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Future prospects of spectroscopic study of Lambda hypernuclei at JLab and J-PARC HIHR

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Abstract. The $(e, e'K^+)$ reaction spectroscopy opened a door to high resolution spectroscopy of Λ hypernuclei at JLab and it is currently the only technique to give sub-MeV energy resolution for reaction spectroscopy of wide-mass range Λ hypernuclei. New experiments from light to heavy hypernuclei are under preparation at JLab to solve hypertriton puzzle, to clarify charge symmetry breaking of Λ hypernuclei and to give a clue to solve the hyperon puzzle or the puzzle of heavy neutron stars.
As a hypernuclear precision spectroscopy experiment with other than electron beams, there is a newly proposed experiment using the (π^+, K^+) reaction at the new HIHR beamline, which is a key facility in the J-PARC hadron experimental facility extension project. The HIHR beamline adopts the momentum dispersion match technique and will enable us to perform sub-MeV resolution spectroscopy for isospin partners to the Λ hypernuclei studied with the electron beams at JLab.

1 Introduction

Neutron star is a quite interesting object which is most dense material in our Universe. It is a compact star with a radius of ~10 km and mass of 1-2 solar masses bound by gravitational force and the strong force prevents it from collapsing to a black hole. Neutron star mergers are now believed to play an important role in production of heavy elements. Astronomical observations of neutron stars by gravitational waves [1, 2] and X-ray hot spot measurement [3, 4] made significant progresses to give information about macroscopic understanding of characteristics of neutron stars. Therefore, microscopic understanding of neutron stars becomes more important than ever and collaborative works between experiments and theories are mandatory.

One of great mysteries of neutron star is the so-called "hyperon puzzle." Naïve discussion about the nucleon Fermi energy and the attractive ΛN two-body interaction, Λ hyperons are expected to appear in the core of neutron star. Inclusion of hyperon such as Λ particle softens the Equation of State (EoS) of neutron stars. Solutions of the Tolman-Oppenheimer-Volkoff equation [5] with the conventional baryon potential models predict maximum masses of neutron stars are limited to be less than about 1.5 solar masses and heavier neutron stars than the limit are considered to collapse into black holes due to their too heavy masses. However, recent astronomical observations clarified the existence of massive neutron stars as heavy as two solar masses [6–8]. The masses of these neutron star are measured by quite

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reliable techniques such as relativistic Shapiro delay and thus, there is no doubt about their existence. The conflict between theoretical predictions of maximum masses of neutron stars with hyperons and astronomical observations of heavy neutron stars is called as the "hyperon puzzle" and it is one of most important problems to be solved for the nuclear physics.

It is common understanding that there is an additional mechanism to make the neutron star EoS stiffer to solve the hyperon puzzle and a promising scenario is inclusion of the three-body repulsive ΛNN force. The repulsive ΛNN force plays significant role in high density environment such as neutron star core. Inclusion of it makes neutron star's EoS stiffer and futhermore it may be so strong that it prevents Λ's appearance [9–12]. Inclusion of the repulsive ΛNN force gives drastic effect to the EoS of neutron stars, however, its gives small effects as small as < 1 MeV to Λ's binding energies of hypernuclei which are able to be studied by the terrestrial experiments with particle accelerators. Therefore, high resolution spectroscopic study of Λ hypernuclei is quite important.

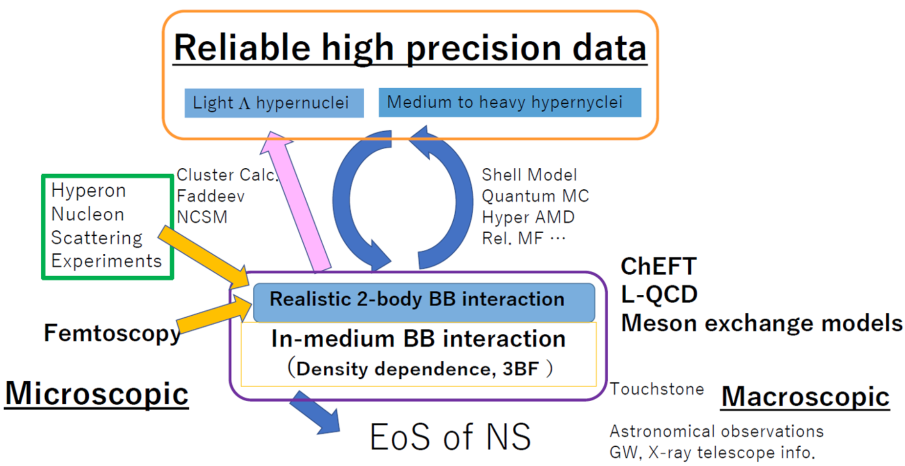

Figure 1. Strategy to solve the hyperon puzzle.

Figure 1 shows the our strategy to solve the hyperon puzzle. Starting point is reliable 2-body baryonic interaction models. Previously these baryonic interaction models have been constructed with SU(3) flavor symmetry and the limited binding energy data of hypenuclei, however, high quality hyperon nucleon scattering experiments was successfully performed recently for ΣN [13] and femotoscopy experiments [14] provides precious information about the p-Σ^0 interaction. In near future, combination of high quality experimental data and progresses of theoretical studies such as chiral effective field theory and lattice QCD will improve greatly our understanding of the 2-body baryonic interaction. The established 2-body baryonic interaction potentials will be converted to the effective in-medium baryonic potentials which include the multi-body forces or density dependences. The validity of such in-medium interaction models will be tested with high precision hypenuclear data. High precision data of hypernuclei provide also precious experimental inputs, for example, low energy constant of the chiral effective field theory. EoS of neutron stars will be deduced from such in-medium baryoic interaction models. Macroscopic information of neutron stars obtained by astronomical observations will serve as a touch stone to test our microscopic understanding of the baryon interaction. Consistent understanding of microscopic as well as macroscopic understandings of neutron stars means the successful solution of the hyperon puzzle.

High precision hypernuclear experiments play an important role in the above scenario. In the following sections, I will explain about future experiments which provide high precision spectroscopy of Λ hypernuclei, namely, a spectroscopic study of electro-produced hypernuclei at Thomas Jefferson National Accelerator Facility (JLab) and a new (π, K^+) reaction spectroscopy of hypernuclei at the HIHR beamline which will be constructed at the extended hadron experimental facility of the Japan Proton Accelerator Research Complex (J-PARC).

2 Spectroscopic study of electro-produced Λ hypernuclei at JLab

2.1 Spectroscopy of Λ hypernuclei with meson beams and electron beams

Conventionally, meson beams have been used for study of Λ hypernuclei. Figures 2 (left) show $n(K^-,\pi^-)\Lambda$ and $n(\pi^+,K^+)\Lambda$ reactions which were used for spectroscopic study of Λ hypernuclei with K^- beams and π^+ beams. The $n(K^-,\pi^-)\Lambda$ reaction is the quark exchange reaction and the $n(\pi^+,K^+)\Lambda$ reaction is the $s\bar{s}$ pair production to convert a neutron to a Λ particle. As shown in figure 2 (right), the $p(e,e'K^+)\Lambda$ reaction is quite similar reaction to the $n(\pi^+,K^+)\Lambda$ reaction while a virtual photon produces $s\bar{s}$ pair in the $p(e,e'K^+)\Lambda$ reaction and a proton is converted to a Λ. Even with the same targets, $p(e,e'K^+)\Lambda$ reaction and $n(K^-,\pi^-)\Lambda$, $n(\pi^+,K^+)\Lambda$ reactions produces different hypernuclei. They are complementary and experiments with both electron beams and meson beams are important to study isospin dependence or charge symmetry breaking of hypernuclei.

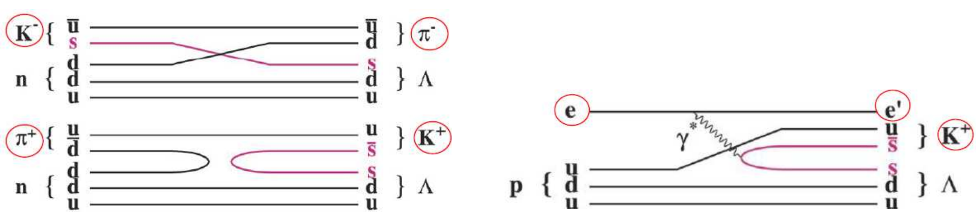

Figure 2. $n(K^-,\pi^-)\Lambda$ and $n(\pi^+,K^+)\Lambda$ reactions (left), and $p(e,e'K^+)\Lambda$ reaction (right)

Though the production cross section of electroproduction of Λ is a few orders of magnitude smaller than that for (π^+,K^+), (K^-,π^-) reactions, the energy resolution of the $p(e,e'K^+)$ experiments is 0.5-0.7 MeV (FWHM) while the resolutions are limited to be a few MeV for the experiments with meson beams due to quality of secondary meson beams. Electrons can be directly accelerated by particle accelerators but meson beams should be produced as secondary beams with proton accelerators. Therefore, momentum information and beam position of electron beams are not necessary to be measured event by event but beam momentum, position measurements are essential for meson beams. Beam detectors limits maximum intensity of meson beams but there is no limitation from beam detectors for electron beams. Therefore meson beam intensity is limited to be several 10^6/s while electron beam intensity can be as high as 100 μA which corresponds to 6×10^{14}/s.

Overcoming lots of experimental difficulties due to high electron background, the $(e,e'K^+)$ reaction spectroscopy of Λ hypernuclei was established in JLab Hall-C [15–23] and Hall-A [24–26].

Table 1. Approved JLab experiments on Λ hypernuclear spectroscopy

Experimental Number	Reaction	Comments on target
E12-15-008 [27]	$^{40,48}\mathrm{Ca}(e, e'K^+)^{40,48}_{\Lambda}\mathrm{K}$	isotopically enriched
E12-19-002 [28]	$^{3,4}\mathrm{He}(e, e'K^+)^{3,4}_{\Lambda}\mathrm{H}$	gaseous cryogenic
E12-20-013 [29]	$^{208}\mathrm{Pb}(e, e'K^+)^{208}_{\Lambda}\mathrm{Tl}$	low melting point

2.2 New JLab experiments of Λ-hypernuclear spectroscopy

Three new experiments on Λ hypernuclei with the $(e, e'K^+)$ reaction were proposed and approved at JLab. Table 1 summarizes these three experiments' information. Neutron star is a nucleon (baryon) many-body system characterized by large mass numbers and very high neutron abundances, *ie.* $A \gg 1$ and asymmetric parameter is close to one ($\delta = (N-Z)/A \sim 1$). So far, most of spectroscopic studies of Λ hypernuclei have been performed with solid targets of stable nuclei. The $(e, e'K^+)$ reaction converts ^{A}Z target to $^{A}_{\Lambda}(Z-1)$ hypernucleus and the (π^+, K^+) reaction does ^{A}Z to $^{A}_{\Lambda}Z$. Therefore A and δ of produced hypernuclei cannot be far from the original targets' A and δ as shown in figure 3. Solid line shows that $A^{-2/3} - \delta$ relation of stable targets and so far produced hypernuclei locate along this line. The newly proposed experiments try to widen our region of hypernuclear study: E12-19-002 tries to study light hypernuclei with cryogenic gaseous targets (^{3,4}He) to study the Charge Symmetry Breaking effects of hypernuclei and to solve the hypertriton puzzle which is the discrepancy between observed short lifetime and shallow binding energy of $^{3}_{\Lambda}$H; E12-20-013 aims to study $^{208}_{\Lambda}$Tl with ^{208}Pb target of which properties can be associated with the uniform nuclear matter; the E12-15-008 experiment investigates the isospin dependence of medium-heavy hypernuclei $^{40,48}_{\Lambda}$K to study isospin dependence of the ΛNN three-body repulsive force. Detail of physics motivations of these three experiments can be found in the experimental proposals and this HYP2022 proceedings [27–29].

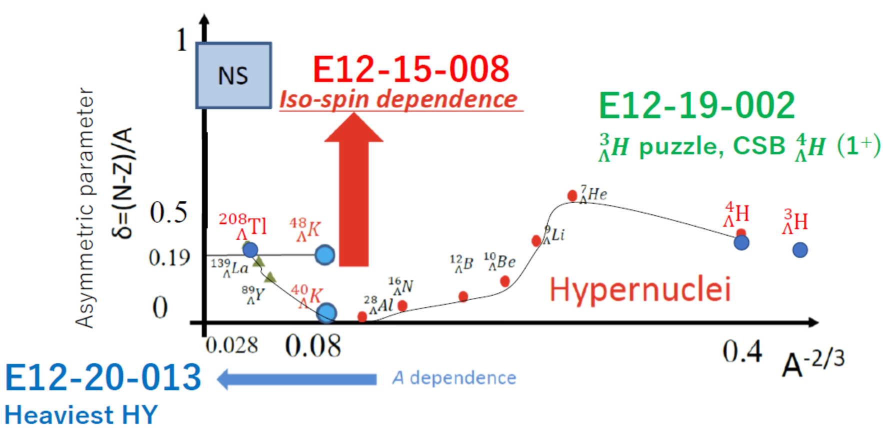

Figure 3. Mass number ($A^{-2/3}$) and asymmetric of parameter ($\delta = (N-Z)/A$) of Λ hypernuclei which are able to be produced from the stable targets by the $(e, e'K^+)$ reaction.

2.3 Experimental setup of JLab hypernuclear experiments

Basic principle of the ${}^{A}Z(e, e'K^{+})^{A}_{\Lambda}(Z-1)$ spectroscopy of hypernuclei is missing mass spectroscopy with the measurement of e' momentum, K^{+} momentum with the known incoming electron beam energy. The cross section of the elementary process, $p(\gamma, K^{+})\Lambda$, becomes maximum in the region of $E_{\gamma} = 1.2 - 1.5$ GeV and recoil momentum of the hypernucleus becomes smaller for higher beam energy in the (γ, K^{+}) reaction which results in larger capture probability of Λ to produce a hypernucleus. Therefore, we chose the virtual photon energy of $E_{\gamma^{*}} = 1.5$ GeV and the central momentum of the kaon spectrometer (HKS) was set at 1.2 GeV/c for the new experiments as we used it in the previous Hall-C hypernuclear experiments.

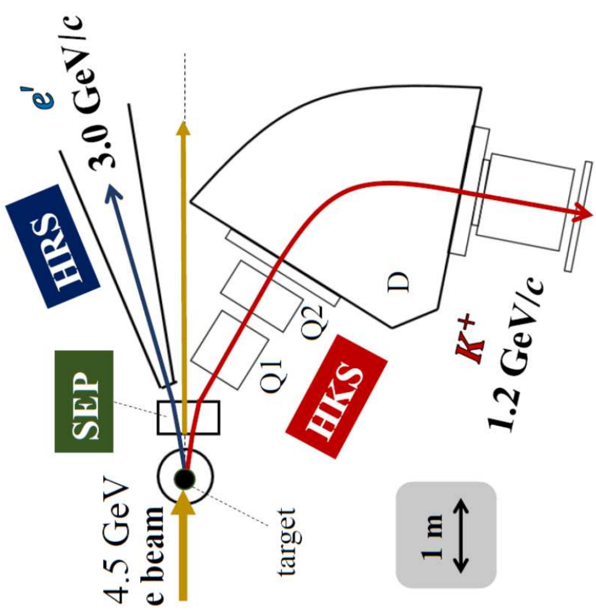

Figure 4. Setup in Hall A. HKS which was used in previous Hall-C experiments will be moved to Hall-A and permanently installed HRS in Hall-A will be used in this option.

There are two possibilities for the experimental site at JLab, namely Hall-A and Hall-C. In Hall-A, we can use the High Resolution Spectrometer (HRS) as the spectrometer for the scattered electrons. It is a vertical bending type spectrometer and it can measure high momentum electrons with an excellent momentum resolution of $\Delta p/p \sim 3 \times 10^{-4}$. Therefore we set the beam energy of $E_{e} = 4.5$ GeV and planned to use HRS with a central momentum of $E_{e'} = 3.0$ GeV which makes the virtual photon energy as $E_{\gamma^{*}} = 1.5$ GeV (Fig. 4). Higher beam energy allows us to make the bremsstrahlung electron background Lorentz boosted to forward angles out of spectrometer's acceptances. Therefore, we can expect good signal-to-noise ratio, though absolute e' momentum resolution which contributes to the missing mass resolution, becomes worse.

Another possibility is the experiment in Hall-C with the scattered electron spectrometer (HES) which was used for the previous hypernuclear experiments in Hall-C. It is designed to measure low energy electrons less than 1 GeV. Therefore, electron beam energy is limited to be less than 2.5 GeV if $E_{\gamma^{*}} = 1.5$ GeV is kept. Lower energy of scattered electron is favored for better energy resolution since smaller momentum (p) gives better absolute resolution for the fixed relative momentum resolution $\Delta p/p$, however, it is disfavored in terms of the signal-to-noise ratio.

In order to avoid physical interference between the forward located kaon and electron spectrometers, we will install a new pair of charge separation magnets (PCS) in both Hall-A and Hall-C options (Fig. 5). Introduction of PCS minimizes effects from the magnetic field to the unused electron beam while previously used a single large dipole (SPLitter) bent the electron beam as well as separated positively charged K^+ and negative e'. Furthermore, the single dipole magnet coupled the kaon spectrometer and the electron spectrometer beam-optically. It resulted in very complicated beam tune and analysis.

Another challenge in Hall-C option is that both HES and HKS are horizontal bending spectrometers which have no vertex resolution along the beamline. It was not a disadvantage for thin solid targets which were used in previous Hall-C experiments since these solid targets' (material) thicknesses were $\sim$ 100 mg/cm^2 or less than $<$ 1 mm. Therefore, we know the reaction point along the beamline with an accuracy of $<$ 1 mm. However, we will use a gaseous target with a length of 20 cm in the new experiment and the reaction point information is necessary for the high-resolution spectroscopy. Therefore, we plan to modify HES to be a vertical bending type spectrometer as shown in Fig. 6. Based on the preliminary simulations, solid angle of vertical HES with the PCS is slightly reduced but optical natures will not be affected much.

Considering the advantages and disadvantages of the spectrometers' performance and the JLab's experimental allocation plan in Halls A and C, we chose to perform the experiments in Hall-C. Detailed simulation and modification design of HES for the safe rotation of 100 ton spectrometer are in progress.

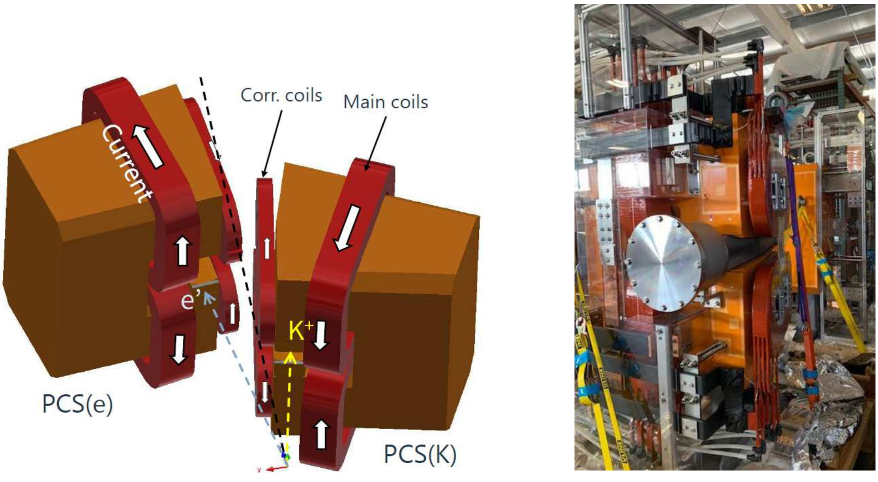

Figure 5. Particle Charge Separator (PCS) consists of a pair of dipole magnets (left). PCS(e) can bend 3 GeV/c electron by 6 degrees and PCS(K) can do 1.2GeV/c K^+ by 18.4 degrees. They were constructed in Japan and shipped to JLab in 2022 (right).

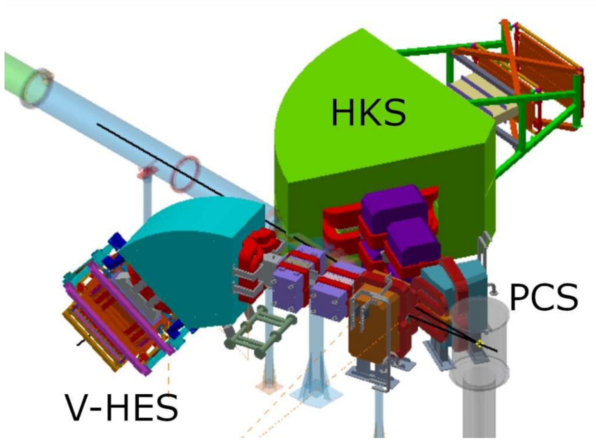

Figure 6. Setup in Hall C. Both HKS and HES were used in previous Hall-C experiments, however, HES will be modified to be a vertical bending spectrometer.

3 Supra-precision (π^+, K^+) reaction spectroscopy of Λ hypernuclei (SπK) at J-PARC HIHR beamline

In the previous section, it was mentioned that high-quality primary electron beam at JLab allows us to avoid measuring momenta and positions of beam electrons. High-resolution kaon and scattered electron spectrometers make it possible to perform the $(e, e'K^+)$ spectroscopy of hypernulei with a resolution of 0.5 MeV.

Until now, the (π^+, K^+) reaction spectroscopy of hypernuclei has been limited not only in yield but also in resolution. Pion beams are produced as secondary particles and their momentum distribution is large; therefore beam momentum analysis is inevitable. The π^+ beam intensity is limited by the operational limits of the beamline detectors. For these reasons, the energy resolution and beam intensity of the (π^+, K^+) hyperuclear spectroscopy have been limited to a few MeV and a few millions of π^+s/sec, respectively. The (π^+, K^+) and $(e, e'K^+)$ spectroscopies, which produce different Λ hypernuclei even with the same target, should provide complementary high-quality data for the detailed isospin dependence or the charge symmetry breaking studies. However, until now, the resolution of (π^+, K^+) spectroscopy has not matched with that of the $(e, e'K^+)$ reaction.

In order to avoid these limitations, a new experiment of supra-precision (π^+, K^+) reaction spectroscopy of Λ hypernuclei (SπK; J-PARC P84) was proposed [30] at the new High Intensity High Resolution beamline (HIHR) which is planned at the extended hadron hall of J-PARC [31, 32].

Introducing HIHR, these limitations of (π^+, K^+) reaction spectroscopy are removed and an excellent energy resolution of 0.4 MeV (FWHM), which is comparable to or even better than the $(e, e'K^+)$ spectroscopy, will be achieved. It enables us to determine the binding energies of Λ for various hypernuclei with a few 10 keV accuracy.

3.1 High Intensity High Resolution beamline (HIHR)

Currently the project to extend the hadron experimental facility is seriously considered and its preparation works are underway. HIHR is a key beamline designed for the Λ hypernuclear spectrscopy with the (π^+, K^+) reaction.

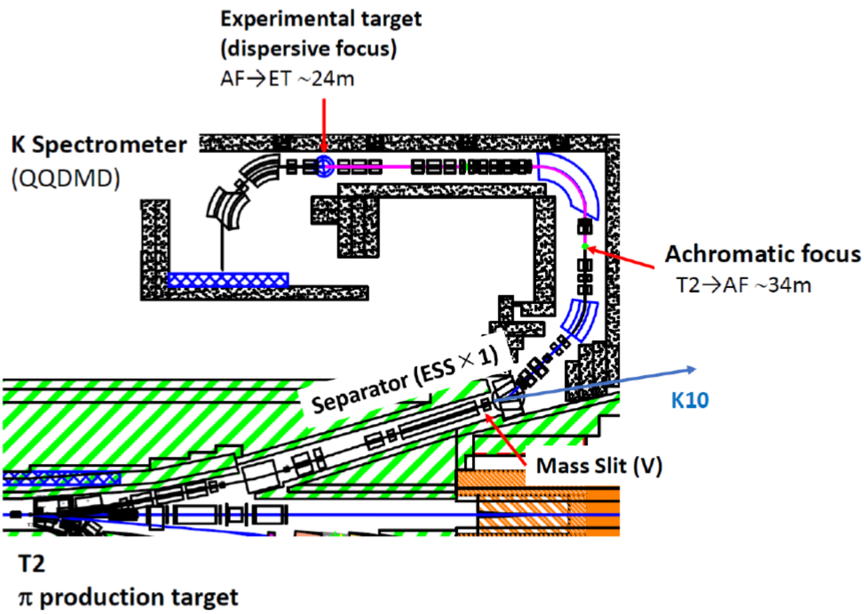

Figure 7. Schematic illustration of the HIHR beamline with a kaon spectrometer [30].

Figure 7 shows a schematic illustration the HIHR beamline with a kaon spectrometer. Primary proton beams bombard a new primary target (T2) to be installed in the hadron extension experimental facility and secondary meson beams are extracted at the production angle of 3 degrees; An Electro-Static Separator (ESS) is used for π, K separation. This beam extraction part is shared with HIHR and K10 beamlines. After this beamline branch, the π beam is focused achromatically and beam profile is redefined here with a beam slit. After the achromatic focus point, the beam is focused with a magnification of 1.1 and a dispersion of 11 cm/%. On the experimental target where the (π^+, K^+) reaction takes place, the momentum dispersion is converted to spatial dispersion and the kaon spectrometer after the experimental target is designed to cancel this spatial dispersion. This technique is called as the dispersion match (Fig. 8) and is well established technique for long-living ions [33–35], however, it is a big challenge to apply this technique to secondary GeV meson beams.

A dispersion match π beamline with an adequately designed kaon spectrometer enable us to avoid any beamline detectors. Because there are no limitations from the beamline detectors, we can use as strong as possible pion beams and can achieve excellent missing mass resolution.

Based on preliminary design of beamline and spectrometer systems, a GEANT4 simulation gives 400 keV (FWHM) resolution and 53/h yield for $^{12}_{\Lambda}$C hypernuclei with $2 \times 10^8 \pi^+$s/spill on the ^{12}C target of 400 mg/cm^2 [30]. Figure 9 shows an expected Λ binding energy spectrum for $^{208}_{\Lambda}$Pb with a beamtime of 60 days based on the above simulation with the theoretical predictions [36, 37]. Major peaks of $i_{13/2}^{-1}$ neutron holes with a Λ in $s, p, d, ..$ angular momentum states are clearly seen, and furthermore sub-major peaks which correspond to $g_{9/2}^{-1}$ neutron holes are also expected to be observed. It should be noted that sub-major peaks filled the valleys between the major peaks and they were treated as background in the previous (π^+, K^+) hypernuclear spectroscopy [38]. The improved resolution at HIHR means not just an improvement in the quality of the spectrum, but takes the study to a different stage. The first campaign of the experiments was proposed for study of ${}^{6,7}_{\Lambda}$Li, ${}^{9}_{\Lambda}$Be, ${}^{10,11}_{\Lambda}$B, ${}^{28}_{\Lambda}$Si, ${}^{40}_{\Lambda}$Ca, ${}^{51}_{\Lambda}$V, ${}^{89}_{\Lambda}$Y, ${}^{139}_{\Lambda}$La and ${}^{208}_{\Lambda}$Pb with a 104 days of beamtime request. This is not the entire program of supra-precision hypernuclear spectroscopy but the first step to realize the “Hypernuclear Factory” at J-PARC.

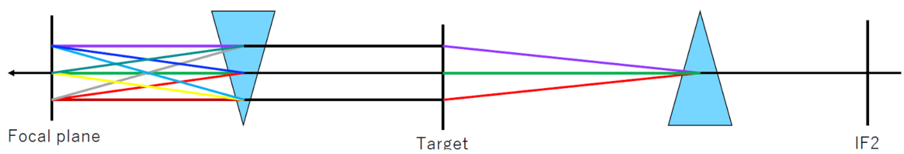

Figure 8. Conceptual illustration of the dispersion match technique. Extracted π beam has a momentum dispersion and it is converted to the spatial dispersion on the experimental target by the HIHR beamline. The kaon spectrometer is designed to cancel this spatial dispersion and the position information on the focal plane gives the excitation energies of the produced hypernuclei.

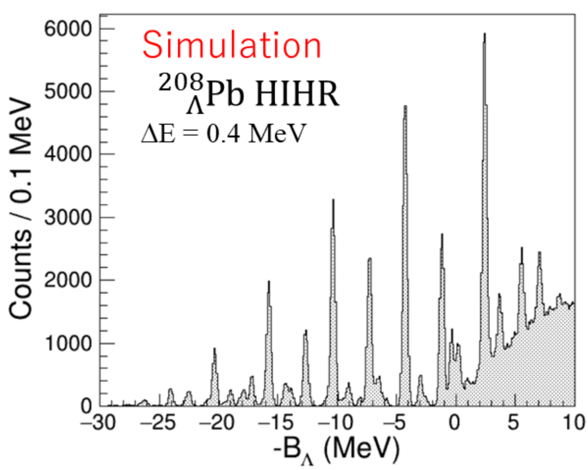

Figure 9. Expected Λ binding energy spectrum of ${}^{208}_{\Lambda}$Pb to be measured at HIHR. Major peaks are clearly observed, and sub-major peaks, which were previously treated as background due to the limited resolution of the experiments, can be also observed.

4 Summary

The $(e, e'K^+)$ reaction spectroscopy of Λ hypernuclei at JLab was established and new experiments from light to heavy hypernuclei are under preparation at JLab. They will provide precious data to give a clue to solve the hyperon puzzle.

At the HIHR beamline in the J-PARC hadron experimental facility extension project, a new precision spectroscopic experiment of Λ hypernuclei (SπK) was proposed using the (π^+, K^+) reaction. With the momentum dispersion match technique, HIHR with a dedicated kaon spectrometer will realize 0.4 MeV (FWHM) resolution spectroscopy to determine the Λ binding energy with an accuracy of a few 10 keV. HIHR will be a hypernuclear factory to study Λ hypernuclei in whole mass range. The high-precision $(e, e'K^+)$ experiments at JLab and new (π^+, K^+) experiments at HIHR are complementary and both of them are essentially important to solve the hyperon puzzle.

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