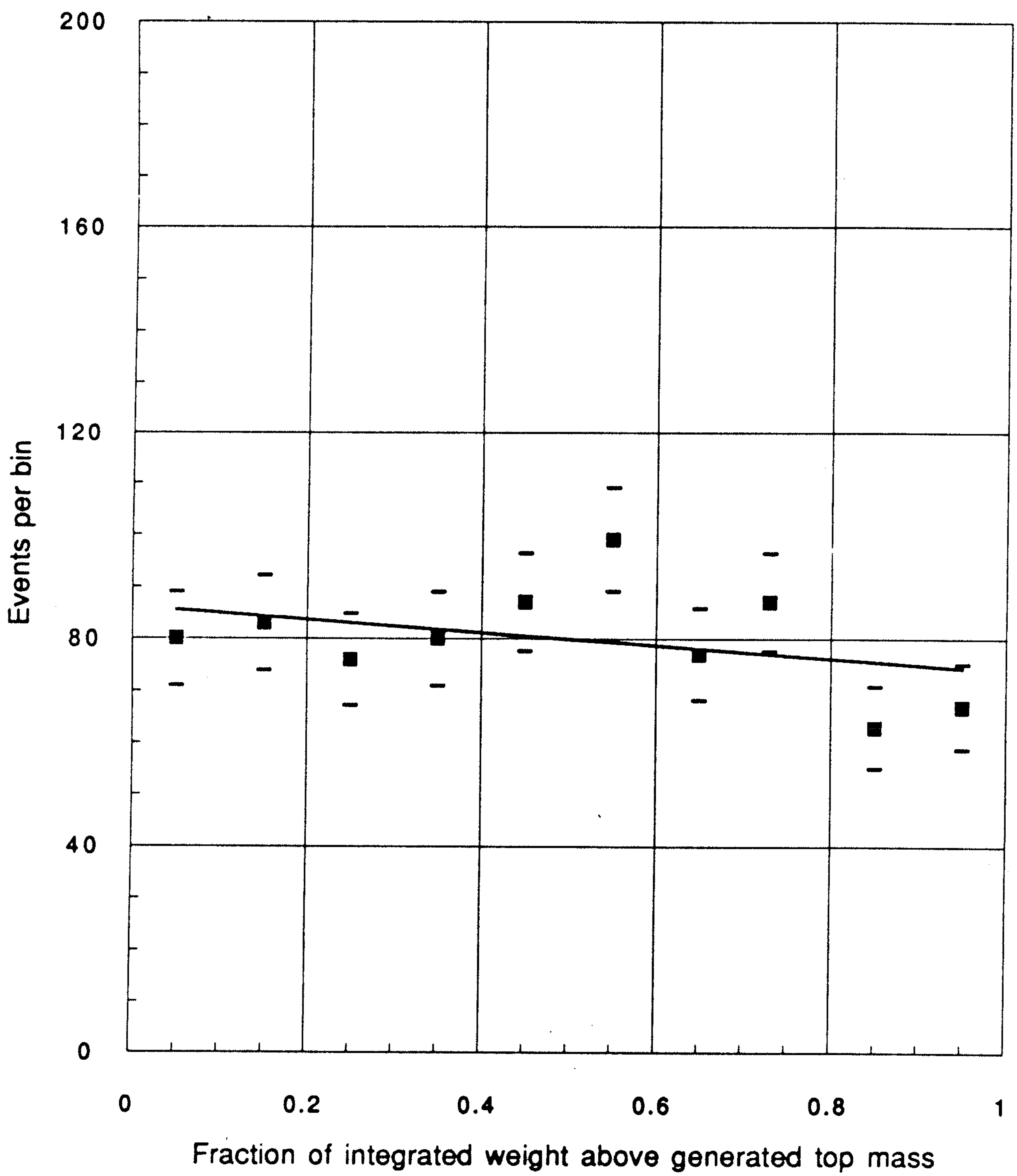
Variables o smearing Smoothed 3X peak M. Strovink 6/1/93

All pairs. Log(Mt), 1/sigT(vis). Ideal measurements.



All pairs. Log(Mt), 1/sigT(vis). Bayesian smearing (ee+emu).

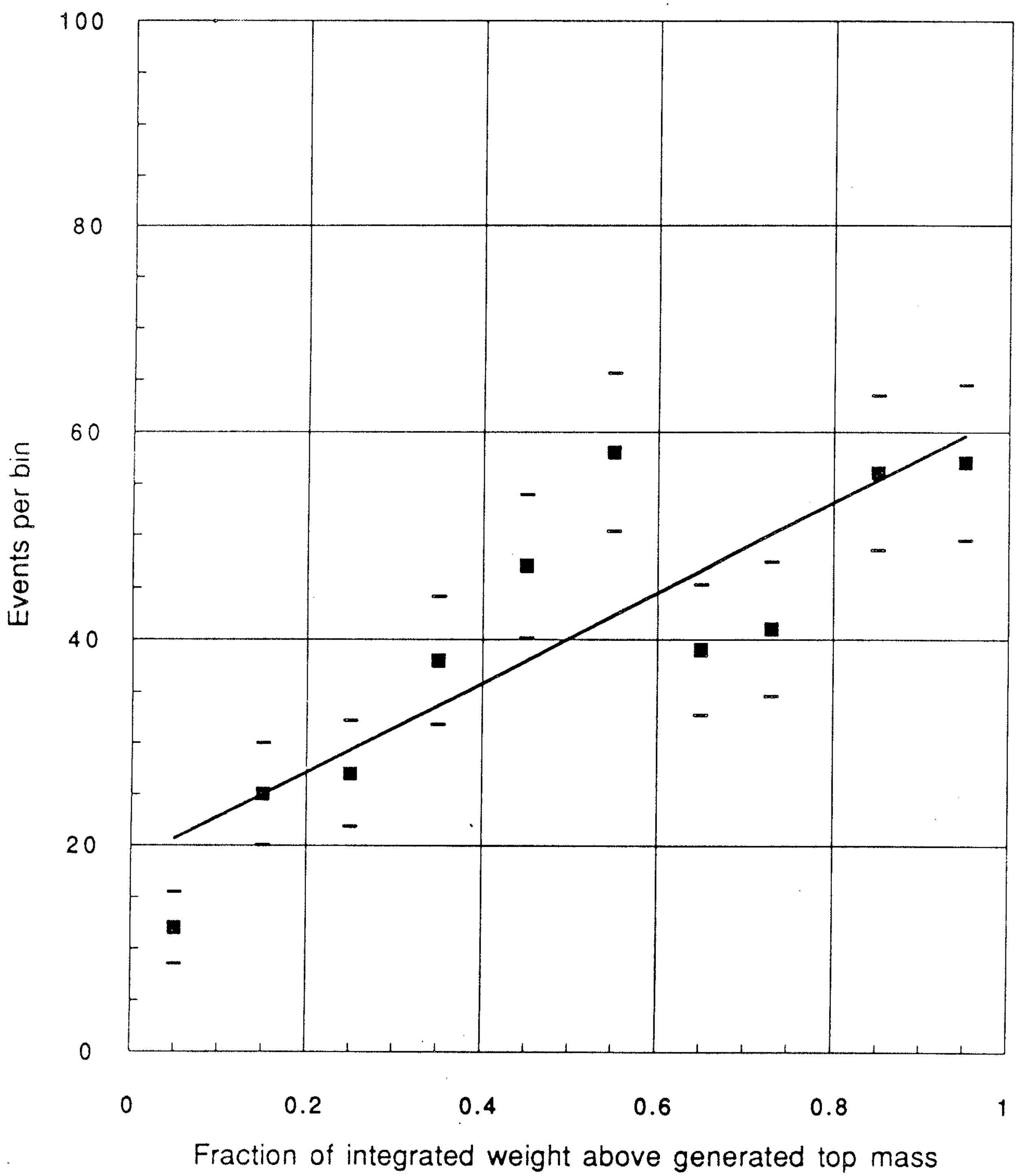
150 GeV Monte Carlo



Variables o invariant. Smoothed, 3X peak. M. Strovink 6/9/93

All pars. Log(Mt). Bayesian smearing (ee+emu).

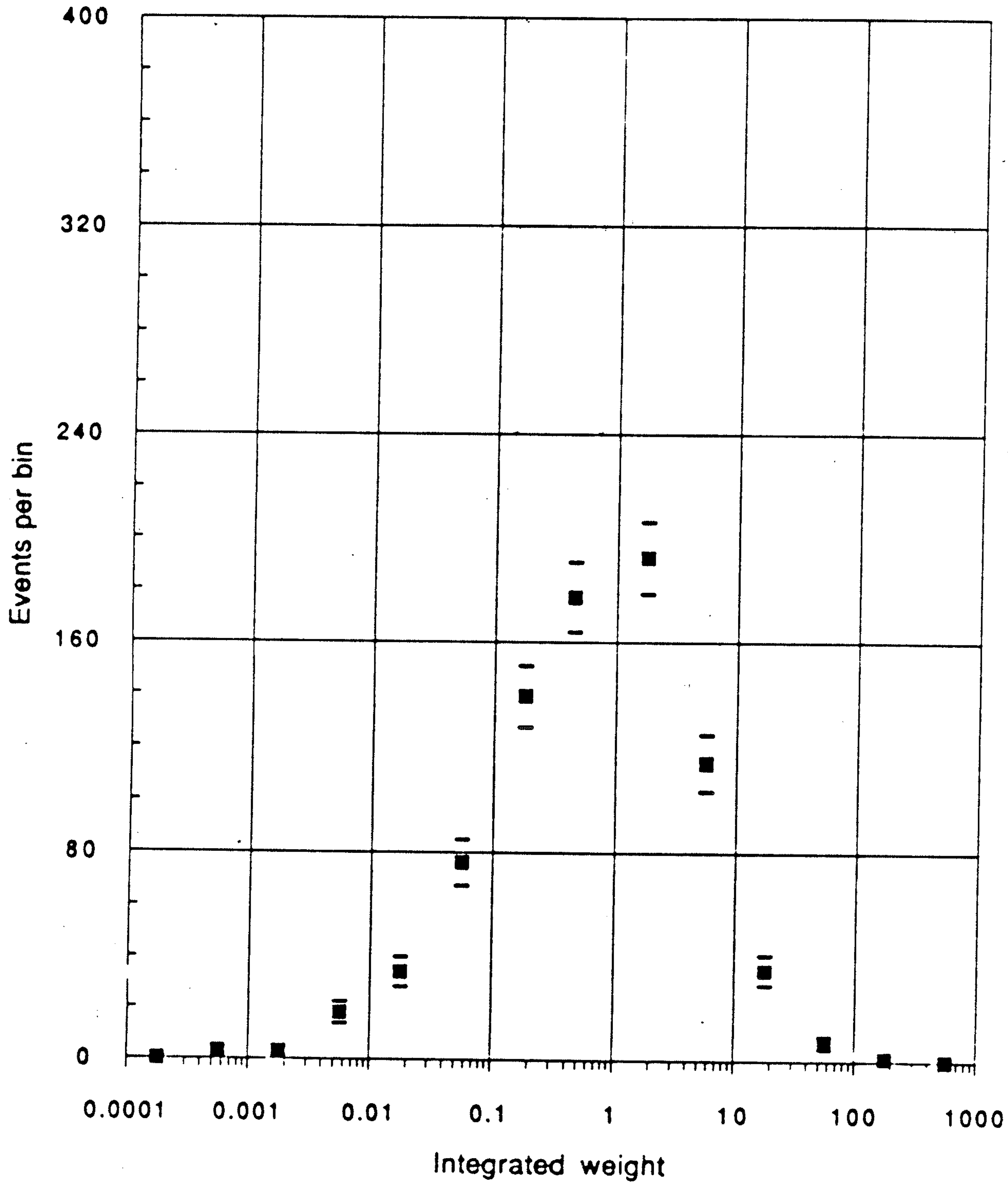
150 GeV Monte Carlo



Dalitz-Goldstein weight. M. Strovink 6/11/93.

All pairs. Log(Mt),1/sigT(vis). Bayesian smearing(ee+emu).

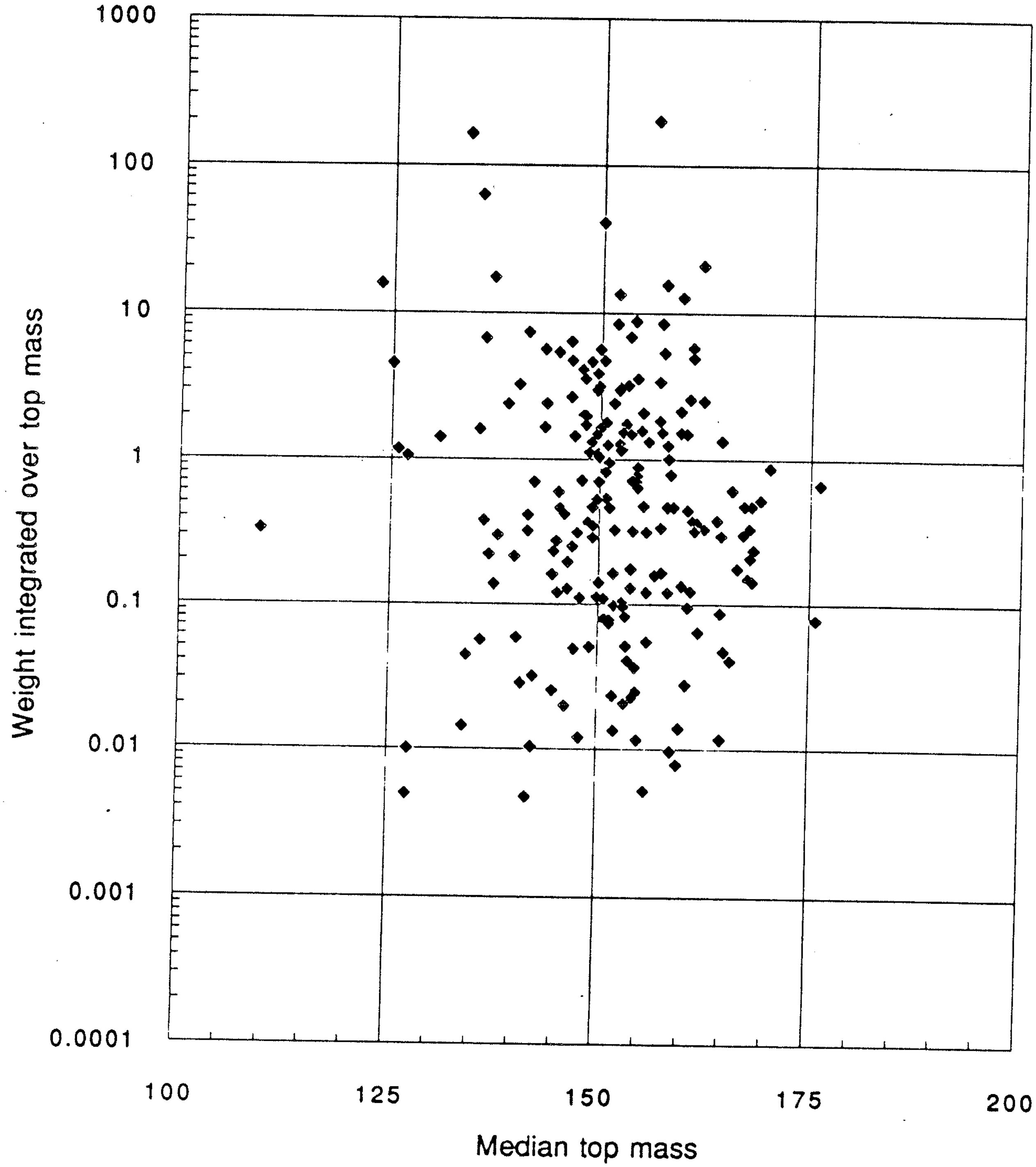
150 GeV Monte Carlo. Median = 0.82.



Correct lepton-jet pairings.

Ideal measurements.

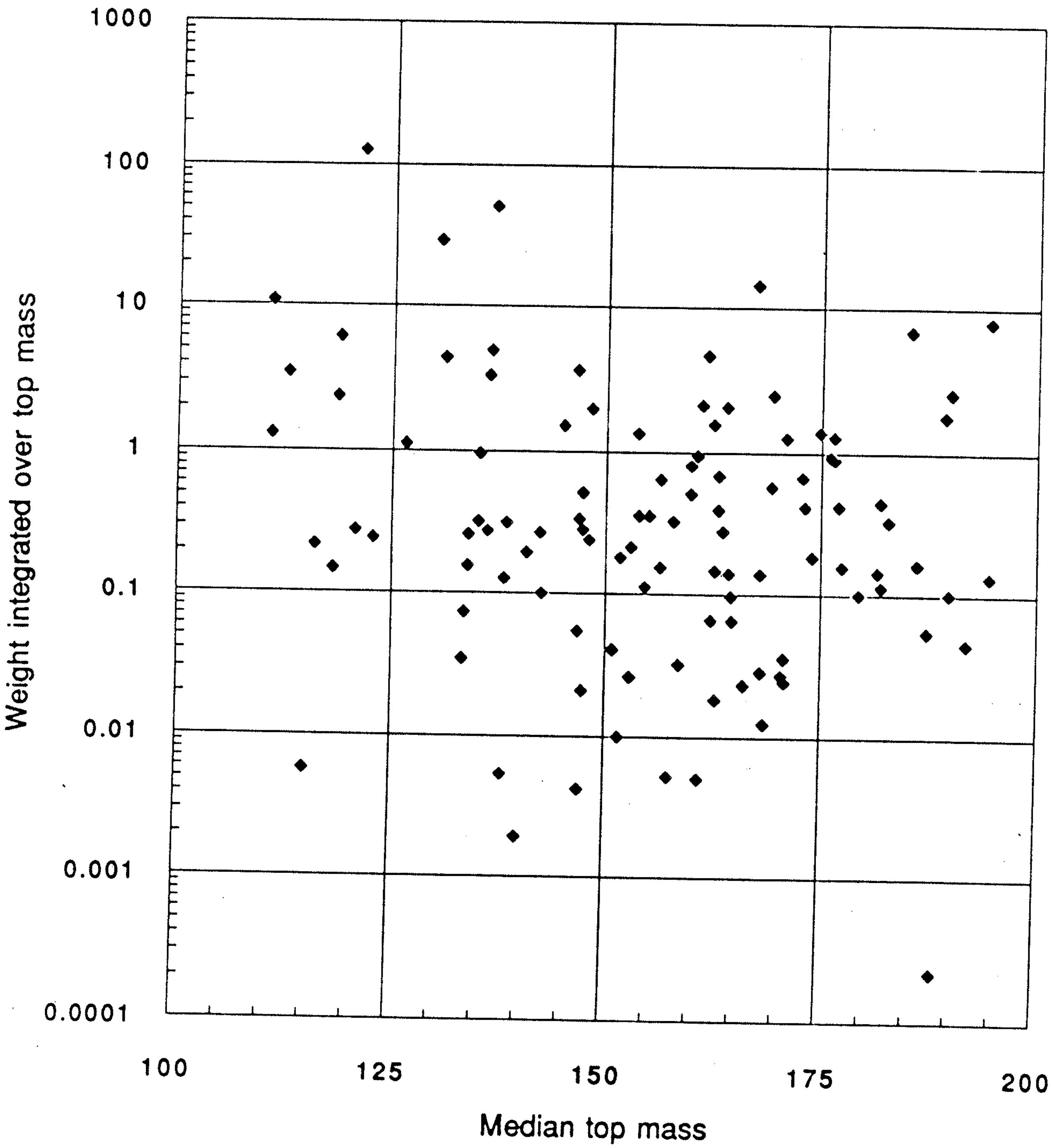
150 GeV MC within D0 acceptance. Median integrated weight = 0.469.
16%-50%-84% of median top masses below 142.8-151.8-160.6 GeV.



WRONG lepton-jet pairings.

Ideal measurements.

150 GeV MC within D0 acceptance. 46% have no solution. 54% have median integrated weight = 0.282, with 16%-50%-84% of median top masses below 134-159-177 GeV.



M. Strovink 5/13/93

Conclusions from Monte Carlo studies

- Well-measured $t\bar{t} \rightarrow$ dilepton events possess surprisingly precise top mass information despite two missing neutrinos.

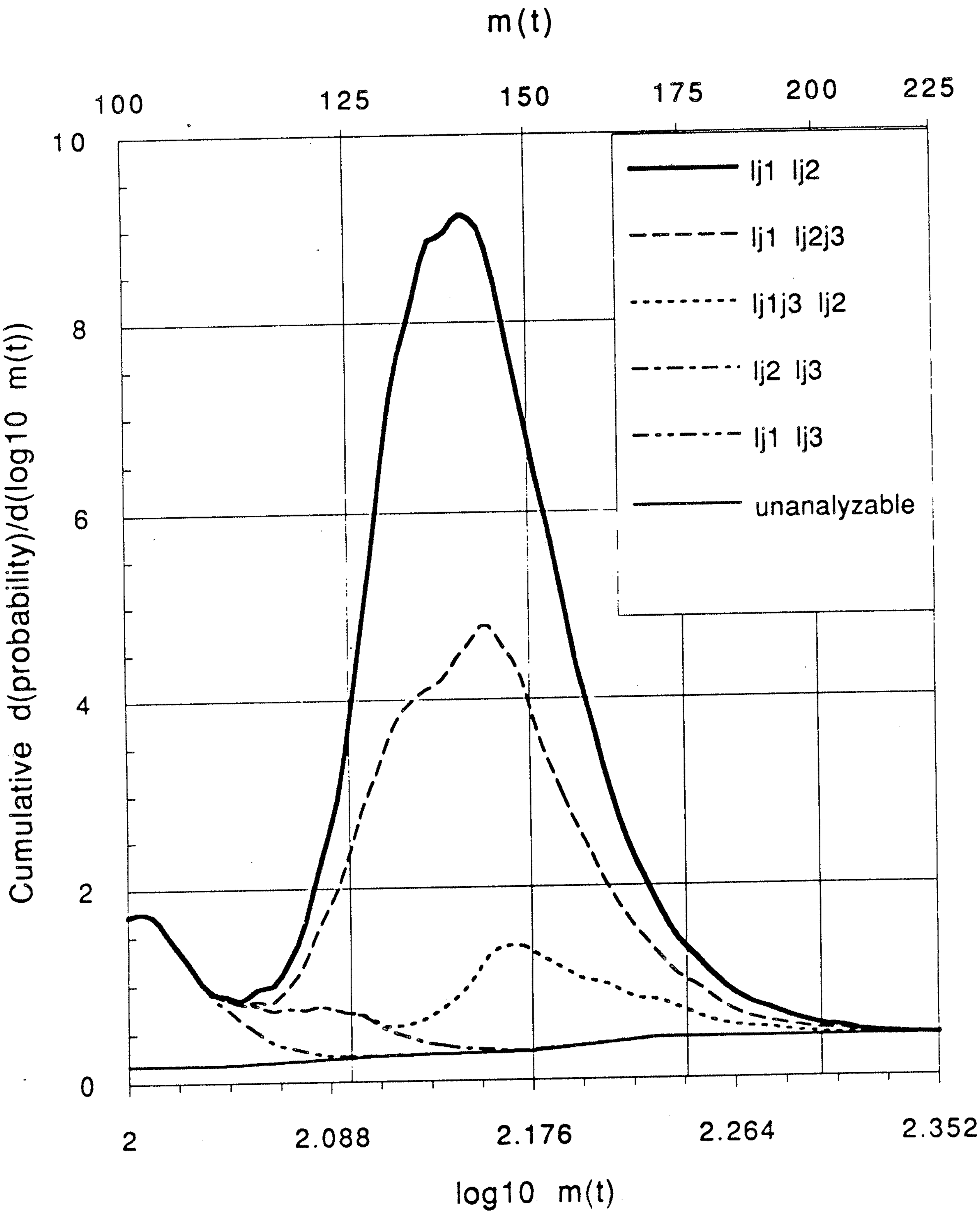
- The DØ top mass analysis extracts this information remarkably well. The best fit m_t tracks the generated m_t to within 1 GeV. The breadth of the mass probability distribution is also reliable, as checked by the uniform "confidence" distribution.

- Resolution smearing causes broadening of the top mass determination that is only about 20% more severe for $e\mu$ than for ee events.

ISAJET top mass	110		130		150		170	
Generated	99889		99787		99709		99622	
Satisfy D0 cuts	60002		68464		71828		74214	
Event 417 "chi-square" cut *	< 0.32		< 0.45		< 0.70		< 0.95	
Satisfy Event 417 cut	1019		978		1050		950	
(A): lj1 lj2	618	60.6%	462	47.2%	394	37.5%	292	30.7%
(B): lj1 lj3	50	4.9%	23	2.4%	16	1.5%	21	2.2%
(C): lj2 lj3	67	6.6%	21	2.1%	12	1.1%	15	1.6%
(D): lj l(min mass jj)	212	20.8%	337	34.5%	410	39.0%	369	38.8%
(E): lj l(med mass jj)	10	1.0%	43	4.4%	98	9.3%	93	9.8%
(F): lj l(max mass jj)	0	0.0%	2	0.2%	12	1.1%	19	2.0%
(G): Unanalyzable	62	6.1%	90	9.2%	108	10.3%	141	14.8%
% analyzable which are (A) or (D)		86.7%		90.0%		85.4%		81.7%

$$* \text{ "}\chi^2\text{"} \equiv \sum_{i=1}^{6} \ln^2\left(\frac{v_i^{ISA}}{v_i^{417}}\right) \quad \text{where } \{v_i\} \equiv \{E_T^{j1}, E_T^{j2}, E_T^{j3}, m_{12}, m_{13}, m_{23}\}$$

00080


m(t)
100
125
150
175
200
225
10
8
6
4
2
0
Cumulative d(probability)/d(log10 m(t))
lj1 lj2
lj1 lj2j3
lj1j3 lj2
lj2 lj3
lj1 lj3
unanalyzable
2
2.088
2.176
2.264
2.352
log10 m(t)

U. Heintz
Oct 12, 1993

Top mass analysis for di-lepton events

· Method III

.... alias method A

.... also known as Uli's method

....

Outline of Mass Analysis

$$
\begin{array}{ccccc}
p\bar{p} & \longrightarrow & t\,\bar{t} + X & & \\
& \swarrow & & \searrow & \\
W^+ b & & & & W^- \bar{b} \\
\downarrow & & & & \downarrow \\
\ell^+ \nu & & & & \ell^- \bar{\nu}_e
\end{array}
$$

$\longrightarrow \quad p(b),\ p(\bar{b}),\ p(\ell^+),\ p(\ell^-),\ \not{E}_T$

14 measured variables

4 mass constraints

$$m(\ell\nu) = m_W$$

$$m(Wb) = m_t$$

18 independent parameters

---

0 constraint fit

$\Rightarrow$ There is a finite number $(0 \ldots 4)$ solutions for $p(t), p(\bar{t})$ that are consistent with the measurements for any given m_t

- weight every solution by a factor, describing how likely it is

$$w(t,\bar{t}) = F_{ud}(x_1)\,F_{ud}(x_2)\;p(E_{e^+}, m_t)\,p(E_{e^-}, m_t)$$

(Dalitz & Goldstein)

- sum weights over the 0…4 solutions for every m_t to get a likelihood to observe this event, given m_t

$$\mathcal{L}(m_t) = \sum_i w_i$$

- $\mathcal{L}$ can be used to assign a probability to the top mass using Bayes' Theorem:

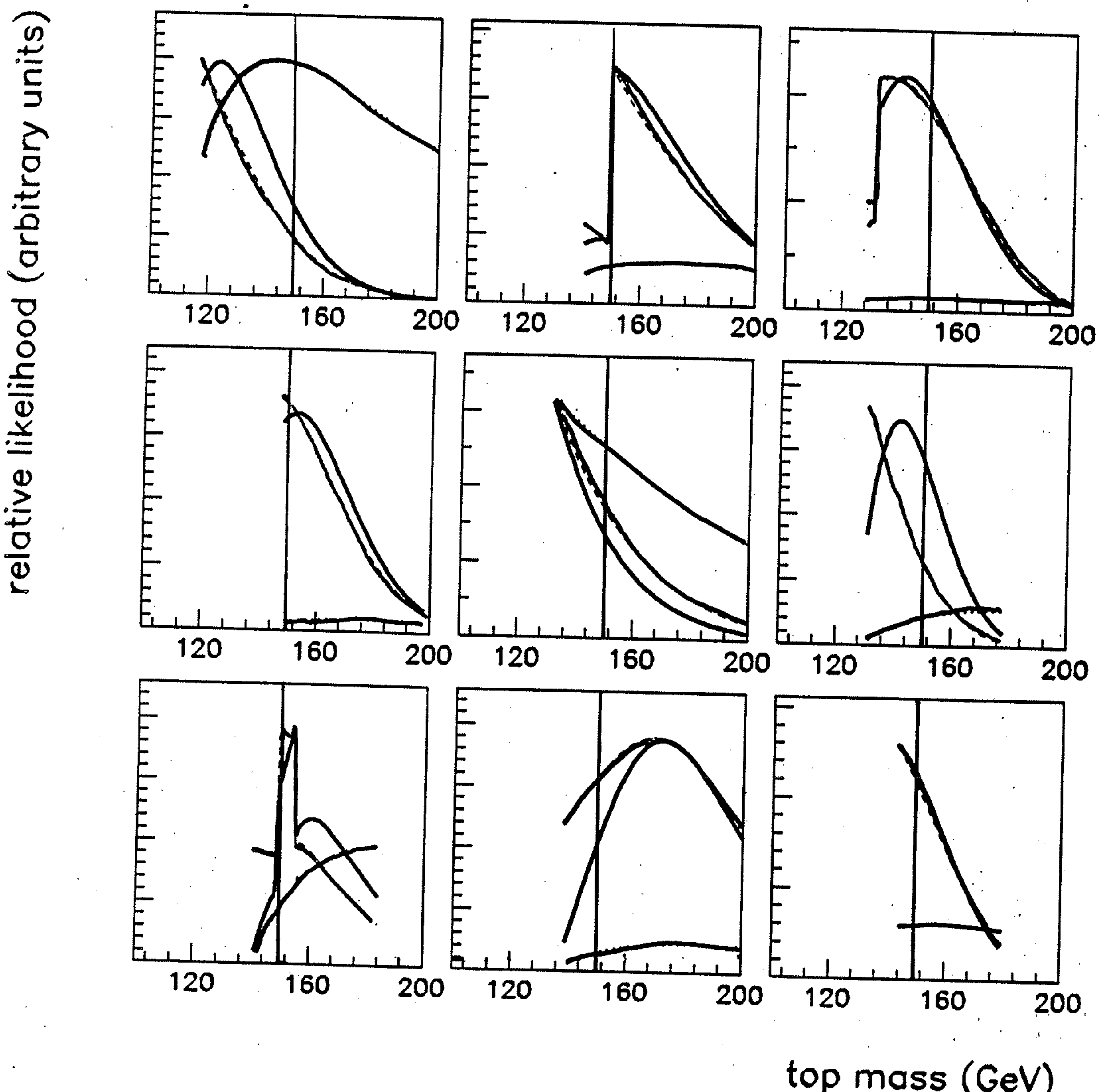
$$p(m_t/\text{event}) = \frac{p(m_t)\,p(\text{event}/m_t)}{\int p(m_t')\,p(\text{event}/m_t')\,dm_t'}$$

$\rightarrow p(m_t/\text{event}) \propto p(\text{event}/m_t) \propto \mathcal{L}(m_t)$

Consistency Check

- use a "naive" Monte Carlo which (Mark Strovink)
  - → generates $p\bar{p} \to t\bar{t}$ using p.d.f's and lowest order Born term
  - → decays $t \to Wb$ enforcing the lepton energy spectrum to conform with the decay weight $p(E_\ell, m_t)$
  - → no effects of initial or final state radiation, hadronization

naive Monte Carlo ($m_{top} = 150$ GeV)

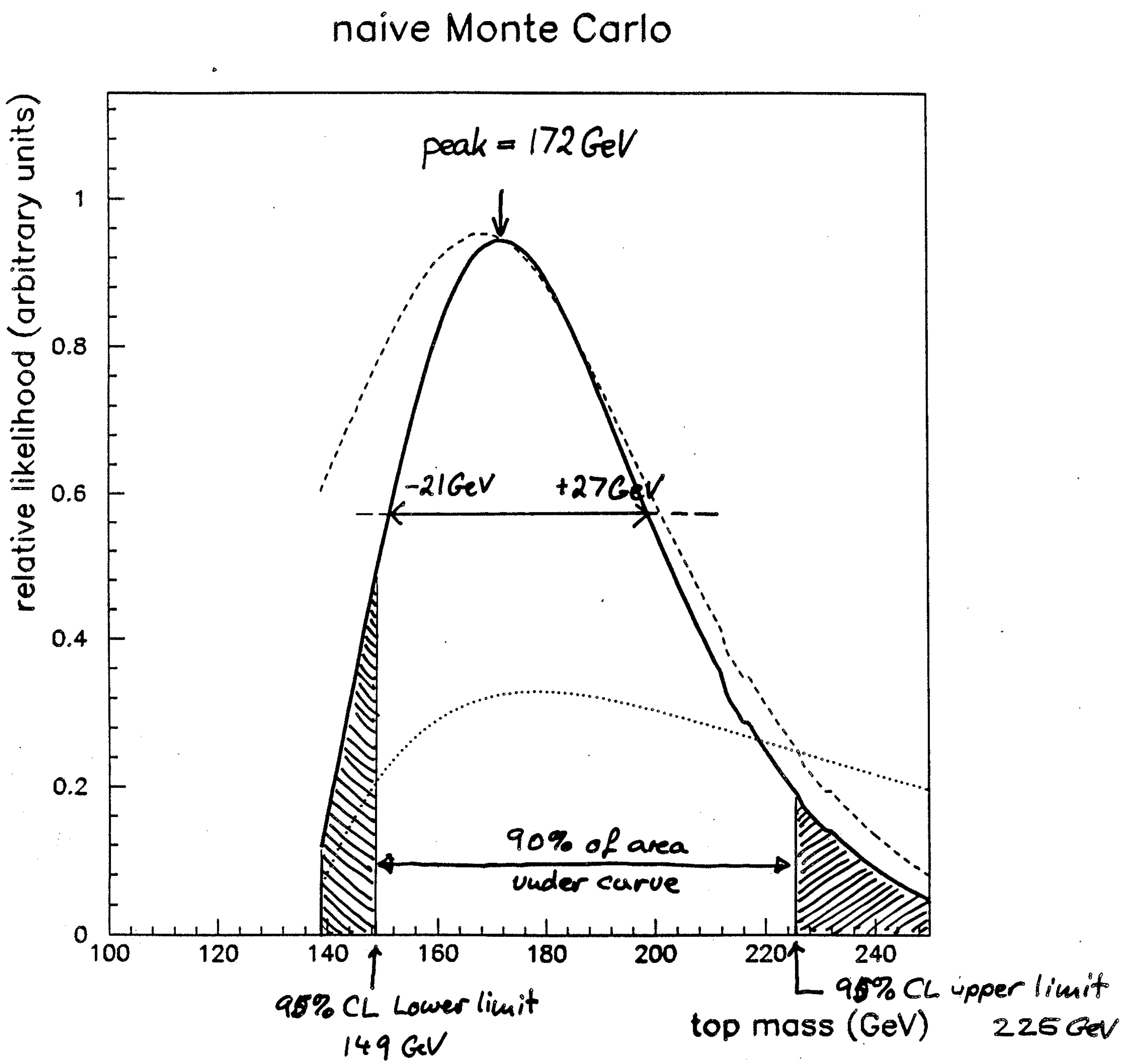


— Decay Weight

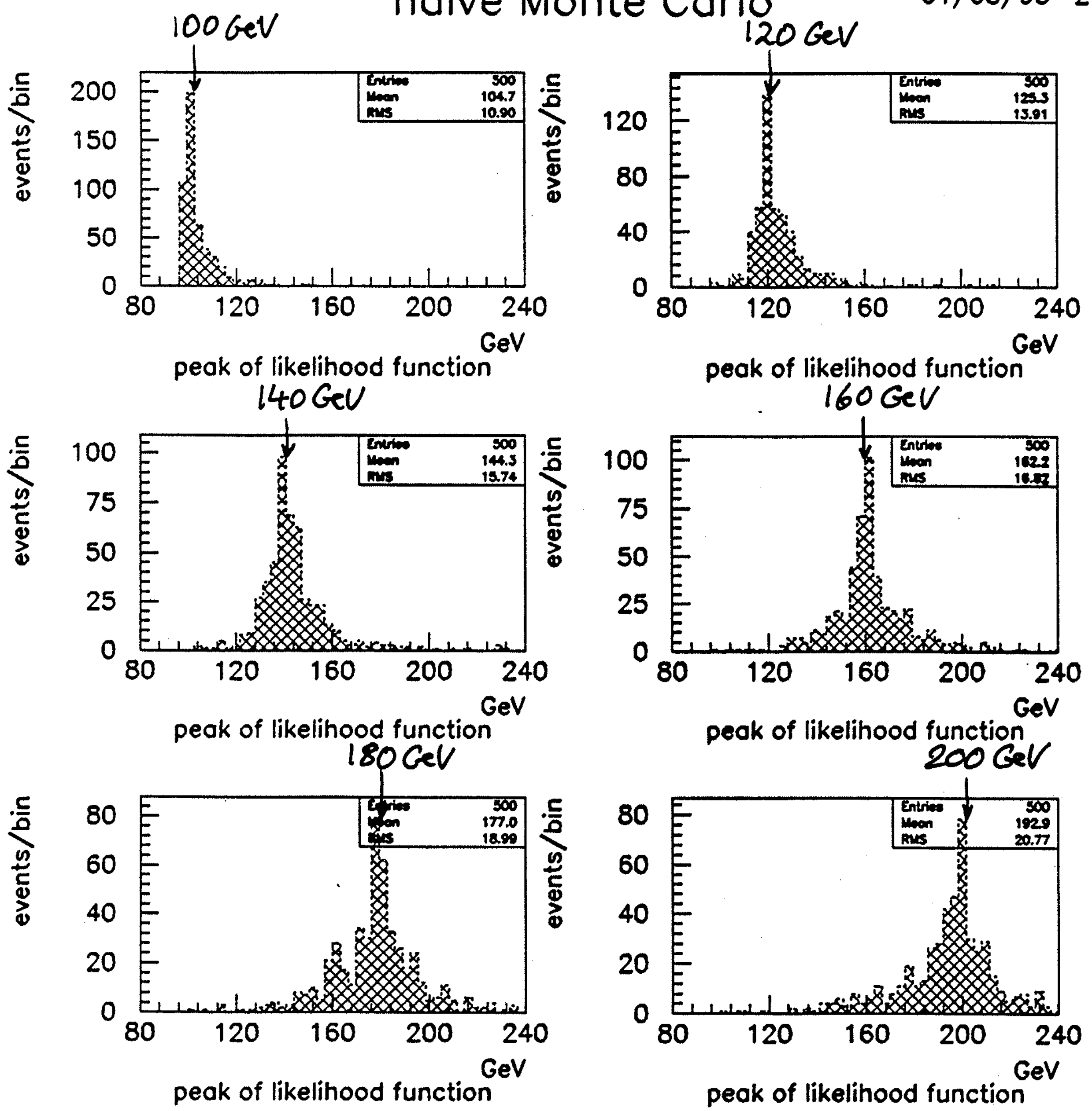
— $F(x_1)\,F(x_2)$

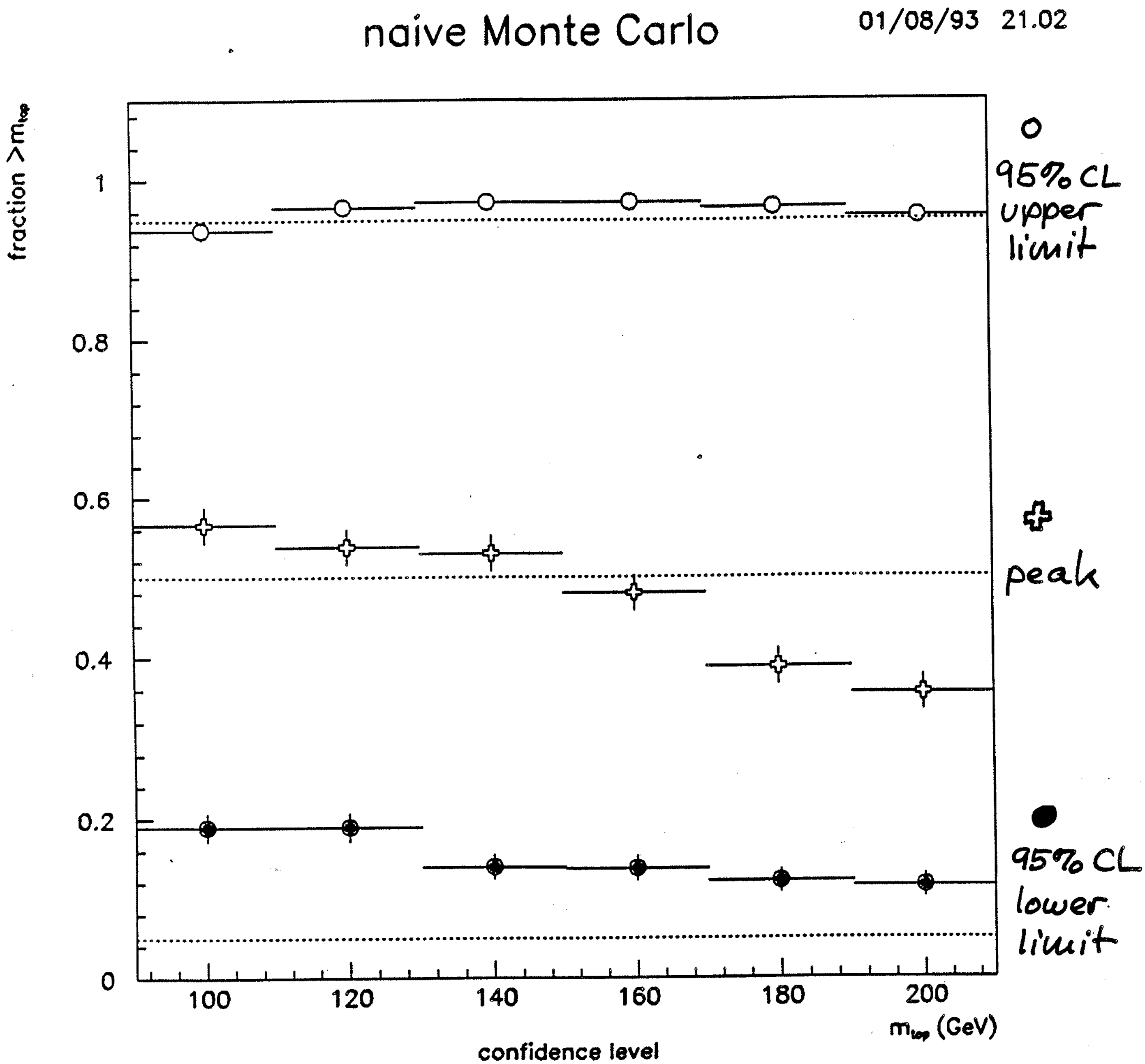
— total Weight (correct b-lepton pairs only)

Some Definitions



naive Monte Carlo 01/08/93 21.03





naive Monte Carlo
01/08/93 21.02
fraction >m_{top}
1
0.8
0.6
0.4
0.2
0
100
120
140
160
180
200
m_{top} (GeV)
confidence level
95% CL upper limit
peak
95% CL lower limit