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Mass-radius ratio bound for horizonless charged compact object in higher dimensions

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ABSTRACT

In this paper, the mass-radius ratio bound for horizonless compact object in higher dimensions is derived. Instead of considering the various matter conditions of the compact objects following Andreasson's approach, we focus on the conditions for the existence of dynamical compact objects. The radius of the outermost circular null geodesic is derived in higher dimensions, which is the lower bound on the minimally allowed radius of dynamically stable horizonless charged compact object. Subsequently, the upper bound on the mass-radius ratio is obtained. Our results are strongly dependent on the dimensions. What's more significant is that the developed bound is proven always to be stronger than the result following Andreasson's approach in higher dimensions.

1. Introduction

Observation of gravitational waves by Advanced LIGO [1] is one of the greatest achievements of this century in physics, it opens a new window to the extreme strong gravitational field environment of coalescing binary systems of compact objects. The detection of the compactness defined as mass-radius ratio of compact object is of central importance for understanding the behavior of astrophysical systems. The LIGO detector will allow us to test the natures of compact objects. It is generally believed that an apparent horizon of spherically isotropic spacetime is formed at $2\mathcal{M} = R$, where $\mathcal{M}$ is the ADM mass of the asymptotically flat spacetime and R is the radius of the object in Schwarzschild coordinates [2]. The well known Buchdahl theorem [3] states that any reasonable static, spherically symmetric interior solution of the object has $2\mathcal{M} \leq 8R/9$. The violation of this inequality would lead to the gravitational object collapses, thus any regular compact object will end up with a black hole.

However, the requirements of Buchdahl's theorem derived in [3] are the non-increasing energy condition and isotropy for the object, which are too restrictive in certain situations. Many efforts have been done to extend the applications of the Buchdahl's theorem. The generalization of Buchdahl's bound was given with a positive cosmology constant Λ in [4–6]. The sharp bound on mass-radius ratio is obtained with deformed restrictions on the energy and pressure of the object [7,8]. In [9–12], the authors show similar bounds on the mass-radius ratio of charged gravitational objects, where the effect of charge $\mathcal{Q}$ is taken into account. For example, in the recent work [12], Andreasson concludes that the inequality of a charged object

$$\frac{2\mathcal{M}}{R} \leq \frac{4}{9} + \frac{2\mathcal{Q}^2}{3R^2} - \frac{2\Lambda R^2}{3} + \frac{4}{9}\sqrt{1 + \frac{3\mathcal{Q}^2}{R^2} + 3\Lambda R^2} \tag{1}$$

exist, if the condition $0 \leq \mathcal{Q}^2/R^2 + \Lambda R^2 \leq 1$ is satisfied. In the framework of Buchdahl approach or Andreasson approach, the mass-radius ratio bounds are obtained for the higher dimensional spheres [13–15]. In these works, it is obvious to see that the values of the mass-radius ratio bounds are significantly affected by the dimensions of spacetimes. In addition, the cases of modified gravity have also attracted many attentions of researchers, and the mass-radius ratio bounds were derived in Gauss-Bonnet gravity [16] and dRGT Massive Gravity theory [17].

Null geodesics are closely related to the characteristic modes of black holes, they play significant roles in reflecting the natures of compact objects. In literature [18,19], the authors concludes that for all spherically symmetric spacetimes, in a geometrical optics approximation, quasinormal modes in any dimension can be interpreted as particles trapped at unstable circular null geodesics and

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slowly leak out. The null circular photon orbits are generally believed to exist outside the horizonless regular objects [20,21]. It is argued that ultracompact object has at least two light rings (i.e., circular null geodesics) if they exist, and one of them is stable [22,23]. What's more, the nonlinear instabilities of ultracompact objects are developed due to the accumulation of the extremely long-lived linear fluctuations on the stable light rings [24–28]. It was suggested that the existence of a dynamically stable horizonless ultracompact object requires either a time scale of the nonlinear instability operation much longer than the Hubble time, or a large radius-mass ratio of the compact object [29–34].

The intriguing studies on the stability of the null geometric motion of compact object inspire researchers to revisit the bound of mass-radius ratio. In the recent study [35], Hod provides an upper bound on the mass-radius ratio by considering the requirement of the dynamically stable horizonless charged compact object. Hod's bound, of the form

$$\frac{2\mathcal{M}}{R} \le \frac{2}{3} + \frac{4\mathcal{Q}^2}{3R^2} \quad \text{for} \quad \frac{\mathcal{M}}{\mathcal{Q}} \ge \frac{8}{9},$$

is proved to be stronger than the degenerated case of Eq. (1), in which the cosmology constant is taken to be zero. On the other hand, it is remarkable that there are various studies on the light rings [36–38] and instabilities of the higher dimensional spacetimes [39–43]. In light of these fantastic results, in this paper, we contribute to this developing tale by extending the Hod scenario to higher dimensional horizonless charged compact objects. We are expected to obtain a dimensionally dependent mass-radius ratio bound of horizonless compact object.

The outline of this paper is as follows. In section 2, we begin with a brief description of the metric of higher dimensional horizonless compact object. In section 3, we analyze the null geodesic motion in higher dimensions, and the radius of the innermost light ring is given. We show that there is a dimensionally dependent mass-radius ratio bound for the dynamical stable horizonless compact object. We conclude our results with a summary in the section 4. Through out of this paper, the units $c = G = k_B = \hbar = 1$ are adopted.

2. Set up

The interior of the horizonless compact object is assumed to be composed of spherically symmetric isotropic charged matter, with energy density ρ, radial pressure P and the proper charge density J^0. We assume the matter satisfies the weak energy condition, i.e., $\rho > |P|$. The geometry we considered is governed by the n-dimensional Einstein-Maxwell field equation

$$G^{\mu}_{\ \nu} = \kappa_n [T_{(\mathrm{em})}{}^{\mu}{}_{\nu} + T_{(\mathrm{m})}{}^{\mu}{}_{\nu}], \tag{2}$$

$$\partial_\nu(\sqrt{-\mathrm{g}}F^{\mu\nu}) = \mu_0 \sqrt{-\mathrm{g}} J^{\mu}, \tag{3}$$

where $\kappa_n = 8\pi G$ and μ_0 are the Newtonian gravitational constant and vacuum permeability in n-dimensional spacetime, respectively. $T_{(\mathrm{em})}{}^{\mu}{}_{\nu}$ is the energy-momentum tensor of the Maxwell electromagnetic field[1]

$$T_{(\mathrm{em})}{}^{\mu\nu} = \frac{1}{\mu_0}[F^{\mu\alpha}F^{\nu}_{\ \alpha} - \frac{1}{4}\mathrm{g}^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}].$$

The energy-momentum tensor of the matter field $T_{(\mathrm{m})}{}^{\mu}{}_{\nu}$ takes the form of a perfect fluid

$$T_{(\mathrm{m})}{}^{\mu\nu} = (\rho + P)u^{\mu}u^{\nu} + P g^{\mu\nu},$$

where the normalized proper velocity satisfying $u^{\mu}u_{\mu} = -1$. Here $g^{\mu\nu}$ is the interior solution of the field equation for the horizonless compact object. The spherically symmetric line element is written as

$$\mathrm{d}s^2 = -e^{2\mu}\mathrm{d}t^2 + e^{2\lambda}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{n-2}, \tag{4}$$

where

$$\mathrm{d}\Omega^2_{n-2} = \mathrm{d}\theta_1^2 + \sum_{j=2}^{n-2}\prod_{i=1}^{j-1}\sin^2\theta_i \mathrm{d}\theta_j^2$$

is the line element of $(n-2)$-dimensional unit sphere. The functions μ and λ in the metric are radial dependent functions.

To solve the field equation, we define a quantity $q(r)$ as the charge within a $(n-2)$-dimensional sphere of radius r

$$q(r) = \int_0^r \mu_0 J^0 \sqrt{-\mathrm{g}}\mathrm{d}r.$$

Then we can obtain the solution of Maxwell equation, and the non-vanishing component of the spherically symmetric electromagnetic field is

$$F^{01} = \frac{1}{\sqrt{-\mathrm{g}}}q(r) = e^{-(\mu+\lambda)}\frac{q(r)}{r^{n-2}},$$

which leads to the non-vanishing components of energy momentum read as

$$T_{(\mathrm{em})}{}^{1}{}_{1} = T_{(\mathrm{em})}{}^{0}{}_{0} = -\frac{1}{2\mu_0}\frac{q^2(r)}{r^{2(n-2)}}.$$

Plugging all these conditions into the Einstein field equation (2), we promptly get the differential form for the functions μ and λ, they are

$$\mu' = -\frac{n-3}{2r}(1 - e^{2\lambda}) + \frac{\kappa_n r}{n-2}e^{2\lambda}\left(P - \frac{q^2(r)}{2\mu_0 r^{2(n-2)}}\right), \tag{5}$$

$$\lambda' = \frac{n-3}{2r}(1 - e^{2\lambda}) + \frac{\kappa_n r}{n-2}e^{2\lambda}\left(\rho + \frac{q^2(r)}{2\mu_0 r^{2(n-2)}}\right), \tag{6}$$

where the prime represents the partial derivative with respect to the radial coordinate r.

The asymptotic flatness requires the asymptotic behaviors of the functions μ and λ near the far-region as

$$\mu(r \to \infty) \longrightarrow 0, \qquad \lambda(r \to \infty) \longrightarrow 0, \tag{7}$$

and the regularity of the spacetime requires

$$\mu(r \to 0) > -\infty, \qquad \lambda(r \to 0) \longrightarrow 0, \tag{8}$$

at $r \to 0$. We integrate the differential equation (6), and the specific form of $e^{-2\lambda}$ inside the compact object arrives at

$$e^{-2\lambda}_{(\mathrm{in})} = 1 - \frac{2\kappa_n m(r)}{(n-2)\mathcal{A}_{n-2}r^{n-3}} - \frac{\kappa_n f(r)}{(n-2)r^{n-3}\mu_0}, \tag{9}$$

where the definition of $m(r)$ and $f(r)$ are

$$m(r) = \mathcal{A}_{n-2}\int_0^r \rho x^{n-2}\mathrm{d}x,$$

$$f(r) = \int_0^r q^2(x)x^{n-4}\mathrm{d}x.$$

[1] The vacuum permeability is usually chosen as $\mu_0 = 1/\varepsilon_0 = 4\pi$ in four dimensions and $\mu_0 = 1/\varepsilon_0 = \mathcal{A}_{n-2}$ in n dimensions, where $\mathcal{A}_{n-2} = 2\pi^{\frac{n-1}{2}}/\Gamma(\frac{n-1}{2})$.

A finite mass $m(r)$ implies that the density ρ shall approach to zero faster than the scale r^{n-1} at $r \to \infty$. Subsequently, the weak energy condition $\rho > |P|$ implies

$$r^{n-1} P \to 0 \quad \text{as} \quad r \to \infty.$$

The metric functions of the well known Tangherlini Reissner-Nordström black hole [44] (i.e., n-dimensional Reissner-Nordström black hole) are

$$e^{2\mu}_{(\text{out})} = e^{-2\lambda}_{(\text{out})} = 1 - \frac{2\kappa_n \mathcal{M}}{(n-2)\mathcal{A}_{n-2} r^{n-3}} + \frac{\kappa_n \mathcal{Q}^2}{(n-2)(n-3)\mathcal{A}^2_{n-2}\varepsilon_0 r^{2(n-3)}},$$

where $\mathcal{M}$ and $\mathcal{Q}$ are the ADM mass and the total charge of the compact object, respectively. And $\mathcal{A}_{n-2}$ is the area of the $(n-2)$-dimensional unit sphere,

$$\mathcal{A}_{n-2} = 2\pi^{\frac{n-1}{2}} / \Gamma(\frac{n-1}{2}).$$

The interior metric should match well with the vacuum solution at the boundary of the compact object, that is to say

$$e^{-2\lambda}_{(\text{out})}|_R = e^{-2\lambda}_{(\text{in})}|_R.$$

We define the gravitational mass within a sphere of radius r to be

$$m_g(r) = m(r) + \frac{\mathcal{A}_{n-2} f(r)}{2\mu_0} + \frac{q^2(r)}{2(n-3)\mathcal{A}_{n-2}\varepsilon_0 r^{n-3}}.$$

Consequently, the form of the interior solution Eq. (9) turns out to be

$$e^{-2\lambda}_{(\text{in})} = 1 - \frac{2\kappa_n m_g(r)}{(n-2)\mathcal{A}_{n-2} r^{n-3}} + \frac{\kappa_n q^2(r)}{(n-2)(n-3)\mathcal{A}^2_{n-2}\varepsilon_0 r^{2(n-3)}}.$$

It is obvious that the gravitational mass and charge functions evaluating at the boundary of the compact object are

$$m_g(R) = \mathcal{M}, \qquad q(R) = \mathcal{Q},$$

which are exactly the ADM mass and the total charge of the compact object. The gravitational mass $m_g(r)$ and function $m(r)$ coincide with those of the situations in the absence of the electromagnetic charge.

3. Upper bound on the mass-radius ratio

In this section, we shall compute the radius of the innermost light rings in higher dimensions. Let us restrict attention to equatorial orbits, namely, $\theta_1 = \theta_2 = \cdots = \theta_{n-3} = \pi/2$, and we denote the angular coordinate $\theta_{n-2} = \varphi$. The Lagrangian of the null circular geodesic is

$$2\mathcal{L} = g_{tt}\dot{t}^2 + g_{rr}\dot{r}^2 + g_{\varphi\varphi}\dot{\varphi}^2, \tag{10}$$

where a dot denotes a derivative with respect to certain affine parameter along the geodesic. According to Eq. (10), the generalized momenta corresponding to the generalized coordinates are

$$P_t = g_{tt}\dot{t} = -E, \quad P_r = g_{rr}\dot{r}, \quad P_\varphi = g_{\varphi\varphi}\dot{\varphi} = L.$$

There are two cyclic coordinates t and φ in the metric (4), corresponding to the stationarity ∂_t and commuting azimuthal ∂_φ Killing vectors, which yield two conserved quantities: the phonon's energy E and the angular momentum L at spatial infinity.

The Hamiltonian of the null circular geodesic is given by

$$2\mathcal{H} = 2\left(P_t\dot{t} + P_r\dot{r} + P_\varphi\dot{\varphi} - \mathcal{L}\right) = \epsilon,$$

where the constant $\epsilon = 1$ for timelike geodesic and $\epsilon = 0$ for null geodesic. The Hamiltonian can be divided into a potential energy ($V(r) \le 0$) part and a kinetic energy ($K \ge 0$) part,

$$V(r) = -E\dot{t} + L\dot{\varphi}, \qquad K = P_r\dot{r}.$$

A null geodesic implies $\mathcal{H} = 0$ and the following two conditions exist

$$V(r) = V'(r) = 0,$$

for the light rings [18,22]. So we get two equations to determine the radius of the light rings, they are

$$g_{rr}\dot{r}^2 = -V(r) = 0, \qquad (g_{rr}\dot{r}^2)' = 0. \tag{11}$$

Substituting Eq. (5) and Eq. (6) into equation Eq. (11), we get the explicit conditions

$$\dot{r}^2 = e^{-2\lambda}(E^2 e^{-2\mu} - \frac{L^2}{r^2}) = 0, \tag{12}$$

and

$$(\dot{r}^2)' = \frac{e^{-2\mu}E^2}{r}\Big[(n-1)e^{-2\lambda} - (n-3) - \frac{2\kappa_n r^2}{n-2}\left(P - \frac{q^2(r)}{2\mu_0 r^{2(n-2)}}\right)\Big] = 0. \tag{13}$$

Before we proceed with the computations of the radius of light rings, it is necessary to introduce a characteristic function

$$\chi(r) = (n-1)e^{-2\lambda} - (n-3) - \frac{2\kappa_n r^2}{n-2}\left(P - \frac{q^2(r)}{2\mu_0 r^{2(n-2)}}\right). \tag{14}$$

For the null circle geodesic, $\chi(r = r_\gamma) = 0$ is the characteristic equation, where r_γ is the radius of light ring. Combine the Eq. (7) and Eq. (8) together with weak energy condition for the matter source, we know that the asymptotic behaviors of $\chi(r)$ are

$$\chi(r = 0) \to 2, \qquad \chi(r \to \infty) \to 2. \tag{15}$$

Additionally, we know that the stability/instability of a null circle geodesic is determined by the second derivative of the potential energy. When $V(r)'' < 0$, the null circle geodesic is unstable, while $V(r)'' > 0$, the corresponding null circle geodesic is stable. Stability of circular null geodesics plays a key role in its applications to astrophysics. It is suggested that the quasinormal modes are determined by the orbital frequency and instability time scale of the unstable null geodesic in any stationary, spherically symmetric and asymptotically flat higher-dimensional spacetime [18,19], so that the unstable photon sphere is responsible for determining the size of a black hole shadow. In the literatures [24,25], the existence of stable light rings in higher dimensions is discussed. While a stable light ring is inferred to generate nonlinear instabilities due to the trapping of time-dependent massless perturbation fields on the circular null geodesics [26–28], which in turn lead to the dynamic instability of the spatially regular compact objects. And also the stable trapping of null geodesics occurs in higher dimensions [45,46].

We can deduce from the Eq. (13) that

$$V''(r = r_\gamma) = -\frac{e^{-2\mu}E^2}{r_\gamma^2}\chi'(r = r_\gamma).$$

Thus the conditions for the existence of a stable null circle geodesic of the horizonless compact object read as

$$\chi(r=r_\gamma)=0, \qquad \chi'(r=r_\gamma)<0.$$

With the asymptotic behaviors Eq. (15) of function $\chi(r)$, we know that the characteristic equation $\chi(r)=0$ has even number of solutions, the light rings always come in pairs. The innermost light ring of the pair is stable and the another one of the pair would be unstable.

The exterior spacetime of the charged compact object is vacuum with vanishing matter density and pressure $\rho=P=0$, and the metric functions are described by the Tangherlini Reissner-Nordström solution

$$e^{2\mu}_{(\mathrm{out})}=e^{-2\lambda}_{(\mathrm{out})}$$
$$=1-\frac{2\kappa_n\mathcal{M}}{(n-2)\mathcal{A}_{n-2}r^{n-3}}+\frac{\kappa_n\mathcal{Q}^2}{(n-2)(n-3)\mathcal{A}_{n-2}^2\varepsilon_0 r^{2(n-3)}}.$$

The characteristic equation of the null circle geodesic in the exterior turns out to be

$$\chi(r_\gamma)=2-\frac{2(n-1)\kappa_n\mathcal{M}}{(n-2)\mathcal{A}_{n-2}r_\gamma^{n-3}}+\frac{\left((n-1)\mu_0+(n-3)\varepsilon_0\mathcal{A}_{n-2}^2\right)\kappa_n\mathcal{Q}^2}{(n-2)(n-3)\mathcal{A}_{n-2}^2 r_\gamma^{2(n-3)}}=0. \qquad (16)$$

In the case of that[2]

$$\frac{\mathcal{M}^2}{\mathcal{Q}^2}\geq\frac{2(n-2)[(n-1)\mu_0+(n-3)\varepsilon_0\mathcal{A}_{n-2}^2]}{(n-1)^2(n-3)\kappa_n}=\frac{4(n-2)^2\mathcal{A}_{n-2}}{(n-1)^2(n-3)\kappa_n},$$

the solutions of Eq. (16) are

$$r_{\pm}^{n-3}=\frac{(n-1)\kappa_n\mathcal{M}}{2(n-2)\mathcal{A}_{n-2}}\pm\frac{1}{2}\sqrt{\frac{(n-1)^2\kappa_n^2\mathcal{M}^2}{(n-2)^2\mathcal{A}_{n-2}^2}-\frac{4\kappa_n\mathcal{Q}^2}{(n-3)\mathcal{A}_{n-2}}}.$$

Remarkably, the inequality implies the relation

$$\frac{r_+^{n-3}}{\mathcal{Q}}\geq\frac{(n-1)\kappa_n\mathcal{M}}{2(n-2)\mathcal{A}_{n-2}\mathcal{Q}}\geq\sqrt{\frac{\kappa_n}{(n-3)\mathcal{A}_{n-2}}}. \qquad (17)$$

As we have mentioned before, the dynamically stable horizonless ultracompact object shall exclude the existence of stable light rings. If the radius of the charged compact object is larger than the radius of the outermost light ring, there won't be any null stable circle geodesic to develop nonlinearity instabilities. That is, the dynamically stable horizonless ultracompact object indicates the absence of light ring, which requires the condition

$$R^{n-3}\geq r_+^{n-3}. \qquad (18)$$

Therefore, the above relation implies that there is an upper bound on the gravitational mass of self-gravitation horizonless charged compact object in higher dimensional spacetime, which shows

$$\mathcal{M}\leq\frac{(n-2)\mathcal{A}_{n-2}}{(n-1)\kappa_n}R^{n-3}+\frac{(n-2)\mathcal{Q}^2}{(n-1)(n-3)R^{n-3}}.$$

[2] Here and after, we adopt the units $\mu_0=1/\varepsilon_0=\mathcal{A}_{n-2}$ in n dimensions.

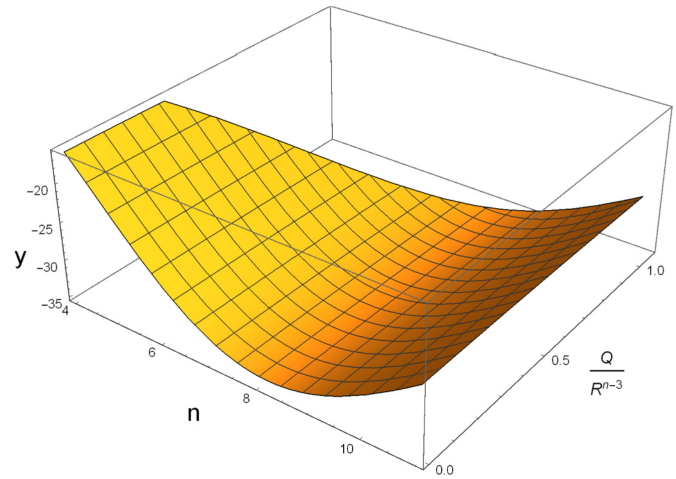

Fig. 1. Comparison between the bounds Eq. (19) and Eq. (20), in regime of $\mathcal{Q}/R^{n-3}$ determined by Eq. (17). y represent the difference between the right side of these two bounds, which show us a negative value with the dimension greater than four.

Naturally, a dimensionless mass-radius ratio $\kappa_n\mathcal{M}/R^{n-3}$ should be the one we concerned in n-dimensions, deducing from the above mass bound, we have

$$\frac{\kappa_n\mathcal{M}}{R^{n-3}}\leq\frac{(n-2)\mathcal{A}_{n-2}}{(n-1)}+\frac{(n-2)\kappa_n\mathcal{Q}^2}{(n-1)(n-3)R^{2(n-3)}}. \qquad (19)$$

The result shows a strong dependence on the dimensions of spacetime. The mass-radius ratio bound is significantly affected by the dimensions of spacetime. In the literature [14], the mass-radius ratio bound of compact object in higher dimensions has also been derived following Andreasson's approach, it suggested that

$$\frac{\kappa_n\mathcal{M}}{R^{n-3}}\leq\frac{(n-2)^2\mathcal{A}_{n-2}}{(n-1)^2}+\frac{\kappa_n\mathcal{Q}^2}{(n-1)(n-3)R^{2(n-3)}}+\frac{(n-1)\mathcal{A}_{n-2}}{n-2}\sqrt{1+\frac{(n-1)\kappa_n\mathcal{Q}^2}{(n-2)\mathcal{A}_{n-2}R^{2(n-3)}}}. \qquad (20)$$

To compare the above two bounds, we take the subtraction between the right hand of the inequality Eq. (19) and the right hand of the inequality Eq. (20), denoting the result as y. We numerically compute the value of y, and the result is plotted in Fig. 1. From the picture it can be seen that as the dimension increase, the difference is always negative in the regime Eq. (17). The right hand side of Eq. (19) is always smaller, so we conclude that the novel bound in the present paper is stronger for all higher dimensions in the regime Eq. (17). The analytical comparison between these two bounds is given in the Appendix A in detail, which support our results.

Meanwhile, when the dimension equal to four, the constants take the values of

$$\mathcal{A}_2=4\pi, \quad \kappa_4=8\pi, \quad \mu_0=\frac{1}{\varepsilon_0}=4\pi,$$

and our bound reduce to

$$\mathcal{M}\leq\frac{R}{3}+\frac{2\mathcal{Q}^2}{3R} \quad \text{for} \quad \frac{\mathcal{M}}{\mathcal{Q}}\geq\sqrt{\frac{8}{9}},$$

which is consistent with the results in [35].

4. Conclusion and discussion

In this paper, we have studied the mass-radius ratio bound for horizonless compact object in higher dimensions. In recent years, particular attentions of researchers have been devoted to the stability/instability of the compact objects. It is stated that the circular null geodesics would come in pairs if they exist, and one of the pair is stable. Moreover, the spacetime with a stable light ring will develop nonlinear instabilities responding to the time-dependent massless perturbations in any dimensions. The intriguing results inspire us to investigate the conditions for the dynamically stable compact objects. In the paper [35], Hod suggested that the radius of the compact objects must larger than the radius of the outermost light rings, which ensures the absence of any stable light ring. Thus the horizonless compact objects would be dynamically stable.

Instead of considering the mass-radius ratio bound in four dimensions, we have generalized the issue into higher dimensions following Hod's approach. Intuitively, the mass-radius ratio we considered in higher dimensions should be the dimensionless parameter $\kappa_n\mathcal{M}/R^{n-3}$. We have analytically derived the upper bound on the mass-radius ratio of stable charged compact objects in n-dimensions, which gives

$$\frac{\kappa_n\mathcal{M}}{R^{n-3}} \leq \frac{(n-2)\mathcal{A}_{n-2}}{(n-1)} + \frac{(n-2)\kappa_n\mathcal{Q}^2}{(n-1)(n-3)R^{2(n-3)}},$$

on the condition of that

$$\frac{\mathcal{M}^2}{\mathcal{Q}^2} \geq \frac{4(n-2)^2\mathcal{A}_{n-2}}{(n-1)^2(n-3)\kappa_n}.$$

The above inequality shows that the dependency of the mass-radius ratio bound on the dimensions of the spacetime is complex. Compare to the previous works on the bound following Andreasson's approach, our results are proven to be stronger and better always in higher dimensions. When the dimension is reduced to four, the bound reduce to the results given in [35].

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Appendix A. Comparison between the bounds

The purpose of this Appendix is to provide a analytical proof that in the regime where the inequality Eq. (17) satisfied, our novel bound on the mass-radius ratio derived in the present paper is stronger than the bound given in the literature [14] in all higher dimensions.

To compare the bounds, we take the subtraction between the right hand of the inequality Eq. (19) and the right hand of the inequality Eq. (20), the result is denoted by symbol y and it shows

$$y = \frac{(n-2)\mathcal{A}_{n-2}}{(n-1)^2}\left(1+\frac{n-1}{n-2}\frac{\kappa_n\mathcal{Q}^2}{\mathcal{A}_{n-2}R^{2n-6}}\right) - \frac{(n-1)\mathcal{A}_{n-2}}{n-2}\sqrt{1+\frac{n-1}{n-2}\frac{\kappa_n\mathcal{Q}^2}{\mathcal{A}_{n-2}R^{2n-6}}}. \qquad \text{(A.1)}$$

The sign of y is crucial for us to determine which bound is stronger. A positive/negative y means that the right hand of inequality Eq. (19) is larger/smaller than the right hand of inequality Eq. (20). If the sign of y is always negative in all higher dimensions, we can declare that our results is stronger and better than others. For convenience, let us introduce a new parameter x defined as $x = \mathcal{Q}/R^{n-3}$, combining with the relations Eq. (17) and Eq. (18), the regime of x is

$$x \in \left\{0, \sqrt{\frac{(n-3)\mathcal{A}_{n-2}}{\kappa_n}}\right\}.$$

Subsequently, after inserting the definition of parameter x into Eq. (A.1), we take the derivation of the function y with respect to x, we arrive at

$$\frac{dy}{dx} = \kappa_n x\left(\frac{2}{n-1} - \frac{(n-1)^2}{(n-2)^2}\frac{1}{\sqrt{1+\frac{(n-1)\kappa_n}{(n-2)\mathcal{A}_{n-2}}x^2}}\right).$$

In the regime of x, we have

$$\begin{aligned}\frac{dy}{dx} &\leq \kappa_n x\left(\frac{2}{n-1} - \frac{(n-1)^2}{(n-2)^2}\frac{1}{\sqrt{1+\frac{(n-1)(n-3)}{(n-2)}}}\right)\\ &< \kappa_n x\left(\frac{2}{n-1} - \frac{(n-1)^2}{(n-2)^2}\frac{1}{\sqrt{n}}\right)\\ &< \kappa_n x\left(\frac{2}{n-1} - \frac{1}{\sqrt{n}}\right) < 0, \qquad \text{for} \quad n \geq 4.\end{aligned}$$

Hence the function y is monotonically decreasing, its maximum evaluate at $x = 0$, which gives

$$y_{\max} = y(x=0) = \frac{(n-2)^2-(n-1)^3}{(n-1)^2(n-2)}\mathcal{A}_{n-2} < 0, \qquad \text{for} \quad n \geq 4$$

Thus, we have proven that our bound is always stronger than the bound given by Eq. (20) in all higher dimensions.

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