Q balls in thermal logarithmic potential [1]

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Abstract

Time evolution of the Q ball in thermal logarithmic potential is studied using lattice simulations. We confirm that the Q ball transforms from the thick-wall type to the thin-wall type when the thermal logarithmic term in the potential is overcome by a mass term with a positive coefficient of radiative corrections as the temperature decreases due to the cosmic expansion. We also discuss the effects of this phenomenon on the detectability of gravitational waves from the Q-ball formation.

1 Introduction

A Q ball is a non-topological soliton, which consists of scalar fields that carry global $U(1)$ charge Q. Its existence and stability are guaranteed by finite Q and it is often generated in the Affleck-Dine (AD) mechanism for baryogenesis. Recently, it is claimed that gravitational waves (GWs) are generated at the Q-ball formation [3], which may be detected by the next generation gravitational wave detectors such as DECIGO and BBO. However, the detailed study of the subsequent evolution and the decay of Q balls revealed it to be difficult even by those next generation gravitational wave detectors [2].

The properties of the AD mechanism and the Q ball depend on the supersymmetry (SUSY) breaking mechanism, because the effective potential of the relevant scalar field (AD field) quite differs for the different mediation mechanism. Consequently, there are various types of Q balls. Among them, the thermal log type Q ball, whose effective potential is dominated by thermal logarithmic term, possesses an interesting feature. As the universe expands, the cosmic temperature decreases and so does the thermal logarithmic potential. Thus, the properties of thermal log type Q ball will change with time. Moreover, zero-temperature potential eventually overcomes the thermal potential and then the type of Q ball changes [2]. If the zero-temperature potential itself allows a Q-ball solution, the type of Q ball changes to the corresponding type.

It may then be naively expected that the Q balls would be destroyed if the zero-temperature potential alone does not allow a Q-ball solution. Recently, however, it was shown that even if the zero-temperature potential alone does not allow a Q-ball solution, the total potential (the thermal logarithmic term and a mass term with a positive radiative correction) *does* allow a Q-ball solution, which would result in the transformation from the thermal log type of the Q ball into the thin-wall type [4]. Since the scenario would be changed in this case, it is important to investigate whether the field configuration dynamically transforms from one type of the Q ball to the other.

Here, we perform numerical simulations on the lattice to see the time evolution of the configuration of the AD field in the potential with a thermal logarithmic term and a mass term with a positive coefficient for radiative corrections, where the latter term alone does not allow a Q-ball solution. We confirm that the thermal log type Q ball transforms to the thin-wall type Q ball found in Ref. [4]. We also find that there is a tiny parameter region where the GWs from Q-ball formation may be detected by future detectors such as DECIGO or BBO only in the case of Q balls in thermal potential [2].

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2 Properties of Q balls

We are interested in the Q-ball properties in the potential with both a thermal logarithmic term and a mass term with a positive coefficient for one-loop radiative correction,

$$V_{\rm tot} = V_{\rm thermal} + V_{\rm grav}, \tag{1}$$

$$V_{\rm thermal} \simeq \begin{cases} T^2|\Phi|^2, & \text{for} \quad |\Phi| \ll T \\ T^4 \log\left(\frac{|\Phi|^2}{T^2}\right), & \text{for} \quad |\Phi| \gg T \end{cases} \tag{2}$$

$$V_{\rm grav} = m_\phi^2 |\Phi|^2 \left[1 + K \log\left(\frac{|\Phi|^2}{\Lambda^2}\right)\right], \tag{3}$$

where Φ is the complex AD field and T is the cosmic temperature. The upper term in $V_{\rm thermal}$ represents the thermal mass from the thermal plasma and the lower one represents the two-loop finite temperature effects coming from the running of the gauge coupling $g(T)$ which depends on the AD field value. Note that even before the reheating from the inflaton decay has not completed, there exists thermal plasma from the partial inflaton decay as a subdominant component of the universe. $V_{\rm grav}$ denotes a soft mass term due to gravity-mediated SUSY breaking, where $m_\phi \sim \mathcal{O}(\text{TeV})$. The second term in the bracket is the one-loop radiative correction, and Λ is the renormalization scale. Here we assume $K > 0$ ($K \simeq 0.01 - 0.1$) so that $V_{\rm grav}$ alone does not allow a Q-ball solution.

At larger temperature when the field starts the oscillation, the potential is dominated by the thermal logarithmic term $V_{\rm thermal}$, and the thermal log type Q balls form [4]. The properties of this type Q ball are similar to those of the gauge-mediation type Q ball [4],

$$\phi_0(T) \sim TQ^{1/4}, \qquad \omega(T) \sim \sqrt{2}\pi T Q^{-1/4}, \qquad E(T) \sim \frac{4\pi\sqrt{2}}{3} T Q^{3/4}, \qquad R(T) \sim \frac{Q^{1/4}}{\sqrt{2}T}, \tag{4}$$

where Q is the charge stored in a Q ball, $\phi_0 = \sqrt{2}|\Phi_0|$ is the AD field value at the center of Q ball, ω is the angular velocity of the AD field, E is the energy stored in a Q ball, and R is its radius. Since the charge Q is the conserved quantity, whose value is determined at the Q-ball formation, the parameters of Q balls change as the temperature decreases according to Eq. (4).

As the temperature decreases further, $V_{\rm grav}$ will eventually dominate the potential at ϕ_0. In our previous study in Ref. [2], we assumed that Q balls are destroyed and turn into almost homogeneous AD field quickly at this moment, because the potential (3) alone does not allow a Q-ball solution. Recently, however, one of the present authors pointed out that a Q-ball solution *does* exist even in this situation [4]. Although the soft mass term overcomes the thermal logarithmic term at large field values, the latter will dominate the potential at smaller field values. As a result, in the light of charge conservation, a thin-wall type Q-ball solution exists. We shall call it the thermal thin-wall type Q ball. The properties of the Q ball are written as

$$\phi_0(T) \sim c(T/m_\phi K^{1/2}) \frac{T^2}{m_\phi K^{1/2}}, \qquad \omega \sim \alpha(T) m_\phi, \qquad E \sim \alpha(T) m_\phi Q, \qquad R \sim \left(\frac{m_\phi K Q}{c\alpha T^4}\right)^{1/3}, \tag{5}$$

where $c(T/m_\phi K^{1/2})$ and $\alpha(T)$ are slowly increasing functions of T and they are of order of unity at the temperature we are interested in. For example, $c(10) \simeq 2.5, c(10^2) \simeq 3.4, c(10^3) \simeq 4.1, c(10^4) \simeq 4.6, c(10^5) \simeq 5.1, c(10^6) \simeq 5.5, c(10^7) \simeq 6.0$ and so on. $\alpha(T)$ is expressed as

$$\alpha^2 = 1 + K\left(\log\left(\frac{c^2T^4}{2m^2K\Lambda^2}\right) + \frac{1}{c^2}\log\left(\frac{c^2T^2}{2m_\phi^2 K}\right)\right), \tag{6}$$

and its temperature dependence is stronger than that of c.

It is true that such a Q-ball solution exists but not clear that the field configuration follows from the thermal log type Q ball to the thermal thin-wall one. In order to tell how the configuration of the AD field evolves, we perform numerical studies on the lattices.

3 Time evolution and type transformation of the Q ball

Here, we investigate the time evolution of the Q ball in the potential of the thermal logarithmic term and the soft mass term with positive radiative corrections by using lattice simulations. Since we are primarily interested in the transformation of a Q ball, here we limit ourselves to a single Q ball assuming the spherical symmetry of the field configuration, and solve the 1-dimensional partial differential equations in the radial direction by using the staggered leapfrog method with second order accuracy both in time and in space.

The time evolution of the field configuration is shown in Fig. 1 for $Q \simeq 1.0 \times 10^9$. The axes are rescaled with respect to the scale factor a so that the rescaled radius is almost constant for the thick-wall type. We can see that the configuration of the Q ball changes from the thick-wall to the thin-wall types. This coincides with the feature of the transformation of the Q-ball solution found in Ref. [4].

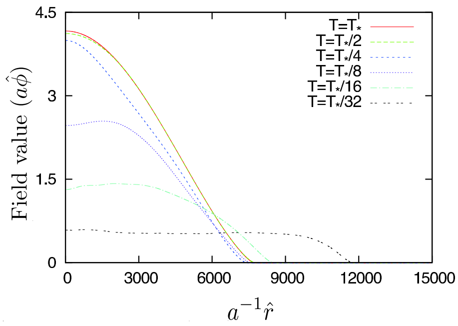


Figure 1: Configurations of the AD field for $Q \simeq 1.0 \times 10^9$ at the time of $T = T_*, T_*/2, T_*/4, T_*/8, T_*/16$, and $T_*/32$ from the top to the bottom, respectively. Q-ball configuration changes from the thick-wall to the thin-wall types.

The temperature dependence of the field value at the Q-ball center and the Q-ball radius with various charges of the Q ball are shown in Fig. 2. Here blue crosses, red x's, and green stars represent numerical results for $Q \simeq 2.5 \times 10^8, 1.0 \times 10^9$, and 3.0×10^9, respectively. Corresponding lines are the analytic estimates (4) and (5) up to numerical coefficients for each case (blue: $Q \simeq 2.5 \times 10^8$, red: $Q \simeq 1.0 \times 10^9$, green: $Q \simeq 3.0 \times 10^9$): At high temperature, $\phi_0 \propto TQ^{1/4}, R \propto T^{-1}Q^{1/4}, \omega \propto TQ^{-1/4}$ for the thermal log type Q ball, while, at low temperature, $\phi_0 \propto T^2, R \propto T^{-4/3}Q^{1/3}, \omega =$ const. for the thermal thin-wall type Q ball. We show the analytical estimates of the field value and the angular velocity in Eq. (5) with purple line in Fig. 2(a), since they are independent of charge Q. We can see that the analytical estimates (4) and (5) are well reproduced by the lattice simulations. One exception is the angular velocity at low temperature. This could be understood by the factor $\alpha(T)$ in Eq. (5), which decreases as T gets lower. We can therefore conclude that the Q ball really transforms from the thermal log type to the thermal thin-wall type in the potential considered here.

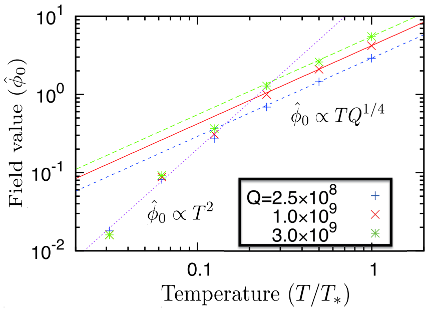
4 Detectability of gravitational waves

Now we see the detectability of the GWs from Q-ball formation. The present density parameter $\Omega^0_{\rm GW}$ and frequency f_0 of the GWs are given by

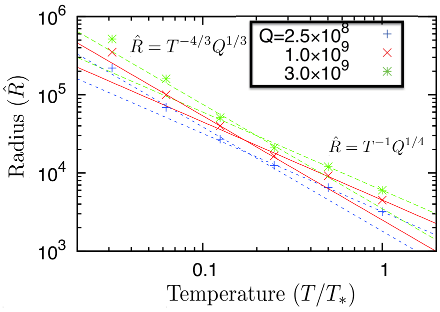
$$\Omega^0_{\rm GW} = \Omega^f_{\rm GW} \left(\frac{a_f}{a_0}\right)^4 \left(\frac{H_f}{H_0}\right)^2, \tag{7}$$

$$f_0 = f_f \left(\frac{a_f}{a_0}\right), \tag{8}$$

where the subscript f and 0 represents the parameters are evaluated at the time of Q-ball formation and the present. They strongly depend on the cosmic history after Q-ball formation. In particular, the



(a) Field value at the center of Q ball



(b) Radius of Q ball

Figure 2: Temperature dependence of Q ball properties. Blue crosses, red x's, and green stars represent the numerical results for $Q \simeq 2.5 \times 10^8, 1.0 \times 10^9$, and 3.0×10^9, respectively. Lines correspond to analytical estimates (4) and (5) up to numerical coefficients.

existence of Q-ball dominated era severely constrains the detectability of GWs, which is determined by the SUSY breaking mechanism, its model parameters and initial conditions. Anyway, we can calculate them by evaluating the Hubble parameter when Q balls dominate the energy density of the Universe and they decay.

As a result, we can find that in the gauge-mediated SUSY-breaking model, if the reheating temperature is $T_R \simeq 10^{10}$ GeV and the initial field value of the AD field is $\phi_{\rm osc} \simeq M_G$ with $m_{3/2} \simeq 10$GeV and $M_F \simeq 10^4$ GeV, the present density parameter of the GWs from the Q-ball formation can be as large as $\Omega^0_{\rm GW} \simeq 10^{-16}$ and their frequency is $f_0 \simeq 10$ Hz [1, 2]. Thus, it is difficult but not impossible to detect them by next-generation gravitational detectors like DECIGO or BBO, but the parameter region for detectable GWs is very small. Moreover, we can find that it is almost impossible to detect GWs from Q-ball formation in other cases. In other cases when the thermal logarithmic potential drives the Q-ball formation, though the present amounts of the GWs from the Q-ball formation can be as large as $\Omega^0_{\rm GW} \simeq 10^{-8}$, the frequencies of such GWs are turned out to be very high. Thus, the identification of such GWs may determine the decay rate of inflaton or the initial condition of the AD mechanism.

We would like to comment on baryogenesis. Generally speaking, including the present case, the amount of produced baryon asymmetry is typically large for the case that AD condensates or Q-balls (almost) dominate the energy density of the Universe so that the present radiations and baryons are attributed to their decays. This is simply because the number densities of radiations and baryons are of the same order unless the (CP-violating) A-terms are suppressed by some symmetry. Thus, it is difficult to explain GWs and baryogenesis simultaneously. Once the GWs from the Q-ball formation are detected, we have the following two possibilities. In the case that such Q-balls are responsible for the present baryon asymmetry, the A-terms are suppressed by symmetry reason. The second option is that Q-balls are irrelevant for baryogenesis, which is realized for the AD fields with $B - L = 0$.

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