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Black holes in Melvin universe

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Abstract A technique was developed by F. J. Ernst in 1976 for immersing black hole spacetimes in Melvin's magnetic universe. This method for magnetizing lack holes uses Harrison's transformations. In this paper we review the earlier work done on magnetizing the Schwarzschild, Reissner-Nordström and Kerr-Newman black holes.

1 Introduction

In 1952 W. B. Bonner gave the solution for empty space having cylindrical symmetry containing electromagnetic field and discussed its physical interpretation [1,2]. This was subsequently rediscovered by M. A. Melvin [3]. It is now usually referred to as the "Melvin universe". The metric that describes Melvin universe is [3, 4]

$$ds^2 = (1 + \tfrac{1}{4}B^2\rho^2)^2(-dt^2 + d\rho^2 + dz^2) + (1 + \frac{1}{4}B^2\rho^2)^{-2}\rho^2 d\phi^2, \quad (1)$$

with $t,\ z \in (-\infty, +\infty)$, $\rho \in [0, \infty)$, $\phi \in [0, 2\pi)$. The electromagnetic field can be described by the Maxwell tensor

$$F = e^{-i\psi}B(dz \wedge dt), \quad (2)$$

where ψ is a real parameter of duality rotation. In particular, for $\psi = 0$, the Maxwell tensor is $F = Bdz \wedge dt$ which describes an electric field pointing along the z-direction, whereas for $\psi = \pi/2$ one obtains $F = B(1 + 1/4B^2\rho^2)^{-2}\rho d\rho \wedge d\phi$, which represents a purely magnetic field oriented along the z-direction. It is a spacetime which is static cylindrically symmetric and in which there exists an axial magnetic and/or electric field aligned with the z-axis, and the magnitude of the field is determined by the parameter B. This solution represents a universe which contains a parallel bundle of electric or magnetic flux held together by its own gravitational pull. Further, for $B = 0$, the metric is Minkowski metric in cylindrical coordinates. If $B \neq 0$, the metric is not asymptotically flat because $1 + 1/4B^2\rho^2$ does not go to 1 at any z.

The above Melvin magnetic solution has been considered as a useful model in, among others, the studies of astrophysical processes, quantum black hole pair creation and gravitational collapse. Its importance derives also from the fact that it appears as a limit in more complicated solutions and is therefore considered as a background for a number of interesting solutions. It was shown already by Melvin [5] and Thorne [6] that the spacetime is, somewhat surprisingly, stable against small radial perturbations, as well as arbitrarily large perturbations which are confined to a finite region about the axis of symmetry.

In 1976, F. J. Ernst using Harrison's transformation [7] presented a procedure for transforming asymptotically flat axially symmetric solutions of the coupled Einstein-Maxwell equations into solutions resembling Melvin's magnetic universe [8,9]. He used this technique for the removal of the nodal singularity of the C-metric [10], and studied the Schwarzschild, Reissner-Nordström and Kerr-Newman black holes in Melvin universe [8,9]. Recently, Ernst's solution generating technique is used by M. Astorino for embedding hairy black holes in Melvin universe [11] and by G. W. Gibbons et. al. for Kerr-Newman black holes [12]. In this paper we present a review of this work and discuss some examples that illustrate how Ernst used Harrison's transformation [7] to generate some electrovac solutions, which

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are of physical interest. In Section 2 we shall discuss how Ernst obtained Melvin universe using Harrison's transformation. Sections 3 and 4 deal with Schwarzschild and Reissner-Nordström black holes in Melvin universe. In Section 5 we review the work [12] done on Kerr-Newman solution having electric as well as magnetic charge in Melvin universe. A brief conclusion is given at the end.

2 Melvin universe by Ernst's technique

The line element of Minkowski space in cylindrical coordination is given by

$$ds^2 = -dt^2 + dz^2 + d\rho^2 + \rho^2 d\phi^2, \quad (3)$$

and the general form of stationary axial symmetric line element can be written as [8]

$$ds^2 = f^{-1}[-2P^{-2}d\xi d\xi^* + \rho^2 dt^2] - f(d\phi - \omega dt)^2. \quad (4)$$

On comparing Eqs. (3) and (4) we have

$$f = -\rho^2, \quad \omega = 0, \quad P = \rho^{-1}, \quad d\xi = \frac{(dz + id\rho)}{\sqrt{2}}. \quad (5)$$

The complex gravitational potential ε associated with gravity is defined by

$$\varepsilon = f - |\Phi|^2 + i\varphi, \quad (6)$$

where Φ is the complex electromagnetic potential, whose real and imaginary parts are electrostatic and magnetostatic potentials, respectively and $|\Phi|$ is the magnitude of complex potential. If E_r, E_θ, H_r and H_θ are the radial and angular components of electric and magnetic fields, the complex electromagnetic potential may be evaluated by

$$H_r + iE_r = P\frac{\partial \Phi}{\partial \theta}, \quad H_\theta + iE_\theta = -P\frac{\partial \Phi}{\partial r}. \quad (7)$$

In Eq. (6) φ is the twist potential. If one defines the symbol

$$\nabla = r\frac{\partial}{\partial r} + i\frac{\partial}{\partial \theta}, \quad (8)$$

the twist potential may be determined by equation

$$-\rho^{-1}f^2\nabla\omega = i\nabla\varphi + \Phi^*\nabla\Phi - \Phi\nabla\Phi^*. \quad (9)$$

Here Φ^* is the conjugate of Φ. Since initially there is no electromagnetic field, so complex gravitational, electromagnetic and twist potentials are given by

$$\varepsilon = f = -\rho^2, \Phi = 0, \quad \varphi = 0. \quad (10)$$

Now by Harrison's transformations new functions are defined as [8]

$$\Lambda = 1 + B\Phi - \tfrac{1}{4}B^2\varepsilon, \quad (11)$$

$$\varepsilon' = \Lambda^{-1}\varepsilon, \quad (12)$$

$$\Phi' = \Lambda^{-1}(\Phi - \tfrac{1}{2}B\varepsilon), \quad (13)$$

and under this transformation functions, f and ω are transformed into new functions f' and ω' as

$$f' = Re\varepsilon' + |\Phi'|^2 = |\Lambda|^{-2} f, \quad (14)$$

$$\nabla\omega' = |\Lambda|^2 \nabla\omega + \rho f^{-1}(\Lambda^*\nabla\Lambda - \Lambda\nabla\Lambda^*), \quad (15)$$

where the operator ∇ is different for different cases, while the function P and ρ are unmodified. From Eqs. (11) - (13)

$$\Lambda = 1 + \frac{1}{4}\rho^2 B^2, \quad (16)$$

$$\varepsilon' = \Lambda^{-1}\varepsilon = -\frac{\rho^2}{1 + \frac{1}{4}\rho^2 B^2}, \quad (17)$$

$$f' = |\Lambda|^{-2} f = -\frac{\rho^2}{(1 + \frac{1}{4}\rho^2 B^2)^2}. \quad (18)$$

As $\Lambda = 1 + \frac{1}{4}\rho^2 B^2$ is real so $\Lambda^*\nabla\Lambda - \Lambda\nabla\Lambda^* = 0$, and from Eq. (5) $\omega = 0$, so $\omega' = 0$. Using the new functions f' and ω' in Eq. (4) the transformed line element is [8]

$$ds^2 = (1 + \frac{1}{4}B^2\rho^2)^2[-dt^2 + dz^2 + d\rho^2] + (1 + \frac{1}{4}B^2\rho^2)^{-2}\rho^2 d\phi^2, \quad (19)$$

with the electromagnetic potential

$$\Phi' = \frac{1}{2}\Lambda^{-1}B\rho^2 = \frac{1}{2}\frac{B\rho^2}{(1 + \frac{1}{4}\rho^2 B^2)}. \quad (20)$$

From the above equation the components of magnetic field are

$$H_z = \Lambda^{-2}B, \quad H_\rho = 0 = H_\phi. \quad (21)$$

This solution is same as Melvin's magnetic universe (1).

3 Schwarzschild black hole in Melvin universe

The Harrison transformation can be used to magnetize the Schwarzschild black hole. The metric of the Schwarzschild black hole is given by

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \quad (22)$$

or

$$ds^2 = -\frac{1}{r^2}(r^2 - 2Mr)dt^2 + \frac{r^2 dr^2}{(r^2 - 2Mr)} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2. \quad (23)$$

Comparing the above equation with Eq.(4) we have

$$f=-r^2\sin^2\theta,\quad \omega=0,\quad \rho=(r^2-2Mr)^{1/2}\sin\theta, \tag{24}$$

$$P=(r^2\sin\theta)^{-1},\quad d\xi=\frac{1}{\sqrt{2}}\left(\frac{dr}{(r^2-2Mr)^{1/2}}+id\theta\right). \tag{25}$$

The radial and angular components of electric and magnetic fields satisfy the equation

$$H_r+iE_r=P\frac{\partial\Phi}{\partial\theta},\quad H_\theta+iE_\theta=-P(r^2-2Mr)^{1/2}\frac{\partial\Phi}{\partial r}. \tag{26}$$

If one defines the symbol

$$\nabla=(r^2-2Mr)^{1/2}\frac{\partial}{\partial r}+i\frac{\partial}{\partial\theta}, \tag{27}$$

the twist potential may be determined by

$$-\rho^{-1}f^2\nabla\omega=i\nabla\varphi+\Phi^*\nabla\Phi-\Phi\nabla\Phi^*. \tag{28}$$

Since initially there is no electromagnetic field, also $\omega=0$, so the complex gravitational, electromagnetic and twist potentials are given by

$$\varepsilon=f=-r^2\sin^2\theta,\quad \Phi=0,\quad \varphi=0, \tag{29}$$

while Eqs. (11)-(13) yield

$$\Lambda=1+B\Phi-\frac{1}{4}B^2\varepsilon=1+\frac{1}{4}B^2r^2\sin^2\theta, \tag{30}$$

$$\varepsilon'=\Lambda^{-1}\varepsilon=-(1+\frac{1}{4}B^2r^2\sin^2\theta)^{-1}r^2\sin^2\theta, \tag{31}$$

$$\begin{aligned}\Phi'&=\Lambda^{-1}(\Phi-\frac{1}{2}B\varepsilon)\\&=\frac{1}{2}(1+\frac{1}{4}B^2r^2\sin^2\theta)^{-1}Br^2\sin^2\theta.\end{aligned} \tag{32}$$

The transformed fields f' and ω' from Eqs. (14) and (15) are

$$f'=|\Lambda|^{-2}f=-\frac{r^2\sin^2\theta}{(1+\frac{1}{4}B^2r^2\sin^2\theta)^2},\quad \omega'=0. \tag{33}$$

Using f' from ω' from the above equations and unmodified functions P and ρ from Eq. (4) we obtain the transformed line element [8]

$$\begin{aligned}ds^2=&|\Lambda|^2\left[-(1-\frac{2M}{r})dt^2+\frac{dr^2}{(1-\frac{2M}{r})}\right.\\&\left.+r^2d\theta^2\right]+|\Lambda|^{-2}r^2\sin^2\theta d\phi^2.\end{aligned} \tag{34}$$

In this case the magnetic field components are given by

$$H_r=\Lambda^{-2}B\cos\theta, \tag{35}$$

$$H_\theta=-\Lambda^{-2}B(1-\frac{2M}{r})^{1/2}\sin\theta. \tag{36}$$

Note that if $M=0$ the above metric becomes Melvin's magnetic universe, while for $M\neq 0$ there is an event horizon at $r=2M$ and the angular component of magnetic field vanishes at the event horizon. Further, the metric has singularity at $r=0$, as in the case of Schwarzschild metric. If we take $B=0$, this reduces to the Schwarzschild solution.

4 Reissner-Nordström black hole in Melvin universe

The application of the procedure to the Reissner-Nordström black hole is not so simple. In this case $\mathbf{E}\times\mathbf{H}$ serves as a source for twist potential, and the transformed metric is not static as in the case of the Schwarzschild black hole, but stationary. The spacetime of Reissner-Nordstr öm black hole is

$$\begin{aligned}ds^2=&-(1-\frac{2M}{r}+\frac{q^2}{r^2})dt^2+(1-\frac{2M}{r}+\frac{q^2}{r^2})^{-1}dr^2\\&+r^2d\theta^2+r^2\sin^2\theta d\phi^2,\end{aligned} \tag{37}$$

or, we can write

$$\begin{aligned}ds^2=&-\frac{1}{r^2}(r^2-2Mr+q^2)dt^2+r^2(r^2-2Mr+q^2)^{-1}dr^2\\&+r^2d\theta^2+r^2\sin^2\theta d\phi^2.\end{aligned} \tag{38}$$

Comparing Eq. (38) with Eq. (4) we note that

$$f=-r^2\sin^2\theta,\ \omega=0,\ \rho=(r^2-2Mr+q^2)^{1/2}\sin\theta, \tag{39}$$

$$P=(r^2\sin\theta)^{-1},\ d\xi=\frac{1}{\sqrt{2}}\left(\frac{dr}{(r^2-2Mr+q^2)^{1/2}}+id\theta\right). \tag{40}$$

The complex electromagnetic potential Φ, whose real and imaginary parts are electrostatic and magnetostatic potentials, respectively, may be evaluated by the equations

$$H_r+iE_r=P\frac{\partial\Phi}{\partial\theta}, \tag{41}$$

$$H_\theta+iE_\theta=-P(r^2-2Mr+q^2)^{1/2}\frac{\partial\Phi}{\partial r}. \tag{42}$$

Solving the above equation we have $\Phi=-\iota q\cos\theta$. If one defines the symbol

$$\nabla=(r^2-2Mr+q^2)^{1/2}\frac{\partial}{\partial r}+i\frac{\partial}{\partial\theta}, \tag{43}$$

the twist potential φ can be determined from

$$-\rho^{-1}f^2\nabla\omega=i\nabla\varphi+\Phi^*\nabla\Phi-\Phi\nabla\Phi^*. \tag{44}$$

Since Φ is pure imaginary so $\Phi^*\nabla\Phi-\Phi\nabla\Phi^*=0$, also $\omega=0$, so the twist potential is also equal to zero i.e. $\varphi=0$. The complex gravitational potential ε associated with gravity is given by

$$\varepsilon=f-|\Phi|^2+i\varphi=-r^2\sin^2\theta-q^2\cos^2\theta, \tag{45}$$

while Eqs. (11) - (14) take the form

$$\begin{aligned}\Lambda&=1+B\Phi-\frac{1}{4}B^2\varepsilon\\&=1+\frac{1}{4}B^2(r^2\sin^2\theta+q^2\cos^2\theta)-iBq\cos\theta,\end{aligned} \tag{46}$$

$$\varepsilon' = \Lambda^{-1}\varepsilon$$
$$= -\left(\frac{r^2\sin^2\theta + q^2\cos^2\theta}{1+\frac{1}{4}B^2(r^2\sin^2\theta + q^2\cos^2\theta) - iBq\cos\theta}\right), \quad (47)$$
$$f' = |\Lambda|^{-2} f$$
$$= -\frac{r^2\sin^2\theta}{(1+\frac{1}{4}B^2(r^2\sin^2\theta+q^2\cos^2\theta))^2 + (Bq\cos\theta)^2}, \quad (48)$$

and ω' can be evaluated by the equation

$$\nabla\omega' = \rho f^{-1}(\Lambda^*\nabla\Lambda - \Lambda\nabla\Lambda^*). \quad (49)$$

Integrating Eq. (49) yields the following expression for ω'

$$\omega' = -2Bqr^{-1} + B^3qr + \frac{1}{2}B^3q^3r^{-1} - \frac{1}{2}B^3qr^{-1} \times (r^2 - 2Mr + q^2)\sin^2\theta + const. \quad (50)$$

Consequently, the transformed metric takes the form [8]

$$ds^2 = |\Lambda|^2\left[-(1-\frac{2M}{r}+\frac{q^2}{r^2})dt^2 + (1-\frac{2M}{r}+\frac{q^2}{r^2})^{-1}dr^2 + r^2d\theta^2\right] + |\Lambda|^{-2}r^2\sin^2\theta(d\phi - \omega' dt)^2, \quad (51)$$

where Λ and ω' are given by Eqs. (46) and (50). The above metric is known as the Reissner-Nordström black hole in Melvin universe which is the charged generalization of Schwarzschild black hole in Melvin universe. If $q = 0$ then this metric reduces to the Schwarzschild black hole in Melvin universe, and if $B = 0$, then the above metric reduces to Reissner-Nordström black hole.

Finally, the components of the electric and magnetic fields may be evaluated from electromagnetic potential Φ'. The results are

$$H_r + iE_r = \Lambda^{-2}[i(\frac{q}{r^2})\{1 - \frac{1}{4}B^2(r^2\sin^2\theta + q^2\cos^2\theta)\} + B(1 - \frac{1}{2}iBq\cos\theta)(1 - \frac{q^2}{r^2})\cos\theta], \quad (52)$$

$$H_\theta + iE_\theta = -B|\Lambda|^2(1 - \frac{1}{2}iq^2\cos\theta) \times (1 - \frac{2M}{r} + \frac{q^2}{r^2})^{1/2}\sin\theta. \quad (53)$$

5 Kerr-Newman black hole in Melvin universe

The spacetime describing magnetized Kerr-Newman black hole of mass M, angular momentum per unit mass a, carrying electric charge q and magnetic charge p, embedded in a Melvin's universe of magnetic field B is [12]

$$ds^2 = H[-fdt^2 + R^2(\frac{dr^2}{\Delta} + d\theta^2)] + \frac{\Sigma\sin^2\theta}{HR^2}(d\phi - \omega dt)^2, \quad (54)$$

where

$$R^2 = r^2 + a^2\cos^2\theta, \quad (55)$$
$$\Delta = (r^2 + a^2) - 2Mr + q^2 + p^2, \quad (56)$$
$$\Sigma = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta, \quad (57)$$
$$f = \frac{R^2\Delta}{\Sigma}, \quad (58)$$
$$H = 1 + \frac{H_{(1)}B + H_{(2)}B^2 + H_{(3)}B^3 + H_{(4)}B^4}{R^2}, \quad (59)$$

with

$$H_{(1)} = 2aqr\sin^2\theta - 2p(r^2 + a^2)\cos\theta,$$
$$H_{(2)} = \frac{1}{2}[(r^2+a^2)^2 - a^2\Delta\sin^2\theta]\sin^2\theta + \frac{3}{2}\overline{q}^2(a^2 + r^2\cos^2\theta),$$
$$H_{(3)} = -\frac{qa\Delta}{2r}[r^2(3-\cos^2\theta)\cos^2\theta + a^2(1+\cos^2\theta)] - \frac{1}{2}p(r^4 - a^4)\sin^2\theta\cos\theta + \frac{q\overline{q}^2a[(2r^2+a^2)\cos^2\theta + a^2]}{2r} - pa^2\Delta\sin^2\theta\cos\theta - \frac{1}{2}p\overline{q}^2(r^2+a^2)\cos^3\theta + \frac{aq(r^2+a^2)^2(1+\cos^2\theta)}{2r},$$

$$H_{(4)} = \frac{1}{16}(r^2+a^2)^2R^2\sin^4\theta + \frac{1}{4}M^2a^2[r^2(\cos^2\theta - 3)^2\cos^2\theta + a^2(1+\cos^2\theta)^2] + \frac{1}{16}\overline{q}^4[r^2\cos^2\theta + a^2(1+\sin^2\theta)]\cos^2\theta + \frac{1}{4}Ma^2r(r^2+a^2)\sin^6\theta + \frac{1}{4}Ma^2\overline{q}^2r(\cos^2\theta - 5)\sin^2\theta\cos^2\theta + \frac{1}{8}\overline{q}^2(r^2+a^2)(r^2+a^2+a^2\sin^2\theta)\sin^2\theta\cos^2\theta.$$

Here $\overline{q}^2 = q^2 + p^2$, and

$$\omega = \frac{1}{\Sigma}[(2Mr - \overline{q}^2)a + \omega_{(1)}B + \omega_{(2)}B^2 + \omega_{(3)}B^3 + \omega_{(4)}B^4], \quad (60)$$

with

$$\omega_{(1)} = -2qr(r^2+a^2) + 2ap\Delta\cos\theta,$$
$$\omega_{(2)} = -\frac{3}{2}a\overline{q}^2(r^2 + a^2 + \Delta\cos^2\theta),$$
$$\omega_{(3)} = 4qM^2a^2r + \frac{1}{2}ap\overline{q}^4\cos^3\theta + \frac{1}{2}qr(r^2+a^2)[r^2 - a^2 + (r^2+3a^2)\cos^2\theta] + \frac{1}{2}ap(r^2+a^2)[3r^2 + a^2 - (r^2 - a^2)\cos^2\theta]\cos\theta - aM\overline{q}^2(2aq + pr\cos^3\theta) - apMr \times [2R^2 + (r^2+a^2)\sin^2\theta]\cos\theta + \frac{1}{2}ap\overline{q}^2[3r^2 + a^2$$

$$+2a^2\cos^2\theta]\cos\theta+\frac{1}{2}q\overline{q}^2r[(r^2+3a^2)\cos^2\theta-2a^2]$$
$$+qM[r^4-a^4+r^2(r^2+3a^2)\sin^2\theta],$$
$$\omega_{(4)}=\frac{1}{2}a^3M^3r(3+\cos^4\theta)-\frac{1}{8}a\overline{q}^4[r^2(2+\sin^2\theta)\cos^2\theta$$
$$+a^2(1+\cos^4\theta)]+\frac{1}{16}a\overline{q}^2(r^2+a^2)[r^2(1-6\cos^2\theta$$
$$+3\cos^4\theta)-a^2(a+\cos^4\theta)]-\frac{1}{4}a^3M^2\overline{q}^2(3+\cos^4\theta)$$
$$-\frac{1}{16}a\overline{q}^6\cos^4\theta+\frac{1}{4}aM^2[r^4(3-6\cos^2\theta+3\cos^4\theta)$$
$$+2a^2r^2(3\sin^2\theta-2\cos^4\theta)-a^4(1+\cos^4\theta)]$$
$$+\frac{1}{8}aM\overline{q}^4r\cos^4\theta+\frac{1}{8}aM\overline{q}^2r[2r^2(3-\cos^2\theta)$$
$$\times\cos^2\theta-a^2(1-3\cos^2\theta-2\cos^4\theta)]+\frac{1}{8}aMr$$
$$\times(r^2+a^2)[r^2(3+6\cos^2\theta-\cos^4\theta)$$
$$-a^2(1-6\cos^2\theta-3\cos^4\theta)].$$

The electromagnetic vector potential is

$$A=(\Phi_0-\omega\Phi_3)dt+\Phi_3d\phi, \tag{61}$$

where

$$\Phi_0=\frac{\Phi_0^{(0)}+\Phi_0^{(1)}B+\Phi_0^{(2)}B^2+\Phi_0^{(3)}B^3}{4\Sigma}, \tag{62}$$

with

$$\Phi_0^{(0)}=4[-qr(r^2+a^2)+ap\Delta\cos\theta,$$
$$\Phi_0^{(1)}=-6a\overline{q}^2(r^2+a^2+\Delta\cos^2\theta),$$
$$\Phi_0^{(2)}=-3q[(r+2M)a^4-(r^2+4Mr+\Delta\cos^2\theta)r^3$$
$$+a^2(2\overline{q}^2(r+2M)-6Mr^2-8M^2r-3r\Delta\cos^2\theta)]$$
$$+3p\Delta[3ar^2+a^3+a(a^2+\overline{q}^2-r^2)\cos^2\theta]\cos\theta,$$
$$\Phi_0^{(3)}=-\frac{1}{2}a[4a^4M^2+12a^2M^2\overline{q}^2+2a^2\overline{q}^4+2a^4Mr$$
$$-24a^2M^3r+4a^2M\overline{q}^2r-24a^2M^2r^2-4a^2Mr^3$$
$$-\overline{q}^2r^4-6Mr^5-6r\Delta\{2M(r^2+a^2)-\overline{q}^2r\}\cos^2\theta$$
$$+a^4\overline{q}^2-12M^2r^4+\Delta(\overline{q}^4-3\overline{q}^2r^2+2Mr^3$$
$$+a^2(4M^2+\overline{q}^2-6Mr))\cos^4\theta],$$

and

$$\Phi_3=\frac{\Phi_3^{(0)}+\Phi_3^{(1)}B+\Phi_3^{(2)}B^2+\Phi_3^{(3)}B^3}{R^2H}, \tag{63}$$

with

$$\Phi_3^{(0)}=aqr\sin^2\theta-p(r^2+a^2)\cos\theta,$$
$$\Phi_3^{(1)}=\frac{1}{2}[\Sigma\sin^2\theta+3\overline{q}^2(a^2+r^2\cos^2\theta)],$$
$$\Phi_3^{(2)}=\frac{3}{4}aqr(r^2+a^2)\sin^4\theta-\frac{3}{4}p(r^2+a^2)^2\sin^2\theta\cos\theta$$
$$+3a^2pMr\sin^2\theta\cos\theta+\frac{3}{2}aqm[r^2(3-\cos^2\theta)$$
$$\times\cos^2\theta+a^2(1+\cos^2\theta)]-\frac{3}{4}aq\overline{q}^2r\sin^2\theta\cos^2\theta$$
$$-\frac{3}{4}p\overline{q}^2[(r^2-a^2)\cos^2\theta+2a^2]\cos\theta,$$
$$\Phi_3^{(3)}=\frac{1}{4}\overline{q}^2(r^2+a^2)[r^2+a^2+a^2\sin^2\theta\cos^2\theta]-\frac{1}{2}a^2\overline{q}^2$$
$$\times Mr(5-\cos^2\theta)\sin^2\theta\cos^2\theta+\frac{1}{2}a^2M^2[r^2(3$$
$$-\cos^2\theta)^2\cos^2\theta+a^2(1+\cos^2\theta)^2]+\frac{1}{2}a^2Mr(r^2$$
$$+a^2)\sin^6\theta+\frac{1}{8}R^2(r^2+a^2)^2\sin^4\theta+\frac{1}{8}\overline{q}^4[r^2\cos^2\theta$$
$$+a^2(2-\cos^2\theta)^2]\cos^2\theta.$$

6 Conclusion

We have described Ernst's solution generating technique [8–10] which uses Harrison's transformations for magnetizing black holes. We have reviewed the earlier work in this direction on Schwarzschild, Reissner-Nordström and Kerr-Newman black holes. Note that if we take electric charge e and magnetic charge p equal to zero the metric for Kerr-Newman black hole in Melvin universe reduces to Kerr black hole in Melvin universe. Further if we take the rotation parameter a equal to zero the metric reduces to the magnetized Schwarzschild black hole. If we put the magnetic field $B=0$ all the metrics reduce to their unmagnetized versions. Other properties of these magnetized black holes and further work in this direction [13] will be reported elsewhere.

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References

1. W. B. Bonner, *Proc. Phys. Soc.* **A 66** (1953) 145.
2. W. B. Bonner, *Proc. Phys. Soc.* **A 67** (1954) 225.
3. M.A. Melvin, *Phys. Lett.* **8** (1965) 65.
4. J. B. Griffiths, J. Podolsky, *Exact Space-Times in Einstein's General Relativity*, Cambridge University Press, Cambridge (2009).
5. M. A. Melvin, *Phys. Rev.* **B 139** (1965) 225.
6. K. S. Thorne, *Phys. Rev.* **B 139** (1965) 244.
7. B. K. Harrison, *J. Math. Phys.* **9** (1968) 1744.
8. F. J. Ernst, *J. Math. Phys.* **17** (1976) 54.
9. F. J. Ernst, W. J. Wild, *J. Math. Phys.* **17** (1976) 182.
10. F. J. Ernst, *J. Math. Phys* **17** (1976) 515.
11. M. Astorino, *Phys. Rev.* **D 87** (2013) 084029.
12. G. W. Gibbons, A. H. Mujtaba and C. N. Pope, *Class. Quant Grav.* **30** (2013) 125008.
13. M. Rizwan, *M.Phil. Thesis*, Quaid-i-Azam University, Islamabad (2015).