Bordisms & Probes of Quantum Gravity

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The following papers, some of them unrelated to the content of this document, were published by the candidate while this thesis was being developed:

1. *Global anomalies & bordism of non-supersymmetric strings*,
   Ivano Basile, Arun Debray, **M. Delgado**, Miguel Montero
   JHEP 02 (2024) 092 arXiv:2310.06895 - Inspire

2. *Emergence of Species Scale Black Hole Horizons*,
   José Calderón-Infante, **M. Delgado**, Angel M. Uranga
   JHEP 01 (2024) 003 arXiv:2310.04488 - Inspire

3. *Black holes as probes of moduli space geometry*,
   M. Delgado, Miguel Montero, Cumrun Vafa
   JHEP 04 (2023) 045 arXiv:2212.08676 - Inspire

4. *Elastic scattering of 3He+4He with SONIK*,
   S.N. Paneru , C.R. Brune, D. Connolly et al.
   Phys.Rev.C 109 (2024) 1, 015802 arXiv:2211.14641 - Inspire

5. *Dynamical Cobordism and the beginning of time: supercritical strings and tachyon condensation*,
   Roberta Angius, **M. Delgado**, Angel M. Uranga
   JHEP 08 (2022) 285 arXiv:2207.13108 - Inspire

6. *At the end of the world: Local Dynamical Cobordism*,
   Roberta Angius, José Calderón-Infante, **M. Delgado**, Jesús Huertas, Angel M. Uranga
   JHEP 06 (2022) 142 arXiv:2203.11240 - Inspire

7. *Dynamical Cobordism and Swampland Distance Conjectures*,
   Ginevra Buratti, José Calderón-Infante, **M. Delgado**, Angel M. Uranga
   JHEP 10 (2021) 037 arXiv:2107.09098 - Inspire

8. *Dynamical Tadpoles, Stringy Cobordism, and the SM from Spontaneous Compactification*,
   Ginevra Buratti, **M. Delgado**, Angel M. Uranga
   JHEP 06 (2021) 170 arXiv:2104.02091 - Inspire

Abstract

This thesis, written as a compendium of articles, investigates some broad consequences of allowing for the dynamical change of spacetime topology in string theory, in the context of the Swampland Program.

In the first four articles presented in this thesis, we investigate dynamical realizations of the cobordisms to nothing predicted by the Cobordism Conjecture. This comes down to considering solutions to the equations of motion where the compact space pinches off at finite distance in spacetime. We studied these setups in a wide variety of examples in string theory, which allowed us to characterize the backreaction of the singular extended objects (cobordism defects) that are located at the point of pinch-off. We found that they always explore infinite distances in moduli space, pointing to a link with the Distance Conjecture. This is not surprising from the perspective that these cobordisms represent dualities between any two effective field theories of quantum gravity in a general sense. It also means that the UV nature of these objects goes well beyond an EFT description. Furthermore, we showed that all of these solutions feature a generic behaviour, locally, near the singularity. These dynamical cobordisms are generally intricately linked to the presence of a scalar (tadpole) potential, which shows the importance of understanding the Distance Conjecture beyond proper moduli spaces. Finally, we discussed dynamical cobordisms where the solution runs along a time- or light-like dimension, and described known vacuum destroying bubbles in supercritical string theories as bordisms to nothing.

In the fifth article we discussed the identification and cancellation of global gauge/diffeomorphism anomalies in effective field theories of quantum gravity, where topology change should be allowed. We described how this can be done through the computation of an appropriate bordism group. We then implemented this for the three ten dimensional non-supersymmetric and non tachyonic string theories, showing that they are free of anomalies. Anomaly inflow arguments also allowed us to shine light on the worldvolume theories of fivebranes in these theories.

Finally, we considered whether the various extended objects in string theory (like D-branes, or the cobordism defects) could be used to probe UV physics, since they naturally warp spacetime and source a potential for the scalars. In the last two articles we discuss how to probe UV physics using black holes. We first showed that large BPS black holes in 4d $\mathcal{N} = 2$ compactifications of type II string theories can allow us to probe the geometry of the underlying compact space solely by measuring its charges and its size. If one wants to probe the physics at the species scale where full-fledged quantum gravity comes into play, one needs to consider instead the smallest possible black hole that one can define in a theory. We considered small, singular black holes and showed that they developed a stretched species scale sized horizon when curvature corrections are taken into account near its core. This had the virtue of unifying the various definitions of the species scale. It also begs the question of whether or not full-fledged quantum gravity is *always* hidden behind a horizon.

After briefly motivating quantum gravity and string theory, we introduce bordism groups and outline the various aspects of the Swampland Program that will be needed in this thesis. After the articles, we finish with an overview of the results published in the papers.

Resumen

Esta tesis, escrita como un compendio de artículos, investiga algunas consecuencias amplias de permitir el cambio dinámico de la topología del espacio-tiempo en la teoría de cuerdas, en el contexto del Programa del Swampland.

En los primeros cuatro artículos presentados en esta tesis, investigamos realizaciones dinámicas de los cobordismos hacia la nada predichos por la Conjetura de Cobordismo. Esto implica considerar soluciones a las ecuaciones de movimiento donde el espacio compacto se pellizca a una distancia finita en el espacio-tiempo. Estudiamos estos escenarios en una variedad de ejemplos en la teoría de cuerdas, lo que nos permite caracterizar los objetos extendidos singulares (defectos de cobordismo) que se encuentran en el punto de pellizco. Encontramos que siempre exploran distancias infinitas en el espacio de módulos, apuntando a un vínculo con la Conjetura de Distancia. Esto no es sorprendente desde la perspectiva de que estos cobordismos representan dualidades entre cualquier par de EFTs de gravitación cuántica en un sentido general. También significa que la naturaleza UV de estos objetos va mucho más allá de la EFT. Además, mostramos que todas estas soluciones presentan un comportamiento genérico, localmente, cerca de la singularidad. Estos cobordismos dinámicos están generalmente vinculados a la presencia de un potencial escalar, lo que muestra la importancia de comprender la conjetura de distancia más allá de los espacios de módulos. Finalmente, discutimos cobordismos dinámicos donde la solución se desarrolla a lo largo de una dimensión temporal o tipo luz, y describimos burbujas que destruyen el vacío en teorías de cuerdas supercríticas como bordismos hacia la nada.

En el quinto artículo discutimos la identificación y cancelación de anomalías globales de gauge/difeomorfismo en teorías efectivas de campos de gravitación cuántica, donde se debería permitir el cambio de topología. Describimos cómo esto puede hacerse mediante el cálculo de un grupo de cobordismo apropiado. Luego lo implementamos para las tres teorías de cuerdas no supersimétricas y no taquiónicas de diez dimensiones, mostrando que están libres de anomalías. Los argumentos de "anomaly inflow" también nos permitieron arrojar luz sobre las teorías de worldvolume de las cincobranas en estas teorías.

Finalmente, consideramos si los diversos objetos extendidos en la teoría de cuerdas (como las D-branas o los defectos de cobordismo) podrían usarse para sondear la física UV, ya que naturalmente deforman el espacio-tiempo y generan un potencial para los escalares. En los dos últimos artículos discutimos cómo sondear la física UV usando agujeros negros. Primero mostramos que los agujeros negros grandes BPS en compactificaciones de 4d $\mathcal{N} = 2$ de teorías de cuerdas de tipo II nos permiten sondear la geometría del espacio compacto subyacente solo midiendo sus cargas y su tamaño. Si uno quiere sondear la física a la escala de especies donde la gravedad cuántica completa entra en juego, es necesario considerar en su lugar el agujero negro más pequeño posible que se pueda definir en una teoría. Consideramos agujeros negros pequeños y singulares y mostramos que desarrollan un horizonte del tamaño de la escala de especies cuando se tienen en cuenta las correcciones de curvatura cerca de su centro. Esto tenía la virtud de unificar las diversas definiciones de la escala de especies. También plantea la pregunta de si la gravedad cuántica completa está *siempre* oculta detrás de un horizonte.

Después de motivar brevemente la gravedad cuántica y la teoría de cuerdas, introducimos grupos de cobordismo y delineamos los diversos aspectos del Programa del Swampland que serán necesarios en esta tesis. Después de los artículos, terminamos con una descripción general de los resultados publicados en los artículos.

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To my family, without whom none of this would have been possible; thank you, merci.

Contents

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Motivation & Context

Quantum gravity (QG) stands at the edge of modern theoretical physics, aiming to reconcile the two main paradigms of modern physics: quantum mechanics and general relativity. While both theories are extremely successful in their respective domains, they conflict when applied together at very small length scales, such as those encountered near the singularities of black holes or during the earliest moments of the universe.

On the one hand, general relativity (GR) describes gravity as the curvature of spacetime caused by the presence of mass and energy. It provides a comprehensive framework for understanding the dynamics of massive objects, such as planets, stars, and galaxies. It has been extensively tested and confirmed through numerous astrophysical observations. On the other hand, quantum mechanics (QM) revolutionized our understanding of the microscopic nature of matter, describing the behavior of particles and fundamental forces with remarkable precision. Most of the time, these two drastically different theories do not clash since they operate at different scales: in most places in the universe, gravity is weak and can be treated classically with GR, providing a background on which to define the QM that describes the other forces. There are however isolated regions of space and time where the two theories clash. Schematically, these a regions where gravity is as strong as the other fundamental forces and the classical description of gravity by GR breaks down. For instance, in the early universe, during periods of extremely high energy and density, the classical description of spacetime by GR breaks down. Similarly, classical descriptions of gravity break down near a black hole's singularity. Understanding the behavior of matter and energy near black holes and at the beginning of the universe thus requires a quantum description of gravity.

The quest for a theory of quantum gravity has therefore become one of the central goals of contemporary theoretical physics. Such a theory would not only shed light on the earliest moments of cosmic evolution and on black hole singularities but more importantly provide a deeper understanding of the fundamental nature of space, time, and gravity. String theory, loop quantum gravity, and other approaches offer promising avenues for exploring these enigmatic questions.

Understanding the unification of GR and QM into a full-fledged quantum gravity theory remains a huge challenge to this day. The first obstacle is due to the fact that there are many conceptual differences between the two frameworks that make such a unification very subtle. We will now go over some of these aspects, in an effort to identify some of the characteristics that a quantum gravity theory should have.

1.1 Aspects of Quantum Gravity

Classical theories of physics are formulated with respect to a fixed background spacetime. For instance, in Newtonian mechanics, time is absolute, and space is considered an unchanging background along which particles move. In special relativity, while the laws of physics are invariant under Lorentz transformations, there is still a fixed Minkowski spacetime background. GR, however, revolutionized our understanding of gravity by introducing the idea that spacetime curvature is *caused* by the distribution of matter and energy. In GR, the gravitational field is encoded in the curvature of spacetime, and particles move along geodesics in this curved spacetime. The key feature of general relativity is its **background independence**. The theory does not presume any fixed spacetime background against which gravitational phenomena occur. Instead, the geometry of spacetime is determined dynamically by the distribution of matter and energy according to Einstein's equations.

In GR, spacetime is treated as a smooth, continuous manifold. In this framework, spacetime can be divided into infinitely small intervals, allowing for precise measurements and predictions of physical phenomena. However, QM introduces the idea of quantization, where physical quantities such as energy, momentum, and angular momentum are divided into discrete quanta. In the context of quantum gravity, the notion of quantized spacetime suggests that spacetime itself might be composed of fundamental building blocks or quanta at the Planck scale, which is the scale at which quantum gravitational effects become important. At that scale, quantized spacetime could give rise to quantum fluctuations in the fabric of spacetime itself. These fluctuations would manifest as **uncertainty in the geometry and topology of spacetime**. The idea that a quantum gravity theory should account for the possibility of changes in the topology of spacetime will play a fundamental role in this thesis.

If spacetime is quantized into discrete little blocks in full-fledged quantum gravity, then why is it that we observe the smooth background spacetime that we know and love? This apparent tension is resolved through the concept of the **emergence** of spacetime. Background independence asserts that the geometry of spacetime should not be imposed from the outside but should emerge from the fundamental principles of the theory itself. In this view, spacetime and gravitational dynamics emerge at low energies as collective phenomena from the underlying quantum degrees of freedom, similar to how fluid dynamics emerges from the collective behavior of molecules. This emergent nature of spacetime and dynamics has been studied in various contexts and there have been some attempts to make it into a quantitative statement in known theories of quantum gravity. We will come back to this in later sections.

The existence of a fundamental length scale associated with the quantization of spacetime nevertheless sets a limit on the resolution of spacetime measurements. Below this scale, the notion of continuous spacetime would lose its meaning. What would it mean to probe quantum gravitational effects in a physical (thought) experiment? In other words, if we could "see" a singularity in spacetime where quantum gravitational effects came into play, what would it look like? Another question is whether it is even possible to have such a naked singularity? Probing length scales smaller than the Planck length would require energies so high that they would generically collapse into a black hole due to the strong gravitational forces involved. This is a heuristic argument, but it goes in the favour of hiding quantum gravitational effects behind a horizon. Characterizing such black holes and naked singularities and studying whether or not they can be used as **probes of quantum gravitational effects** will be a large part of this thesis.

Finally, one last aspect of quantum gravity that ties in with all that is mentioned above is **the holographic principle**, which suggests that spacetime itself may be an emergent phenomenon arising from the entanglement structure of a quantum field theory living on its boundary [1, 2]. This principle was initially motivated by the study of black holes. Black holes possess an entropy proportional to their horizon area, suggesting a deep connection between the information content of a black hole and its surface area. This led to the proposal that the entropy of a black hole could be thought of as encoded on the horizon surface rather than within its volume. The holographic principle found its most profound realization in the AdS/CFT correspondence [3]. This correspondence states that the dynamics of a gravitational theory in an Anti de-Sitter spacetime can be fully described by the behavior of a quantum field theory on a fixed and flat spacetime, effectively encoding five-dimensional gravity in terms of a four-dimensional quantum field theory.

We have just described some features that a theory of quantum gravity should have. We saw that the notion of a background spacetime on which one can define quantum field theories and interactions should arise as a low energy description, at scales well bellow the Planck mass. This leads to a physical theory that one can compare to the laws of our universe, and perhaps even attempt to make predictions. In the next section, we will detail how this can be done in the context of a specific theory of quantum gravity: string theory. While loop quantum gravity offers intriguing insights and has made significant progress in addressing certain issues related to the quantization of gravity, it is hard to connect to real world physics since there are many subtleties in identifying a low-energy limit in which one can recover GR [4]. For this reason, our focus being on the low energy theories that stem from quantum gravity, we will base our discussion in string theory for the rest of this thesis.

1.2 Aspects of String Theory and the String Landscape

This section is meant to give a glimpse into the structure of string theory and outline how it exhibits the special features of quantum gravity theories described in the previous section. We will then describe how we can connect string theory to low-energy, four-dimensional physics. For a more in-depth introduction to string theory, see [5, 6].

String theory is a theory of quantum gravity with renowned success. In its essence, it posits that the fundamental building blocks of the universe are not point-like particles but rather tiny, one-dimensional entities known as strings. String theory thus only has one fundamental scale, the string length l_s. These strings are assumed to propagate in a d-dimensional spacetime. In perturbative string theory, we assume that spacetime is flat Minkowski space and we quantize the oscillatory modes of the string and find its spectrum. All of the modes of the string describe particle states in spacetime. The massless modes lead (at the very least) to a d-dimensional graviton, a 2-form gauge field and scalar field ϕ, the *dilaton*, that parametrizes the coupling of the string $g_s = e^{\phi}$. All of the heavy modes of the string have masses at the string scale, they therefore decouple from physics at energy scales much lower than $m_s = l_s^{-1}$. We therefore obtain a d-dimensional low-energy effective field theory (EFT) that describes these three fields propagating on a flat spacetime.

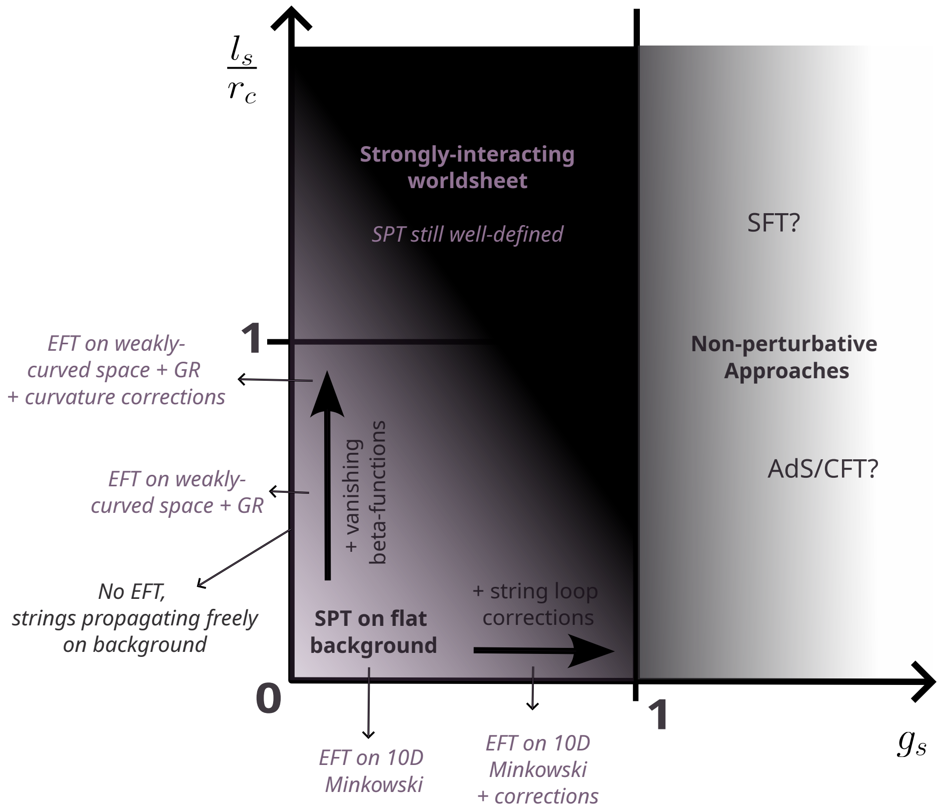


Figure 1.1: A diagram describing our understanding of string theory according to the value of the string coupling g_s and the curvature of spacetime relative to the string scale. At weak coupling and low curvatures, we have 10D EFTs of string theory, describing various particles moving on the background described by general relativity. At strong curvatures, the worldsheet theory becomes strongly coupled but string perturbation theory is still well defined (the path integral over the moduli space of Riemann surfaces still makes sense). At strong coupling, string perturbation theory completely breaks down and we have to resort to other methods.

We can incorporate weakly curved spacetime into this discussion as a perturbation around the flat background. The big surprise that is responsible for some of the fame of string theory is that, in order for the worldsheet theory to remain consistent in curved spacetime, the metric has to satisfy Einstein's equations. In this way, GR arises naturally in the low-energy limit of string theory. Therefore, at low energies $E \ll l_s^{-1}$ and as long as spacetime is weakly curved, string theory is described by a low-energy effective theory that describes the dynamics of the 2-form gauge field, the dilaton and all other massless modes of the string, propagating on a background spacetime dictated by GR. In this limit, string theories thus reduce to theories that look like our world, though in a higher number of dimensions and possibly with a lot more supersymmetry.

For the simplicity of this discussion, we will mostly consider the eight known ten-dimensional non-tachyonic string theories, five of which lead to supersymmetric spacetimes [7–12]. There are other consistent string theories but they always have a tachyon in their spectrum, which signals an instability of the ten-dimensional vacuum. All eight of these theories, shown in black in Figure 1.2, reduce at low energies to ten-dimensional EFTs with matter and gauge fields minimally coupled to gravity.

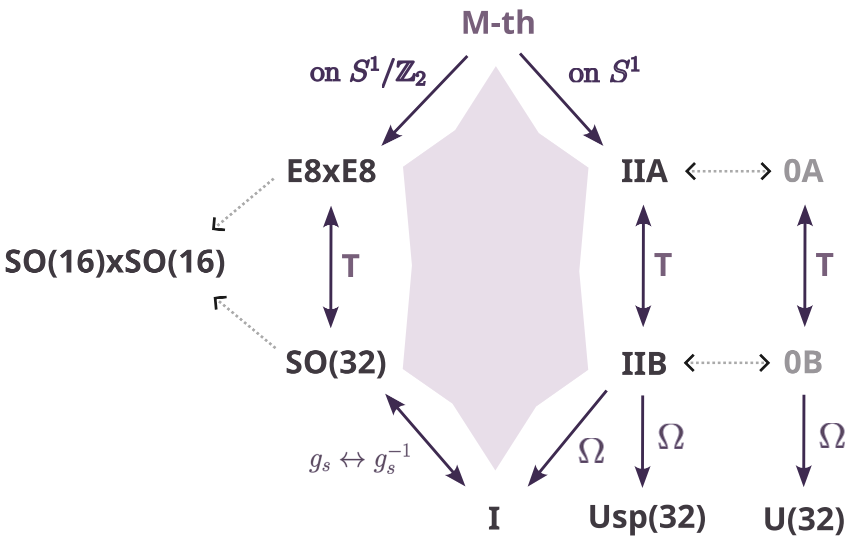


Figure 1.2: A diagram representing the eight ten-dimensional non-tachyonic string theories in black and some arrows demonstrating how they are related to each other, to M-theory (in purple) and to tachyonic type 0 string theories (in gray).

Emergent Spacetime At low energies $E \ll l_s^{-1}$, a massless oscillatory mode of the string gives rise to a graviton in spacetime, which satisfies the equations of motion of GR in weakly curved spacetime. When one goes up in energies and considers string theory at the string scale, this effective description completely breaks down and one needs to take into account all of the massive oscillatory modes of the string. That is, you can not only consider the massless excitations which lead to familiar particle states in spacetime, you need to consider the string itself as the primary object of interest. In this sense, spacetime and GR only emerge at low energies. Importantly, at the string scale, you consider strings themselves moving in a flat "background", but in no shape or form is this theory describing GR. The better description would be in terms of some gas of strings, perhaps in the spirit of [13], or in terms of string field theory [14]. Even in the limit $g_s \to 0$, one would have to consider a non-interacting theory of strings moving around in flat space. When talking about the emergence of a *spacetime*, we always refer to the one that is governed by general relativity and that only exists in the low-energy limit of string theory.

Background Independence in String Theory Above, we discussed how we quantized the oscillations of the string on a flat background and showed at low-energies that this could be considered to be a flat (or weakly-curved) spacetime. This was for practical reasons, as quantizing the string on curved backgrounds is a very daunting task. From the perspective of the worldsheet, it is not obvious that string theory is background independent: because we picked a background to quantize it, background independence of the quantum theory is not manifest. However, in the low-energy EFTs, there are subtle ways in which the background independence of the full theory can be seen. For instance, there are dualities between string theories that map a string theory on a background X to another one (possibly the same one) on a different background Y, as is schematically depicted in Figure 1.2. The duality states that the spectrum of the two theories are exactly the same. This hints to the fact that the choice of background may be somewhat irrelevant. Furthermore, the most studied realization of the holographic principle is the AdS/CFT correspondence which equates the partition function of a string theory on an AdS spacetime with the partition function of a conformal field theory in flat space, in one dimension less. This means that all of the gravity in AdS can be recast in terms of CFT correlators in flat space. While the original formulation of the AdS/CFT correspondence involves AdS spacetime, there have been extensions and generalizations to other spacetime backgrounds. Furthermore, in holography the geometry of the extra (holographic) dimension is not meant to be fixed, and one technically has to sum over all geometries (even topologically different ones) that are compatible with the asymptotic boundary conditions. This versatility suggests that the essential features of the correspondence are not tied to any specific background geometry but rather reflect the holographic principle in a profound way. Finally, in non-perturbative formulations of string theory, such as string field theory, background independence could play a more prominent role (see e.g. [15]).

The String Landscape In order to make contact with our four-dimensional world, we need to compactify some of the extra dimensions. This comes down to wrapping some of them on a compact manifold of size l_{KK} and making $l_{KK}^{-1} \gg E$. This will have the effect of decomposing all of the ten-dimensional fields into their Fourrier modes: the massless modes survive the compactification and appear in the lower-dimensional theory whilst the heavy modes, often called Kaluza-Klein replicas, decouple from the lower-dimensional physics due to their mass $m_{KK} = l_{KK}^{-1} \gg E$. During the process of compactification, the parts of the ten-dimensional fields that propagate in the internal dimensions lead to new dynamical scalars in the lower-dimensional theory, called *moduli*. They parametrize the geometry of the compact manifold.

Through this process, from the ten-dimensional effective field theories that arise as the low-energy limit of string theories, we can recover four-dimensional effective field theories with gravity, gauge and matter fields, and an array of scalar fields that are either the dilaton or the moduli associated to the compact space. Do these theories look like our world? can we use them to make predictions from string theory?

The fact is that there are many (perhaps infinitely) different ways of performing this compactification, depending on which compact manifold we choose and what extra ingredients we add in along the way. The bigger problem is that only a few of these choices are simple enough to make the full compactification tractable mathematically and they all yield very different four dimensional low energy theories. These theories all describe general relativity, but have different particle contents, vacuum energy density and different symmetries. They are said to populate the Landscape of string theory. Indeed, although compactifying string theory to four dimensions is well understood conceptually, it is very hard to perform in mathematical detail except for some isolated cases that preserve a lot of (super)symmetries. A lot of progress has been made in recent years to find detailed, top-down examples of compactifications that lead to realistic universes, but it is fair to say that very few, if any at all, are under full computational control.

1.3 The Swampland Program

Faced with this computational difficulty, one can ask whether or not it is possible to obtain *any* low energy theory from string theory. In recent years, this has been shown not to be true. There are some theories that simply cannot be obtained from string theory at low energies. Identifying the characteristics that separate these theories from the ones that can has been the whole premise of the Swampland Program [16]. The Swampland Program aims to distinguish between effective theories that arise from a consistent theory of quantum gravity - those in the "Landscape" - and those that do not, that are said to be in the "Swampland". In other words, the Swampland Program provides criteria that effective field theories must satisfy to be considered as low-energy limits of a consistent theory of quantum gravity, such as string theory. EFTs that are in the Swampland are therefore thought to be inconsistent with fundamental principles of a UV-complete theory of quantum gravity. The Swampland Program has generated significant interest and research activity within the string theory community and beyond, as it provides valuable insights into the structure of quantum gravity and the landscape of effective field theories that truly stem from it.

The Swampland Conjectures, which form the basis of the program, are the criteria that segregate the EFTs in the landscape from those in the swampland. They are obtained in general by noticing patterns in theories that are known to stem from top-down constructions in string theory. Sometimes we can trace back these patterns to a fundamental aspect of quantum gravity theories such as background independence. There are also occasions where they can be motivated by black hole physics. A subset of these conjectures are on such good theoretical standing that they can be proven in (near-)generality in string theory. We will now outline some of the conjectures and Swampland-related themes that will be relevant for this thesis. Along the way we will point out how they resonate with some of the fundamental features of quantum gravity theories discussed above. For a more complete review of the Swampland, see e.g. [17, 18].

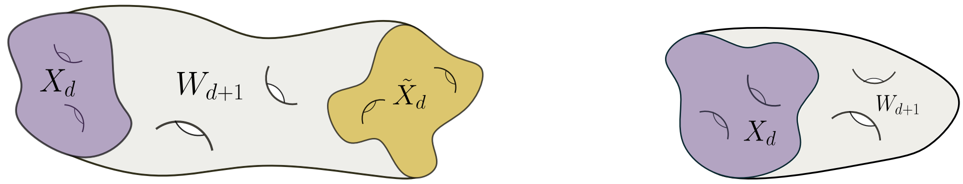
1.3.1 Bordisms and the Swampland

In this section, we first introduce bordism groups for the layman, in the hopes of providing a bit of clarity on what kind of information these groups contain. Then, we will use them in the context of the cobordism conjecture [19] and later in the context of identifying and classifying global anomalies in theories with dynamical gravity.

1.3.1.1 Bordism Groups for dummies

In this chapter we provide a brief introduction to bordism groups for readers that have little (or no) knowledge of algebraic topology. For a mathematical introduction on bordism groups see [20]. For more in-depth discussions with a special focus on physical applications, see [19, 21, 22].

Bordism groups are elaborate mathematical objects used in algebraic topology to classify closed manifolds up to a certain equivalence relation called bordism. Put simply, a bordism groups collect equivalence classes of closed manifolds of a given dimension, considering two manifolds X_d into $\tilde{X}_d$ to be equivalent if there exists a higher-dimensional manifold W_{p+1} whose boundary is the disjoint union of X_d into $\tilde{X}_d$. This is depicted schematically in Figure 1.3a. This higher dimensional manifold parametrizes the deformation of X_d into $\tilde{X}_d$.



(a) A bordism between two manifolds

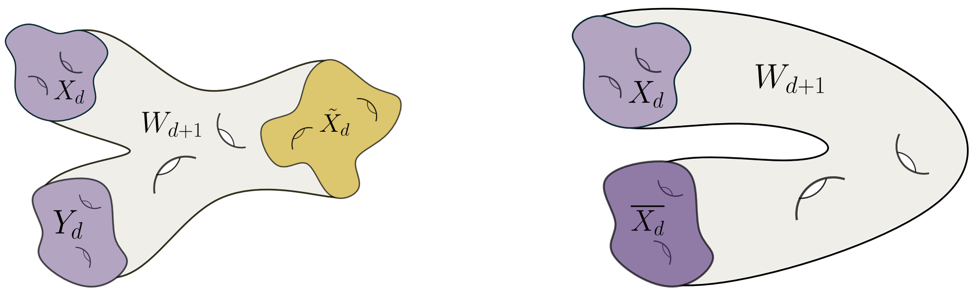
(b) A bordism to nothing

Figure 1.3: a) X_d and $\tilde{X}_d$ are bordant to eachother, they are in the same equivalence class. b) X_d is in the same equivalence class as the neutral element. X_d is said to be bordant to nothing.

Schematically, the bordism group Ω_d is the set of equivalence classes of d-dimensional manifolds, where two manifolds are considered equivalent if they are bordant to eachother:

$$\Omega_d \cong \{\text{d-dimensional manifolds } X_d\}/\text{bordism} \tag{1.1}$$

Group Structure Bordism groups Ω_d have an abelian group structure, with addition under the disjoint union of manifolds. There is therefore a neutral element, the equivalence class of "nothing". This means that some manifolds X_d can be bordant to nothing, which is to say that they are boundaries. In that case, there is a higher dimensional manifold W_{p+1} that parametrizes how X_d can be shrunk into a point, as depicted in Figure 1.3b. The inverse is given by an appropriate orientation-reversed version of X_d such that their disjoint union is bordant to nothing, as depicted in Figure 1.4b.



(a) The bordism of a disjoint union

(b) The inverse

Figure 1.4: a) The disjoint union of X_d and Y_d is bordant to $\tilde{X}_d$. b) The inverse of X_d is defined as the manifold that is such that the disjoint union of X_d and its inverse is bordant to nothing.

Structure Things get a bit more complicated when you want to consider bordisms between manifolds that satisfy extra conditions. So far, we have not specified anything about X_d except for the fact that it is a manifold, this is the most basic form of bordism, known as unoriented bordism. We can also consider the bordism of oriented manifolds in which case we call it oriented bordism. More generally, we can consider the bordisms of manifolds with a specific *structure*. When there is additional structure, the notion of cobordism must be formulated more precisely: the structure has to extend to the W_{d+1} dimensional manifolds that defines the bordism.

Defining a structure intuitively comes down to finding a way to translate a bundle of a topological group G into a vector bundle on the entire tangent space of X_d. For instance, for $G = SO$, a structure is equivalent to a choice of orientation of X_d. For $G = \text{Spin}$, it is a choice of spin structure. Let us denote a generic structure by a ξ-structure. When defined formally, a $\xi-$structure on W_{d+1} is known to induce a structure on its boundary. In that way, we can define bordisms between manifolds X_d and $\tilde{X}_d$ with ξ-structure through a manifold W_{d+1} with a ξ-structure.

It can help to think of adding structure as trivializing a new class of topological invariants. For instance, for a d-manifold to be orientable, its first Stiefel-Whitney class [23] must vanish, and on top of that, for it to be a spin manifold, its second Stiefel-Whitney class must vanish as well. Once the vanishing of these classes is imposed, the manifold under consideration can be shown to admit a spin structure. The point is that there are various spin structures for each dimension d, and this is exactly what the spin bordism groups quantify.

Generators and Invariants If the bordism group under consideration Ω_d^ξ vanishes, then all d-manifolds with ξ-structure can be deformed into one another and into nothing. However, when they are not trivial, they generically take the form:

$$\Omega_d^\xi \cong \underbrace{\mathbb{Z} \times \cdots \times \mathbb{Z}}_{r \text{ times}} \times \underbrace{\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2} \cdots}_{p \text{ times}} \tag{1.2}$$

for p^i non-negative integers. This decomposition is true in all cases of interest in this thesis but there is no theorem proving that is it true in general. The first part correspond to the free part of the group, whilst the second part corresponds to torsional classes. Manifolds that are part of a torsional class indexed by $\mathbb{Z}_{p^i}$ are such that only the disjoint union of p^i copies of them is bordant to nothing. Equation (1.2) means that any d-manifold with ξ-structure is in an equivalence class of bordism that can be indexed by r integers and p fractions of integers given by $1/p^i$.

To see where these integers (and fractions of integers) are coming from, we need to consider bordism invariants. These are topological invariants that, when evaluated on a manifold, yield (one of) the aforementioned indices. With a complete set of $r + p$ bordism invariants, one can identify precisely in what bordism equivalence class a manifold belongs to. In particular, these invariants help in identifying the generators of the bordism group: the set of d-manifolds that are representative of the bordism class under consideration. For instance, Pontryagin numbers and Stiefel-Whitney numbers [23] are oriented cobordism invariants; they determine an oriented manifold's oriented cobordism class completely.

Determining the bordism invariant is in general easier said than done, they are not always simple integrals of characteristic classes of some tangent or gauge bundle. For instance, for spin bordism they are more complicated quantities known as eta-invariants of Dirac operators. In such cases, finding the relevant generator is not always easy.

Bordisms and Cohomology To a bordism theory Ω^ξ, we can associate a generalized cohomology theory, with homology groups $\Omega_d^\xi(W)$ for any space W. $\Omega_d^\xi(W)$ describes the set of equivalence classes of d-manifolds X_d equipped with a map into W. If W is a point then we fall back on the bordism group described above. Generalized cohomology theories share all of the axioms of a cohomology theory except for the dimension axiom: the bordism groups of a point are not trivial for $d > 0$.

In physical setups, we often have to describe the bordisms of d-manifolds with ξ-structure and with a non trivial G-bundle, which one can imagine to be some sort of gauge bundle. In this case, the bordism groups one has to consider are $\Omega_d^\xi(BG)$ where BG is the classifying space of G which classifies the various physically distinct G-bundles. Importantly, even if we know all of the $\Omega_d^\xi = \Omega_d^\xi(pt)$ and the homology of the space W, this does not mean that we can compute $\Omega_d^\xi(W)$. The Atiyah-Hirzebruch spectral sequence however gives a starting point for these calculations by reducing the computation of $\Omega_d^\xi(W)$ to that of various homology groups on W (for more information on this, see e.g. [24]).

Final Remarks

- For many types of ξ-structure (but not all of them) $\Omega_*^\xi = \bigoplus_d \Omega_d^\xi$ is a $\mathbb{Z}$-graded ring. This is the case for oriented bordism, spin bordism, string bordism etc. but not for pin$^\pm$ bordism. That means that on top of abelian group structure described above, they allow for multiplication under the cartesian product of two manifolds. The ring is $\mathbb{Z}$-graded by the dimensions of the manifolds.

- Given the discussion above, one might wonder what is the difference between considering $\Omega_d^\xi(BG)$ and considering $\Omega_x^{\xi+BG}(pt)$ where the G-bundle is added to the structure. The difference is that adding structure can in general involve intertwining the G-bundle with the tangent bundle of X_d. Intuitively, the difference is the same as the difference between considering a manifold with spin structure and considering a spinor bundle on a manifold. One is a characteristic of the tangent bundle everywhere, and the other is just a localized bundle.

In the next sections we will see how bordism groups can be used in the context of QG and the Swampland Program.

1.3.1.2 The Cobordism Conjecture

The cobordism conjecture [19] applies to string theories compactified on a p-dimensional manifolds X_p. The conjecture states that any two backgrounds must equivalent in a precise sense described by the mathematical construction of bordism groups. In this way, it speaks to the fundamental nature of quantum gravity: that its observables should be *background independent*. It quantifies the fact that there should always exist a physical process in quantum gravity that can deform any background into any other.

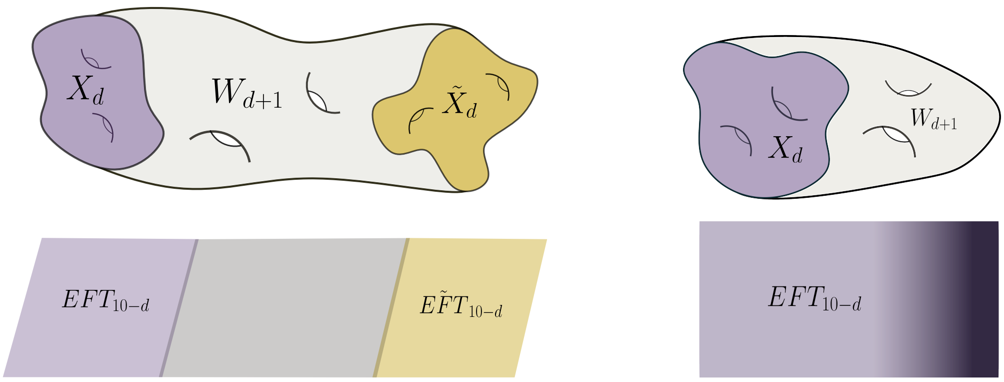
Saying that one should be able to deform any and all p-dimensional backgrounds into one another in quantum gravity comes down to saying that all p-dimensional backgrounds are part of the same equivalence class in bordism. If all p-dimensional manifolds are in the same bordism class, then they have to be in the class of “nothing”, i.e. the trivial class. The cobordism conjecture can therefore be stated as:

The Cobordism Conjecture

Consider a D-dimensional QG theory compactified on a d-dimensional compact manifold, all bordism classes must be trivial:

$$\forall d \text{ such that } d \leq D, \quad \Omega_d^{QG} = 0\,. \tag{1.3}$$

From the perspective of the lower-dimensional effective field theories obtained from the compactification, this means that any two of these theories can be connected by a domain wall as in Figure 1.5a and that any such theory should admit a boundary, as in Figure 1.5b.



(a) A bordism between two manifolds

(b) A bordism to nothing

Figure 1.5: Bordisms from the perspective of the lower dimensional EFTs one gets after compactifying on X_d and $\tilde{X}_d$. A bordism between X_d and $\tilde{X}_d$ looks like a domain wall (a) and a bordim to nothing looks like a boundary (b).

The difficulty here is in defining what are the topology-changing processes allowed in full-fledged quantum gravity. Of course, we do not know the whole set of such processes. In mathematics, bordism are usually considered between manifolds with specific structures. For instance, spin bordism groups Ω_d^{spin} describe the bordisms between d-dimensional manifolds equipped with a spin structure. Ideally, we would know what kind of structure is appropriate to describe quantum gravity backgrounds but for that we would need a full-fledged, non-perturbative description of string theory. Instead, we consider much simpler bordism groups that are only meant to approximate those of full-fledged quantum gravity. For instance, we know that the low-energy EFTs we get from string theory generically contain fermions. This means that they should only be compactified on manifolds that admit a spin structure. We can therefore consider spin bordism as a first approximation of Ω_d^{QG}. As we go along and further constrain the type of structure that our theories should admit, we can refine our description in terms of more elaborate bordism groups that account for this added structure.

If one of these approximate bordism groups are non-trivial, it only signals the fact that the bordism groups we consider are too poor an approximation of Ω_d^{QG} and that new structure needs to be added to the theory in order to trivialize these classes. In this way, the cobordism is predicting the existence of new structure in the theory that should trivialize all the classes:

$$\Omega_d^{approx.} \neq 0 \Longrightarrow \Omega_d^{approx+new\ structure} = 0\,. \tag{1.4}$$

More precisely, the presence of a non-trivial bordism class would signal the presence of an uncancelled global topological charge in the compact space, given by the bordism invariants, that prevent you from shrinking it to a point:

$$Q = \int_{X_d} [\text{topological OP}]_d \tag{1.5}$$

where we have represented the bordism invariant as an integral of some sort of characteristic class over the compact space, but it can be something more exotic, as discussed in the previous section. The presence of this topological operator supported on X_d signals the presence of a $(D-d-1)$-form global symmetry (see e.g. [25] for a review on higher-form symmetries). This global charge has the effect of putting a "label" on the background X_d, which should not be allowed in a background independent theory. The statement that $\Omega_d^{QG} = 0$ thus means that there must be new structure in quantum gravity that gets rid of this global symmetry.[1] How can we get rid of a global symmetry? There are two options:

1. **Breaking the symmetry:** this amounts to introducing a defect that is charged under this symmetry and can thus absorb the uncancelled topological charge in the compactification. The objects that are charged under the $(D-d-1)$-form global symmetry generated by this charge are $(D-d-1)$-dimensional defects.

   A simple example of this was discussed in [19]. It is the case of the one-dimensional spin bordism group
   $$\Omega_1^{spin} = \mathbb{Z}_2\,, \tag{1.6}$$
   which is generated by the circle with periodic spin structure. A fermion with anti-periodic boundary conditions on this circle creates a configuration that you cannot deform to a point. In the case of the ten-dimensional heterotic string theories, their compactification on the circle with periodic string structure should be perfectly consistent, so this symmetry has to be broken by an eight-dimensional defect, which has yet to be identified.

2. **Gauging the symmetry:** this amounts to introducing a dynamical $(D-d)$-form that couples to (the Hodge dual of) the $(D-d)$-form current associated to the symmetry. Doing so in a consistent way eventually leads to new consistency conditions on the theory.

   A simple example of this was discussed in [19]. It is the case of the four-dimensional spin bordism group
   $$\Omega_4^{spin} = \mathbb{Z}\,, \tag{1.7}$$
   which is generated by the K3 surface. Gauging this symmetry means to couple it to a 6-form gauge field. Consider heterotic string theories turning off all gauge bundles

[1] As a side note, this shows how the cobordism conjecture can thus be seen as a direct implication of a different, more general Swampland conjecture: the no global symmetries conjecture [26]. In a few words, this conjecture states that any global symmetry should be gauged or broken in quantum gravity. When applied to topological symmetries such as that generated by (1.5), it reduces to the cobordism conjecture.

and forgetting about the branes. Consistency of the theory requires this 6-form to be the magnetic NSNS field, B_6, and leads to the observation that K3 carries five units of NS5 brane charge. This is the known fact that compactifying heterotic string theories on K3 is inconsistent on its own. Therefore, one can either choose to ignore K3 completely, or to refine the description by considering heterotic string theories with gauge bundles and/or five-branes that can cancel this charge. The first option comes down to considering twisted-string bordism $\Omega_4^{twisted-string} = 0$ and the second comes down to breaking the symmetry by introducing NS5 branes in the compactification to cancel the charge. The second option seems the most natural, given that we know heterotic string theories have gauge fields and five-branes in their spectrum.

The new $(D-d-1)$-dimensional defects predicted by the cobordism conjecture can in general be highly non-supersymmetric and unstable and will most likely back-react on the geometry in a singular way. Isolating the properties of these objects from the EFT's perspective is thus a very daunting task. For examples of direct applications of the cobordism conjecture (at the topological level), see for instance [22, 27–36]).

In particular, even in cases where the bordism classes are trivial and we need not introduce a new defect, describing how the domain walls between EFTs (or equivalently, the boundaries, since you can glue two boundaries to make a domain wall) predicted by the cobordism conjecture should appear in spacetime is difficult. This can be seen from the simple fact that you can have two completely different string theories with different degrees of freedom on each side. The domain wall has to be such that it converts all of the degrees of freedom of one theory into that of the other. In this way, the domain wall is implementing a duality in a general sense. This means that its physics are highly non-perturbative and therefore out of reach of the low-energy physics of the EFT. From this, one can already see that such objects will generally be singular from the perspective of the EFT.

The work in Chapter 2 of this thesis takes a step in the direction of describing these new defects and how they backreact on spacetime. We now turn to another corner of the Swampland Program where bordism groups play and important role.

1.3.1.3 Bordisms and Global Anomalies

Bordism groups also play an important role in the identification of global gauge and diffeomorphism anomalies in EFTs of quantum gravity. Anomaly cancellation is important in any consistent theory, but there are specific types of anomalies that arise when topology-changing processes should be allowed.

Consider an anomaly in a theory on a spacetime X_d, that is, consider a gauge transformation or a diffeomorphism under which the path integral is not invariant. When this anomalous transformation can be made arbitrarily small (close to the identity), we call the anomaly "local". Local anomalies are the most problematic type of gauge/gravitational anomaly since the path integral is not invariant under an infinitesimal transformation. They are the sort of anomaly that can be computed using triangle Feynman diagrams (in four dimensions). These anomalies are reviewed in [37], using the modern formalism of the anomaly polynomial in two dimensions more.

When local anomalies are shown to vanish, we are left with global anomalies. These are anomalies associated to gauge transformations and diffeomorphisms that cannot be deformed to the identity. A famous example of a global gauge anomaly is Witten's SU(2) anomaly [38]. The modern way of computing global anomalies is through what is known as an anomaly theory, which lives on a (d+1)-dimensional manifold whose boundary is the spacetime manifold X_d. This theory is not meant to be describing a physical theory in one dimension more, instead, it is thought of as a means to parametrize the anomalous global gauge transformation. Indeed, the anomaly theory is engineered to give exactly the opposite anomaly of that of the path integral, such that when coupled together, the total anomaly cancels (for a pedagogical review, see [21]).

It turns out that is it can be easier to compute the anomaly using the anomaly theory than it is to compute it in the partition function. The reason, is that when local anomalies cancel, global anomalies become bordism invariants of the $(d+1)$-dimensional spaces. This means that detecting a potential global anomaly can be done "simply" by computing the relevant bordism groups in $d+1$ dimensions. Of course, this can be easier said than done, depending on the structure we have to impose on the bordism groups.

The anomaly theory on the higher-dimensional manifold Y_{d+1} parametrizes the space of gauge and metric configurations on X_d. In particular, the space Y_{d+1} can have holes and accounts for topology change of X_d. Evaluating the path integral on a topology changing transition is impossible without a non-pertubative description of QG since these processes are usually singular from the perspective of the EFT. The bordism groups thus shed light on the consistency conditions that arise from global anomaly cancellation in theories with dynamical gravity. This can be viewed as a bottom-up approach to the Swampland Program.

Chapter 3 will describe how this works in more detail. In particular, we will show how bordism groups were used to assess whether or not global anomalies cancel in the three ten-dimensional non-supersymmetric and non-tachyonic string theories in Figure 1.2. This ends our introduction to bordism groups in a Swampland context, we now turn to introducing another seemingly unrelated Swampland conjecture.

1.3.2 The Distance Conjecture

Before stating the distance conjecture, let us recall that there are two kinds of scalar fields in EFTs that come from string theory.

Firstly there is the dilaton, that exists already in ten dimensions and therefore also in any EFT obtained from compactifying the ten-dimensional ones. It is linked to the string coupling as $g_s = e^{\phi}$. When the dilaton goes to $\pm\infty$, our string theory becomes infinitely weakly or strongly coupled. These are limits that are usually linked to string dualities. In these limits, the string theory we started with is leaving its regime of validity and the degrees of freedom of the dual string become the appropriate degrees of freedom to describe the process at low energies. When $g_s \to 0$, they are the modes of the string theory we started with, and when $gs \to \infty$ they are usually the modes of a dual string. For example, one can see in Figure 1.2 that type I at strong coupling is dual to SO(32) string theory at infinitely weak coupling. In both cases, there is a tower of heavy string modes that becomes light in both of these limits, that of the string at strong coupling, and that of the dual string at weak coupling.

Then, there are the moduli that arise only after compactification, that parametrize the geometry of the compact manifold. For instance, if you compactify one of the ten dimensional theories on a circle or radius R, you will obtain a dynamical scalar in the nine-dimensional theory that is linked to R as follows: $\sigma \sim \log R$. This scalar can also go to $\pm\infty$ in the nine-dimensional spacetime. We see that if $\sigma \to +\infty$, $R \to \infty$ and so we observe a decompactification of the circle. In this limit, the nine-dimensional EFT breaks down completely as all of the KK modes become light. If $\sigma \to -\infty$, then $R \to 0$ and the circle is pinching off. In these limits, stringy physics are at play, since the better description is that of the T-dual string theory compactified on a radius $R^{-1} \to \infty$. Therefore the pinching off in the original frame is a decompactification in the T-dual frame, and the tower of states becoming light is that of the KK modes in the T-dual frame. These modes correspond to winding states in the original frame. T-duality is where the stringy physics comes in to play: the string being an extended objects can not only propagate in internal dimensions but also wrap around them.

The moral of the story is that no matter what scalars you consider in your EFTs of string theory, there is always an infinite tower of states becoming light when you go to infinite distance in scalar field space,[2] that signal the break down of the EFT. This is exactly the statement of the distance conjecture. It is motivated by a wide variety of examples in string theory, where in these limits, there is always an infinite tower of massive states getting exponentially light. The conjecture is stated as follows:

The Distance Conjecture

In any effective field theory of quantum gravity, there is a tower of states becoming light as one goes to infinite distance in field space $\varphi \to \infty$ with characteristic mass m [39]:

$$\frac{m}{M_p} \sim e^{-\lambda\varphi}, \quad \lambda \in O(1)\,. \tag{1.8}$$

The reason for the exponential dependence on distance in field space is motivated by the two types of towers that we observe in string theory: KK towers and string towers, which both have masses that depend exponentially on the corresponding moduli. There is in fact a conjecture that those are the only two options in string theory [40]. The SDC has been discussed in many examples in string theory, see for instance [39–61].

From the discussion above, it is clear that infinite distance limits in field space are where the UV physics happen. They are always taking us to regions of the parameter space of our theory where the EFT is at the limit of its range of validity. When they are not simple decompactification limits, these infinite distance limits generically take us to a frame where the dual theory is a better low-energy description than the one we started with.

1.3.3 Probes of Quantum Gravity

It is of obvious interest to quantum gravity phenomenologists to be able to probe quantum gravitational effects in a physical experiment. One option would be to exploit the extended objects that naturally exist in string theory. These can be black holes, (black-) branes,

[2] Importantly, note that in the context of the distance conjecture, we always consider canonically normalized kinetic terms for the scalar fields, in the Einstein frame.

they can be singular, or have a horizon. A simple example of this are D-branes in type II theories. In full-fledged string theory they correspond to the end points of open strings and are very well understood. In the low-energy EFTs of string theory, they arise as non-perturbative objects that back-react on gravity in a very singular way. All branes except the D3 are the location where both the curvature and the dilaton blow up, reflecting the fact that D-branes cannot be fully described in the EFT.

Interestingly, these D-branes as well as other examples of UV (cobordism) defects (see Chapter 2) are generally located at infinite distance in moduli space, showcasing the fact that they are better described in a dual frame. This also explains how the domain walls between theories predicted by the cobordism conjecture back-react so singularly on spacetime, they are intrinsically linked to dualities, which in turn comes from the background independence of String Theory.

Being that they are singular at their core, these objects are by definition not controlled probes of UV physics. It is tempting to take a black hole instead or a black brane, whose horizon is large enough such that curvatures are small and corrections can be ignored. These large black holes can in fact constitute controlled probes, and they will be used in Chapter 4 this thesis to uncover topological properties about the underlying compactification.

However, if one wants to probe physics at such high energies that the true effects of full-fledged string theory come into play, one ought to use more singular probes. In fact one might even wonder whether it makes sense to probe such an energy scale, since spacetime and dynamics completely break down there. Let us first ask what this scale actually is. The naive answer is to assume it is the Planck scale, since that is when loop corrections to the graviton propagator become relevant. In reality, there is a subtlety to this argument: in a theory with light particle spaces, this scale is actually [62, 63]:

$$\Lambda_{sp} = \frac{M_{pl}}{N^{\frac{1}{d-2}}} \tag{1.9}$$

This energy scale has been dubbed "Species scale" in the literature. The point is, that in the presence of a large number of light species, the scale at which full-fledged quantum gravity comes into effect can be a lot lower than the Planck scale. With the distance conjecture in mind, we see that the species scale gets lowered at infinite distance in moduli space. This means that the singular probes discussed above (D-branes and other cobordism defects) are regions of spacetime where the species scale is greatly lowered. Identifying the tower and how fast the species scale becomes light in these limits has been studied in [61, 64–66]. The species scale has also been studied in the interior of moduli space in [67–69].

One may wonder then whether or not we could use these probes to ride our way up all the way to the Species scale. In Chapter 4 we will discuss this and see that curvature corrections near the singular core might always hide the full-fledged quantum gravity effects behind a species-scale sized horizon.

1.4 Outline of the thesis

After this general introduction, the second part of the thesis contains the collection of seven articles in the published versions. Four of them should be read one after the other, as they pertain to the same subject: dynamical realizations of cobordisms in spacetime.

They are presented in Chapter 2. Chapter 3 contains one stand-alone article, on the topic of bordisms and global gauge and gravitational anomalies in non-supersymmetric string theories. The last two articles are presented together in Chapter 4 as they both deal with probing UV effects with extended objects. We leave the conclusions for the last part of the thesis, in Chapter 6 where the main results of these works are summarized.

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2
Dynamical Cobordism

This chapter contains the articles:

- *Dynamical Tadpoles, Stringy Cobordism, and the SM from Spontaneous Compactification*,
  Ginevra Buratti, **M. Delgado**, Angel M. Uranga
  JHEP 06 (2021) 170 arXiv:2104.02091 - Inspire

- *Dynamical Cobordism and Swampland Distance Conjectures*,
  Ginevra Buratti, José Calderón-Infante, **M. Delgado**, Angel M. Uranga
  JHEP 10 (2021) 037 arXiv:2107.09098 - Inspire

- *At the end of the world: Local Dynamical Cobordism*,
  Roberta Angius, José Calderón-Infante, **M. Delgado**, Jesús Huertas, Angel M. Uranga
  JHEP 06 (2022) 142 arXiv:2203.11240 - Inspire

- *Dynamical Cobordism and the beginning of time: supercritical strings and tachyon condensation*,
  Roberta Angius, **M. Delgado**, Angel M. Uranga
  JHEP 08 (2022) 285 arXiv:2207.13108 - Inspire

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Dynamical tadpoles, stringy cobordism, and the SM from spontaneous compactification

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Abstract: We consider string theory vacua with tadpoles for dynamical fields and uncover universal features of the resulting spacetime-dependent solutions. We argue that the solutions can extend only a finite distance Δ away in the spacetime dimensions over which the fields vary, scaling as $\Delta^n \sim \mathcal{T}$ with the strength of the tadpole $\mathcal{T}$. We show that naive singularities arising at this distance scale are physically replaced by ends of spacetime, related to the cobordism defects of the swampland cobordism conjecture and involving stringy ingredients like orientifold planes and branes, or exotic variants thereof. We illustrate these phenomena in large classes of examples, including $\mathrm{AdS}_5 \times T^{1,1}$ with 3-form fluxes, 10d massive IIA, M-theory on K3, the 10d non-supersymmetric USp(32) strings, and type IIB compactifications with 3-form fluxes and/or magnetized D-branes. We also describe a 6d string model whose tadpole triggers spontaneous compactification to a semirealistic 3-family MSSM-like particle physics model.

Keywords: Flux compactifications, Superstring Vacua, Supersymmetry Breaking

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Contents

1 Introduction and conclusions

Supersymmetry breaking string vacua (including 10d non-supersymmetric strings) are generically affected by tadpole sources for dynamical fields, unstabilizing the vacuum [1, 2]. We refer to them as *dynamical tadpoles* to distinguish them from *topological tadpoles*, such as RR tadpoles, which lead to topological consistency conditions on the configuration (note however that dynamical tadpoles were recently argued in [3] to relate to violation of swampland constraints of quantum gravity theories). Simple realizations of dynamical tadpoles arose in early models of supersymmetry breaking using antibranes in type II (orientifold) compactifications [4–7], or in 10d non-supersymmetric string theories [8].

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Dynamical tadpoles indicate the fact that equations of motion are not obeyed in the proposed configuration, which should be modified to a spacetime-dependent solution (more precisely, solution in which some fields do not preserve the maximal symmetry in the corresponding spacetime dimension, but we stick to the former nomenclature), e.g. rolling down the slope of the potential. This approach has been pursued in the literature (see e.g. [9–13]), although the resulting configurations often contain metric singularities or strong coupling regimes, which make their physical interpretation difficult.

In this work we present large classes of spacetime[1] dependent field configurations sourced by dynamical tadpoles, which admit a simple and tractable smoothing out of such singularities. Remarkably, these examples reveal a set of notable physical principles and universal scaling behaviours. We argue that the presence of a dynamical tadpole implies the appearance of ends of spacetime (or walls of nothing) at a finite spacetime distance, which is (inversely) related to the strength of the tadpole. These ends of spacetime moreover correspond to cobordism defects (or end of the world branes) of the theory implied by the swampland cobordism conjecture [14, 15]. In most setups the cobordism defects end up closing off the space into a compact geometry (possibly decorated with branes, fluxes or other ingredients), thus triggering spontaneous compactification.

We can sum up the main features described above, and illustrated by our examples, in two lessons:

Finite distance. *In the presence of a dynamical tadpole controlled by an order parameter $\mathcal{T}$, the spacetime-dependent solution of the equations of motion cannot be extended to spacetime distances beyond a critical value Δ scaling inversely proportional to $\mathcal{T}$, with a scaling relation*

$$\Delta^{-n} \sim \mathcal{T}\,. \tag{1.1}$$

In our examples, $n = 1$ or $n = 2$ for setups with an underlying AdS-like or Minkowski vacuum, respectively.

Dynamical cobordism. *The physical mechanism cutting off spacetime dimensions at scales bounded by the Δ above, is a cobordism defect of the initial theory (including the dynamical tadpole source).*

To be precise, when there are multiple spacetime directions to be closed off, the actual defect is the cobordism defect corresponding to circle or toroidal compactifications of the initial theory, with suitable monodromies on non-trivial cycles. This is analogous to the mechanism by which F-theory on half a $\mathbf{P}_1$ provides the cobordism defect for type IIB on $\mathbf{S}^1$ with $\mathrm{SL}(2,\mathbf{Z})$ monodromy [14] (see also [16]).

As explained, we present large classes of models illustrating these ideas, including (susy and non-susy) 10d string theories and type II compactifications with D-branes, orientifold planes, fluxes, etc. For simplicity, we present models based on toroidal examples (and orbifolds and orientifolds thereof), although many of the key ideas easily extend to more

[1] Actually, we restrict to configurations of fields varying over spatial dimensions (rather than time); yet we abuse language and often refer to them as spacetime-dependent.

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general setups. This strongly suggests that they can apply to general string theory vacua. Very remarkably, the tractability of the models allows to devise spontaneous compactification whose endpoint corresponds to some of the (supersymmetric extensions of the) SM-like D-brane constructions in the literature. As will be clear, our examples can often be regarded as novel reinterpretations of models in the literature.

Although our examples are often related to supersymmetric models, supersymmetry is not a crucial ingredient in our discussion. Dynamical tadpoles correspond to sitting on the slope of potentials, which, even in theories admitting supersymmetric vacua, correspond to non-supersymmetric points in field space. On the other hand, supersymmetry of the final spacetime-dependent configuration is a useful trick to guarantee that dynamical tadpoles have been solved, but it is possible to build solutions with no supersymmetry but equally solving tadpoles.

Our results shed new light on several features observed in specific examples of classical solutions to dynamical tadpoles, and provide a deeper understanding of the appearance of singularities, and the stringy mechanism smoothing them out and capping off dimensions to yield dynamical compactification. In particular, we emphasize that our discussion unifies several known phenomena and sheds new light on the strong coupling singularities of type I' in [17] and in heterotic M-theory [18] (and its lower bound on the 4d Newton's constant). There are several directions which we leave for future work, for instance:

- As is clear from our explicit examples, many constructions of this kind can be obtained via a reinterpretation of known compactifications. This strongly suggests that our lessons have a general validity in string theory. It would be interesting to explore the discussion of tadpoles, cobordism and spontaneous compactifications in general setups beyond tori.

- A general consequence of (1.1) is a non-decoupling of scales between the geometric scales controlling the order parameter of the dynamical tadpole and the geometric size of the spontaneously compactified dimensions. This is reminiscent of the swampland AdS distance conjecture [19]. It would be interesting to explore the generation of hierarchies between the two scales, possibly based on discrete $\mathbf{Z}_k$ gauge symmetries as in [20].

- Our picture can be regarded as belonging to the rich field of swampland constraints on quantum gravity [21] (see [22–24] for reviews). It would be interesting to study the interplay with other swampland constraints. In particular, the relation between the strength of the dynamical tadpole and the size of the spacetime dimensions is tantalizingly reminiscent of the first condition on $|\nabla V|/V$ of the de Sitter conjecture [25–27], with $\mathcal{T} = |\nabla V|$ and if we interpret V as the inverse Hubble volume and hence a measure of size or length scale in the spacetime dimensions. It would be interesting to explore cosmological setups and a possible role of horizons as alternative mechanisms to cut off spacetime. Also, the inequality admittedly works in different directions in the two setups, thus suggesting they are not equivalent, but complementary relations.

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- It would be interesting to apply our ideas to the study of other setups in which spacetime is effectively cut off, such as the capping off of the throat in near horizon NS5-branes due to strong coupling effects, or the truncation in [28] of throats of the euclidean wormholes in pure Einstein+axion theories [29].

- Finally, we have not discussed time-dependent backgrounds.[2] These are obviously highly interesting, but their proper understanding is likely to require new ingredients, such as *end (or beginning) of time* defects (possibly as generalization of the spacelike S-branes [30, 31]).

Until we come back to these questions in future work, the present paper is organized as follows. In section 2 we reinterpret the Klebanov-Strassler (KS) warped throat supported by 3-form fluxes as a template illustrating our two tadpole lessons. Section 2.1 explains that the introduction of RR 3-form flux in type IIB theory on $\mathrm{AdS}_5 \times T^{1,1}$ produces a tadpole. The varying field configuration is the Klebanov-Tseytlin solution, which leads to a metric singularity at a finite distance scaling as (1.1), as we show in section 2.2. In section 2.3 we relate the KS smoothing of this singularity with cobordism defects. In section 2.4 we extend the discussion to other warped throats. In section 3 we present a similar discussion in toroidal compactifications with fluxes. Section 3.1 introduces a $\mathbf{T}_5$ compactification with RR 3-form flux, whose tadpole backreacts producing singularites at finite distance as we show in section 3.2. In section 3.3 we argue they are smoothed out by capping off dimensions and triggering spontaneous compactification. In section 4 we build examples in the context of magnetized D-branes. In section 4.1 we describe the tadpole backreaction and its singularities, which are removed by spontaneous compactification in section 4.2. In section 5 we turn to the dilaton tadpole of several 10d strings. In section 5.1 we consider massive type IIA theory, where the running dilaton solutions produce dynamical cobordisms by introduction of O8-planes as cobordism defects of the IIA theory, eventually closely related to type I' compactifications. In section 5.2 we discuss a similar picture for M-theory on K3 with G_4 flux, and a Horava-Witten wall as its cobordism defect. In section 5.3 we consider the 10d non-supersymmetric USp(32) theory, in two different approaches. In section 5.3.1 we build on the classical solution in [9] and discuss its singularities in the light of the cobordism conjecture. In section 5.3.2 we describe an explicit (and remarkably, supersymmetry preserving) configuration solving its tadpole via magnetization and spontaneous compactification on $\mathbf{T}^6$. In section 6 we discuss an interesting application, describing a 6d model with tadpoles, which upon spontaneous compactification reproduces a semi-realistic MSSM-like brane model. Finally, appendix A discusses the violation of swampland constraints of type IIB on $\mathrm{AdS}_5 \times T^{1,1}$ when its tadpole is not duly backreacted, in a new example of the mechanism in [3].

2 The fluxed conifold: KS solution as spontaneous cobordism

In this section we consider the question of dynamical tadpoles and their consequences in a particular setup, based on the gravity dual of the field theory of D3-branes at a coni-

[2] For classical solutions of tadpoles involving time dependence, see e.g. [11].

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fold singularity. The discussion is a reinterpretation, in terms useful for our purposes, of the construction of the Klebanov-Tseytlin (KT) solution [32] and its deformed avatar, the Klebanov-Strassler (KS) solution [33]. This reinterpretation however provides an illuminating template to discuss dynamical tadpoles in other setups in later sections.

We consider type IIB on $\mathrm{AdS}_5 \times T^{1,1}$, where $T^{1,1}$ is topologically $\mathbf{S}^2 \times \mathbf{S}^3$ [34]. This is the near horizon geometry of D3-branes at the conifold singularity [34] (see also [35–37]), which has been widely exploited in the context of holographic dualities. The vacuum is characterized by the IIB string coupling $e^{\phi} = g_s$ and the RR 5-form flux N. The model has no scale separation, since the $T^{1,1}$ and AdS_5 have a common scale R, given by

$$R^4 = 4\pi\, g_s N \alpha'^2 \,. \tag{2.1}$$

In any event, we will find useful to discuss the model, and its modifications, in terms of the (KT) 5d effective theory introduced in [32]. This is an effective theory not in the Wilsonian sense but in the sense of encoding the degrees of freedom surviving a consistent truncation. In particular, it includes the dilaton ϕ (we take vanishing RR axion for simplicity), the NSNS axion $\Phi = \int_{\mathbf{S}^2} B_2$ and the $T^{1,1}$ breathing mode q (actually, stabilized by a potential arising from the curvature and the 5-form flux), which in the Einstein frame enters the metric as

$$ds_{10}^2 = R^2 \left(e^{-5q}\, ds_5^2 + e^{3q} ds_{T^{1,1}}^2 \right) \,. \tag{2.2}$$

This approach proved useful in [38] in the discussion of the swampland distance conjecture [39] in configurations with spacetime-dependent field configurations (see [19] for a related subsequent development, and [40, 41]).

2.1 The 5d tadpole and its solution

Let us introduce M units of RR 3-form flux in the $\mathbf{S}^3$, namely

$$F_3 = M\, \omega_3 \,, \tag{2.3}$$

where ω_3 is defined in eq. (27) in [33]. We do not need its explicit expression, it suffices to say that it describes a constant field strength density over the $\mathbf{S}^3$. The introduction of this flux sources a backreaction on the dilaton and the metric, namely a dynamical tadpole for ϕ and q. In addition, as noticed in [38], it leads to an axion monodromy potential for Φ [42–45]. The situation is captured by the KT effective action (with small notation changes) for the 5d scalars ϕ, Φ and q, collectively denoted by φ^a

$$S_5 = -\frac{2}{\kappa_5^2} \int d^5x\, \sqrt{-g_5} \Big[\frac{1}{4} R_5 - \frac{1}{2} G_{ab}(\varphi) \partial\varphi^a \partial\varphi^b - V(\varphi) \Big], \tag{2.4}$$

with the kinetic terms and potential given by

$$G_{ab}(\varphi)\partial\varphi^a\partial\varphi^b = 15(\partial q)^2 + \frac{1}{4}(\partial\phi)^2 + \frac{1}{4} e^{-\phi-6q} (\partial\Phi)^2 \,, \tag{2.5}$$

$$V(\varphi) = -5e^{-8q} + \frac{1}{8} M^2\, e^{\phi-14q} + \frac{1}{8}(N + M\Phi)^2 e^{-20q} \,. \tag{2.6}$$

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Clearly $g_s M^2$ is an order parameter of the corresponding dynamical tadpole. In the following we focus on the case[3] of N being a multiple of M.

Ignoring the backreaction of the dynamical tadpole (i.e. considering constant profiles for the scalars over the 5d spacetime) is clearly incompatible with the equations of motion. Furthermore, as argued in [3], it can lead to violations of swampland constraints. In particular, since the introduction of F_3 breaks supersymmetry, if the resulting configuration was assumed to define a stable vacuum, it would violate the non-susy AdS conjecture [46]; also, as we discuss in appendix A, it potentially violates the Weak Gravity Conjecture [47].

Hence, we are forced to consider spacetime-dependent scalar profiles to solve the equations of motion. Actually, this problem was tackled in [33], with the scalars running with r, as we now review in the interpretation in [38]. There is a non-trivial profile for the axion Φ, given by

$$\Phi = 3 g_s M \, \log(r/r_0) \, . \tag{2.7}$$

This implies the cancellation of the dilaton tadpole, which can be kept constant $e^\phi = g_s$, as follows from its equation of motion from (2.5), (2.6)

$$\nabla\phi \sim -e^{-6q-\phi}(\partial\Phi)^2 + e^{-14q+\phi} M^2 \, . \tag{2.8}$$

2.2 Singularity at finite distance

The varying Φ corresponds to the introduction of an NSNS 3-form flux in the configuration

$$H_3 = -g_s \, *_{6d} \, F_3 \, , \tag{2.9}$$

where the 6d refers to $T^{1,1}$ and the AdS_5 radial coordinate r, and the Hodge duality is with the $\mathrm{AdS}_5 \times T^{1,1}$ metric. This is precisely such that the complexified flux combination $G_3 = F_3 - \tau H_3$ satisfies the imaginary self duality (ISD) constraint making it compatible with 4d Poincaré invariance in the remaining 4d coordinates (and in fact, it also preserves supersymmetry). The backreaction on the metric thus has the structure in [48, 49]. The metric (2.2) takes the form

$$ds^2_{10} = Z^{-\frac{1}{2}} \eta_{\mu\nu} dx^\mu dx^\nu + Z^{\frac{1}{2}} \left(dr^2 + r^2 ds^2_{T^{1,1}} \right) , \tag{2.10}$$

where Z obeys a Laplace equation in AdS_5, sourced by the fluxes, and reads

$$Z(r) = \frac{1}{4r^4} \, (g_s M)^2 \, \log(r/r_0) \, . \tag{2.11}$$

The warp factor also enters in the RR 5-form flux, which decreases with r as

$$N(r) = \int_{\mathbf{S}^5} F_5 = g_s M^2 \log(r/r_0) \, . \tag{2.12}$$

This matches nicely with the monodromy for the axion Φ as it runs with r [38]. These features (as well as some other upcoming ones) were nicely explained as the gravity dual of a Seiberg duality cascade in [33].

[3]This implies that the configuration is uncharged under a discrete $\mathbf{Z}_M$ symmetry, measured by N mod M, and associated to the redundancy generated by transformation $\phi \to \phi + 1$, $N \to N - M$, see footnote 5.

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This 5d running solution in [32] solves the dynamical tadpole, but is not complete, as it develops a metric singularity at $r = r_0$. This is a physical singularity at finite distance in spacetime, whose parametric dependence on the parameters of the initial model is as follows

$$\begin{aligned}\Delta(r) &= \int_{r_0}^{r} Z(r)^{\frac{1}{4}}\, dr \sim \int_{r_0}^{r} (g_s M)^{\frac{1}{2}} \left[\log(r/r_0)\right]^{\frac{1}{4}} \frac{dr}{r} \\ &\sim (g_s M)^{\frac{1}{2}} \left[\log(r/r_0)\right]^{\frac{5}{4}} = (g_s N)^{\frac{1}{4}} \frac{N}{g_s M^2} \sim R\, \frac{N}{g_s M^2}\,. \end{aligned} \tag{2.13}$$

In the last equalities we used (2.12), (2.1). Hence, starting with an $\mathrm{AdS}_5 \times T^{1,1}$ theory with N units of RR 5-form flux, the introduction of M units of RR 3-form flux leads to a breakdown of the corresponding spacetime-dependent solution at a distance scaling as $\Delta \sim M^{-2}$. Recalling that the dynamical tadpole is controlled by an order parameter $\mathcal{T} = g_s M^2$, this precisely matches the scaling relation (1.1) of the Finite Distance Lesson.

2.3 Dynamical cobordism and the KS solution

As is well known, the singularity in the KT solution is smoothed out in the KS solution [33]. This is given by a warped version of the deformed conifold metric, instead of the conical conifold singularity, with warp factor again sourced by an ISD combination of RR 3-form flux on $\mathbf{S}^3$ and NSNS 3-form flux on $\mathbf{S}^2$ times the radial coordinate. At large r the KS solution asymptotes to the KT solution, but near $r \sim r_0$, the solutions differ and the KT singularity is replaced by the finite size $\mathbf{S}^3$ of the deformed conifold.

Hence, the Finite Distance Lesson still applies even when the singularity is removed, and the impossibility to extend the coordinate r to arbitrary distances is implemented by a smooth physical end of spacetime. The purpose of this section is to highlight a novel insight on the KS solution, as a non-trivial realization of the swampland cobordism conjecture [14, 15]. The latter establishes that any consistent quantum gravity theory must be trivial in (a suitably defined version of) cobordism. Namely in an initial theory given by an n-dimensional internal compactification space (possibly decorated with additional ingredients, like branes or fluxes), there must exist configurations describing an $(n+1)$-dimensional (possibly decorated) geometry whose boundary is the initial one. The latter describes an end of the world defect (which we will refer to as the 'cobordism defect') for the spacetime of the initial theory. Since the arguments about the swampland cobordism conjecture are topological, there is no claim about the unprotected properties of the cobordism defect, although in concrete examples it can preserve supersymmetry; for instance, in maximal dimensions, the Horava-Witten boundary is the cobordism defect for 11d M-theory, and similarly the O8-plane is the cobordism defect of type IIA theory.[4]

In our setup, the initial theory is $\mathrm{AdS}_5 \times T^{1,1}$ with N units of RR 5-form flux and M units of RR 3-form flux on $\mathbf{S}^3$. From the above discussion, it is clear that the KS solution

[4]Other 10d theories are conjectured to admit cobordism branes, but they cannot be supersymmetric and their nature is expected to be fairly exotic, and remains largely unknown. We will come back to this point in section 5.3.1.

is just the cobordism defect of this theory.[5] The remarkable feature is that the end of spacetime is triggered dynamically by the requirement of solving the equations of motion after the introduction of the RR 3-form flux, hence it is fair to dub it dynamical cobordism. Hence, this is a very explicit illustration of the Dynamical Cobordism Lesson.

This powerful statement will be realized in many subsequent examples in later sections, and will underlie the phenomenon of spontaneous compactification, when the cobordisms close off the spacetime directions bounding them into a compact variety.

2.4 More general throats

A natural question is the extension of the above discussion to other $\mathrm{AdS}_5 \times \mathbf{X}_5$ vacua with 3-form fluxes. This question is closely related to the search for general classes of gravity duals to Seiberg duality cascades and their infrared deformations, for which there is a concrete answer if $\mathbf{X}_5$ is the real base of a non-compact toric CY threefold singularity $\mathbf{Y}_6$, which are very tractable using dimer diagrams [50, 51] (see [52] for a review).

From our perspective, the result in [53] is that the $\mathbf{X}_5$ compactification with 3-form flux F_3 admits a KS-like end of the world (cobordism defect[6]) if $\mathbf{Y}_6$ admits a complex deformation which replaces its conical singularity by a finite-size 3-cycle corresponding to the homology dual of the class $[F_3]$. In cobordism conjecture terms, in these configurations the corresponding global symmetry is broken, and spacetime may close off without further ado (as the axion monodromy due to the 3-form fluxes allows to eat up the RR 5-form flux before reaching the end of the world). Such complex deformations are easily discussed in terms of the web diagram for the toric threefold, as the splitting of the web diagram into consistent sub-diagrams [53]. Simple examples include the deformation of the complex cone over dP_2 to a smooth geometry, or the deformation of the complex cone over dP_3 to a conifold, or to a smooth geometry.

There are however singularities (or 3-form flux assignments), for which the complex deformations are simply not available. One may then wonder about how our Dynamical Cobordism lesson applies. The answer was provided in particular examples in [54–56]: the infrared end of the throat contains an explicit system of fractional D-branes, which in the language of the cobordism conjecture kill the corresponding cobordism classes, and allow the spacetime to end. As noticed in these references, the system breaks supersymmetry, and in [55] it was moreover noticed (as later revisited in [57]) to be unstable and lead to a runaway behaviour for the field blowing up the singularity. Hence, this corresponds to an additional dynamical tadpole, requiring additional spacetime dependence, to be solved. Simple examples include the complex cone over dP_1, and the generic $Y^{p,q}$ theories. We will not enter the discussion of possible mechanisms to stabilize these models, since following [58] they are likely to require asymptotic modifications of the warped throat ansatz (i.e. at all positions in the radial direction, including the initial one).

[5]Recalling footnote 3, the case of N multiple of M implies the vanishing of a $\mathbf{Z}_M$ charge, and allows the cobordism defect to be purely geometrical; otherwise the cobordism defect ending spacetime must include explicit D3-branes, which are the defect killing the corresponding cobordism class [14].

[6]We note in passing that the regions between different throats in the multi-throat configurations [53] can be regarded as domain walls interpolating between two different, but bordant, type IIB vacua.

3 Type IIB fluxes and spontaneous compactification

In this section we construct an explicit 5d type IIB model with a tunable dynamical tadpole, and describe the spacetime-dependent solution solving its equations of motion, which is in fact supersymmetry preserving. The configuration displays dynamical cobordism resulting in spontaneous compactification to 4d. The resulting model is a simple toroidal compactification with ISD NSNS and RR 3-form fluxes [48, 49], in particular it appeared in [59, 60]. With this perspective in hindsight, one can regard this section as a reinterpretation of the latter flux compactification. Our emphasis is however in showing the interplay of the dynamical tadpoles in the 5d theory and the consequences in the spacetime configuration solving them.

3.1 The 5d tadpole and its solution

Consider type IIB on $\mathbf{T}^5$, which for simplicity we consider split as $\mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{S}^1$. We label the coordinates of the $\mathbf{T}^2$'s as (x^1, y^1) and (x^2, y^2), with periodicity 1, and introduce complex coordinates as $z^1 = x^1 + \tau_1 y^1$, $z^2 = x^2 + \tau_2 y^2$. We also use a periodic coordinate $x^3 \simeq x^3 + 1$ to parametrize the $\mathbf{S}^1$. For simplicity, we do not consider moduli deviating from this rectangular structure,[7] and also take the $\mathbf{T}^5$ to have an overall radius R,

$$ds^2 = R^2[(dz^1)^2 + (dz^2)^2 + (dx^3)^2] \,. \tag{3.1}$$

The result so far is a standard 5d supersymmetric $\mathbf{T}^5$ compactification.

We introduce a non-trivial dynamical tadpole source by turning on an RR 3-form flux (using conventions in [49])

$$F_3 = (2\pi)^2 \alpha' \, N \, dx^1 \, dx^2 \, dx^3 \,. \tag{3.2}$$

The introduction of this flux does not lead to RR topological tadpoles, but induces dynamical tadpoles for diverse fields. In the following we focus on the dynamics of the 5d light fields R, τ_1, τ_2, the dilaton ϕ and the NSNS axion Φ defined by

$$B_2 = \Phi \, dy^1 \, dy^2 \,. \tag{3.3}$$

The discussion of the dynamical tadpole is similar to the $T^{1,1}$ example in section 2, so we sketch the result. There is a dilaton tadpole, arising from the dimensional reduction of the 10d kinetic term for the 3-form flux,

$$\nabla^2 \phi = \frac{1}{12} e^{\phi} \, (F_3)^2 \,. \tag{3.4}$$

Since $(F_3)^2$ is a constant source density, which does not integrate to zero over $\mathbf{T}^5$, there is no solution for this Laplace equation if we assume the solution to be independent of the 5d spacetime coordinates. One possibility would be to allow for 5d spacetime dependence of ϕ (at least in one extra coordinate, as in [9]). Here we consider a different possibility, which

[7] As usual, they can be removed in orbifold models, although we will not focus on this possibility.

is to let the NSNS axion Φ acquire a dependence on one of the 5d coordinates, which we denote by y, as follows

$$\Phi = -(2\pi)^2\alpha'\,\frac{N}{t_3}\,y \quad \Rightarrow \quad H_3 = -(2\pi)^2\alpha'\,\frac{N}{t_3}\,dy^1\,dy^2\,dy\,. \tag{3.5}$$

We have thus turned on NSNS 3-form field strength in the directions y^1, y^2 in $\mathbf{T}^5$ and the 5d spacetime coordinate y. Here the sign has been introduced for later convenience, and t_3 is a positive real parameter allowing to tune the field strength density, whose meaning will become clear later on.

Including this new source, the dilaton equation of motion becomes

$$\nabla^2\,\phi = \frac{1}{12}\left[e^{\phi}\,(F_3)^2 - e^{-\phi}\,(H_3)^2\right]. \tag{3.6}$$

Hence, the spacetime-dependent profile (3.5) can cancel the right hand side and solve the dilaton tadpole when

$$e^{2\phi}\,(F_3)^2 = (H_3)^2\,. \tag{3.7}$$

We can thus keep the dilaton constant $e^{\phi} = g_s$. Taking for simplicity purely imaginary $\tau_1 = it_1$ and $\tau_2 = it_2$, the condition (3.7) is simply

$$g_s\,t_1\,t_2\,t_3 = 1\,. \tag{3.8}$$

In addition to the dilaton, the 3-form fluxes backreact on the metric and other fields, which we discuss next.

3.2 The singularities

We now discuss the backreaction on the metric and other fields. For convenience, we use the complex coordinates z^1, z^2 and $z^3 = x^3 + iy$. In terms of these, we can write the combination

$$G_3 = F_3 - \tau H_3 = \frac{(2\pi)^2}{4}\alpha'\,N(d\bar{z}_1\,dz_2\,dz_3 + dz_1 d\bar{z}_2 dz_3 + dz_1 dz_2 d\bar{z}_3 + d\bar{z}_1 d\bar{z}_2 d\bar{z}_3)\,. \tag{3.9}$$

Regarding $\mathbf{T}^5 \times \mathbf{R}^1_y$ as a (non-compact) CY, this is a combination of $(2,1)$ and $(0,3)$ components, which is thus ISD. There is a backreaction on the metric and RR 4-form of the familiar black 3-brane kind. In particular, the metric includes a warp factor Z

$$ds^2_{10} = Z^{-\frac{1}{2}}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + Z^{\frac{1}{2}}\,R^2[dz^1 d\bar{z}^1 + dz^2 d\bar{z}^2 + dz^3 d\bar{z}^3]\,, \tag{3.10}$$

where x^{μ} runs through the four Poincaré invariant spacetime coordinates. The warp factor is determined by the Laplace equation

$$-\,\tilde{\nabla}^2 Z = \frac{g_s}{12} G_3 \cdot \overline{G}_3 = \frac{g_s}{6}\,(F_3)^2\,, \tag{3.11}$$

with the tilde indicating the Laplacian is computed with respect to the unwarped, flat metric, and in the last equation we used (3.7).

Note that, since y parametrizes a non-compact dimension, there is no tadpole problem in solving (3.11) i.e. we need not add background charge. One may then be tempted to conclude that this provides a 5d spacetime-dependent configuration solving the 5d tadpole. However, the solution is valid locally in y, but cannot be extended to arbitrary distances in this direction. Since the local flux density in $\mathbf{T}^5$ is constant, we can take Z to depend only on[8] y, hence leading to a solution

$$-\frac{d^2Z}{dy^2} = \frac{g_s}{6}(F_3)^2 \quad \Rightarrow \quad Z = 1 - \frac{g_s}{12}(F_3)^2\, y^2\,, \tag{3.12}$$

where we have set an integration constant to 1. The solution hits metric singularities at

$$y^{-2} = \frac{1}{12} g_s (F_3)^2\,, \tag{3.13}$$

showing there is a maximal extent in the direction y. Let us introduce the quantity $\mathcal{T} = \frac{1}{12} g_s (F_3)^2$, which controls the parametric dependence of the tadpole. Then, the distance between the singularities is

$$\Delta = \int_{-\mathcal{T}^{-1/2}}^{\mathcal{T}^{-1/2}} Z^{\frac{1}{4}}\, dy = \frac{2}{\sqrt{\mathcal{T}}} \int_0^1 (1-t^2)^{\frac{1}{4}}\, dt\,, \tag{3.14}$$

with $t = \sqrt{\mathcal{T}}y$. We thus recover the scaling (1.1) with $n = 2$,

$$\Delta^{-2} \sim \mathcal{T}\,. \tag{3.15}$$

Hence the appearance of the singularities as a consequence of the dynamical tadpole is as explained in the introduction.

3.3 Cobordism and spontaneous compactification

The appearance of singularities is a familiar phenomenon. In this section we argue that they must be smoothed out, somewhat analogously to the KS solution in section 2. The fact that it is possible follows from the swampland cobordism conjecture [14, 15], namely there must exist an appropriate cobordism defect closing off the extra dimension into nothing. Since there are two singularities, the formerly non-compact dimension becomes compact, in an explicit realization of spontaneous compactification.[9]

In the following, we directly describe the resulting geometry, which turns out to be a familiar $\mathbf{T}^6$ (orientifold) compactification with ISD 3-form fluxes. Consider type IIB theory on $\mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$, with

$$F_3 = (2\pi)^2 \alpha' \, N \, dx^1\, dx^2\, dx^3\,, \quad H_3 = (2\pi)^2 \alpha' \, N \, dy^1\, dy^2\, dy^3\,. \tag{3.16}$$

We use $z^i = x^i + it_i y^i$, hence the above defined t_3 is the complex structure modulus for the $\mathbf{T}^2$ involving the newly compact dimension. For moduli satisfying (3.8) the $\mathbf{T}^6$ flux combination G_3 is given by (3.9), which is ISD and indeed compatible with 4d Poincaré

[8] In fact, this is the leading behaviour at long distances, compared with the $\mathbf{T}^5$ size scale R.
[9] Spontaneous compactification has been discussed in the context of dynamical tadpoles in [9].

invariance as usual. Notice that in this case, it is possible to achieve a large size for the new compact dimension $t_3 \gg 1$ by simply e.g. taking small g_s. This corresponds to the regime of small 5d tadpole, with the relation

$$t_3^{-2} \sim g_s^2 \sim \mathcal{T}^2 \,, \tag{3.17}$$

in agreement with the maximal distance relation in the previous section.

Consistency, in the form of C_4 RR tadpole cancellation, requires the introduction of O3-planes at fixed points of the involution $\mathcal{R} : z^i \to -z^i$ (together with mobile D3-branes). From the perspective of the 5d theory, the additional dimension is compactified on an interval, with two end of the world defects given by the O3-planes, which constitute the cobordism defects of the configuration (possibly decorated with explicit D3-branes if needed).

4 Solving dynamical tadpoles via magnetization

In this section we consider a further setup displaying dynamical tadpoles, based on compactifications with magnetized D-branes [61–65]. In toroidal setups, these have been (either directly or via their T-dual intersecting brane world picture) widely used to realize semi-realistic particle physics models in string theory. In more general setups, magnetized 7-branes are a key ingredient in the F-theory realization of particle physics models [66–68].

4.1 Solving dynamical tadpoles of magnetized branes

We consider a simple illustrative example. Consider type IIB theory compactified on $\mathbf{T}^2 \times \mathbf{T}^2$ (labelled 1 and 2, respectively) and mod out by $\Omega\mathcal{R}_1(-1)^{F_L}$, where $\mathcal{R}_1 : z_1 \to -z_1$. This introduces 4 $\mathrm{O7}_1$-planes spanning $(\mathbf{T}^2)_2$ and localized at the fixed points on $(\mathbf{T}^2)_1$. We also have 32 D7-branes (as counted in the covering space), split as 16 D7-branes (taken at generic points) and their 16 orientifold images. This model is related by T-duality on $(\mathbf{T}^2)_1$ to a type I toroidal compactification, but we proceed with the D7-brane picture.

We introduce M units of worldvolume magnetic flux along $(\mathbf{T}^2)_2$ for the U(1) of a D7-brane[10]

$$\frac{1}{2\pi\alpha'} \int_{\mathbf{T}^2} F_2 = M \,. \tag{4.1}$$

The orientifold requires we introduce $-M$ units of flux on the image D7-brane.[11] This also ensures that there is no net induced $\mathbf{Z}$-valued D5-brane charge in the model, and hence no associated RR tadpole, in agreement with the fact that the RR 6-form is projected out. In addition, there is a $\mathbf{Z}_2$ K-theory charge [70] which is cancelled as long as $M \in 2\mathbf{Z}$.

The introduction of the worldvolume flux leads to breaking of supersymmetry. As is familiar in the discussion of supersymmetries preserved by different branes [71], we introduce the angle

$$\theta_2 = \arctan(2\pi\alpha' F) = \arctan\left(M\chi\right) , \tag{4.2}$$

where F is the field strength and χ is the inverse of the $\mathbf{T}^2$ area, in string units.

[10] If N the D7-branes are coincident, it is also possible to use the overall $\mathrm{U}(1) \subset \mathrm{U}(N)$. We will stick to the single D7-brane for the moment, but such generalization will arise in later examples.

[11] For simplicity we consider vanishing discrete NSNS 2-form flux [69], although such generalization will arise in later examples.

This non-supersymmetric configuration introduces dynamical tadpoles. For small θ_2, the extra tension can be described in effective field theory as an FI term controlled by θ [72–74]. In fact, in [75] a similar parametrization was proposed for arbitrary angles. By using the DBI action, the extra tension has the structure

$$V \sim \frac{1}{g_s}\left(\sqrt{1+(\tan\theta_2)^2}-1\right). \tag{4.3}$$

This leads to a tadpole for the dilaton and the $(\mathbf{T}^2)_2$ Kähler modulus.

We now consider solving the tadpole by allowing for some spacetime-dependent background. Concretely, we allow for a non-trivial magnetic field $-F$ on two of the non-compact space coordinates, parametrized by the (for the moment, non-compact) coordinate z_3. In fact this leads to a configuration preserving supersymmetry since, defining the angle θ_3 in analogy with (4.2), we satisfy the SU(2) rotation relation $\theta_3+\theta_2=0$ [71]. In other words, the field strength flux has the structure

$$F_2 = F(dz_2 d\bar{z}_2 - dz_3 d\bar{z}_3)\,, \tag{4.4}$$

which is $(1,1)$ and primitive (i.e. $J\wedge F_2=0$), which are the supersymmetry conditions for a D-brane worldvolume flux.

Hence, it is straightforward to find spacetime-dependent solutions to the tadpole of the higher-dimensional theory, at the price of breaking part of the symmetry of the lower-dimensional spacetime. In the following we show that, as in earlier examples, this eventually also leads to spontaneous compactification.

4.2 Backreaction and spontaneous compactification

The spacetime field strength we have just introduced couples to gravity and other fields, so we need to discuss its backreaction.

In fact, this is a particular instance of earlier discussions, by considering the F-theory lift of the D7-brane construction. This can be done very explicitly by taking the configuration near the $\mathrm{SO}(8)^4$ weak coupling regime [76]. The configuration without magnetic flux $M=0$ simply lifts to F-theory on $\mathrm{K3}\times\mathbf{T}^2\times\mathbf{R}^2$, where the $(\mathbf{T}^2)_1$ (modulo the $\mathbf{Z}_2$ orientifold action) is the $\mathbf{P}_1$ base of K3, and the $\mathbf{T}^2$ and $\mathbf{R}^2$ explicit factors correspond to the directions z_2 and z_3, respectively. As is familiar, the 24 degenerate fibers of the K3 elliptic fibration form 4 pairs, reproducing the 4 orientifold planes, and 16 D7-branes in the orientifold quotient. Actually, the discussion below may be carried out for F-theory on K3 at generic points in moduli space, even not close to the weak coupling point.

The introduction of magnetization for one 7-brane corresponds to the introduction of a G_4 flux along the local harmonic $(1,1)$-form supported at an I_1 degeneration (or enhanced versions thereof, for coincident objects), of the form

$$G_4 = \omega_2\wedge F(dz_2 d\bar{z}_2 - dz_3 d\bar{z}_3)\,. \tag{4.5}$$

This flux is self-dual, and in fact $(2,2)$ and primitive, which is the supersymmetry preserving condition for 4-form fluxes in M/F-theory [48, 77]. The backreacted metric is described by a warp factor satisfying a Laplace equation sourced by the fluxes, similar to (3.11). Considering the regime in which the warp factor is taken independent of the internal space and depends only on the coordinates in the $\mathbf{R}^2$ parametrized by z_3, the constant flux density leads to singularities at a maximal length scale Δ

$$\Delta^{-2} \sim F^2\,. \tag{4.6}$$

This is another instance of the universal relation (1.1) with $\mathcal{T} \sim F^2$, hence $n = 2$.

This is in complete analogy with earlier examples. Hence, we are led to propose that the smoothing out of these singularities is provided by the compactification of the corresponding coordinates, e.g. on a $\mathbf{T}^2$, with the addition of the necessary cobordism defects, namely orientifold planes and D-branes.[12]

To provide an explicit solution, we introduce the standard notation (see e.g. [63, 64]) of (n, m) for the wrapping numbers and the magnetic flux quanta on the $(\mathbf{T}^2)_i$'s for the directions $i = 1, 2, 3$. In this notation, the $O7_1$-planes and unmagnetized $D7_1$-branes are associated to $(0, 1) \times (1, 0) \times (1, 0)$, while the magnetized $D7_1$-branes[13] correspond to $(0, 1) \times (1, M) \times (1, -M)$, and $(0, 1) \times (1, -M) \times (1, M)$ for the orientifold images. In other words, we require a flux quantization condition on $(\mathbf{T}^2)_3$ as in (4.1), up to a sign flip.

Since now the last complex dimension is compact, there is an extra RR tadpole cancellation condition, which requires the introduction of 16 $O7_3$-planes, wrapped on $(\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2$ and localized at fixed points in $(\mathbf{T}^2)_1$, namely with wrapping numbers $(1, 0) \times (1, 0) \times (0, 1)$. This introduces an extra orbifold action generated by $(z_1, z_2, z_3) \to (z_1, -z_2, -z_3)$, so the model can be regarded as a (T-dual of a) magnetized version of the D9/D5-brane $\mathbf{T}^4/\mathbf{Z}_2$ orientifolds in [78, 79]. Allowing for n additional mobile $D7_3$-branes (as counted in the covering space, and arranged in orbifold and orientifold invariant sets), the RR tadpole cancellation conditions is

$$2M^2 + n = 32\,. \tag{4.7}$$

The supersymmetry condition is simply that the $\mathbf{T}^2$ parameters satisfy $\chi_2 = \chi_3$.

From the perspective of the original 6d configuration, the tadpole in the initial $\mathbf{T}^2 \times \mathbf{T}^2$ configuration has triggered a spontaneous compactification. Since the additional O-planes and D-branes required to cancel the new RR tadpoles are localized in z_3, they can be interpreted as the addition of I-branes to cancel the cobordism charge of the original model.

It should be possible to generalize the above kind of construction to global K3-fibered CY threefolds with O7-planes. The local fibration in a small neighbourhood of a generic point of the base provides a local 6d model essentially identical to our previous one. On

[12] To be precise, the cobordism defects of an $\mathbf{S}^1$ compactification of the model. This is analogous to the mechanism by which F-theory on half a $\mathbf{P}_1$ provides the cobordism defect for type IIB on $\mathbf{S}^1$ with $\mathrm{SL}(2, \mathbf{Z})$ monodromy [14] (see also [16]). In fact, since magnetized branes often lead to chiral theories in the bulk, this extra circle compactification allows them to become non-chiral and admit an end of the world describable at weak coupling, see the discussion below (5.14) in section 5.3. We will nevertheless abuse language and refer as cobordism defect to the structures involved in the final spontaneous compactification under discussion.

[13] If the magnetization is in the $\mathrm{U}(1) \subset \mathrm{U}(N)$ of a stack of N coincident branes, see footnote 10, the corresponding wrapping goes as (N, M).

the other hand, the global geometry defining how the two extra dimensions compactify would correspond to another possible spontaneous compactification, with the ingredients required for the cancellation of the new RR tadpoles.

However, a general drawback of this class of models is that the scales of the compact spaces in the directions 2 and 3 are of the same order.[14] Thus, there is no separation of scales, and no reliable regime in which the dynamics becomes that of a 6d model. This is easily avoided in more involved models, as we will see in the examples in coming sections.

5 Solving tadpoles in 10d strings

In this section we consider dynamical tadpoles arising in several 10d string theories, and confirm the general picture. We illustrate this with various examples, with superymmetry (massive type IIA and M-theory on K3), and without it (non-supersymmetric 10d USp(32) theory).

5.1 Massive IIA theory

We consider 10d massive type IIA theory [80]. This can be regarded as the usual type IIA string theory in the presence of an additional RR 0-form field strength $F_0 \equiv m$. The string frame effective action for the relevant fields is

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi}[R + 4(\partial\phi)^2] - \frac{1}{2}(F_0)^2 - \frac{1}{2}(F_4)^2 \right\} + S_{\text{top}} \,, \tag{5.1}$$

where S_{top} includes the Chern-Simons terms. In the Einstein frame $G_E = e^{-\frac{\phi}{2}}G$, we have

$$S_{10,E} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G_E} \left\{ \left[R - \frac{1}{2}(\partial\phi)^2 \right] - \frac{1}{2} e^{\frac{5}{2}\phi} m^2 - \frac{1}{2} e^{\frac{1}{2}\phi}(F_4)^2 \right\} . \tag{5.2}$$

Here we have used m to emphasize this quantity is constant. This theory is supersymmetric, but at a given value of ϕ, it has a tadpole controlled by

$$\mathcal{T} \sim e^{\frac{5}{2}\phi} m^2 \,. \tag{5.3}$$

This is in particular why the massive IIA theory does not admit 10d maximally symmetric solutions. In the following we discuss two different ways of solving it, leading to Minkowski or AdS-like configurations.

5.1.1 Solution in 9d and type I' as cobordism

To solve the tadpole (5.3) we can consider a well-known 1/2 BPS solution with the dilaton depending on one coordinate x^9. Since the flux m can be regarded as generated by a set of m distant D8-branes, this is closely related to the solution in [81]. We describe it in

[14] In the toroidal example, if the magnetization along $(\mathbf{T}^2)_2$ is on the overall $\mathrm{U}(1) \subset \mathrm{U}(16)$ of 16 coincident D7-branes, the magnetic field along z_2 is $F \sim M/16$; this weakened tadpole implies an increase of the critical size of the spontaneously compactified dimensions by a factor of 4.

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conventions closer to [17], for later use. In the Einstein frame, the metric and dilaton background have the structure

$$(G_E)_{MN} = Z(x^9)^{\frac{1}{12}}\, \eta_{MN}\,, \quad e^{\phi} = Z(x^9)^{-\frac{5}{6}}\,, \quad \text{with } \; Z(x^9) \sim B - m x^9\,, \tag{5.4}$$

where B is some constant (in the picture of flux generated by distant D8-branes, it relates to the D8-brane tensions). The solution hits a singularity at $x^9 = B/m$. Starting at a general position x^9, the distance to the singularity is

$$\Delta = \int_{x^9}^{\frac{B}{m}} Z(x^9)^{\frac{1}{24}}\, dx^9 \sim Z(x^9)^{\frac{25}{24}}\, m^{-1} \sim m^{-1} e^{-\frac{5}{4}\phi}\,, \tag{5.5}$$

where in the last equality we have traded the position for the value the dilaton takes there. Recalling (5.3), this reproduces the Finite Distance scaling relation (1.1) with $n = 2$,

$$\Delta^{-2} \sim \mathcal{T}\,. \tag{5.6}$$

It is easy to propose the stringy mechanism capping off spacetime before or upon reaching this singularity, according to the Dynamical Cobordism lesson. This should be the cobordism defect of type IIA theory, which following [14] is an O8-plane, possibly with D8-branes.

In fact, this picture is implicitly already present in [17], which studies type I' theory, namely type IIA on an interval, namely IIA on $\mathbf{S}^1$ modded out by $\Omega\mathcal{R}$ with $\mathcal{R} : x^9 \to -x^9$, which introduces two O8$^-$-planes which constitute the interval boundaries. There are 32 D8-branes (in the covering space), distributed on the interval, which act as domain walls for the flux $F_0 = m$, which is piecewise constant in the interval. The metric and the dilaton profile are controlled by a piecewise linear function $Z(x^9)$. The location of the boundaries at points of strong coupling was crucial to prevent contradiction with the appearance of certain enhanced symmetries in the dual heterotic string (the role of strong coupling at the boundaries for the enhancements was also emphasized from a different perspective in [82, 83]). In our setup, we interpret the presence of (at least, one) O8-plane as the cobordism defect triggered by the presence of a dynamical tadpole in the bulk theory.

5.1.2 A non-supersymmetric Freund-Rubin solution

We now consider for illustration a different mechanism to cancel the dynamical tadpole, which in fact underlies the spontaneous compactification to (non-supersymmetric[15]) AdS$_4\times$ $\mathbf{S}^6$ in [80]). The idea is that, rather than solving for the dilaton directly, one can introduce an additional flux F_4 along three space dimensions and time (or its dual F_6 on six space dimensions) to balance off the dilaton sourced by F_0. This can be used to fix ϕ to a constant, and following [80] leads to a scaling

$$F_4 \sim m^2\, d(\mathrm{vol})_4\,, \tag{5.7}$$

[15]Thus, it should be unstable according to [46]. However, being at a maximum of a potential is sufficient to avoid dynamical tadpoles, so the solution suffices for our present purposes.

where $d(\mathrm{vol})_4$ is the volume form in the corresponding 4d. Using arguments familiar by now, the constant F_4 backreaction on the metric is encoded in a solution of the 4d Laplace equation with a constant source, leading to a solution quadratic in the coordinates (to avoid subtleties, we take solutions depending only on the space coordinates). This develops a singularity at a distance scaling as

$$\Delta^2 \sim |F_4|^{-2} \sim m^{-4} \sim \mathcal{T}\,, \tag{5.8}$$

where in comparison with (5.3) we have taken constant dilaton.

The singularities are avoided by an $\mathrm{AdS}_4 \times \mathbf{S}^6$ compactification, whose curvature radius is $R \sim m^2$, in agreement with the above scaling. From our perspective, the compactification should be regarded as a dynamical cobordism (where the cobordism is actually that of the 10d theory on an $\mathbf{S}^5$ (i.e. equator of $\mathbf{S}^5$)).

5.2 An aside on M-theory on K3

In this section we relate the above system to certain compactifications of M-theory and to the Horava-Witten end of the world branes as its cobordism defect. Although the results can be obtained by direct use of M-theory effective actions, we illustrate how they can be recovered by applying simple dualities to the above system.

Consider the above massive IIA theory with mass parameter m, and compactify on $\mathbf{T}^4/\mathbf{Z}_2$. This introduces O4-planes, and requires including 32 D4-branes in the configuration, either as localized sources, or dissolved as instantons on the D8-branes. Actually this can be considered as a simple model of K3 compactifications, where in the general K3 the O4-plane charge is replaced by the contribution to the RR C_5 tadpole arising from the CS couplings of D8-branes and O8-planes to $\mathrm{tr}\, R^2$.

We now perform a T-duality in all the directions of the $\mathbf{T}^4/\mathbf{Z}_2$ (Fourier-Mukai transform in the case of general K3). We obtain a similar model of type I' on $\mathbf{T}^4/\mathbf{Z}_2$, but now with the tadpole being associated to the presence of m units of non-trivial flux of the RR 4-form field-strength over $\mathbf{T}^4$ (namely, K3). Also, the dilaton of the original picture becomes related to the overall Kähler modulus of K3. Finally, we lift the configuration to M-theory by growing an extra $\mathbf{S}^1$ and decompactifying it. We thus end up with a 7d compactification of M-theory on K3, with m units of G_4 flux,

$$\int_{\mathrm{K3}} G_4 = m\,. \tag{5.9}$$

This leads to a dynamical tadpole, cancelled by the variation of the overall Kähler modulus (i.e. the K3 volume) along one the 7d space dimensions, which we denote by x^{11}. As in previous sections, this will trigger a singularity at a finite distance in x^{11}, related to the tadpole by $\Delta^{-2} \sim \mathcal{T}$. The singularity is avoided by the physical appearance of a cobordism defect, which for M-theory is a Horava-Witten (HW) boundary [84]. This indeed can support the degrees of freedom to kill the G_4 flux, as follows. From [85], the 11d G_4 is sourced by the boundary as

$$dG_4 = \delta(x^{11}) \left(\mathrm{tr}\, F^2 - \frac{1}{2} \mathrm{tr}\, R^2 \right), \tag{5.10}$$

where $\delta(x^{11})$ is a bump 1-form for the HW brane, and F is the field-strength for the E_8 gauge fields in the boundary. Hence, the m units of G_4 in the K3 compactification can be absorbed by a HW boundary with an E_8 bundle with instanton number $12 + m$ (the 12 coming from half the Euler characteristic of K3 $\int_{\mathrm{K3}} \mathrm{tr}\, R^2 = 24$).

The above discussion is closely related to the picture in [18], which discusses compactification of HW theory (namely, M-theory on an interval with two HW boundaries) on K3 and on a CY threefold. It includes a Kähler modulus varying over the interval according to a linear function[16] and the appearance of a singularity at finite distance. In that case, the HW brane was located at the strong coupling point, based on heuristic arguments, and this led, in the CY_3 case, to a lower bound on the value of the 4d Newton's constant.

Our perspective remarkably explains that the location of the HW wall is not an arbitrary choice, but follows our physical principle of Dynamical Cobordism, and the bound on the Newton's constant is a consequence of that of Finite Distance!

5.3 Solving tadpoles in the non-supersymmetric 10d USp(32) theory

The previous examples were based on an underlying supersymmetric vacuum, on top of which the dynamical tadpole is generated via the introduction of fluxes or other ingredients. In this section we consider the opposite situation, in which the initial theory is strongly non-supersymmetric and displays a dynamical tadpole from the start. In particular we consider the non-supersymmetric 10d USp(32) theory constructed in [4], in two different ways: first, we use our new insights to revisit the spacetime-dependent solution proposed in [9] (see also [10] for other proposals); then we present a far more tractable solution involving magnetization, which in fact provides a supersymmetric compactification of this non-supersymmetric 10d string theory.

5.3.1 The Dudas-Mourad solution and cobordism

The non-supersymmetric 10d USp(32) theory in [4] is obtained as an Ω orientifold of type IIB theory. The closed string sector is as in type I theory, except that the $\mathrm{O9}^-$-plane is replace by an $\mathrm{O9}^+$-plane. Cancellation of RR tadpoles requires the introduction of open strings, which must be associated to 32 $\overline{\mathrm{D9}}$-branes. The closed string sector is a 10d $\mathcal{N} = 1$ supergravity multiplet; the orientifold action on the D9-branes breaks supersymmetry, resulting in an open string sector with USp(32) gauge bosons and gauginos in the two-index antisymmetric representation. All anomalies cancel, a remarkable feat from the field theory viewpoint, which is just a consequence of RR tadpole cancellation from the string viewpoint.

Although the RR tadpoles cancel, the NSNS tadpoles do not, implying that there is no maximally symmetric 10d solution to the equations of motion. In particular there is a dynamical dilaton tadpole of order the string scale, as follows from the terms in the 10d (Einstein frame) action

$$S_E = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} \left[R - \frac{1}{2} (\partial\phi)^2 \right] - T_9^E \int d^{10}x \sqrt{-G}\, 64\, e^{\frac{3\phi}{2}} \,, \tag{5.11}$$

[16] In the presence of explicit M5-branes, it is a piecewise linear function. It is straightforward to include them in our cobordism description if wished, with explicit branes considered as part of the cobordism defect.

where T_9^E is the (anti)D9-brane tension. The tadpole scales as $\mathcal{T} \sim T_9^E g_s^{3/2}$, with the dilaton dependence arising from the fact that the supersymmetry breaking arises from the Moebius strip worldsheet topology, with $\chi = 3/2$.

Ref. [9] proposed solutions of this dynamical tadpole with 9d Poincaré invariance, an d the dilaton varying over one spacetime dimension (see also [86, 87] for more recent, related work). In the following we revisit the solution with dependence on one spatial coordinate y, from the vantage point of our Lessons.

The 10d solution is, in the Einstein frame,

$$\phi = \frac{3}{4}\alpha_E y^2 + \frac{2}{3}\log|\sqrt{\alpha_E}y| + \phi_0\,,$$
$$ds_E^2 = |\sqrt{\alpha_E}y|^{\frac{1}{9}}\, e^{-\frac{\alpha_E y^2}{8}}\eta_{\mu\nu}dx^\mu dx^\nu + |\sqrt{\alpha_E}y|^{-1}e^{-\frac{3\phi_0}{2}}e^{-\frac{9\alpha_E y^2}{8}}\,dy^2\,, \tag{5.12}$$

where $\alpha_E = 64k^2T_9$. There are two singularities, at $y = 0$ and $y \to \infty$, which despite appearances are separated by a finite distance

$$\Delta \sim \int_0^\infty |\sqrt{\alpha_E}y|^{-\frac{1}{2}}e^{-\frac{3\phi_0}{4}}e^{-\frac{9\alpha_E y^2}{16}}\,dy \sim e^{-\frac{3\phi_0}{4}}\alpha_E^{-\frac{1}{2}}\,. \tag{5.13}$$

The fact that the solution has finite extent in the spatial dimension on which the fields vary is in agreement with the Finite Distance Lesson, and in fact satisfying its quantitative bound (1.1)

$$\Delta^{-2} \sim \mathcal{T}\,. \tag{5.14}$$

We can now consider how the Dynamical Cobordism Lesson applies in the present context. Following it, we expect the finite extent in the spatial dimensions to be physically implemented via the cobordism defect corresponding to the 10d USp(32) theory. In general the cobordism defect of bulk chiral 10d theories are expected to be non-supersymmetric, and in fact rather exotic, as their worldvolume dynamics must gap a (non-anomalous) set of chiral degrees of freedom. In fact, on general grounds they can be expected to involve strong coupling.[17] An end of the world defect imposes boundary conditions on bulk supergravity fields, which at weak coupling should be at most linear in the fields, to be compatible with the superposition principle. A typical example are boundary conditions that pair up bulk fermions of opposite chiralities. However, the anomaly cancellation in the 10d USp(32) theory involves fields of different spins, which cannot be gapped by this simple mechanism, and should require strong coupling dynamics (a similar phenomenon in a different context occurs in [88]).

This strong coupling fits nicely with the singularity at $y \to \infty$, but the singularity at $y = 0$ lies at weak coupling. The simplest way out of this is to propose that the singularity at $y = 0$ is actually smoothed out by perturbative string theory (namely, α' corrections, just like orbifold singularities are not singular in string theory), and does not turn into an end of the world defect. Hence the solution (5.12) extends to $y < 0$, and, since the background is even in y, develops a singularity at $y \to -\infty$. This is still at finite distance Δ scaling as (5.14), and lies at strong coupling, thus allowing for the possibility that the singularity is turned into the cobordism defect of the 10d USp(32) theory.

[17] We are indebted to Miguel Montero for this argument, and for general discussions on this section.

It would be interesting to explore this improved understanding of this solution to the dilaton tadpole. Leaving this for future work, we turn to a more tractable solution in the next section.

5.3.2 Solving the tadpole via magnetization

We now discuss a more tractable alternative to solve the dynamical tadpole via magnetization, following section 4.

Stabilizing the tadpole via magnetization is, ultimately, equivalent to finding a compactification (on a product of $\mathbf{T}^2$'s) which is free of tadpoles, for instance by demanding it to be supersymmetric. Hence we need to construct a supersymmetric compactification of the non-supersymmetric 10d USp(32) theory [4].

As explained above, the 10d model is constructed with an O9$^+$-plane and 32 $\overline{\text{D9}}$-branes. Hence, we need to introduce worldvolume magnetic fields in different 2-planes, in such a way that the corresponding angles add up to 0 mod 2π. It is easy to convince oneself that this requires magnetization in at least three complex planes, ultimately triggering a $\mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$ compactification. In order to preserve supersymmetry, we need the magnetization to induce D5-brane charges, rather than $\overline{\text{D5}}$-brane charge, hence we need the presence of three independent kinds of negatively charged O5$_i^-$-planes, where $i = 1, 2, 3$ denotes the $\mathbf{T}^2$ wrapped by the corresponding O5-plane. We are thus considering an orientifold of $\mathbf{T}^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ with an O9$^+$-plane, and 8 O5$_i^-$-planes.[18]

The wrapping numbers for the O-planes, and for one simple solution of all constraints for the D9-branes (and their explicitly included orientifold image D9-branes), are

Object	N_α	(n_α^1, m_α^1)	(n_α^2, m_α^2)	(n_α^3, m_α^3)
O9$^+$	32	$(1,0)$	$(1,0)$	$(1,0)$
O5$_1^-$	-32	$(1,0)$	$(0,1)$	$(0,-1)$
O5$_2^-$	-32	$(0,1)$	$(1,0)$	$(0,-1)$
O5$_3^-$	-32	$(0,1)$	$(0,-1)$	$(1,0)$
D9	16	$(-1,1)$	$(-1,1)$	$(-1,1)$
D9$'$	16	$(-1,-1)$	$(-1,-1)$	$(-1,-1)$

It obeys the RR tadpole conditions for the $\mathbf{Z}$-valued D9- and D5-brane charges, and the discrete $\mathbf{Z}_2$ RR tadpole conditions for D3- and D7$_i$-brane charges [70].

The supersymmetry condition determined by the O-plane wrappings is

$$\sum_i \arctan(-\chi_i) \equiv \theta_1 + \theta_2 + \theta_3 = 0 \text{ mod } 2\pi\,. \tag{5.15}$$

The model is in fact T-dual (in all $\mathbf{T}^6$ directions) to that in section 5 of [59].

It is easy to see that the above condition forces at least one of the $\mathbf{T}^2$ to have $\mathcal{O}(1)$ area in α' units. From our perspective, this a mere reflection of the fact that the 10d dynamical

[18] For such combinations of orientifold plane signs, see the analysis in [89], in particular its table 6. We will not need its detailed construction for our purposes.

tadpole to be canceled is of order the string scale, hence it agrees with the scaling $\Delta^{-2} \sim \mathcal{T}$. Happily, the use of an α' exact configuration, which is moreover supersymmetric, makes our solution reliable. This is an improvement over other approaches e.g. as in section 5.3.1.

Although we have discussed the compactification on (an orientifold of) $\mathbf{T}^6$ directly, we would like to point out that it is easy to describe it as a sequence of $\mathbf{T}^2$ spontaneous compactifications, each eating up a fraction of the initial 10d tadpole until it is ultimately cancelled upon reaching $\mathbf{T}^6$. However, this picture does not really correspond to a physical situation, given the absence of decoupling of scales. This is true even in setups which seemingly allow for one $\mathbf{T}^2$ of parametrically large area. Indeed, consider for instance the regime $\chi_3 \sim 2\lambda$ and $\chi_1, \chi_2 \sim \lambda^{-1}$, for $0 < \lambda \ll 1$, which corresponds to $\theta_1, \theta_2 \sim \frac{\pi}{2} + \lambda$, $\theta_3 \sim \pi - 2\lambda$. This corresponds to a compactification on substringy size $(\mathbf{T}^2)_1 \times (\mathbf{T}^2)_2$ and a parametrically large $(\mathbf{T}^2)_3$. However, the fact that the $(\mathbf{T}^2)_1$, $(\mathbf{T}^2)_2$ can be T-dualized into large area geometries shows that there is not true decoupling of scales: in the original picture, the small sizes imply that there are towers of light winding modes, whose scale is comparable with the KK modes of $(\mathbf{T}^2)_3$. Hence, the lack of decoupling is still present, as expected from our general considerations in the introduction.

6 The SM from spontaneous compactification

In this section we explore an interesting application of the above mechanism, and provide an explicit example of a 6d theory with brane-antibrane pairs, and a dynamical tadpole triggering spontaneous compactification to a 4d (MS)SM-like particle physics model. Interestingly, the complete chiral matter and electroweak sector, including the Higgs multiplets, are generated as degrees of freedom on cobordism branes. Only the gluons are present in some form in the original 6d models.

Consider the type IIB orientifold of $\mathbf{T}^4/\mathbf{Z}_2$ with orientifold action Ω constructed in [78, 79], possibly with magnetization. To describe it, we introduce the notation in [69, 90] of wrapping numbers (n^i_α, m^i_α), where n^i_α and m^i_α provide the wrapping number and magnetic flux quantum of the D-brane α on the i^{th} $\mathbf{T}^2$, respectively. We consider the following stacks of D-branes (and their orientifold images, not displayed explicitly)

N_α	(n^1_α, m^1_α)	(n^2_α, m^2_α)
$N_{a+d} = 6 + 2$	$(1, 3)$	$(1, -3)$
$N_{h_1} = 4$	$(1, -3)$	$(1, -4)$
$N_{h_2} = 4$	$(1, -4)$	$(1, -3)$
40	$(0, 1)$	$(0, -1)$

The O9- and O5-planes correspond to the wrapping numbers $(1, 0) \times (1, 0)$ and $(0, 1) \times (0, -1)$ respectively. The stacks a and d are taken different and separated by Wilson lines, but they can be discussed jointly for the time being. They correspond to 8 D9-branes with worldvolume magnetic fluxes 72 units of D5-brane charge. The stacks h_1 and h_2 correspond to 8 additional D9-branes, with 96 with units of induced $\overline{\text{D5}}$-branes charge.

The addition of 40 explicit D5-branes leads to RR tadpole cancellation (once orientifold images are included). In terms of the wrapping numbers, we have

$$\sum_\alpha N_\alpha n_\alpha^2 n_\alpha^3 = 16\,, \quad \sum_\alpha N_\alpha m_\alpha^2 m_\alpha^3 = -16\,. \tag{6.1}$$

The model is far from supersymmetric due to the presence of D5-$\overline{\text{D5}}$ pairs, and in fact has a decay channel to supersymmetric model by their annihilation. On the other hand, even at the top of the tachyon potential, the theory is not at a critical point of its potential due to dynamical tadpole for the closed string moduli, namely the area moduli of the $\mathbf{T}^2$'s. In other words, the excess tension depends on these, as they enter the angles determining the deviation from the supersymmetry condition

$$\arctan\left(\frac{m_\alpha^1}{n_\alpha^1}\chi_1\right) + \arctan\left(\frac{m_\alpha^2}{n_\alpha^2}\chi_2\right) = 0\,. \tag{6.2}$$

For instance, we can make the stacks a, d supersymmetric, by choosing

$$\chi_1 = \chi_2\,, \tag{6.3}$$

but the D-branes h_1 and h_2 break supersymmetry. Hence, there is a dynamical tadpole associated to the excess tension of these latter objects.

The dynamical tadpole can be solved by introducing magnetization along two of the 6d spacetime dimensions. The backreaction of this extra flux forces these two dimensions to be compactified on a $\mathbf{T}^2$, with the addition of cobordism I-branes [15], which in general includes orientifold planes and D-branes, as in the examples above. We take these extra branes to be arranged in two new stacks b and c. Overall, we end up with an orientifold of $\mathbf{T}^6/(\mathbf{Z}_2\times\mathbf{Z}_2)$, with D-brane stacks and topological numbers given by

N_α	(n_α^1, m_α^1)	(n_α^2, m_α^2)	(n_α^3, m_α^3)
$N_{a+d} = 6+2$	$(1,3)$	$(1,-3)$	$(1,0)$
$N_b = 2$	$(0,1)$	$(1,0)$	$(0,1)$
$N_c = 2$	$(-1,0)$	$(0,-1)$	$(0,1)$
$N_{h_1} = 2$	$(1,-3)$	$(1,-4)$	$(2,-1)$
$N_{h_2} = 2$	$(1,-4)$	$(1,-3)$	$(2,-1)$
40	$(0,1)$	$(0,-1)$	$(0,1)$

The model satisfies the RR tadpole conditions

$$\begin{aligned} \sum_\alpha N_\alpha n_\alpha^1 n_\alpha^2 n_\alpha^3 &= 16\,, \qquad & \sum_\alpha N_\alpha n_\alpha^1 m_\alpha^2 m_\alpha^3 &= 16\,, \\ \sum_\alpha N_\alpha m_\alpha^1 n_\alpha^2 m_\alpha^3 &= 16\,, \qquad & \sum_\alpha N_\alpha m_\alpha^1 m_\alpha^2 n_\alpha^3 &= -16\,. \end{aligned} \tag{6.4}$$

This corresponds to O9-planes along $(1,0)\times(1,0)\times(1,0)$, and O5-planes along $(0,1)\times(0,-1)\times(1,0)$, as already present in the 6d theory, and cobordism O5-planes along $(0,1)\times(1,0)\times(0,1)$ and $(1,0)\times(0,1)\times(0,1)$.

The model still contains only 3 stacks of D-branes with non-trivial angles, so that they are just enough to fix the 2 parameters χ_i of the $\mathbf{T}^2$'s. The O-planes fix the supersymmetry condition signs to

$$\arctan\left(\frac{m_\alpha^1}{n_\alpha^1}\chi_1\right)+\arctan\left(\frac{m_\alpha^2}{n_\alpha^2}\chi_2\right)-\arctan\left(\frac{m_\alpha^3}{n_\alpha^3}\chi_3\right)=0\,. \tag{6.5}$$

Using the branes above, we get

$$\chi_1=\chi_2\,,\quad \chi_3=\frac{14\chi_1}{1-12\chi_1^2}\,. \tag{6.6}$$

The regime of large $(\mathbf{T}^2)_3$ corresponds to small χ_3, which is also attained for small χ_1. Note that in this context the last condition $\chi_1 \sim \chi_3$ encodes the relation between the 6d tadpole and the inverse area of the spontaneously compactified $\mathbf{T}^2$.

The model is, up to exchange of directions in the $\mathbf{T}^6$ and overall sign flips, precisely one of the examples of 4d MSSM-like constructions in [59, 60]. The gauge group is $\mathrm{U}(3)_a \times \mathrm{USp}(2)_b \times \mathrm{U}(1)_c \times \mathrm{U}(1)_d$, where we break the naive $\mathrm{USp}(2)_c$ by Wilson lines or shifting off the O-plane for the corresponding D5-branes. Taking into account the massive U(1)'s due to BF couplings, this reproduces the SM gauge group. In addition, open strings between the different brane stacks reproduce a 3-family (MS)SM chiral matter content, and the MSSM Higgs doublet pair. Hence, we have described the spontaneous compactification of a 6d model to a semi-realistic MSSM-like 4d theory.

A fun fact worth emphasizing is that most of the SM spectrum is absent in the original 6d model, and arises only after the spontaneous compactification. In particular, all the MSSM matter and Higgs chiral multiplets, as well as the electroweak gauge sector, arise from open string sectors involving the b and c branes, which arises as cobordism branes. It is remarkable that cobordism entails that spontaneous compactification implies not just the removal of spacetime dimensions, but also the dynamical appearance of novel degrees of freedom. It is tantalizing to speculate on the potential implications of these realizations in cosmological or other dynamical setups.

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A Dynamical tadpoles and swampland constraints

In this appendix we use the model in section 2 to illustrate the result in [3] that, in theories with a dynamical tadpole which is not duly backreacted on the field configuration, the mistreatment can show up as violations of swampland constraints.

We consider type IIB theory on $\mathrm{AdS}_5 \times T^{1,1}$ and introduce M units of RR 3-form flux. In the coordinates in [33, 34], it reads

$$F_3 = \frac{1}{2}M[\sin\theta_1(\cos\theta_2 d\theta_1 d\phi_1 d\phi_2 + d\theta_1 d\phi_1 d\psi) + \sin\theta_2(\cos\theta_1 d\theta_2 d\phi_1 d\phi_2 - d\theta_2 d\phi_2 d\psi)]\,.$$

It has constant coefficients in terms of *fünf*-bein 1-forms g^i in [33] $F_3 = \frac{1}{2}Mg^5(g^1g^2 + g^3g^3)$, hence its kinetic term $|F_3|^3$ is constant over the $T^{1,1}$ geometry. This acts as a constant background source for e.g. the Laplace equation for the dilaton, which has no solution over the compact $T^{1,1}$ geometry. This inconsistency of the equations of motion, assuming no backreaction on the underlying geometry, signals the dynamical tadpole in the configuration. In the following we will argue that it moreover can lead to violation of the Weak Gravity Conjecture [47].

For concreteness we focus on the simplest set of states, corresponding to 5d BPS particle states in the original theory ($M = 0$), with the BPS bound corresponding to the WGC bound, for the gauge interaction associated to the KK U(1) dual to the $\mathrm{U}(1)_R$ symmetry of the dual CFT. For small R-charge $n \ll N$, these particle states are dual to chiral primary single-trace mesonic operators of the $\mathrm{SU}(N)^2$ theory, e.g. $\mathrm{tr}\,(A_1B_1 \ldots A_1B_1)$; in the AdS side, they correspond to KK gravitons with momentum n on the $\mathbf{S}^1$. For very large R-charge, the KK gravitons polarize due to Myers' effect [91] into giant gravitons [92], and their dual operators are determinant or sub-determinant operators [93]. Note that on $T^{1,1}$ we have D3-branes wrapped on homologically trivial 3-cycles (but sustained as BPS states by their motion on $\mathbf{S}^1$), hence they are different from (di)baryonic operators, which correspond to D3-branes wrapped on the non-trivial $\mathbf{S}^3$ [94].

Our strategy is to consider these states in the presence of F_3, but still keeping the geometry as $\mathrm{AdS}_5 \times T^{1,1}$ (i.e. with no backreaction of the dynamical tadpole), and show that the interaction of F_3 makes these states non-BPS, hence violating the WGC bound. This analysis will be quite feasible in the giant graviton regime $1 \ll n \sim N$, by using the wrapped D3-brane worldvolume action. Admittedly, proving a full violation of the WGC would require showing the violation of the BPS condition for all values of n; we nevertheless consider the large n result as a compelling indication that the WGC is indeed violated in this configuration, thus making its inconsistency manifest.

Supersymmetric 3-cycles for D3-branes are easily obtained from holomorphic 4-cycles in the underlying CY threefold [95, 96] (see also [97]). Describing the conifold as $z^1z^2 - z^3z^4 = 0$, any holomorphic function of these coordinates $f(x, y, z, w) = 0$ defines a holomorphic 4-cycle corresponding to a giant graviton D3-branes, i.e. wrapped on a trivial[19] 3-cycle in $T^{1,1}$. We focus on a simple class of D3-branes studied in detail in [98]. They are defined by the 4-cycle $z^1 = \sqrt{1-\alpha^2}$, with $\alpha \in [0, 1]$ being a real constant, encoding the size of the 3-cycle (with $\alpha = 0, 1$ corresponding to the pointlike KK graviton and the maximal giant graviton, respectively). We will follow the analysis in [98] with the inclusion of the effect of F_3 on the D3-brane probe.

[19]Di-baryonic D3-branes are on the other hand associated to non-Cartier divisors in the conifold, i.e. 4-cycles which can be defined in terms of the a_i, b_i homogeneous coordinates of the linear sigma model, but cannot be expressed as a single equation $f(z^i) = 0$.

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It is convenient to change to new coordinates $\{\chi_1, \chi_2, \chi_3, \alpha, \nu\}$

$$\begin{cases} \chi_1 = \frac{1}{3}(\psi - \phi_1 - \phi_2) \\ \chi_2 = \frac{1}{3}(\psi + 3\phi_1 - \phi_2) \\ \chi_3 = \frac{1}{3}(\psi - \phi_1 + 3\phi_2) \end{cases} \qquad \begin{cases} \sqrt{1-\alpha^2} = \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2} \\ \nu = \frac{2u}{\alpha^2+u^2} \quad \text{with} \quad u = \cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2} \end{cases} \tag{A.1}$$

These are adapted to the D3-brane embedding, which simply reads

$$\sigma^0 = t, \quad \sigma^1 = \nu \text{ (doubly-covered)}, \quad \sigma^2 = \chi_2, \quad \sigma^3 = \chi_3\,.$$

The double covering is very manifest for the maximal giant graviton, $\alpha = 1$, $z^1 = 0$. It corresponds to the defining equation $z^3 z^4 = 0$, which splits in two components, corresponding to two (oppositely oriented) copies of the non-trivial[20] $\mathbf{S}^3$. The double covering remains even for non-maximal giants, even though they correspond to irreducible 4-cycles.

The RR 3-form field strength in these coordinates is

$$\begin{aligned} F_3 = M(&[a_{12}\, d\chi_1\wedge d\chi_2 + a_{13}\, d\chi_1\wedge d\chi_3 + a_{23}\, d\chi_2\wedge d\chi_3]\wedge d\alpha \\ &+ [v_{12}\, d\chi_1\wedge d\chi_2 + v_{13}\, d\chi_1\wedge d\chi_3 + v_{23}\, d\chi_2\wedge d\chi_3]\wedge d\nu)\,, \end{aligned} \tag{A.2}$$

with

$$\begin{cases} a_{12} = \frac{9}{4}\alpha(1 \pm \frac{\sqrt{1-\nu^2}}{1-c}) \\ a_{13} = \frac{9}{4}\alpha(-1 \pm \frac{\sqrt{1-\nu^2}}{1-c}) \\ a_{23} = \frac{9}{4}\alpha(\frac{-c}{1-c}) \end{cases} \qquad \begin{cases} v_{12} = \mp\frac{9}{4}\frac{c(\nu^2-c)}{\nu^3\sqrt{1-\nu^2}(1-c)} \\ v_{13} = \mp\frac{9}{4}\frac{c(\nu^2-c)}{\nu^3\sqrt{1-\nu^2}(1-c)} \\ v_{23} = -\frac{9}{4}\frac{c^2}{\nu^3(1-c)} \end{cases}$$

where we have introduced $c = 1 - \sqrt{1-\alpha^2\nu^2}$. We can fix a gauge and find the RR 2-form

$$C_2 = M(c_{12} d\chi_1\wedge d\chi_2 + c_{13} d\chi_1\wedge d\chi_3 + c_{23} d\chi_2\wedge d\chi_3)\,, \tag{A.3}$$

with

$$\begin{cases} c_{12} = -\frac{9}{8}(-\alpha^2 \mp \frac{2\sqrt{1-\nu^2}c}{\nu^2}) \\ c_{13} = -\frac{9}{8}(\alpha^2 \mp \frac{2\sqrt{1-\nu^2}c}{\nu^2}) \\ c_{23} = \frac{9}{8}\frac{(\alpha^2\nu^2-2c)}{\nu^2} \end{cases}$$

Its pullback on the D3-brane worldvolume is

$$P[C_2] = M\dot{\chi}_1(c_{12} dt\wedge d\chi_2 + c_{13} dt\wedge d\chi_3) + M c_{23} d\chi_2\wedge d\chi_3\,. \tag{A.4}$$

We can now compute the effect of this background on the D3-brane by using its worldvolume action. This is easy in the S-dual frame, in which the RR 2-form couples to the D3-brane just like the NSNS 2-form in the original DBI+CS D3-brane action.[21] After integrating over χ_2, χ_3, this reads

$$\begin{aligned} &S = S_{\rm BDI} + S_{\rm CS} = \frac{64\pi^2}{9}\int dt L\,, \\ \text{with} \quad &L = \int_0^1 d\nu\, 2\Big(-T_3\sqrt{-\det\left(P[G]_{\mu\nu} + P[C_2]_{\mu\nu}\right)} + \mu_3 R^4 c_4\dot{\chi}_1\Big)\,, \end{aligned} \tag{A.5}$$

[20] In terms of the linear sigma model coordinates we have $z^1 = a_1 b_1$, $z^2 = a_2 b_2$, $z^3 = a_1 b_2$, $z^4 = a_2 b_1$, and the two components correspond to $a_1 = 0$ and $b_1 = 0$, which are non-Cartier divisors.

[21] Related to this, one can check that the above background is neither pure gauge on the D3, nor cannot be removed by a change in the worldvolume gauge field strength flux.

where the factor of 2 of the double-covering of ν has been added, and the last term arises from the CS coupling to the RR 4-form as in [98].

We are interested in focusing on the angular momentum of the state $P_{\chi_1} = \frac{\partial L}{\partial \dot{\chi}_1}$ conjugate to the angular coordinate χ_1. This reads

$$P_{\chi_1} = \frac{3}{2}\int_0^1 d\nu \left(\frac{\sqrt{3\pi} A g_\nu \, N T_3 (\dot{\chi}_1)}{\sqrt{N \, g_\nu (B - A(\dot{\chi}_1)^2)}} + 9\pi c_4 \mu_3 \, N \right), \tag{A.6}$$

with

$$\begin{aligned} A &= \frac{81}{64\nu^4}\{-4\, M^2[(2(1-\alpha^2\nu^2)c - \alpha^2\nu^2)c] - 3\pi N(\alpha^2-1)\nu^2 c^2\} \equiv A_{M^2}M^2 + A_N N \,, \\ B &= \frac{81}{64\nu^4}\{4M^2\left[(4-\alpha^2\nu^2)c^2 - 2\alpha^2\nu^2 c\right] \\ &\qquad + \pi N\left[2\alpha^4\nu^4 + (\alpha^2\nu^2 - 2c)(-3\alpha^2\nu^2 - 3\nu^2 + 8)\right]\} \\ &\equiv B_{M^2}M^2 + B_N N \,. \end{aligned} \tag{A.7}$$

In the last equalities we have highlighted the parametric dependence on N and M.

Despite the fact that we have not managed to find a closed form for the result, since $M \ll N$ we can find an expansion for the integrand in the form

$$n = p_0(\alpha, \nu, \dot{\chi}_1)\, N + p_2(\alpha, \nu, \dot{\chi}_1)\, M^2 + \mathcal{O}(M^4)\,, \tag{A.8}$$

where the coefficient functions are computable, but we will not need their explicit expressions.

The coefficient p_0 is the survivor for the $M = 0$ case, and leads to an integer momentum. On the other hand, the subleading correction p_2 produces a momentum which is not integer. This already signals a problem, since (as the geometry is considered undeformed even after introducing F_3) the gauge coupling of the KK U(1) is as in the $M = 0$ case, hence charges under it should be integer in the same units. Hence one can directly claim that the assumption of ignoring the dynamical tadpole backreaction lead to violation of charge quantization, in contradiction with common lore for consistency with quantum gravity [99].

The above discussion however seems to contradict the fact that any quantum excitation on a periodic $\mathbf{S}^1$ direction must have quantized momentum to have a well-defined wavefunction. In fact, an alternative interpretation of the above mismatch is that the D3-brane probe computation assumes a well-defined worldvolume embedding, in particular well-defined (hence classical) trajectories for the 5d particle. It is only for BPS states in supersymmetric vacua that such a computation is guaranteed to end up producing quantized momenta. The fact that our holomorphic embedding ansatz fails to do so is just a reflection that the actual integer-quantized states are *not* described by holomorphic equations. Since the latter condition is the one ensuring the match between the particle mass and charge, it is clear that non-holomorphic embeddings will produce larger masses for the same charge, hence violating the BPS/WGC bound.

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Dynamical Cobordism and Swampland Distance Conjectures

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Abstract: We consider spacetime-dependent solutions to string theory models with tadpoles for dynamical fields, arising from non-trivial scalar potentials. The solutions have necessarily finite extent in spacetime, and are capped off by boundaries at a finite distance, in a dynamical realization of the Cobordism Conjecture. We show that as the configuration approaches these cobordism walls of nothing, the scalar fields run off to infinite distance in moduli space, allowing to explore the implications of the Swampland Distance Conjecture. We uncover new interesting scaling relations linking the moduli space distance and the SDC tower scale to spacetime geometric quantities, such as the distance to the wall and the scalar curvature. We show that walls at which scalars remain at finite distance in moduli space correspond to domain walls separating different (but cobordant) theories/vacua; this still applies even if the scalars reach finite distance singularities in moduli space, such as conifold points.

We illustrate our ideas with explicit examples in massive IIA theory, M-theory on CY threefolds, and 10d non-supersymmetric strings. In 4d $\mathcal{N} = 1$ theories, our framework reproduces a recent proposal to explore the SDC using 4d string-like solutions.

Keywords: Flux compactifications, Superstring Vacua, Supersymmetry Breaking

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1 Introduction and conclusions

A remarkable proposal in the Swampland Program of quantum gravity constraints on effective field theories [1] (see [2–4] for reviews) is the Cobordism Conjecture [5], that is based on the expected absence of exact global symmetries in quantum gravity. In short, it states that any configuration in a consistent theory of quantum gravity should not carry any cobordism charge. In practice, it implies that any configuration in a consistent theory of quantum gravity should admit, at the topological level, the introduction of a boundary ending spacetime into nothing,[1] in the sense of [6] (see [7, 8] for recent related discussions). Accordingly, we will refer to such boundaries as *walls of nothing*. Equivalently, it implies that any two consistent theories of quantum gravity must admit, at the topological level, an interpolating configuration connecting them, as a generalized domain wall separating the two theories. We will refer to such configurations as *interpolating domain walls*.

[1]This boundary may be dressed by additional defects, such as D-branes or O-planes in string setups, to absorb the relevant charges.

The Cobordism Conjecture is topological in nature. However, it can lead to remarkable breakthroughs when supplemented by additional assumptions. For instance, the extra ingredient of supersymmetry of the theory (and possibly of its walls) has led to highly non-trivial constraints in lower dimensional theories, see e.g. [9, 10].

An important step forward in endowing cobordism walls with dynamics was taken in [11], in the study of theories with tadpoles for dynamical fields (dubbed *dynamical tadpoles*, as opposed to topological tadpoles, such as RR tadpoles, which lead to topological consistency conditions on the configuration[2]). These are ubiquitious in the presence of scalar potentials, and in particular in non-supersymmetric string models. In theories with dynamical tadpoles the solutions to the equations of motion vary over the non-compact spacetime dimensions. Based on the behaviour of large classes of string models, it was proposed in [11] that such spacetime-dependent running solutions must hit cobordism walls of nothing at a finite distance Δ in spacetime[3] (as measured in the corresponding Einstein frame metric), scaling as $\Delta^{-n} \sim \mathcal{T}$ with the strength of the tadpole $\mathcal{T}$. These examples included holographic $\mathrm{AdS}_5 \times T^{1,1}$ compactifications with RR 3-form flux, type IIB 3-form flux compactifications, magnetized D-brane models, massive IIA theory, M-theory on K3 with G_4 flux, and the 10d non-supersymmetric USp(32) string theory. On the other hand, interpolating cobordism walls connecting different theories were not discussed. One of the motivations of this work is to fill this gap.

We argue that, when a running solution in theories with dynamical tadpoles hits a wall, the behaviour of the configuration across the wall, and in particular the sharp distinction between interpolating domain walls and walls of nothing, is determined by the behaviour of scalar fields as one reaches the wall, via a remarkable correspondence:

- When scalars remain at finite distance points in moduli space as one hits the wall, it corresponds to an interpolating domain wall, and the solution continues across it in spacetime (with jumps in quantities as determined by the wall properties);
- On the other hand, when the scalars run off to infinity in moduli space as one reaches the wall (recall, at a finite distance in spacetime), it corresponds to a wall of nothing, capping off spacetime beyond it.

We also argue that scalars reaching singular points at finite distance in moduli space upon hitting the wall still define interpolating domain walls, rather than walls of nothing; hence, walls of nothing are not a consequence of general singularities in moduli space, but actually to those at infinity in moduli space. This suggests that, in the context of dynamical solutions,[4] the walls of nothing of the Cobordism Conjecture are closely related to the Swampland Distance Conjecture.[5] We indeed find universal scaling relations between the

[2]Note however that dynamical tadpoles were recently argued in [12] to relate to violation of swampland constraints of quantum gravity theories.

[3]For related work on dynamical tadpoles in non-supersymmetric theories, see [13–20].

[4]Note that, in setups with no dynamical tadpole, one can still have e.g. cobordism walls of nothing without scalars running off to infinity: for instance, 11d M-theory, which does not even have scalars, admits walls of nothing defined by Horava-Witten boundaries; similar considerations may apply to potential theories with no moduli (or with all moduli stabilized at high enough scale).

[5]The status of the SDC in spacetime dependent running solutions was addressed in [21].

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(finite) distance to the wall in spacetime and the scale of the SDC tower [22]. In addition, we uncover a universal scaling relation between the curvature scalar in running solutions and the SDC tower scale that is reminiscent of the Anti de Sitter Distance Conjecture (ADC) [23].

We illustrate these ideas in several large classes of string theory models, including massive IIA, and M-theory on CY threefolds. Moreover, we also argue that our framework encompasses the recent discussion of EFT string solutions in 4d $\mathcal{N}=1$ theories in [24] (see also [25]), where saxion moduli were shown to attain infinity in moduli space at the core of strings magnetically charged under the corresponding axion moduli. We show that EFT string solutions are the cobordism walls of nothing of $\mathbf{S}^1$ compactifications of the 4d $\mathcal{N}=1$ theory with certain axion fluxes on the $\mathbf{S}^1$. Our scalings also relate to those between EFT string tensions and the SDC tower scale in [24].

The paper is organized as follows. In section 2 we present the main ideas in the explicit setup of running solutions in massive IIA theory, and their interplay with type I' solutions [26]. In section 3 we carry out a similar discussion for M-theory on CY threefolds with G_4 flux (in section 3.1) and their relation to strongly coupled heterotic strings [27]. In section 3.2 we use it to discuss domain walls across singularities at finite distance in moduli space, following [28]. In section 4 we discuss the $\mathbf{S}^1$ compactification of general 4d $\mathcal{N}=1$ theories. In section 4.1 we introduce dynamical tadpoles from axion fluxes, whose running solutions hit walls of nothing at which saxions run off to infinity. In section 4.2 we relate the discussion to the EFT strings of [24]. In section 5 we discuss the moduli space distances in walls of nothing and interpolating walls in 4d $\mathcal{N}=1$ theories with non-trivial superpotentials of the kind arising in flux compactifications. In section 6 we discuss our proposal in non-supersymmetric string theories, in particular the 10d USp(32) string. In section 7 we offer some final remarks and outlook. Appendix A provides some observations on cobordism walls in holographic throats.

2 Cobordism walls in massive IIA theory

Walls of nothing and infinite moduli space distance. In this section we consider different kinds of cobordism walls in massive IIA theory [29], extending the analysis in [11]. The Einstein frame 10d effective action for the relevant fields is

$$S_{10,E} = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{-G_E}\left\{\left[R-\frac{1}{2}(\partial\phi)^2\right]-\frac{1}{2}e^{\frac{5}{2}\phi}F_0^2-\frac{1}{2}e^{\frac{1}{2}\phi}(F_4)^2\right\}, \tag{2.1}$$

where the Romans mass parameter is denoted by F_0 to suggest it is a 0-form field strength flux. This theory is supersymmetric, but has a dilaton tadpole

$$\mathcal{T}\sim e^{\frac{5}{2}\phi}F_0^2\,, \tag{2.2}$$

so the theory does not admit 10d maximally symmetric solutions. The solutions with maximal (super)symmetry are 1/2 BPS configurations with the dilaton depending on one coordinate x^9, closely related to that in [30]. In conventions closer to [26], the Einstein frame metric and dilaton are

$$(G_E)_{MN}=Z(x^9)^{\frac{1}{12}}\,\eta_{MN}\,,\quad e^{\phi}=Z(x^9)^{-\frac{5}{6}}\,,\quad \text{with}\;\; Z(x^9)\sim -F_0\,x^9\,, \tag{2.3}$$

where we have set some integration constant to zero. The solution hits a singularity at $x^9 = 0$. The spacetime distance from a general position x^9 to the singularity is [11]

$$\Delta = \int_{x^9}^{0} Z(x^9)^{\frac{1}{24}}\, dx^9 \sim Z(x^9)^{\frac{25}{24}}\, F_0^{-1} \sim F_0^{-1} e^{-\frac{5}{4}\phi} \sim \mathcal{T}^{-\frac{1}{2}}\,, \tag{2.4}$$

in agreement with the scaling relation $\Delta^{-2} \sim \mathcal{T}$, that was dubbed Finite Distance lesson in [11]. Following the Dynamical Cobordism proposal therein, the singularity is resolved in string theory into a cobordism wall of nothing, defined by an O8-plane (possibly dressed with D8-branes to match the F_0 flux to be absorbed),[6] ending the direction x^9 as a boundary.

We now notice that, since $Z \to 0$ implies $\phi \to \infty$ as $x^9 \to 0$, the dilaton runs off to infinity in moduli space as one hits the wall, as befits a wall of nothing from our discussion in the introduction. According to the SDC, there is an infinite tower of states becoming massless in this region, with a scale decaying exponentially with the moduli space distance D as

$$M_{\rm SDC} \sim e^{-\lambda D}\,, \tag{2.5}$$

with some positive $\mathcal{O}(1)$ coefficient λ.

It is interesting to find a direct relation between these quantities and the spacetime distance to the wall. The distance in moduli space is given by $\phi = \sqrt{2}\, D$, as can be seen from the kinetic term for ϕ in (2.1). From (2.4) we have

$$\Delta \sim e^{-\frac{5}{2\sqrt{2}} D}\,, \quad M_{\rm SDC} \sim \Delta^{\frac{2\sqrt{2}}{5}\lambda}\,. \tag{2.6}$$

Hence the SDC tower scale goes to zero with the distance to the wall with a power-like scaling.

It is a natural question to ask if this tower of states becomes light in the actual dynamical configuration (rather than in the adiabatic framework of the standard formulation of the SDC). In this particular setup, the SDC tower corresponds to D0-branes which end up triggering the decompactification of the M-theory eleventh dimension. In the dynamical solution, there are a finite number of extra massless states, responsible for the enhancement of the perturbative open string gauge group to the exceptional symmetries which are known to arise from the heterotic dual theory [26] (see also [31]). On the other hand, there is no signal of an infinite tower of states becoming massless simultaneously. The appearance of the SDC in the dynamical context has thus different implications as compared with the usual adiabatic formulation.

Let us now turn to another novel, and tantalizing, scaling. The scalar curvature for the running solution reads

$$|R| \sim (-x^9)^{-\frac{25}{12}} \sim e^{\frac{5}{\sqrt{2}} D}\,. \tag{2.7}$$

Using this, we can write the SDC tower scale in terms of the scalar curvature as

$$M_{\rm SDC} \sim e^{-\lambda D} \sim |R|^{-\frac{\sqrt{2}}{5}\lambda}\,. \tag{2.8}$$

[6]This imposes a swampland bound on the possible values of F_0 that are consistent in string theory.

This scaling is highly reminiscent of the Anti de Sitter Distance Conjecture (ADC) of [23],[7] even though the setup under consideration is very different.[8] Note however that, as in the ADC, it signals a failure of the decoupling of scales, and hence a breakdown of the effective field theory near the wall of nothing. This fits nicely with our observation that the wall can only be microscopically defined in the UV complete theory, and works as a boundary condition defect at the level of the effective theory.

Interpolating domain walls. There is a well known generalization of the above solutions, which involves the inclusion of D8-branes acting as interpolating domain walls across which F_0 jumps by one unit. The general solution of this kind is provided by (2.3) with a piecewise constant F_0 and a piecewise continuous function Z [26].

The D8-brane domain walls are thus (a very simple realization of) cobordism domain walls interpolating between different Romans IIA theories (differing just in their mass parameter). The point we would like to emphasize is that, since Z remains finite across them, the dilaton remains at finite distance in moduli space, as befits interpolating domain walls from our discussion in the introduction.

3 Cobordism walls in M-theory on CY3

In this section we recall results from the literature on the strong coupling limit of the heterotic string, also known as heterotic M-theory [27, 32–34] (see [35, 36] for review and additional references). They provide straightforward realizations of the different kinds of cobordism walls in M-theory compactifications on CY threefolds. The discussion generalizes that in [11], and allows to study the behaviour at singular points at finite distance in moduli space, in particular flops at conifold points.

3.1 M-theory on CY3 with G_4 flux

We consider M-theory on a CY threefold $\mathbf{X}$, with G_4 field strength fluxes on 4-cycles. For later convenience, we follow the presentation in [28]. We introduce dual basis of 2- and 4-cycles $C^i \in H_2(\mathbf{X})$ and $D_i \in H_4(\mathbf{X})$, and define

$$\int_{D_i} G_4 = a_i\,, \qquad \int_{C^i} C_6 = \tilde{\lambda}^i\,. \tag{3.1}$$

We also denote by b_i the 5d vector multiplet of real Kähler moduli, with the usual Kähler metric and the 5d $\mathcal{N}=1$ prepotential

$$G_{ij} = -\frac{1}{2}\frac{\partial^2}{\partial b_i \partial b_j}\ln\mathcal{K}\,, \qquad \mathcal{K} \equiv \frac{1}{3!}d_{ijk}b^i b^j b^k\,, \tag{3.2}$$

with d_{ijk} being the triple intersection numbers of $\mathbf{X}$. We have the familiar constraint $\mathcal{K}=1$ removing the overall modulus V, which lies in a hypermultiplet.

[7] It is possible that the result is ultimately linked to the generalized distance conjectures in [23]; we leave this as an open question for future work.

[8] In contrast to the ADC, that considers the limit of vanishing curvature of a family of AdS vacua, in our setup the scalar curvature blows up as the singularity is approached. However, we do find a power-like scaling similar to the ADC one.

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The 5d effective action for these fields is

$$S_5 = -\frac{M_{p,11}^9}{2} L^6 \Big[\int_{M_5} \sqrt{-g_5} \left(R + G_{ij}(b)\partial_M b^i \partial^M b^j + \frac{1}{2V^2}\partial_M V \partial^M V + \lambda(\mathcal{K}-1) \right) \\ + \frac{1}{4V^2} G^{ij}(b) a_i \wedge \star a_j + d\tilde{\lambda}^i \wedge a_i \Big] - \sum_{n=0}^{N+1} \alpha_i^{(n)} \int_{M_4^{(n)}} \left(\tilde{\lambda}^i + \frac{b^i}{V}\sqrt{g_4} \right). \quad (3.3)$$

Here λ is a Lagrange multiplier, and L the reference length scale of the Calabi-Yau. With hindsight, we include 4d localized terms which correspond to different walls in the theory, with induced 4d metric g_4.

The G_4 fluxes a_i induce dynamical tadpoles for the overall volume and the Kähler moduli b_i. There are 1/2 supersymmetric solutions running in one spacetime coordinate, denoted by y, with the structure

$$\begin{aligned} ds_5^2 &= e^{2A} ds_4^2 + e^{8A} dy^2 \,, \\ V &= e^{6A} \,, \quad b^i = e^{-A} f^i \,, \\ e^{3A} &= \left(\frac{1}{3!} d_{ijk} f^i f^j f^k \right) \,, \\ (d\tilde{\lambda}^i)_{\mu\nu\rho\sigma} &= \epsilon_{\mu\nu\rho\sigma} e^{-10A} \left(-\partial_{11} b^i + 2 b^i \partial_{11} A \right) . \end{aligned} \quad (3.4)$$

The whole solution is determined by a set of one-dimensional harmonic functions. They are given in terms of the local values of the G_4 fluxes,

$$d_{ijk} f^j f^k = H_i \,, \qquad H_i = a_i y + c_i \,. \quad (3.5)$$

Here the c_i are integration constants set to have continuity of the H_i, and hence of the f_i, across the different interpolating domain walls in the system, which produce jumps as follows. Microscopically, the interpolating domain walls correspond to M5-branes wrapped on 2-cycles $[C] = \sum n_i C^i$, leading to jumps in the fluxes that in units of M5-brane charge are given by

$$\Delta a_i = n_i \,. \quad (3.6)$$

Hence, interpolating domain walls maintain the theory at finite distance in moduli space. This is not the case for cobordism walls of nothing, which arise when $e^A \to 0$, and hence $V \to 0$, which sits at infinity in moduli space. This regime was already discussed (in the simpler setup of K3 compactifications) in [11], where the cobordism domain was argued to be given by a Horava-Witten boundary (dressed with suitable gauge bundle degrees of freedom, as required to absorb the local remaining G_4 flux), in agreement with the strong coupling singularity discussed in [27]. The wall appears at a finite spacetime distance Δ following the scaling $\Delta^{-2} \sim \mathcal{T}$ in [11]. In what follows, we describe the scaling relations of the moduli space distance and the SDC tower at these walls of nothing.

Since they are characterized by the vanishing of the overall volume of $\mathbf{X}$, it is enough to follow the behaviour of V and the discussion simplifies. Restriction to this sector amounts to setting all $f_i \equiv f$ in (3.4), and all $H_i \equiv H$. Also, since the wall of nothing arises when $H \to 0$, we can take this location as $y = 0$ and write

$$e^{2A} \sim H(y) \sim \alpha y \,. \quad (3.7)$$

Using the metric in (3.4), the spacetime distance from a point $y > 0$ is

$$\Delta = \int_0^y (\alpha y)^2 dy = \frac{1}{3}\alpha^2 y^3 \,. \tag{3.8}$$

We are also interested in the traversed distance in moduli space D. Using the kinetic term in (3.3), the relevant integral to compute is

$$D = -\int \frac{1}{\sqrt{2}V}\frac{dV}{dy}dy \,. \tag{3.9}$$

Using $V \sim H^3$, we get as leading behavior near the singularity

$$D \simeq -\frac{3}{\sqrt{2}}\log y = -\frac{1}{\sqrt{2}}\log\frac{3\Delta}{\alpha^2} \,, \tag{3.10}$$

where in the last equality we used (3.8). This implies

$$\Delta \sim e^{-\sqrt{2}D} \,, \tag{3.11}$$

and leads to a power-like scaling of the SDC tower mass

$$M_{\rm SDC} \sim \Delta^{\frac{\lambda}{\sqrt{2}}} \,. \tag{3.12}$$

Computing the curvature scalar from (3.4), we get

$$|R| \sim e^{2\sqrt{2}D} \,. \tag{3.13}$$

So the SDC tower scale can be expressed, in an ADC-like manner, as

$$M_{\rm SDC} \sim |R|^{-\frac{\lambda}{2\sqrt{2}}} \,. \tag{3.14}$$

We thus recover a similar behaviour to the examples in section 2.

3.2 Traveling across finite distance singularities in moduli space

The setup of M-theory on a CY3 $\mathbf{X}$ allows to address the question of whether walls of nothing could arise at finite distance in moduli space, if the scalars hit a singular point in moduli space. This is actually not the case, as can be explicitly shown by following the analysis in [28] for flop transitions.

Specifically, they considered the flop transition between two Calabi-Yau manifolds with $(h_{1,1}, h_{2,1}) = (3, 243)$, in the setup of a CY3 compactification of the Horava-Witten theory, namely with two boundaries restricting the coordinate y to an interval. In our more general setup, one may just focus on the dynamics in the bulk near the flop transition as one moves along y. Hence we are free to locate the flop transition point at $y = 0$.

In terms of the Kähler moduli $t^i = V^{\frac{1}{3}}b_i$ of $\mathbf{X}$, and changing to a more convenient basis

$$t^1 = U \,, \quad t^2 = T - \frac{1}{2}U - W \,, \quad t^3 = W - U \,, \tag{3.15}$$

and similar (proper transforms under the flop) for $\tilde{\mathbf{X}}$, the Kähler cones of $\mathbf{X}$ and $\tilde{\mathbf{X}}$ are defined by the regions

$$\mathbf{X}: \quad W > U > 0\,, \quad T > \frac{1}{2}U + W\,, \tag{3.16}$$

$$\tilde{\mathbf{X}}: \quad U > W > 0\,, \quad T > \frac{3}{2}U\,. \tag{3.17}$$

This shows that the flop curve is C_3, and the area is $W - U$, changing sign across the flop.

Near the flop point $y = 0$, the harmonic functions for the two CYs $\mathbf{X}$ and $\tilde{\mathbf{X}}$ have the form

$$\begin{array}{ll} \mathbf{X} \text{ at } y \leq 0 & \tilde{\mathbf{X}} \text{ at } y \geq 0 \\ \quad H_T = -18y + k_T\,, & \quad \tilde{H}_T = 18y + k_T\,, \\ \quad H_U = -25y + k_0\,, & \quad \tilde{H}_U = 24y + k_0\,, \\ \quad H_W = 6y + k_0\,, & \quad \tilde{H}_W = -5y + k_0\,. \end{array} \tag{3.18}$$

Hence

$$\begin{array}{ll} \mathbf{X} \text{ at } y \leq 0 & \tilde{\mathbf{X}} \text{ at } y \geq 0 \\ H_{W-U} = 31y\,, & \tilde{H}_{W-U} = -29y\,. \end{array} \tag{3.19}$$

Even though the flop point is a singularity in moduli space, and despite the sign flip for $W - U$, the harmonic functions are continuous and the solution remains at finite distance in moduli space. This agrees with the picture that it corresponds to an interpolating domain wall. In fact, as discussed in [28], the discontinuity in their slopes (and the related change in the G_4 fluxes) makes the flop point highly analogous to the above described interpolating domain walls associated to M5-branes.

The above example illustrates a further important aspect. It provides an explicit domain wall intepolating between two different (yet cobordant) topologies. It would be extremely interesting to extend this kind of analysis to other topology changing transitions, such as conifold transitions[9] [38]. This would allow for a further leap for the dynamical cobordism proposal, given that moduli spaces of all CY threefolds are expected to be connected by this kind of transitions [39].

We have thus established that physics at finite distance in moduli space gives rise to interpolating domain walls, rather than walls of nothing, even at singular points in moduli space. The implication is that the physics of walls of nothing is closely related to the behaviour near infinity in moduli space and hence to the SDC. In the following section we explore further instances of this correspondence in general 4d $\mathcal{N} = 1$ theories.

4 $\mathbf{S^1}$ compactification of 4d $\mathcal{N} = 1$ theories and EFT strings

In this section we study a systematic way to explore infinity in moduli space in general 4d $\mathcal{N} = 1$ theories. This arises in a multitude of string theory constructions, ranging from

[9]For a proposal to realize conifold transitions dynamically in a time-dependent background, see [37].

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heterotic CY compactifications to type II orientifolds on CY spaces [40]. Our key tool is an $\mathbf{S}^1$ compactification to 3d with certain axion fluxes. We will show that the procedure secretly matches the construction of EFT strings in [24] (see also [25]). Actually, this correspondence was the original motivation for this paper.

4.1 Cobordism walls in 4d $\mathcal{N}=1$ theories on a circle

We want to consider general 4d $\mathcal{N}=1$ theories near infinity in moduli space. According to [41–43], the moduli space in this asymptotic regime is well approximated by a set of axion-saxion complex fields, with metric given by hyperbolic planes. We start discussing the single-field case, and sketch its multi-field generalization at the end of this section.

Consider a 4d $\mathcal{N}=1$ theory with complex modulus $S=s+ia$, where a is an axion of unit periodicity and s its saxionic partner. We take a Kähler potential

$$K=-\frac{2}{n^2}\log(S+\bar{S})\,. \tag{4.1}$$

The 4d effective action is

$$\begin{aligned} S&=\frac{M_{P,4}^2}{2}\int d^4x\sqrt{-g_4}\left\{R_4-\frac{n^{-2}}{s^2}\left[(\partial s)^2+(\partial a)^2\right]\right\},\\ &=\frac{M_{P,4}^2}{2}\int d^4x\sqrt{-g_4}\left\{R_4-(\partial\phi)^2-e^{-2n\phi}\,(\partial a)^2\right\}, \end{aligned} \tag{4.2}$$

where in the last equation we have defined $\phi=\frac{1}{n}\log ns$.

We now perform an $\mathbf{S}^1$ compactification to 3d with the following ansatz for the metric[10] and the scalars

$$\begin{aligned} &ds_4^2=e^{-\sqrt{2}\sigma}ds_3^2+e^{\sqrt{2}\sigma}R_0^2d\theta^2\,,\\ &\phi=\phi(x^\mu)\,,\qquad a=\frac{\theta}{2\pi}q+a(x^\mu)\,, \end{aligned} \tag{4.3}$$

where x^μ denote the 3d coordinates and $\theta\sim\theta+2\pi$ is a periodic coordinate. Regarding the axion as a 0-form gauge field, the ansatz for a introduces q units of its field strength flux (we dub it axion flux) on the S^1. We allowed for a general saxion profile to account for its backreaction, as we see next.

The dimensional reduction of the action (4.2) gives (see e.g. [44])

$$S_3=\frac{M_{P,3}}{2}\int d^3x\sqrt{-g_3}\left\{R_3-G_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b-V(\varphi)\right\}, \tag{4.4}$$

where

$$G_{ab}\partial_\mu\varphi^a\partial^\mu\varphi^b=(\partial\sigma)^2+(\partial\phi)^2+e^{-2n\phi}\,(\partial a)^2\,, \tag{4.5a}$$

$$V(\varphi)=e^{-2\sqrt{2}\sigma-2n\phi}\left(\frac{q}{2\pi R_0}\right)^2, \tag{4.5b}$$

and $M_{P,3}=2\pi R_0M_{P,4}^2$ is the 3d Planck mass.

[10] We omit the KK U(1) because it will not be active in our discussion.

The last term in the 3d action corresponds to a dynamical tadpole for a linear combination of the saxion and the radion, induced by the axion flux. We thus look for running solutions of the 3d equations of motion. We focus on solutions with constant axion in 3d $a(x^\mu) = 0$, for which the equations of motion read

$$\frac{1}{\sqrt{-g_3}}\partial_\nu\left(\sqrt{-g_3}g^{\mu\nu}\partial_\mu\sigma\right) = -\sqrt{2}\,e^{-2\sqrt{2}\sigma-2n\phi}\left(\frac{q}{2\pi R_0}\right)^2 , \tag{4.6a}$$

$$\frac{1}{\sqrt{-g_3}}\partial_\nu\left(\sqrt{-g_3}g^{\mu\nu}\partial_\mu\phi\right) = -n\,e^{-2\sqrt{2}\sigma-2n\phi}\left(\frac{q}{2\pi R_0}\right)^2 . \tag{4.6b}$$

We consider solutions in which the fields run with one of the coordinates x^3 (which with hindsight we denote by $r \equiv x^3$). We focus on a particular 3d axion-saxion ansatz

$$s(r) = s_0 - \frac{q}{2\pi}\log\frac{r}{r_0}\,, \quad a(r) = a_0\,, \tag{4.7}$$

for which the radion can be solved as

$$\sqrt{2}\sigma = \frac{2}{n}(\phi-\phi_0) + 2\log\frac{r}{R_0} = \frac{2}{n^2}\log\left(1-\frac{q}{2\pi s_0}\log\frac{r}{r_0}\right) + 2\log\frac{r}{R_0}\,. \tag{4.8}$$

This, together with (4.7), provides the scalar profiles solving the dynamical tadpole. The motivation for this particular solution is that it preserves 1/2 supersymmetry, as we discuss in the next section in the context of its relation with the 4d string solutions in [24].

Note that as $r \to 0$, the radion blows up as $\sigma \to -\infty$, implying that the $\mathbf{S}^1$ shrinks to zero size, and the metric becomes singular. As one hits this singularity, the saxion goes to infinity, so we face a wall at which the scalars run off to infinity in moduli space. According to our arguments, it must correspond to a cobordism wall of nothing, capping off spacetime so that the $r < 0$ region is absent; hence the suggestive notation to regard this coordinate as a radial one, an interpretation which will become more clear in the following section. The finite distance Δ to the wall can be shown to obey the scaling $\Delta^{-2} \sim \mathcal{T}$ introduced in [11].

Note that the asymptotic regime near infinity in moduli space $s \gg 1$ corresponds to the regime

$$r \ll r_0 e^{\frac{2\pi}{q}(s_0-1)}\,. \tag{4.9}$$

Hence the exploration of the SDC's implications requires zooming into the region close to the wall of nothing.

Let us emphasize that the microscopic structure of the wall of nothing cannot be determined purely in terms of the effective field theory, and should be regarded as provided by its UV completion.[11] On the other hand, we can use effective field theory to obtain the scaling relations between different quantities, as in the string theory examples in the previous sections.

[11] In particular, possible constraints on q could arise from global consistency of the backreaction.

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The scaling relations. We can now study the scaling relations between spacetime and moduli space distances, and the SDC tower scale. From the spacetime profiles for σ and ϕ, it is easy to check that the contribution from the radion dominates in the $r \to 0$ limit. The resulting scaling between the moduli space distance D and r is

$$r \simeq e^{-D}\,, \tag{4.10}$$

showing again that $D \to \infty$ as $r \to 0$. On the other hand, the spacetime distance Δ in the same limit gives

$$d\Delta \simeq \frac{r}{R_0}\left(-\frac{q}{2\pi s_0}\log\left(\frac{r}{r_0}\right)\right)^{\frac{2}{n^2}} \simeq \frac{1}{R_0}\left(-\frac{q}{2\pi s_0}\right)^{\frac{2}{n^2}} D^{2/n^2} e^{-2D} dD\,. \tag{4.11}$$

Upon integration one gets an incomplete gamma function that, after keeping the leading order in $D \to \infty$, finally gives

$$\Delta \sim e^{-2D+\frac{2}{n^2}\log D}\,. \tag{4.12}$$

This is an exponential behaviour up to logarithmic corrections. It would be interesting to relate this to existing results on log corrections to Swampland conjectures (see [45]), but we skip them for now. The resulting relation allows to write the scalings of the SDC tower scale as

$$M_{\rm SDC} \sim e^{-\lambda D} \sim \Delta^{\frac{\lambda}{2}}\,, \tag{4.13}$$

that is again a power-like relation with $\mathcal{O}(1)$ exponents.

Let us turn to computing the scaling of the SDC scale with the scalar curvature R. The general expression for R is rather complicated, but simplifies in the leading order approximation at $r = 0$

$$\log|R| \simeq -4\log r \simeq 4\,D\,. \tag{4.14}$$

Hence, the SDC tower mass scales as

$$M_{\rm SDC} \sim e^{-\lambda D} \sim |R|^{-\frac{1}{4}\lambda}\,. \tag{4.15}$$

Amusingly, we again recover a power-like scaling highly reminiscent of the ADC.

Multi-field generalization. Let us end this section by mentioning that the above simple model admits a straightforward generalization to several axion-saxion moduli a^i, s^i. One simply introduces a vector of axion fluxes q^i and generalizes the above running solution to

$$a^i = a_0^i + \frac{\theta}{2\pi}q^i\,, \quad s^i(r) = s_0^i - \frac{q^i}{2\pi}\log\frac{r}{r_0}\,. \tag{4.16}$$

The corresponding backreaction on σ is

$$\sqrt{2}\,\sigma = -K(r) + K_0 + 2\log\frac{r}{R_0}\,. \tag{4.17}$$

We leave this as an exercise for the reader, since the eventual result is more easily recovered by relating our system to the 4d string-like solutions in [24], to which we now turn.

4.2 Comparison with EFT strings

The ansatz (4.7) is motivated by the relation of our setup with the string-like solutions to 4d $\mathcal{N}=1$ theories discussed in [24], which we discuss next. This dictionary implies that those results can be regarded as encompassed by our general understanding of cobordism walls of nothing and the SDC.

In a 4d perspective, (4.7) corresponds to a holomorphic profile $z=re^{i\theta}$

$$S = S_0 + \frac{q}{2\pi}\log\frac{z}{z_0}\,. \tag{4.18}$$

The axion flux in (4.3) implies that there is a monodromy $a\to a+q$ around the origin $z=0$. Hence, the configuration describes a BPS string with q units of axion charge. The solution for the metric can easily be matched with that in [24]. The 4d metric takes the form

$$ds_4^2 = -dt^2 + dx^2 + e^{2Z}dzd\bar{z}\,, \tag{4.19}$$

with the warp factor

$$2Z = -K + K_0 = \frac{2}{n^2}\log\frac{s}{s_0}\,. \tag{4.20}$$

This matches the 3d metric (4.19) by writing

$$ds_3^2 = e^{\sqrt{2}\,\sigma}\left(-dt^2+dx^2\right) + e^{2Z+\sqrt{2}\,\sigma}dr^2\,, \tag{4.21}$$

and (4.8) ensures the matching of the S^1 radion with the 4d angular coordinate range.

$$\int_0^{2\pi} d\theta e^{\sigma/\sqrt{2}}R_0 = \int_0^{2\pi} d\theta e^{Z} r\,. \tag{4.22}$$

Hence, in 4d $\mathcal{N}=1$ theories there is a clear dictionary between running solutions in $\mathbf{S}^1$ compactifications with axion fluxes and EFT string solutions. The compactification circle maps to the angle around the string; the axion fluxes map to string charges; the coordinate in which fields run (semi-infinite, due to the wall of nothing) maps to the radial coordinate away from the string; the saxion running due to the axion flux induced dynamical tadpole maps to the string backreaction on the saxion, i.e. the string RG flow; the scalars running off to infinity in moduli space as one hits the wall of nothing map to the scalars running off to infinity in moduli space as one reaches the string core. Note that the fact that the wall of nothing is not describable within the effective theory maps to the criterion for an EFT string, i.e. it is regarded as a UV-given defect providing boundary conditions for the effective field theory fields.

This dictionary allows to extend the interesting conclusions in [24] to our context. For instance, the relation between the string tension and its backreaction on the geometry provides a scaling with the spacetime distance. This is the counterpart of the scaling relations we found in our 3d dynamical cobordism discussion in the previous section.

On another line, the Distant Axionic String Conjecture in [24] proposes that every infinite field distance limit of a 4d $\mathcal{N}=1$ effective theory consistent with quantum gravity can be realized as an RG flow UV endpoint of an EFT string. We can thus map it into the

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proposal that every infinite field distance limit of a 4d $\mathcal{N}=1$ effective theory consistent with quantum gravity can be realised as the running into a cobordism wall of nothing in some axion fluxed $\mathbf{S}^1$ compactification to 3d. It is thus natural to extend this idea to a general conjecture

Cobordism Distance Conjecture. *Every infinite field distance limit of a effective theory consistent with quantum gravity can be realized as the running into a cobordism wall of nothing in (possibly a suitable compactification of) the theory.*

The examples in the previous sections provide additional evidence for this general form of the conjecture, beyond the above 4d $\mathcal{N}=1$ context.

5 4d $\mathcal{N}=1$ theories with flux-induced superpotentials

In the previous section we discussed cobordism walls in compactifications of 4d $\mathcal{N}=1$ theories on $\mathbf{S}^1$ with axion fluxes. Actually, it is also possible to study running solutions and walls in these theories without any compactification. This requires additional ingredients to introduce the dynamical tadpoles triggering the running. Happily, there is a ubiquitous mechanism, via the introduction of non-trivial superpotentials, such as those arising in flux compactifications. We discuss those vacua and their corresponding walls in this section. The discussion largely uses the solutions constructed in [46], whose notation we largely follow.

Let us consider a theory with a single axion-saxion complex modulus $\Phi = a + iv$. The 4d effective action, in Planck units, is

$$S = -\int d^4x\,\sqrt{-g}\left[\frac{1}{2}R + \frac{|\partial\Phi|^2}{4(\mathrm{Im}\,\Phi)^2} + V(\Phi,\overline{\Phi})\right] \tag{5.1}$$

with the $\mathcal{N}=1$ scalar potential

$$V(\Phi,\overline{\Phi}) = e^K\left(K^{\Phi\overline{\Phi}}\,|D_\Phi W|^2 - 3|W|^2\right). \tag{5.2}$$

We focus on theories of the kind considered in [46], where the superpotential is induced from a set of fluxes m^I, e_I, with $I=0,1$, and is given by

$$W = e_I f^I(\Phi) - m^I \mathcal{G}_I(\Phi) \tag{5.3}$$

for f^I, $\mathcal{G}_I$ some holomorphic functions whose detailed structure we do not need to specify.

In general, these fluxes induce a dynamical tadpole for Φ, unless it happens to sit at the minimum of the potential. The results in [46] allow to build 1/2 BPS running solutions depending on one space coordinate y with

$$ds^2 = e^{2Z(y)}dx_\mu dx^\mu + dy^2\,. \tag{5.4}$$

For the profile of the scalar, the solution has constant axion a, but varying saxion. Defining the 'central charge' $\mathcal{Z} = e^{\mathcal{K}/2}\,W$ and $\mathcal{Z}_*$ its value at the minimum of the potential (and similarly for other quantities), the profile for the scalar v is

$$v(y) = v_*\,\coth^2\left(\frac{1}{2}|\mathcal{Z}_*|\,y\right). \tag{5.5}$$

Note that in [46] this solution was built as 'the left hand side' of an interpolating domain wall solution (more about it later), but we consider it as the full solution in our setup. Note also that we have shifted the origin in y with respect to the choice in [46].

The backreaction of the scalar profile on the metric is described by

$$Z(y) = d + e^{-\frac{1}{2}\hat{\mathcal{K}}_0} \left[\log \left(-\sinh \left(\frac{1}{2} |\mathcal{Z}_*| \, y \right) \right) + \log \cosh \left(\frac{1}{2} |\mathcal{Z}_*| y \right) \right] , \tag{5.6}$$

where d is just an integration constant and $\hat{\mathcal{K}}_0$ is an additive constant in the Kähler potential.

The solution exhibits a singularity at $y = 0$, which (since the metric along y is flat) is at finite distance in spacetime from other points. On the other hand it is easy to see that the scalar v runs off to infinity as we hit the wall, since

$$v(y) \to 4\, v_* \, |\mathcal{Z}_*|^{-2} \, y^{-2} \quad \text{as } y \to 0 \, . \tag{5.7}$$

We can obtain the scaling of the moduli space distance with the spacetime distance. Using the kinetic term in (5.1),

$$D = - \int \frac{1}{\sqrt{2} v} \frac{dv}{dy} \, dy \simeq -\sqrt{2} \log y \simeq -\sqrt{2} \log \Delta \, . \tag{5.8}$$

In the last two equalities we have used (5.7) and (5.4) respectively. We thus get a familiar power-like scaling for the SDC scale

$$M_{\rm SDC} \sim \Delta^{\sqrt{2}\lambda} \, . \tag{5.9}$$

We also recover the ADC-like scaling with the scalar curvature. At leading order in $y \to 0$ one finds

$$\log |R| \simeq -2 \log y \simeq \sqrt{2} D \, , \tag{5.10}$$

which gives

$$M_{\rm SDC} \sim |R|^{-\frac{1}{\sqrt{2}}\lambda} \, . \tag{5.11}$$

This all fits very nicely with our picture that the solution is describing a cobordism wall of nothing, and that the solution for $y > 0$ is unphysical and not realized. This provides an effective theory description of the cobordism defects for general 4d $\mathcal{N} = 1$ theories, in a dynamical framework. It would be interesting to find explicit microscopic realizations of this setup.

Let us conclude this section by mentioning that it is possible to patch together several solutions of the above kind and build cobordism domain walls interpolating between different flux vacua. In particular in [46] the solution provided 'the left hand side' of one such interpolating domain wall solution whose 'right hand side' was glued before reaching (in our choice of origin) $y = 0$, hence before encountering the wall of nothing. The particular solution on the right hand side was chosen to sit at the minimum of the corresponding potential, for which there is no tadpole and thus the functions D and v are simply set to constants, fixed to guarantee continuity. Consequently, the solutions remain at finite distance in moduli space, in agreement with our picture for interpolating domain walls. In some sense, the flux changing membrane is absorbing the tadpole, thus avoiding the appearance of the wall of nothing. We refer the reader to [46] (see also [25]) for a detailed discussion.

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6 Walls in 10d non-supersymmetric strings

The above examples all correspond to supersymmetric solutions, and even the resulting running solutions preserve some supersymmetry. This is appropriate to establish our key results, but we would like to illustrate that they are not restricted to supersymmetric setups. In order to illustrate that these ideas can apply more generally, and can serve as useful tools for the study of non-supersymmetric theories, we present a quick discussion of the 10d non-supersymmetric USp(32) theory [47], building on the solution constructed in [13] and revised in [11].[12]

The 10d (Einstein frame) action reads

$$S_E = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}\left[R - \frac{1}{2}(\partial\phi)^2\right] - T_9^E \int d^{10}x\sqrt{-G}\, 64\, e^{\frac{3\phi}{2}}\,, \tag{6.1}$$

where T_9^E is the (anti)D9-brane tension. The theory has a dynamical dilaton tadpole $\mathcal{T} \sim T_9^E g_s^{3/2}$, and does not admit maximally symmetric solutions. The running solution in [13] preserves 9d Poincaré invariance, and reads

$$\begin{aligned} \phi &= \frac{3}{4}\alpha_E y^2 + \frac{2}{3}\log|\sqrt{\alpha_E}y| + \phi_0\,, \\ ds_E^2 &= |\sqrt{\alpha_E}y|^{\frac{1}{9}}\, e^{-\frac{\alpha_E y^2}{8}}\eta_{\mu\nu}dx^\mu dx^\nu + |\sqrt{\alpha_E}y|^{-1}e^{-\frac{3\phi_0}{2}}e^{-\frac{9\alpha_E y^2}{8}}\, dy^2\,, \end{aligned} \tag{6.2}$$

where $\alpha_E = 64k^2T_9$. There are two singularities, at $y = 0$ and $y \to \infty$, which despite appearances are located at finite spacetime distance, satisfying the scaling $\Delta^{-2} \sim \mathcal{T}$ introduced in [11]. In this case, there is no known microscopic description for the underlying cobordism defect, but we can still consider the effective theory solution to study the theory as we hit the walls.

We consider the two singularities at $y = 0, \infty$, and look at the behaviour of the solution near them. The distance from a generic point y to the singularites is given by the integral [11]

$$\Delta \sim \int |\sqrt{\alpha_E}y|^{-\frac{1}{2}}e^{-\frac{3\phi_0}{4}}e^{-\frac{9\alpha_E y^2}{16}}\, dy\,, \tag{6.3}$$

on the intervals $[y, 0]$ when $y \to 0$, and $[y, \infty]$ when $y \to \infty$. They give (lower and upper) incomplete gamma functions

$$\Delta_0 \sim \gamma\left(\frac{1}{4}, \frac{9\alpha_E y^2}{16}\right) \qquad \text{and} \qquad \Delta_\infty \sim \Gamma\left(\frac{1}{4}, \frac{9\alpha_E y^2}{16}\right)\,. \tag{6.4}$$

By expanding at leading order as $y \to 0$ and $y \to \infty$, one gets

$$\Delta_0 \sim y^{\frac{1}{2}} \qquad \text{and} \qquad \Delta_\infty \sim y^{-\frac{3}{2}}e^{-\frac{9\alpha_E y^2}{16}}\,. \tag{6.5}$$

The moduli space distance is $\phi = \sqrt{2}D$. Its leading behavior is $D \simeq -\frac{\sqrt{2}}{3}\ln y$ as $y \to 0^+$ and $D \simeq \frac{3\alpha_E}{4\sqrt{2}}y^2$ as $y \to \infty$. This leads to the scaling relations

$$\begin{aligned} y \to 0^+ : &\quad \Delta_0 \sim e^{-\frac{3}{2\sqrt{2}}D}\,, \\ y \to \infty : &\quad \Delta_\infty \sim D^{-\frac{3}{4}}e^{-\frac{3}{2\sqrt{2}}D} \sim e^{-\frac{3}{2\sqrt{2}}D - \frac{3}{4}\ln D}\,. \end{aligned} \tag{6.6}$$

[12] For other references related to dynamical tadpoles in non-supersymmetric theories, see [14–20].

In both cases we have the moduli space distance running off to infinity as we approach the wall. This is in agreement with their interpretation as cobordism walls of nothing.[13] Moreover, we recover the a familiar power-like scaling of the SDC mass scale with the same numerical factors in both cases

$$M_{\rm SDC} \sim e^{-\lambda D} \sim \Delta^{\frac{2\sqrt{2}}{3}\lambda} . \tag{6.7}$$

It is interesting to see that one can also recover a standard power-like scaling for both singularities if the moduli space distance D is compared with the spacetime curvature scalar R. The latter reads

$$|R| = \sqrt{\alpha_E}\, e^{\frac{3\phi_0}{2}} \left(\frac{2}{9}y^{-1} + \frac{7}{2}\alpha_E y + \frac{9}{8}\alpha_E^2 y^3\right) e^{\frac{9\alpha_E}{8}y^2} . \tag{6.8}$$

Let us start with the $y \to 0$ singularity. We can approximate the logarithm of the scalar curvature as

$$\log|R| \simeq -\log y \simeq \frac{3}{\sqrt{2}} D . \tag{6.9}$$

This allows to rewrite the SDC scaling in the form of the ADC-like scaling

$$M_{\rm SDC} \sim e^{-\lambda\Delta} \sim |R|^{-\frac{\sqrt{2}}{3}\lambda} . \tag{6.10}$$

Let us now turn to the $y \to \infty$ limit. In this case the logarithm of the scalar curvature is approximated to

$$\log|R| \simeq \frac{9\alpha_E}{8} y^2 \simeq \frac{3}{\sqrt{2}} D , \tag{6.11}$$

thus recovering the same behavior as for the other singularity.

As announced, we find a nice power-like scaling, reminiscent as usual of the ADC relations. It is amusing that the precise coefficient arises in both the strong and weak coupling singularities, which may hint towards some universality or duality relation in this non-supersymmetric 10d model.

7 Final remarks

In this work we have considered running solutions solving the equations of motion of theories with tadpoles for dynamical fields. These configurations were shown to lead to cobordism walls of nothing at finite distance in spacetime [11], in a dynamical realization of the Cobordism Conjecture. We have also discussed interpolating domain walls across which we change to a different (but cobordant) theory/vacuum. We have shown that the key criterion distinguishing both kinds of walls is related to distance in field space: walls of nothing are characterized by the scalars attaining infinite distance in moduli space, while interpolating domain walls remain at finite distance in moduli space.

[13] The interpretation of the $y \to 0$ singularity as a wall of nothing was deemed unconventional, since it would arise at weak coupling. It is interesting that we get additional support for this interpretation from the moduli space distance behaviour.

Hence, cobordism walls of nothing provide excellent probes of the structure of the effective theory near infinite distance points, and in particular the Swampland Distance Conjectures. This viewpoint encompasses and generalizes that advocated for EFT strings in 4d $\mathcal{N} = 1$ theories in [24]. We have found interesting new general scaling relations linking, for running solutions, the moduli space distance and the SDC tower mass scale to geometric spacetime quantities, such as the distance to the wall or the scalar curvature. The latter takes a form tantalizingly reminiscent of the Anti de Sitter Distance Conjecture (ADC), suggesting it may relate to the generalized distance in [23].

We have illustrated the key ideas in several large classes of string models, most often in supersymmetric setups (yet with nontrivial scalar potentials to produce the dynamical tadpole triggering the running); however, we emphasize that we expect similar behaviours in non-supersymmetric theories, as we have shown explicitly for the 10d non-susy USp(32) theory.

There are several interesting open question that we leave for future work:

- We have mainly focused on space-dependent running solutions. It is clearly interesting to consider time-dependent solutions, extending existing results in the literature [13–20], and exploit them in applications, in particular with an eye on possible implications for inflationary models or quintessence.

- A particular class of time-dependent solutions are dynamical bubbles. In particular, a tantalizing observation is that in the original bubble of nothing in [6], the 4d radion modulus goes to zero size (which lies at infinite distance in moduli space of the $\mathbf{S}^1$ compactification) as one hits the wall. Although the setup is seemingly unrelated, it would be interesting to understand universal features of bubbles of nothing along the lines considered in our work.

- The appearance of ADC-like scaling relations in our running solutions possibly signals an underlying improved understanding of infinite distance limits in dynamical (rather than adiabatic) configurations. For instance, as shown in [21], the $r \to \infty$ limit in the Klebanov-Strassler solution [48] avoids the appearance of a tower of states becoming massless exponentially with the distance. This was related to having a non-geodesic trajectory in moduli space (see [49] for a general discussion about non-geodesics and the SDC). However, as dictated by the lack of separation of scales in this model, an ADC-like scaling is yet respected as the scalar curvature goes to zero in this limit. This could point to a more universal way of writing the SDC in dynamical configurations.

- In all the examples we find precise numbers relating the parameter in the SDC λ to the power in the ADC-like scaling. It would certainly be interesting to find a pattern in these values and possibly relate them to properties of the infinite distance limits along the lines of [41–43]. On a similar line of thought, it has been argued that in supersymmetric cases the ADC's scaling parameter should be 1/2 [23], assuming this applies to our setup, it would be interesting to extract the SDC's parameter λ from our supersymmetric examples with an ADC-like scaling. It would be remarkable that they match the existing proposals for the value of λ.

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- The ADC-like scaling may also signal some potential interplay with the Gravitino Distance Conjecture [50, 51]. One expects to find a power relation between the mass of the gravitino and the scalar curvature of the solution, it would be certainly interesting to test this and to look for some pattern in the corresponding powers.

- The trajectory in moduli space in spacetime-dependent solutions has a strong presence in the study of black holes, in particular attractor equations and flows. The attempts to relate them to the SDC (see e.g. [52]) can have an interesting interplay with our general framework.

- We certainly expect interesting new applications of our results to the study of non-supersymmetric strings, and to supersymmetry-breaking configurations in string theory.

We hope to report on these problems in the near future.

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A Holographic examples

In [11] it was shown that Dynamical Cobordism underlies the structure of the gravity dual of the $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$ conifold theory, namely fractional brane deformation of $\mathrm{AdS}_5 \times T^{1,1}$. This in fact explains the appearance of a singularity at finite radial distance [53] and its smoothing out into a configuration capping of the warped throat [48], as a cobordism wall of nothing. In this appendix we provide some examples of other warped throat configurations which illustrate the appearance of other cobordism walls of nothing, and cobordism domain walls interpolating between theories corresponding to compactification on horizons of different topology. The discussion is strongly inspired by the constructions in [54] (see also [55]).

A.1 Domain wall to a new vacuum

As a first example we consider a configuration in which a running of the conifold theory hits a wall (given by the tip of a KS throat) interpolating to an $\mathrm{AdS}_5 \times \mathbf{S}^5$ vacuum. The latter is the maximally symmetric solution of a theory at the bottom of its potential, i.e. with no dynamical tadpole. We carry out the discussion in terms of the dual field theory, which translates easily into the just explained gravity picture. The dilaton is constant in the whole configuration, so we skip factors of g_s.

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Consider the conifold theory with $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$ at some scale, i.e. at some position r there are N units of 5-form flux and M units of 3-form flux. The Klebanov-Tseytlin solution [53] gives a running for the effective flux

$$N(r) = N + M^2 \log(r)\,, \tag{A.1}$$

and we get a singu at a value r_0 defined by

$$N + M^2 \log r_0 = 0 \quad \Rightarrow \quad r_0 = e^{-N/M^2}\,. \tag{A.2}$$

Naively, the singularity would seem to be smoothed out into a purely geometric background with a finite size $\mathbf{S}^3$. Indeed, this is the full story if N is multiple of M, namely $N = KM$: in the field theory, the $\mathrm{SU}(KM) \times \mathrm{SU}(KM + M)$ theory suffers a cascade of K Seiberg dualities in which K decreases by one unit in each step. Morally, the cascade ends when the effective $K = 0$ and then we just have a pure $\mathrm{SU}(M)$ SYM, which confines and develops a mass gap. This is the end of the RG flow, with no more running, hence the spacetime in capped off in the IR region of the dual throat.

However, as also noticed in [48], the story is slightly different if $N = KM + P$. After the K steps in the duality cascade, one is left over with an $\mathrm{SU}(P)$ gauge theory with three complex scalar degrees of freedom parametrizing a deformed mesonic moduli space corresponding to (the symmetrization of P copies of) the deformed conifold. This gauge theory flows to $\mathcal{N} = 4$ $\mathrm{SU}(P)$ SYM in the infrared, which is a conformal theory. In the parameter range $1 \ll P \ll M \ll N$, the whole configuration admits a weakly coupled supergravity dual given by a KS throat at which infrared region we have a finite size $\mathbf{S}^3$, at which P D3-branes (which we take coincident) would be located; however, since P is large, they backreact and carve out a further $\mathrm{AdS}_5 \times \mathbf{S}^5$ with P units of RR 5-form flux, which continues the radial direction beyond the KS throat endpoint region. Hence, this region acts as an interpolating domain wall between two different (but cobordant) theories, namely the conifold throat (with a dynamical tadpole from the fractional brane charge), and the $\mathrm{AdS}_5 \times \mathbf{S}^5$ vacuum (with no tadpole, and preserving maximal symmetry). The picture is summarized in figure 1.

A.2 Domain wall to a new running solution

Running can lead to an interpolating domain wall, across which we find not a vacuum, but a different running solution (subsequently hitting a wall of nothing, other interpolating domain walls, or just some AdS vacuum). We now illustrate this idea with an example of a running solution A hitting a domain wall interpolating to a second running solution B, which subsequently hits a wall of nothing. The example is based on the multi-flux throat construction in [54] (whose dimer picture is given in [56]). It is easy to devise other generalizations displaying the different behaviours mentioned above.

Consider the system of D3-branes at the singularity given by the complex cone over dP_3. The gauge theory is described by the quiver and dimer diagrams[14] in figure 2.

[14] For references, see [55, 57–59].

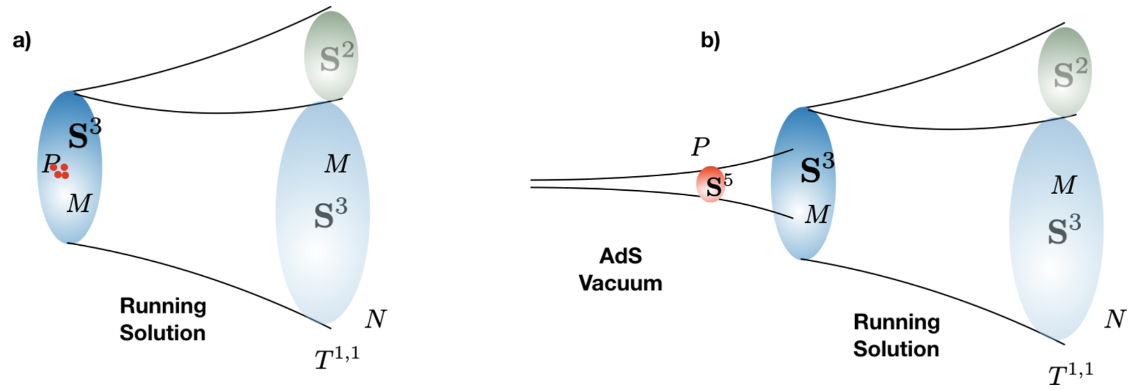

Figure 1. Domain wall interpolating between the conifold theory with fractional branes, and an AdS vacuum. Figure a) shows a heuristic intermediate step of a KS solution with a number P of left-over probe D3-branes. If P is large, the appropriate description requires including the backreaction of the D3-branes, which lead to a further AdS throat, to the left of the picture in figure b). Hence the running of the dynamical tadpole in the right hand side ends in a domain wall separating it from an AdS vacuum.

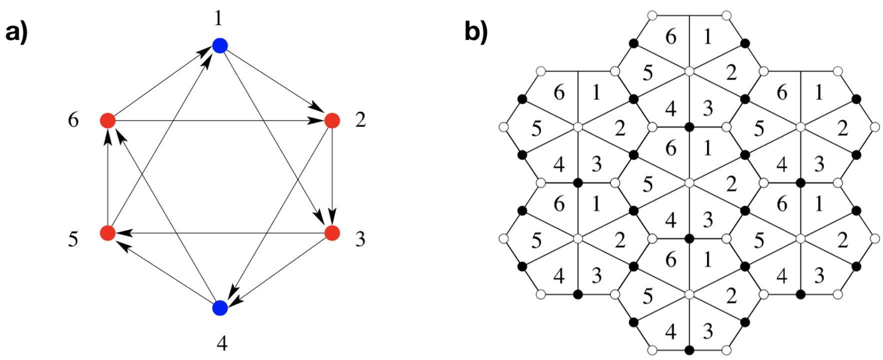

Figure 2. The quiver and dimer diagrams describing the gauge theory on D3-branes at the tip of the complex cone over dP_3.

We can add fractional branes, i.e. rank assignments compatible with cancellation of non-abelian anomalies. There are several choices, corresponding to different fluxes on the 3-cycles in the dual gravitational theory. Some of them correspond to 3-cycles which can be grown out of the singular origin to provide a complex deformation of the CY. These are described as the splitting of the web diagram into sub-webs in equilibrium, see [56]. In particular we focus on the complex deformation of complex cone over dP_3 to a conifold, see the web diagram in figure 3.

There are two kinds of fractional branes, associated to M and P. In the gravity dual, these correspond to RR 3-form fluxes on 3-cycles (obtained by an $\mathbf{S}^1$ fibration over a 2-cycle on dP_3), and there are NSNS 3-form fluxes in the dual 3-cycles. These are non-compact, namely they span a 2-cycle (dual to the earlier 2-cycle in dP_3) and the radial direction. For more details about the quantitative formulas of this kind of solution, see section 5 of [60].

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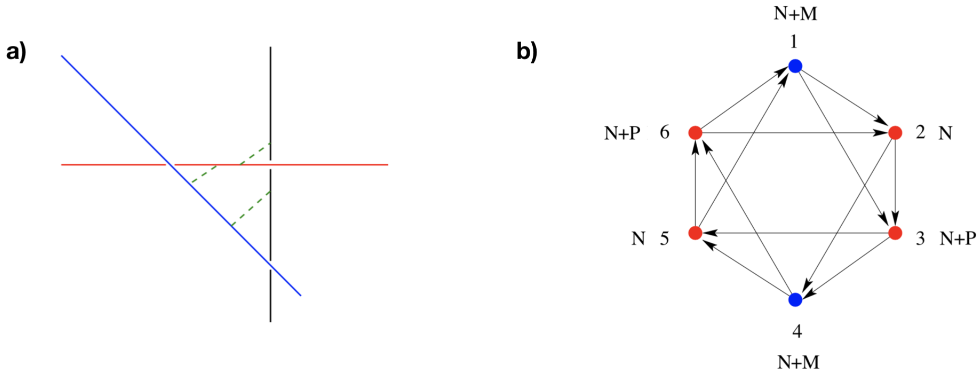

Figure 3. a) Web diagram of the complex cone over dP_3 splitting into three sub-webs. b) Rank assignment (fractional branes) that trigger those complex deformations.

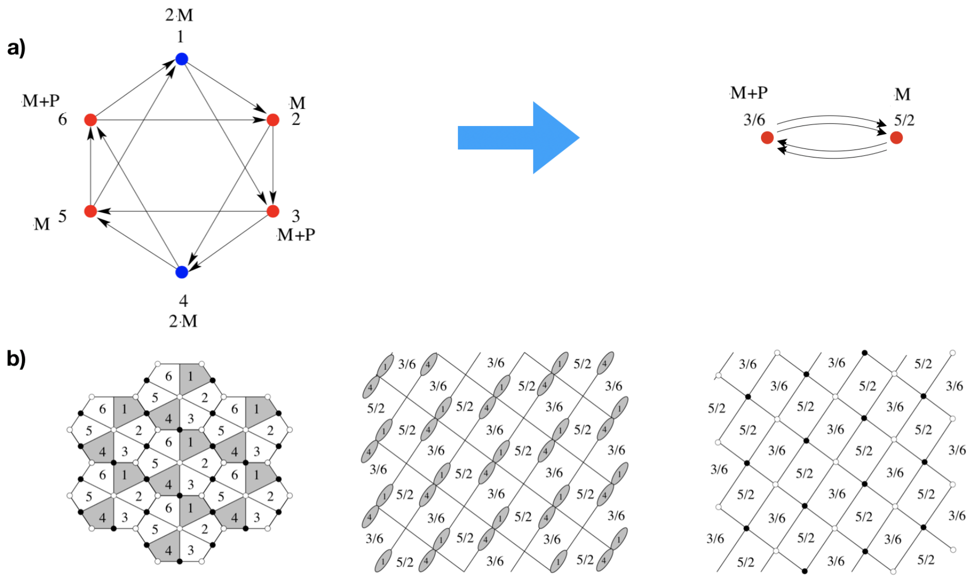

Figure 4. a) Quiver of the dP_3 theory in the last step of the first cascade, which turns into the conifold upon strong dynamics of the nodes 1 and 4. b) Same story in the dimer picture.

If we focus in the regime[15] $P \ll M$, then the larger flux M implies a larger corresponding component of the H_3 flux, which means a faster running of the corresponding 5d NSNS axion. The axion associated to the flux P also runs, but more slowly. In the field theory, the duality cascade is controlled by M, so that N is reduced in multiples of M (at leading order in P/M). When N is exhausted we are left with a rank assignment as given in figure 4a. The result of the strong dynamics triggered by M can be worked out in field theory as in [54] or using dimers as in [56]. All the info about this last description is in figure 4b.

The result is a conifold theory with M regular branes and P fractional branes. This is the standard KS story (with just different labels for the branes): M decreases in sets of P until it is exhausted, then the running stops due to strong dynamics. In the gravity dual, we

[15] Note that in [54] the regime is the opposite, but both kinds of fractional branes are similar, so the result is the same up to relabeling.

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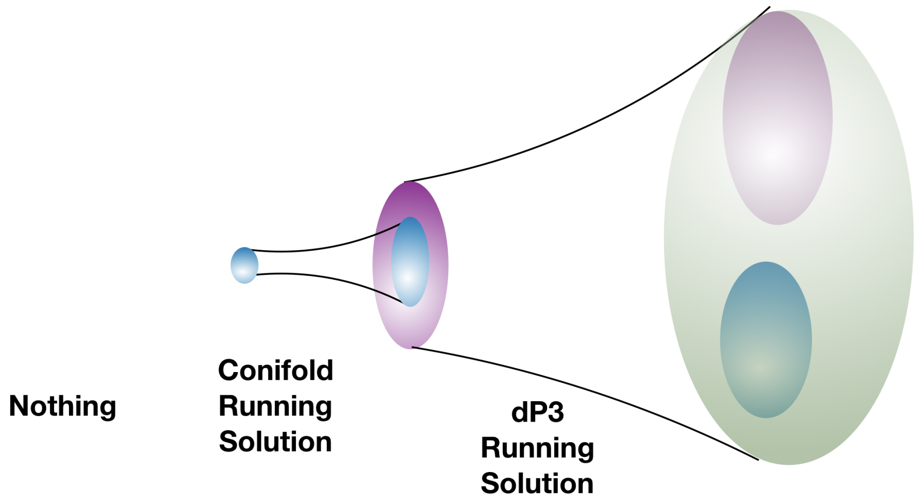

Figure 5. Domain wall interpolating between the theory on dP_3 with $(M+P)$ fractional branes, and a conifold theory with M regular branes and P fractional ones. The running of one of the dynamical tadpoles in the dP_3 theory stops at the wall but the other continues running until it reaches the $\mathbf{S}^3$ at the bottom of the KS throat.

have a KS throat sticking out and spacetime ends on the usual $\mathbf{S}^3$ (alternatively, if M is not a multiple of P, there is a number P of leftover D3-branes, which, if large, can trigger a further AdS throat as in section A.1. A sketch of the gravity dual picture is shown in figure 5.

Note that this kind of domain wall interpolates into two topologically different compactifications. As we cross it, the compactification space changes, and the spectrum of light fields changes (at the massless level, one of the axions ceases to exist). In this sense, it is a cobordism domain wall connecting two different quantum gravity theories [5].

A.3 Cobordism domain walls to disconnected solutions

The construction of singularities admitting complicated patterns of complex deformations (or resolutions) can be carried out systematically for toric singularities, using the techniques in [55]. This can be used to build sequences of domain walls realizing a plethora of possibilities. For our last class of examples, we consider cobordism domain walls to disconnected theories.

This has already been realized in the geometry used in [61] to build a bifid throat, i.e. two throats at the bottom of a throat, see figure 6. These had been proposed in [62] as possible hosts of axion monodromy inflation models (see [63–67] for additional references).

Actually, a far simpler way of getting a running solution with a domain wall to a disconnected set of e.g. vacua is to consider the KS setup in section A.1, with the P leftover D3-branes split into two stacks P_1 and P_2 of D3-branes at separated locations on the $\mathbf{S}^3$ (with $P_1, P_2 \gg 1$). This corresponds to turning on a vev v for a Higgsing $\mathrm{SU}(P) \to \mathrm{SU}(P_1) \times \mathrm{SU}(P_2)$ (with $P_1 + P_2 = P$) with a scale for v much smaller than the scale of confinement Λ of the original $\mathrm{SU}(KM+P) \times \mathrm{SU}(KM+M+P)$ theory. In the gravity dual, we have a running solution in the holographic direction, towards low energies; upon reaching Λ, we have the $\mathbf{S}^3$ domain wall, out of which we have one $\mathrm{AdS}_5 \times S^5$-like

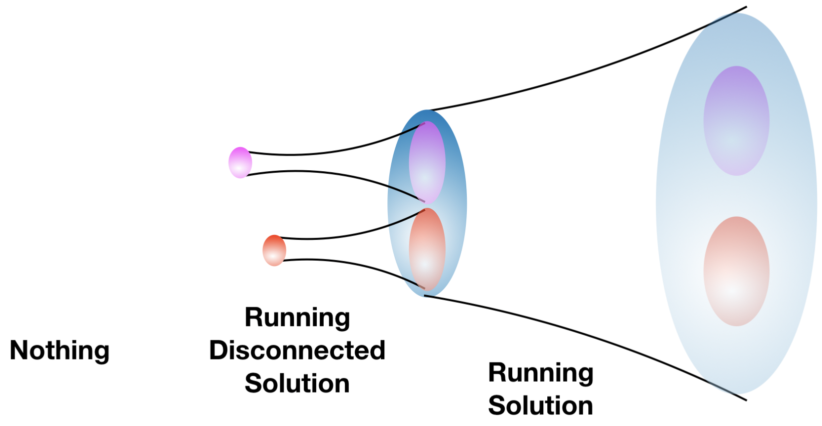

Figure 6. Picture of a bifid throat. It represents a domain wall implementing a cobordism between one theory and a disconnected set of two quantum gravity theories.

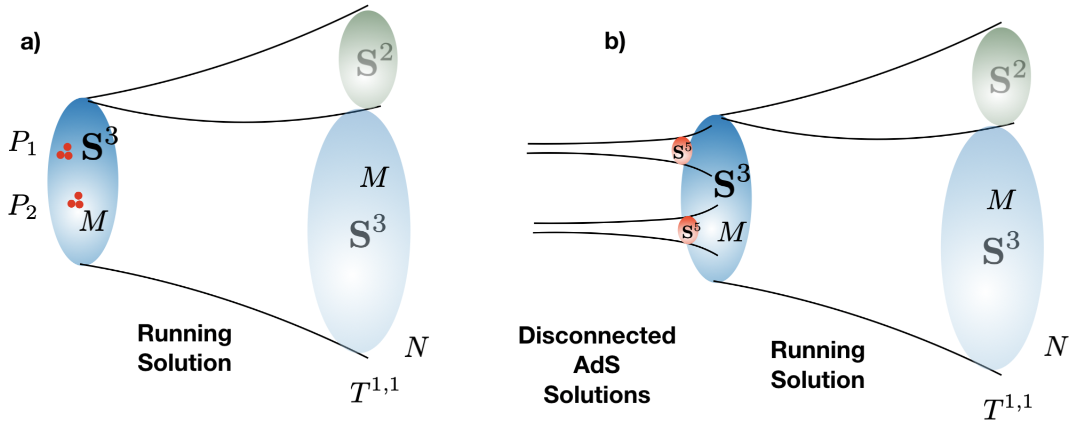

Figure 7. Picture of a bifid throat with two AdS tongues. It represents a domain wall implementing a cobordism between one theory and a disconnected set of two AdS theories.

vacuum (with flux P), until we hit the scale v, and the single throat splits into two $\mathrm{AdS}_5 \times \mathbf{S}^5$ throats (with fluxes P_1, P_2). If $v \simeq \Lambda$, the splitting of throats happens in the same regime as the domain wall ending the run of the initial solution. This is depicted in figure 7.

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At the end of the world: Local Dynamical Cobordism

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Abstract: The Cobordism Conjecture states that any Quantum Gravity configuration admits, at topological level, a boundary ending spacetime. We study the dynamical realization of cobordism, as spacetime dependent solutions of Einstein gravity coupled to scalars containing such end-of-the-world 'branes'. The latter appear in effective theory as a singularity at finite spacetime distance at which scalars go off to infinite field space distance. We provide a local description near the end-of-the-world branes, in which the solutions simplify dramatically and are characterized in terms of a critical exponent, which controls the asymptotic profiles of fields and the universal scaling relations among the spacetime distance to the singularity, the field space distance, and the spacetime curvature. The analysis does not rely on supersymmetry. We study many explicit examples of such Local Dynamical Cobordisms in string theory, including 10d massive IIA, the 10d non-supersymmetric USp(32) theory, Bubbles of Nothing, 4d $\mathcal{N}=1$ cosmic string solutions, the Klebanov-Strassler throat, Dp-brane solutions, brane configurations related to the D1/D5 systems, and small black holes. Our framework encompasses diverse recent setups in which scalars diverge at the core of defects, by regarding them as suitable end-of-the-world branes. We explore the interplay of Local Dynamical Cobordisms with the Distance Conjecture and other swampland constraints.

Keywords: D-Branes, Flux Compactifications, Superstring Vacua

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1 Introduction

The Cobordism Conjecture [1] states that in any consistent theory of quantum gravity all cobordism classes are trivial. In simple terms, it must admit at the topological level a configuration ending spacetime.[1] Such end-of-the-world configuration may correspond to a boundary (such as the 10d Horava-Witten boundary of 11d M-theory [2, 3]), a bubble of nothing in which some compactification space shrinks to zero size [4] (see [5–8] for some recent works), or more exotic possibilities, and may possibly be dressed with charged objects, such as branes, orientifold planes or generalizations (dubbed I-folds in [9]). The cobordism conjecture, already at this topological level, has produced interesting results, see [6, 9–13] for some references.[2]

An exploration of the Cobordism Conjecture beyond the topological level was undertaken in [21, 22] via the study of spacetime varying solutions to the equations of motion in theories with dynamical tadpoles, namely, potentials which do not have a minimum and thus do not admit maximally symmetric solutions (see [23–26] for early work and [27–30] for related recent developments, and [31, 32] for a complementary approach to cobordism solutions). In the solutions in [21, 22], which we refer to as Dynamical Cobordisms, the fields run along a spatial coordinate until the solution hits a singularity at finite distance in spacetime, which (once resolved in the full UV theory) ends spacetime.

These solutions exhibit sharp features in the region near the singularity. For instance, the scalars go off to infinite distance in moduli (or field) space at the spacetime singularity. Moreover, in the effective field theory description, the field space distance D, the spacetime curvature R and the spacetime distance Δ to the singularity are related by interesting scaling laws, namely (in Planck units)

$$\Delta \sim e^{-\frac{1}{2}\delta D}, \qquad |R| \sim e^{\delta D} \tag{1.1}$$

for suitable positive coefficient δ.

The singularities in these solutions are resolved in the full UV description, in terms of the corresponding cobordism configuration. In string theory examples, the latter often admits a tractable microscopic description involving geometries closing-off spacetime, possibly dressed with defects, as explained above. In this spirit, they were dubbed 'cobordism defects' or 'walls of nothing' in [21, 22]. In this work we will mainly focus on the effective field theory description, where they remain as singular sources, which we refer to as End-of-The-World (ETW) branes.[3]

The universal form of the scaling relations (1.1) was found by inspecting several explicit examples, but it suggests that a simple universal local description near the ETW branes should be possible in the effective theory. In this paper we provide this local description by studying Dynamical Cobordisms near walls at which the scalars run off to infinite field space

[1] Equivalently, any two quantum gravity theories admit, at the topological level, a domain wall connecting them. For this paper we will emphasize the formulation as in the main text.

[2] Spacetimes with boundaries have also been considered in the holographic setup, see [14–20] for some recent approaches.

[3] This follows the nomenclature in some of the references in footnote 2.

distance. In this local description, the solutions simplify dramatically and are characterized in terms of a critical exponent δ, which controls the asymptotic profiles of fields and the scaling relations (1.1) in a very direct way. The analysis does not rely on supersymmetry and can be applied to non-supersymmetric setups.

This provides a powerful universal framework to describe ETW branes within effective field theory, as we illustrate in many different examples. We exploit it to describe Dynamical Cobordisms in several 10d string theories, including non-supersymmetric cases. We also use it to characterize warped throats [33, 34] as Dynamical Cobordisms. We moreover show that the familiar 10d Dp-brane supergravity solutions can be regarded as Dynamical Cobordisms of sphere compactifications with flux, and are described by our local analysis with the D-branes playing the role of ETW branes. Finally, we argue that 4d small black hole solutions (see [35, 36] for some reviews), including those of the recent work [37], can be similarly regarded as Dynamical Cobordisms of $\mathbf{S}^2$ compactifications with flux, with the small black hole core playing the role of ETW brane.

Our models provide setups in which scalars explore large field space distances in a dynamical setup (as pioneered in [38], see also [39]), in contrast with the alternative adiabatic approach. Hence our description of Local Dynamical Cobordisms is the natural arena for the dynamical realization of swampland proposals[4] dealing with infinity in scalar moduli/field space.

The paper is organized as follows. In section 2 we present the general formalism for the local description of Dynamical Cobordisms. In section 2.1 we present the general equations of motion, and in section 2.2 we apply them to describe the local dynamics near ETW branes, and derive the universal scaling relations. In section 3 we apply the local description to several 10d examples, including massive IIA theory in section 3.1 and the non-supersymmetric USp(32) theory of [40] in section 3.2. In section 4 we interpret D-brane supergravity solutions as Dynamical Cobordisms (section 4.1) and express them as ETW branes in the local description (section 4.2). Similar ideas are applied in section 4.3 to the EFT string in 4d $\mathcal{N} = 1$ theories in [41], and in section 4.4 to the Klebanov-Strassler warped throat [33, 34]. In section 5 we discuss small black holes as Dynamical Cobordisms. In section 5.1 we warm up by expressing the supergravity solution of D2/D6-branes on $\mathbf{T}^4$ as a Dynamical Cobordism, and in section 5.2 we relate it to small black holes via a further $\mathbf{T}^2$ compactification. In section 5.3 we consider more general small black holes, such as those in [37], and derive scaling relations despite the absence of a proper Einstein frame in 2d. In section 6 we discuss the interplay of Swampland constraints with the results of our local description for the behaviour of several quantities near infinity in field space. In section 6.1 we consider the Distance Conjecture, the de Sitter conjecture and the Transplanckian Censorship Conjecture. In section 6.2 we discuss potentially large backreaction effects when the UV description of the ETW branes involve a large number of degrees of freedom, suggesting mechanisms to generate non-trivial minima near infinity in field space. In section 7 we offer some final thoughts. In appendix A we generalize the ansatz in the main text to allow for non-zero constant curvature in the ETW brane

[4] See [38] for a related viewpoint.

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worldvolume directions (section A.1), and apply it to describe Witten's bubble of nothing as a 4d Dynamical Cobordism and provide its local description (section A.2). In appendix B we discuss subleading corrections to the local description, specially relevant in cases where the leading contributions vanish.

2 Local dynamical cobordisms

In this section we formulate our local effective description near End of The World (ETW) branes, in terms of gravity coupled to a scalar field. We would like to emphasize that we consider a general scalar potential, but remarkably derive non-trivial results for its asymptotic behaviour near infinity in field space. The key input is just that the dynamics should allow for the scalar to go off to infinity in field space in a finite spacetime distance.

Interestingly, the scalar potential generically behaves as an exponential near infinity in moduli/field space, suggesting a first-principles derivation of the 'empirical' evidence for such exponential potentials, coming from string theory and other swampland considerations (see [42–44] for reviews). In particular, exponential potentials and constraints on them have been discussed in [31, 32], for the restricted case of bubbles of nothing (i.e. UV completed to a purely geometrical higher dimensional configuration, à la [4]). In contrast, our analysis holds for fully general ETW branes (and hence, allows for more general potentials, including cases without this asymptotic growth).

We focus on the case of a single scalar; however, our discussion also applies to setups with several scalars, by simply combining them into one effective scalar encapsulating the dynamics of the solutions (as illustrated in several of our examples in later sections).

2.1 General ansatz

Consider d-dimensional Einstein gravity coupled to a real scalar[5] field with a potential,

$$S = \int d^d x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right) , \tag{2.1}$$

where we are taking $M_{Pl} = 1$ units. We focus on $d > 2$, and deal with the $d = 2$ case in some explicit examples in section 5.

ETW branes define boundaries of the d-dimensional theory, hence they are described as real codimension 1 solutions. We take the ansatz

$$\begin{aligned} ds^2 &= e^{-2\sigma(y)} ds^2_{d-1} + dy^2 , \\ \phi &= \phi(y) , \end{aligned} \tag{2.2}$$

where y parametrizes the coordinate transverse to the ETW brane.

We consider flat metric in the $(d-1)$-dimensional slices. The corresponding analysis for general non-zero constant curvature, carried out in the same spirit and leading to essentially similar results, is presented in appendix A.

[5]Even though our analysis holds for general potential, we often refer to the scalar as modulus, and its field space as moduli space.

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The equations of motion are

$$\phi'' - (d-1)\sigma'\phi' - \partial_\phi V = 0\,, \tag{2.3}$$

$$\frac{1}{2}(d-1)(d-2)\sigma'^2 + V - \frac{1}{2}\phi'^2 = 0\,, \tag{2.4}$$

$$(d-2)\sigma'' - \phi'^2 = 0\,, \tag{2.5}$$

where prime denotes derivative with respect to y. The first one is the equation of motion for the scalar; for the Einstein equations, they split into transverse and longitudinal components to the ETW brane, giving two independent equations, subsequently combined into the last two equations.

The analysis of these equations is more amenable in terms of a new quantity, the tunneling potential introduced in [45, 46] (see also [47–52])

$$V_t(\phi) \equiv V(\phi) - \frac{1}{2}\phi'^2\,. \tag{2.6}$$

Using it to eliminate the scalar from the eoms we get

$$(d-1)\sqrt{2\,(V-V_t)}\,\sigma' - \partial_\phi V_t = 0\,, \tag{2.7}$$

$$\frac{1}{2}(d-1)(d-2)\sigma'^2 + V_t = 0\,, \tag{2.8}$$

$$(d-2)\sigma'' - 2\,(V-V_t) = 0\,. \tag{2.9}$$

Finally, combining the first two equations to eliminate σ we get

$$\frac{1}{4}(d-2)\,(\partial_\phi V_t)^2 + (d-1)\,(V-V_t)\,V_t = 0\,. \tag{2.10}$$

This is a d-dimensional generalization of a condition found in [50] in the context of domain walls.

Now, given a potential $V(\phi)$, one can use this equation to solve for the tunneling potential $V_t(\phi)$, and then use (2.6) and (2.8) to solve for $\phi(y)$ and $\sigma(y)$ respectively. In addition, one should check that (2.9) is also satisfied.

Before moving on, let us comment on the implications that these equations have for the signs of the relevant quantities. From equation (2.8) we learn that $V_t \le 0$. In addition, from (2.6) we get that $V - V_t \ge 0$. Notice that these two facts are consistent with equation (2.10). Finally, combining the last inequality with (2.9) we learn that $\sigma'' \ge 0$. When solving our system of equations we will systematically pick signs so that these inequalities are satisfied.

A nice way of parametrizing the freedom of choosing the potential is by writing

$$V(\phi) = a(\phi)V_t(\phi)\,, \tag{2.11}$$

where we have to impose that $a(\phi) \le 1$ for the reason explained above. Plugging this into (2.10) one can easily get to the solution

$$\log\left(\frac{V_t}{V_t^0}\right) = \pm 2\sqrt{\frac{d-1}{d-2}}\int_{\phi_0}^{\phi}\sqrt{1-a\left(\tilde{\phi}\right)}\,d\tilde{\phi}\,, \tag{2.12}$$

where we are taking $V_t^0 \equiv V_t(\phi_0)$ as boundary condition.

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2.2 Local description of end of the world branes

As explained in the introduction, we are interested in solutions for which the scalar attains infinity in field space i.e. $\phi \to \pm\infty$ at a point at finite distance in spacetime, defining an ETW brane. Without loss of generality we take this boundary to be $y = 0$, and the infinity in field space as $\phi \to \infty$.

From (2.12), it is clear that the asymptotic behavior as $y \to 0$, $\phi \to \infty$ is controlled by the asymptotic profile of $a(\phi)$. We know from the previous section that $a(\phi) \leq 1$ and we restrict our analysis to the cases where $a(\phi)$ has a well-defined and constant limit $a < 1$ as $\phi \to \infty$ (we briefly remark on the behavior $a \to 1$ below (2.15)). Indeed, although one can cook up potentials realizing other possibilities, we have not encountered them in any of the string theory examples in later sections. We therefore ignore other possibilities in what follows, leaving for future work the question about the consistency of such behaviors from the viewpoint of UV completions. Note that the constraint $a < 1$ includes $a = 0$, which corresponds to solutions with potential negligible with respect to the kinetic energy for the scalar (at least asymptotically).

Taking constant a, (2.12) gives

$$V_t(\phi) \simeq -c\, e^{\delta\,\phi}\,, \tag{2.13}$$

where $c > 0$ is related to the boundary condition used before. As explained in appendix B, we also allow c to hide some ϕ-dependence, corresponding to subleading corrections. The leading behaviour is an exponential controlled by the critical exponent δ, given by

$$\delta = 2\sqrt{\frac{d-1}{d-2}\,(1-a)}\,. \tag{2.14}$$

Here we choose the plus sign for δ. As we will see later this will imply that ETW brane explores $\phi \to \infty$ as explained above.

The critical exponent δ controls the structure of the local solution, in particular the asymptotic profile of fields as $y \to 0$, and the scaling relations among different physical local quantities.

Recall that the freedom of choosing a potential is parametrized by a. It is then interesting to ask how the potential itself looks like when approaching the end of the world. Plugging (2.13) into (2.11) we find

$$V(\phi) \simeq -a\, c\, e^{\delta\,\phi}\,. \tag{2.15}$$

Note that we get an exponential dependence, for any value of $a < 1$. As a side-note, for $a = 1$, the potential V may take different forms e.g. power-like, growing strictly slower than exponentials.

Also notice that, since $c > 0$, the sign of the potential is completely determined by that of a. Moreover, using the relation between a and the critical exponent δ in (2.14), we can put bounds on the latter depending on the sign of the potential. Namely, for $V > 0$ we must have $a < 0$, which implies $\delta > 2\sqrt{\frac{d-1}{d-2}}$, while if $V < 0$ then $0 < a < 1$, yielding $\delta < 2\sqrt{\frac{d-1}{d-2}}$. We thus neither have negative potentials whose exponential behaviour is arbitrarily strong, nor positive potentials whose exponential behaviour is arbitrarily mild. The explanation is that such exponentials would lead to $\phi'^2 \gg V$ as we approach the ETW brane, and therefore they correspond to the $a = 0$ case of our analysis.

It is interesting that we have derived fairly generically an exponential shape of the potential near infinity in moduli space, from the requirement that the theory contains ETW branes, namely configurations reaching infinity in moduli space at finite spacetime distance. In section 6.1 we will study its interplay with a variety of swampland constraints on scalar potentials. We note however that theories with milder growth of the potential (most prominently, theories with vanishing potential and exact moduli spaces) are still included in the analysis, and correspond to $a(\phi) \to 0$. The corresponding statement that $V \to 0$ in this case actually means that the theory can have any potential as long as it grows slower than ϕ'^2.

From (2.6) we can obtain the asymptotic profile of ϕ as $y \to 0$

$$\phi(y) \simeq -\frac{2}{\delta} \log \left(\frac{\delta^2}{4} \sqrt{2c\, \frac{d-2}{d-1}}\, y \right) . \tag{2.16}$$

Here we are ignoring an additive integration constant, irrelevant in the $\phi \to \infty$ limit. We have also fixed another integration constant by demanding that the function blows up for $y \to 0$. The leading term as $y \to 0$ is

$$\phi(y) \simeq -\frac{2}{\delta} \log y \, . \tag{2.17}$$

Hence the scalar goes off to infinity as we approach the end of the world. This motivates the appearance of a lowered cutoff as we approach the wall, above which a more complete microscopic description simply resolves the singularity; this resonates with the swampland distance conjecture, as we discuss in section 6.1.

Plugging (2.16) into (2.8) we can also solve for $\sigma(y)$. The final result is

$$\sigma(y) \simeq \pm \frac{4}{(d-2)\delta^2} \log y \, . \tag{2.18}$$

Here we ignore an integration constant which can be reabsorbed by a change of coordinates. Note that, to comply with (2.9), we only need to pick the minus sign.

Furthermore, the d-dimensional scalar curvature is given by

$$R = (d-1) \left(2\sigma'' - d\sigma'^2 \right) \sim \frac{1}{y^2} \, . \tag{2.19}$$

We thus recover that the curvature blows up as we approach the end of the world, leading to a naked singularity in the effective field theory description.

Notice that we have ignored a prefactor that, interestingly, vanishes for the special case $\delta^2 = \frac{2d}{d-2}$. For that value one should consider the next-to-leading order term in the $y \to 0$ expansion. In what follows we ignore this case and keep the generic one.

Since the scalar ϕ is normalized canonically, the field space distance D as $y \to 0$ is (2.17). Also, the distance in spacetime to the singularity is given by y. Hence from (2.17) and (2.19) we obtain the universal relations

$$\Delta \sim e^{-\frac{\delta}{2} D} , \qquad |R| \sim e^{\delta D} . \tag{2.20}$$

The solutions provides a simple universal description of dynamical cobordism in terms of the effective field theory. The microscopic description of the cobordism defect is available only in the UV complete theory, and is thus model-dependent (but known in many cases, see our explicit examples in later sections). From our present perspective, the only microscopic information we need is the very existence of such defects, guaranteed by the swampland cobordism conjecture [1].[6] It is thus remarkable that, the simple requirement that scalars go to infinity at finite spacetime distance leads to a complete local description of the EFT behaviour near a dynamical cobordism. Moreover, it constrains the structure of the theory, in particular it naturally yields an exponential behavior of the scalar potential near infinity in field space.

The above local description can be used to prove a general relation, introduced in [21], between the dynamical tadpole (defined as the derivative of the potential $\mathcal{T} = \partial_\phi V(\phi)$) at a given point and the spacetime distance Δ to the ETW brane, which in our examples is given by

$$\Delta \sim (\mathcal{T})^{-\frac{1}{2}} . \tag{2.21}$$

Indeed, using (2.15) and (2.17), we obtain $\mathcal{T}$ evaluated at a point y^*:

$$\mathcal{T}|_{y=y^*} = \partial_\phi V|_{y=y^*} = -a\, c\, \delta\, e^{\delta\phi}|_{y=y^*} = -a\, c\, \delta\, (y^*)^{-2} , \tag{2.22}$$

Δ is constructed as the distance from a point y^* to the singularity at $y = 0$, we therefore have $\Delta = y^*$. We hence have a general relation[7]

$$\Delta = \left(\frac{-\mathcal{T}}{a\, c\, \delta} \right)^{-\frac{1}{2}} \sim (\mathcal{T})^{-\frac{1}{2}} . \tag{2.23}$$

This relation places a bound on the spacetime extent of a solution whose running is induced by a dynamical tadpole, as emphasized in [21, 22], due the dynamical appearance of an end of spacetime. We would nevertheless mention that there exist solutions with spacetime boundaries even in situations with no dynamical tadpole. The simplest example is Horava-Witten theory, which corresponds to M-theory on an interval with two boundaries. Even in our present context of scalars running off to infinity at finite spacetime distance, it is possible to find ETW branes in cases with vanishing potential $V = 0$ (or asymptotically negligible potentials, $a = 0$).

[6] To be more precise, there are theories in which the cobordism higher-form symmetry is gauged, rather than broken by the existence of the defects. In such cases, the gauging imposes the constraint that the total charge cancels in the configuration; our analysis applies to those cases as well, with the ETW brane corresponding to a mere ending of spacetime with no explicit charged defects, similar to a bubble of nothing, see appendix A.2.

[7] For the particular case of the warped throat in 4.4 this corrects the statement in [21].

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3 Some 10d examples

In this section we consider examples of 10d theories with Dynamical Cobordism solutions in [21, 22], and use the above local description to easily derive their structure. The results nicely match the asymptotic behavior of the complete solutions in the literature.

3.1 The 10d massive type IIA theory

We consider the 10d massive type IIA theory. The effective action in the Einstein frame for the relevant fields is

$$S_{10,E} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left\{R-(\partial\phi)^2-\frac{1}{2}e^{\frac{5}{2}\sqrt{2}\phi}F_0^2-\frac{1}{2}e^{\frac{\sqrt{2}}{2}\phi}|F_4|^2\right\}, \tag{3.1}$$

where F_0 denotes the Romans mass parameter. The $\sqrt{2}$ factors in the exponents ensure that the normalization of the scalar agrees with our conventions.

This theory has a potential

$$V=\frac{1}{2}e^{\frac{5}{\sqrt{2}}\phi}F_0^2, \tag{3.2}$$

hence it does not admit 10d maximally symmetric solutions. On the other hand there are 9d Poincaré invariant (and in fact 1/2 supersymmetric) running solutions of the equations of motion in which the dilaton (and other fields) depend on a space coordinate, e.g. x^9. The metric and dilaton profile read

$$\begin{aligned} ds_{10}^2 &= Z\left(x^9\right)^{1/12}\eta_{\mu\nu}dx^\mu dx^\nu\,, \\ e^{\sqrt{2}\phi} &= Z\left(x^9\right)^{-5/6}\,, \end{aligned} \tag{3.3}$$

where the coordinate function is $Z\left(x^9\right) = -F_0x^9$. This solution hits a singularity at $x^9=0$, which was proposed to correspond to an end of the world brane in [21, 22]. In the microscopic theory, it corresponds to an O8-plane (possibly with D8-branes), as in one of the boundaries of the interval of type I' theory [53].

In the following we show how the local structure of the Dynamical Cobordism can be obtained from the analysis in the previous section.

The only input of the local analysis is the potential (3.2). Matching it with the local analysis expression (2.15), we obtain the following values for δ and, using (2.14) for a:

$$\delta=\frac{5}{\sqrt{2}}\,,\qquad a=-\frac{16}{9}. \tag{3.4}$$

Plugging this into (2.17) we obtain the dilaton profile

$$\phi\simeq-\frac{2\sqrt{2}}{5}\log y. \tag{3.5}$$

We can now obtain the profile for σ (2.18)

$$\sigma\simeq-\frac{1}{25}\log y\,, \tag{3.6}$$

which determines the metric via (2.2). As usual, the local description predicts the scalings

$$\Delta \sim e^{-\frac{5}{2\sqrt{2}}D}, \quad |R| \sim e^{\frac{5}{\sqrt{2}}D}. \tag{3.7}$$

These results from the local analysis are in agreement with the scaling relations obtained in the paper [21] from the complete solution. In fact, this can be done very easily from (3.3), by a change of coordinates

$$y = \int_0^{x^9} \left(-F_0 \tilde{x}^9\right)^{1/24} d\tilde{x}^9, \tag{3.8}$$

in terms of which the solution acquires the form of (2.2)

$$ds_{10}^2 = \left[\frac{25}{24}(-F_0)\, y\right]^{2/25} ds_9^2 + dy^2,$$
$$e^{\sqrt{2}\phi} = \left[\frac{25}{24}(-F_0)\, y\right]^{-4/5}.$$

This indeed corresponds to profiles for σ (via (2.2)) and ϕ in agreement with (3.6) and (3.5) respectively.

3.2 The 10d non-supersymmetric USp(32) string

Let us consider a second example in the same spirit, but in the absence of supersymmetry. We consider the 10d non-supersymmetric USp(32) theory, built in [40] as a type IIB orientifold with a positively charged O9-plane and 32 anti-D9-branes. The 10d Einstein frame action for the relevant fields is

$$S_E = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}\,\left\{R - (\partial\phi)^2\right\} - T_9^E \int d^{10}x\sqrt{-G}\, 64\, e^{\frac{3}{\sqrt{2}}\phi}. \tag{3.9}$$

We have introduced factors of $\sqrt{2}$ relative to the conventions in [40], to normalize the scalar as in previous sections.

This theory has a dilaton tadpole, due to the uncanceled NSNS tadpoles, and hence does not admit maximally symmetric 10d solution. On the other hand, there are 9d Poincaré invariant running solutions of its equations of motion [23], given by

$$ds_E^2 = |\sqrt{\alpha_E} r|^{\frac{1}{9}}\, e^{-\frac{\alpha_E r^2}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + |\sqrt{\alpha_E} r|^{-1} e^{-\frac{3\phi_0}{\sqrt{2}}} e^{-\frac{9\alpha_E r^2}{8}}\, dr^2,$$
$$\phi = \frac{3}{4\sqrt{2}}\alpha_E r^2 + \frac{\sqrt{2}}{3}\log|\sqrt{\alpha_E} r| + \phi_0, \tag{3.10}$$

where $\alpha_E = 64\kappa^2 T_9^E$, and ϕ_0 is a reference value for the dilaton. The coordinate r was denoted by y in [23] but here, we preserve y for the coordinate of the local analysis near end of the world branes.

The solution hits two singularities, at $r \to 0$ and at $r \to +\infty$, which are at finite spacetime distance, yet the scalar attains infinity in fields space ($\phi \to -\infty$ at $r \to 0$, and $\phi \to \infty$ at $r \to \infty$, respectively). As discussed in [21, 22], it thus describes a Dynamical Cobordism with two end of the world branes. The existence of two boundaries, and hence a

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finite size spacetime coordinate, arises in this example, but is not a general feature of running solutions, as we have seen in previous sections. It would be interesting to understand a general criterion discriminating between the two possibilities, but we leave this question for future work. In any event, even in setups with two boundaries, our local analysis applies to each of them individually, as we discuss next. Indeed, let us now exploit the local analysis to display the scalings near these walls, with the scalar potential in (3.9) as sole input.

3.2.1 $r \to 0$

From equation (3.10), we see that $r \to 0$ corresponds to the limit $\phi \to -\infty$. The potential in (3.9) vanishes in that limit. As a consequence, we have an ETW brane in which the potential becomes negligible, i.e., the critical exponents for the local model are

$$\delta = \frac{3}{\sqrt{2}}\,, \qquad a = 0. \tag{3.11}$$

The local analysis then leads to the dilaton and radion profiles

$$\phi \simeq \frac{2\sqrt{2}}{3}\log y\,, \qquad \sigma \simeq -\frac{1}{9}\log y\,. \tag{3.12}$$

Note that we have chosen the sign of $\phi \to -\infty$ as $y \to 0$.

These results allow to obtain the universal scalings for the curvature and spacetime distance with the field space distance (2.20), namely

$$\Delta \sim e^{-\frac{3}{2\sqrt{2}}D}\,, \qquad |R| \sim e^{\frac{3}{\sqrt{2}}D}\,. \tag{3.13}$$

It is easy to check that the above profiles and scaling reproduce the behaviour of the complete solution (3.10). This can be shown by the following coordinate change to bring it into the ansatz (2.2):

$$y = \int \sqrt{|\sqrt{\alpha_E}r|^{-1} e^{-\frac{3\phi_0}{\sqrt{2}}} e^{-\frac{9\alpha_E r^2}{8}}}\, dr \sim \left[\Gamma\left(\frac{1}{4}, \frac{9\alpha_E}{16}r^2\right) - \Gamma\left(\frac{1}{4}, 0\right)\right] \sim \sqrt{r}\,. \tag{3.14}$$

In the last step we have taken the leading behaviour as $r \to 0$. By also taking the leading behaviour in (3.10), plugging in y, and reading off σ as it appears in (2.2) we finally recover the profiles predicted by the local analysis in (3.12).

3.2.2 $r \to \infty$

This should be described by a local model where $\phi \to +\infty$ at $y \to 0$, i.e. the origin of a new local coordinate (which corresponds to $r \to \infty$). In this case the potential in (3.9) is blowing up, hence via (2.15) and (2.14), we get $\delta = 3/\sqrt{2}$, $a = 0$, just as in (3.11). The result $a = 0$ may seem puzzling, since from (2.15) this would seem to imply $V \to 0$. However, one should recall that in the local description $a = 0$ simply means that $V \ll \phi'^2$. Indeed, it may happen that c blows up as $\phi \to \infty$ in such a way that it compensates having $a \to 0$ in this same limit. We will explicitly check this later on.

The dilaton and radion profiles read

$$\phi \simeq -\frac{2\sqrt{2}}{3}\log y\,, \qquad \sigma \simeq -\frac{1}{9}\log y\,. \tag{3.15}$$

The dilaton sign differs from (3.12) in order to have $\phi \to +\infty$ as $y \to 0$. We also recover the scalings for Δ and R with D, which are again given by (3.13).

Let us now show that the above local model indeed reproduces the $r \to \infty$ regime of (3.10). The required change of variables is now

$$y = \int_r^\infty |\sqrt{\alpha_E}\tilde{r}|^{-1/2} e^{-\frac{3}{4}\phi_0} e^{-\frac{9\alpha_E r^2}{16}} d\tilde{r} \sim \Gamma\left(\frac{1}{4}, \frac{9\alpha_E}{16} r^2\right) \sim r^{-\frac{3}{2}} e^{-\frac{9\alpha_E}{16} r^2}. \tag{3.16}$$

The integration limits are chosen so that the finite distance singularity at $r \to \infty$ is located at the origin for the new coordinate. In the last step we have taken the leading behaviour of the Gamma function as $r \to \infty$.

Taking the logarithm of this expression and keeping the leading behaviour we get

$$\log y \simeq -\frac{9\alpha_E}{16} r^2 . \tag{3.17}$$

Finally, by also taking the leading behaviour in (3.10), reading off σ as it appears in (2.2) and plugging in our previous expression for y, we recover the profiles anticipated by the local analysis in (3.15).

Let us now come back to the issue of having $a = 0$ while not having vanishing potential. First, let us check that indeed $\phi'^2/V \to \infty$ as we approach the ETW brane. We can compute it, with no approximations, as

$$\frac{\phi'^2}{V} \sim \left(\frac{3\alpha_E}{2\sqrt{2}} r + \frac{\sqrt{2}}{3}\frac{1}{r}\right)^2 , \tag{3.18}$$

where we are ignoring irrelevant numerical prefactors. Importantly, for this computation one has to remember that ϕ' is the derivative with respect to y, not with respect to r. As advanced, we find that this blows up to infinity in both $r \to 0$ and $r \to \infty$ limits.

Moreover, using this result one can compute the tunneling potential as $\phi \to \infty$ as

$$V_t \simeq \frac{\phi'^2}{2} \sim r^2 V \sim \phi\, e^{\frac{3}{\sqrt{2}}\phi} \sim e^{\frac{3}{\sqrt{2}}\phi + \log\phi} , \tag{3.19}$$

where we have plugged in the value of V from (3.9) and $r^2 \sim \phi$ from the $r \to \infty$ limit of $\phi(r)$ in (3.10). As advertised, we find a case in which the coefficient c in (2.13) blows up as we approach the wall of nothing. This is consistent with our local analysis because, as we see in the last equality, c does not blow up faster than the exponential, i.e., it gives subleading corrections to $\log V_t$ (see appendix B for more details).

4 Branes as cobordism defects

The local analysis of section 2 provides a general framework to describe effective ETW branes, encapsulating Dynamical Cobordisms of the underlying theory. An interesting observation is that, in compactified theories with fluxes, the cobordism requires the introduction of charged objects. Namely, those required to break the corresponding cobordism charge to avoid a global symmetry, which should be absent in Quantum Gravity. A typical example

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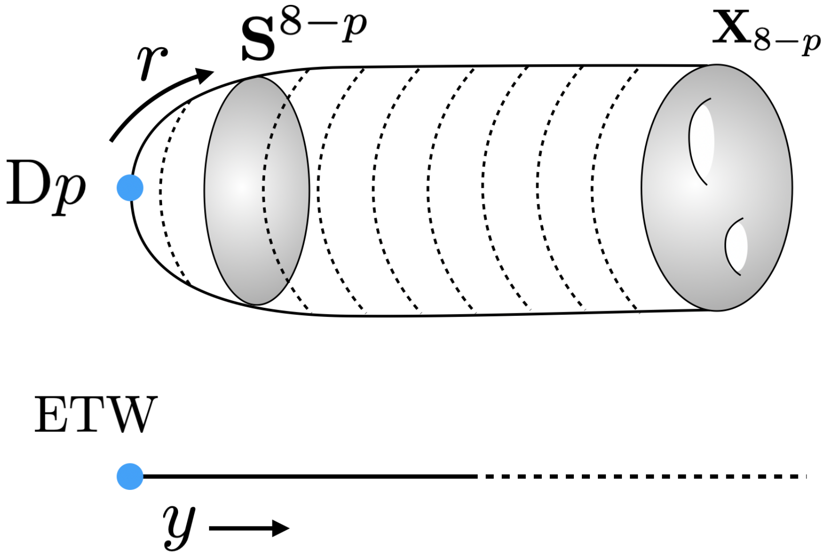

Figure 1. Dp-branes as cobordism defect in theories with $(8-p)$ compact dimensions from the higher and lower dimensional perspective. Our local $(p+2)$-dimensional description of the $\mathbf{S}^{8-p}$-truncation corresponds to the local structure of a Dynamical Cobordism of a more general compactification on $\mathbf{X}_{8-p}$.

is the introduction of NS5- and D-branes in bubbles of nothing in compactifications with NSNS and RR fluxes (see [6] for a recent discussion on bubbles of nothing).

Therefore it is interesting to explore the description of such objects in the local picture of section 2. As a simple illustrative setup, in this section we describe the geometry around a stack of Dp-branes in the language of the local analysis of section 2. In local terms, it corresponds to regarding the Dp-brane supergravity solution as a compactification of the 10d theory on $\mathbf{S}^{8-p}$ with flux, yielding a $d = (p+2)$-dimensional running solution along one of the coordinates (morally the radial coordinate), which has finite extent and end on an effective ETW brane. The microscopic description of the latter is actually given by the Dp-brane in the UV.

The above idea generalizes the description in [22] of the EFT strings solutions in [41] as cobordism defects of $\mathbf{S}^1$ compactifications of the underlying 4d $\mathcal{N} = 1$ theory with axion flux along the $\mathbf{S}^1$.

We note that the compactification of the 10d theory on the $\mathbf{S}^{8-p}$ around a Dp-brane actually corresponds to a truncation onto the SO$(9-p)$-invariant sector. Sphere truncations have long been studied in the literature, in particular in the holographic context, see [54] for a discussion for Dp-brane solutions. However, in our context we should regard the sphere truncation as a fair local description of Dynamical Cobordisms in actual compactifications, including those with scale separation, allowing for a more physical notion of lower-dimensional effective theory. Our local analysis should be regarded as part of the latter. This is depicted in figure 1, and is illustrated quantitatively in a similar example for Witten's bubble of nothing in appendix A.2.

Finally, although we phrase our discussion in terms of Dp-branes, notice that other string theory branes admit similar analysis; in fact, the NS5-brane is essentially the same as the D5-brane, since we are working in the Einstein frame, in which S-duality acts manifestly.

4.1 Compactification to a running solution

Let us begin with a precise description of the general procedure of compactifying a codimension $(n+1)$ brane-like solution in $d+n$ dimensions down to a running solution (codimension 1) in d dimensions. In next sections, we will apply this reasoning to the Dp-branes as cobordism defects of $\mathbf{S}^{8-p}$ compactifications.

Take the general metric of a codimension n object in $d+n$-dimensions:

$$ds^2 = e^{-2\mu(r)} ds_{d-1}^2 + e^{2\nu(r)} \left(dr^2 + r^2 d\Omega_n^2 \right) . \tag{4.1}$$

The directions in ds_{d-1}^2 span the worldvolume of the object, while we have split the transverse directions into radial and angular ones.

We want to perform an $\mathbf{S}^n$ truncation to look at this solution from the d-dimensional perspective. We thus take the compactification ansatz

$$ds^2 = e^{-2\alpha\omega(r)} ds_d^2 + e^{2\beta\omega(r)} r_0^2 ds_n^2 , \tag{4.2}$$

where r_0 is a reference scale. By requiring that the d-dimensional action is in the Einstein frame and has canonically normalized kinetic term for the radion ω we get the following constraints for α and β:

$$\gamma \equiv \frac{\alpha}{\beta} = \frac{n}{d-2} \qquad \beta^2 = \frac{d-2}{n(d+n-2)} . \tag{4.3}$$

The first one implements the Einstein frame requirement, while in the second one we already apply both conditions. Note that for $d=2$ we recover the familiar statement that there is no Einstein gravity in 2 dimensions. We will deal with reductions to 2d in section 5, and consider $d>2$ in what follows.

By matching the compactification ansatz (4.2) with the metric in (4.1) we obtain the profile for the radion

$$e^{2\beta\omega(r)} = e^{2\nu(r)} \left(\frac{r}{r_0} \right)^2 , \tag{4.4}$$

as well as the lower-dimensional metric

$$ds_d^2 = e^{2\alpha\omega(r)} \left(e^{-2\mu(r)} ds_{d-1}^2 + e^{2\nu(r)} dr^2 \right) . \tag{4.5}$$

In order to put solutions in the general form (2.2) used for the local description in section 2, we introduce a new coordinate

$$y = \int e^{\alpha\omega(r)} e^{\nu(r)} dr , \tag{4.6}$$

in terms of which we can borrow the results (2.15)–(2.20) from the local description.

From the viewpoint of the d-dimensional theory, there is a non-trivial potential arising from the curvature of the $\mathbf{S}^n$, and possibly other sources (such as fluxes, etc). Generically the total potential does not have a minimum, hence the running solutions can be regarded as induced by a dynamical tadpole. Applying the results in [21, 22], the d-dimensional solution must describe a Dynamical Cobordism ending on an ETW brane, to which we can apply the local analysis in section 2.

Note that however there are cases with a non-trivial minimum. A prominent example is the $\mathbf{S}^5$ compactification with a large number N of RR 5-form field string flux units (see [55] for a discussion in similar terms). The minimum corresponds to a setup with no tadpole, and admits a maximally symmetric solution, namely the celebrated $\mathrm{AdS}_5 \times \mathbf{S}^5$. Because of this, we will not consider the D3-branes in our discussion, and focus on genuinely running solutions.

4.2 D-branes as Dynamical Cobordisms

In this section we regard the 10d Dp-brane solutions as $\mathbf{S}^{8-p}$ compactifications and re-express them in terms of the local description of ETW branes of the $(p+2)$-dimensional theory of section 2. Note that, in contrast with section 3, we do not intend to *derive* the local solutions from a $(p+2)$-dimensional scalar potential; rather we take the familiar 10d solutions and express their near brane asymptotics as local $(p+2)$-dimensional ETW brane solutions.

Consider the Dp-brane solution in the 10d Einstein frame, with $0 \leq p \leq 8$. The 10d metric and dilaton profile take the form

$$ds_{10}^2 = Z\,(r)^{\frac{p-7}{8}}\,\eta_{\mu\nu}dx^\mu dx^\nu + Z\,(r)^{\frac{p+1}{8}}\left(dr^2 + r^2 d\Omega_{8-p}^2\right)\,, \tag{4.7}$$

$$\Phi = \frac{(3-p)}{4\sqrt{2}}\log\left(Z(r)\right)\,, \tag{4.8}$$

where the warp factor is given by the harmonic functions

$$Z(r) = 1 + \left(\frac{\rho}{r}\right)^{7-p} \qquad \text{for } 0 \leq p \leq 6\,, \tag{4.9}$$

$$Z(r) = 1 - \frac{N}{2\pi}\log\left(\frac{r}{\rho}\right) \qquad \text{for } p = 7\,, \tag{4.10}$$

$$Z(r) = 1 - \frac{|r|}{\rho} \qquad \text{for } p = 8\,. \tag{4.11}$$

Here $\rho > 0$ is a length scale. For the cases $p \neq 7$ it depends on the number of Dp-branes, N, while for $p = 7$ this dependence does not enter in ρ but has been made explicit in the solution.

As we have explained, these formulas should be regarded as the local description near the D-branes in possibly more general compactifications, namely the above $Z(r)$ should be though of as local expansions around the D-brane location of the warp factor in more general compactification spaces, cf. figure 1.

We immediately see that for $p \neq 3$, the dilaton reaches infinite values near the point $r = 0$, the core of the Dp-brane. As explained above, we do not consider the case $p = 3$, since it relates to AdS minimum of the theory. The solution does not run towards an ETW brane but towards a minimum in the potential. Similarly, for $p = 8$, the dilaton reaches finite values at $r = 0$. This fits with the identification of $D8$-branes as interpolating walls instead of walls of nothing in [22]. In the following we restrict to $p \neq 3$ and $1 \leq p \leq 7$, the lower bound to avoid reduction to 2d (postponed until section 5), and the upper bound to have non-trivial sphere compactification.

The Dp-brane is a solution of the following generic type II theory with a dilaton and RR field:

$$S_{10} \sim \frac{1}{2} \int d^{10}x \sqrt{-g_{10}} \left\{ R_{10} - (\partial\Phi)^2 - \frac{1}{2n!} e^{a\Phi} |F_n|^2 \right\} . \tag{4.12}$$

where $n = 8 - p$. This 10d theory does not have a scalar potential. However, once compactified on $\mathbf{S}^{8-p}$ with N units of F_{8-p} flux, the curvature of the sphere as well as the flux itself will generate dynamical tadpoles for the ensuing radion and $(p + 2)$-dimensional dilaton. Indeed, let us perform this compactification explicitly and show that we find ourselves in an end-of-the-world scenario, reproducing the associated scaling relations of [22].

Taking a compactification ansatz of the form (4.2) we obtain the $d = (p+2)$-dimensional Einstein frame metric:

$$ds_d^2 = \left(\frac{r^2}{r_0^2} Z(r)^{\frac{p+1}{8}} \right)^{\frac{8-p}{p}} \left\{ Z(r)^{\frac{p-7}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + Z(r)^{\frac{p+1}{8}} dr^2 \right\} , \tag{4.13}$$

where the Greek indices correspond to directions along the world volume of the p-brane. The $(p+2)$-dimensional dilaton inherits the same profile as the original one and one obtains the radion's profile through matching:

$$e^{2\beta\omega(r)} = \frac{r^2}{r_0^2} Z(r)^{\frac{p+1}{8}} . \tag{4.14}$$

The radion is canonically normalized if $\beta^2 = \frac{p}{8(8-p)}$.

The solution has a spacetime singularity at $r = 0$, at which both the dilaton and radion blow up. We can now compute the relevant scaling quantities, namely the spacetime distance Δ_d to the singularity, the curvature scalar $|R_d|$ near the singularity, and the distance D traversed in field space. For the former two we obtain:

$$\Delta_d \sim \begin{cases} r^{\frac{(p-3)^2}{2p}} & \text{for } p \in [1,6] \text{ and } p \neq 3\,, \\ r^{8/7} & \text{for } p = 7\,. \end{cases} \tag{4.15}$$

$$|R_d| \sim \begin{cases} r^{-\frac{(p-3)^2}{p}} & \text{for } p \in [1,6] \text{ and } p \neq 3\,, \\ r^{-16/7} & \text{for } p = 7\,. \end{cases} \tag{4.16}$$

For the field space distance near the singularity, we obtain the following by plugging in the profiles of the radion (4.14) and dilaton (4.7):

$$D(r) = \int (d\omega^2 + d\Phi^2)^{1/2} dr \simeq \begin{cases} -\frac{|3-p|}{2} \sqrt{\frac{9-p}{p}} \log r & \text{for } p \in [1,6] \text{ and } p \neq 3\,, \\ -\frac{4}{\sqrt{14}} \log r & \text{for } p = 7\,. \end{cases} \tag{4.17}$$

The solution thus describes Dynamical Cobordisms with the following scaling relations:

$$\Delta_d \sim e^{\frac{|p-3|}{p}\sqrt{\frac{p}{9-p}}\,D}\,, \qquad |R_d| \sim e^{\frac{2|p-3|}{p}\sqrt{\frac{p}{9-p}}\,D} \qquad \text{for } p \in [1,6] \text{ and } p \neq 3\,, \tag{4.18}$$

and

$$\Delta_9 \sim e^{-\frac{2\sqrt{14}}{7}\,D}\,, \qquad |R_9| \sim e^{\frac{4\sqrt{14}}{7}\,D} \qquad \text{for } p = 7\,. \tag{4.19}$$

This shows that Dp-brane are cobordism defects, which reduced on the surrounding $\mathbf{S}^{8-p}$ can be described as ETW branes. In the following we describe their structure in terms of the local description of section 2.2. This will allow us a much simpler computation of the above scaling relations.

The objective is to put the d-dimensional metric in domain-wall form (2.2). In the notation of section 4.1, one obtains:

$$\sigma(r) = -\alpha\omega(r) + \mu(r) = -\frac{8-p}{p}\log\left(Z(r)^{\frac{p+1}{16}}\left(\frac{r}{r_0}\right)\right) - \frac{p-7}{16}\log\left(Z(r)\right)\,. \tag{4.20}$$

The new coordinate y is obtained as

$$y = \int^r e^{\alpha\omega(r)}e^{\nu(r)}dr = \int^r Z(r)^{\frac{p+1}{2p}}\left(\frac{r}{r_0}\right)^{\frac{8-p}{p}} dr\,. \tag{4.21}$$

For a general Dp-brane with $p \neq 3, 7$, in the limit $r \to 0$ we have

$$y = \int^r \left(\frac{\rho}{r}\right)^{(7-p)\frac{p+1}{2p}}\left(\frac{r}{r_0}\right)^{\frac{8-p}{p}} dr \sim r^{\frac{(p-3)^2}{2p}}\,. \tag{4.22}$$

Using equation (4.20), this yields

$$\sigma(r) \simeq \sigma\left(y^{\frac{2p}{(p-3)^2}}\right) \simeq -\frac{(9-p)}{(p-3)^2}\log y\,. \tag{4.23}$$

We may compare this to the profile for σ put forward by the local description described in section 2.2:

$$\sigma(y) \simeq -\frac{4}{p\,\delta^2}\log y\,. \tag{4.24}$$

We can thus extract the value of δ and, for completeness, that of a:

$$\delta^2 = \frac{4(p-3)^2}{p(9-p)}\,, \qquad 1 - a = -\frac{(p-3)^2}{(p-9)(p+1)}\,. \tag{4.25}$$

Thus, we have, from equation (2.17):

$$D(y) \simeq -\frac{2}{\delta}\log y \simeq -\frac{|p-3|\sqrt{9-p}}{2\sqrt{p}}\log r\,. \tag{4.26}$$

We have thus recovered exactly the profile (4.17), without having to use the explicit scalar profile. From (2.20), we also recover the scaling relations (4.18), namely:

$$\Delta_d = y \sim e^{-\frac{|p-3|}{p}\sqrt{\frac{p}{9-p}}\,D}\,, \qquad |R_d| \sim e^{2\frac{|p-3|}{p}\sqrt{\frac{p}{9-p}}\,D}\,. \tag{4.27}$$

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Hence, in this case we have used the local description to recover the field-space distance and scaling relations near the singularity without knowing the full details of the d-dimensional theory. In fact, we can use the local description to derive the asymptotic behaviour of interesting d-dimensional quantities. For instance, the scalar potential scales near the singularity as (2.15):

$$V(D) = -c \left(1 - \frac{(p-3)^2}{(9-p)(p+1)}\right) e^{\frac{2(p-3)}{\sqrt{p(9-p)}} D} . \tag{4.28}$$

This is a very interesting bottom-up approach. In the actual d-dimensional action, the potential would depend on the radion and dilaton with contributions from the curvature of the sphere and the flux traversing it. However, the local description encapsulates only the dependence on the effective scalar dominating the field distance D near the ETW brane, erasing any other irrelevant UV information. From the previous equation we find that the potential is negative as we approach the ETW brane (recall $c > 0$). With the extra input that the curvature and the flux contributions to the potential are negative and positive respectively, the local description is then telling us that it is the curvature term the one that dominates in this limit.

For the D7-brane, the coordinate y is given by

$$y = \int^r e^{\alpha\omega(r)} e^{\nu(r)} dr = \int^r \left(-\frac{N}{2\pi} \log\left(\frac{r}{\rho}\right)\right)^{\frac{4}{7}} \left(\frac{r}{r_0}\right)^{\frac{1}{7}} dr \sim r^{\frac{8}{7}} , \tag{4.29}$$

where we have neglected the logarithmic contribution compared to the polynomial one. Similarly, we have:

$$\sigma(r) \simeq \sigma\left(y^{\frac{7}{8}}\right) \simeq -\alpha\omega\left(y^{\frac{7}{8}}\right) \simeq -\frac{1}{8} \log y . \tag{4.30}$$

Hence, comparing this to equation (2.18), we find:

$$\delta^2 = \frac{32}{7} , \qquad a = 0 . \tag{4.31}$$

This means that the asymptotic potential vanishes, in the sense of $\phi'^2 \gg V$. Plugging this value of δ^2 into equation (2.17) and (2.20), we recover the same field space distance and scaling relations as in the computations of the previous section:

$$D(y) \simeq -\sqrt{\frac{7}{8}} \log y \simeq -\frac{4}{\sqrt{14}} \log r , \tag{4.32}$$

$$\Delta_9 = y \sim e^{-\sqrt{\frac{8}{7}} D} , \qquad |R_9| \sim e^{2\sqrt{\frac{8}{7}} D} . \tag{4.33}$$

4.3 Revisiting the EFT strings

In [41, 56] it was proposed that in 4d $\mathcal{N} = 1$ theories the limits in which saxionic scalars go to infinity in moduli space can be studied as radial flows in 4d supersymmetric EFT string solutions magnetically charged under the corresponding axionic partners. In [22] the result was recovered by considering running solution of the compactification of the theory to 3d with axion fluxes along the $\mathbf{S}^1$: the solutions implement a Dynamical Cobordism

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ending spacetime along the running direction, and the EFT string arises as the cobordism defect required to get rid of the axion flux. In this section we revisit the analysis in [22] from the local description, with the EFT string becoming an ETW brane. As expected, the analysis is fairly similar to the 10d D7-brane example in the previous section; indeed, upon compactification of the 10d theory on a CY3, the wrapped D7-branes turns into the simplest avatar of the EFT strings in [41, 56].

In the 4d EFT string solution [41, 56], the profile for the scalars is given by

$$s(r) = s_0 - \frac{q}{2\pi} \log \frac{r}{r_0} \,, \tag{4.34}$$

$$a(\theta) = a_0 + \frac{\theta}{2\pi} q \,. \tag{4.35}$$

In our 3d interpretation, equation (4.35) describes the axionic flux over the $\mathbf{S}^1$, and equation (4.34) solves the dynamical tadpole for the saxion.

The 4d metric takes the form

$$ds_4^2 = -dt^2 + dx^2 + e^{2D} dz d\bar{z} \,, \tag{4.36}$$

with $z = re^{i\theta}$. The warp factor is given by

$$2D = -K + K_0 = \frac{2}{n^2} \log \frac{s}{s_0} \,, \tag{4.37}$$

where the Kähler potential is $K = -\frac{2}{n^2} \log s$. This D should not be confused with the field space distance, and we trust the reader to distinguish them by the context.

Matching the 4d metric (4.36) to the setup in section 4.1 with $n = 1$, we obtain the 3d coordinate y:

$$y = \int^r e^{\alpha\omega(r)} e^{\nu(r)} dr = \int^r \left(1 - \frac{q}{2\pi s_0} \log \frac{r}{r_0}\right)^{\frac{2}{n^2}} \frac{r}{r_0} dr \sim r^2 \,, \tag{4.38}$$

where we have once more neglected the logarithm compared to the polynomial contribution. Then, we can put the 3d metric in the domain-wall form (2.2), in the $r \to 0$ limit, with:

$$\sigma\left(y^{\frac{1}{2}}\right) = -\gamma\beta\omega\left(y^{\frac{1}{2}}\right) \simeq -\log\left(\left(1 - \frac{q}{2\pi s_0} \log \frac{y^{\frac{1}{2}}}{r_0}\right)^{\frac{1}{n^2}} \frac{y^{\frac{1}{2}}}{r_0}\right) \simeq -\frac{1}{2} \log y \,. \tag{4.39}$$

Comparing this to (2.18), we obtain

$$\delta^2 = 8 \,, \quad a = 0 \,. \tag{4.40}$$

We can use these parameters to recover the profiles and scaling of the local solution. For instance, we obtain that $\phi'^2 \gg V$, as in the D7-brane case. We also obtain the field-space profile and scaling relations[8] from (2.17) and (2.20):

$$D(y) \simeq -\sqrt{\frac{1}{2}} \log y \,, \tag{4.41}$$

$$\Delta = y \sim e^{-\sqrt{2}\, D} \,, \qquad |R| = e^{2\sqrt{2}\, D} \,. \tag{4.42}$$

We thus find that the full solution can be described in terms of the local description, with the EFT string described in terms of an ETW brane.

[8]This result corrects a factor of $\sqrt{2}$ arising from $|D| \simeq |\sigma_p| \simeq -\sqrt{2} \log(r)$, which was missing in [22].

4.4 The Klebanov-Strassler throat

In the previous examples we have shown that D-branes can play the role of ETW branes in running solutions of compactifications with fluxes. We would like to mention, however, an alternative mechanisms in which Dynamical Cobordisms can get rid of fluxes in the compactification, namely when the running involves axion monodromy.[9] This is most clearly illustrated in the celebrated Klebanov-Strassler (KS) solution [34], related to the compactification of type IIB theory on the 5d Sasaki-Einstein space $T^{1,1}$ with N units of RR 5-form flux and M units of RR 3-form flux on an $\mathbf{S}^3 \subset T^{1,1}$.

As shown in [21], the KS solution can be regarded as a Dynamical Cobordism, in which the tip of the throat ends spacetime at finite spacetime distance in the radial direction, smoothing out (or UV completing) the singularity of the related Klebanov-Tseytlin (KT) solution [33]. In this section we show that the structure of the KT solution is indeed that of an ETW brane from the viewpoint of the 5d effective theory.

Consider the KT solution [33], whose 10d Einstein frame metric reads:

$$ds_{10}^2 = h^{-1/2}(r)\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(r)\left(dr^2 + r^2 ds_{T^{1,1}}^2\right)\,, \tag{4.43}$$

with

$$h(r) = b_0 + \frac{M^2 \log\left(r/r_*\right)}{4r^4}\,. \tag{4.44}$$

The singularity is at r_s such that $h(r_s) = 0$, signalling the location of the ETW brane. One can show that $\partial_r h \neq 0$ at $r = r_s$, hence we may expand this harmonic function near this point as

$$h(r) \sim r - r_s \equiv \tilde{r}\,. \tag{4.45}$$

We now take the compactification ansatz

$$ds_{10}^2 = L^2\left(e^{-5q}ds_5^2 + e^{3q}ds_{T^{1,1}}^2\right) \tag{4.46}$$

with L an overall scale. Matching with (4.43) we get the profile for the breathing mode

$$q(r) = \frac{1}{6}\log\left(\left(\frac{r}{L}\right)^4 h(r)\right) \simeq \frac{1}{6}\log\tilde{r}\,, \tag{4.47}$$

where in the last equality we have taken the near ETW limit. We also get the 5d Einstein frame metric:

$$L^2 ds_5^2 = \left(\left(\frac{r}{L}\right)^2 h^{\frac{1}{2}}\right)^{\frac{5}{3}}\left(h^{-\frac{1}{2}}\eta_{\mu\nu}dx^\mu dx^\nu + h^{\frac{1}{2}}dr^2\right)\,. \tag{4.48}$$

From it we can derive the relation between $\tilde{r}$ and the radial coordinate y in the local analysis, which is

$$\tilde{r} \sim y^{\frac{3}{5}}\,. \tag{4.49}$$

Reading off the warp factor

$$e^{-2\sigma} = \left(\left(\frac{r}{L}\right)^2 h^{\frac{1}{2}}\right)^{\frac{5}{3}} h^{-\frac{1}{2}} \sim \tilde{r}^{\frac{1}{3}} \sim y^{\frac{1}{5}}\,, \tag{4.50}$$

[9] For axion monodromy in inflation, see [57–64].

we finally find

$$\sigma(y) \simeq -\frac{1}{10}\log y\,. \tag{4.51}$$

Hence, the 5d KT solution near the singularity fits with the form of an ETW brane in our local description with

$$\delta = \frac{2\sqrt{30}}{3}\,, \qquad a = -\frac{3}{2}\,. \tag{4.52}$$

We can also check that the solution for the scalars also fits in the local model description. The NSNS axion is given by

$$T(r) = \tilde{T} + M\log r \simeq T_s + \frac{M}{r_s}\tilde{r}\,, \tag{4.53}$$

again in the near ETW brane limit. Here $T_s = T(r_s)$, which we can keep arbitrary. The field space metric from the 5d action in [33] is given by

$$dD^2 = 30(\partial q)^2 + \frac{1}{2}g_s^{-1}e^{-6q}(\partial T)^2\,. \tag{4.54}$$

Using the profiles for q and T in the $\tilde{r} \to 0$ limit, we have

$$(\partial q)^2 \simeq \frac{1}{36\tilde{r}^2}\,, \qquad e^{-6q}(\partial T)^2 \simeq \left(\frac{M}{r_s}\right)^2\frac{1}{\tilde{r}}\,. \tag{4.55}$$

For $\tilde{r} \to 0$, the breathing modes dominates the field space distance in field-space. Following [65], it is then an asymptotically geodesic trajectory. This is in contrast with the $r \to \infty$ limit, for which the field-space trajectory was shown to be highly non-geodesic in [38]. Hence we have

$$dD^2 \simeq 30(\partial q)^2 \simeq \frac{5}{6}\tilde{r}^{-2}\,. \tag{4.56}$$

Upon integration and using (4.49) we obtain

$$D(y) \simeq -\frac{\sqrt{30}}{10}\log y\,. \tag{4.57}$$

This again takes the form found in our local analysis, for the above coefficients (4.52).

Finally, we also check that the 5d scalar potential from [33] scales as predicted by the local model. The complete potential is

$$V(\phi) = -5e^{-8q} + \frac{1}{8}g_s M^2 e^{-14q} + \frac{1}{8}(N + MT)^2 e^{-20q}\,. \tag{4.58}$$

Plugging in $T = T_s$ and $D \simeq -\sqrt{30}\,q$ as dictated by (4.56), we get

$$V(D) = -5e^{\frac{4\sqrt{30}}{15}D} + \frac{1}{8}g_s M^2 e^{\frac{7\sqrt{30}}{15}D} + \frac{1}{8}(N + MT_s)^2 e^{\frac{2\sqrt{30}}{3}D}\,. \tag{4.59}$$

For $N + MT_s \neq 0$, we find that the last term dominates as $D \to \infty$. As predicted by our local analysis, it has an exponential behaviour with D with the coefficient δ given in (4.52). Moreover, as predicted by finding $a < 0$, the coefficient in front of this exponential is positive.

We hope these examples suffice to convince the reader that the local description provides a simple and efficient framework to discuss the structure of Dynamical Cobordisms near the ETW brane.

5 Small black holes as Dynamical Cobordisms

The analysis of the previous section for single-charge D-brane solutions can be similarly carried out for systems of multiple charges, namely combining D-branes of different dimensionalities. Such systems have been extensively employed in the construction and microscopic understanding of black holes, both with finite horizon, starting with [66], or with vanishing classical horizon area (small black holes) (see [35, 36] for some reviews). In this section we describe brane configurations, closely related to the celebrated D1/D5 system, leading to small black holes, and describe them as cobordism defects of suitable sphere compactifications of the underlying theory. The resulting dimensionally truncated theory corresponds to a 2d theory of gravity and an effective scalar (2d dilaton gravity), for which we find scaling relations analogous to the higher dimensional cases. This description relates the Dynamical Cobordisms to the realization of the Swampland Distance Conjecture in small black holes[10] in [37].

5.1 The D2/D6 system on $\mathbf{T}^4$

We consider a configuration of D6- and D2-branes in the following (1/4 susy preserving) configuration

$$D6:\ 0\ \ 1\ \ 2 \times \times \times 6\ \ 7\ \ 8\ \ 9 \tag{5.1}$$

$$D2:\ 0\ \ 1\ \ 2 \times \times \times \times \times \times \tag{5.2}$$

where the numbers correspond to directions spanned by the brane worldvolumes and $\times$'s mark transverse directions. We consider all branes to coincide in the mutually transverse directions 345. We moreover smear the D2-branes in the direction 6789. Eventually these directions will be taken to be compact, so the smeared description is valid for small compactification size.

In the 10d Einstein frame the metric and dilaton profile are given by harmonic superposition (see [70] for background)

$$\begin{aligned}ds^2 &= Z_6(r)^{-\frac{1}{8}} Z_2(r)^{-\frac{5}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + Z_6(r)^{\frac{7}{8}} Z_2(r)^{\frac{3}{8}} (dr^2 + r^2 d\Omega_2^2) + Z_6(r)^{-\frac{1}{8}} Z_2(r)^{\frac{3}{8}} dx^m dx^m ,\\ \Phi(r) &= \frac{1}{2\sqrt{2}} \log\left(Z_6(r)^{-\frac{3}{2}} Z_2(r)^{\frac{1}{2}} \right) ,\end{aligned} \tag{5.3}$$

where r is the radial coordinate in 345, $d\Omega_2^2$ is the volume of a unit $\mathbf{S}^2$ in this $\mathbf{R}^3$, and $m = 6, 7, 8, 9$. The harmonic functions are

$$Z_6(r) = 1 + \frac{\rho_6}{r} , \qquad Z_2(r) = 1 + \frac{\rho_2}{r} . \tag{5.4}$$

As announced, we now consider compactifying the directions 6789 on a $\mathbf{T}^4$ (similar results hold for K3 compactification, as usual), with the compactification ansatz

$$ds^2 = e^{-\frac{t}{\sqrt{2}}} ds_6^2 + e^{\frac{t}{\sqrt{2}}} ds_{T^4}^2 . \tag{5.5}$$

[10]For other approaches to Swampland constraints using (large and small) black holes, see e.g. [67–69].

Matching this ansatz to (5.3), we obtain the canonically normalized radion

$$t(r) = \sqrt{2}\log\left(Z_6(r)^{-\frac{1}{8}}Z_2(r)^{\frac{3}{8}}\right) . \tag{5.6}$$

The 6d Einstein frame metric reduces to:

$$\begin{aligned} ds_6^2 &= e^{\frac{t}{\sqrt{2}}}\left(Z_6\left(r\right)^{-\frac{1}{8}} Z_2\left(r\right)^{-\frac{5}{8}} \eta_{\mu\nu}dx^\mu dx^\nu + Z_6\left(r\right)^{\frac{7}{8}} Z_2\left(r\right)^{\frac{3}{8}}\left(dr^2 + r^2 d\Omega_2^2\right)\right) \\ &= Z\left(r\right)^{-\frac{1}{4}} \eta_{\mu\nu}dx^\mu dx^\nu + Z\left(r\right)^{\frac{3}{4}}\left(dr^2 + r^2 d\Omega_2^2\right) \end{aligned} \tag{5.7}$$

where $Z(r) = Z_6(r)Z_2(r)$.

One can see that the dilaton and radion are both blowing up upon reaching the point $r = 0$, which is at finite spacetime distance, hence the configuration can be dubbed a 6d small black 2-brane.

As in section 4, we can describe the configuration as a Dynamical Cobordism of the 6d theory compactified on an $\mathbf{S}^2$ with suitable 2-form fluxes (for the RR 2-form field strength and the $\mathbf{T}^4$ reduction of the RR 6-form field strength). To implement this, we take the general ansatz:

$$ds_6^2 = e^{-2\alpha\sigma}ds_4^2 + r_0^2 e^{2\beta\sigma}d\Omega_2^2 . \tag{5.8}$$

In the resulting 4d theory, there are non-trivial potential terms for the new radion σ arising from the curvature of $\mathbf{S}^2$ and the 2-form fluxes. Imposing the Einstein frame in 4d comes down to setting $\gamma = \frac{\alpha}{\beta} = 1$. One can then choose β such that the radion $\sigma(r)$ has a canonically normalized kinetic term and one obtains $\beta = \frac{1}{2}$. From matching this compactification ansatz to equation (5.7), we obtain the canonically normalized radion σ,

$$\sigma(r) = \log\left(\frac{r^2}{r_0^2}Z(r)^{\frac{3}{4}}\right) , \tag{5.9}$$

and the following 4d Einstein frame metric

$$ds_4^2 = e^{\sigma}\left(Z(r)^{-\frac{1}{4}}\eta_{\mu\nu}dx^\mu dx^\nu + Z(r)^{\frac{3}{4}}dr^2\right) = \left(\frac{r}{r_0}\right)^2\left(Z(r)^{\frac{1}{2}}\eta_{\mu\nu}dx^\mu dx^\nu + Z(r)^{\frac{3}{2}}dr^2\right) .$$

This solution is a 4d Dynamical Cobordism, with the D2/D6-brane system playing the role of cobordism defect. The solution has the structure of an ETW brane; there are 3 running scalars going off to infinite distance at the singularity at $r = 0$, which is straightforward to show lies at finite spacetime distance. Indeed, near $r = 0$, we have

$$\Delta = \int_0^r \left(\frac{r}{r_0}\right)\left(Z_6(r)Z_2(r)\right)^{\frac{3}{4}}dr \sim \sqrt{r} . \tag{5.10}$$

Furthermore, near the singularity, the distance in field space goes like:

$$dD^2 = d\Phi^2 + d\sigma^2 + dt^2 \simeq \frac{1}{2}\frac{dr^2}{r^2} \to D \simeq -\frac{1}{\sqrt{2}}\log(r) \tag{5.11}$$

Near the singularity, the Ricci scalar in 4d behaves as:

$$|R| \sim r^{-1} \tag{5.12}$$

These lead to the familiar scaling relations near $r=0$:

$$|R|^{-\frac{1}{2}} \sim \Delta \sim e^{-\frac{1}{\sqrt{2}}|D|} \,. \tag{5.13}$$

Since the above full solution has the structure of a Dynamical Cobordism, it should be possible to express it in the framework of our local description, with the D2/D6-brane system playing the role of the ETW brane. Let us define the new coordinate:

$$y = \int^r \left(\frac{r}{r_0}\right) \left(Z_6(r) Z_2(r)\right)^{\frac{3}{4}} dr \sim \sqrt{r}\,, \tag{5.14}$$

where we have considered the leading behaviour near $r=0$.

Using equation (4.4), we have:

$$\sigma\left(y^2\right) = -\frac{1}{2}\log\left(\frac{y^4}{r_0^2} Z\left(y^2\right)^{\frac{1}{2}}\right) \simeq -\log y\,. \tag{5.15}$$

Matching this to the profile in (2.17), we see that $\delta^2 = 2$ and $a = \frac{2}{3}$. Then we automatically fall back on the previous field-space distance and scaling relations using equations (2.17) and (2.20):

$$D(y) \simeq -\sqrt{2}\log y\,, \tag{5.16}$$

$$\Delta = y \sim e^{-\frac{1}{\sqrt{2}}D} \sim |R|^{-\frac{1}{2}}\,. \tag{5.17}$$

This gives yet another nice check of the usefulness of the local analysis.

Beyond the Einstein frame. One last remark that will be relevant in the next sections is that scaling relations similar to those of (5.13) can be found, independent of the frame chosen during the compactification. Indeed, if one insists on keeping γ (and thus, also β) general and tracking it throughout the computations, one obtains the new coordinate near $r=0$:

$$\Delta = y \sim r^{\frac{1}{4}(\gamma+1)} \quad \text{and} \quad |R| \sim r^{-\frac{1}{2}(\gamma+1)}\,. \tag{5.18}$$

Note that, if $\gamma < -1$, then these scalings behave opposite to those we have seen for ETW branes. This illustrates that the scalings mentioned rely on using the Einstein frame metric to describe the ETW brane.

In setups where one needs (or finds convenient) to use general frames, the condition for an ETW brane is that the picture of a scalar going off to infinity at finite spacetime distance can be attained by a suitable change of frame. In this respect, we note that there is an extra subtlety in dealing with the field space distance in general frames. Indeed, not being in the Einstein frame implies that the radion is multiplying the Einstein-Hilbert term in the action:

$$S_4 \supset \frac{1}{2}\int d^4x\sqrt{-g_4}\, e^{2\beta\sigma(1-2\gamma)}\left\{e^{2\beta\gamma\sigma}\left(R_4 - (\partial t)^2 - (\partial\Phi)^2 - \beta^2\left(6\gamma^2 - 8\gamma + 6\right)(\partial\sigma)^2\right)\right\}\,. \tag{5.19}$$

It thus makes sense to define the field space distance measured in units set by this coefficient of the Ricci scalar in the action. This field space distance near the singularity in this general frame reads:

$$
\begin{aligned}
dD^2 &= d\Phi^2 + \beta^2 \left(6\gamma^2 - 8\gamma + 6\right) d\sigma^2 + dt^2 \\
D &\simeq -\frac{\sqrt{6\gamma^2 - 8\gamma + 10}}{4} \log r \,.
\end{aligned}
\tag{5.20}
$$

Hence, we can derive the following universal scaling relations in a general frame:

$$
\Delta \sim e^{-\frac{\gamma+1}{\sqrt{6\gamma^2-8\gamma+10}}|D|} \sim |R|^{-\frac{1}{2}} \,. \tag{5.21}
$$

Note that these reduce to those of (5.13) when setting $\gamma = 1$, as required by the Einstein frame. As a side note, one cannot recover this result in the local description detailed in section 2.2 as it was constructed in the Einstein frame. We leave such a more general formulation of the local construction for future work.

5.2 Small black holes from the D2/D6 system on $\mathbf{T}^4 \times \mathbf{T}^2$

Let us now consider turning our D6/D2-brane systems into a (small) black hole, by a further compactification on $\mathbf{T}^2$.

We take the ansatz

$$
ds_6^2 = e^{-q} ds_4^2 + e^q ds_{T^2}^2 \,. \tag{5.22}
$$

By matching this ansatz to the 6d metric obtained previously (5.7), we get the 4d Einstein frame metric:

$$
\begin{aligned}
e^{q(r)} &= Z\left(r\right)^{-\frac{2}{8}} \,, \\
ds_4^2 &= \left(g_4\right)_{ij} dx^i dx^j = e^{q(r)} \left(-Z\left(r\right)^{-\frac{2}{8}} dt^2 + Z\left(r\right)^{\frac{6}{8}} \left(dr^2 + r^2 d\Omega_2^2\right)\right) \\
&= -Z\left(r\right)^{-\frac{1}{2}} dt^2 + Z\left(r\right)^{\frac{1}{2}} \left(dr^2 + r^2 d\Omega_2^2\right) \,.
\end{aligned}
\tag{5.23}
$$

This solution describes a small black hole (in fact, equivalent to the celebrated D1/D5-brane one, by T-duality in one of the $\mathbf{T}^2$ directions), of the kind considered in [37].

To motivate the relation with the more general discussion in the next section, let us make the following heuristic argument. Although our solution has three scalar fields, the radial evolution can be reduced to one effective scalar as follows. Near $r = 0$, all three scalars have the same profile, so we may combine them in one effective scalar D whose effective action near $r = 0$ is of the form

$$
S_4 \sim \frac{1}{2} \int d^4x \sqrt{g_4} \left\{ R_4 - (\partial D)^2 - \frac{1}{4} e^{\frac{1}{\sqrt{2}} D} |F_2|^2 \right\} \tag{5.24}
$$

where we have restricted to the U(1) linear combination under which the D2/D6 system is charged.

With this proviso, we can frame this particular example with the more general class of small Black Holes considered in [37], to be discussed next.

5.3 General small black holes

In the context of the swampland program, [37] proposed the use of 4d small black hole solutions to provide further evidence for a number of a number of Swampland conjectures. A particularly important property is that the 4d solutions contain scalars going off to infinite field space distance at the black hole core. In the spirit of previous sections, in this section we show that these 4d solutions can be turned into 2d Dynamical Cobordisms upon reducing on the $\mathbf{S}^2$, with the small black hole playing the role of the ETW brane. In fact we will check that the 2d running solution satisfies the familiar scaling relations (for a general frame, since there is no Einstein frame in 2d).

Let us briefly review the key features of such solutions. We consider 4d Einstein-Maxwell coupled to a scalar controlling the gauge coupling. We take the action

$$S_{4d} \sim \frac{1}{2}\int d^4x\,\sqrt{-g_4}\,\left(R_4 - (\partial\phi)^2 - e^{2a\phi}|F_2|^2\right)\,. \tag{5.25}$$

We focus on exponential dependence, since it provided the most explicit class considered in [37]. It also fits with the special role of exponential functions in local descriptions of ETW branes.

Without loss of generality, we take $a > 0$ so that $\phi \to \infty$ corresponds to weak coupling for the U(1) gauge field. Note that this a should not be confused with the parameter in (2.11), and we trust the reader to distinguish them by the context.

In this theory, electrically charged extremal black holes take the form

$$ds_4^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2R(r)^2d\Omega_2^2\,, \tag{5.26}$$

where

$$R(r) = \left(1 - \frac{r_h}{r}\right)^{\frac{a^2}{1+a^2}}\,,\qquad f(r) = \left(1 - \frac{r_h}{r}\right)^{\frac{2}{1+a^2}}\,. \tag{5.27}$$

In addition, the profile for the scalar is given by

$$\phi(r) = \phi_0 - \frac{\sqrt{2}\,a}{1+a^2}\log\left(1 - \frac{r_h}{r}\right)\,. \tag{5.28}$$

The scalar goes off to infinity at the horizon $r = r_h$, which is however not smooth, since the $\mathbf{S}^2$ shrinks to zero size, leading to a small black hole.

In the string theory context, small black holes can be easily built by using D-branes. In fact, we now recast the above solution in a form closer to the solution (5.23), which described our system of D2- and D6-branes on $\mathbf{T}^4 \times \mathbf{T}^2$. This was already anticipated when we obtained (5.24), which has the structure of (5.25) (for $a = \frac{1}{2\sqrt{2}}$).

Carrying out the coordinate change $r \to r + r_h$, the metric (5.26) becomes

$$ds_4^2 = -\left(1 + \frac{r_h}{r}\right)^{-\frac{2}{1+a^2}}dt^2 + \left(1 + \frac{r_h}{r}\right)^{\frac{2}{1+a^2}}\left(dr^2 + r^2d\Omega_2^2\right)\,. \tag{5.29}$$

Similarly, the scalar reads

$$\phi(r) = \phi_0 + \frac{\sqrt{2}\,a}{1+a^2}\log\left(1 + \frac{r_h}{r}\right)\,. \tag{5.30}$$

This has the structure of (5.23) with $Z(r) = (1 + r_h/r)^{\frac{4}{1+a^2}}$. Note that the core of the small black hole now lies at $r = 0$.

We now perform the reduction on $\mathbf{S}^2$ to express these solutions as 2d running solutions describing a local Dynamical Cobordism, with the small black hole playing the role of the ETW brane. We will also recover the corresponding (general frame) scaling relations.

Since there is no Einstein frame in 2d, we perform the $\mathbf{S}^2$ reduction with the following general ansatz:

$$ds_4^2 = e^{-2\alpha\omega} ds_2^2 + e^{2\beta\omega} r_0^2 d\Omega_2 \,. \tag{5.31}$$

The 2d action obtained from the compactification contains the terms

$$S_{2d} \supset \frac{1}{2} \int d^2x \sqrt{-g_2}\, e^{2\beta\omega} \left(R_2 - (\partial\phi)^2 - 6\beta^2 (\partial\omega)^2 \right) \,. \tag{5.32}$$

These expressions already show the impossibility to define an Einstein frame: it would require $\beta = 0$, and this would kill the radion's kinetic term. We therefore keep β general, so we deal with a dilaton-gravity theory. By matching the ansatz (5.31) with the 4d metric (5.29) we get the profile for the radion

$$\omega(r) = \frac{1}{\beta} \log \left(\frac{r}{r_0} \left(1 + \frac{r_h}{r} \right)^{\frac{1}{1+a^2}} \right) \,, \tag{5.33}$$

and the 2d metric

$$ds_2^2 = \left(\frac{r}{r_0} \right)^{2\gamma} \left(- \left(1 + \frac{r_h}{r} \right)^{-\frac{2(1-\gamma)}{1+a^2}} dt^2 + \left(1 + \frac{r_h}{r} \right)^{\frac{2(1+\gamma)}{1+a^2}} dr^2 \right) \,, \tag{5.34}$$

where $\gamma = \frac{\alpha}{\beta}$.

Computing the 2d Ricci scalar and taking the leading order in $r \to 0$ we get

$$|R| \sim r^{-2\frac{(\gamma+1)a^2}{1+a^2}} \,, \tag{5.35}$$

where we are ignoring a constant prefactor.[11]

Similarly, the spacetime distance from a given r to the singularity, at leading order in $r \to 0$, scales as

$$\Delta \sim r^{\frac{(\gamma+1)a^2}{(1+a^2)}} \,. \tag{5.36}$$

We note that, as expected, the scaling is the familiar ETW one if $\gamma > -1$. As explained above, the fact that 2d gravity is topological means that the criterion for an ETW brane in a solution should be that the usual relations hold in *some* suitable frame.

Let us now recover the usual scalings with the field distance. Recalling the latter is measured in units set by the coefficient of the Ricci scalar in the action, we can it read off from (5.32) as:

$$dD^2 = d\phi^2 + 6\beta^2 d\omega^2 \,. \tag{5.37}$$

[11]This prefactor vanishes for either $a^2 = 1$ or $a^2 = -2\gamma$. We will skip these cases without further discussion.

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Plugging the profiles (5.28) and (5.33) at leading order as $r \to 0$ and integrating the line element we recover

$$D(r) \simeq -\frac{a\sqrt{2+6a^2}}{1+a^2}\, \log r\,. \tag{5.38}$$

Finally, together with the previous results for the distance to the end of the world and the curvature, we obtain the scalings

$$\Delta \sim e^{-\frac{\delta}{2}\,D}\,, \qquad |R| \sim e^{\delta\, D}\,, \tag{5.39}$$

with

$$\delta = \frac{2(\gamma+1)a}{\sqrt{2+6a^2}}\,. \tag{5.40}$$

Hence, we recover the general frame scaling relations introduced in section 5.1. This shows that small black hole solutions can be regarded as just another instance of Dynamical Cobordism, and that they admit local scaling relations identifying the small black hole core with ETW branes in 2d.

6 Swampland constraints and surprises from the UV

In this section we discuss interesting interplays of the scalar running off to infinity in field space in Local Dynamical Cobordisms and the Swampland constraints.

6.1 Swampland distance conjecture and other constraints

Many studies of Swampland constraints are related to infinity in scalar moduli/field space (see [42–44] for reviews). Since Dynamical Cobordisms explore infinite field space distances, in this section we discuss the interplay with different Swampland constraints, especially the Distance Conjecture [71] (see [38, 39, 65, 72–83] and the reviews above for other approaches).

Let us focus on the simplest expression of the Distance Conjecture, which states that, when the scalars are taken to infinite field space distance D (in an adiabatic approach, namely, by changing the spacetime independent vevs), there is a tower of states becoming exponentially light, and thus the cutoff of the effective theory is lowered as

$$\Lambda \sim e^{-\alpha D}\,, \tag{6.1}$$

with some positive order 1 coefficient α.

This scaling can be combined in an interesting way with our scalings near ETW branes. For instance, using (2.20), we have

$$\Lambda \sim \Delta^{\frac{2\alpha}{\delta}}\,. \tag{6.2}$$

This matches with our intuition that the full description of the ETW brane requires UV completing the effective theory. It is important to note that the appearance of an infinite tower in the adiabatic version of the Distance Conjecture does not necessarily imply the appearance of a tower in the present Dynamical Cobordism context. On the other hand, the lowered cutoff certainly signals that there could be situations where the naive ETW brane

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picture as described in effective theory may be corrected. We will see explicit examples in section 6.2.

Using also (2.20), we get that the cutoff scale relates to the spacetime curvature as

$$|R| \sim \Lambda^{-\frac{\delta}{\alpha}}\,, \tag{6.3}$$

(where we have taken the generic case $\delta \neq (2d/(d-2))^{1/2}$ for concreteness). This relation, already noted in [22] is reminiscent of (although admittedly different in spirit from) that in [39] for AdS vacua.

From this perspective, the correlation between the appearance of the naked singularity and the running of the scalar going off to infinity suggests that the lowered cutoff of the swampland distance conjecture is responsible for regulating the singularity, which would be resolved in a more complete microscopic UV description. This remark is in the spirit of [41] (see also [56]) and [37], where the singular behaviour of certain defects (EFT strings or small black holes, respectively) is related to scalars going off to infinite distance.

From our perspective, the relation follows from the Dynamical Cobordism Distance Conjecture in [22]. In our present terms: every infinite field distance limit of an effective theory consistent with quantum gravity can be realized as a solution running into a cobordism ETW brane (possibly in a suitable compactification of the theory).

In particular, in sections 4 and 5 we provided a description of general defects as ETW branes of Dynamical Cobordisms. This general framework encompasses the defects in [37, 41] as particular examples.

An interesting spin-off of our local analysis is that it constrains the asymptotic form of the potential. Namely, whenever it is not vanishing (actually, negligible as compared with the scalar kinetic energy) it has an exponential form with a critical exponent δ, cf. (2.15). It is thus interesting to compare this asymptotic form of the potential with Swampland constraints expected to hold near infinity in scalar field space.

Let us consider the de Sitter conjecture in the version of [84] (see [76, 85] for the refined one), namely $|\nabla V|/V > \mathcal{O}(1)$. From (2.15) we have

$$\frac{V'}{V} = \delta\,. \tag{6.4}$$

Since in general the critical exponent $\delta \sim \mathcal{O}(1)$, the potential satisfies the de Sitter conjecture. This fits nicely with the idea that the latter is expected to hold near infinity in moduli/field space.

Moreover, let us compare with the Transplanckian Censorship Conjecture [86]

$$|\nabla V| \geq \frac{2}{\sqrt{(d-1)(d-2)}}\, V\,. \tag{6.5}$$

When $V < 0$, the constraint is trivial; on the other hand, when $V > 0$, in our setup we must have $a < 0$, and the expression (2.14) for δ guarantees that the above inequality is satisfied. A caveat for the above statements is that both the de Sitter and the Transplanckian Censorship conjectures involve the gradient ∇V, whereas our local description provides the potential only along one direction, the effective scalar dominating the running near the ETW brane. Hence, the comments above would hold under the assumption that the effective scalar in the local description follows a gradient flow. It would be interesting to assess this point in explicit models, and we leave this as an open question for future work.

6.2 Large N surprises from the UV

In the previous section we have discussed that the Distance Conjecture implies a lowered cutoff as one approaches the ETW brane. Indeed, as mentioned at several points, the microscopic description of the ETW branes lies in the underlying UV completion. In most of our examples, the corresponding cobordism defect is known, so that the end of the world picture can be confirmed in the full theory. However, it is conceivable that in some specific cases there exist UV effects hidden at the core of the ETW brane potentially modifying this picture. In this section we present two examples, where such corrections exist and lead to large backreactions, ultimately turning the candidate ETW brane into a domain wall interpolating to a new region beyond the apparent singularity. A further interesting observation is that both examples are related to large N physics and holography.

Large number of M2-branes. Consider as our first example a stack of N D2-branes in flat 10d spacetime (or at a smooth point in any other compactification). Locally around the D2-brane location the $\mathbf{S}^6$ truncation yields a 4d theory with an ETW brane, at which a scalar (a combination of the radion and the dilaton) goes to infinity in field space. One may follow the theory in this limit and, as noted in [54], realize that the strong coupling is solved by lifting to M-theory, and turning the D2-branes into M2-branes. For small N, the UV completion of the effective ETW brane is thus merely a stack of M2-branes removing the flux and allowing spacetime to end, as befits a Dynamical Cobordism.

On the other hand, for N large we have a different behavior: the large number of M2-branes backreact on the geometry and generate an infinite $\mathrm{AdS}_4 \times \mathbf{S}^7$ throat. The effective theory ETW brane has a UV description with so many degrees of freedom that it actually generates a gravity dual beyond the wall.

From the perspective of the running scalars, the $\mathrm{AdS}_4 \times \mathbf{S}^7$ represents a minimum of the ($\mathbf{S}^7$ radion) potential. Hence the full D2/M2 solution describes the running of the theory from the slope of the potential down to a stable minimum, at which the theory relaxes to a maximally symmetric solution, instead of hitting an end of the world. The location of the minimum in field space is hidden near infinity in the original D2-brane effective description. Hence, the large N allows for the appearance of a minimum at strong coupling, which is nevertheless tractable.[12]

Moreover, the full D2/M2 solution describes a dynamical cobordism from the M-theory perspective. Far away from the stack of branes we can use the description in terms of D2-branes. As described above the 4d theory would be obtained by compactifying Type IIA on an $\mathbf{S}^6$. This would be further lifted to M-theory on $\mathbf{S}^6 \times \mathbf{S}^1$. On the other hand, we have just argued that close to the stack of branes the 4d theory is given by M-theory on $\mathbf{S}^7$. We then see that this solution describes a dynamical cobordism between to different

[12] This is reminiscent of the argument [87] that the scale separation (and hence the tractability) of the AdS minima in [88, 89] is controlled by a large number of flux units.

compactifications. Notice that this is not a cobordism to nothing, described by ETW brane solutions.

Warped KS throat with large number of D3-branes. Our second example is based on the warped throat considered in section 4.4. Recall we have type IIB theory compactified on $T^{1,1}$ with N units of RR 5-form flux and M units of RR 3-form flux on the $\mathbf{S}^3$, and we focus on the choice of parameters $N = KM + P$. At the level of the 4d effective theory, we recover a KT solution with a singularity at a finite spacetime distance, at which a scalar (a combination of the $T^{1,1}$ radion and the dilaton, but dominated by the former) goes off to infinite field space distance.

The UV smoothing of this singularity is slightly trickier than the $N = KM$ case of section 4.4. It involves the smoothing of the singular conifold geometry into a deformed conifold, with a finite size $\mathbf{S}^3$, but there remain P D3-branes at the tip of the throat. This can be shown using the holographic dual field theory, as follows. There is a Seiberg duality cascade from the initial $\mathrm{SU}(N) \times \mathrm{SU}(N + M)$ theory in which N effectively decreases in multiples of M; hence, in the last step of the cascade we have an $\mathrm{SU}(P) \times \mathrm{SU}(M + P)$ gauge theory, whose strong coupling dynamics leads to an remnant $\mathcal{N} = 4$ $\mathrm{SU}(P)$ theory, as befits the above mentioned P probe D3-branes.

Hence, for small P the ETW brane of the 5d theory is microscopically described by the smooth Klebanov-Strassler throat dressed with P explicit D3-branes, required to absorb the remnant 5-form flux and allow spacetime to end.

On the other hand, for P large we have a different behavior: the large number of D3-branes backreact on the geometry and generate an infinite $\mathrm{AdS}_5 \times \mathbf{S}^5$ throat. The effective theory ETW brane has a UV description with so many degrees of freedom that it actually generates a gravity dual beyond the wall. The interpretation of this strong correction in terms of the running scalars is similar to the one mentioned above, as the apperance of an AdS minimum hidden near the infinite field space distance limit of the effective description.

We have seen two examples in which a naive ETW brane in the effective description has a UV description encoding large backreactions on the geometry recreating a geometry beyond the wall. Alternatively, the corrections generate minima in the scalar potential in the region near field space infinity of the effective description. It would be interesting to explore in more detail these and other possible classes of examples exhibiting this phenomenon. We hope to report on this in the future.

7 Conclusions

In this paper we have studied Dynamical Cobordism solutions in which theories of gravity coupled to scalars develop an end of spacetime. The latter is encoded in the effective theory as the appearance of a singularity at finite spacetime distance, at which some scalars run off to infinite field space distance. We have provided a local description of the configurations in the near ETW brane regime, and shown that the solutions are largely simplified, and fall in universality classes characterized by a critical exponent δ, which controls the profiles of

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Example	d	δ	a
Massive IIA	10	$\frac{5}{\sqrt{2}}$	$-\frac{16}{5}$
Non-susy USp(32) *string*	10	$\frac{3}{\sqrt{2}}$	0
D7 branes	9	$\frac{4\sqrt{14}}{7}$	0
D6 branes	8	$\sqrt{2}$	$\frac{4}{7}$
D5 branes	7	$\frac{2}{\sqrt{5}}$	$\frac{5}{6}$
D4 branes	6	$\frac{1}{\sqrt{5}}$	$\frac{24}{25}$
Klebanov-Strassler	5	$\frac{2\sqrt{30}}{3}$	$-\frac{3}{2}$
Bubble of Nothing	4	$\sqrt{6}$	0
D2 branes	4	$\frac{\sqrt{14}}{7}$	$\frac{20}{21}$
D2/D6 on $T^4 \times S^2$	4	$\sqrt{2}$	$\frac{2}{3}$
D1 branes	3	$\sqrt{2}$	$\frac{3}{4}$
EFT string	3	$2\sqrt{2}$	0

Table 1. Table of examples in this paper, with the corresponding parameters for the local description near the ETW brane.

the different fields and the scaling relations among the field space distance D, spacetime distance Δ and scalar curvature R.

We have studied several explicit models of ETW branes and characterized them in the local description, computing their critical exponent. The different examples and their key parameters are displayed in table 1. This list is intended to illustrate typical values of these parameters. It would be interesting to explore more examples and to explore possible connections among ETW branes described by the same parameters.

We have moreover shown that small black holes can also be regarded as Dynamical Cobordisms, and satisfy similar scaling laws. It would be interesting to explore from the cobordism perspective the recent applications of small black holes to the derivation of swampland constraints.

There are several interesting open directions for the future:

- We have focused on solutions with spatial dependence. It would certainly be interesting to explore time-dependent backgrounds, and their possible application to cosmology.

- In our local analysis we have focused on certain particular choices. For instance, we have not considered solutions where $|V| \gg |V_t|$, and we have moreover taken solutions controlled by a constant parameter $a < 1$. More general possibilities are in principle allowed from a mere effective field theory perspective, but they are not realized in any of the string theory examples we have explored. It is thus an interesting question if there are UV complete models realizing them, or on the contrary, they are excluded by some further arguments of consistency with Quantum Gravity.

- Finally, it would be interesting to get a better understanding of the possible appearance of non-trivial corrections in the large field region near the ETW branes, in particular

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those leading to large backreactions signalling the existence of new minima of the scalar potential. This could lead to further insights into the stabilization of moduli in asymptotic regions of moduli/field space. The two examples mentioned in our work signal an interesting interplay with large N limits and holography, which may provide an extra leverage on these configurations.

We hope our work motivates interesting results in this and other directions.

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A Local Dynamical Cobordisms with curved $(d-1)$-dimensional slices

A.1 General analysis for curved slices

We can generalize the discussion in section 2 to the case in which the ETW brane has constant internal curvature R_d. Namely we take the foliation ansatz (2.2) with ds^2_{d-1} describing a constant curvature $(d-1)$-dimensional metric. The equations of motion read

$$(d-1)\sqrt{2\,(V-V_t)}\,\sigma' - \partial_\phi V_t = 0\,, \tag{A.1}$$

$$\frac{1}{2}(d-1)(d-2)\sigma'^2 + V_t - \frac{1}{2}e^{2\sigma}R_d = 0\,, \tag{A.2}$$

$$(d-2)\sigma'' - 2\,(V-V_t) - \frac{1}{d-1}e^{2\sigma}R_d = 0\,, \tag{A.3}$$

where we have again introduced the tunneling potential defined in (2.6).

For $R_d \neq 0$, it is still possible to eliminate σ by combining the first two equations (and their derivatives):

$$(d\,\partial_\phi V_t - (d-1)\partial_\phi V)\,\partial_\phi V_t = 2(d-1)(V_t - V)\left[\partial^2_\phi V_t + \frac{2}{d-2}((d-1)V-(d-2)V_t)\right]. \tag{A.4}$$

Importantly, in this derivation we need to assume $R_d \neq 0$, so that we do not expect to necessarily recover the results in section 2.

Restricting to the case $V = aV_t$, with a a constant, we find that the solution to this equation is

$$V_t(\phi) = -c\left(\cosh\left(\frac{(a(d-1)+2-d)\phi}{\sqrt{(1-a)(d-2)(d-1)}}\right)\right)^{2-\frac{2}{a(d-1)+2-d}}\,, \tag{A.5}$$

where we have ignored an integration constant that is irrelevant for the $\phi \to \infty$ limit.

Notice that, for $a > 1$, the coefficient in front of ϕ becomes imaginary and then what we have is a cosine, rather than a hyperbolic cosine. As we are not interested in this behaviour we from now on require $a < 1$. From computing ϕ'^2 from this solution and requiring that it must be positive, we then learn that we must have $c > 0$.

In addition, as we are interested in ETW branes, we want to require that ϕ'^2 blows up as $\phi \to \infty$. This is equivalent to having $|V_t| \to \infty$ in this same limit, which in turn implies that the power in (A.5) must be positive. This gives us that the only ETW brane solutions are for $a < \frac{d-2}{d-1}$. For this range of a, we can approximate the hyperbolic cosine by an exponential (as we are interested in the limit $\phi \to \infty$) and we have

$$V_t(\phi) \simeq -c\left(\exp\left(\frac{(a(d-1)+2-d)\phi}{\sqrt{(1-a)(d-2)(d-1)}}\right)\right)^{2-\frac{2}{a(d-1)+2-d}} = -c\,e^{\delta\,\phi}\,. \tag{A.6}$$

The coefficient δ is

$$\delta = 2\sqrt{\frac{d-1}{d-2}(1-a)}\,. \tag{A.7}$$

So for $a < \frac{d-2}{d-1}$ the case of a ETW brane with internal curvature coincides with the case studied in the paper. Interestingly, this case turns out to be more restrictive than the $R_d = 0$ one, for which any $a < 1$ described an ETW brane.

This solution was also assuming that $a \neq \frac{d-2}{d-1}$. Plugging that particular value in (A.4), we find that the equation of motion simplifies to

$$(\partial_\phi V_t)^2 = V_t \cdot \partial_\phi^2 V_t\,. \tag{A.8}$$

This equation has the solution

$$V_t = -c\,e^{\delta\,\phi}\,, \tag{A.9}$$

with c and δ arbitrary constants. In order to describe an ETW brane we require $\delta > 0$. Interestingly, for this special value of a with $R_d \neq 0$, we find that we recover the exponential behaviour, but with the freedom of choosing the critical exponent δ.

In both cases we find the same exponential behaviour for V_t. Therefore, just as in section 2.2, we find that the potential takes the form

$$V(\phi) \simeq -a\,c\,e^{\delta\,\phi}\,. \tag{A.10}$$

However, here we uncover that, for a given potential of this form, the setup with $R_d \neq 0$ allows for two possible values of a, namely the $a < \frac{d-2}{d-1}$ given in (A.7)), or the value $a = \frac{d-2}{d-1}$, with δ and a independent. For this reason, from now on we keep a and δ as different variables when solving the rest of the equations, and at the end we comment on the two possibilities.

Using (2.6) we can obtain the profile for ϕ

$$\phi(y) \simeq -\frac{2}{\delta} \log \left(\frac{\delta}{2} \sqrt{2(1-a)c\, y} \right) . \tag{A.11}$$

Notice that this is the equivalent to (2.16), but with a and δ kept independent. The leading behaviour is then given by

$$\phi(y) \simeq -\frac{2}{\delta} \log y \, , \tag{A.12}$$

and thus the field only depends on the critical exponent.

We can now use (A.1) to get the profile for the warp factor σ:

$$\sigma \simeq -\frac{1}{(d-1)(1-a)} \log y \, , \tag{A.13}$$

where we have set an integration constant to zero without loss of generality. We recover the equivalent to (2.18), albeit with a and δ kept independent. We see that the warp factor doesn't depend on δ, but specifically on the prefactor a of the potential.

Finally, we have to check that the solution is compatible with (A.3). From it we obtain the condition

$$\frac{4}{\delta^2} - \frac{d-2}{(d-1)(1-a)} + \frac{R_d}{d-1} \, y^{2-\frac{2}{(d-1)(1-a)}} = 0 \, . \tag{A.14}$$

Let us now apply it for the two possible values for a:

- For $a < \frac{d-2}{d-1}$, the power of y in the last term is positive, so that it is subleading in the $y \to 0$ limit. Moreover, recall that in this case δ relates to a via (A.7), which is the precise the value for which the first two terms cancel each other. In conclusion, for $a < \frac{d-2}{d-1}$ having $R_d \neq 0$ becomes irrelevant as we approach the ETW and we basically recover the same results as in the $R_d = 0$ case.

- For $a = \frac{d-2}{d-1}$, the exponent of y vanishes, and hence the R_d term is relevant. In this case, consistency of the equations requires

$$\delta = 2 \left(d - 2 - \frac{R_d}{d-1} \right)^{-\frac{1}{2}} . \tag{A.15}$$

  Therefore, for this case δ is also fixed, but in terms of R_d. Notice that this quantity must satisfy $R_d < (d-2)(d-1)$. Provided this condition, we find that δ can take any positive value.

This case corresponds to a metric $ds^2 = dy^2 + y^2 ds^2_{d-1}$, hence it describes a conical singularity. The singularity is absent in the case $R_d = (d-1)(d-2)$, namely the curvature of ds^2_{d-1} is that of $\mathbf{S}^{d-1}$, and the geometry is locally smooth, and we have $\delta = 0$ and no exponential growth of the potential. Also, in order to have an ETW brane, the $(d-1)$-dimensional curvature must be lower than that of $\mathbf{S}^{d-1}$.

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In conclusion, given a potential with an exponential behaviour as $\phi \to \infty$, in the $R_d \neq 0$ case there exist two different kind of solutions. In the first one the value of R_d is irrelevant and we recover the same behaviour as in the $R_d = 0$ case (but with a more constrained critical exponent, $\delta > \frac{2}{\sqrt{d-2}}$). In the second, the curvature R_d is relevant and it must be fixed by the critical exponent by (A.15).

A.2 Witten's bubble of nothing

To illustrate the above general formulation for curved $(d-1)$-dimensional slices, we consider the example of the celebrated Witten's bubble of nothing [4] (see [5–8] for other recent realization of bubbles of nothing). We show it admits a description in an effective 4d theory of gravity coupled to a scalar with zero potential, as a 4d Dynamical Cobordism, and characterize its local description and critical exponent δ.

Related discussion of a 4d effective description of the configuration have appeared in [90] (recently revisited in the context of bubbles in de Sitter space in [31, 32]).

Since we have restricted our discussion to dependence on spatial coordinates, we actually consider the euclidean 5d Schwarzschild black hole solution, before the Wick rotation to the expanding bubble solution. The 5d metric reads

$$ds^2 = \left(1 - \frac{R^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2 + \left(1 - \frac{R^2}{r^2}\right) d\phi^2 \,. \tag{A.16}$$

Here ϕ parametrizes an $\mathbf{S}^1$ fibered over the radial coordinate r, times and $\mathbf{S}^3$; the radial coordinate is constrained to the range $r \geq R$, and the $\mathbf{S}^1$ shrinks to zero size at the euclidean horizon $r = R$ (in a smooth way for the periodicity $\phi \sim \phi + 2\pi R$).

We would like to perform a reduction to 4d along the $\mathbf{S}^1$. This is a sphere reduction analogous to those in section 4.1. Hence, we match this metric with (4.2), for $n = 1$, $d = 4$, and, using (4.3), $\alpha = -\sqrt{1/6}$ and $\beta = -\sqrt{2/3}$. We obtain that the radion ω in (4.2) is:

$$\omega = -\sqrt{\frac{3}{8}} \log\left(1 - \frac{R^2}{r^2}\right) \,. \tag{A.17}$$

The 4d metric is given in (4.5) and reads

$$ds_4^2 = \left(1 - \frac{R^2}{r^2}\right)^{-\frac{1}{2}} dr^2 + \left(1 - \frac{R^2}{r^2}\right)^{\frac{1}{2}} r^2 d\Omega_3^2 \,. \tag{A.18}$$

We would now like to zoom into the location of the ETW brane, the euclidean horizon $r = R$. So we introduce the coordinate $\tilde{r} = 1 - \frac{R^2}{r^2}$. Near $r \to R$ the metric scales as

$$ds_4^2 \sim \tilde{r}^{-\frac{1}{2}} d\tilde{r}^2 + \tilde{r}^{\frac{1}{2}} d\Omega_3^2 \,. \tag{A.19}$$

Now, we make the change (4.6):

$$y = \int \frac{d\tilde{r}}{\tilde{r}^{1/4}} \simeq \tilde{r}^{3/4} \,. \tag{A.20}$$

Replacing $\tilde{r} \simeq y^{\frac{4}{3}}$ in (A.19) we get the 4d metric as a foliation of $\mathbf{S}^3$ slices:

$$ds_4^2 \sim dy^2 + y^{\frac{2}{3}} d\Omega_3^2 \,. \tag{A.21}$$

This corresponds to a metric of the kind (2.2) for curved 3d slices, namely of the kind studied in appendix A.1. Using (A.13) we can see that $a = 0$, and from (A.7) $\delta = \sqrt{6}$. Interestingly, this corresponds to the case in which the curvature of the slices is irrelevant, and the solution is similar to the $R_d = 0$ case.

We could have also obtained the same result from the profile for the radion,

$$\omega = -\sqrt{\frac{3}{8}} \log \tilde{r} \simeq -\sqrt{\frac{2}{3}} \log y \,. \tag{A.22}$$

By using (A.12), $\omega \simeq -\frac{2}{\delta} \log y$, we read that $\delta = \sqrt{6}$, hence $a = 0$.

Hence Witten's bubble of nothing is described by a 4d Dynamical Cobordism running solution with the scalar reaching off to infinite distance in fields space at a rate controlled by the critical exponent $\delta = \sqrt{6}$. This provides a simple local description in terms of an ETW brane. From this perspective, the 5d solution provides the UV completion of the ETW brane, which in this case is purely a geometrical closing-off of the geometry.

We would like to emphasize that this example provides an explicit realization of the picture discussed in section 4, in particular figure 1 (albeit, with no brane dressing at the tip). Namely, the complete solution involves a genuine compactification on a finite size $\mathbf{S}^1$, yet it is described by a local EWT brane model identical to that obtained as an $\mathbf{S}^1$ reduction on a flat $\mathbf{R}^2$ (which, given the vanishing potential, straightforwardly leads to $a = 0$, hence $\delta = \sqrt{6}$). This supports the picture in section 4 that the sphere reductions in the flat space transverse to the D-branes suffices to provide the local description even in the (physically more interesting case) in which the transverse space is globally given by a more involved geometry, implementing the actual compactification to the lower-dimensional theory.

B Subleading corrections to the local description

In section 2.2 we took constant a as a proxy for the leading behaviour of $a(\phi)$ as $\phi \to \infty$. Here we consider the role of possible subleading corrections. We notice that these corrections do not necessarily go to zero as $\phi \to \infty$ in (2.12). For example, let us take

$$\sqrt{1 - a(\phi)} = \sqrt{1 - a} + \frac{b}{\phi} \,. \tag{B.1}$$

It is clear that $a(\phi)$ asymptotes to a as $\phi \to \infty$, but after doing the integral in (2.12) the correction to the leading behaviour given by the second term behaves as $\log \phi$. Indeed, ignoring constant prefactors we get

$$V_t \sim \phi^{2\sqrt{\frac{d-1}{d-2}}\, b} e^{\delta\, \phi} \,, \tag{B.2}$$

with δ defined in (2.14). Comparing with (2.13) we see that we can describe this example with our leading order analysis if we allow for $c \sim \phi^{2\sqrt{\frac{d-1}{d-2}}b}$. Notice that the example in section 3.2 precisely realise this behaviour (see equation (3.19)).

As a general lesson, we can include these kind of corrections that do not vanish in the $\phi \to \infty$ limit by promoting c from just a constant to a ϕ-dependent quantity that may hide subleading corrections. In this way, it may happen that $c \to \infty$ as $\phi \to \infty$ as long as it blows-up slower than an exponential (otherwise it would not represent a subleading behaviour).

This remark is specially interesting in the $a(\phi) \to 0$ case. From (2.15) we would conclude that $V \to 0$ if c is a finite constant. However, if allowing $c \to \infty$ because of possible subleading terms, it can happen that a times c remains finite in the $\phi \to \infty$ limit. In this way, we describe a solution in which $\phi'^2 \gg V$ (i.e., $a(\phi) \to 0$) without requiring that V vanishes asymptotically.

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Dynamical Cobordism and the beginning of time: supercritical strings and tachyon condensation

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Abstract: We describe timelike linear dilaton backgrounds of supercritical string theories as time-dependent Dynamical Cobordisms in string theory, with their spacelike singularity as a boundary defining the beginning of time. We propose and provide compelling evidence that its microscopic interpretation corresponds to a region of (a strong coupling version of) closed tachyon condensation. We argue that this beginning of time is closely related to (and shares the same scaling behaviour as) the bubbles of nothing obtained in a weakly coupled background with lightlike tachyon condensation. As an intermediate result, we also provide the description of the latter as lightlike Dynamical Cobordism.

Keywords: String and Brane Phenomenology, Tachyon Condensation, Bosonic Strings

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1 Introduction

One of the outstanding questions in string theory is the understanding of time-dependent backgrounds and in particular the resolution of cosmological (i.e. spacelike) singularities (see [1, 2] for reviews). On general grounds, and in analogy with timelike singularities, one may expect that stringy effects smooth out the singularity, thus providing a microscopic description of the beginning of time.

This is a natural proposal from the perspective of the Swampland Cobordism Conjecture [3], which states that any consistent theory of quantum gravity should admit configurations ending spacetime, namely boundaries or general cobordism defects leading to walls of nothing.[1] This also resonates with (a Lorentzian version of) the no-boundary proposal for the Hartle-Hawking wavefunction of the universe [9].

[1] A prototype is the infinite volume limit of bubbles of nothing in higher-dimensional compactifications in which the internal space shrinks to zero size [4] (see [5–8] for some recent work).

From this perspective, such cosmological solutions would correspond to dynamical time-dependent configurations with a beginning of time given by a cobordism defect extending in the spatial directions. This appealing picture is however hampered by the general lack of understanding of the microscopic structure of spacelike singularities.

The cobordism conjecture has been exploited at the topological level with interesting results, see e.g. [6, 10–15]. On the other hand, there is substantial progress in understanding the implications of the cobordism conjecture at the dynamical level. The configurations dubbed Dynamical Cobordisms in [16–18] (see also [19])[2] describe spacetime dependent solutions in which the fields run until they hit a real-codimension 1 singularity at finite distance in spacetime, at which certain scalars run off to infinite distance in field space. In several examples of such spatially varying solutions, the timelike singularities had a known string theory UV description, which displayed an end of spacetime. Remarkably, [18] showed that in the effective theory description these singularities (dubbed end-of-the-world (ETW) branes) follow universal scaling laws, and are characterized by a single critical exponent.

In this paper we take the natural next step of starting the study of time-dependent Dynamical Cobordism with spacelike singularities and of shedding some light on their resolution. The particular arena to explore these ideas are timelike linear dilaton backgrounds in supercritical bosonic string theory.

Supercritical string theories provide consistent versions of string theory in a general number D of spacetime dimensions, provided a suitable timelike linear dilaton background is turned on [30–33]. They provide an excellent testing ground for general features of string theory (see [34] for a recent example). In particular, and as will be relevant to our discussion, they constitute a setup in which closed string tachyon physics has been subject to quantitative analysis (see e.g. [35–41]).[3] For our purposes, the main property of these theories is that the timelike linear dilaton background makes them one of the simplest time-dependent setups in string theory.

We express these backgrounds as time-dependent Dynamical Cobordisms, exhibiting their beginning of time singularity and characterizing it as an ETW brane, with a precise critical exponent. We moreover propose, providing non-trivial support for it, that the stringy resolution of the singularity involves a region of (the strong coupling version of) bulk tachyon condensation. This is a realization of the mechanism in [45] in a different setup which, as promised, provides a stringy analogue of the Hartle-Hawking proposal.

Our approach is based on the realization that the beginning of time singularity, and the walls of nothing described via lightlike tachyon condensation in [35] (see also [36–38, 46–48] for related results) admit an ETW brane description in the effective theory with exactly the same critical exponent. Moreover, we show that these configurations, which seemingly contain two intersecting ETW walls, actually contain a single recombined one with two

[2]For the related topic of solutions in theories with dynamical tadpoles, see [20–23] for early work and [24–29] for related recent developments.

[3]See [42–44] and references therein for discussion of the fate of localized closed tachyons and related instabilities.

different asymptotic regions, a lightlike one corresponding to tachyon condensation at weak coupling and a spacelike one at strong coupling corresponding to the beginning of time.

A potential caveat to our analysis is the use of effective theories to describe tachyon condensation phenomena, which involve stringy scales and are not fully understood for closed tachyons (see [49] and references therein for further discussion). We however encounter that the main feature of the ETW wall, the critical exponent, is surprisingly robust under corrections of the effective action. This suggest that the main results may survive beyond the validity of the tools used to extract it in the present work. The same considerations apply to the study of the beginning of time singularity, which lies at strong coupling.

The paper is organized as follows. In section 2 we recall the Dynamical Cobordisms of [16–18], and the structure of the ETW branes in terms of their critical exponent. In section 3 we discuss the timelike linear dilaton background as a time-dependent solution: in section 3.1 we express it as a Dynamical Cobordism with a beginning of time; in section 3.2 we describe the singularity at the beginning of time as an ETW brane; and in section 3.3 we explore its UV description in terms of a timelike tachyon condensate. In section 4 we discuss walls of nothing arising in lightlike tachyon condensation and show that they correspond to Dynamical Cobordisms with a lightlike ETW brane: in section 4.1 we recall the worldsheet description, and in section 4.2 we provide their spacetime description and characterize their ETW brane and critical exponent. In section 5 we combine results and formulate our proposal that the UV description of the beginning of time in the linear dilaton background is (a strong coupling version of) closed tachyon condensation. In section 6 we offer some final thoughts. In appendix A we mention that the dimension quenching mechanism in [36, 37] can be described as a dynamical cobordism describing an interpolating wall [17] between theories of different dimension. Some calculational details have been postponed to appendices B, C.

2 Overview of Dynamical Cobordisms

In a series of papers [16–18] the analysis of dynamical spacetime-dependent solutions realizing cobordisms to nothing was initiated (see also [19]). Such solutions, from the perspective of the lower-dimensional effective field theory, present universal features that allow them to be described in a general framework as follows.

Consider the lower-dimensional EFT to be D-dimensional[4] Einstein gravity coupled to a scalar with arbitrary potential (in $M_{Pl} = 1$ units):

$$S = \int d^D x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right) . \tag{2.1}$$

We consider solutions in which the metric and scalar vary along one coordinate, denoted by y. The ansatz for the metric is

$$ds^2 = e^{-2\sigma(y)} ds^2_{D-1} + dy^2 . \tag{2.2}$$

[4]We use D for the spacetime dimension and $\mathcal{D}$ for the field space distance.

Here we follow earlier references and considered space-dependent running solutions. The sign flips necessary for time-dependent ones will be taken into account in the concrete examples of later sections.

In the following we take a flat metric for the $D-1$ dimensional slices. All solutions that describe a cobordism to nothing present a spacetime singularity at finite spacetime distance Δ where the scalars explore an infinite distance $\mathcal{D}$ in field space, this is the location of the ETW wall. Indeed, the solution does not extend beyond this point which, without loss of generality, we choose to be $y=0$.

One of the highlights of the analysis in [18] is that the solutions near ETW branes behave in a simple way. We quote some of the main expressions encapsulating this

$$\phi(y) \simeq -\frac{2}{\delta}\log y \quad , \quad \sigma(y) \simeq -\frac{4}{(D-2)\delta^2}\log y \quad , \quad |R| \simeq \frac{1}{y^2}\,. \tag{2.3}$$

with δ a scaling coefficient which characterizes the local solution near the ETW brane, and $|R|$ is the spacetime scalar curvature. Although [18] focused on space-dependent running solutions, it is straightforward to extend the discussion to time-dependent ones, and recover the same scaling laws.

From the above profiles, all solutions describing ETW walls present universal scaling relations between Δ, $\mathcal{D}$ and the spacetime scalar curvature $|R|$, as follows

$$\Delta \sim e^{-\frac{\delta}{2}\mathcal{D}} \quad , \quad |R| \sim e^{\delta\mathcal{D}}\,. \tag{2.4}$$

We also get that the scalar potential behaves as[5]

$$V(\phi) \simeq -a\,c\,e^{\delta\,\phi}\,, \tag{2.5}$$

for a constant $a<1$ related to δ by

$$\delta = 2\sqrt{\frac{D-1}{D-2}\,(1-a)}\,. \tag{2.6}$$

3 Supercritical strings as time-dependent Dynamical Cobordism

In this section we discuss the maximally symmetric configuration of supercritial strings, and interpret the necessary linear dilaton background as a running solution which satisfies the properties of a time-dependent Dynamical Cobordism. The local behaviour is hence that of an ETW brane.

In this work we focus on the supercritical bosonic theory. We expect similar ideas to apply to other supercritical theories, including supercritical type 0 or heterotic superstrings [36, 37].

[5]Note that if y is timelike, the overall sign of the potential changes. The quantity c is a positive constant related to the boundary condition used when solving the equations of motion. Subleading corrections to the potential can be included by promoting c to a function $c=c(\phi)$ with slower growth than an exponential.

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3.1 Linear timelike dilaton as Dynamical Cobordism

Consider bosonic string theory in D-dimensional Minkowski space (in the string frame). In order to satisfy the central charge constraint for the theory, there is a linear dilaton background

$$\Phi = v_M X^M \,, \tag{3.1}$$

with

$$v \cdot v = -\frac{D-26}{6\alpha'} \,, \tag{3.2}$$

with contractions defined with respect to the flat Minkowski metric.

Hence, supercritical strings require a timelike dilaton gradient, whereas subcritical strings require an spacelike one. The critical $D = 26$ theory does not require a dilaton profile for consistency, but does admit a lightlike dilaton background. We thus expect our discussion to extend this background of the critical theory as well.

These linear dilaton theories define conformal theories exactly in α', which implies that they satisfy the equations of motion of the spacetime (string frame) action

$$S_{\text{str.}} = \frac{1}{2}\int d^D x \sqrt{-G_{(s)}}\, e^{-2\Phi}\left[-\frac{2(D-26)}{3\alpha'} + R_{(s)} + 4(\partial\Phi)^2\right] . \tag{3.3}$$

We explicitly denote string frame quantities with an s subindex, while quantities with no subindex are implicitly defined in the Einstein frame.

In the following we focus on supercritical strings and timelike dilaton background

$$\Phi = -qX^0 \,, \tag{3.4}$$

where $q \equiv v^0$. Here we have absorbed a possible additive constant by shifting time, so that the dilaton vanishes at $X^0 = 0$. From the two solutions of (3.2) we choose the one leading to weak coupling $g_s = e^{\Phi}$ in the future $X^0 \to \infty$, namely

$$q = \sqrt{\frac{D-26}{6\alpha'}} \ . \tag{3.5}$$

In the next section, we reinterpret this linear dilaton background as a running solution with an ETW wall at the origin of time.

3.2 The ETW brane at the beginning of time

The spacetime physics of the singularity was considered from a cosmological perspective in [35]. Here we instead study it from the perspective of the ETW branes of dynamical cobordism in section 2.

To discuss the spacetime physics, we focus on the Einstein frame, so the metric reads

$$ds^2 = \exp\left(\frac{4qX^0}{D-2}\right)\eta_{MN}dx^M dx^N \,. \tag{3.6}$$

We see that at $X^0 \to -\infty$ the warp factor goes to zero, and we hit a singularity. We can introduce a time coordinate y giving the invariant interval to the singularity as

$$y = \frac{D-2}{2q} \exp\left(\frac{2qX^0}{D-2}\right) , \tag{3.7}$$

in terms of which (3.6) is recast as the time-dependent version of (2.2):

$$ds^2 = -dy^2 + \frac{4q^2y^2}{(D-2)^2}\, dx^m dx^m\,, \tag{3.8}$$

for $m = 1, \ldots, D-1$. We thus obtain

$$\sigma(y) = -\log y\,. \tag{3.9}$$

Comparing this to (2.3) gives:

$$\delta = \frac{2}{\sqrt{D-2}}\,. \tag{3.10}$$

Expressing the dilaton in terms of a scalar ϕ with canonical kinetic term in the Einstein frame as

$$\phi = \frac{2}{\sqrt{D-2}}\,\Phi \sim -\sqrt{D-2}\,\log y\,. \tag{3.11}$$

This is precisely the scaling relation for the scalar (2.3) for the value of δ in (3.10).

We also get the expected scaling of the potential. The Einstein frame action gives

$$S = \frac{1}{2}\int d^Dx\sqrt{-G}\left[R - \frac{4}{D-2}\,(\partial\Phi)^2 - \frac{2(D-26)}{3\alpha'}\exp\left(\frac{4\Phi}{D-2}\right)\right], \tag{3.12}$$

which, comparing with (2.5) and using the normalized dilaton (3.11) yields the precise value of δ in (3.10).

To conclude, we recover the scaling relations (2.4), which state that the configuration hits an ETW singularity at finite time in the past at which the scalar runs off to infinite distance in field space. According to the cobordism interpretation of such singularities in [17, 18], it defines a beginning of time, a boundary in the time direction, for this solution.

The microscopic description of the ETW brane requires some understanding of spacelike defects in string theory, which remains mostly *terra incognita*.[6] In our particular example, this is even more so since it lies at strong coupling.[7] Despite these difficulties, we find compelling evidence that the microscopic description of our spacelike ETW brane is the strong coupling avatar of tachyon condensation. We propose a direct approach to this proposal in the next section, and a further indirect, but quantitatively more reliable, route to support this picture in section 4.

[6] See e.g. [50] for attemps invoking S-branes.

[7] For the critical type IIA with a lightlike dilaton background, a microscopic description for the analogous singularity was proposed in [51], based on M(atrix) theory [52]. Such a description does not seem feasible in our case.

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3.3 The timelike tachyon case

The resolution of spacelike singularities in a tractable worldsheet approach was addressed in [45] in a setup with a shrinking 1-cycle, in terms of the condensation of a closed string tachyon in the winding sector (see [53, 54] for proposed higher-genus generalizations). In short, the regime near the singularity was proposed to be coated by a longer duration region in which the tachyon condenses with an exponential profile. The latter describes an effective Liouville wall in the time direction, beyond which no string excitation can propagate. This was argued to be a stringy definition of the *nothing* in the Hartle-Hawking description of the wavefunction of the universe [9]. In this picture, spacetime emerges smoothly as the tachyon turns off. In our terms, it describes a cobordism to nothing in the time direction.

In this section we explore a similar interpretation for the spacelike singularity encountered in our timelike linear dilaton setup. The idea is to consider an exponential profile for the closed string tachyon of supercritical bosonic theory. The tachyon couples to the worldsheet as a 2d potential. The condition for this deformation to be marginal, to linear order in conformal perturbation theory, or equivalently, the linearized spacetime equation of motion for the tachyon is

$$\partial^2 T(X) - 2\, v^M\, \partial_M T(X) + \frac{4}{\alpha'} T(X) = 0\ . \tag{3.13}$$

We will discuss corrections to this later on.

For a general tachyon exponential profile

$$T(X^M) = \mu\, \exp(\beta_M\, X^M)\ . \tag{3.14}$$

we obtain a condition on β:

$$\beta \cdot \beta - 2v \cdot \beta + \frac{4}{\alpha'} = 0\,. \tag{3.15}$$

We now focus on a timelike tachyon profile

$$T = \mu \exp(-\beta^0 X^0)\,, \tag{3.16}$$

with the condition

$$-(\beta^0)^2 + 2q\beta^0 + \frac{4}{\alpha'} = 0\,. \tag{3.17}$$

There are two solutions to this quadratic equation. A possibility is to choose $\beta^0 < 0$, so that the tachyon grows for late times $X^0 \to \infty$. This is a good strategy to study the process of tachyon condensation in a weakly coupled regime, see e.g. [38]. In fact, it is closely related to our approach (albeit for lightlike tachyons) in section 4.

Here, instead, we are interested in having tachyon condensation at the beginning of time, to provide a resolution of the spacelike ETW brane at $y = 0$, hence we need the tachyon to grow in the past $X^0 \to -\infty$, we thus require $\beta^0 > 0$. Using (3.5) we have

$$\beta^0 = \frac{\sqrt{D-26} + \sqrt{D-2}}{\sqrt{6\alpha'}}\ . \tag{3.18}$$

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We may now compare the relative growth of the string coupling $g_s = e^{\Phi}$ and of the tachyon as $X^0 \to -\infty$ to assess if the tachyon condensation could be studied using worldsheet techniques. We have

$$T/g_s = \mu \exp\left(-\sqrt{\frac{D-2}{6\alpha'}}X^0\right) . \tag{3.19}$$

This shows that the tachyon grows parametrically faster than the string coupling as $X^0 \to -\infty$. This leads to the expectation that the worldsheet analysis provides a reliable description of the physics at early times. In analogy with [45], and based on the extensive analysis in [35–37], the presence of the worldsheet potential creates a Liouville wall expelling all string excitations, providing a microscopic definition of an ETW brane in time.

The drawback of this approach is that it relied on trusting the linearized deformation approximation, which is expected to experience strong higher order corrections.[8] Therefore the scenario can be at most regarded as a qualitative description. In the next section we turn to a different approach, involving α' exact solutions.

4 Lightlike tachyon condensation

We are thus led to consider solutions under better control. In this section we consider an α'-exact solution of the supercritical linear dilaton theory with tachyon profile along a lightlike direction. As established in [35, 36] for the bosonic theory, at late times this leads to a wall of nothing moving at the speed of light, analogous to the asymptotic behaviour of a bubble of nothing. After recalling the argument, we carry out a new spacetime analysis that shows that at late times the background corresponds to a lightlike ETW brane, and show that its critical exponent is exactly the same as for the beginning of time ETW brane of the previous section. This tantalizing relation is a strong support for our interpretion of the beginning of time is (a strongly coupled version of) a closed string tachyon condensation phase, discussed in section 5.

4.1 Lightlike tachyon in the worldsheet description

Consider introducing an exponential tachyon background (3.14) along a lightlike direction $X^+ = (X^0 + X^1)/\sqrt{2}$

$$T = \mu \exp(\beta X^+) . \tag{4.1}$$

The linearized tachyon marginality condition (3.13) is satisfied for

$$\beta = \frac{2\sqrt{2}}{q\alpha'} . \tag{4.2}$$

At late times $X^0 \to \infty$, the string coupling is small and one may perform a reliable worldsheet analysis. As shown in [35] and contrary to the timelike tachyon case, the

[8] For the $\beta^0 < 0$ solution, corrections are interestingly expected to be suppressed in a large D approximation, as exploited in [55]. Although large D could be interesting in our $\beta^0 > 0$ case to increase the hierarchy in (3.19), it does not lead to a similar suppression of corrections.

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deformation by the operator (4.1) is exact, as higher order corrections in the perturbation vanish, since the lightlike nature of the insertions prevent the existence of non-trivial Wick contractions. Furthermore, in light-cone coordinates the propagator of the $X^{+/-}$ fields is oriented from X^+ to X^- and we know that all interaction vertices introduced by the tachyon potential only depend on X^+. These two facts combined show that there are no possible Feynman diagrams beyond tree-level, which implies the solution is exact in α'. One can thus conclude that the linearized tree-level solution (4.1) is exactly conformally invariant.

The tachyon couples as a worldsheet potential, which grows infinitely at $X^+ \to \infty$. This 2d potential prevents any string modes from entering the corresponding region, which thus becomes a region of nothing. The physical interpretation of this is that the tachyon configuration describes a wall of nothing propagating at the speed of light, which effectively ends spacetime at an effective value of X^+.

The finite range in X^+ can be estimated by e.g. cutting off X^+ when the $T = 1$. This gives

$$\Delta X^+ = -\beta^{-1} \log \mu/\mu^* \,, \tag{4.3}$$

where μ^* defines a reference position from which we measure the range to the wall. A more precise derivation follows from the *gedanken* experiment of solving the motion of classical strings incoming into the tachyon wall [35]. The initial speed reduces to zero at a turning point, after which the string is pushed back by the tachyon wall and its speed asymptotes to that of light. The turning point position in X^+ in the formulas in [35] gives back the result (4.3).

Given the importance of the notion of finiteness on the location of the tachyon wall, we provide an alternative derivation, carried out by adapting the techniques in [45]. We briefly sketch the results here and give more computational details in appendix B. Decomposing the field $X^+(\tau,\sigma)$ into its zero and nonzero modes $X^+(\tau,\sigma) = X_0^+ + \widehat{X}^+(\tau,\sigma)$ and performing a Wick rotation, the Euclidean partition function reads:

$$Z(\mu) = \int dX_0^+ \int \mathcal{D}\widehat{X}^+ \mathcal{D}X^- \mathcal{D}X^i \mathcal{D}g \mathcal{D}(\text{ghosts}) e^{-S_E^{\text{deformed}}} \,, \tag{4.4}$$

with the Euclidean action:

$$\begin{aligned} S_E^{\text{deformed}} =& \frac{1}{2\pi\alpha'} \int d^2\sigma_E \sqrt{g} \left[\partial_{\sigma^0}\widehat{X}^+ \partial_{\sigma^0} X^- + \partial_{\sigma^1} X^- \partial_{\sigma^1}\widehat{X}^+ + \partial_\alpha X^i \partial_\alpha X^i\right] \\ &+ \frac{1}{2\pi} \int d^2\sigma_E \sqrt{g} R_2 \Phi(X) + \frac{i\mu_E}{2\pi} \int d^2\sigma_E \sqrt{g} e^{\beta X^+} \,. \end{aligned} \tag{4.5}$$

From this we see that when the tachyon condenses, at large X^+, the path integral becomes suppressed. This results in a truncation of contributions to the integral coming from string oscillations with $X^+ \mapsto \infty$. This is the same mechanism as that of a Liouville wall in Liouville theory: no physical degrees of freedom exist in this region. In fact, one can show that the partition function in (4.4) can be directly related to that of the free theory (with no tachyon deformation) as follows.

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After integrating out the zero-mode X_0^+, one can show that:

$$\frac{\partial Z}{\partial \mu_E} = -\frac{1}{\beta \mu_E} \int \mathcal{D}\widehat{X}^+ \mathcal{D}X^- \mathcal{D}X^i \mathcal{D}g \mathcal{D}(\text{ghosts}) e^{-S_E^{\text{free}}} \,, \tag{4.6}$$

where S_E^{free} is the euclidean action of the worldsheet theory without the tachyon potential. Integrating with respect to μ and fixing a cutoff for X^+ such that $\mu_* = e^{\beta X_*^+}$, we obtain:

$$Z_1 = -\frac{\log(\mu_E/\mu_*)}{\beta} \widehat{Z} \,, \tag{4.7}$$

where $\widehat{Z}$ is the partition function for the 2d theory without the tachyon insertion. Hence the partition function Z in the presence of the tachyon background related to that of the theory without the tachyon $\hat{Z}$ via the factor $\frac{\log(\mu_E/\mu_*)}{\beta}$, which thus provides an effective "size" of the direction X^+, which matches that of (4.3).

The interpretation of the exponential tachyon as a wall of nothing receives further support from the dimension quenching mechanism in [36]. In appendix A we review it from the perspective of dynamical cobordisms in the spacetime perspective, to be discussed next.

4.2 Spacetime description and lightlike ETW brane

In this section we study the spacetime description of the wall of nothing corresponding to the lightlike tachyon, and show that it satisfies the properties of (the lightlike version of) and ETW brane. This nicely confirms the worldsheet arguments of the previous section.

4.2.1 Effective action

In order to describe the spacetime dynamics of the lightlike tachyon configuration, we need an effective spacetime action for the relevant fields, in particular for the tachyon. This is already a subtle point, since tachyon condensation processes may in principle backreact on the whole tower of stringy states, hence the validity of the truncation to an effective theory with a finite set of fields is to some extent questionable.

In any event, this approach has been successful enough in open string tachyon effective actions, and we may venture into its use for the closed case, hoping that fortune favors the brave.

The construction of the most general 2-derivative effective action for the metric, dilaton and tachyon in supercritical string theory has been discussed in [35] and [38], whose discussion we follow. In the string frame it has the structure

$$S = \frac{1}{2} \int d^D x \sqrt{-G_{(s)}}\, e^{-2\Phi} \left[f_1 R_{(s)} + 4 f_2 (\nabla\Phi)^2 - f_3 (\nabla T)^2 - 2 f_4 - f_5 \nabla T \cdot \nabla \Phi \right] \,. \tag{4.8}$$

where the $f_i(T)$ are general functions of the tachyon. By demanding that the equations of motion are compatible with the linear dilaton background with an exponential tachyon profile, one can show that the $f_i(T)$ can be expressed in terms of $f_1(T)$:

$$\begin{aligned} f_2(T) &= f_1(T), \qquad f_3(T) = -f_1''(T) - \frac{f_1'(T)}{T}, \qquad f_5(T) = 4\, f_1'(T)\,, \\ f_4 &= \frac{1}{2}\left[4\, f_1(T) \left(\frac{D-26}{6\alpha'} \right) + T\, f_1'(T) \left(\beta \cdot \beta + \frac{8}{\alpha'} \right) - T^2\, f_1''(T)\, \beta \cdot \beta \right] \,. \end{aligned} \tag{4.9}$$

For general exponential tachyon profiles, the tachyon background is only a solution at linearized order, hence we expect the above relations to receive corrections. For lightlike tachyons, however, the solution is exact in worldsheet perturbation theory, hence the above relations hold, and the corrections at most modify the behaviour of f_1 at large T. Note also that for lightlike tachyon profiles $\beta\cdot\beta = 0$ and the tachyon potential $V_{(s)} = f_4$ becomes β-independent.

Going to the Einstein frame, we redefining the metric to absorb the f_1 prefactor as well as the usual dilaton factor,

$$(G_{(s)})_{MN} = e^{\frac{4}{D-2}\Phi} f_1^{-\frac{2}{D-2}} G_{MN}\,, \tag{4.10}$$

the spacetime action is

$$\begin{aligned} S = \frac{1}{2}\int d^Dx\sqrt{-G}\Bigg[R &- \frac{4}{D-2}(\partial_M\Phi\partial^M\Phi - \frac{f_1'}{f_1}\partial_M\Phi\partial^M T) \\ &- \left[\frac{D-1}{D-2}\frac{f_1'^2}{f_1^2} - \frac{f_1''}{f_1} - \frac{f_1'}{f_1 T}\right]\partial_M T\partial^M T - \frac{2}{3\alpha'}e^{\frac{4\Phi}{D-2}} f_1^{\frac{-D}{D-2}}((D-26)f_1 + 12Tf_1')\Bigg]. \end{aligned} \tag{4.11}$$

The complete expression for f_1 is actually not known, beyond its expansion around $T = 0$

$$f_1 = 1 - T^2 + \ldots \tag{4.12}$$

Nevertheless, [38] proposed a set of regularity conditions on the effective action, which to some extent constrain f_1 further, and several explicit solutions were proposed, concretely $f_1 = \exp(-T^2)$ and $f_1 = 1/\cosh(\sqrt{2}T)$. Interestingly, in the large T regime (which is near the wall of nothing, our main focus), both can be parametrized as

$$f_1 \sim A \exp\left(-b\,T^k\right). \tag{4.13}$$

Actually, an outcome of [38] is that the behaviour of the system is not particularly sensitive to the precise form f_1. In the following we focus on the dependence (4.13), but later show that the same results hold, even at quantitative level, for very general forms of f_1.

4.2.2 The local scalings

We propose that the lightlike tachyon background, in the weak coupling regime, corresponds to a dynamical cobordism in X^+, and that the tachyon wall corresponds to an ETW brane, namely a singularity in effective theory at finite spacetime distance, and at which some scalar runs of to infinite field theory distance. In the following we show that the scalings derived from the Einstein frame spacetime solution are indeed of the ETW kind.

Note that, because the dynamical cobordism takes place via dependence on the lightlike coordinate X^+, in order to discuss spacetime *distance*, we choose slices of constant X^0, and measure spatial distance along X^1, along which the dilaton remains constant.

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Again, recall that we focus on the dependence (4.13), but similar conclusions hold for a very general class of profiles of f_1. The running scalar along X^1 is only the tachyon, hence the distance in field space as we approach the wall is given by:

$$\mathcal{D} = \int^{T_{\rm ETW}} \left(\sqrt{\frac{f_1''}{f_1} + \frac{f_1'}{f_1}\left(\frac{1}{T} - \frac{D-1}{D-2}\frac{f_1'}{f_1}\right)}\right) dT \sim \frac{b}{\sqrt{D-2}}\, T_{\rm ETW}^k\,, \tag{4.14}$$

which diverges (for $k > 1$) since the tachyon goes to infinity at the ETW brane at $X^1 \to \infty$.

Let us now check that the wall is indeed at finite spacetime distance in the Einstein frame. The length along X^1 is

$$\begin{aligned}\Delta =& \int^{\rm ETW} f_1^{\frac{1}{D-2}}\, dx^1 = A^{\frac{1}{D-2}} \int^{\rm ETW} \exp\left(-\frac{bT^k}{D-2}\right) dx^1 \\ =& \frac{\sqrt{2}}{bk} A^{\frac{1}{D-2}} \int^{\rm ETW} \exp\left(-\frac{\mathcal{D}}{\sqrt{D-2}}\right) \frac{d\mathcal{D}}{\mathcal{D}} = \frac{\sqrt{2}}{bk} A^{\frac{1}{D-2}}\, \mathrm{Ei}\left(-\frac{\mathcal{D}}{\sqrt{D-2}}\right)\,,\end{aligned} \tag{4.15}$$

where this last function is the exponential integral, and is clearly convergent, showing that the tachyon background behaves as dynamical cobordism ending at an ETW brane, where the (tachyon) scalar runs off to infinite field space distance at a finite spacetime distance.

We can check the scaling relations of ETW branes of section 2. We can expand (4.15) for $\mathcal{D} \to \infty$ as

$$\mathrm{Ei}\left(-\frac{\mathcal{D}}{a}\right) \sim e^{-\mathcal{D}/a}\left(\frac{a}{\mathcal{D}} + \ldots\right)\,, \tag{4.16}$$

and get

$$\Delta \sim \exp\left[-\frac{\mathcal{D}}{\sqrt{D-2}} - \log\left(\frac{\mathcal{D}}{\sqrt{D-2}}\right)\right]\,. \tag{4.17}$$

Comparing this with (2.4) gives a value

$$\delta = \frac{2}{\sqrt{D-2}}\,. \tag{4.18}$$

Namely, we recover an exponential relation. It is interesting to point out that the log correction is reminiscent of that encountered in [17, 18] for the EFT strings in [56]. Also note that, restricting to the leading exponential scaling, the critical exponent is independent of k. This is a particular case of the claimed robustness of the results under changes of f_1, and will be explored in general in section 4.2.3.

We can also compute the scaling of the Ricci scalar, which, upon direct computation gives

$$|R| \sim \exp\left[\frac{2\mathcal{D}}{\sqrt{D-2}} + \log\left(\frac{\mathcal{D}^2}{D-2}\right)\right]\,. \tag{4.19}$$

Again, the leading terms gives the scaling corresponding to an ETW brane, with δ given by (4.18), again remarkably independent of k.

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The potential, computed in the limit $T \mapsto \infty$ with (4.11), also agrees nicely with the general formula provided by the local analysis in (2.5):

$$V(T) = -\frac{8}{\alpha'} A^{-\frac{2}{D-2}} kb\, T^k e^{\frac{4\phi}{D-2}} e^{\delta \mathcal{D}} = -a\, c(T)\, e^{\delta \mathcal{D}}\,, \tag{4.20}$$

with $a \in [0,1]$ and the subleading polynomial correction can be absorbed by the function $c(T)$.[9]

Even more remarkably, the value of δ (4.18) for the lightlike tachyon agrees with the critical exponent (3.10) of the ETW brane at the beginning of time of the linear dilaton solution. This shows that both kinds of ETW branes are very similar, and is strongly suggestive that they may admit similar microscopic descriptions. Hence, we claim that the singularity at the beginning of time is a dynamical cobordism to nothing triggered by (the strong coupling version of) the condensation of the closed string tachyon. We look deeper into this argument in section 5 but before then, we show that this surprising matching of the critical exponents holds for general profiles of f_1.

4.2.3 General f_1

Let us now show that the above structure, and in particular the same value for the critical exponent δ, holds for general f_1 under very mild conditions. In particular we demand that f_1 decays at large T faster then $1/T$. This is a very reasonable requirement, in particular notice that this ensures the convergence of the integral for the spacetime distance Δ to the ETW brane. Hence it implements the intuition that the wall of nothing propagating at the spped of light hits in finite time any point at finite spacetime distance.

Consider now the integral for the field space distance

$$\mathcal{D} = \int \left[\frac{f_1''}{f_1} + \frac{f_1'}{f_1}\left(\frac{1}{T} - \frac{D-1}{D-2}\frac{f_1'}{f_1}\right)\right]^{\frac{1}{2}} dT\,. \tag{4.21}$$

We start massaging the integrand of $\mathcal{D}$, by noticing that

$$\frac{f_1''}{f_1} + \frac{f_1'}{f_1}\left(\frac{1}{T} - \frac{D-1}{D-2}\frac{f_1'}{f_1}\right) = \left(\frac{f_1'}{f_1}\right)' + \frac{f_1'}{T\,f_1} + \frac{1}{D-2}\left(\frac{f_1'}{f_1}\right)^2\,, \tag{4.22}$$

it is easy to show that for f_1 decaying faster than $1/T$, the dominant term is the last one. In fact one can see by considering different profiles (e.g. power-law, exponential, exponential of an exponential, etc) that, the faster the decay, the more the last terms dominates. Then

$$\mathcal{D} \sim \int \frac{1}{\sqrt{D-2}} \frac{f_1'}{f_1}\, dT\,. \tag{4.23}$$

We may write this as

$$d\mathcal{D} = \frac{1}{\sqrt{D-2}}\frac{f_1'}{f_1}\, dT = \frac{1}{\sqrt{D-2}} d\log f_1 = \sqrt{D-2}\; d\log f_1^{\frac{1}{D-2}}\,. \tag{4.24}$$

[9] More details about such subleading corrections can be found in the appendix B of [18].

Namely

$$f_1^{\frac{1}{D-2}} = \exp(-\mathcal{D}/\sqrt{D-2})\,, \tag{4.25}$$

where we have chosen the appropriate sign for the distance to be positive (recall that f_1 is a function that decreases to zero). This allow to express the spacetime distance as

$$\Delta \sim \int f_1^{1/(D-2)} dx^1 \sim \int \exp(-\mathcal{D}/\sqrt{D-2}) \frac{dT}{T}\,. \tag{4.26}$$

This has a similar structure to the intermediate expression in (4.15). Similar to the exponential integral there, the above integral behaves just like the exponential in the integrand, leading to the scaling

$$\Delta = \exp(-\mathcal{D}/\sqrt{D-2})\,, \tag{4.27}$$

which reproduces the value of δ in (4.18). Indeed, one can check that the additional terms in the integrand lead to subleading corrections, of the kind in (4.17) (for a proof of this statement under mild assumptions, we refer the reader to appendix C).

5 The strong coupling region and the origin of time

In this section we argue that the microscopic description of the ETW brane at the beginning of time is a region of (the strong coupling version of) closed string tachyon condensation.

The ETW brane recombination. Let us now consider the full lightlike tachyon configuration, including the strongly coupled region, and consider the interplay of the two ETW branes we have encountered.

In the string frame variables there are two asymptotic regions, controlled by seemingly different physics. The first is the region $X^0 \to -\infty$, with X^1 finite (hence $X^+ \to -\infty$), which corresponds to a linear timelike dilaton configuration, with negligible tachyon background. The second is the region $X^+ \to \infty$ at finite X^1 (hence $X^0 \to \infty$), which corresponds to a lightlike tachyon configuration at weak string coupling. Both regions are disjoint, as they only coincide at infinity in $X^0 \to -\infty$, $X^+ \to \infty$ (hence we need $X^1 \to \infty$).

In the Einstein frame, these asymptotic regimes turn into singularities at finite distance in spacetime, triggered by the running off of suitable scalars (the tachyon or the dilaton) to infinite distance in field space. Following the dynamical cobordism interpretation advocated in [16–18], these are ETW branes chopping off the region of spacetime beyond them.

An important observation is that the effective theory in which one describes ETW branes is not valid at arbitrarily short distances to the singularity. The singularity is expected to be smoothed out by new UV physics which implies the existence of a cutoff in the applicability of the effective theory. This translates into cutting of a strip of spacetime around the singularities, hence providing a notion of 'strechted' ETW brane in effective theory. This can be obtained in different ways, for instance by imposing a maximal bound on the scalar curvature. Instead, we use a criterion directly inspired by the swampland distance conjecture [57], as follows.

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The distance conjecture states that when an effective theory reaches to a large distance $\mathcal{D}$ in field space, its effective cutoff scales as

$$\Lambda \sim e^{-\alpha \mathcal{D}}\,, \tag{5.1}$$

for some order 1 coefficient α. The actual distance conjecture moreover claims that there is an infinite tower of states becoming light with Λ, but this formulation corresponds to an adiabatic motion in moduli space, and such towers may actually not arise in dynamical situations with spacetime dependence of the scalars [58].[10] Hence we stick to the milder statement that a cutoff is developed, whose origin in our context would stem from the UV completion of the ETW brane.

In our configuration we hence consider the slice of spacetime at which the field space distance (in the combined tachyon-dilaton system) reaches a large but finite value. From the Einstein frame action (4.11), the relevant kinetic terms read

$$\frac{4}{D-2}\partial\Phi\cdot\partial\Phi \;-\; \frac{4}{D-2}\frac{f_1'}{f_1}\partial\Phi\cdot\partial T \;+\; \left(-\frac{f_1''}{f_1}-\frac{f_1'}{Tf_1}+\frac{D-1}{D-2}\frac{f_1'^{\,2}}{f_1^2}\right)\partial T\cdot\partial T\,. \tag{5.2}$$

We are interested in the behaviour near the intersection of the two singularities. Since this lies at large T, we can simplify the last term using the argument in section 4.2.3. Using $(f_1'/f_1)\partial T = \partial \log f_1$, the kinetic term may be written

$$\frac{1}{D-2}(\,2\partial\Phi - \partial\log f_1\,)^2\,, \tag{5.3}$$

so that the slices of constant distance are defined by

$$\mathcal{D} \sim \frac{1}{\sqrt{D-2}}(2\Phi - \log f_1) = \text{const}\,. \tag{5.4}$$

Note that interestingly, the swampland distance cutoff (5.1) is

$$\Lambda \sim e^{-\alpha\mathcal{D}} \sim \exp\left(-\frac{\alpha\delta}{2}\,(2\Phi - \log f_1)\right) = \left(e^{-2\Phi} f_1\right)^{\alpha\delta/2}, \tag{5.5}$$

where δ is given by (4.18). The factor inside brackets is the prefactor of the Einstein term in the string frame action. The fact that it relates to the cutoff scale shows that one gets the same spacetime slices if one uses a bound in the scalar curvature to limit the applicability of the effective theory, rather than in the field space distance. Indeed, this is expected from the scaling (2.3) of R with $\mathcal{D}$ near ETW branes.

The curve in the (x^0, x^1)-plane defined by (5.4) asymptotes to constant X^0 on one side and to constant X^+ on the other. For illustration we may consider f_1 as in (4.13) and get

$$2qx^0 - b\mu^k e^{\beta k x^+} = \text{const.} \tag{5.6}$$

[10] A simple example is the Taub-NUT geometry, which can be regarded as a spacetime-dependent solution of $\mathbf{S}^1$ compactifications, in which the circle shrinks to zero size at a point in the base. Hence, it attains infinite distance in the naive circle compactification moduli space, but no tower of light particles or other disasters arise.

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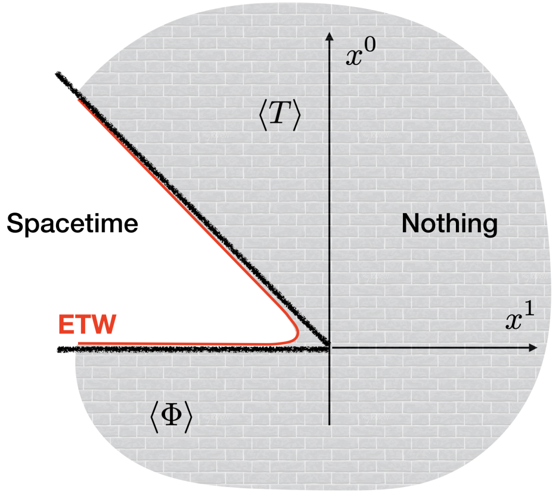

Figure 1. Depiction of a spacetime slice at which the solutions attain a large fixed field space distance in the tachyon-dilaton space. The asymptotes near $x^0 \sim 0$ and $x^+ \sim 0$, should be regarded as corresponding to the physical location of the ETW branes, which are hence recombined in the middle region, and which for a boundary of spacetime, with nothing beyond them. The spray line signal the location of the singularities.

This leads to slices of the form

$$x^1 = \frac{\sqrt{2}}{k\beta}\log(x^0 + cst) - x^0 + \frac{\sqrt{2}}{k\beta}\log\frac{2q}{b\mu^k}\,, \tag{5.7}$$

for some constant *cst* related to the cutoff. In figure 1 we depict the structure of such curves and of the resulting spacetime picture.

From this it is clear that what seemed to be an intersection between the two ETW walls is in fact a smooth region that interpolates between the two. This hence strongly motivates that this solution describes a one and only recombined ETW wall. Another interesting indication for this is the fact that the dilaton-tachyon mixing in the effective action (scaling like f_1'/f_1 times powers of the string couplings) gets large for large tachyon and strong coupling, namely near the naive intersection of the singularities. Hence, the two walls, which asymptotically correspond to the dilaton or the tachyon running off to infinite distance in their field space, become of a very similar nature in that intersection region.

As a side note, one may wonder if our analysis extends to the case of critical $D = 26$ bosonic string theory. Indeed, this theory is tachyonic and, as mentioned in section 3.1, it admits a lightlike dilaton background. It is then clear from the equation of motion for the tachyon (3.13) that, if we take the dilaton along the direction X^+, the only option is to take the tachyon along X^-. This means that the ETW branes corresponding to large tachyon and strong coupling are of the intersecting kind, exactly like those described in this section. It would be interesting to explore these lightlike linear dilaton backgrounds in other critical theories, and make contact with existing proposals concerning the resulting singularities [36, 51].

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6 Conclusions

In this paper we have studied timelike linear dilaton backgrounds of supercritical string theories as time-dependent solutions in string theory, and addressed the question of the resulting spacelike singularity, from the perspective of the cobordism conjecture. We have quantitatively characterized the solution as a Dynamical Cobordism in which the dilaton rolls until it hits infinite field space distance at a singularity at finite time in the past. We have shown that the singularity in effective theory follows the scaling behaviour of ETW branes. In order to clarify its microscopic description, we have considered lightlike tachyon condensation backgrounds, whose microscopic description had been argued to correspond to a stringy version of a bubble of nothing. Using an effective theory approach, we have characterized them as ETW branes and have encountered precisely the same scaling exponent as the beginning of time singularity. Together with the fact that both ETW branes join smoothly from the effective theory perspective, this has motivated our proposal that the spacelike singularity should correspond in string theory to a region of (a strong coupling version of) closed tachyon condensation, giving rise to a cobordism boundary defining the beginning of time.

There are several open questions and new directions:

- We have used an effective theory for the tachyon in terms of the undetermined function f_1. Even though our results are robust under changes of the precise form of this function, it would be interesting to determine it, or at least its asymptotic behavior for large T.
- Conversely, it would be interesting to understand if the criterion that the theory allows for a resolution of spacelike singularities can be used as a constraint on effective theories. For instance, there exist choices of f_1 which lead to lightlike tachyon ETW branes with scalings different from the beginning of time one. Such theories may not be compatible with a microscopic description of the latter, as the ETW brane may not be compatible for recombination. It is thus tantalizing to claim that this can be used as a criterion to exclude such choices of f_1. It would also be interesting to understand this possibility, possibly invoking other swampland constraints or physical considerations.
- Although we have focused on the bosonic theory, there is a rich set of phenomena arising in lightlike tachyon backgrounds in other string theories. We expect our ideas to lead to interesting new insights into this web of transitions.
- Finally, an interesting corner in this circle of ideas is that of lightlike dilaton backgrounds in critical string theories. They are toy models of cosmological singularities, which in certain supersymmetric cases admit interesting proposals for their microscopic description [51]. It would be exciting to use cobordism ideas to make progress on the understanding of such backgrounds.

We hope to report on these and other questions in the near future.

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A Dimension quenching as an interpolating domain wall

In [17] it was argued that, when the scalars remain at finite distance in field space as one hits the wall, the corresponding configuration described an interpolating wall between different QG theories. This scenario is built in contrast with the end-of-the-world walls where the fields reach infinite distance in field-space at the wall. Instead of the solution ending abruptly at the location of a singularity, these interpolating solutions continue across the wall into another theory. On each side of the interpolating wall, the field spaces may have different structures but the location of the wall itself is at finite distance in both of them. As a result, the interpolating wall must have all the right properties for communicating between the two theories, whatever they may be. It is clear from this that the microscopic nature of these walls can be hard to describe; it may in general be non-supersymmetric and may involve strong-coupling physics. The existence of such objects is one of the predictions of the Cobordism Conjecture [3].

The examples of interpolating walls in [17] were simple enough to be described by standard supersymmetric objects. For example, D8 branes in massive type IIA string theory were identified as such walls interpolating between "different" massive IIA theories with different units of 0-form flux. Here, we re-interpret the dimension-quenching bubbles of [36, 37] as interpolating walls between bosonic string theories of different dimensions.

Throughout this paper we have dealt with tachyons with exponential profiles along one light-like direction. These solutions were shown to lead to bubbles of nothing, which fit in the wall-of-nothing description. There are, however, slightly more complicated solutions to the tachyon equation of motion (3.13). Following [36], we can consider a profile with oscillatory dependence on another coordinate, denoted by X^2:

$$T(X) = \mu_0^2 \exp(\beta X^+) - \mu_k^2 \cos(k X^2) \exp(\beta_k X^+) \,. \tag{A.1}$$

This is a solution to the equation of motion with a timelike linear dilaton $\Phi = -qX^0$ background if:

$$q\beta_k = \sqrt{2}\left(\frac{2}{\alpha'} - \frac{1}{2}k^2\right) . \tag{A.2}$$

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Since the tachyon couples to the worldsheet as a potential, the theory has a vacuum at $X^2 = 0$. One can show that expanding around this vacuum in the limit where the wavelength of oscillations k^{-1} is much larger than the string length l_s yields:

$$T(X^+, X^2) = \frac{\mu^2}{2\alpha'} \exp(\beta X^+) : (X^2)^2 : , \tag{A.3}$$

where $\mu^2 = \alpha' k^2 \mu_k^2$, and dots denote normal ordering. We refer the reader to [36] for additional aspects of the detailed derivation.

The physical interpretation of this is clear. Before the tachyon condenses, at $X^+ \to -\infty$, the string propagates in $D-1$ spatial dimensions. As the string reaches a regime where $T \sim 1$ (namely $X^+ \sim \beta^{-1} \log \mu$), the potential confines the string to the region where it is vanishing, at $X^2 = 0$. Strings that oscillate along the X^2 dimension will be expelled from the region of large tachyon condensate. This bubble thus interpolates between a region of $D-1$ spatial dimensions to one where the string can effectively propagate in $D-2$ dimensions. These types of bubbles were dubbed dimension-quenching or dimension-changing bubbles.

Turning now to the dynamical cobordism perspective, one can see in (A.3) that the tachyon field remains at a finite value (hence at a finite distance in field space) at the location of this bubble at $X^2 = 0$. This fits perfectly with the description of an interpolating wall as described in [17]. We thus interpret these dimension-quenching bubbles as examples of dynamical cobordism interpolating walls between bosonic theories of different dimensions.

As a side note, one can construct similar bubbles that kill more than one dimension by granting oscillatory dependence of the tachyon on extra dimensions. Furthermore, this dimension-quenching mechanism also extends to superstring theories and can be used to draw connections between supercritical Type 0 theories and their 10-dimensional critical counterparts [36].

B The partition function with a lightlike tachyon background

The computation of the partition function in the presence of the lightlike tachyon background is obtained evaluating the path integral without vertex operator insertions:

$$Z(\mu) = \int \mathcal{D}X^+ \mathcal{D}X^- \mathcal{D}X^i \mathcal{D}g \mathcal{D}(\text{ghosts}) e^{iS^{\text{deformed}}} , \tag{B.1}$$

where we have emphasized the fact that the integration along the lightlike directions does not affect the spacelike directions ($i = 2, \ldots, D-1$). Being at weak coupling, we only consider the one-loop contribution and so we have to evaluate the 2d action on a genus one worldsheet:

$$\begin{aligned} S^{\text{deformed}} =& -\frac{1}{2\pi\alpha'} \int_{\mathcal{M}_1} d^2\sigma \sqrt{g} g^{\alpha\beta} \left[-\partial_\alpha X^+ \partial_\beta X^- - \partial_\alpha X^- \partial_\beta X^+ + \partial_\alpha X^i \partial_\beta X_i \right] + \\ &+ \frac{1}{2\pi} \int_{\mathcal{M}_1} d^2\sigma \sqrt{g} R_2 \phi(X) - \frac{1}{2\pi} \mu \int_{\mathcal{M}_1} d^2\sigma \sqrt{g} e^{\beta X^+} . \end{aligned} \tag{B.2}$$

In analogy with the procedure in [45], we decompose the field $X^+(\tau,\sigma)$ into its zero and nonzero modes:

$$X^+(\tau,\sigma) = X_0^+ + \widehat{X}^+(\tau,\sigma)\,, \tag{B.3}$$

we get a standard integration for the zero mode:

$$\mathcal{D}X^+ = dX_0^+\mathcal{D}\widehat{X}^+. \tag{B.4}$$

Choosing the following convention to perform a Wick rotation:

$$\tau \mapsto i\tau_E \qquad X^i \mapsto iX_E^i \qquad \mu \mapsto -i\mu_E\,, \tag{B.5}$$

equation (B.1) becomes:

$$Z\left(\mu\right) = \int dX_0^+ \int \mathcal{D}\widehat{X}^+\mathcal{D}X^-\mathcal{D}X^i\mathcal{D}g\mathcal{D}(\text{ghosts})e^{-S_E^{\text{deformed}}}\,. \tag{B.6}$$

where the tachyonic potential gives an oscillating contribution to the integral in the condensate region. Such a behavior produces a truncation of the contributions to the integral coming from configurations with $X^+ \mapsto \infty$. The Euclidean 2d action is:

$$\begin{aligned} S_E^{\text{deformed}} =& \frac{1}{2\pi\alpha'}\int d^2\sigma_E\sqrt{g}\left[\partial_{\sigma^0}\widehat{X}^+\partial_{\sigma^0}X^- + \partial_{\sigma^1}X^-\partial_{\sigma^1}\widehat{X}^+ + \partial_\alpha X^i\partial_\alpha X^i\right] \\ &+ \frac{1}{2\pi}\int d^2\sigma_E\sqrt{g}R_2\phi(X) + \frac{i\mu_E}{2\pi}\int d^2\sigma_E\sqrt{g}e^{\beta X^+}\,. \end{aligned} \tag{B.7}$$

Using the variable change:

$$y = e^{\beta X_0^+} \qquad \longrightarrow \qquad dX_0^+ = \frac{dy}{\beta y}, \tag{B.8}$$

and making the dependence of the integrand on X_0^+ explicit, we obtain:

$$Z(\mu_E) = \int \mathcal{D}\widehat{X}^+\mathcal{D}X^-\mathcal{D}X^i\mathcal{D}g\mathcal{D}(\text{ghosts})\int_0^\infty \frac{dy}{\beta y}e^{-S_E^{\text{kinetic}}-S_E^{\text{dilaton}}-\frac{i\mu_E}{2\pi}\int d^2\sigma_E\sqrt{g}ye^{\beta\widehat{X}^+}}\,, \tag{B.9}$$

where $S_E^{\text{kinetic}} + S_E^{\text{dilaton}} = S_E^{\text{free}}$ are respectively the kinetic and the dilaton contributions in the Euclidean action (B.7).

Now, let us consider the following quantity:

$$\frac{\partial Z}{\partial\mu_E} = \int \mathcal{D}\widehat{X}^+\mathcal{D}(\text{others})\int_0^\infty \frac{dy}{\beta}e^{-S_E^{\text{free}}+\frac{-i\mu_E}{2\pi}\int d^2\sigma_E\sqrt{g}ye^{\beta\widehat{X}^+}}\left(\frac{-i}{2\pi}\int d^2\sigma_E\sqrt{g}e^{\beta\widehat{X}^+}\right)\,. \tag{B.10}$$

Let us finally perform the integration on the zero mode. We obtain:

$$\frac{\partial Z}{\partial\mu_E} = -\frac{1}{\beta\mu_E}\int \mathcal{D}\widehat{X}^+\mathcal{D}X^-\mathcal{D}X^i\mathcal{D}g\mathcal{D}(\text{ghosts})e^{-S_E^{\text{free}}}\,, \tag{B.11}$$

where S_E^{free} is the euclidean action of the world-sheet theory without the tachyon potential. Integrating with respect to μ and fixing a cutoff for X^+ such that $\mu_* = e^{\beta X_*^+}$, we obtain:

$$Z_1 = -\frac{\log(\mu_E/\mu_*)}{\beta}\widehat{Z}\,, \tag{B.12}$$

where $\widehat{Z}$ is the partition function for the free 2d theory, namely without the tachyon insertion and without integrating the zero modes of X^+. Note that the tachyon's contribution to the partition function is entirely encoded in the zero modes.

We can interpret this factor as a "size" of the direction X^+. Indeed, because of the potential barrier created by the condensation of the tachyon, no physical degrees of freedom penetrate inside the bubble wall, beyond $X^+ \sim 1$. As mentioned previously, the path integral is suppressed in this region. The direction X^+ thus has an effectively finite "size" that agrees with the estimate in (4.3).

C The critical exponent for general f_1

In this appendix we provide more details regarding how we obtain the scaling relation (4.27) for a general f_1 decaying faster than T^{-1}. The starting point is (4.26), which we rewrite as follows:

$$\Delta \sim \int \exp\big(-\frac{\mathcal{D}}{\sqrt{D-2}} - \log(T\mathcal{D}')\big) d\mathcal{D}\,, \tag{C.1}$$

where the prime stands for derivation with respect to T. Proving that the first term in the exponential is the dominant one in the limit $T \to \infty$ comes down to comparing the two terms:

$$-\frac{\mathcal{D}}{\sqrt{D-2}} \sim \log f_1 \quad \text{and} \quad -\log T\mathcal{D}' \sim \log\left(\frac{f_1}{|f_1'|T}\right). \tag{C.2}$$

Notice that in the special case where f_1 is power-like $f_1 = T^{-k}$, with $k > 0$, the second term in the exponential is constant so one automatically obtains the scaling relation (4.27). For other choices of f_1, we wish to check that in the limit $T \to \infty$,

$$\frac{|\log(\frac{f_1}{|f_1'|T})|}{|\log f_1|} \to 0\,. \tag{C.3}$$

We consider a positive and monotonically decreasing function f_1, we require:

$$\left|\log\left(\frac{f_1}{|f_1'|T}\right)\right| \ll |\log f_1| \quad \text{as} \quad T \to \infty\,. \tag{C.4}$$

We know for a fact that $\log f_1$ is negative when $T \to \infty$. As we will show shortly, $\log(\frac{f_1}{|f_1'|T})$ is negative, then one can easily show that (C.4) implies:

$$|f_1'| \ll T^{-1}\,, \tag{C.5}$$

which is true for any function f_1 under consideration since $f_1 \ll T^{-1}$.

The question of whether (C.3) is verified is thus recast into the question of the sign of $\log(\frac{f_1}{|f_1'|T})$. In order to determine if this term is negative, we can write it as follows,

$$\log\left(\frac{f_1}{|f_1'|T}\right) = \log\left(\frac{f_1}{T}\right) - \log(|f_1'|)\,. \tag{C.6}$$

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One clearly sees that the first term is negative whilst the second is positive. We would therefore like to show that the first one dominates:

$$|\log\left(\frac{f_1}{T}\right)| \gg |\log(|f_1'|) \quad \to \quad \frac{f_1}{T} \ll |f_1'| \tag{C.7}$$

This is condition is verified for all valid choices of f_1 that are not power-like. Indeed, we can use the trivial fact that $f_1 \ll T$ as $T \to \infty$ to show that $\frac{f_1}{|f_1'|} \ll T$. In the case where f_1 is a power-like function the inequality is not strict as $\frac{f_1}{T} \sim |f_1'|$, but we still recover the right scaling relations as mentioned previously.

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3
Global Anomalies

This chapter contains the article:

- *Global anomalies & bordism of non-supersymmetric strings*,
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Global anomalies & bordism of non-supersymmetric strings

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Abstract: The three tachyon-free non-supersymmetric string theories in ten dimensions provide a handle on quantum gravity away from the supersymmetric lamppost. However, they have not been shown to be fully consistent; although local anomalies cancel due to versions of the Green-Schwarz mechanism, there could be global anomalies, not cancelled by the Green-Schwarz mechanism, that could become fatal pathologies. We compute the twisted string bordism groups that control these anomalies via the Adams spectral sequence, showing that they vanish completely in two out of three cases (Sugimoto and $\mathrm{SO}(16)^2$) and showing a partial vanishing also in the third (Sagnotti 0'B model). We also compute lower-dimensional bordism groups of the non-supersymmetric string theories, which are of interest to the classification of branes in these theories via the Cobordism Conjecture. We propose a worldvolume content based on anomaly inflow for the $\mathrm{SO}(16)^2$ NS5-brane, and discuss subtleties related to the torsion part of the Bianchi identity. As a byproduct of our techniques and analysis, we also reprove that the outer $\mathbb{Z}_2$ automorphism swapping the two E_8 factors in the supersymmetric heterotic string is also non-anomalous.

Keywords: String and Brane Phenomenology, Superstrings and Heterotic Strings

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Contents

1 Introduction

It is widely known that there are five supersymmetric string theories in ten dimensions [1]. It is slightly less known that there are several other non-supersymmetric string theories in ten dimensions, many of which have tachyons; and there are just three known models in ten dimensions which are both non-supersymmetric in spacetime and tachyon free: the SO(16) × SO(16) string [2, 3], the Sugimoto model [4], and the Sagnotti 0'B model [5, 6]. These non-supersymmetric models and their compactifications have been the subject of a renovated interest in the recent literature (see e.g. [7–35]), presumably because they constitute a promising arena to study quantum gravity away from the supersymmetric lamppost.

In spite of this recent surge of work, we still know remarkably little about the three tachyon-free non-supersymmetric models in ten dimensions, particularly when compared with their supersymmetric counterparts. In particular, the spectrum of all three non-supersymmetric models is chiral in ten dimensions, and so it is potentially anomalous. Local anomalies have long been known to cancel via non-supersymmetric versions of the Green-Schwarz mechanism [2–6]. However, to our knowledge, except for an inconclusive analysis in [36] these models have not been shown to be free of global anomalies, as was done early on in [37–39] for the supersymmetric chiral theories.[1] A global anomaly could lead to an inconsistency of the model, and having to discard it, or to new topological couplings that cancel it [40].

The purpose of this paper is twofold: on one hand, we compute the potential global anomaly of the three tachyon-free non-supersymmetric string theories in quantum gravity (albeit only a subclass of anomalies for the Sagnotti 0'B model), showing that it vanishes. We do this using cobordism theory, and computing the relevant twisted string bordism groups. This is a standard technique for proving anomaly cancellation results; see [40–47] for recent anomaly cancellation theorems in string and supergravity theories using this technique.

The second important result of our paper is precisely the calculation of these twisted string bordism groups (which have not appeared in the literature before), which are summarized in table 1 in the conclusions. The physics use of these bordism groups is that they can be used to predict new, singular configurations (branes) of the corresponding non-supersymmetric string theories, by means of the Cobordism Conjecture [48] (see [49–52] for similar recent work in type II and supersymmetric heterotic string theories). Furthermore, the calculation of the bordism groups themselves by means of the Adams spectral sequence is interesting in its own right, and we expect that similar techniques can be used to compute string bordism groups of e.g. six-dimensional compactifications, and more generally, to study anomalies of any theory with a 2-group symmetry or a Green-Schwarz mechanism.

Along the way, we will encounter and comment on issues such as whether the heterotic Bianchi identity can be taken to take values on the free part of cohomology or the torsion piece must be included, or the connections between anomaly cancellation in eleven-dimensional backgrounds and anomaly inflow on non-supersymmetric NS5 branes on these theories. We also include a quick introduction to the Green-Schwarz mechanism in the modern formalism of anomaly theory, providing for the first time a candidate for the worldvolume degrees of freedom for the NS5 brane in the $\mathrm{SO}(16)\times\mathrm{SO}(16)$ string. We also study global anomalies in the $\mathbb{Z}_2$ outer automorphism swapping the two factors of the $\mathrm{SO}(16)\times\mathrm{SO}(16)$ string, showing that anomalies vanish.

The upshot of our paper is:

- The bordism group controlling anomalies of the Sugimoto string, $\Omega_{11}^{\text{String-Sp}(16)}$, vanishes (theorem 3.48), and therefore the theory is anomaly-free.[2]

- The bordism group $\Omega_{11}^{\text{String-SU}(32)\langle c_3\rangle}$, controlling the anomaly of the Sagnotti 0'B model, is isomorphic to 0 or $\mathbb{Z}_2$ (theorem 3.63). We do not know whether the anomaly vanishes, although it does in all specific backgrounds we looked into.

- For the $\mathrm{SO}(16)\times\mathrm{SO}(16)$ heterotic string, (where the identity component of the global form of the gauge group is actually $\mathrm{Spin}(16)\times\mathrm{Spin}(16)$, since the massless spectrum

[1]Even in this case, only gravitational and global anomalies in the identity component of the gauge group have been considered.

[2]Modulo potential subtleties regarding the global structure of the gauge group, that we comment on in the Conclusions.

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contains both spinors and vectors[3]), the bordism group $\Omega_{11}^{\text{String-Spin}(16)^2}$ controlling the anomaly vanishes (theorem 3.78), and therefore this theory is anomaly-free.

- There is also a $\mathbb{Z}_2$ gauge symmetry swapping the two factors of Spin(16), whose anomaly we also studied. The bordism group controlling the anomaly has order 64 — but nevertheless (theorem 4.30), the anomaly vanishes.

As a consequence of our calculations, we also can cancel an anomaly in a supersymmetric string theory.

- When one takes into account the $\mathbb{Z}_2$ symmetry of the $E_8 \times E_8$ heterotic string swapping the two copies of E_8, the anomaly vanishes (corollary 4.34).

The cancellation of this anomaly is not a new result: it is a special case of the more general work of [42]. Our argument rests on different physical assumptions and is a different mathematical result; for example, we do not assume the Stolz-Teichner conjecture. Thus we answer a question of [46], who showed the bordism group controlling this anomaly has order 64 but did not address the anomaly, and asked for a bordism-theoretic argument that the anomaly vanishes.

One key application of this $\mathbb{Z}_2$ symmetry of the $E_8 \times E_8$ heterotic string is constructing the CHL string [54], a nine-dimensional string theory obtained by compactifying the $E_8 \times E_8$ heterotic string on a circle, where the monodromy around the circle is the $\mathbb{Z}_2$ symmetry we discussed above. An anomaly in the $\mathbb{Z}_2$ symmetry would have implied an inconsistency in the CHL string. We find that the anomaly vanishes, in agreement with the results in [42], which showed this from a worldsheet perspective; by contrast, we approach the question from a pure spacetime perspective.

One can make an analogous construction for the $\mathrm{SO}(16) \times \mathrm{SO}(16)$ heterotic string, compactifying it on a circle whose monodromy exchanges the two bundles. The result is a nine-dimensional non-supersymmetric string theory whose gauge group is (perhaps a quotient of) Spin(16). Studying this theory would be an interesting extension of similar constructions in the $E_8 \times E_8$ case [55] and in the $\mathrm{SO}(16) \times \mathrm{SO}(16)$ case without the monodromy [35]. Analogously to the CHL string, an anomaly in the $\mathbb{Z}_2$ symmetry of the $\mathrm{SO}(16) \times \mathrm{SO}(16)$ heterotic string would lead to an inconsistency of this new theory, and our anomaly cancellation result implies a consistency check for this theory on backgrounds where the gauge group is Spin(16). It would be interesting to study this theory on more general backgrounds.

We have also identified a plethora of non-trivial bordism classes on these theories. It is a natural direction to explore the nature and physics of the bordism defects associated to these branes [49–51], a task we will not pursue in this paper. Furthermore, representatives of the bordism classes we encountered provide natural examples of interesting compactification manifolds for these non-supersymmetric strings to various dimensions; studying these, finding out whether moduli are stabilized (including SUSY-breaking stringy corrections to the potential) etc. is another important open direction to study.

[3] See [53] for an analysis detailing some possibilities for a global quotient in the $\mathrm{SO}(16)^2$ gauge group. In this paper, we assume the simply-connected global form $\mathrm{Spin}(16)^2$ (so "$\mathrm{SO}(16)^2$" is an abuse of notation); this has the advantage that all anomalies we find also exist for any other possibility (although with a nontrivial global quotient, there could be more anomalies than the ones that we study here).

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This paper is organized as follows: in section 2 we provide a lightning review of modern methods to study anomalies and how these cancel via the Green-Schwarz mechanism, as well as a detailed description of how this happens for each of the three tachyon-free, non-supersymmetric string theories. We believe this is the first time these important results are collected together in a single reference, and with a unified notation. Section 3 contains our main result — the calculation of bordism groups for these theories using the Adams spectral sequence — together with a discussion of the natural cohomology theory for the Bianchi identity to take values in. We also study in detail the relationship of higher-dimensional anomaly cancellation to the worldvolume theory of magnetic NS branes. Section 4 extends the anomaly calculation to the $\mathrm{SO}(16)\times\mathrm{SO}(16)$ string including the (gauged) automorphism swapping the two $\mathbb{Z}_2$ factors. This multiplies the number of interesting bordism classes, but anomalies still cancel. Finally, section 5 presents a table with our results, conclusions, and potential further directions, including a few comments on how these anomalies might be studied from a worldsheet point of view, in the line of [42, 45].

2 Local anomalies and the Green-Schwarz mechanism

The word "anomaly" describes the breaking of a classical symmetry by quantum effects. In a Lagrangian theory, anomalies correspond to a lack of invariance of the path integral under a symmetry transformation. They can arise for both global and gauge symmetries in field theories. Anomalies in global symmetries only point to the fact that the symmetry cannot be gauged; they can lead to anomaly matching conditions that heavily constrain the RG-flow and strong coupling dynamics of the theory [56]. In contrast, anomalies in gauge theories point to true inconsistencies: a gauge symmetry is by definition a redundancy of the theory and as such can never be broken. In this paper, we will only consider anomalies in gauge symmetries.

The anomalies under consideration arise in field theories when they are coupled to gauge fields and dynamical gravity. They then correspond to a lack of invariance of the path integral under a gauge transformation/diffeomorphism (for a review, see [57]). When this transformation can be continuously connected to the identity, we speak of local anomalies. Their cancellation heavily constrains a theory; for example the gauge group of $\mathcal{N}=1$ supergravities in ten dimensions is constrained by anomaly cancellation to be either one of four gauge groups: (a quotient of) $\mathrm{Spin}(32)$, $E_8\times E_8$, $\mathrm{U}(1)^{496}$ or $\mathrm{U}(1)^{248}\times E_8$. The last two can be ruled out as low-energy EFTs of a consistent theory of quantum gravity by demanding the consistency of the worldvolume theory of brane probes in [58] (see also [11] for developments in the context of orientifold models) and using more general arguments in [59].

When the anomalous symmetry transformation cannot be continuously connected to the identity, then we speak of global anomalies (not to be confused with anomalies in global symmetries!). Global anomalies and their cancellation will be at the heart of this paper. It only makes sense to study them once local anomalies cancel. We therefore review local anomaly cancellation in the remainder of this section, before discussing global anomalies in the next ones.

The most direct way to study local anomalies is to compute certain one-loop Feynman diagrams involving external gauge bosons and/or gravitons, and chiral fermions in the internal legs; for ten-dimensional theories, the relevant diagram has 6 external legs [60].

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There is, however, a much more concise way of studying such anomalies, through what is called an anomaly polynomial [57]. This is a certain formal polynomial in the gauge-invariant quantities $\mathrm{tr}F^m$ and $\mathrm{tr}R^m$, which are certain contractions of Riemann and gauge field strength tensors that do not involve the metric. If the theory we wish to study lives in d spacetime dimensions, the anomaly polynomial is of degree $(d+2)$. Although the anomaly polynomial is often discussed in the physics literature directly in terms of $\mathrm{tr}F^m$ and $\mathrm{tr}R^m$, we find it more natural and convenient to write it down in terms of (the free part of) Chern and Pontryagin characteristic classes, more common in the mathematical literature. These can be written as linear combinations of $\mathrm{tr}F^m$ and $\mathrm{tr}R^m$ via Chern-Weil theory as follows. The i-th Chern class $c_{\mathbf{r},i}$ is associated to a complex vector bundle, in some representation $\mathbf{r}$ of the gauge group. Via Chern-Weil theory, they are represented in cohomology by the following characteristic polynomial of the field strength (here, t is just a dummy variable):[4]

$$\sum_i c_{\mathbf{r},i}\, t^i = \det\left(\frac{iF}{2\pi}t+1\right), \tag{2.1}$$

or, expanding the determinant,

$$\sum_i c_{\mathbf{r},i}\, t^i = 1+\frac{i\,\mathrm{tr}_{\mathbf{r}}(F)}{2\pi}t+\frac{\mathrm{tr}_{\mathbf{r}}(F^2)-\mathrm{tr}_{\mathbf{r}}(F)^2}{8\pi^2}t^2+\cdots \tag{2.2}$$

The traces are over the gauge indices and as such the i-th Chern class $c_{\mathbf{r},i}$ is a $2i$-form. The Pontryagin classes are characteristic classes associated to a real vector bundle, which we will always take to be the spacetime tangent bundle. One way to define them is in terms of the Chern classes of the complexification of the vector bundle. The total Pontryagin class is the sum of the Pontryagin classes and its first few terms are as follows:

$$p = 1+p_1+p_2+\cdots \tag{2.3}$$

$$p = 1-\frac{\mathrm{tr}(R^2)}{8\pi^2}+\frac{\mathrm{tr}(R^2)^2-2\,\mathrm{tr}(R^4)}{128\pi^4}\cdots \tag{2.4}$$

Similarly to the case of the Chern classes, the Pontryagin class p_i is a $4i$-form. The reason we prefer these characteristic classes over the trace notation $\mathrm{tr}F^m$ and $\mathrm{tr}R^m$ is that, as will be clear later, the anomaly polynomial is a sum of Atiyah-Singer indices, and these indices are written in terms of these classes in the mathematical literature.

The precise relationship between local anomalies and the anomaly polynomial is as follows. The anomalous variation of the quantum effective action $\delta_\Lambda\Gamma$ can be related to the $(d+2)$-dimensional anomaly polynomial through what is called the Wess-Zumino descent procedure, which we briefly outline here. Since the characteristic classes are (locally) exact, the anomaly polynomial itself is also (locally) exact. We can therefore locally write it as

$$P_{d+2} = dI_{d+1} \tag{2.5}$$

[4] Representing characteristic classes with differential forms misses any torsional components of integral cohomology, which is the more natural domain of characteristic classes. This subtlety will play an important role when discussing certain global anomalies in later parts of this paper, but it is immaterial in the present discussion.

where I_{d+1} is called the (Lagrangian density of the) anomaly theory and locally satisfies the descent property $\delta_\Lambda I_{d+1} = dI_d$. It is related to an anomalous variation of the effective action $\delta_\Lambda\Gamma$, which is local, by extending the spacetime manifold X_d into a $(d+1)$-dimensional manifold Y_{d+1} whose boundary is X_d. One finds

$$\delta_\Lambda\Gamma = \int_{X_d=\partial Y_{d+1}} I_d = \int_{Y_{d+1}} dI_d = \delta_\Lambda\left[\int_{Y_{d+1}} I_{d+1}\right] \equiv \delta_\Lambda\left[\alpha(Y_{d+1})\right], \tag{2.6}$$

where the anomaly I_d is only defined up to a closed form and $\alpha(Y_{d+1})$ is called the anomaly theory. The anomaly corresponding to a given gauge transformation is then computed as the integral of I_d over the spacetime manifold X_d. It follows that local anomalies vanish if and only if the anomaly polynomial vanishes.

The anomaly polynomial can be written as a sum of contributions from all of the fields in the theory. Each contribution is given by an index density in $(d+2)$-dimensions, whose integral over a compact manifold (with suitable structure, e.g. spin for fermions) gives the index of the corresponding Dirac operator via the Atiyah-Singer index theorem. We now list some of these contributions that will be relevant in what follows. The index density associated to a left-handed Weyl fermion in the representation $\mathbf{r}$ of a gauge group with field strength F is:

$$\mathcal{I}_{1/2} = \left[\hat{A}(R)\,\mathrm{tr}_{\mathbf{r}}\, e^{iF/2\pi}\right]_{d+2}, \tag{2.7}$$

where the term $\mathrm{tr}_{\mathbf{r}}\, e^{iF/2\pi}$ is sometimes referred to as the Chern character. The notation $[\cdots]_{d+2}$ means that one should select the $(d+2)$-form part of the enclosed expression. $\hat{A}(R)$ is called the *A-roof polynomial*, and it can be expanded as:

$$\hat{A}(R) = 1 - \frac{p_1}{24} + \frac{(7p_1^2 - 4p_2)}{5760} + \frac{-31p_1^3 + 44p_1p_2 - 16p_3}{967680}\cdots \tag{2.8}$$

Note that expression (2.7) can be easily applied to a fermion that is a singlet under the gauge group, in which case the Chern character reduces to 1. The contribution to the anomaly polynomial corresponding to a left-handed fermion singlet is therefore simply:

$$\mathcal{I}_{\text{Dirac}} = \left[\hat{A}(R)\right]_{d+2}. \tag{2.9}$$

The index density associated to a left-handed Weyl gravitino in $(d+2)$ dimensions is:

$$\mathcal{I}_{3/2} = \left[\hat{A}(R)\left(\mathrm{tr}\, e^{iR/2\pi} - 1\right)\mathrm{tr}_{\mathbf{r}}\, e^{iF/2\pi}\right]_{d+2}. \tag{2.10}$$

Finally, a self-dual tensor gives a contribution:

$$\mathcal{I}_{\text{SD}} = \left[-\frac{1}{8}L(R)\right]_{d+2}, \tag{2.11}$$

where $L(R)$ is called the *L-polynomial* or *Hirzebruch genus*, and it can be expanded as follows:

$$L(R) = 1 + \frac{p_1}{3} + \frac{-p_1^2 + 7p_2}{45} + \frac{2p_1^3 - 13p_1p_2 + 62p_3}{945} + \cdots \tag{2.12}$$

Armed with these index densities, we can now review anomalies in ten-dimensional $\mathcal{N} = 1$ supergravities. With this supersymmetry, the only multiplets are the gravity and vector

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multiplets [60]. There are pure gravitational anomalies coming from the chiral gravitini in the gravity multiplet as well as Majorana-Weyl spinors in both gravity and vector multiplets. There can also be gauge anomalies involving the fermions in the vector multiplet, which transform in the adjoint representation of the gauge group (they are gaugini). The anomaly polynomial of these theories is computed using (2.7)–(2.11) summing over all chiral fields (dilatino, gravitino, and gaugini), and reduces to the following expressions for the $\mathrm{Spin}(32)/\mathbb{Z}_2$ and the $E_8 \times E_8$ theories respectively:

$$P_{12}^{\mathrm{Spin}(32)/\mathbb{Z}_2} = -\frac{p_1 + c_{\mathbf{32},2}}{2} \times \frac{1}{192}\left(16\, c_{\mathbf{32},2}^2 - 32\, c_{\mathbf{32},4} + 4\, c_{\mathbf{32},2}\, p_1 + 3\, p_1^2 - 4\, p_2\right), \tag{2.13}$$

$$\begin{aligned} P_{12}^{E_8 \times E_8} = &-\frac{p_1 + c^{(1)}_{\mathbf{16},2} + c^{(2)}_{\mathbf{16},2}}{2} \\ &\times \frac{1}{192}\left(8\,(c^{(1)}_{\mathbf{16},2})^2 + 8\,(c^{(2)}_{\mathbf{16},2})^2 + 4\,(c^{(1)}_{\mathbf{16},2} + c^{(2)}_{\mathbf{16},2})\, p_1 - 8\, c^{(1)}_{\mathbf{16},2}\, c^{(2)}_{\mathbf{16},2} + 3\, p_1^2 - 4\, p_2\right). \end{aligned} \tag{2.14}$$

Here, the Chern classes are expressed in the vector representations of $\mathrm{Spin}(32)$ and the $\mathrm{SO}(16)$ subgroups of each E_8 respectively, and the (1) and (2) indices differentiate between each of the two E_8 gauge groups. These expressions agree with the ones in [60] once (2.1)–(2.4) are used to put Chern and Pontryagin classes back into traces. The anomaly polynomial does not vanish, and at first sight this would mean that the theories are inconsistent. However, both anomaly polynomials factorize in the following schematic form:

$$P_{12} = X_4 X_8\,, \tag{2.15}$$

where the X_4 is a four-form and X_8 is an eight-form. This factorization property is the key to cancelling the anomaly, through what is known as the Green-Schwarz mechanism [60, 61]. There is another field in these theories that can contribute to the aforementioned diagrams: the massless Kalb-Ramond B-field. Consistency of the 10-dimensional supergravity requires that the B-field not be invariant under gauge and gravitational interactions, and in fact it must satisfy an identity of the form

$$dH_3 = X_4\,, \tag{2.16}$$

where $H_3 \equiv dB_2 - \omega_{\mathrm{CS}}$ is the gauge-invariant curvature of the B-field built from the Chern-Simons forms of gauge and spin connections that appears in the (super)gravity couplings. $X_4 = d\omega_{\mathrm{CS}}$ is a linear combination of characteristic classes of gauge and gravitational bundles. There is more to the Bianchi identity at the global level, a subject which we will discuss in section 3.1.

In this case, the anomaly corresponding to a factorized anomaly polynomial (2.15) can be cancelled by introducing the following term in the action:

$$-\int B_2 \wedge X_8\,. \tag{2.17}$$

This term in the action is actually generated by string perturbation theory, as shown in [62] in the heterotic case, and it contributes a term $-dB_2 \wedge X_8$ to the Lagrangian density of the anomaly theory. Using the Bianchi identity and (2.5), we see that this adds a term $-X_4 X_8$ to

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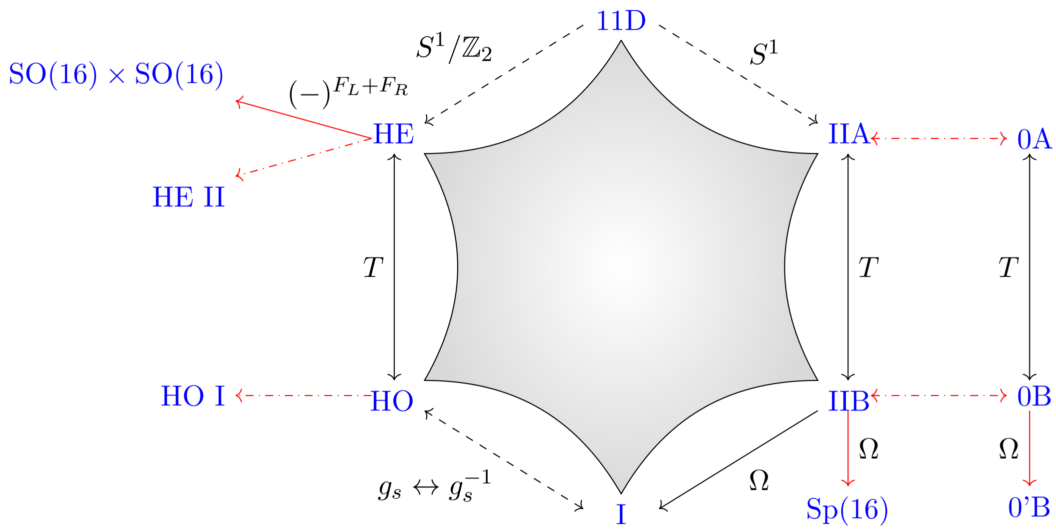

Figure 1. A diagram showing how the three tachyon-free non-supersymmetric string theories relate to the supersymmetric ones and M-theory, via various worldsheet orbifolds. For instance, the $\mathrm{SO}(16)^2$ is obtained from the E_8^2 heterotic by orbifolding spacetime and gauge group fermion number, $F = F_L + F_R$. We also list some tachyonic examples via red dot-dashed arrows, although we are not exhaustive.

the anomaly polynomial, exactly canceling the term in (2.15) so that the total local anomaly vanishes. This is the basic principle of the Green-Schwarz mechanism where the anomaly is cancelled by introducing an extra term in the action.

The main focus of this paper is however the three known[5] ten-dimensional non-tachyonic, non-supersymmetric string theories, which also feature a B-field and a Green-Schwarz mechanism to cancel local anomalies. The diagram in figure 1 sums up how these non-supersymmetric theories are related to eachother and to the supersymmetric ones via gaugings of various worldsheet symmetries. We will now briefly describe the matter content of these theories as well as their anomalies.

- *The Sugimoto model*

  The Sugimoto string [4] can be thought of as the non-supersymmetric sibling of the supersymmetric type I $\mathrm{Spin}(32)/\mathbb{Z}_2$ string. The main departure for us is that the gauge algebra is $\mathfrak{sp}(16)$ instead, and thus the Chern classes are taken in the fundamental representation of this group.[6] This distinction arises from the different kind of orientifold projection of type IIB, which introduces anti-D9 branes and an O9 plane with positive Ramond-Ramond charge *and* tension. The sign change in the reflection coefficients for unoriented strings scattering off the O9 is such that the Chan-Paton degeneracies reconstruct representations of the symplectic group Sp(16).

  As in the type I case, the closed-string sector arranges into an $\mathcal{N} = 1$ supergravity multiplet, while the chiral fermions from the open-string sector arrange into the anti-

[5]It was recently proven that there are no other examples in the heterotic context [63].
[6]Note that we use $\mathfrak{sp}(16)$ and Sp(16) instead of the notation $USp(32)$ that is often employed in the literature.

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symmetric rank-two representation of the gauge group, leading to the same anomaly polynomial formally. This representation is however reducible and contains a singlet; this is nothing but the Goldstino that accompanies the breaking of supersymmetry [64–67]. The low-energy interactions comply with the expected Volkov-Akulov structure of nonlinear supersymmetry [68–70], although there is no tunable parameter that recovers a linear realization. All in all, since the anomaly polynomial is formally identical to the type I case, it factorizes as follows:

$$P_{12}^{\text{Sugimoto}} = -\frac{p_1 + c_{\mathbf{32},2}}{2} \times \frac{1}{192}\left(16\, c_{\mathbf{32},2}^2 - 32\, c_{\mathbf{32},4} + 4\, c_{\mathbf{32},2}\, p_1 + 3 p_1^2 - 4 p_2\right). \tag{2.18}$$

- *The Sagnotti model*

  The type 0'B string [5, 6] of Sagnotti is built from an orientifold projection of the tachyonic type 0B string, where the unique tachyon-free choice involves an O9 plane with zero tension. The resulting gauge group is U(32), Ramond-Ramond p-form potentials with $p = 0\,, 2\,, 4$ (the latter having a self-dual curvature) survive the projection and they get anomalous Bianchi identities for the gauge-invariant curvatures,

$$\begin{aligned} dH_1 &= X_2\,, \\ dH_3 &= X_4\,, \\ dH_5 &= X_6 \end{aligned} \tag{2.19}$$

  where $X_2 = c_{\mathbf{32},1}$, X_4 is formally identical to the one in Sugimoto and type I strings, and X_6 is a polynomial in p_1, $c_{\mathbf{32},1}$, $c_{\mathbf{32},2}$ and $c_{\mathbf{32},3}$. The Bianchi identity for X_2 tells us that the low-energy gauge group reduces to SU(32), since $c_{\mathbf{32},1}$ is set to zero. Physically, the anomalous Bianchi identity for the RR axion induces the kinetic term $|dC_0 + A|^2$, with A the gauge field of the diagonal $\mathfrak{u}(1)$. This is just the Stückelberg mass term for A. All these RR fields with anomalous Bianchi identities play a crucial role in the cancellation of local anomalies via a more complicated Green-Schwarz mechanism involving a decomposition of the anomaly polynomial [6] of the form

$$P_{12}^{0\text{'B}} = X_2\, X_{10} + X_4\, X_8 + X_6\, X_6\,. \tag{2.20}$$

  As we will see, setting X_6 to zero implies that $c_{\mathbf{32},3}$ is also trivial, and so we shall impose this condition when studying global anomalies of this theory.

- *The heterotic model*

  The case of the $\mathrm{SO}(16) \times \mathrm{SO}(16)$ string [2, 3] is slightly different. Along with its two supersymmetric counterparts, it is the unique ten-dimensional heterotic model that is devoid of tachyons. It is built from a projection of either of the two heterotic models, most directly the $E_8 \times E_8$ one under the projector built from a combination of spacetime fermion number and an E_8 lattice symmetry. As a result, it does not have any chiral fields that are uncharged under the gauge symmetry, and in particular it does not have

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a gravitino. Its anomaly polynomial was derived in [2, 3] and factorizes as:

$$\begin{aligned} P_{12}^{\mathrm{SO}(16)^2} = & -\frac{p_1 + c^{(1)}_{\mathbf{16},2} + c^{(2)}_{\mathbf{16},2}}{2} \\ & \times \frac{1}{24}\left((c^{(1)}_{\mathbf{16},2})^2 + (c^{(2)}_{\mathbf{16},2})^2 + c^{(1)}_{\mathbf{16},2}\, c^{(2)}_{\mathbf{16},2} - 4\, c^{(1)}_{\mathbf{16},4} - 4\, c^{(2)}_{\mathbf{16},4} \right) , \end{aligned} \tag{2.21}$$

where the (1) and (2) indices differentiate between each of the two SO(16) gauge groups. The Green-Schwarz mechanism is carried by the Kalb-Ramond field, which survives the projection as befits a heterotic model [71].

All in all, local anomalies vanish for all three non-supersymmetric string theories, by the Green-Schwarz mechanism (or a more complicated version of it). This was already known in the literature, but leaves open the possibility for the presence of global anomalies. Global anomalies are those that arise in gauge/diffeomorphism transformations that cannot be continuously connected to the identity. These anomalies are not detected by the anomaly polynomial. In the following section we detail how one can study these anomalies and we evaluate them for the case of the three non-supersymmetric tachyon-free string theories.

3 Global anomalies and bordism groups

In the previous section, we have summarized prior results in the literature regarding anomaly cancellation of ten-dimensional non-supersymmetric string theories via the Green-Schwarz mechanism. Importantly, the Green-Schwarz mechanism only guarantees cancellation of *local* anomalies — it guarantees that the (super)gravity path integral is gauge invariant as long as we only consider gauge transformations infinitesimally close to the identity. More generally, one also need discuss *global* anomalies, namely anomalies in gauge transformations that cannot be continuously deformed to the identity. The archetypal example of such a global anomaly is Witten's SU(2) anomaly [72]. If one includes topology-changing transitions, one has even more general anomalies (dubbed Dai-Freed anomalies in [73]), involving a combination of gauge transformations and spacetime topology change. In this paper, we will take the point of view that such anomalies should cancel in a consistent quantum theory of gravity, where spacetime topology is supposed to fluctuate.

The framework of anomaly theories introduced briefly in the previous section (2.5) can also be used to study global anomalies of Lagrangian theories such as the ones we are interested in. Given a d-dimensional quantum field theory, an anomaly on a manifold X_d (possibly decorated with gauge field, spin structure, etc.) means that the partition function $Z(X_d)$ is not invariant under gauge transformations (or diffeomorphisms, for the case of a gravitational anomaly). In a modern understanding (see [73–77] for detailed reviews, and also [78, 79] for a discussion in the context of the 6d Green-Schwarz mechanism), the anomaly can be captured by an invertible $(d+1)$-dimensional field theory α with the property that, when evaluated on a manifold with boundary Y_{d+1} with $\partial Y_{d+1} = X_d$, the product

$$Z(X_d) \cdot e^{-2\pi i \alpha(Y_{d+1})} \tag{3.1}$$

is invariant under gauge transformations. The d-dimensional QFT arises as a boundary mode of the $(d+1)$-dimensional invertible field theory α, and the anomaly is re-encoded

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in the fact that (3.1) is not the partition function of a d-dimensional quantum field theory — its value depends in general on Y_{d+1} and the particular way on which the fields on X_d are extended to Y_{d+1}.

In general, it may be very difficult to determine α. However, in weakly coupled Lagrangian theories, we have a prescription to associate an anomaly theory to each of the chiral degrees of freedom involved. For instance, the anomaly theory for a Weyl fermion in d-dimensions (d even) is given by the so-called eta invariant of a $(d+1)$-dimensional Dirac operator with the same quantum numbers as the fermion we started with [75, 80],

$$e^{2\pi i\alpha_{\text{fermion}}(Y_{d+1})} = e^{2\pi i\eta_{d+1}(Y_{d+1})}. \tag{3.2}$$

If one has several fermions, the total anomaly theory is simply the product of these (so that the η invariants add up). There are other topological couplings that can also contribute to the anomaly theory, as we will see below.

Two different open manifolds Y_{d+1} and Y'_{d+1}, both having X_d as a boundary, will yield values for the partition function (3.1) differing by a factor

$$e^{2\pi i\alpha(Y_{d+1})}e^{-2\pi i\alpha(Y'_{d+1})} = e^{2\pi i\alpha(Y_{d+1}\cup\overline{Y'}_{d+1})}. \tag{3.3}$$

The manifold $Y_{d+1}\cup\overline{Y'}_{d+1}$ is just a general closed $(d+1)$-dimensional manifold. In an anomaly free-theory, the partition function in (3.1) should not depend on the choice of extension; therefore, in this picture, anomaly cancellation is simply the statement that the anomaly theory $\alpha(\tilde{Y}_{d+1})$ be trivial when evaluated on a closed manifold $\tilde{Y}_{d+1}$.

The particular case in which $\tilde{Y}_{d+1}$ itself is a boundary, $\tilde{Y}_{d+1} = \partial Z_{d+2}$, corresponds to local anomalies, which allows us to connect the discussion to the preceding section. The η invariants introduced above, that give the anomalies for chiral fermions, can in this case be evaluated by means of the APS index theorem [81],

$$\eta(\tilde{Y}_{d+1}) = \text{Index} - \int_{Z_{d+2}} P_{d+2}, \tag{3.4}$$

where P_{d+2} is the anomaly polynomial of the previous subsection, and the "Index" is an integer. We thus recover the usual, perturbative, anomaly cancellation condition in terms of the anomaly theory. In theories where anomalies are cancelled via the Green-Schwarz mechanism, another ingredient is necessary. The ten-dimensional action has an extra Green-Schwarz term (2.17), which is the boundary mode of an 11d invertible field theory

$$\alpha_{\text{GS}}(Y_{11}) = \int_{Y_{11}} H \wedge X_8. \tag{3.5}$$

The total anomaly theory is therefore the sum of the fermion anomaly and $\alpha_{\text{GS}}(Y_{11})$. On a manifold which is itself a boundary, $\tilde{Y}_{11} = \partial Z_{12}$,

$$\int_{\tilde{Y}_{11}} H \wedge X_8 = \int_{Z_{12}} dH \wedge X_8 = \int_{Z_{12}} X_4 \wedge X_8, \tag{3.6}$$

where in the last equality we used the constraint that we are restricting to twisted string manifolds satisfying the (anomalous) Bianchi identity $dH = X_4$. Taking this last contribution into account, we see that the local anomaly coming from the GS term can cancel that of the fermions, provided that the anomaly polynomial factorizes as discussed in section 2.

In the rest of this paper, we will assume that local anomalies cancel, and ask what is the value of the total anomaly theory,

$$e^{2\pi i\alpha_{\text{tot}}} = e^{2\pi i\alpha_{\text{fermions}}}e^{2\pi i\alpha_{\text{GS}}} \tag{3.7}$$

when evaluated on 11-dimensional closed manifolds which are *not* boundaries. This task seems daunting at first, since, depending on the collection of background fields, there can be infinitely many such manifolds. Fortunately, one can prove[7] [75] that the partition function of the anomaly theory α_{tot} (mod 1) is a *bordism invariant*,[8]

$$e^{2\pi i\alpha_{\text{tot}}(Y_{11}^{(1)})} = e^{2\pi i\alpha_{\text{tot}}(Y_{11}^{(2)})} \quad \text{if} \quad Y_{11}^{(1)} \cup \overline{Y_{11}^{(2)}} = \partial Z_{d+2}. \tag{3.8}$$

This reduces the problem significantly: since α_{tot} (mod 1) is a bordism invariant, one need only evaluate it on a single representative per bordism class. Furthermore, these classes form an abelian group, the bordism group (of manifolds suitably decorated with a twisted string structure and gauge bundle). Bordism groups have appeared prominently in the field theory and quantum gravity literature, and there are many techniques available for their computation (see [51] for a detailed introduction). Thus, to compute these anomalies, we will just compute the relevant bordism groups and evaluate the anomaly theory on generators. Notice that if it happens that the relevant bordism group Ω_{11} is 0, there are no global anomalies to check! That this happens was in fact shown by Witten in [37, 38] for the $E_8 \times E_8$ string when one does not take into account the $\mathbb{Z}_2$ symmetry switching the two E_8 gauge fields.[9] See [39] for an analysis of type I string.

More recently, [42, 45] used the Stolz-Teichner conjecture to analyze global anomalies in supersymmetric, heterotic string theory even in stringy backgrounds, lacking a geometric description. In this paper we content ourselves with the target space treatment described above, which may miss anomalies of non-geometric backgrounds. In the following, we present the calculation and results for the three ten-dimensional non-supersymmetric string theories described in section 2. But before that, we will describe and justify more carefully the precise structure that will be assumed in our bordism calculations.

3.1 Bianchi identities and twisted string structures

As described in the previous subsection, the computation of global anomalies can be organized in terms of a bordism calculation and an anomaly theory, which is just a homomorphism from the bordism group to U(1). The precise bordism group to be used (i.e. the particular

[7]The proof is a straightforward application of the APS index theorem (3.4), see [75].

[8]There is also a more theoretical and more general proof that the partition function of the anomaly theory is a bordism invariant, due to Freed-Hopkins-Teleman [82] and Freed-Hopkins [83]; they show that up to a deformation, which is irrelevant for anomaly calculations, the partition function of *any* reflection-positive invertible field theory is a bordism invariant.

[9]If one does want to take this $\mathbb{Z}_2$ symmetry into account, for example to study the CHL string, the relevant Ω_{11} is nonzero [46, Theorem 2.62], and it was not known whether the global anomaly cancels. We will show that it does cancel in this paper, in section 4.

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structure that our manifolds are required to have) depends on the theory we are interested in. For instance, all heterotic string theories under consideration include fermions, so we will consider only manifolds (and bordism between them) carrying a spin structure; the anomaly theory is related to the η invariant for a certain Dirac operator on this manifold. This means that the second Stiefel-Whitney class of the allowed manifolds where the anomaly theory is to be evaluated will vanish,

$$w_2 = 0. \tag{3.9}$$

In heterotic string theories, also the Bianchi identity (2.16) needs to be taken into account. Equation (3.6) illustrates that cancellation of perturbative anomalies requires us to assume $dH = X_4$ even off-shell.[10] Therefore, we will restrict our bordism groups to consist of 11-dimensional manifolds in which (2.16) is satisfied. In particular, we will set

$$\int_{M_4} X_4 = 0 \tag{3.10}$$

for any closed 4-manifold M_4. The precise expression of X_4 in terms of characteristic classes depends on the particular theory under study. The particular case

$$X_4 = \frac{p_1}{2} \tag{3.11}$$

has been studied in the mathematical literature, and receives the name of a *string* structure. The X_4's that appear in heterotic string theories are always of the form

$$X_4 = a\,\frac{p_1}{2}(\text{Tangent bundle}) + b\,c_2(\text{Gauge bundle}), \quad a, b \in \mathbb{Z}, \tag{3.12}$$

and we will refer to the data of a solution to this equation for chosen a and b as a *twisted* string structure. This notion appeared in the mathematical literature in [84, Definition 8.4].

The bordism groups related to the three non-supersymmetric string theories we are going to consider are

$$\Omega_{11}^{\text{String}-Sp(16)}, \quad \Omega_{11}^{\text{String}-\text{SU}(32)\langle c_3\rangle}, \quad \Omega_{11}^{\text{String}-Spin(16)^2}, \tag{3.13}$$

for the Sugimoto, Sagnotti, and SO(16) × SO(16) heterotic theories, respectively; these are the bordism groups of twisted string manifolds where the particular choice of twisted string structure is spelled out by the Green-Schwarz mechanisms for these theories as we discussed in section 2.

Remark 3.14. Before presenting the results for the bordism groups, we must discuss an important subtlety, which affects the bordism calculation. Up to this point in this paper, we have been cavalier when writing down characteristic classes such as "p_1" or "c_2", and defined these characteristic classes as closed differential forms (e.g. in (2.4)) by way of Chern-Weil theory. However, these differential forms have quantized periods, as is the case for data coming out of any quantum theory, and a proper treatment of the Green-Schwarz mechanism should take this into account. There are two ways to do this.

[10] If we insist on keeping the B-field as a background; see the discussion at the end of this section.

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1. The simplest approach is to lift to $\mathbb{Z}$-valued cohomology: the quantized periods are a reminder that the de Rham classes of the Chern-Weil forms of p_1, c_2, etc., lift canonically to classes in $H^4(BG;\mathbb{Z})$ for various Lie groups G, and on many manifolds M, these integer-cohomology lifts of these characteristic classes can be torsion! Thus it is natural to wonder whether the B-field should be an element of $H^3(-;\mathbb{Z})$ and the Bianchi identity (3.12) should take place in $H^4(-;\mathbb{Z})$. The definitions of string structure and twisted string structure in mathematics assume this lift has taken place.

2. Alternatively, one could lift to differential cohomology $\check{H}^4(-;\mathbb{Z})$, which amounts to observing that it is not just the $\mathbb{Z}$-cohomology lift which is natural, but also the data of the Chern-Weil form; differential cohomology is a toolbox for encoding both of these pieces of data. Indeed, for any compact Lie group G and class $c \in H^*(BG;\mathbb{Z})$, there is a canonical differential refinement $\check{c} \in \check{H}^*(B_\nabla G;\mathbb{Z})$ [85, 86], where $B_\nabla G$ is the classifying stack of G-connections.[11] Thus we could instead ask: should we begin with a B-field in $\check{H}^3(-;\mathbb{Z})$ and ask for the Bianchi identity to take place in $\check{H}^4(-;\mathbb{Z})$? This combines the two other formalisms we considered, differential forms and integral cohomology.

The answer in the mathematics literature is often the second option, beginning with Freed [39, section 3] and continuing in, for example, [46, 87–104]. In particular, [94, 95] interpret the data entering into the Green-Schwarz mechanism as specifying a connection for a Lie 2-group built as an extension of the gauge group by $B\mathrm{U}(1)$, providing an appealing physical interpretation of the lift to differential cohomology.

We are interested in classifying anomalies, and while there is an interesting differential refinement of the story of the bordism classification of anomalies due to Yamashita-Yonekura [105–107], the deformation classification of anomalies ultimately can proceed without differential-cohomological information, because it boils down to studying bordism groups. Because of this, we will work with characteristic classes in integral cohomology, noting here that the correct setup of the Green-Schwarz mechanism taking torsion and Chern-Weil forms into account uses differential cohomology, and that for our computations it makes no difference.

Note that cancellation of perturbative anomalies around (3.6) only requires the free part of $X_4 \in H^4(-;\mathbb{Z})$ to be trivial in a compact manifold, and poses no obvious restriction on torsion. Reference [38] studies a particular example suggesting that this should be the case, but does not attempt to make a general argument. To ascertain whether the torsion piece of X_4 must also be trivialized or not, consider the physical origin of the Bianchi identity, which is itself a two-dimensional version of the Green-Schwarz mechanism described above (see e.g. [58, 108]). Consider a worldsheet wrapped on a 2-manifold Σ_2 of the ambient ten-dimensional spacetime manifold M_{10}. The configuration should be invariant under spacetime diffeomorphisms, and gauge transformations, which are manifested as global symmetries of the worldsheet. However, in heterotic or type I theories, the worldvolume degrees of freedom are chiral, and anomalous under these transformations. The anomaly theory, which we denote $\alpha^{\text{worldsheet}}$, is encoded by

[11] If you do not want to think about stacks, this statement is essentially equivalent to the notion that for a principal G-bundle $P \to M$ with connection Θ, the differential characteristic class $\check{c}(P,\Theta) \in \check{H}^*(M;\mathbb{Z})$ is natural in (P,Θ).

a three-dimensional η invariant. Applying the APS index theorem (3.4), we obtain

$$\exp(2\pi i\alpha^{\text{worldsheet}}) = \exp\left(2\pi i \int X_4\right), \tag{3.15}$$

where X_4 is a certain differential form built out of characteristic classes, and which is precisely the X_4 appearing above (indeed, (3.15) is usually taken to give the definition of X_4). As things stand, any configuration with an insertion of a fundamental string worldsheet on Σ_2 has a gravitational anomaly; however, the worldsheet also has an electric coupling to the B-field,

$$\exp\left(2\pi i \int_{\Sigma_2} B_2\right), \tag{3.16}$$

whose anomaly theory is simply

$$\alpha_B = \int H. \tag{3.17}$$

Now, the total worldsheet anomaly is

$$\exp\left(2\pi i\alpha^{\text{worldsheet}}_{\text{total}}\right) = \exp\left(2\pi i\alpha^{\text{worldsheet}}\right)\exp\left(2\pi i\alpha_B\right). \tag{3.18}$$

The physical consistency condition is that the total anomaly is trivial

$$\exp\left(2\pi i\alpha^{\text{worldsheet}}_{\text{total}}\right) = 1, \quad \text{for all } M_3. \tag{3.19}$$

When M_3 is a boundary, anomaly cancellation is achieved, as above, by setting $dH = X_4$, precisely the Bianchi identity described above. However, this is not all there is to (3.19). Assuming that anomalies vanish when M_3 is a boundary, $\exp(2\pi i\alpha^{\text{worldsheet}}_{\text{total}}(M_3))$ is actually only dependent on the integer homology class of M_3. In fact, since it is a map that assigns a phase to each 3-cycle in the ambient 10-dimensional manifold M_{10}, it can be regarded as an element of $H^3(M_{10};\mathrm{U}(1))$, with U(1) coefficients. Using the long exact sequence in cohomology associated to the short exact sequence of groups $\mathbb{Z} \to \mathbb{R} \to \mathrm{U}(1)$, we obtain that [76]

$$H^3(M_{10};\mathbb{R}) \to H^3(M_{10};\mathrm{U}(1)) \to H^4(M_{10};\mathbb{Z}) \to H^4(M_{10};\mathbb{R}), \tag{3.20}$$

where the third map is taking the free part of the integer cohomology class. In general, $\exp(2\pi i\alpha^{\text{worldsheet}}_{\text{total}})$ will have pieces both in the image of the first map and in its cokernel. An example where the anomaly theory has a non-trivial piece in the image of the first map of (3.20) can be obtained by compactifying heterotic string theory on a Bieberbach 3- manifold, a fixed-point free quotient of the torus T^3.[12] Since T^3 is Riemann-flat, a quick analysis would suggest that the Bianchi identity is satisfied automatically with no gauge bundle or B-field turned on. However, trying to implement this manifold directly in the worldsheet results in a theory which is not level-matched. The problem is that $\exp(2\pi i\alpha^{\text{worldsheet}}_{\text{total}})$ with no gauge bundle turned on is nontrivial for most Bieberbach manifolds, and so the anomaly theory is a nontrivial class in $H^3(M_{10};\mathbb{R})$. Cancelling this anomaly forces either a B-field (discrete torsion) to be turned on, or a non-trivial flat gauge bundle to be present.

[12] We thank Cumrun Vafa for pointing out this example to one of us.

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The rest of the anomaly theory is in the image of the second map in (3.20), and can therefore be represented by a certain torsion integer cohomology class in $H^4(M_{10};\mathbb{Z})$, whose free part vanishes. We will now show that this is in fact the torsion part of $X_4 - dH$. Consider a torsion 3-cycle M_3 of order k, i.e. such that kM_3 is the boundary of some 4-manifold N_4. Let us see how to compute the anomaly in this case. First, the anomaly theory $\alpha^{\text{worldsheet}}_{\text{total}}$ is a linear combination of η invariants, which in this particular case can be re-expressed as linear combinations of gravitational and gauge Chern-Simons numbers as discussed above. The Chern-Simons invariant is additive on disconnected sums, and so, we have

$$\alpha^{\text{worldsheet}}_{\text{total}}(M_3) = \frac{1}{k}\alpha^{\text{worldsheet}}_{\text{total}}(kM_3). \tag{3.21}$$

Next, we can use the fact that $kM_3 = \partial N_4$, to write (after exponentiation)[13]

$$\exp\left(2\pi i \alpha^{\text{worldsheet}}_{\text{total}}(M_3)\right) = \exp\left(\frac{2\pi i}{k}\left(\text{Index}_{N_4} + \int_{N_4}(X_4 - dH)\right)\right). \tag{3.22}$$

This expression is not obviously independent of the choice of N_4, but when $(X_4 - dH)$ is pure torsion, it actually is. The reason is that the quantity $\int_{N_4}(X_4 - dH)$ may be rewritten as a linking pairing in homology [109]. If we Poincaré dualize $(X_4 - dH)_{\text{tor}}$ to a torsion 6-cycle M_6, the linking pairing between M_6 and M_3 is constructed by choosing a boundary N_4 for kM_3 and computing $\int_{N_4}(X_4 - dH)$ modulo k. Importantly, the result does not depend on the choice of N_4 (see [109] for a review and proof of these facts).

In short, the full analog of the Bianchi identity is (3.19). Unpacking this condition, we recover that:

- There is the condition on any 3-manifold M_3 that

$$\int_{M_3} H = \int_{M_3} CS_3^{X_4}, \tag{3.23}$$

  where $CS_3^{X_4}$ is a (local) Chern-Simons form obeying $dCS_3^{X_4} = X_4$. This will force discrete B-fields to be turned on in certain situations, such as on Bieberbach manifolds (these were referred to as "worldsheet discrete theta angles" in [79]).

- As a consequence of the previous point, when M_3 is a boundary, we get that the Bianchi identity $dH = X_4$ must hold over the integers.

The general analysis we just carried out is somewhat abstract; in the next subsection, we will verify its correctness by explicitly checking, in a variety of backgrounds, that anomalies in ten dimensions only cancel if the torsional part of the Bianchi identity holds.

Finally, we comment on another possible way in which the anomaly calculation could have been set, avoiding the calculation of string bordism groups altogether, as in [79]. Anomalies are always studied with respect to a choice of background fields. The approach we have followed here takes the metric g, the gauge field A, and the 2-form field B as background fields, and imposes the Bianchi identity as a restriction on the allowed backgrounds. However,

[13]The anomaly theory is a linear combination of real-valued eta invariants, thus division by k is well-defined and there is no phase ambiguity.

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in a quantum theory of gravity, there are no global symmetries, and therefore, there are no background fields either. This is manifested in the fact that all three of g, A, B are actually dynamical fields that we are supposed to path-integrate over. Treating these as backgrounds is justified if there is some sort of weak coupling limit in which the fields become frozen. This is automatically the case at low energies in any ten-dimensional string theory, since the couplings of all of g, A, B are dimensionful and become irrelevant in the deep IR. It is not the case e.g. in six dimensions, where antisymmetric tensor fields are often strongly coupled and cannot be treated perturbatively. In such cases, the only approach available is to explicitly perform the path integral over the tensor fields, compute their effective action, and verify that the resulting path integral indeed cancels against the contributions of other chiral fields. There is no meaningful analog of the notion of having a string structure, since no weak coupling notion is available. The anomaly theory (as a function of the metric and background gauge fields) can then studied on general spin manifolds (and not just string manifolds), and anomalies cancel in a standard way, because the B-field (which is integrated over) couples to background 4 and 8-forms X_4 and X_8, and has a mixed anomaly captured by the anomaly polynomial $\int X_4 X_8$, just what is needed to cancel the anomaly of the fermions. From a perturbative string worldsheet point of view, we feel it is more natural to keep B as a background field; furthermore, the techniques we use in this paper can be extended to compute lower-dimensional string bordism groups, which control solitonic objects in these non-supersymmetric theories via the Cobordism Conjecture [48].

3.2 Evidence for torsional Bianchi identities

In the previous subsection, we gave an argument that the Bianchi identity holds at the level of torsion, too. The argument relies heavily on string perturbation theory, and one may worry e.g. that it does not capture strongly coupled situations. In this section, we provide independent evidence, which does not rely on the worldsheet at all, that the Bianchi identity holds at the level of integer cohomology. We do so by computing Dai-Freed anomalies of supersymmetric and non-supersymmetric string theories on simple eleven-dimensional lens spaces. Lens spaces are quotients of spheres by $\mathbb{Z}_p$ groups; they are the simplest examples of manifolds whose cohomology is purely torsional (except in bottom and top degrees, as usual). In particular, their first Pontryagin classes are torsion; the upshot of the calculation in this section is that spacetime anomalies on lens spaces seem to vanish if and only if the Bianchi identities are satisfied at the level of integral cohomology, including torsional classes.

Now we turn to the details of evaluating anomalies on lens spaces; we refer the reader to [40, 51] for more on lens spaces and the corresponding expressions for eta invariants. (Eleven-dimensional) lens spaces are defined to be quotients of the form

$$L_p^{11} = S^{11}/\mathbb{Z}_p, \tag{3.24}$$

where the $\mathbb{Z}_p$ action acts as scalar multiplication by $e^{2\pi i/p}$ on the six complex coordinates $\mathbb{C}^6$ and where we embed the covering S^{11} as the unit sphere. An important property of these lens spaces is that the Green-Schwarz term,

$$H \wedge X_8, \tag{3.25}$$

will automatically vanish on lens spaces, since $H^3(S^{11}/\mathbb{Z}_n;\mathbb{Z}) = 0$. As a result, the calculation of the full anomalies of string theories on lens spaces reduces to determining the anomalies of the chiral fields. We will now evaluate the anomaly theory of the Type I and the Heterotic string theories (supersymmetric and non-supersymmetric) on certain eleven-dimensional lens spaces.

3.2.1 Type I and HO heterotic theories

As the Green-Schwarz (GS) contribution to the anomaly theory vanishes on lens spaces, the remaining fermion anomaly theory of the type I and HO heterotic theory is given by

$$\alpha(L_p^{11}) = \eta_0^{\mathrm{RS}}(L_p^{11}) - 3\,\eta_0^{\mathrm{D}}(L_p^{11}) + \eta_{\mathbf{adj}}^{\mathrm{D}}(L_p^{11})\,. \tag{3.26}$$

The Rarita-Schwinger eta invariant η^{RS} arises from the anomaly theory of a ten-dimensional gravitino according to $\alpha_{\text{gravitino}} = \eta^{\mathrm{RS}} - 2\eta^{\mathrm{D}}$ [40]. In order to evaluate this anomaly theory on L_p^{11}, one can derive the branching rules for the adjoint representation of Spin(32) in terms of the charge-q irreducible $\mathbb{Z}_p$ representations $\mathcal{L}^q$. This branching depends on how $\mathbb{Z}_p$ is included in the gauge group. We choose a family of inclusions of the form

$$\mathbb{Z}_p \quad \hookrightarrow \quad \mathrm{U}(1) \quad \overset{k}{\hookrightarrow} \quad \mathrm{SU}(N) \quad \hookrightarrow \quad \mathrm{Spin}(2N)\,, \tag{3.27}$$

according to which the (complexified) vector representation of $\mathrm{Spin}(2N)$ splits as

$$\mathbf{2N} \quad \longrightarrow \quad \mathbf{N} \oplus \mathbf{N}^*\,. \tag{3.28}$$

The parameter k denotes an inclusion that places the U(1) fundamental representation $\mathcal{L}$ in k diagonal blocks, in pairs $L \equiv \mathcal{L} \oplus \mathcal{L}^{-1}$, and the rest in the trivial representation $\mathcal{L}^0$. Then, the vector representation of $\mathrm{Spin}(2N)$ further splits into

$$\mathbf{2N} \quad \longrightarrow \quad \mathbf{N} \oplus \mathbf{N}^* \quad \longrightarrow \quad [kL \oplus (N-2k)\mathcal{L}^0] \oplus [kL \oplus (N-2k)\mathcal{L}^0]\,. \tag{3.29}$$

In order to find the branching rules for other representations, it is convenient to use Chern characters. Letting $x \equiv c_1(\mathcal{L})$, the Chern character of $\mathbf{N}$ (and $\mathbf{N}^*$) decomposes into

$$\mathrm{ch}(\mathbf{N}) \quad \longrightarrow \quad k\left(e^x + e^{-x}\right) + (N-2k)\,. \tag{3.30}$$

Then we can build the characters for adjoint, symmetric and antisymmetric $\mathrm{SU}(N)$ representations, from which we can reconstruct the characters for $\mathrm{Spin}(2N)$ representations of interest, such as the adjoint (antisymmetric) and spinorial. The resulting branching rules involve the representations $L^q \equiv \mathcal{L}^q \oplus \mathcal{L}^{-q}$. In particular, the adjoint of $\mathrm{Spin}(2N)$ branches according to

$$\mathbf{adj} \longrightarrow k(2k-1)L^2 \oplus 4k(N-2k)L \oplus [N(2N-1) - 2k(2k-1) - 8k(N-2k)]\,\mathcal{L}^0\,, \tag{3.31}$$

which gives the corresponding eta invariant. Using the expressions

$$\begin{aligned} \eta_q^{\mathrm{D}}(L_p^{11}) &= \frac{2p^6 + 21p^4 + 168p^2 - 191 - 42p^4q^2 + 210p^2q^4 - 630p^2q^2}{60480p} \\ &\quad + \frac{-252pq^5 + 1260pq^3 - 1008pq + 84q^6 - 630q^4 + 1008q^2}{60480p}\,, \\ \eta_0^{\mathrm{RS}}(L_p^{11}) &= \frac{22p^6 - 273p^4 - 3192p^2 + 3443}{60480p}\,, \end{aligned} \tag{3.32}$$

the anomaly simplifies to

$$\alpha^{(k)}_{\mathrm{Spin}(32)}(L^{11}_p) = \frac{(p^2-1)(p^4+(11-5k)p^2+10(k-3)^2)}{60p}\,. \tag{3.33}$$

In order to compare the cases in which $\alpha = 0$ mod 1 to the Bianchi identity, let us recall that the total Pontryagin class of L^{2k-1}_p is $p(L^{2k-1}_p) = (1+y)^k$ with y a generator of $H^4(L^{2k-1}_p;\mathbb{Z}) \cong \mathbb{Z}_p$. Thus $p_1(L^{11}_p) = 6y$, and one can show that the canonical choice of $\frac{p_1}{2}$ afforded by the spin structure is $\frac{p_1}{2} = 3y$. On the other hand, according to the branching rule (3.29), the total Chern class of the associated vector bundle is $c = (1-y)^{2k}$, and thus $c_2 = -2k\,y$. Therefore,

$$\frac{p_1+c_2}{2} = (3-k)\,y \tag{3.34}$$

vanishes if and only if $k = 3$ mod p. Plugging in $k = 3 + mp$ with m integer, the anomaly does vanish (mod 1), and it does not vanish otherwise.

3.2.2 $E_8 \times E_8$ theory

The calculation for the $E_8 \times E_8$ theory is almost the same as in the preceding case. The anomaly theory has the same form of (3.26), the only difference being the branching of the adjoint representation $\mathbf{adj} = (\mathbf{248},\mathbf{1}) \oplus (\mathbf{1},\mathbf{248})$. We employ the same construction as before, embedding $\mathbb{Z}_p$ into the Spin(16) subgroup of E_8. The general construction is thus specified by a pair (k_1,k_2) pertaining to the two E_8 factors. One then has to compute the branching for the **120** and the spinorial **128** of Spin(16) which compose the adjoint representation of E_8. The former has been presented in the preceding section, now with $N = 8$, while the latter can be constructed computing Chern characters of antisymmetric representations of SU(8) whose direct sum gives the branching of the spinorial representation:

$$\mathbf{128}_+ \oplus \mathbf{128}_- \quad\longrightarrow\quad \bigoplus_{m=0}^{8} \binom{\mathbf{8}}{\mathbf{m}}\,. \tag{3.35}$$

The Chern character for the various antisymmetric representations can be found by expanding the graded Chern character for the exterior algebra $\Lambda(V) = \oplus_n \Lambda^n(V)$

$$\mathrm{ch}(\Lambda(V)) \equiv \sum_n t^n\,\mathrm{ch}(\Lambda^n(V))\,, \tag{3.36}$$

which can be computed exploiting the property $\Lambda(U \oplus V) \simeq \Lambda(U) \otimes \Lambda(V)$ and that, for line bundles $\mathcal{L}$,

$$\mathrm{ch}(\Lambda(\mathcal{L})) = 1 + t\,e^{c_1(\mathcal{L})}\,. \tag{3.37}$$

Thus, (3.30) gives

$$\mathrm{ch}(\Lambda(\mathbf{N})) \quad\longrightarrow\quad (1+t\,e^{x})^k\,(1+t\,e^{-x})^k\,(1+t)^{N-2k}\,. \tag{3.38}$$

For instance for $N = 8$ and $k = 1$, summing the even or odd rank characters leads to

$$\mathrm{ch}(\mathbf{128}) \quad\longrightarrow\quad 64 + 32\,(e^{x}+e^{-x})\,, \tag{3.39}$$

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which means that the spinorial representations branch according to $\mathbf{128} \to 32L \oplus 64\mathcal{L}^0$. Analogously, $\mathbf{120} \to L^2 \oplus 24L \oplus 70\mathcal{L}^0$, so that all in all

$$\mathbf{248} \quad \longrightarrow \quad L^2 \oplus 56L \oplus 134\mathcal{L}^0 \,. \tag{3.40}$$

The anomaly for this particular choice $(k_1, k_2) = (1, 0)$ then simplifies to

$$\alpha^{(1,0)}_{E_8\times E_8}(L^{11}_p) = \frac{p^6 + 5p^4 + 34p^2 - 40}{60p} \tag{3.41}$$

which vanishes (mod 1) for $p = 2$. Let us now look at the Bianchi identity. The Chern class of the adjoint $E_8 \times E_8$ associated bundle is

$$c = (1 - 4y)\,(1 - y)^{56}\,, \tag{3.42}$$

with y a generator of $H^4(L^{11}_p;\mathbb{Z})$, and thus $c_2 = -60y$. For $E_8 \times E_8$ we have to divide $\frac{c_2}{2}$ by 30 in the Bianchi identity, thus getting

$$\frac{p_1}{2} + \frac{c_2}{60} = 2\,y\,. \tag{3.43}$$

This class only vanishes if $p = 2$, which is the same value for which the anomaly vanishes! Similarly, for $(k_1, k_2) = (1, 1)$ the Bianchi class is y, which never vanishes (except for the trivial case $p = 1$), and accordingly the anomaly never vanishes either.

One can carry on with more complicated embeddings computing the spinorial branching of $\mathbf{128}$: for $(k_1, k_2) = (2, 1)$ the Bianchi class vanishes, and indeed the anomaly turns out to always vanish mod 1. At first glance, the case $(k_1, k_2) = (2, 0)$ appears to present an exception, since the Bianchi class is $y \neq 0$ but the anomaly vanishes for $p = 5$. However, in order to find the relationship between torsional Bianchi identities and anomalies, for given torsion the anomaly should vanish for all allowed backgrounds, and the $(1, 1)$ embedding has the same Bianchi class but nonvanishing anomaly for $p = 5$.

The general expression for any (k_1, k_2) for the $E_8 \times E_8$ theory is more involved due to how the spinorial representations branch, but the procedure to compute the anomaly is systematic.

3.2.3 Non-supersymmetric theories

Let us now address the non-supersymmetric cases. An immediate consequence of the above result for the supersymmetric heterotic theories is that the anomaly on lens spaces satisfying the torsional Bianchi identity also vanishes for the non-SUSY heterotic theory, since its chiral matter content is in the virtual difference of the corresponding representations [71]. This fact will turn out to be useful when discussing fivebrane anomaly inflow in section 3.4.2.

For the Sagnotti model, the anomaly theory can be written as [5, 6]

$$\begin{aligned}\alpha_{0'\mathrm{B}}(L^{11}_p) &= \alpha_{\text{self-dual}}(L^{11}_p) - \eta^{\mathrm{D}}_{\mathbf{antisym}}(L^{11}_p) \\ &= -\alpha^{\mathrm{RS}}_0(L^{11}_p) + 3\,\eta^{\mathrm{D}}_0(L^{11}_p) - \eta^{\mathrm{D}}_{\mathbf{antisym}}(L^{11}_p)\end{aligned} \tag{3.44}$$

since it contains a four-form RR field with self-dual curvature, similarly to type IIB. Following the same procedure as before, now with the simpler inclusion $\mathbb{Z}_p \hookrightarrow \mathrm{U}(1) \hookrightarrow \mathrm{SU}(32)$, one can evaluate the fermionic anomalies; for the self-dual field, in the second line of eq. (3.44)

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we have used anomaly cancellation in type IIB supergravity to recast its anomaly theory in terms of fermionic contributions, along the lines of [40]. Thus we obtain

$$\alpha^{(k)}_{0'\mathrm{B}}(L^{11}_p) = -\frac{(p^2-1)(5k^2-5k(p^2+12)+2(p^4+11p^2+90))}{120p}\,. \tag{3.45}$$

The Chern class of the associated fundamental bundle is now $c = (1-y)^k$, so that $c_2 = -k\,y$ and the Bianchi class

$$\frac{p_1+c_2}{2} = \left(3-\frac{k}{2}\right)y \tag{3.46}$$

vanishes for $k = 6 \bmod p$. Notice that $c_3 = 0$ as well for these bundles, since the total Chern class only contains powers of $y \in H^4(L^{11}_p, \mathbb{Z})$. Substituting $k = 6 + mp$ for integer m, the anomaly vanishes as expected, but not otherwise.

The calculation for the Sugimoto model is essentially identical: the anomaly theory is simply $\alpha_{\mathrm{Sugimoto}} = -\alpha_{0'\mathrm{B}}$, since the antisymmetric fermion has now positive chirality and the gravitino and dilatino contribute the opposite of the self-dual tensor. The inclusion we employ is $\mathbb{Z}_p \hookrightarrow \mathrm{U}(1) \hookrightarrow \mathrm{Sp}(1) \simeq \mathrm{SU}(2) \overset{k}{\hookrightarrow} \mathrm{Sp}(16)$, under which the **32** representation branches according to

$$\mathbf{32} \quad \longrightarrow \quad kL \oplus (32-2k)\mathcal{L}^0\,. \tag{3.47}$$

Since the resulting Bianchi class is also the same, one obtains the same result: the anomalies cancel on lens backgrounds which satisfy the Bianchi identity at the torsional level.

3.3 Vanishing bordism classes

We now turn to the main results of this paper — the calculation of string bordism groups with twisted string structures corresponding to the non-supersymmetric strings, by means of homotopy theory. These sections cover in some detail the mathematical aspects of the calculation; a table summarizing the results can be found in the Conclusions.

3.3.1 Sp(16)

At this point we make our first bordism computation: that every closed, spin 11-manifold M with a principal Sp(16)-bundle P satisfying the Green-Schwarz identity $\frac{1}{2}p_1(M) + c_2(P) = 0$ is the boundary of a compact spin 12-manifold on which the Sp(16)-bundle and Green-Schwarz data extend. This implies that the anomalies we study in this paper vanish for the Sugimoto string.

To make these computations, we use the Adams and Atiyah-Hirzebruch spectral sequences. By now these are standard tools in the mathematical physics literature, so we point the reader to [51, 110] for background and many example computations written for mathematical physicists. The computations in this paper are a little more elaborate: twisted string bordism rather than twisted spin bordism. There are fewer such calculations in the literature, but we found the references [46, 111–114] helpful.

On to business. The data of a G-gauge field and a B-field satisfying a Bianchi identity is expressed mathematically as a principal bundle for a Lie 2-group extension of G by

$BU(1)$. Such extensions are classified by $H^4(BG;\mathbb{Z})$ [115, Corollary 97]. Let String(n)-Sp(16) denote the Lie 2-group which is the extension of $\mathrm{Spin}(n) \times \mathrm{Sp}(16)$ by $BU(1)$ classified by $\frac{1}{2}p_1 + c_2 \in H^4(B(\mathrm{Spin}(n) \times \mathrm{Sp}(16));\mathbb{Z})$, and String-Sp(16) be the colimit as $n \to \infty$ as usual. A string-Sp(16)-structure on a manifold M is data of a spin structure, a Sp(16)-bundle P, and a trivialization of the Green-Schwarz term $\frac{1}{2}p_1(M) + c_2(P)$: exactly what we need for the Sugimoto string.

Though we are primarily interested in showing $\Omega_{11}^{\text{String-Sp}(16)} = 0$, the lower-dimensional bordism groups are barely more work.

Theorem 3.48. *The low-dimensional* String-Sp(16) *bordism groups are:*

$$\begin{aligned}
\Omega_0^{\text{String-Sp}(16)} &\cong \mathbb{Z} & \Omega_6^{\text{String-Sp}(16)} &\cong \mathbb{Z}_2\\
\Omega_1^{\text{String-Sp}(16)} &\cong \mathbb{Z}_2 & \Omega_7^{\text{String-Sp}(16)} &\cong \mathbb{Z}_4\\
\Omega_2^{\text{String-Sp}(16)} &\cong \mathbb{Z}_2 & \Omega_8^{\text{String-Sp}(16)} &\cong \mathbb{Z}^{\oplus 3} \oplus \mathbb{Z}_2\\
\Omega_3^{\text{String-Sp}(16)} &\cong 0 & \Omega_9^{\text{String-Sp}(16)} &\cong (\mathbb{Z}_2)^{\oplus 3}\\
\Omega_4^{\text{String-Sp}(16)} &\cong \mathbb{Z} & \Omega_{10}^{\text{String-Sp}(16)} &\cong (\mathbb{Z}_2)^{\oplus 3}\\
\Omega_5^{\text{String-Sp}(16)} &\cong \mathbb{Z}_2 & \Omega_{11}^{\text{String-Sp}(16)} &\cong 0.
\end{aligned}$$

Proof. Let $V \to B\mathrm{Sp}(16)$ be the vector bundle associated to the defining representation; it is rank 64 as a real vector bundle. Then by an argument analogous to [51, section 10.4], there is an isomorphism $\Omega_*^{\text{String-Sp}(16)} \cong \Omega_*^{\text{String}}((B\mathrm{Sp}(16))^{V-64})$, where $(B\mathrm{Sp}(16))^{V-64}$ is the *Thom spectrum* of the virtual vector bundle $V - \underline{\mathbb{R}}^{64} \to B\mathrm{Sp}(16)$. The Thom spectrum X^V of $V \to X$ is a homotopy-theoretic object whose homotopy groups can be expressed as certain kinds of bordism groups by the Pontrjagin-Thom theorem; the upshot is that string bordism groups of X^V are isomorphic to (X, V)-twisted string bordism groups of a point. See [51, section 10.4] for more information and references.

If *tmf* denotes the spectrum of connective topological modular forms, then it follows that the map $\Omega_*^{\text{String}}(X) \to \mathit{tmf}_*(X)$ is an isomorphism in degrees 15 and below whenever X is a space or connective spectrum [112, Theorem 2.1] (the latter condition includes all Thom spectra we study in this paper). Therefore for the rest of the proof we focus on $\mathit{tmf}_*((B\mathrm{Sp}(16))^{V-64})$. These are finitely generated abelian groups, so we may work one prime at a time (see [51, section 10.2]).

As input, we will need the following calculation of Borel.

Proposition 3.49 (Borel [116, section 29])**.** $H^*(B\mathrm{Sp}(16);\mathbb{Z}) \cong \mathbb{Z}[c_2, c_4, \ldots, c_{32}]$*, where* c_i *is the pullback of the* i^{th} *Chern class under the map* $B\mathrm{Sp}(16) \to B\mathrm{U}(32)$.

For large primes p (i.e. $p \geq 5$), we want to show that $\mathit{tmf}_*((B\mathrm{Sp}(16))^{V-64})$ lacks p-torsion in degrees 11 and below. This follows because when $p \geq 5$, the homotopy groups of the p-localization $\mathit{tmf}_{(p)}$ are free and concentrated in even degrees [117, section 13.1], and the $\mathbb{Z}_{(p)}$ cohomology of $B\mathrm{Sp}(16)$ (hence also of $(B\mathrm{Sp}(16))^{V-64}$, by the Thom isomorphism) is always free and concentrated in even degrees as a consequence of proposition 3.49 and the universal coefficient theorem, so the Atiyah-Hirzebruch spectral sequence computing p-localized $\mathit{tmf}_*((B\mathrm{Sp}(16))^{V-64})$ collapses with only free summands on the E_∞-page, preventing p-torsion in $\mathit{tmf}_*((B\mathrm{Sp}(16))^{V-64})$ in the range we care about.

For $p = 3$, the 3-localized Atiyah-Hirzebruch spectral sequence does not immediately collapse, so we use the Adams spectral sequence (and we will see that this Adams spectral sequence does immediately collapse). The Adams spectral sequence takes the form

$$E_2^{s,t} = \mathrm{Ext}_{\mathcal{A}}^{s,t}(H^*(X;\mathbb{Z}_p),\mathbb{Z}_p) \Longrightarrow \pi_{t-s}^s(X)_p^\wedge. \tag{3.50}$$

Let us explain this notation. We pick a prime p; then $\mathcal{A}$ is the p-primary Steenrod algebra, the $\mathbb{Z}_p$-algebra of all natural transformations $H^*(-;\mathbb{Z}_p) \to H^{*+t}(-;\mathbb{Z}_p)$ that commute with the suspension isomorphism. The mod p cohomology of any space or spectrum X is thus naturally a $\mathbb{Z}$-graded $\mathcal{A}$-module, so we may apply $\mathrm{Ext}_{\mathcal{A}}$, the derived functor of $\mathrm{Hom}_{\mathcal{A}}$. This gives us two gradings: the original $\mathbb{Z}$-grading on cohomology is the one labeled t, and the grading arising from the derived functors is the one labeled s. On the right-hand side of (3.50), π_*^s denotes stable homotopy groups, and $(-)_p^\wedge$ denotes p-completion. We will not need to worry in too much detail about p-completion: we will only ever p-complete finitely generated abelian groups A, for which the p-completion carries the same information as the free summands and the p-power torsion summands of A. Thus we will typically be implicit about p-completion — in particular, $\mathbb{Z}_p$ always denotes the cyclic group of order p, not the p-adic integers.

We are interested in *tmf*-homology (or really string bordism), rather than stable homotopy, which means replacing X with $tmf \wedge X$ in (3.50); then the Adams spectral sequence converges to $tmf_{t-s}(X)_p^\wedge$.

By work of Henriques and Hill (see [111, 117]), building on work of Behrens [118] and unpublished work of Hopkins-Mahowald, there is a change-of-rings theorem for the 3-primary Adams spectral sequence for *tmf* simplifying (3.50) to

$$E_2^{s,t} = \mathrm{Ext}_{\mathcal{A}^{tmf}}^{s,t}(H^*(X;\mathbb{Z}_3),\mathbb{Z}_3) \Longrightarrow tmf_*(X)_3^\wedge. \tag{3.51}$$

Here $\mathcal{A}^{tmf}$ is the graded $\mathbb{Z}_3$-algebra

$$\mathcal{A}^{tmf} = \mathbb{Z}_3\langle \beta, \mathcal{P}^1\rangle/(\beta^2, (\mathcal{P}^1)^3, \beta(\mathcal{P}^1)^2\beta - (\beta\mathcal{P}^1)^2 - (\mathcal{P}^1\beta)^2), \tag{3.52}$$

with $|\beta| = 1$ and $|\mathcal{P}^1| = 4$. For the Adams E_2-page, $\mathcal{A}^{tmf}$ acts on $H^*(X;\mathbb{Z}_3)$ by sending β to the Bockstein for $0 \to \mathbb{Z}_3 \to \mathbb{Z}_9 \to \mathbb{Z}_3 \to 0$ and $\mathcal{P}^1$ to the first mod 3 Steenrod power. See [111–114] for more information and some example computations with this variant of the Adams spectral sequence.

As input, we need to know how β and $\mathcal{P}^1$ act on $H^*((B\mathrm{Sp}(16))^{V-32};\mathbb{Z}_3)$. This is determined in [114, Corollary 2.37] from the input data of the action of the images of β and $\mathcal{P}^1$ on the mod 3 Steenrod algebra on $H^*(B\mathrm{Sp}(16);\mathbb{Z}_3)$. As the cohomology of $B\mathrm{Sp}(16)$ is concentrated in even degrees, β must act trivially, and thus likewise for the Thom spectrum $(B\mathrm{Sp}(16))^{V-64}$. Shay [119] computes the action of $\mathcal{P}^1$ on mod 3 Chern classes;[14] the formula implies that in $H^*(B\mathrm{Sp}(16);\mathbb{Z}_3)$, $\mathcal{P}^1(c_2) = c_4 + c_2^2$ and $\mathcal{P}^1(c_4) = c_4c_2$. For the Thom class, $\mathcal{P}^1(U) = Uc_2$ [114, Theorem 2.28]. Using the Cartan formula, we can compute the $\mathcal{A}^{tmf}$-module structure on $H^*((B\mathrm{Sp}(16))^{V-64};\mathbb{Z}_3)$.

Definition 3.53. If M is a $\mathbb{Z}$-graded module over a $\mathbb{Z}$-graded algebra A, we will let $\Sigma^k M$ denote the same underlying A-module with the grading shifted up by k, i.e. if $x \in M$ is homogeneous of degree m, then $x \in \Sigma^k M$ has degree $m + k$. We will write Σ for Σ^1.

[14] We also found Sugawara's explicit calculations of this formula in [120, section 5] helpful.

The notation Σ^k is inspired by the suspension of a topological space, which has the effect of increasing the degrees of elements in cohomology by 1.

Definition 3.54. Let N_3 denote the nontrivial $\mathcal{A}^{tmf}$-module extension of $C\nu$ by $\Sigma^8\mathbb{Z}_3$, where $C\nu$ is the $\mathcal{A}^{tmf}$-module defined in [114, section 3.2].

Then, there is an $\mathcal{A}^{tmf}$-module isomorphism

$$H^*((B\mathrm{Sp}(16))^{V-64};\mathbb{Z}_3) \cong N_3 \oplus \Sigma^8 N_3 \oplus P, \tag{3.55}$$

where P is concentrated in degrees 12 and above (so we can ignore it). We draw the decomposition (3.55) in figure 3, left.

We need to compute $\mathrm{Ext}_{\mathcal{A}^{tmf}}(N_3, \mathbb{Z}_3)$. To do so, we use the fact that the short exact sequence of $\mathcal{A}^{tmf}$-modules (which we draw in figure 2, top)

$$0 \longrightarrow \Sigma^8\mathbb{Z}_3 \longrightarrow N_3 \longrightarrow C\nu \longrightarrow 0 \tag{3.56}$$

induces a long exact sequence on Ext groups; traditionally one draws the Ext of the first and third terms of a short exact sequence in the same Adams chart, so that the boundary maps have the same degree as a d_1 differential. See Beaudry-Campbell [110, section 4.6, section 5] for more information and some examples for modules over a different algebra $\mathcal{A}(1)$, and [114, figures 2, 3, and 5] for some $\mathcal{A}^{tmf}$-module examples.

We will draw the long exact sequence in Ext corresponding to (3.56) in figure 2. To do so, we need $\mathrm{Ext}_{\mathcal{A}^{tmf}}(\mathbb{Z}_3)$, which is due to Henriques-Hill [111, 117], and $\mathrm{Ext}_{\mathcal{A}^{tmf}}(C\nu)$, which is computed in topological degree 14 and below in [114, figure 2]. Our notation for names of Ext classes follows [114, section 3]; $\mathrm{Ext}_{\mathcal{A}^{tmf}}(\Sigma^8\mathbb{Z}_3)$ is a free $\mathrm{Ext}_{\mathcal{A}^{tmf}}(\mathbb{Z}_3)$-module on a single generator, so call that generator z.[15] Most boundary maps are nonzero for "degree reasons," meaning that their domain or codomain is the zero group. For $t-s \le 14$, there are two exceptions: $\partial(z)$ could be $\pm\alpha y$ or 0, and $\partial(\alpha z)$ could be $\pm\beta x$ or 0. Since the boundary maps commute with the $\mathrm{Ext}_{\mathcal{A}^{tmf}}(\mathbb{Z}_3)$-action and $\alpha(\alpha y) = \beta x$,[16] these two boundary maps are either both zero or both nonzero. To see that they are both nonzero, we use that $\mathrm{Ext}^{0,8}_{\mathcal{A}^{tmf}}(N_3) \cong \mathrm{Hom}_{\mathcal{A}^{tmf}}(N_3, \Sigma^8\mathbb{Z}_3) = 0$, so $z \in \mathrm{Ext}^{0,8}_{\mathcal{A}^{tmf}}(\Sigma^8\mathbb{Z}_3)$ is not the image of an Ext class for N_3, so $\partial(z) \neq 0$.

With this Ext in hand, we can draw the E_2-page of the Adams spectral sequence in figure 3, right. The spectral sequence collapses at E_2 in the range we study for degree reasons. The straight lines denote actions by $h_0 \in \mathrm{Ext}^{1,1}_{\mathcal{A}^{tmf}}(\mathbb{Z}_3, \mathbb{Z}_3)$, which lift to multiplication by 3, so we see there is no 3-torsion in degrees 11 and below.

Lastly, for $p = 2$, we use the Adams spectral sequence again; the outline of the proof is quite similar to the $p = 3$ case, but the details are different. Specifically, we will once again

[15] There are two classes which generate $\mathrm{Ext}_{\mathcal{A}^{tmf}}(\Sigma^8\mathbb{Z}_3)$ as an $\mathrm{Ext}_{\mathcal{A}^{tmf}}(\mathbb{Z}_3)$-module, and one is -1 times the other. For the purposes of this paper, it does not matter which one we call z and which one we call $-z$.

[16] The equation $\alpha(\alpha y) = \beta x$ is stated in [114, Remark 3.21], but not proven there. One way to prove it is to compare with the equivalent α-action $\alpha y \mapsto \beta x$ in $\mathrm{Ext}_{\mathcal{A}^{tmf}}(N_1)$ in the long exact sequence in (*ibid.*, figure 5): because $\partial(\alpha y) = \pm\beta w$ and $\alpha\beta w \neq 0$, and because $\alpha(\partial(-)) = \partial(\alpha\cdot -)$, $\alpha(\alpha y) \neq 0$, hence must be $\pm\beta x$, and we can choose the generator x so that we obtain βx and not $-\beta x$. The calculation of $\partial(\alpha x)$ in (*ibid.*, Lemma 3.24) does not use any information about $\alpha(\alpha y)$.

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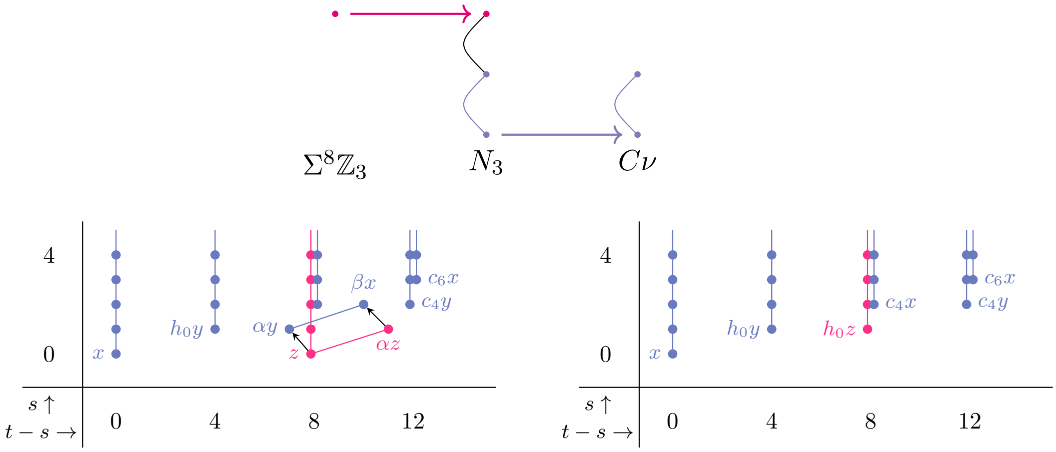

Figure 2. Top: the short exact sequence (3.56) of $\mathcal{A}^{tmf}$-modules. Lower left: the induced long exact sequence in Ext. Lower right: $\mathrm{Ext}_{\mathcal{A}^{tmf}}(N_3)$ as computed by the long exact sequence.

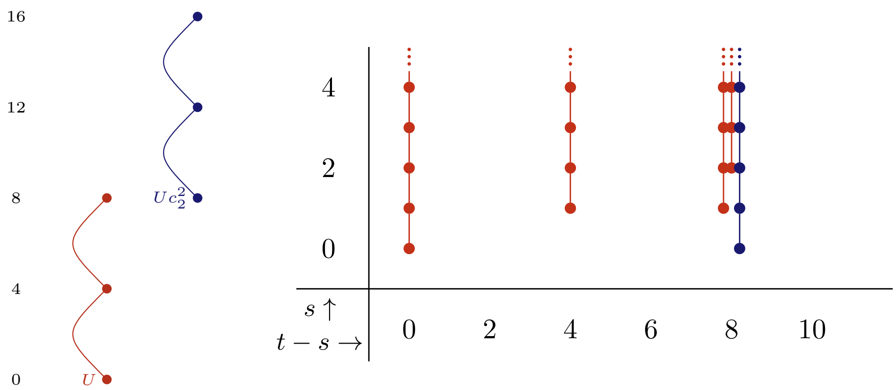

Figure 3. Left: the $\mathcal{A}^{tmf}$-module structure on $H^*((B\mathrm{Sp}(16))^{V-64};\mathbb{Z}_3)$ in low degrees; the pictured submodule contains all elements in degrees 11 and below. Right: the E_2-page of the Adams spectral sequence computing $tmf_*((B\mathrm{Sp}(16))^{V-64})^\wedge_3$.

use the Adams spectral sequence and a standard change-of-rings theorem to simplify the calculation of the E_2-page, but the algebra of cohomology operations is different.

Let $\mathcal{A}(2)$ be the subalgebra of the mod 2 Steenrod algebra generated by Sq^1, Sq^2, and Sq^4. There is an isomorphism $H^*(tmf;\mathbb{Z}_2) \cong \mathcal{A} \otimes_{\mathcal{A}(2)} \mathbb{Z}_2$ [121, 122], which by a standard argument simplifies the E_2-page of the 2-primary Adams spectral sequence to

$$E_2^{s,t} = \mathrm{Ext}^{s,t}_{\mathcal{A}(2)}(H^*(X;\mathbb{Z}_2),\mathbb{Z}_2) \Longrightarrow tmf_*(X)^\wedge_2. \tag{3.57}$$

The next thing to do is to determine how $\mathcal{A}(2)$ acts on $H^*((B\mathrm{Sp}(16))^{V-64};\mathbb{Z}_2)$. Since this cohomology ring vanishes in degrees not divisible by 4, Sq^1 and Sq^2 act trivially. For Sq^4, [110, section 3.3] says $\mathrm{Sq}^4(U) = Uw_4(V) = c_2$, and the Wu formula computes the Steenrod squares in $H^*(B\mathrm{Sp}(16);\mathbb{Z}_2)$, using that the mod 2 reductions of the generators in proposition 3.49 are Stiefel-Whitney classes. This allows us to completely describe the $\mathcal{A}(2)$-action on $H^*((B\mathrm{Sp}(16))^{V-64};\mathbb{Z}_2)$ in the degrees we need: $\mathrm{Sq}^4(U) = Uc_2$, $\mathrm{Sq}^4(Uc_2^2) = Uc_2^3$,

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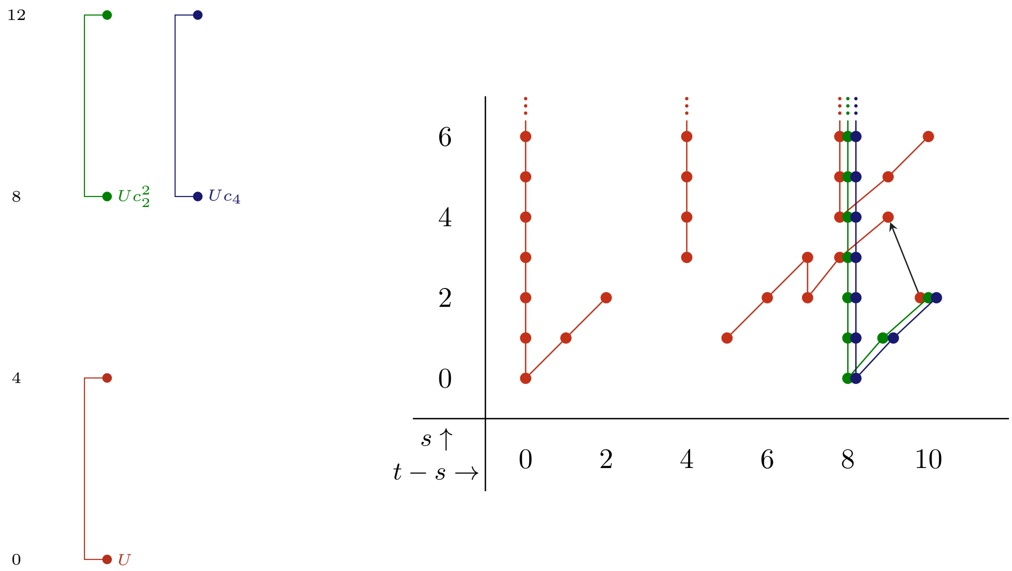

Figure 4. Left: the $\mathcal{A}(2)$-module structure on $H^*((B\mathrm{Sp}(16))^{V-64};\mathbb{Z}_2)$ in low degrees; the pictured submodule contains all elements in degrees 11 and below. Right: the E_2-page of the Adams spectral sequence computing $tmf_*((B\mathrm{Sp}(16))^{V-64})^\wedge_2$.

$\mathrm{Sq}^4(Uc_4) = Uc_6$, and all other actions by Sq^1, Sq^2, or Sq^4 starting in degree 11 or below vanish. Thus, if M_4 denotes the $\mathcal{A}(2)$-module consisting of two $\mathbb{Z}_2$ summands in degrees 0 and 4 connected by a Sq^4, there is an isomorphism

$$H^*((B\mathrm{Sp}(16))^{V-64};\mathbb{Z}_2) \cong M_4 \oplus \Sigma^8 M_4 \oplus \Sigma^8 M_4 \oplus P, \tag{3.58}$$

where P contains no elements in degrees 11 or below, and hence will be irrelevant to our calculations. We draw (3.58) in figure 4, left.

Bruner-Rognes [113, section 4.4] compute $\mathrm{Ext}_{\mathcal{A}(2)}(M_4)$; using their result, we give the E_2-page of the Adams spectral sequence computing $tmf_*((B\mathrm{Sp}(16))^{V-64})^\wedge_2$ in figure 4, right.

Looking at the E_2-page, most differentials are ruled out by degree considerations or the fact that they must commute with the action of h_0 or h_1. The only options left are d_2 and d_3 out of $E_r^{0,8}$ and $d_2\colon E_2^{2,12} \to E_2^{4,13}$.

Lemma 3.59. *All classes in $E_2^{0,8} \cong (\mathbb{Z}_2)^{\oplus 2}$ survive to the E_∞-page.*

Proof. Classes $x \in E_2^{0,\bullet}$ of an Adams spectral sequence for G-bordism correspond naturally to (a subset of) $\mathbb{Z}_2$-valued characteristic classes c_x for manifolds with G-structure, and x survives to the E_∞-page if and only if there is a closed manifold M with G-structure such that $\int_M c_x = 1$; see [41, section 8.4].

For the Adams spectral sequence for string-Sp(16) bordism at $p = 2$, the two classes corresponding to a basis of $E_2^{0,8}$ are the mod 2 reductions of c_2^2 and c_4. To finish this lemma, we will find closed string-Sp(16) 8-manifolds on which these classes do not vanish.

- The quaternionic projective plane $\mathbb{HP}^2$ has a tautological principal Sp(1)-bundle $P := S^{11} \to \mathbb{HP}^2$; let $P^\vee \to \mathbb{HP}^2$ be the same space with the quaternion-conjugate Sp(1)-action, and let $Q \to \mathbb{HP}^2$ be the principal Sp(16)-bundle induced from $P^\vee$ by the

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inclusion $i\colon \mathrm{Sp}(1) \to \mathrm{Sp}(16)$. Using the fact that i pulls c_2 back to c_2 and Borel and Hirzebruch's calculation of the characteristic classes of $\mathbb{HP}^2$ [123, section 15.5, section 15.6] (see also [41, section 5.2] for a good review), the reader can verify that $(\mathbb{HP}^2, Q)$ has a unique string-Sp(16) structure, meaning in particular that $c_2(Q) = -\frac{1}{2}p_1(\mathbb{HP}^2)$, and that $\int_{\mathbb{HP}^2} c_2(Q)^2 = 1$.

- For c_4, take S^8 with principal Sp(16)-bundle $P \to S^8$ classified by either generator of

$$[S^8, B\mathrm{Sp}(16)] = \pi_8(B\mathrm{Sp}(16)) \xrightarrow{\cong} \pi_8(BSp) = \pi_0(B\mathrm{Sp}) = \mathbb{Z}, \tag{3.60}$$

using Bott periodicity. Since $H^4(S^8;\mathbb{Z}) = 0$, $c_2(P)$ and $\frac{1}{2}p_1(S^8)$ vanish and therefore (S^8, P) is string-Sp(16); and $\int_{S^8} c_4(P) = 1$ essentially by definition. ☐

The differential out of $E_2^{2,12}$ does not vanish — to see this, consider the map

$$f\colon \mathbb{HP}^1 \longrightarrow \mathbb{HP}^\infty \simeq B\mathrm{Sp}(1) \longrightarrow B\mathrm{Sp}(16) \tag{3.61a}$$

and the induced map on Thom spectra

$$f_*\colon \mathit{tmf}_*((\mathbb{HP}^1)^{f^*V-64}) \longrightarrow \mathit{tmf}_*((B\mathrm{Sp}(16))^{V-64}). \tag{3.61b}$$

The map f_* induces a pullback map on mod 2 cohomology and on Adams spectral sequences; the map on mod 2 cohomology is the quotient by all elements of degree greater than 4, so the effect on Adams spectral sequences is to kill all summands in Ext except for the red summands. As $H^*((\mathbb{HP}^1)^{f^*V-64};\mathbb{Z}_2)$ consists of two $\mathbb{Z}_2$ summands in degrees 0 and 4, joined by a Sq^4, the Adams spectral calculating its 2-completed *tmf*-homology is worked out by Bruner-Rognes [113, Theorem 8.1], who show that $d_2\colon E_2^{2,12} \to E_2^{4,13}$ is an isomorphism. Thus this differential persists to the Sp(16) Adams spectral sequence. ☐

3.3.2 U(32)

Now we discuss the Sagnotti string, whose gauge group is U(32). The Green-Schwarz mechanism for this theory involves three classes in degrees 2, 4, and 6 canceling c_1, c_2, and c_3 of the gauge bundle, respectively.

We may impose the degree-2, 4, and 6 conditions on $B\mathrm{U}(32)$ in any order. Starting with c_1, we obtain $B\mathrm{SU}(32)$; then, let $B\mathrm{SU}(32)\langle c_3\rangle$ denote the fiber of the map

$$c_3\colon B\mathrm{SU}(32) \longrightarrow K(\mathbb{Z}, 6). \tag{3.62}$$

A map $X \to B\mathrm{SU}(32)\langle c_3\rangle$ is equivalent data to a rank-32 complex vector bundle $V \to X$ with SU-structure and a trivialization of $c_3(V)$. There is a tautological such vector bundle $V_t \to B\mathrm{SU}(32)\langle c_3\rangle$, which is the pullback of the tautological bundle over $B\mathrm{SU}(32)$.

Finally, the degree-4 condition for a U(32)-bundle V over a manifold M asks for a trivialization of $\frac{1}{2}p_1(M)+c_2(V)$. Thus, we ask for a $(B\mathrm{SU}(32)\langle c_3\rangle, V_t)$-twisted string structure on M, i.e. a map $f\colon M \to B\mathrm{SU}(32)\langle c_3\rangle$ and a string structure on $TM \oplus f^*V_t$; the Whitney sum formula for $\frac{1}{2}p_1$ unwinds this into the usual Green-Schwarz condition. We will be a little casual with the notation and call a $(B\mathrm{SU}(32)\langle c_3\rangle, V_t)$-structure a String-SU(32)$\langle c_3\rangle$-structure, even though we do not construct a Lie 2-group String-SU(32)$\langle c_3\rangle$ realizing this twisted string structure (and indeed, there is no guarantee one exists).

Theorem 3.63. *$\Omega_{11}^{\text{String-SU}(32)\langle c_3\rangle}$ is isomorphic to either 0 or $\mathbb{Z}_2$.*

The ambiguity is in a differential we were not able to resolve. Unfortunately, this means we were not able to use bordism-theoretic methods alone to calculate the anomaly of the Sagnotti string. Our proof also yields partial information on lower-dimensional bordism groups; there is ambiguity due to Adams spectral sequence differentials, some of which we suspect are nonzero.

Proof. Before we start our analysis, we need to understand $H^*(B\mathrm{SU}(3)\langle c_3\rangle; A)$ for various coefficient rings A. As $B\mathrm{SU}(32)\langle c_3\rangle$ is the fiber of $c_3\colon B\mathrm{SU}(32) \to K(\mathbb{Z}, 6)$, the fiber of $B\mathrm{SU}(32)\langle c_3\rangle \to B\mathrm{SU}(32)$ is $\Omega K(\mathbb{Z}, 6) \simeq K(\mathbb{Z}, 5)$; moreover, this fibration pulls back from the universal fibration with fiber $K(\mathbb{Z}, 5)$, namely the loop-space-path-space fibration for $K(\mathbb{Z}, 6)$:

$$\begin{tikzcd} K(\mathbb{Z},5) \arrow[r, "="] \arrow[dr] & K(\mathbb{Z},5) \arrow[drr] & & \\ & B\mathrm{SU}(32)\langle c_3\rangle \arrow[rr] \arrow[d] & & * \arrow[d] \\ & B\mathrm{SU}(32) \arrow[rr, "c_3"] & & K(\mathbb{Z},6). \end{tikzcd} \tag{3.64}$$

We will compute $H^*(B\mathrm{SU}(32)\langle c_3\rangle; A)$ for various A using the Serre spectral sequence, along with some information gained from the map of Serre spectral sequences induced by (3.64).

As for the Sugimoto string, we work one prime at a time.

Lemma 3.65. *For $p \geq 5$, there is no p-torsion in $H^*(B\mathrm{SU}(32)\langle c_3\rangle; \mathbb{Z})$ in degrees 12 and below, and all free summands are concentrated in even degrees.*

Proof. It suffices to work with cohomology valued in the ring $\mathbb{Z}[1/6]$ of rational numbers whose denominators in lowest terms are of the form $2^m 3^n$, as tensoring with $\mathbb{Z}[1/6]$ preserves all p-power torsion for $p \geq 5$.

Cartan [124] and Serre [125] computed $H^*(K(\mathbb{Z}, n); \mathbb{Z}[1/6])$; their formulas imply that when $p \geq 5$, $H^k(K(\mathbb{Z}, 5); \mathbb{Z}[1/6])$ is torsion-free for $k \leq 12$, and vanishes apart from $H^5(K(\mathbb{Z}, 5); \mathbb{Z}[1/6]) \cong \mathbb{Z}[1/6]$.

Now consider the Serre spectral sequence for the fibration on the left in (3.64) using cohomology with $\mathbb{Z}[1/6]$ coefficients. The map of fibrations (3.64) induces a map of Serre spectral sequences, and this map is an isomorphism on $E_2^{0,\bullet}$. Since this map commutes with differentials, this means the fate of all classes in $E_2^{0,\bullet}$ is determined by their preimages in the spectral sequence for $K(\mathbb{Z}, 5) \to * \to K(\mathbb{Z}, 6)$. For example, we know thanks to Serre [126, section 10] that in that spectral sequence, E transgresses to the mod 2 reduction of the tautological class F of $K(\mathbb{Z}, 6)$. Therefore in the spectral sequence for $B\mathrm{SU}(32)\langle c_2\rangle$, E transgresses to the pullback of F, which is c_3. The Leibniz rule then tells us $d_6(xE) = xc_3$ for $x \in H^*(B\mathrm{SU}(32); \mathbb{Z}[1/6])$; since this cohomology ring is polynomial, $xc_3 \neq 0$ as long as $x \neq 0$, so these differentials never vanish. Therefore the nonzero part of the E_∞-page, at least in total degree 12 and below, is a quotient of $E_2^{*,0} = H^*(B\mathrm{SU}(32); \mathbb{Z}[1/6])$. Since $H^*(B\mathrm{SU}(32); \mathbb{Z}[1/6])$ is free and concentrated in even degrees. This implies the E_∞-page is also free and concentrated in even degrees in total degree 12 and below, which implies the lemma statement. ☐

Corollary 3.66. *For $p \geq 5$, $\Omega_k^{\text{String-SU}(32)\langle c_3\rangle}$ lacks p-torsion for $k \leq 11$.*

Proof. We want to compute $\Omega_*^{\text{String}}((B\text{SU}(32)\langle c_3\rangle)^{V_t-64})$, and as noted above, we may replace Ω_*^{String} with *tmf* for the degrees k in the corollary statement. Because of lemma 3.65 and the fact that $tmf_{(p)}$ has homotopy groups concentrated in even degrees and lacks p-torsion, the Atiyah-Hirzebruch spectral sequence computing $tmf_*((B\text{SU}(32)\langle c_3\rangle)^{V_t-64})_p^\wedge$ collapses with no p-torsion in the range 11 and below. ☐

As usual, $p = 2$ and $p = 3$ are harder.

Lemma 3.67.

2. *$H^*(B\text{SU}(32)\langle c_3\rangle;\mathbb{Z}_2) \cong \mathbb{Z}_2[c_2, G, c_4, H, J, K, c_6, L, \dots]/(\dots)$, with $|c_i| = 2i$, $|G| = 7$, $|H| = 8$, $|J| = 10$, $|K| = 11$, and $|L| = 12$; all missing generators and relations are in degrees 13 and above. In addition, we have the following Steenrod squares:*
   - *Sq^1 vanishes on the named generators except $\text{Sq}^1(G) = H + \lambda_1 c_4$ and $\text{Sq}^1(K) = L + \lambda_2 c_2 c_4 + \lambda_3 c_6$ for some $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{Z}_2$.*
   - *Sq^2 vanishes on the named generators except for $\text{Sq}^2(H) = J$ and possibly on c_6, K, and L.*
   - *$\text{Sq}^4(c_2) = c_2^2$, $\text{Sq}^4(c_4) = c_2c_4 + c_6$, $\text{Sq}^4(G) = K$, and $\text{Sq}^4(H) = L$.*
3. *$H^*(B\text{SU}(32)\langle c_3\rangle;\mathbb{Z}_3) \cong \mathbb{Z}_3[c_2, c_4, J, c_6, \dots]/(\dots)$ with $|c_i| = 2i$ and $|J| = 10$, and with all missing generators and relations in degrees 13 and above; c_i denotes the pullback of the mod 3 reduction of the i^{th} Chern class along $B\text{SU}(32)\langle c_3\rangle \to B\text{SU}(32)$, and $\mathcal{P}^1(c_2) = c_2^2 + c_4$ and $\mathcal{P}^1(c_4) = c_2c_4$.*

Proof. This is a standard argument with the Serre spectral sequence for the fibration on the left in (3.64), so we sketch the details.

For the mod 2 cohomology, we need as input $H^*(B\text{SU}(32);\mathbb{Z}_2) \cong \mathbb{Z}_2[c_2, c_3, \dots, c_{32}]$ with $|c_i| = 2i$ [116, section 29]: these are the mod 2 reductions of the Chern classes. We also need $H^*(K(\mathbb{Z}, 5);\mathbb{Z}_2)$, which was computed by Serre [126, section 10]. This is a polynomial ring on infinitely many generators; the six in degrees below 13 are $E \in H^5$, the mod 2 reduction of the tautological class; $G := \text{Sq}^2(E)$; $H := \text{Sq}^1(H)$; $I := \text{Sq}^4(E)$; $K := \text{Sq}^4(G)$; and $L := \text{Sq}^5(G)$.

In the Serre spectral sequence, the class E transgresses to c_3, and the proof is the same as in the proof of lemma 3.65. Similarly, we divine the fate of the other classes on the line $p = 0$:

- In the Serre spectral sequence for the rightmost fibration in (3.64), G transgresses via d_8 to $\text{Sq}^2(F)$ by the Kudo transgression theorem [127], so in the leftmost fibration, $d_r(G) = 0$ for $r \leq 7$, and $d_8(G) = \text{Sq}^2(c_3) = 0$. Thus G is a permanent cycle.
- In a similar way, H transgressing to $\text{Sq}^3(F)$ via d_9 pulls back to imply $d_9(H) = \text{Sq}^3(c_3) = 0$, so H is also a permanent cycle. Likewise, E^2 is a permanent cycle, because in the fibration over $K(\mathbb{Z}, 6)$, it supports the transgressing $d_{11}(E^2) = \text{Sq}^5(F)$, and $\text{Sq}^5(c_3) = 0$, and similarly K and L are permanent cycles.
- $I = \text{Sq}^4(E)$ transgresses to $\text{Sq}^4(c_3) = c_5 + c_2c_3$.

The Leibniz rule then cleans up the rest of the spectral sequence in degrees 12 and below. This gives us the ring structure. For the Steenrod squares, we use the information of the $\mathcal{A}$-action on $H^*(B\mathrm{SU}(32);\mathbb{Z}_2)$ coming from the Wu formula, together with the $\mathcal{A}$-actions we gave when describing $H^*(K(\mathbb{Z},5);\mathbb{Z}_2)$ above. There is ambiguity in the Steenrod squares in $H^*(B\mathrm{SU}(32)\langle c_3\rangle;\mathbb{Z}_2)$ coming from the loss of information passing to the associated graded on the E_∞-page, which is the source of λ_1, λ_2, and λ_3 in the theorem statement. However, by pulling back to the analogous fibration over $B\mathrm{SU}(2)$, where the fiber bundle admits a section (as c_3 of an SU(2)-bundle is canonically trivial), so we can use the Künneth formula to compute Steenrod squares. Pulling back to SU(2) loses all information about c_i for $i > 2$, so this leaves ambiguity in c_4 and c_6 as described in the theorem statement, but resolves the ambiguity involving c_2. Some ambiguity can be erased by redefining generators, which is how we disambiguate $\mathrm{Sq}^4(H) = L$, but this still leaves the choices listed in the theorem statement.

For $\mathbb{Z}_3$ cohomology, we begin with $H^*(K(\mathbb{Z},5);\mathbb{Z}_3) \cong \mathbb{Z}_3[E, \mathcal{P}^1(E), \beta\mathcal{P}^1(E), \mathcal{P}^2(E), \dots]$, with the remaining generators in degrees 15 and above, where $E \in H^5(K(\mathbb{Z},5);\mathbb{Z}_3)$ is the mod 3 reduction of the tautological class. Just like for $\mathbb{Z}_2$ cohomology, E transgresses via d_5 to c_3; then the Kudo transgression theorem [127] tells us

- $\mathcal{P}^1(E)$ transgresses via d_{10} to $\mathcal{P}^1(c_3) = c_2c_3 - c_5 = -c_5$ by the E_{10}-page (as $d_5(c_2E) = c_2c_3$), and
- $\beta\mathcal{P}^1(E)$ is a permanent cycle (it transgresses to $\beta(-c_5) = 0$, which we know for degree reasons).

We obtain $\mathcal{P}^1(c_i)$ from Sugawara's calculations [120, section 5] of Shay's formula [119]. With the fate of these classes known, the Leibniz rule cleans up the rest of the spectral sequence in total degrees 12 and below to obtain the theorem statement. □

Now, just as in the proof of theorem 3.48, we run the Adams spectral sequences at $p = 3$ and $p = 2$. The twist by V_t twists the action of $\mathcal{P}^1$ at $p = 3$, and the action of Sq^4 at $p = 2$, in an analogous way. Reusing names of $\mathcal{A}^{tmf}$-modules from section 3.3.1, we conclude that there is an $\mathcal{A}^{tmf}$-module isomorphism

$$H^*((B\mathrm{SU}(32)\langle c_3\rangle)^{V_t-64};\mathbb{Z}_3) \cong N_3 \oplus \Sigma^8 N_3 \oplus \Sigma^{10} N_3 \oplus P, \tag{3.68}$$

where P is concentrated in degrees 12 and above, so will not affect us. We draw (3.68) in figure 5, left. We calculated $\mathrm{Ext}_{\mathcal{A}^{tmf}}(N_3)$ in figure 2; using this, we discover that, like for the Sugimoto string, in degrees 11 and below, the E_2-page consists only of h_0-towers in degrees 0, 4, and 8, so there can be no 3-torsion. See figure 5, right, for a picture of this Adams spectral sequence.

Last, $p = 2$. The ambiguity in the Steenrod actions is not severe enough to get in the way of the existence of an isomorphism of $\mathcal{A}(2)$-modules

$$H^*((B\mathrm{SU}(32)\langle c_3\rangle)^{V_t-64};\mathbb{Z}_2) \cong M_4 \oplus \Sigma^7 N_1 \oplus \Sigma^8 M_4 \oplus \Sigma^8 M_4 \oplus \Sigma^{11} N_2 \oplus P \tag{3.69}$$

where P is concentrated in degrees 12 and above, so will be irrelevant for us, and:

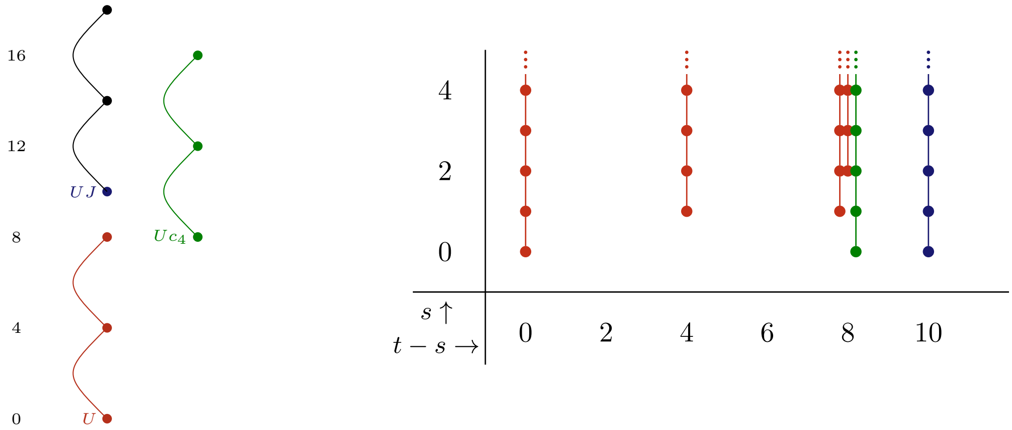

Figure 5. Left: the $\mathcal{A}^{tmf}$-module structure on $H^*((B\mathrm{SU}(32)\langle c_3\rangle)^{V_t-64};\mathbb{Z}_3)$ in low degrees; the pictured submodule contains all elements in degrees 11 and below. Right: the E_2-page of the Adams spectral sequence computing $tmf_*((B\mathrm{SU}(32)\langle c_3\rangle)^{V_t-64})^\wedge_3$. This figure is part of the proof of theorem 3.63.

- N_1 is isomorphic to $\mathcal{A}(2)\otimes_{\mathcal{A}(1)} Q$ in degrees 6 and below (i.e. the quotients of these two $\mathcal{A}(2)$-modules by their submodules of elements in degrees 7 and above are isomorphic); and
- N_2 is isomorphic to $\mathcal{A}(2)\otimes_{\mathcal{A}(1)} Q$ in degrees 3 and below.

Here Q is the "question mark," the $\mathcal{A}(1)$-module which has a $\mathbb{Z}_2$-vector space basis $\{x_0,x_1,x_3\}$ with $|x_i|=i$, and with $\mathrm{Sq}^1(x_0)=x_1$, $\mathrm{Sq}^2(x_1)=x_3$ (all other $\mathcal{A}(1)$-actions are trivial for degree reasons). The module $\Sigma^{11}N_2$ is generated by Uc_2G. We draw the decomposition (3.69) in figure 6, left.

Bruner-Rognes [113, section 4.44] calculate $\mathrm{Ext}_{\mathcal{A}(2)}(M_4)$. For N_1 and N_2, we use that an isomorphism of $\mathcal{A}(2)$-modules in degrees k and below implies the existence of an isomorphism of Ext groups in topological degrees $k-1$ and below, so it is good enough to know $\mathrm{Ext}_{\mathcal{A}(2)}(\mathcal{A}(2)\otimes_{\mathcal{A}(1)} Q)$; then the change-of-rings theorem (see, e.g., [110, section 4.5]) implies

$$\mathrm{Ext}_{\mathcal{A}(2)}(\mathcal{A}(2)\otimes_{\mathcal{A}(1)} Q)\cong \mathrm{Ext}_{\mathcal{A}(1)}(Q), \tag{3.70}$$

and Adams-Priddy [128, table 3.11] compute $\mathrm{Ext}_{\mathcal{A}(1)}(Q)$. Putting all this together, we can draw the E_2-page in figure 6, right. In the range relevant to us, the E_2-page is generated as an $\mathrm{Ext}_{\mathcal{A}(2)}(\mathbb{Z}_2)$-module by the following ten summands.

1. Coming from $\mathrm{Ext}(M_4)$: $a_1\in\mathrm{Ext}^{0,0}$, $a_2\in\mathrm{Ext}^{3,7}$, $a_3\in\mathrm{Ext}^{1,6}$, $a_4\in\mathrm{Ext}^{2,9}$, and $a_5\in\mathrm{Ext}^{2,12}$.
2. Coming from $\mathrm{Ext}(\Sigma^7 N_1)$: $b_1\in\mathrm{Ext}^{7,0}$ and $b_2\in\mathrm{Ext}^{2,12}$.
3. Coming from $\mathrm{Ext}(\Sigma^8 M_4)$: $c\in\mathrm{Ext}^{0,8}$.
4. Coming from $\mathrm{Ext}(\Sigma^8 M_4)$: $d\in\mathrm{Ext}^{0,8}$.
5. Coming from $\mathrm{Ext}(\Sigma^{11} N_2)$: $e\in\mathrm{Ext}^{0,11}$.

These are subject to various relations: notably, if x is any one of these generators, $h_2x=0$.

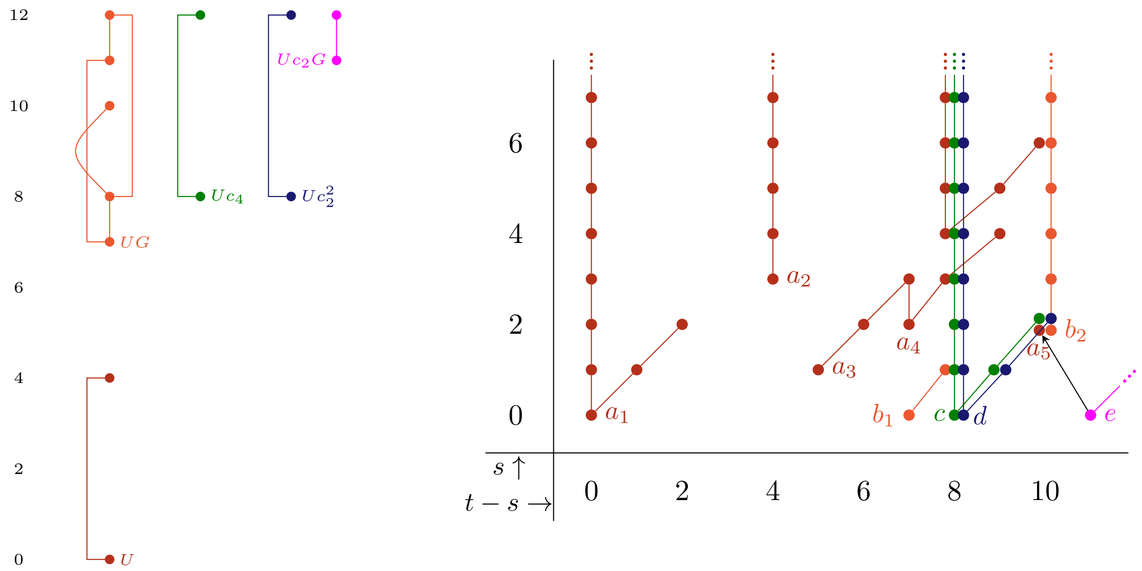

Figure 6. Left: the $\mathcal{A}(2)$-module structure on the quotient of $H^*((B\mathrm{SU}(32)\langle c_3\rangle)^{V_t-64};\mathbb{Z}_2)$ by its submodule of elements in degrees greater than 12; the pictured submodule contains all elements in degrees 11 and below. Right: the E_2-page of the Adams spectral sequence computing $tmf_*((B\mathrm{SU}(32)\langle c_3\rangle)^{V_t-64})^\wedge_2$. This figure is part of the proof of theorem 3.63. We were unable to determine the value of $d_2(e)$: it is either 0 or $h_1^2 d$.

Once we take into account the fact that differentials commute with h_0 and h_1, we still need to determine $d_2(b_1)$, $d_r(c)$, $d_r(d)$, $d_2(a_5)$, $d_2(b_2)$, and $d_r(e)$.

Lemma 3.71. *For all r, $d_r(c) = 0$ and $d_r(d) = 0$; $d_2(a_5) = h_1^2 a_4$.*

The values of these differentials are the same as for the corresponding classes in the Adams spectral sequence for the Sugimoto string, and the proofs are the same as we gave for them in section 3.3.1. ☐

Ultimately we need to address $d_2\colon E_2^{0,11} \to E_2^{2,13}$. If this differential vanishes, there is also potential for $d_6\colon E_6^{0,11} \to E_6^{6,16}$ to be nonzero. The fate of these two differentials determines whether $\Omega_{11}^{\text{String-SU}(32)\langle c_3\rangle}$ is nonzero, so it is unfortunate that the techniques we applied were unable to resolve them.

We are able to obtain some partial information, though.

Lemma 3.72. *If $d_2(e) = 0$, so that $d_6(e)$ is defined, then $d_6(e) = 0$.*

Proof. $d_6(e) \in E_6^{6,16}$. No other nonzero differentials have source or target $E_r^{6,16}$, so $E_6^{6,16} \cong E_2^{6,16} \cong (\mathbb{Z}_2)^{\oplus 2}$, spanned by the classes $w_1 h_1^2 a_1$ and $h_0^4 b_2$. Here $w_1 \in \mathrm{Ext}_{\mathcal{A}(2)}^{4,12}(\mathbb{Z}_2)$ is the class whose image in $\mathrm{Ext}_{\mathcal{A}(1)}(\mathbb{Z}_2)$ is the Bott periodicity class. Thus there are $\lambda_1, \lambda_2 \in \mathbb{Z}_2$ such that

$$d_6(e) = \lambda_1 w_1 h_1^2 a_1 + \lambda_2 h_0^4 b_2. \tag{3.73}$$

Because d_6 commutes with h_0 and $h_0 e = 0$, $\lambda_2 = 0$.

To show $\lambda_1 = 0$, consider the map of Adams spectral sequences induced by the map from the *tmf*-homology to the *ko*-homology of $(B\mathrm{SU}(32)\langle c_3\rangle)^{V_t-32}$. The map on E_2-pages is the map

$$\mathrm{Ext}_{\mathcal{A}(2)}(H^*((B\mathrm{SU}(32)\langle c_3\rangle)^{V_t-32};\mathbb{Z}_2),\mathbb{Z}_2) \longrightarrow \mathrm{Ext}_{\mathcal{A}(1)}(H^*((B\mathrm{SU}(32)\langle c_3\rangle)^{V_t-32};\mathbb{Z}_2),\mathbb{Z}_2) \tag{3.74}$$

induced by the inclusion $\mathcal{A}(1) \to \mathcal{A}(2)$ of algebras. It is possible to compute the right-hand Ext groups using the decomposition (3.69) and the techniques in [110]; one learns that e and $w_1 h_1^2 a_1$ both remain nonzero after (3.74), so it suffices to compute $d_6(e)$ in the *ko*-homology Adams spectral sequence. There, though, the submodule M_{a_1} of the Ext groups generated by a_1 splits off: because $V_t - 32$ is spin, there is a Thom isomorphism $ko_*((B\mathrm{SU}(32)\langle c_3\rangle)^{V_t-32}) \cong ko_*(B\mathrm{SU}(32)\langle c_3\rangle)$, so $ko_*(\mathrm{pt})$ splits off; as this splitting lifts to the level of spectra, it also splits M_{a_1} off of the Adams spectral sequence, so all differentials into M_{a_1} from any other summand vanish. Thus $d_6(e)$ cannot be $h_1^2 w_1 a_1$ in the *ko*-homology Adams spectral sequence, so the same is true in the *tmf*-homology Adams spectral sequence. $\square$

Likewise, since $E_2^{2,12} \cong (\mathbb{Z}_2)^{\oplus 4}$, spanned by the classes a_5, b_2, $h_1^2 c$, and $h_1^2 d$, then there are $\lambda_1, \dots, \lambda_4 \in \mathbb{Z}_2$ such that

$$d_2(e) = \lambda_1 a_5 + \lambda_2 b_2 + \lambda_3 h_1^2 c + \lambda_4 h_1^2 d. \tag{3.75}$$

Lemma 3.76. *In* (3.75), $\lambda_1 = 0$, $\lambda_2 = 0$, *and* $\lambda_3 = 0$.

Proof. Because $h_0 b_2 \neq 0$ but $h_0 h_1^2 = 0$, if $\lambda_2 \neq 0$, then $h_0 d_2(e) \neq 0$. However, since d_2 commutes with h_0-multiplication, and $h_0 e = 0$, λ_2 must vanish and $d_2(e) \in \mathrm{span}(a_5, h_1^2 c, h_1^2 d)$.

By lemma 3.71, $d_2(c) = 0 = d_2(d) = 0$, so $d_2(h_1^2 c) = d_2(h_1^2 d) = 0$, and $d_2(a_5) = h_1^2 a_4$. Therefore $d_2 \colon \mathrm{span}(a_5, h_1^2 c, h_1^2 d) \to E_2^{4,13}$ is nonzero on a class $\mu_1 a_5 + \mu_2 h_1^2 c + \mu_3 h_1^2 d$ if and only if $\mu_1 \neq 0$. Thus $\lambda_1 = 0$: otherwise $d_2(d_2(e)) \neq 0$, and it is always true that $d_2 \circ d_2 = 0$.

For λ_3, consider the map $r \colon B\mathrm{SU}(3)\langle c_3\rangle \to B\mathrm{SU}(32)\langle c_3\rangle$ and the map r_* it induces of Adams spectral sequences. The pullback r^* on cohomology kills c_4 but leaves c_2 and G alone; therefore on Ext groups, $e \in \mathrm{Im}(r_*)$ (because e is the filtration 0 class corresponding to $c_2 G$), $h_1^2 c \in \mathrm{Im}(r_*)$ (because c is the filtration 0 element corresponding to c_2^2), and $h_1^2 d \notin \mathrm{Im}(r_*)$ (because d corresponds to c_4). The map r_* commutes with differentials, so $d_2(e) \in \mathrm{Im}(r_*)$, which is only consistent if $\lambda_3 = 0$. $\square$

Determining whether $\lambda_4 = 0$ appears to be difficult. This would be a good problem to address because if $\lambda_4 \neq 0$, so that $d_2(e) \neq 0$, then the bordism group controlling the anomaly of the Sagnotti string would vanish, and the anomaly would cancel, at least on the class of backgrounds we studied.

Because the class e potentially causing a nonzero bordism group is in Adams filtration 0, the corresponding bordism invariant is the integral of a modulo 2 characteristic class, explicitly

$$\int c_2 G. \tag{3.77}$$

The class $G \in H^7(B\mathrm{SU}(32)\langle c_3\rangle;\mathbb{Z}_2)$ is a little mysterious, so we go into some more detail; it is an example of a *secondary characteristic class* in the sense of Peterson-Stein [129].

Recall that by a trivialization of a cohomology class $z \in H^k(X;A)$, where A is an abelian group, we mean a null-homotopy of a map $f_z\colon X \to K(A,k)$ whose homotopy class represents z. There is a space of such trivializations, and a standard result in obstruction theory implies that its set of path components is a torsor over $H^{k-1}(X;A)$. In other words, given two trivializations of f_z, their difference is well-defined as an element of $H^{k-1}(X;A)$.

The Wu formula implies $\mathrm{Sq}^2(c_3) = 0$ in $H^8(B\mathrm{SU}(32);\mathbb{Z}_2)$, and in fact provides a canonical trivialization for $\mathrm{Sq}^2(c_3)$. Pulling back to $B\mathrm{SU}(32)\langle c_3\rangle$ trivializes c_3, and therefore provides a second trivialization of $\mathrm{Sq}^2(c_3)$. The difference between these two trivializations is the class $G \in H^7(B\mathrm{SU}(32)\langle c_3\rangle;\mathbb{Z}_2)$.

As a final comment, if we knew of a manifold with a non-trivial integral of c_2G, it would by definition be a generator of the bordism group. We could evaluate the anomaly theory on it in order to determine whether or not the anomaly vanishes. Regrettably, we do not know of such a manifold.

3.3.3 Spin(16) × Spin(16)

Next we discuss the symmetry type of the non-supersymmetric heterotic string with gauge Lie algebra $\mathfrak{so}(16)\oplus\mathfrak{so}(16)$. Although usually called $\mathrm{SO}(16)^2$, there are fields transforming in spinor representations in the massless spectrum of the theory which means that we should instead consider $\mathrm{Spin}(16)^2$. There is a further subtlety: according to [53], the gauge group G is the quotient of $\mathrm{Spin}(16)\times\mathrm{Spin}(16)$ by the diagonal $\mathbb{Z}_2$ subgroup $\langle(k,k)\rangle$, where $k \in \mathrm{Spin}(16)$ is either central element not equal to ± 1.[17]

As the computation of $H^*(BG)$ is complicated, we will make a simplifying assumption: only working with the double cover $\mathrm{Spin}(16)\times\mathrm{Spin}(16)$, as we mentioned above. Thus our anomaly cancellation results are only partial information: if we found an anomaly for $\mathrm{Spin}(16)^2$, it would imply the existence of an anomaly for the actual gauge group G. However, we found that anomalies cancel for $\mathrm{Spin}(16)^2$, which is only partial information: there could be an anomaly of the theory which vanishes when restricted to gauge fields induced from a $\mathrm{Spin}(16)^2$ gauge field. It would be interesting to address the more general question of the anomaly for G.[18]

Let String-$\mathrm{Spin}(16)^2$ be the Lie 2-group which is the string cover of $\mathrm{Spin}\times\mathrm{Spin}(16)\times\mathrm{Spin}(16)$ corresponding to the degree-4 cohomology class $\frac{1}{2}p_1^{(1)} - \frac{1}{2}p_1^{(2)} - \frac{1}{2}p_1^{(3)}$, where $c^{(i)}$ refers to the cohomology class c coming from the i^{th} factor of $B\mathrm{Spin}$ or $B\mathrm{Spin}(n)$.[19] Quotienting String-$\mathrm{Spin}(16)^2$ by the $\mathrm{Spin}(16)^2$ factor produces a map to Spin; composing with $\mathrm{Spin}\to O$ we obtain a tangential structure as usual.

[17] Strictly speaking, the analysis of [53] does not take into account the full string spectrum. Therefore, *a priori* the correct gauge group G may differ from this particular quotient of $\mathrm{Spin}(16)^2$.

[18] Like for any double cover, for any odd prime p, the quotient $B\mathrm{Spin}(16)\times B\mathrm{Spin}(16)\to BG$ is a p-primary equivalence, so the lack of p-primary torsion we establish for $\mathrm{Spin}(16)\times\mathrm{Spin}(16)$ remains valid for G.

[19] Elsewhere in the paper we have referred to $\frac{1}{2}p_1^L$ and $\frac{1}{2}p_1^R$ as Chern classes, and indeed they are Chern classes of the representations that play a role in the Green-Schwarz mechanism for this string theory. However, the bordism computation we perform in this section only depends on the characteristic class, not the representation (this is the thesis of [114]), so to emphasize this independence, we use the more intrinsic name $\frac{1}{2}p_1$, as this class is one-half of the first Pontrjagin class of the vector representation of $\mathrm{Spin}(n)$.

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Theorem 3.78. *In degrees* 11 *and below, the* String-Spin$(16)^2$ *bordism groups are:*

$$\begin{aligned}
\Omega_0^{\text{String-Spin}(16)^2} &\cong \mathbb{Z} & \Omega_6^{\text{String-Spin}(16)^2} &\cong 0\\
\Omega_1^{\text{String-Spin}(16)^2} &\cong \mathbb{Z}_2 & \Omega_7^{\text{String-Spin}(16)^2} &\cong 0\\
\Omega_2^{\text{String-Spin}(16)^2} &\cong \mathbb{Z}_2 & \Omega_8^{\text{String-Spin}(16)^2} &\cong \mathbb{Z}^2 \oplus {\color{red}\mathbb{Z}^3} \oplus {\color{blue}\mathbb{Z}}\\
\Omega_3^{\text{String-Spin}(16)^2} &\cong 0 & \Omega_9^{\text{String-Spin}(16)^2} &\cong (\mathbb{Z}_2)^{\oplus 2} \oplus {\color{red}(\mathbb{Z}_2)^{\oplus 2}} \oplus {\color{blue}\mathbb{Z}_2}\\
\Omega_4^{\text{String-Spin}(16)^2} &\cong \mathbb{Z} \oplus {\color{red}\mathbb{Z}} & \Omega_{10}^{\text{String-Spin}(16)^2} &\cong (\mathbb{Z}_2)^{\oplus 3} \oplus {\color{red}(\mathbb{Z}_2)^{\oplus 3}} \oplus {\color{blue}\mathbb{Z}_2}\\
\Omega_5^{\text{String-Spin}(16)^2} &\cong 0 & \Omega_{11}^{\text{String-Spin}(16)^2} &\cong 0.
\end{aligned}$$

The colors in the theorem statement will be explained below; they correspond to different summands in (an approximation to) $MT(\text{String-Spin}(16)^2)$.

Proof. The inclusion $i\colon \text{Spin}(16) \to \text{Spin}$ induces a map $Bi\colon B\text{Spin}(16) \to B\text{Spin}$ which is 15-connected, because it is an isomorphism on cohomology in degrees 15 and below. This map sends $\frac{1}{2}p_1$ to $\frac{1}{2}p_1$, so is compatible with the construction of String-Spin$(16)^2$ — that is, if String-Spin^2 is defined in the same way as String-Spin$(16)^2$ but using Spin instead of Spin(16), then i induces a map of tangential structures

$$i_2\colon B(\text{String-Spin}(16)^2) \to B(\text{String-Spin}^2), \tag{3.79}$$

as well as the analogous map on bordism groups. Because i is 15-connected, i_2 is also 15-connected, so the induced map of Thom spectra is also 15-connected (e.g. check on cohomology, where it follows from 15-connectivity of i_2 via the Thom isomorphism). Therefore for $k \le 15$, the map $\Omega_k^{\text{String-Spin}(16)^2} \to \Omega_k^{\text{String-Spin}^2}$ induced by i is an isomorphism. Therefore for the rest of this proof, we can work only with String-Spin^2 bordism without affecting the results.

Concretely, a string-Spin^2 structure on a vector bundle $E \to X$ is data of a spin structure on E and two virtual spin vector bundles $V^L, V^R \to X$ and a trivialization of $\frac{1}{2}p_1(E) - \frac{1}{2}p_1(V^L) - \frac{1}{2}p_1(V^R)$. Since $\frac{1}{2}p_1$ is additive in direct sums [46, Lemma 1.6], this is equivalent to a trivialization of $\frac{1}{2}p_1(E - V^L - V^R)$, meaning that a string-Spin^2 structure is equivalent to the data of V^L and V^R and a string structure on $W := E - V^L - V^R$.

The data (E, V^L, V^R) and (E, W, V^R) are equivalent, as $V^L = E - W - V^R$, and the spin structure on V^L can be recovered from the spin structures on E, W, and V^R by the two-out-of-three property (the string structure on W includes data of a spin structure). Therefore the data of a string-Spin^2 structure on $E \to X$ is equivalent to the following data:

- a spin structure on E,
- a virtual string vector bundle $W \to X$, and
- a virtual spin vector bundle $V^R \to X$.

Taking bordism groups, we learn

$$\Omega_*^{\text{String-Spin}^2} \xrightarrow{\cong} \Omega_*^{\text{Spin}}(B\text{Spin} \times B\text{String}). \tag{3.80}$$

For any spaces A and B, the stable splitting $\Sigma^\infty_+(A) \simeq \Sigma^\infty A \vee \mathbb{S}$ and its analogue for B together imply a stable splitting

$$\Sigma^\infty_+(A \times B) \simeq \mathbb{S} \vee \Sigma^\infty A \vee \Sigma^\infty B \vee \Sigma^\infty(A \wedge B), \tag{3.81a}$$

implying that for any generalized homology theory h,

$$h_*(A \times B) \cong h_*(\mathrm{pt}) \oplus \widetilde{h}_*(A) \oplus \widetilde{h}_*(B) \oplus \widetilde{h}_*(A \wedge B). \tag{3.81b}$$

Here $\widetilde{h}(X)$ denotes "reduced h-homology" of a space X, meaning the quotient $h(X)/i_*(h(\mathrm{pt}))$ induced by a choice of basepoint $i\colon \mathrm{pt} \to X$. Thus for example $\widetilde{\Omega}_*^{\mathrm{Spin}}(X)$ denotes reduced spin bordism, etc. Apply (3.81b) for $h = \Omega_*^{\mathrm{Spin}}$, $A = B\mathrm{Spin}$, and $B = B\mathrm{String}$:

$$\begin{aligned}\Omega_*^{\text{String-Spin}^2} &\cong \Omega_*^{\mathrm{Spin}}(B\mathrm{Spin} \times B\mathrm{String})\\ &\cong \Omega_*^{\mathrm{Spin}} \oplus \widetilde{\Omega}_*^{\mathrm{Spin}}(B\mathrm{Spin}) \oplus \widetilde{\Omega}_*^{\mathrm{Spin}}(B\mathrm{String}) \oplus \widetilde{\Omega}_*^{\mathrm{Spin}}(B\mathrm{Spin} \wedge B\mathrm{String}).\end{aligned} \tag{3.81c}$$

The colors in (3.81c) indicating the pieces of this direct-sum decomposition correspond to the colors in the theorem statement displaying which pieces of the bordism groups come from which summands in (3.81c).

The final step is to determine the four summands in (3.81c).

- Ω_*^{Spin} was calculated by Milnor [130, section 3] and Anderson-Brown-Peterson [131].
- $\widetilde{\Omega}_*^{\mathrm{Spin}}(B\mathrm{Spin})$ was calculated by Francis [132, section 2.2].
- $\widetilde{\Omega}_*^{\mathrm{Spin}}(B\mathrm{String})$ was computed by Davis [133] at $p = 2$. At odd primes, these groups are easy to calculate in the range we need: because $B\mathrm{String}$ is 7-connected, $\widetilde{\Omega}_k^{\mathrm{Spin}}(B\mathrm{String})$ vanishes for $k < 8$; for $8 \leq k \leq 11$, use the Atiyah-Hirzebruch spectral sequence. Work of Stong [134] and Giambalvo [135] implies that in degrees 11 and below, $\widetilde{H}^*(B\mathrm{String};\mathbb{Z})$ consists of a single summand isomorphic to $\mathbb{Z}$ in degree 8, and the remaining groups vanish. This suffices to collapse the Atiyah-Hirzebruch spectral sequence into the blue groups in the theorem statement.
- For $A = \mathbb{Z}$ or $\mathbb{Z}_2$, $\widetilde{H}^k(B\mathrm{Spin};A)$ vanishes for $k < 4$, and $H^k(B\mathrm{String};A)$ vanishes for $k < 8$, so by the Künneth formula, $\widetilde{H}^k(B\mathrm{Spin} \wedge B\mathrm{String};A)$ vanishes for $k < 12$. Therefore the Atiyah-Hirzebruch spectral sequence for $\widetilde{\Omega}_*^{\mathrm{Spin}}(B\mathrm{Spin} \wedge B\mathrm{String})$ vanishes in degrees 11 and below. $\square$

Remark 3.82 (Analogy with $E_8 \times E_8$). The two-step simplification of String-Spin$(16)^2$ (first replace Spin(16) with Spin, then recast as spin bordism of a space) is directly analogous to Witten's [136, section 4] simplification of the symmetry type of the $E_8 \times E_8$ heterotic string: first, there is a 15-connected map $BE_8 \to K(\mathbb{Z}, 4)$, so in dimensions relevant to string theory we may replace the former with the latter; then Witten recast the data of the two maps to $K(\mathbb{Z}, 4)$ and the twisted string structure given by the Green-Schwarz procedure as a spin structure and a single map to $K(\mathbb{Z}, 4)$.

Remark 3.83 (Analogy with Spin(32) and detecting a non-supersymmetric 0-brane). The same two-step procedure also works for the $Spin(32)/\mathbb{Z}_2$ heterotic string when one restricts to Spin(32)-bundles, showing that the relevant twisted string bordism groups coincide with $\Omega_*^{\mathrm{Spin}}(B\mathrm{String})$, which vanishes in dimension 11. As with $\mathrm{Spin}(16) \times \mathrm{Spin}(16)$, this is only partial information towards a complete anomaly cancellation result.

However, the partial information provided by these bordism groups is already useful: combined with the Cobordism Conjecture [48], it detects Kaidi-Ohmori-Tachikawa-Yonekura's non-supersymmetric 0-brane [50]. To see this, consider $\Omega_8^{\mathrm{Spin}}(B\mathrm{String}) \cong \mathbb{Z}^3$: two of the $\mathbb{Z}$ summands come from $\Omega_*^{\mathrm{Spin}}(\mathrm{pt})$, and as such are generated by $\mathbb{HP}^2$ and the Bott manifold; the third $\mathbb{Z}$ summand is represented by S^8 with the map to $B\mathrm{String}$ given by the generator of $[S^8, B\mathrm{String}] = \pi_8(B\mathrm{String}) \cong \mathbb{Z}$. Tracing through the simplification from twisted string bordism of $B\mathrm{Spin}(32)$ to the spin bordism of $B\mathrm{String}$, we see that this S^8 has the Spin(32)-bundle arising from the generator of $\pi_8(\mathrm{Spin}(32)) \cong \mathbb{Z}$, which is detected by p_2.

The Cobordism Conjecture predicts that associated to this bordism class (or rather its image in the corresponding bordism group for $Spin(32)/\mathbb{Z}_2$), there is a 0-brane in $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic string theory whose link is S^8 with this Spin(32)-bundle and twisted string structure. This is precisely the 0-brane discovered by Kaidi-Ohmori-Tachikawa-Yonekura [50]. Those authors also discuss a 6-brane in the $Spin(32)/\mathbb{Z}_2$ heterotic string, but its description uses $\pi_1(Spin(32)/\mathbb{Z}_2) \cong \mathbb{Z}_2$, so it is invisible to the Spin(32) computation we made here.

3.4 Physical intuition from fivebrane anomaly inflow

As we have just seen, the relevant bordism groups vanish, and therefore there are no Dai-Freed anomalies (except possibly for the Sagnotti string). It is instructive to study the vanishing of anomalies more explicitly in particular examples, to better understand the physics at play. Let us recall from section 3 the structure of the anomaly theory for ten dimensional theories that feature a Green-Schwarz mechanism:

$$\alpha_{\mathrm{GS}}(Y_{11}) = \int_{Y_{11}} H \wedge X_8. \tag{3.84}$$

The boundary mode of this eleven dimensional field theory gives exactly the contribution of the Green-Schwarz term to the classical action:

$$S_{\mathrm{GS}} = \int_{Y_{10}} B_2 \wedge X_8. \tag{3.85}$$

In this section, we consider simple backgrounds of the factorized form $Y_{11} = S^3 \times M_8$ for the anomaly theory (3.84). We will also take one unit of three-form H flux threading the sphere, so that the Green-Schwarz term gives a nontrivial contribution, and M_8 a spin manifold equipped with a gauge bundle E such that $\frac{p_1(M_8)+c_2(E)}{2}$ is trivial in integer cohomology. Unlike more general backgrounds, these factorized ones allows for an intuitive understanding of how anomalies are cancelled, via inflow.

On these backgrounds, the eta invariant contribution to the anomaly theory (coming from the fermions) vanishes on account of the factorization property

$$\eta(A \times B) = \eta(A)\,\mathrm{index}(B), \tag{3.86}$$

where A is odd-dimensional. The eta invariant of fermions on S^3_H vanishes modulo 1, as it is the same as the eta invariant on a three-sphere, which is the boundary of $\mathbb{R}^4$. As a result, the anomaly theory simplifies to the Green-Schwarz term

$$\alpha(S^3_H \times M_8) = \int_{M_8} X_8 \,. \tag{3.87}$$

If we can now show that this quantity is always an integer, Dai-Freed anomalies will vanish on all such factorized backgrounds. This result does not hinge on the precise bordism groups computed in the preceding section.

In order to prove that eq. (3.87) is always an integer, we can connect it with the anomaly inflow mechanism on a fivebrane. Specifically, S^3_H is a non-trivial bordism class, and one possible boundary for it in string theory is a fivebrane. The fivebrane is a codimension four object, and it is characterized precisely by the fact that the angular S^3 in the transverse space is threaded by one unit of H-flux. These fivebranes are precisely D5-branes in the orientifold models and NS5-branes in the heterotic model. We will now show that X_8 coincides with the anomaly polynomial of such a fivebrane, up to terms which vanish when the Bianchi identity holds. In the presence of fivebranes coupling to B_6, the dual of B_2, the classical gauge variation of the effective action is compensated by the quantum anomaly of the chiral worldvolume degrees of freedom. For this inflow mechanism to work, the anomaly polynomial of a single fivebrane has to be $I_8 = X_8$ (up to terms that vanish on a twisted String manifold). To see this, notice that the bulk action receives additional worldvolume contributions of the form

$$S = S_{\text{bulk}} + S_{\text{GS}} + S_{\text{wv}} + \mu \int_W B_6 \,, \tag{3.88}$$

where W denotes the worldvolume of the fivebrane(s) and B_6 the dual of B_2. The bulk action S_{bulk}, which describes the ten-dimensional effective (super)gravity theory, is accompanied by the Green-Schwarz term S_{GS} of eq. (2.17) to cancel bulk anomalies. The brane is instead described by the worldvolume DBI action S_{wv} accompanied by the magnetic coupling to B_6, which is the relevant coupling in the following argument. The equation of motion for B_6 and the corresponding dual Bianchi identity are

$$\begin{aligned} d \star dB_6 &= \mu\, \delta(W \hookrightarrow M_{10}) \,, \\ dH_3 &= \mu\, \delta(W \hookrightarrow M_{10}) \,, \end{aligned} \tag{3.89}$$

where $H_3 = dB_2$ and the δ is a distribution-valued four-form that describes the embedding of W in spacetime. Correspondingly, the Bianchi identity for the gauge invariant field strength $H \equiv \widetilde{H}_3 = dB_2 - \omega_{\text{CS}}$, which ordinarily reads $dH = X_4$, also receives a new localized contribution. Because of this Bianchi identity, there is a new classical contribution to the gauge variations. Using descent, $X_8 = dX_7^{(0)}$, $\delta X_7^{(0)} = dX_6^{(1)}$, one finds

$$\begin{aligned} \delta_{\text{new}} S_{\text{GS}} &= -\int_{M_{10}} dB_2 \wedge \delta X_7^{(0)} \\ &= -\int_{M_{10}} dH_3 X_6^{(1)} \\ &= -\mu \int_W X_6^{(1)} \,, \end{aligned} \tag{3.90}$$

which cancels by inflow provided that the worldvolume theory of the D5-brane has an anomaly polynomial $I_8 = \mu X_8$ [137, 138]. With our choice of units, the elementary charge $\mu = n_5 \in \mathbb{Z}$ counts the number of fivebranes.

As described in section 2, the anomaly polynomial is a sum of indices, given by the APS index theorem for each one of the anomalous degrees of freedom propagating on the fivebrane. As such, we know that I_8 is an integer and we can conclude from the previous discussion that X_8 must also be an integer. This anomaly inflow argument thus allows one to show that the anomaly (3.87) always vanishes.

For the orientifold models, this mechanism can be implemented explicitly, since these theories have D5-branes whose worldvolume degrees of freedom are known. We describe this in detail in section 3.4.1.

On the heterotic side, although the $\mathrm{SO}(16)\times\mathrm{SO}(16)$ theory is known to have NS5 branes, their worldvolume degrees of freedom are not known and so we have to resort to other arguments to prove that the anomaly (3.87) vanishes. The proof can be found at the end of section 3.4.2. There is, however, a more physical way of understanding why anomalies cancel in the heterotic case: one can show that the anomaly polynomial of $\mathrm{SO}(16)\times\mathrm{SO}(16)$ can be directly related to that the supersymmetric heterotic theories. Therefore, one use this connection to show that anomalies cancel for $\mathrm{SO}(16)\times\mathrm{SO}(16)$ by showing that they cancel in the supersymmetric cases. This is done explicitly in section 3.4.2.

Finally, since we have proven that anomalies vanish in the heterotic case, we can reverse the anomaly inflow argument above to speculate about the worldvolume degrees of freedom of the NS5 brane. Indeed, we identify what kind of degrees of freedom give rise to the correct anomaly polynomial so as to have $X_8 = I_8$. We do so away from strong coupling effects, in the puffed-up instanton limit of the NS5 brane. This is detailed in section 3.4.3.

3.4.1 Sp(16) and U(32)

Let us begin with the orientifold models. This cancellation of anomalies by inflow was first constructed for the case of $\mathrm{Spin}(32)/\mathbb{Z}_2$ in [137, 138]. The chiral fermions on the worldvolume of the D5-brane consist of one vector multiplet of $\mathrm{Spin}(32)/\mathbb{Z}_2$ and two gauge singlets, such that,

$$(X_8)_{\mathrm{Spin}(32)/\mathbb{Z}_2} - I_{\mathrm{Spin}(32)/\mathbb{Z}_2} = -\frac{1}{24} p_1 (X_4)_{\mathrm{Spin}(32)/\mathbb{Z}_2}\,, \tag{3.91}$$

where $(X_8)_{\mathrm{Spin}(32)/\mathbb{Z}_2}$ and $(X_4)_{\mathrm{Spin}(32)/\mathbb{Z}_2}$ can be read off from (2.13). This shows how one recovers $I_8 = X_8$ up to a term that vanishes on a twisted string manifold.

One may wonder where the extra term in (3.91) comes from, even if we know it to vanish on a twisted string manifold. This can be understood as follows; from the perspective of the ten-dimensional supergravity action, a D5-brane amounts to introducing a delta function localized on the brane. The D5-brane gives a localized contribution to the 10d action of the form $B_2 \wedge Y_4 \wedge \delta_4$ where Y_4 is some 4-form which in this case is reduces to $Y_4 = -\frac{1}{24} p_1$, expanding the A-roof genus in the Chern-Simons effective worldvolume action. Indeed, using the Bianchi identity for the H_3 flux, we see that this term contributes to the anomaly polynomial as:

$$\int_{Z_{12}} X_4 \wedge Y_4 \wedge \delta_4 = \int_{X_8} X_4 \wedge Y_4\,. \tag{3.92}$$

Therefore, the appearance of the extra term in (3.91) can be traced down to not properly taking into account the delta-function source that corresponds to the localized D5-brane.

The same mechanism happens in the two non-supersymmetric orientifold models, as was found by [139] along the lines of [138, 140]. The worldvolume degrees of freedom on D5-branes can be extracted from one-loop open-string amplitudes [141], and the chiral fermions arrange in the virtual representation

$$\left(\frac{\mathbf{N(N+1)}}{\mathbf{2}},\mathbf{1}\right)-\left(\frac{\mathbf{N(N-1)}}{\mathbf{2}},\mathbf{1}\right)-(\mathbf{N},\mathbf{32}) \tag{3.93}$$

of $\mathrm{SO}(N)\times\mathrm{Sp}(16)$ (for the Sugimoto model[20]) or $\mathrm{U}(N)\times\mathrm{U}(32)$ (for the Sagnotti model). In order to compare the anomaly polynomial I_8 (without worldvolume gauge field) with the bulk X_8, one needs to decompose characteristic classes of the bulk tangent bundle in terms of the worldvolume tangent bundle TW and normal bundle N. In detail,

$$\begin{aligned} p_1(TM_{10}) &= p_1(TW)+p_1(N)\,, \\ p_2(TM_{10}) &= p_2(TW)+p_1(TW)\,p_1(N)+p_2(N)\,, \\ p_1(N) &= c_1(N)^2-2\,c_2(N)\,, \\ p_2(N) &= c_2(N)^2=\chi(N)^2\,. \end{aligned} \tag{3.94}$$

When the normal bundle of the worldvolume is trivial, one obtains

$$I_8-X_8\propto p_1(TM_{10})\,X_4\,, \tag{3.95}$$

and therefore the inflow mechanism implies that X_8 integrates to an integer on any spin 8-manifold with $X_4=0$. When the normal bundle is non-trivial, there are additional contributions to the above expression, proportional to the Euler class of N. However, the full brane action also contains another term [137, 138] proportional to B_2 rather than B_6, which induces another classical variation to be canceled by inflow. As a result, the anomaly polynomial of the fivebrane worldvolume theory is not quite the above I_8, but has an additional contribution that cancels the normal bundle terms [138]. In more detail, adding a coupling of the type $\int_W B_2 Y_4$ to the fivebrane worldvolume action contributed a new classical variation to the effective action, which arises by descent from $\Delta I_8=-(X_4+n_5\,\chi(N))Y_4$. Therefore, the full anomaly polynomial of the fivebrane worldvolume ought to be $I_8=n_5\,X_8-\Delta I_8$, again up to terms that vanish on twisted String backgrounds. This additional coupling can be shown to cancel the normal bundle terms in the anomaly [138] (see also [142] for a discussion in the context of M-theory).

3.4.2 $\mathrm{SO}(16)\times\mathrm{SO}(16)$

For the heterotic model, no such result is available, since the worldvolume degrees of freedom of NS5-branes are not understood without supersymmetry or dualities at one's disposal. However, one can nonetheless express X_8 as an index of six-dimensional chiral fields; since index are manifestly integers, this will be enough to establish that anomalies cancel. In order

[20] In this case, a single brane corresponds to $N=2$.

to do so, let us observe that the formal difference of representations of the chiral fermions of the non-supersymmetric heterotic model can be rewritten as[21]

$$\begin{aligned}&(\mathbf{128},\mathbf{1})+(\mathbf{1},\mathbf{128})-(\mathbf{16},\mathbf{16})\\&=(\mathbf{128},\mathbf{1})+(\mathbf{1},\mathbf{128})+(\mathbf{120},\mathbf{1})+(\mathbf{1},\mathbf{120})\\&\quad-(\mathbf{120},\mathbf{1})-(\mathbf{1},\mathbf{120})-(\mathbf{16},\mathbf{16})\,,\end{aligned}\tag{3.96}$$

The matter fields in the first line after the equal correspond precisely to the decomposition of the adjoint of $\mathfrak{e}_8\oplus\mathfrak{e}_8$ into representations of the $\mathfrak{so}_{16}\oplus\mathfrak{so}_{16}$ subalgebra; they are the field content that would arise after giving a vev to an adjoint $\mathfrak{e}_8\oplus\mathfrak{e}_8$ field. Similarly, the fields in the second line are (with reversed chirality) those fields that would arise after adjoint Higgsing from the $\mathfrak{so}(32)$ algebra to its $\mathfrak{so}_{16}\oplus\mathfrak{so}_{16}$ subalgebra. What we are seeing here is that, at a formal level (as far as the chiral spectrum is concerned), the $\mathrm{SO}(16)^2$ is equivalent to one copy of the $E_8\times E_8$ string stacked on top of a copy of the $\mathrm{Spin}(32)/\mathbb{Z}_2$ string, with opposite chirality, and Higgsed to a common subgroup with algebra $\mathfrak{so}_{16}\oplus\mathfrak{so}_{16}$. Therefore, we can write, at the level of anomaly polynomials, the equality

$$P_{12}^{E_8\times E_8}|_{\mathrm{SO}(16)^2}-P_{12}^{\mathrm{Spin}(32)/\mathbb{Z}_2}|_{\mathrm{SO}(16)^2}=P_{12}^{\mathrm{SO}(16)^2}\,,\tag{3.97}$$

where we have merely restricted to $\mathrm{SO}(16)^2$ bundles inside of the two groups above. Since each of the supersymmetric string theories are anomaly-free by themselves, the formal linear combination will also be. This argument, which can be carried out at the level of eta invariants etc. and not just anomaly polynomials, is yet another proof of the fact that the $\mathrm{SO}(16)^2$ theory is anomaly free,[22] without relying explicitly on bordism calculations. Furthermore, in particular, this holds for the Green-Schwarz terms, which are

$$\begin{aligned}&(X_8)_{\mathrm{Spin}(32)/\mathbb{Z}_2}|_{\mathrm{SO}(16)^2}-(X_8)_{E_8\times E_8}|_{\mathrm{SO}(16)^2}\\&=\frac{1}{24}\left((c^{(1)}_{\mathbf{16},2})^2+(c^{(2)}_{\mathbf{16},2})^2+c^{(1)}_{\mathbf{16},2}\,c^{(2)}_{\mathbf{16},2}-4\,c^{(1)}_{\mathbf{16},4}-4\,c^{(2)}_{\mathbf{16},4}\right)=(X_8)_{\mathrm{SO}(16)^2}\,.\end{aligned}\tag{3.98}$$

It is unclear whether this connection between the non-supersymmetric $\mathrm{SO}(16)\times\mathrm{SO}(16)$ theory and the supersymmetric theories persists beyond a formal equality at the level of (super)gravity, or whether on the contrary it has a deeper meaning. Some previous work [143, 144] (see also [145]) identified connections between supersymmetric and non-supersymmetric strings via interpolating models, which are nine-dimensional compactifications recovering either supersymmetric or non-supersymmetric strings in different decompactification limits. In particular, an interpolating model was constructed between the $\mathrm{SO}(16)\times\mathrm{SO}(16)$ theory and the supersymmetric $\mathrm{Spin}(32)/\mathbb{Z}_2$ theory, matching the worldsheet CFT descriptions and solitons in between the two.[23]

The cancellation of anomalies by fivebrane inflow for the $\mathrm{SO}(16)\times\mathrm{SO}(16)$ theory thus follows from that of the two supersymmetric heterotic theories. The anomaly inflow in the

[21]To our knowledge, this was first explicitly stated in the literature in [71], though we learned from Luis Álvarez-Gaumé that the authors of [2] were also aware of this fact.

[22]Even with the right global quotient.

[23]Since the non-supersymmetric theories have NS-NS tadpoles and would-be moduli run in the absence of (large) stabilizing fluxes [146, 147] and/or spacetime warping [148, 149], there may be additional subtleties in understanding the dynamics of this duality.

case of $\mathrm{Spin}(32)/\mathbb{Z}_2$ was discussed above (3.91). The case of $E_8 \times E_8$ is slightly more involved and we will discuss it now. The anomaly inflow of the NS5-brane was famously discussed in [150] where the limit in which an instanton in $E_8 \times E_8$ becomes point-like was matched to the world volume theory of the NS5 brane at strong coupling. For our purposes, we can ignore strong coupling dynamics and focus on matching a 6-dimensional anomaly theory of chiral fermions to $(X_8)_{E_8\times E_8}$ which can be read off from (2.14). This guarantees that $(X_8)_{E_8\times E_8}$ is an integer and that local anomalies cancel in 10d. We now detail how this can be done.

One can show that $(X_8)_{E_8\times E_8}$ can be decomposed as follows:

$$(X_8)_{E_8\times E_8} = \frac{(c^{(1)}_{\mathbf{16},2} - c^{(2)}_{\mathbf{16},2})^2}{32} + \frac{1}{24}X_4^2 + \mathcal{I}_{\mathrm{SD}} + 2\,\mathcal{I}_{\mathrm{Dirac}} \tag{3.99}$$

where $\mathcal{I}_{\mathrm{SD}} + 2\,\mathcal{I}_{\mathrm{Dirac}}$ is the index of a self-dual form field and 2 fermion singlets in 8 dimensions. The index of a self-dual form field in 8 dimensions can be shown to be an integer over 8 [42]. Indeed, it can be written in terms of the signature of the 8-manifold as follows [103]:

$$\mathcal{I}_{\mathrm{SD}} = -\frac{\sigma}{8}\,. \tag{3.100}$$

On the other hand, the index of chiral fermions is always an integer. In order to simplify the first term in (3.99), we can rewrite the Chern classes in an embedded SU(2) subgroup of each E_8, which are known to be integer-valued. Therefore, on a twisted string manifold, X_8 reduces to:

$$X_8 = \frac{(c^{(1)}_{\mathbf{2},2} - c^{(2)}_{\mathbf{2},2})^2}{8} - \frac{\sigma}{8} + n \quad \text{with} \quad n \in \mathbb{Z} \tag{3.101}$$

where $\frac{1}{2}c^{(i)}_{\mathbf{16},2} \to c^{(i)}_{\mathbf{2},2}$ are the 2nd Chern classes in the fundamental of the SU(2) subgroup of the i-th E_8. The Bianchi identity $(X_4)_{E_8\times E_8} = 0$ gives us

$$c^{(2)}_{\mathbf{2},2} = -\frac{p_1}{2} - c^{(1)}_{\mathbf{2},2}\,. \tag{3.102}$$

Plugging this into (3.101), we see that the condition for anomalies to vanish comes down to showing that the following quantity is an integer:

$$\frac{(c^{(1)}_{\mathbf{2},2})^2}{2} + \frac{c^{(1)}_{\mathbf{2},2}\,p_1}{4} + \frac{p_1^2}{32} - \frac{\sigma}{8}\,. \tag{3.103}$$

As it happens, it was shown in [103] that the last two terms give (28 times) an integer. Indeed, one can show that:

$$28\,\mathcal{I}_{\mathrm{Dirac}} = \frac{p_1^2}{32} - \frac{\sigma}{8}\,. \tag{3.104}$$

Now, to show that the first two terms of (3.103) are an integer, one can note that on a twisted string manifold, $\frac{p_1}{2}$ is a characteristic vector of $H^4(X;\mathbb{R})$. This, in particular, means that:

$$\frac{p_1}{2}c^{(i)}_{\mathbf{2},2} \mod 2 = (c^{(i)}_{\mathbf{2},2})^2 \mod 2\,. \tag{3.105}$$

Therefore we have shown that on a twisted string manifold, X_8 is always an integer; and so there can never be an anomaly.

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Given that the X_8 of $\mathrm{SO}(16)\times\mathrm{SO}(16)$ is a linear combination of those of $E_8\times E_8$ and $Spin(32)/\mathbb{Z}_2$, we can infer that $(X_8)_{\mathrm{SO}(16)^2}$ is an integer and so that all anomalies vanish for this non-supersymmetric theory. Nevertheless, for completeness, let us detail explicitly how $(X_8)_{\mathrm{SO}(16)^2}$ can be proven to be an integer. One can write $(X_8)_{\mathrm{SO}(16)^2}$ as follows:

$$
\begin{aligned}
(X_8)_{\mathrm{SO}(16)^2} = & -\frac{1}{32}(c^{(1)}_{\mathbf{16},2}-c^{(2)}_{\mathbf{16},2})^2-\mathcal{I}_{\mathrm{SD}}-4\,\mathcal{I}_{\mathrm{Dirac}} \\
& -\frac{1}{48}(X_4)_{\mathrm{SO}(16)^2}(c^{(1)}_{\mathbf{16},2}+c^{(2)}_{\mathbf{16},2}+3p_1)+\mathcal{I}^{\mathbf{16}_{(1)}}_{\mathrm{Dirac}}+\mathcal{I}^{\mathbf{16}_{(2)}}_{\mathrm{Dirac}}
\end{aligned}
\tag{3.106}
$$

where $\mathcal{I}^{\mathbf{16}_{(i)}}_{\mathrm{Dirac}}$ is the contribution of a fermion that transforms in the $\mathbf{16}$ of $\mathrm{SO}(16)_i$, which is known to be integer-valued. Therefore, on a twisted string manifold, the cancellation of anomalies comes down to showing that the following quantity is an integer:

$$
-\frac{1}{32}(c^{(1)}_{\mathbf{16},2}-c^{(2)}_{\mathbf{16},2})^2-\mathcal{I}_{\mathrm{SD}}=-\frac{1}{32}(c^{(1)}_{\mathbf{16},2}-c^{(2)}_{\mathbf{16},2})^2+\frac{\sigma}{8}
\tag{3.107}
$$

Given that one can put the 2nd Chern classes in the SU(2) subgroup of SO(16) as $c^{(i)}_{\mathbf{16},2}\to 2c^{(i)}_{\mathbf{2},2}$; the proof goes exactly as in the $E_8\times E_8$ case.

One can sometimes read-off the chiral field content of a theory from the anomaly polynomial. For instance, for the $E_8\times E_8$ case, the anomaly polynomial (3.99) suggests that the chiral field content of the NS5 brane is a self-dual form field and 2 fermion singlets. There are no chiral fields charged under the gauge group, since all the gauge-dependent parts of (3.99) are in the factorized piece. As it happens, this exactly the chiral field content of a 6d $(1,0)$ tensor multiplet, which is precisely the worldvolume field content of the NS5 brane in $E_8\times E_8$ string theory. This answer is essentially determined by anomalies together with supersymmetry. In the non-supersymmetric case of the $\mathrm{SO}(16)^2$ string, reading off the chiral field content from (3.106) in the same way suggests that the chiral field content of the $\mathrm{SO}(16)^2$ NS5 brane is:

- Four fermion singlets,
- A fermion transforming in the $(\mathbf{16},\mathbf{1})\oplus(\mathbf{1},\mathbf{16})$ of SO(16),
- A self-dual 2-form field.

There are some subtleties in assessing whether or not these are truly the chiral degrees of freedom propagating on this non-supersymmetric brane. First of all, there is no supersymmetry to constrain the worldvolume theory of the NS5 brane which can therefore carry any kind of chiral degrees of freedom. As we have seen from (3.104) and (3.100), indices can sometimes be exchanged for one another and yet give the same integer. This means that the X_8 does not completely fix the worldvolume content of the NS5 brane, and that any chiral field content with the same anomaly as the one proposed above remains a possibility. Another reason why we cannot be sure that (3.106) correctly describes the degrees of freedom propagating on the NS5 brane is that we cannot be sure that an NS5 brane (understood as a small instanton where the full spacetime gauge group symmetry gets restored) exists to begin with. Unlike in the supersymmetric case, in general we expect that the size modulus of the instanton, being

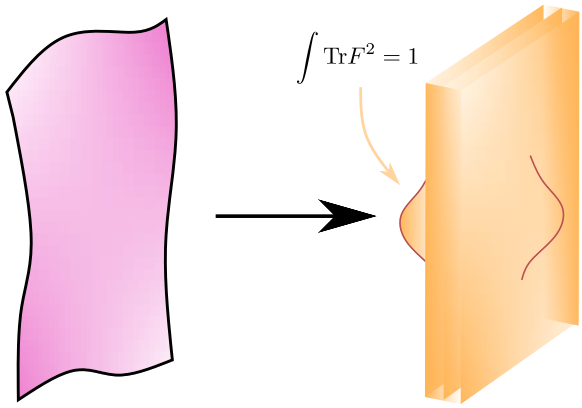

Figure 7. A sketch of a fivebrane puffing up into an instanton which can be described within the effective field theory.

non-supersymmetric, receives a potential due to quantum effects that may lead to the small instanton limit being obstructed. The study of the strong coupling effects near the small instanton limit is beyond the validity of effective field theory, and thus beyond the scope of this paper (although it may be amenable to a version of the constructions in [50]), but we point out that, if the limit does exist and the small instanton transition does survive, the transition point would be a natural place to look for a non-supersymmetric interacting CFT, a cousin of the E_8 SCFT. It would be interesting to explore this further. On the other hand, studying the anomaly inflow on the worldvolume of the puffed-up NS5 brane instanton is accessible within the effective field theory (see figure 7). We do this explicitly in the next section.

3.4.3 Anomaly inflow on puffed-up fivebrane instantons

The above result shows that there are no Dai-Freed anomalies on factorized backgrounds of the form $S^3_H \times M_8$, since X_8 integrates to an integer. Anomaly inflow on fivebranes dictates that X_8 be the anomaly polynomial associated to the worldvolume theory on a single fivebrane, possibly up to terms that vanish when the Bianchi identity is satisfied. As explained above, without direct access to the relevant degrees of freedom on the fivebrane worldvolume, studying the anomaly inflow on the point-like NS5 brane is impossible. Luckily, one can still examine the anomaly inflow on the worldvolume of puffed-up fivebrane instantons, which can be described in the low-energy approximation. Puffing-up the fivebrane corresponds to delocalizing it along its transverse dimensions, making it look like a four dimensional gauge instanton.

Introducing an instanton Higgses one of the SO(16) factors, say the first $\mathrm{SO}(16)^{(1)}$, according to $\mathrm{SO}(16) \to \mathrm{SU}(2)^{(a)} \times \mathrm{SU}(2)^{(b)} \times \mathrm{SO}(12)$, so that the vector and spinor representations branch into

$$\mathbf{16} = (\mathbf{1}^{(\mathbf{a})}, \mathbf{1}^{(\mathbf{b})}, \mathbf{12}) + (\mathbf{2}^{(\mathbf{a})}, \mathbf{2}^{(\mathbf{b})}, \mathbf{1}) \,, \tag{3.108a}$$

$$\mathbf{128} = (\mathbf{2}^{(\mathbf{a})}, \mathbf{1}^{(\mathbf{b})}, \mathbf{32}) + (\mathbf{1}^{(\mathbf{a})}, \mathbf{2}^{(\mathbf{b})}, \overline{\mathbf{32}}) \,. \tag{3.108b}$$

If the instanton bundle only involves $\mathrm{SU}(2)^{(a)}$, there an $\mathrm{SU}(2)^{(b)} \times \mathrm{SO}(12) \times \mathrm{SO}(16)^{(2)}$ unbroken symmetry and the background has fermion zero modes (fzm) arising from the representations $(\mathbf{8_s}, \mathbf{16^{(1)}}, \mathbf{16^{(2)}})$ and $(\mathbf{8_c}, \mathbf{128^{(1)}}, \mathbf{1^{(2)}})$ of the spacetime isometries and the original gauge group. As a result, one has

$$\begin{cases} 1 \text{ fzm in the rep } (\mathbf{8_s}, \mathbf{2^{(b)}}, \mathbf{1}, \mathbf{16^{(2)}})\,, \\ 1 \text{ fzm in the rep } (\mathbf{8_c}, \mathbf{1^{(b)}}, \mathbf{32}, \mathbf{1^{(2)}})\,. \end{cases} \tag{3.109}$$

The two types of fermion zero modes have different chirality, and thus the corresponding worldvolume anomaly polynomial reads

$$P_8 = \frac{1}{2}[\hat{A}(R)(\mathrm{ch}(F)_{(\mathbf{2^{(b)}},\mathbf{1},\mathbf{16^{(2)}})} - \mathrm{ch}(F)_{(\mathbf{1^{(b)}},\mathbf{32},\mathbf{1^{(2)}})})]_8\,, \tag{3.110}$$

which evaluates to

$$\begin{aligned} P_8 = \frac{1}{24}\big[& -2p_1 c_{\mathbf{12},2} + p_1(c^{(2)}_{\mathbf{16},2} + 8c^{(b)}_{\mathbf{2},2}) - c^2_{\mathbf{12},2} - 4c_{\mathbf{12},4} \\ & + 2(c^{(2)}_{\mathbf{16},2} + 2c^{(b)}_{\mathbf{2},2})(c^{(2)}_{\mathbf{16},2} + 4c^{(b)}_{\mathbf{2},2}) - 4c^{(2)}_{\mathbf{16},4}\big]\,. \end{aligned} \tag{3.111}$$

The next step is to evaluate X_8 on this background. This amounts to decomposing characteristic classes according to the branching rules, and one finds

$$\begin{aligned} & c^{(1)}_{\mathbf{16},2} \to c_{\mathbf{12},2} + 2c^{(a)}_{\mathbf{2},2} + 2c^{(b)}_{\mathbf{2},2}\,, \\ & c^{(1)}_{\mathbf{16},4} \to 2c_{\mathbf{12},2}c^{(a)}_{\mathbf{2},2} + 2c_{\mathbf{12},2}c^{(b)}_{\mathbf{2},2} + c_{\mathbf{12},4} - 2c^{(a)}_{\mathbf{2},2}c^{(b)}_{\mathbf{2},2} + (c^{(a)}_{\mathbf{2},2})^2 + (c^{(b)}_{\mathbf{2},2})^2\,. \end{aligned} \tag{3.112}$$

Finally, $(c^{(a)}_{\mathbf{2},2})^2$ should be replaced by zero, since it is proportional to the square of the worldvolume current $\delta(W)$ of the fivebrane. All in all, when the dust settles one arrives at

$$X_8 - P_8 = \frac{1}{24}\left(2c_{\mathbf{12},2} - c^{(2)}_{\mathbf{16},2} - 8c^{(b)}_{\mathbf{2},2}\right)\left(c_{\mathbf{12},2} + 2c^{(b)}_{\mathbf{2},2} + c^{(2)}_{\mathbf{16},2} + p_1\right)\,, \tag{3.113}$$

where the second factor corresponds to X_4 for the unbroken piece of the gauge group. The inflow therefore works when X_8 and P_8 are equal on manifolds where the Bianchi identity holds. A similar argument works for more general choices of instanton bundles.

For the orientifold models, one expects the small limit of the "fat" fivebrane instantons to yield the worldvolume degrees of freedom of D5-branes. This is a nice crosscheck that we detail now. For the Sugimoto model (the calculation is identical in the Sagnotti model), the anomaly polynomial P_8 associated to the fermion zero modes of the instanton is

$$P_8 = [\hat{A}(R)\mathrm{ch}(F)_{\mathbf{30}}]_8 = \frac{1}{192}\left(8p_1 c_{\mathbf{30},2} - 4\,(8c_{\mathbf{30},4} + p_2) + 16c^2_{\mathbf{30},2} + 7p_1^2\right)\,, \tag{3.114}$$

since under the branching $\mathrm{Sp}(16) \to \mathrm{SU}(2) \times \mathrm{USp}(30)$ the adjoint representation, containing the gauginos, decomposes according to

$$\mathbf{495} = (\mathbf{2}, \mathbf{30}) + (\mathbf{1}, \mathbf{434}) + (\mathbf{1}, \mathbf{1})$$

where the only charged contribution comes from the first term on the right-hand side. In the small limit, the SU(2) Chern classes vanish and the remaining ones are enhanced to Sp(16) classes, ending up with

$$P_8^{\text{small}} = \frac{1}{192}\left(8p_1 c_{\mathbf{32},2} - 4\,(8c_{\mathbf{32},4} + p_2) + 16c^2_{\mathbf{32},2} + 7p_1^2\right) = X_8 + \frac{1}{24}\,X_4\,p_1. \tag{3.115}$$

Thus reproducing the anomaly polynomial of a D5-brane worldvolume up to terms that vanish on the allowed backgrounds.

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4 Anomalies and bordism for the swap $\mathbb{Z}_2$ action

4.1 Overview and the bordism computation

The $E_8 \times E_8$ heterotic string theory has a $\mathbb{Z}_2$ symmetry given by swapping the two copies of E_8, so it is possible to expand the gauge group of the theory to $(E_8 \times E_8) \rtimes \mathbb{Z}_2$. To our knowledge, this fact first appears in [53, section I] (see also [151, section 2.1.1]). The question of anomaly cancellation for this string theory is completely different in the absence versus in the presence of this extra $\mathbb{Z}_2$: without it, the anomaly is known to vanish, as Witten [136, section 4] showed it is characterized by a bordism invariant $\Omega_{11}^{\mathrm{Spin}}(BE_8) \to \mathbb{C}^\times$, and Stong [152] showed $\Omega_{11}^{\mathrm{Spin}}(BE_8) \cong 0$. But with the $\mathbb{Z}_2$ swapping symmetry turned on, the relevant bordism group has order 64 [46, Theorem 2.62] courtesy of a harder computation; even though we cannot determine this group exactly, we will show that the anomaly vanishes, in accordance with the results in [107] obtained from a worldsheet perspective.

In this section, we discuss a closely analogous story for the $\mathrm{Spin}(16) \times \mathrm{Spin}(16)$ non-supersymmetric heterotic string. The gauge group $\mathrm{Spin}(16) \times_{\mathbb{Z}_2} \mathrm{Spin}(16)$ (where the diagonal $\mathbb{Z}_2$ we quotient by corresponds to either of the subgroups in each $\mathrm{Spin}(16)$ whose quotient is *not* $SO(16)$) admits a $\mathbb{Z}_2$ automorphism switching the two $\mathrm{Spin}(16)$ factors, enlarging the gauge group of this theory to $(\mathrm{Spin}(16) \times_{\mathbb{Z}_2} \mathrm{Spin}(16)) \rtimes \mathbb{Z}_2$; see [53, section III].

In this paper, we chose to work with $\mathrm{Spin}(16) \times \mathrm{Spin}(16)$, which simplifies the bordism computations at the expense of applying to only some backgrounds. The $\mathbb{Z}_2$ symmetry enlarges the gauge group to $G_{16,16} := (\mathrm{Spin}(16) \times \mathrm{Spin}(16)) \rtimes \mathbb{Z}_2$. The Green-Schwarz mechanism is analogous: if $x \in H^*(B\mathrm{Spin}(16); A)$ for some coefficient group A, let x^L and x^R denote the copies of x in $H^*(B(\mathrm{Spin}(16) \times \mathrm{Spin}(16)); A)$ coming from the first, resp. second copies of $\mathrm{Spin}(16)$ via the Künneth formula. Then the class $\frac{1}{2}p_1^L + \frac{1}{2}p_1^R$, which was the characteristic class of the Green-Schwarz mechanism in the absence of the $\mathbb{Z}_2$ symmetry, descends through the Serre spectral sequence for the fibration

$$\begin{tikzcd} B(\mathrm{Spin}(16) \times \mathrm{Spin}(16)) \arrow[r] & BG_{16,16} \arrow[d] \\ & B\mathbb{Z}_2 \end{tikzcd} \tag{4.1}$$

to define a class in $H^*(BG_{16,16}; \mathbb{Z})$, and the Green-Schwarz mechanism asks, on a spin manifold M with a principal $G_{16,16}$-bundle $P \to M$, for a trivialization of

$$\tfrac{1}{2}p_1(M) - (\tfrac{1}{2}p_1^L + \tfrac{1}{2}p_1^R)(P). \tag{4.2}$$

Let $\mathbb{G}_{16,16}$ denote the Lie 2-group corresponding to this data, i.e. the string cover of $\mathrm{Spin} \times G_{16,16}$ corresponding to the class (4.2). Quotienting by $G_{16,16}$ defines a map to Spin and therefore a tangential structure in the usual way; a $\mathbb{G}_{16,16}$-structure on a vector bundle $E \to M$ is a spin structure on E, a double cover $\pi\colon M' \to M$, a pair of rank-16 spin vector bundles V^L and V^R on M' identified under the deck transformation of M', and a trivialization of $\frac{1}{2}p_1(E) - (\frac{1}{2}p_1(V^L) - \frac{1}{2}p_1(V^R))$ (the class $\frac{1}{2}p_1(V^L) + \frac{1}{2}p_1(V^R)$ descends from M' to M). If the double cover $M' \to M$ is trivial, this is equivalent to a Spin-$\mathrm{Spin}(16)^2$ structure as defined in section 3.3.3.

Theorem 4.3.

$$\begin{aligned} \Omega_0^{\mathbb{G}_{16,16}} &\cong \mathbb{Z} & \Omega_6^{\mathbb{G}_{16,16}} &\cong \mathbb{Z}_2 \\ \Omega_1^{\mathbb{G}_{16,16}} &\cong (\mathbb{Z}_2)^{\oplus 2} & \Omega_7^{\mathbb{G}_{16,16}} &\cong \mathbb{Z}_{16} \\ \Omega_2^{\mathbb{G}_{16,16}} &\cong (\mathbb{Z}_2)^{\oplus 2} & \Omega_8^{\mathbb{G}_{16,16}} &\cong \mathbb{Z}^{\oplus 3} \oplus (\mathbb{Z}_2)^{\oplus i} \\ \Omega_3^{\mathbb{G}_{16,16}} &\cong \mathbb{Z}_8 & \Omega_9^{\mathbb{G}_{16,16}} &\cong (\mathbb{Z}_2)^{\oplus j} \\ \Omega_4^{\mathbb{G}_{16,16}} &\cong \mathbb{Z} \oplus \mathbb{Z}_2 & \Omega_{10}^{\mathbb{G}_{16,16}} &\cong (\mathbb{Z}_2)^{\oplus k} \\ \Omega_5^{\mathbb{G}_{16,16}} &\cong 0 & \Omega_{11}^{\mathbb{G}_{16,16}} &\cong A, \end{aligned}$$

where either $i = 1$, $j = 4$, and $k = 4$, or $i = 2$, $j = 6$, and $k = 5$, and A is an abelian group of order 64 isomorphic to one of $\mathbb{Z}_8 \oplus \mathbb{Z}_8$, $\mathbb{Z}_{16} \oplus \mathbb{Z}_4$, $\mathbb{Z}_{32} \oplus \mathbb{Z}_2$, or $\mathbb{Z}_{64}$.

The fact that $\Omega_{11}^{\mathbb{G}_{16,16}} \neq 0$ implies that the Spin(16) $\times$ Spin(16) heterotic theory with its $\mathbb{Z}_2$ swapping symmetry could have an anomaly; we will nevertheless be able to cancel it later in this section.

Proof. The proof is nearly identical to the analogous calculation for the $E_8 \times E_8$ heterotic string, which is done in [46, section 2.2, section 2.3]; therefore we will be succinct and direct the reader there for the details.

Let $V \to B\mathrm{Spin}(16) \times B\mathrm{Spin}(16)$ be the direct sum of the tautological vector bundles on the two factors. The $\mathbb{Z}_2$ swapping action on $B\mathrm{Spin}(16) \times B\mathrm{Spin}(16)$ lifts to make V into a $\mathbb{Z}_2$-equivariant vector bundle, so V descends to a vector bundle we will also call V over $BG_{16,16}$. Since the action of $\mathbb{Z}_2$ is compatible with the spin structures on the two tautological bundles, $V \to BG_{16,16}$ is spin, so $w_1(V) = 0$ and $w_2(V) = 0$; and essentially by definition, $\frac{1}{2}p_1(V) = \frac{1}{2}p_1^L + \frac{1}{2}p_1^R$. Therefore just as for the other theories we studied, there is an isomorphism

$$\Omega_*^{\mathbb{G}_{16,16}} \xrightarrow{\cong} \Omega_*^{\mathrm{String}}((BG_{16,16})^{V-32}). \tag{4.4}$$

This is the biggest difference between the computations for the Spin(16) $\times$ Spin(16) and $E_8 \times E_8$ theories: see [46, Lemma 2.2]. Much of the theory developed in [46, section 2] and in [114] and applied to the $E_8 \times E_8$ theory in *loc. cit.* can therefore be avoided for the Spin(16) $\times$ Spin(16) case; nevertheless, the calculation is pretty similar.

First we must establish the absence of p-torsion for primes $p > 3$. This is analogous to the other twisted string bordism computations in this paper, and we do not go into detail.

At $p = 3$, we follow [114, section 3.2]. First we need $H^*(BG_{16,16}; \mathbb{Z}_3)$; the Serre spectral sequence for $\mathbb{Z}_3$ cohomology and the fibration (4.1) collapses to an isomorphism

$$H^*(BG_{16,16}; \mathbb{Z}_3) \xrightarrow{\cong} H^*(B(\mathrm{Spin}(16) \times \mathrm{Spin}(16)); \mathbb{Z}_3)^{\mathbb{Z}_2}. \tag{4.5}$$

In the degrees relevant to us, $H^*(B\mathrm{Spin}(16); \mathbb{Z}_3)$ is generated by the Pontrjagin classes p_1 and p_2 with no relations in degrees 11 and below, so we obtain the following additive basis for $H^*(BG_{16,16}; \mathbb{Z}_3)$ in degrees 11 and below: 1, $p_1^L + p_1^R$, $(p_1^L)^2 + (p_1^R)^2$, $p_2^L = p_2^R$, and $p_1^L p_1^R$. Using this, we determine the $\mathcal{A}^{tmf}$-module structure on $H^*((BG_{16,6})^{V-32}; \mathbb{Z}_3)$ using [114, Corollary 2.37]: if U denotes the Thom class, $\beta(U) = 0$ and $\mathcal{P}^1(U) = -\mathrm{U}(p_1^L + p_1^R)$ (as

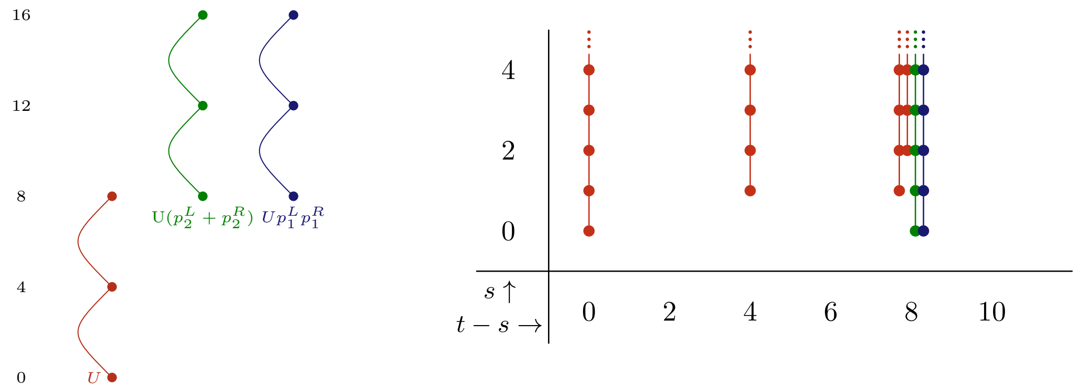

Figure 8. Left: the $\mathcal{A}^{tmf}$-module structure on $H^*((BG_{16,16})^{V-32}; \mathbb{Z}_3)$ in low degrees; the pictured submodule contains all elements in degrees 11 and below. Right: the E_2-page of the Adams spectral sequence computing $tmf_*((BG_{16,16})^{V-32})^\wedge_3$.

$\frac{1}{2}x = -x$ in a $\mathbb{Z}_3$-vector space). Using this and the Cartan formula, we find an $\mathcal{A}^{tmf}$-module isomorphism

$$H^*((BG_{16,16})^{V-32}; \mathbb{Z}_3) \cong N_3 \oplus \Sigma^8 N_3 \oplus \Sigma^8 N_3 \oplus P, \tag{4.6}$$

where N_3 is as in definition 3.54 and P is concentrated in degrees 12 and above, and will be irrelevant for us. We draw (4.6) in figure 8, left. Using the calculation of $\mathrm{Ext}_{\mathcal{A}^{tmf}}(N_3)$ from figure 2, we can draw the E_2-page of the Adams spectral sequence in figure 8, right; it collapses to show there is no 3-torsion in degrees 11 and below.

Finally $p = 2$. First, we need $H^*(BG_{16,16}; \mathbb{Z}_2)$; Evens' generalization [153] of a theorem of Nakaoka [154, Theorem 3.3] gives us the following additive basis for these cohomology groups in degrees 13 and below:

- classes of the form $c^L + c^R$, where c ranges over a basis of $H^*(B\mathrm{Spin}(16); \mathbb{Z}_2)$ in degrees 13 and below;
- the classes $w_4^L w_4^R$, $w_6^L w_6^R$, $w_4^L w_k^R + w_k^L w_4^R$ for $k = 6, 7, 8$, and $(w_4^L)^2 w_4^R + w_4^L (w_4^2)^R$; and
- finally, we have classes of the form x^m, $w_4^L w_4^R x^m$, and $w_6^L w_6^R x^m$, where x is the generator of $H^1(B\mathbb{Z}_2; \mathbb{Z}_2)$, pulled back by the quotient $G_{16,16} \to \mathbb{Z}_2$ by the normal $\mathrm{Spin}(16) \times \mathrm{Spin}(16)$ subgroup.

Quillen's detection theorem [155, Proposition 3.1] computes the $\mathcal{A}(2)$-action on these classes. Since V has vanishing w_1 and w_2, but $w_4(V) = w_4^L + w_4^R$, $\mathrm{Sq}^1(U) = 0$, $\mathrm{Sq}^2(U) = 0$, and $\mathrm{Sq}^4(U) = \mathrm{U}(w_4^L + w_4^R)$. Using this, we can obtain $\mathcal{A}(2)$-module structure on $H^*((BG_{16,16})^{V-32}; \mathbb{Z}_2)$ by direct computation with the Cartan formula similarly to [46, Proposition 2.41].

Proposition 4.7. *Let $\mathcal{M}$ be the quotient of $H^*((BG_{16,16})^{V-32}; \mathbb{Z}_2)$ by all elements in degrees 14 and above. Then $\mathcal{M}$ is the direct sum of the following submodules.*

1. *M_1, the summand containing U.*
2. *$M_2 := \widetilde{H}^*(\mathbb{RP}^\infty; \mathbb{Z}_2)$ (modulo elements in degrees 14 and above).*

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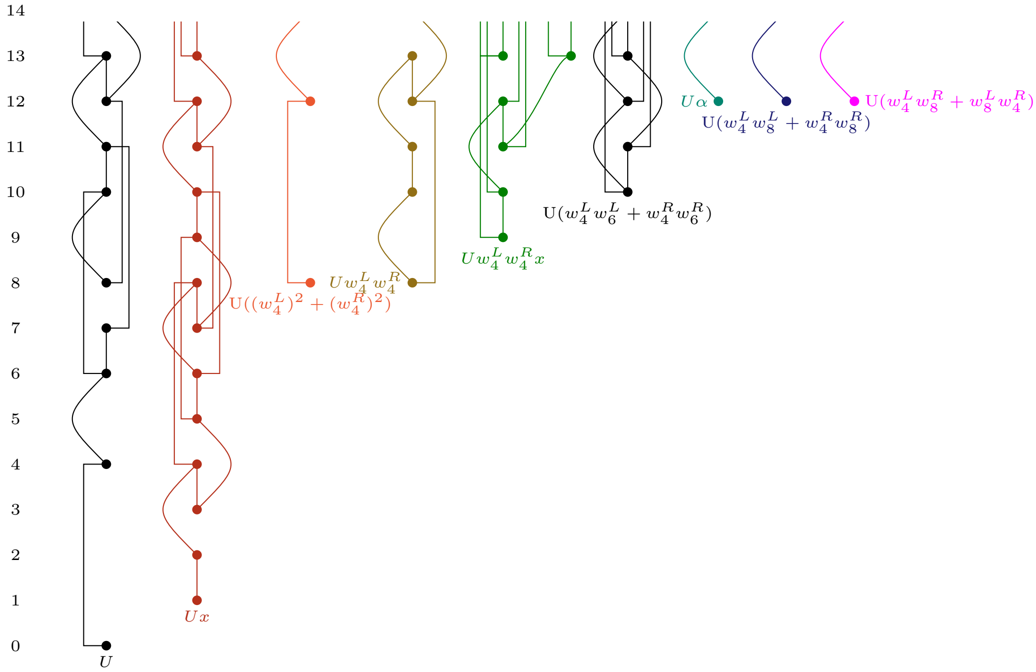

Figure 9. The $\mathcal{A}(2)$-module structure on $H^*(B((\mathrm{Spin}(16) \times \mathrm{Spin}(16)) \rtimes \mathbb{Z}_2)^{V-32}; \mathbb{Z}_2)$ in low degrees. The figure includes all classes in degrees 13 and below. Here $\alpha := (w_4^L)^2 w_4^R + w_4^L (w_4^R)^2$.

3. M_3, *the summand containing* $\mathrm{U}((w_4^L)^2 + (w_4^R)^2)$.
4. M_4, *the summand containing* $Uw_4^L w_4^R$.
5. M_5, *the summand containing* $Uw_4^L w_4^R x$.
6. M_6, *the summand containing* $\mathrm{U}(w_4^L w_6^L + w_4^R w_6^R)$.
7. M_7, *the summand containing* $\mathrm{U}((w_4^L)^2 w_4^R + w_4^L (w_4^R)^2)$.
8. M_8, *the summand containing* $\mathrm{U}(w_4^L w_8^L + w_4^R w_8^R)$.
9. M_9, *the summand containing* $\mathrm{U}(w_4^L w_8^R + w_8^L w_4^R)$.

We draw this decomposition in figure 9.

The next step is to split off some of these summands in a manner similar to [46, Corollary 2.36]. Morally this is exactly the same simplification we used in theorem 3.78 and discussed further in remark 3.82, but the details are a little more complicated.

Definition 4.8. Let $\xi\colon B\mathbb{G}_{16,16'} \to BO$ be the tangential structure defined analogously to $B\mathbb{G}_{16,16}$, but with Spin in place of Spin(16).

Lemma 4.9. *The map* $\mathrm{Spin}(16) \hookrightarrow \mathrm{Spin}$ *induces a map* $\Omega_k^{\mathbb{G}_{16,16}} \to \Omega_k^{\mathbb{G}'_{16,16}}$ *which is an isomorphism for* $k \le 14$.

This means that, for our string-theoretic applications, it does not matter whether we use $B\mathbb{G}_{16,16}$ or $B\mathbb{G}'_{16,16}$.

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Proof. We want to show that the map $MT\mathbb{G}_{16,16} \to MT\mathbb{G}'_{16,16}$ of bordism spectra is an isomorphism on π_k for $k \leq 14$. By the Whitehead theorem we may equivalently use $H^k(-;\mathbb{Z})$, and by the Thom isomorphism, it suffices to show the map $B\mathbb{G}_{16,16} \to B\mathbb{G}'_{16,16}$ is an isomorphism on $\mathbb{Z}$-cohomology in degrees 14 and below. The cohomology rings of these spaces can be computed in two steps: first the Serre spectral sequence for the fibration $B(\mathrm{Spin}(16) \times \mathrm{Spin}(16)) \to BG_{16,16} \to B\mathbb{Z}_2$, then the Serre spectral sequence for the fibration $B^2\mathrm{U}(1) \to B\mathbb{G}_{16,16} \to BG_{16,16}$; and analogously for $B\mathbb{G}'_{16,16}$ with Spin in place of Spin(16). For each of these two steps, the map $\mathrm{Spin}(16) \to \mathrm{Spin}$ induces a map of Serre spectral sequences. and because $H^*(B\mathrm{Spin};\mathbb{Z}) \to H^*(B\mathrm{Spin}(16);\mathbb{Z})$ is an isomorphism in degrees 15 and below, we learn that at each of the two steps, the two spectral sequences are isomorphic in degrees 14 and below, which implies the map $B\mathbb{G}_{16,16} \to B\mathbb{G}'_{16,16}$ induces an isomorphism on cohomology in degrees 14 and below. ☐

Proposition 4.10. *There is a spectrum $\mathcal{Q}$ and a splitting*

$$MT\mathbb{G}'_{16,16} \xrightarrow{\simeq} MTSpin \vee \mathcal{Q}, \tag{4.11}$$

such that the pullback map on cohomology corresponding to the projection $MT\mathbb{G}'_{16,16} \to \mathcal{Q}$ is a map

$$H^*(\mathcal{Q};\mathbb{Z}_2) \cong \mathcal{A} \otimes_{\mathcal{A}(2)} \mathcal{L} \longrightarrow H^*(MT\mathbb{G}'_{16,16};\mathbb{Z}_2) \cong \mathcal{A} \otimes_{\mathcal{A}(2)} H^*((BG_{16,16})^{V-32};\mathbb{Z}_2) \tag{4.12}$$

given by the inclusion of an $\mathcal{A}(2)$-module $\mathcal{L} \hookrightarrow H^*((BG_{16,16})^{V-32};\mathbb{Z}_2)$, followed by applying $\mathcal{A} \otimes_{\mathcal{A}(2)} -$; the quotient of $\mathcal{L}$ by all classes in degrees 14 and above is isomorphic to

$$M_2 \oplus M_3 \oplus M_4 \oplus M_5 \oplus M_7 \oplus M_8 \oplus M_9. \tag{4.13}$$

Proof. The idea is the same as [46, Corollary 2.36]: show that a spin structure induces a $\mathbb{G}'_{16,16}$-structure, such that forgetting back down to BSpin recovers the original spin structure.

Any spin vector bundle $E \to M$ has a canonical $\mathbb{G}'_{16,16}$-structure with a trivial double cover $M' := M \amalg M$, V^L equal to E on one copy of M inside M' and equal to 0 on the other copy of M, and V^R the image of V^L under the deck transformation, as $\frac{1}{2}p_1(V^L) + \frac{1}{2}p_1(V^R)$ (descended to M) is canonically identified with $\frac{1}{2}p_1(E)$. Composing with the forgetful map $B\mathbb{G}'_{16,16} \to B\mathrm{Spin}$ gives a map $B\mathrm{Spin} \to B\mathrm{Spin}$ homotopy equivalent to the identity and therefore maps of spectra $MTSpin \to MT\mathbb{G}'_{16,16} \to MTSpin$, yielding the splitting as promised.

To see the statement on cohomology, one can look at the edge morphism in the Serre spectral sequence for $B^2\mathrm{U}(1) \to B\mathbb{G}'_{16,16} \to BG'_{16,16}$. ☐

As we already know spin bordism groups in the dimensions we need, we focus on computing $\pi_*(\mathcal{Q})^\wedge_2$. Because the cohomology of $\mathcal{Q}$ is of the form $\mathcal{A} \otimes_{\mathcal{A}(2)} \mathcal{L}$, the change-of-rings theorem simplifies the Adams spectral sequence for $\mathcal{Q}$ to the form

$$E_2^{s,t} = \mathrm{Ext}^{s,t}_{\mathcal{A}(2)}(\mathcal{L},\mathbb{Z}_2) \Longrightarrow \pi_{t-s}(\mathcal{Q})^\wedge_2; \tag{4.14}$$

we will then add on the summands coming from Ω_*^{Spin} to obtain the groups in the theorem statement. The first thing we need is $\mathrm{Ext}_{\mathcal{A}(2)}$ of M_2, M_3, M_4, M_5, M_7, M_8, and M_9.

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1. Davis-Mahowald [156, table 3.2] compute $\mathrm{Ext}(M_2)$.
2. In degrees 14 and below, M_3 is isomorphic to $\Sigma^8\mathcal{A}(2)\otimes_{\mathcal{A}(1)}\mathbb{Z}_2$ (meaning the quotients of these modules by their submodules of elements in degrees 15 and above are isomorphic). Therefore when $t-s\leq 14$, there is an isomorphism

$$\mathrm{Ext}^{s,t}_{\mathcal{A}(2)}(M_3,\mathbb{Z}_2)\cong\mathrm{Ext}^{s,t}_{\mathcal{A}(2)}(\mathcal{A}(2)\otimes_{\mathcal{A}(1)}\mathbb{Z}_2,\mathbb{Z}_2), \tag{4.15a}$$

and the change-of-rings theorem (see, e.g., [110, section 4.5]) implies that in all degrees,

$$\mathrm{Ext}_{\mathcal{A}(2)}(\mathcal{A}(2)\otimes_{\mathcal{A}(1)}\mathbb{Z}_2,\mathbb{Z}_2)\cong\mathrm{Ext}_{\mathcal{A}(1)}(\mathbb{Z}_2,\mathbb{Z}_2). \tag{4.15b}$$

Liulevicius [157, Theorem 3] first calculated the algebra $\mathrm{Ext}_{\mathcal{A}(1)}(\mathbb{Z}_2,\mathbb{Z}_2)$.
3. As an $\mathrm{Ext}(\mathbb{Z}_2)$-module, $\mathrm{Ext}(M_4)\cong\mathbb{Z}_2[h_0]$ with $h_0\in\mathrm{Ext}^{1,1}$ [46, (2.43)].
4. $\mathrm{Ext}(M_5)$ is computed in [46, figure 2].
5. Finally, for M_7, M_8, and M_9, we only need to know their Ext groups in degrees 12 and below. For $i=7,8,9$, there is a surjective map $M_i\to\Sigma^{12}\mathbb{Z}_2$ whose kernel is concentrated in degrees 14 and above, so (e.g. using the long exact sequence in Ext associated to a short exact sequence of $\mathcal{A}(2)$-modules [110, section 4.6]) for $t-s\leq 12$, Ext of each of these modules is isomorphic to $\mathrm{Ext}(\Sigma^{12}\mathbb{Z}_2)$, which was computed by May (unpublished) and Shimada-Iwai [158, section 8].

These assemble into a description of the E_2-page of (4.14) (compare [46, Proposition 2.46]).

Proposition 4.16. *The E_2-page of the Adams spectral sequence for $\mathcal{Q}$ in degrees $t-s\leq 12$ is as displayed in figure 10. In this range, the E_2-page is generated as an $\mathrm{Ext}_{\mathcal{A}(2)}(\mathbb{Z}_2)$-module by ten elements:*

- $p_1\in\mathrm{Ext}^{0,1}$, $p_3\in\mathrm{Ext}^{0,3}$, $p_7\in\mathrm{Ext}^{0,7}$, *and* $b\in\mathrm{Ext}^{2,10}$, *coming from* $\mathrm{Ext}(M_2)$*;*
- $a_1\in\mathrm{Ext}^{0,8}$ *and* $a_3\in\mathrm{Ext}^{3,15}$, *coming from* $\mathrm{Ext}(M_3)$*.*
- $a_2\in\mathrm{Ext}^{0,8}$, *coming from* $\mathrm{Ext}(M_4)$*.*
- $c\in\mathrm{Ext}^{0,9}$ *and* $d\in\mathrm{Ext}^{0,11}$, *coming from* $\mathrm{Ext}(M_5)$*.*
- $e\in\mathrm{Ext}^{0,12}$, *coming from* $\mathrm{Ext}(M_7)$*.*
- $f\in\mathrm{Ext}^{0,12}$, *coming from* $\mathrm{Ext}(M_8)$*.*
- $g\in\mathrm{Ext}^{0,12}$, *coming from* $\mathrm{Ext}(M_9)$*.*

The next step is to evaluate the differentials. Unlike the other Adams spectral sequences we considered in this paper, there are several differentials to address, even after using that differentials commute with the action of h_0, h_1, and h_2:

- d_2 on a_1, a_2, a_3, c, d, e, f, and g,
- d_3 on a_1, a_2, and a_3, and
- d_4, d_5, and d_6 on e, f, and g.

The argument is nearly the same as in [46, Lemmas 2.47, 2.50, and 2.56].

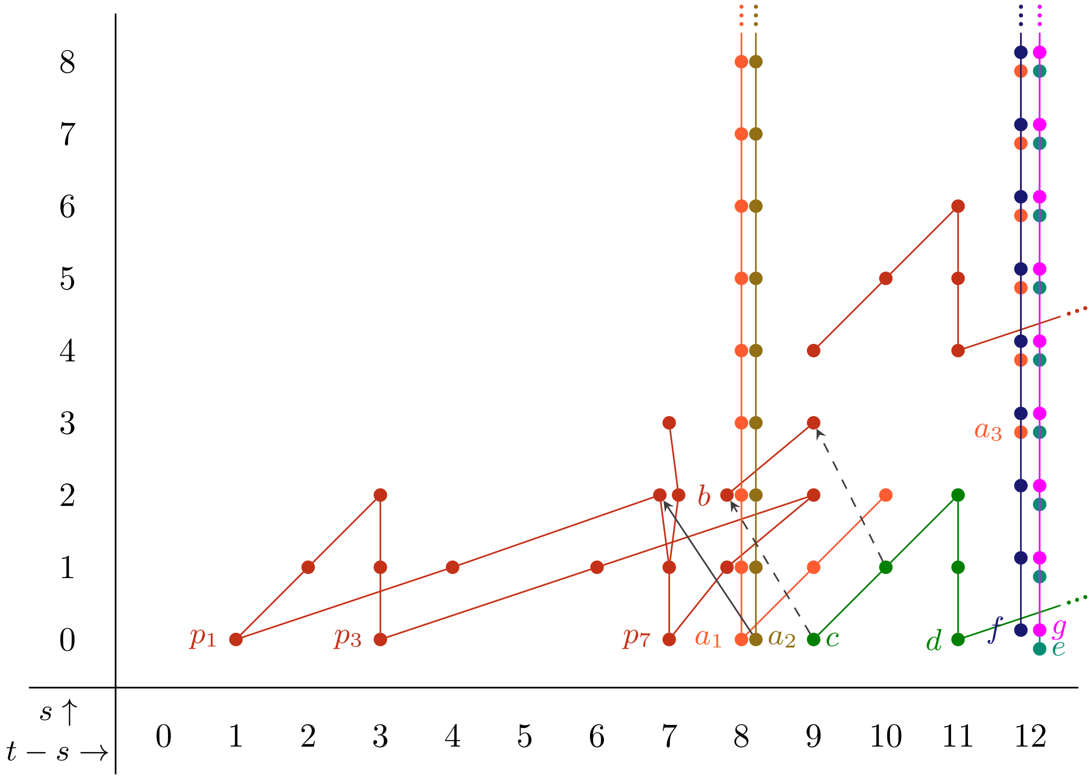

Figure 10. The E_2-page of the Adams spectral sequence computing $tmf_*((BG_{16,16})^{V-32})^\wedge_2$. In lemma 4.17 we show that $d_2(a_2) = h_2^2 p_1$ and that many other differentials vanish. We do not know the values of $d_2(c)$ or $d_2(h_1 c)$, which is why those differentials are denoted with dotted lines.

Lemma 4.17. *$d_2(a_2) = h_2^2 p_1$, and all differentials vanish on a_1, a_3, e, f, and g.*

Proof. If ξ' denotes the tangential structure identical to $\mathbb{G}_{16,16}$ except with $K(\mathbb{Z},4)$ in place of $B\mathrm{Spin}(16)$, then the class $\frac{1}{2}p_1$, interpreted as a map $B\mathrm{Spin}(16) \to K(\mathbb{Z},4)$, induces a map of tangential structures from $\mathbb{G}_{16,16}$-structure to ξ'-structure, hence also a map of Thom spectra, hence a map of Adams spectral sequences. The E_2-page for $\Omega_*^{\xi'}$ is computed in [46, figure 3] in the range $t - s \leq 12$, and looks very similar to our E_2-page in figure 10; using the comparison map between these two spectral sequences, we conclude the differentials in the lemma statement. $\square$

The comparison map would also tell us $d_2(c)$, except that the fate of this differential in ξ'-bordism is not known.

Lastly, we address the class $d \in E_2^{0,11}$. Since d has topological degree 11, its fate affects the size of $\Omega_{11}^{\mathbb{G}_{16,16}}$, hence the possible anomaly theories for the $\mathrm{Spin}(16) \times \mathrm{Spin}(16)$ theory.

Definition 4.18. Embedding each $S^k \hookrightarrow \mathbb{R}^{k+1}$ and using the notation $(\vec{x}, \vec{y}, \vec{z})$ for a vector in $\mathbb{R}^5 \times \mathbb{R}^5 \times \mathbb{R}^4$, let $\mathbb{Z}_2$ act on $S^4 \times S^4 \times S^3$ by the involution

$$(\vec{x}, \vec{y}, \vec{z}) \longmapsto (-\vec{y}, -\vec{x}, -\vec{z}). \tag{4.19}$$

This action is free on $S^4 \times S^4 \times S^3$; let Y_{11} denote the quotient.

Y_{11} is an $(S^4 \times S^4)$-bundle over $\mathbb{RP}^3$.

Lemma 4.20. *Y_{11} has a spin structure.*

Proof. To prove this, we will stably split the tangent bundle of Y_{11}. This is a standard technique; for more examples from a similar perspective, see [41, section 5.2, section 5.5.2], [51, Examples 14.51 and 14.54; Lemma 14.56; Propositions 14.74, 14.83, and 14.101], and [46, Lemma 2.68].

Recall that, since the normal bundle to $S^k \hookrightarrow \mathbb{R}^{k+1}$ is trivialized by the unit outward normal vector field $\vec{v}$, there is an isomorphism $\phi\colon TS^k \oplus \underline{\mathbb{R}} \cong \underline{\mathbb{R}}^{k+1}$; since $\vec{v}$ is $O(k+1)$-invariant, ϕ promotes to an isomorphism of $O(k+1)$-equivariant vector bundles, where $O(k+1)$ acts trivially on the normal bundle and via the defining representation on $\underline{\mathbb{R}}^{k+1}$.

Applying this thrice, we have an isomorphism of vector bundles

$$T(S^4 \times S^4 \times S^3) \oplus \underline{\mathbb{R}}^3 \xrightarrow{\cong} \underline{\mathbb{R}}^5 \oplus \underline{\mathbb{R}}^5 \oplus \underline{\mathbb{R}}^4. \tag{4.21}$$

The $\mathbb{Z}_2$-action on $S^4 \times S^4 \times S^3$ we used to define in Y_{11} in definition 4.18 extends to a linear action on $\mathbb{R}^5 \times \mathbb{R}^5 \times \mathbb{R}^4$, upgrading (4.21) to an isomorphism of $\mathbb{Z}_2$-equivariant vector bundles. In a little more detail:

- $\mathbb{Z}_2$ acts on $T(S^4 \times S^4 \times S^3)$ as the derivative of the involution (4.19).
- $\mathbb{Z}_2$ acts on $\underline{\mathbb{R}}^5 \oplus \underline{\mathbb{R}}^5 \oplus \underline{\mathbb{R}}^4$ as the $\mathbb{Z}_2$-representation described by the same formula (4.19).
- $\mathbb{Z}_2$ acts on the normal $\underline{\mathbb{R}}^3$ by inverting and swapping the first two coordinates, and inverting the third: $(x, y, z) \mapsto (-y, -x, -z)$.[24]

The isomorphism (4.21) of $\mathbb{Z}_2$-equivariant vector bundles descends through the quotient by $\mathbb{Z}_2$ to an isomorphism of vector bundles on Y_{11}; trivial bundles made equivariant by a $\mathbb{Z}_2$-representation descend to vector bundles associated to that representation and the principal $\mathbb{Z}_2$-bundle $\pi\colon S^4 \times S^4 \times S^3 \to Y_{11}$.

In particular, if $\sigma_\pi \to Y_{11}$ denotes the line bundle associated to π and the sign representation σ of $\mathbb{Z}_2$ on $\mathbb{R}$ and $\mathbb{R}$ denotes the trivial representation, then the $\mathbb{Z}_2$-representation $(x, y) \mapsto (-y, -x)$ on $\mathbb{R}^2$ is isomorphic to $\sigma \oplus \mathbb{R}$. Using this, we obtain an isomorphism of vector bundles

$$TY_{11} \oplus \sigma_\pi \oplus \underline{\mathbb{R}}^2 \xrightarrow{\cong} \sigma_\pi^{\oplus 5} \oplus \underline{\mathbb{R}}^5 \oplus \sigma_\pi^{\oplus 4}. \tag{4.22a}$$

Therefore we have an isomorphism of *virtual* vector bundles

$$TY_{11} \xrightarrow[\text{virt.}]{\cong} \sigma_\pi^{\oplus 8} + \underline{\mathbb{R}}^3. \tag{4.22b}$$

For any vector bundle V, $V^{\oplus 4}$ is spin, as can be verified with the Whitney sum formula, and the existence of a spin structure is an invariant of the virtual equivalence class of a vector bundle, so we can conclude. ☐

Proposition 4.23. *Y_{11} admits a $\mathbb{G}_{16,16}$-structure such that the bordism invariant*

$$\int_{Y_{11}} w_4^L w_4^R x^3 = 1. \tag{4.24}$$

[24] In particular, unlike most of the standard examples of the stable splitting technique, the normal bundle is *not* equivariantly trivial. This is because the image of the $\mathbb{Z}_2$-representation in $O(14)$ is not contained in the subgroup $O(5) \times O(5) \times O(4)$.

Proof. The following data describes a $\mathbb{G}_{16,16}$-structure on Y_{11}: identify $S^4 = \mathbb{HP}^1$ and consider the tautological quaternionic line bundle $L \to \mathbb{HP}^1$ on the first S^4 factor, and $L^* := \mathrm{Hom}_{\mathbb{H}}(L, \mathbb{H})$ on the second S^4 factor. These have associated $\mathrm{Sp}(1) = \mathrm{Spin}(3)$ bundles; inflate via $\mathrm{Spin}(3) \hookrightarrow \mathrm{Spin}(16)$ to obtain a $(\mathrm{Spin}(16) \times \mathrm{Spin}(16))$-bundle on $S^4 \times S^4 \times S^3$. The two $\mathrm{Spin}(16)$-bundles are switched when one applies the involution (4.19), so on the quotient Y_{11}, we obtain a principal $G_{16,16}$-bundle $P \to Y_{11}$.

To verify the claim in the first sentence of our proof, we need to check that a spin structure on Y_{11} and the principal $G_{16,16}$-bundle $P \to Y_{11}$ satisfy the Green-Schwarz condition $\frac{1}{2}p_1(TY_{11}) + \frac{1}{2}p_1(V^L) + \frac{1}{2}p_1(V^R) = 0$. In fact, the two parts of this expression vanish separately.

- In (4.22b), we learned that TY_{11} is virtually equivalent to $\sigma_\pi^{\oplus 8} \oplus \mathbb{R}^3$. This bundle turns out to admit a string structure, meaning $\frac{1}{2}p_1(TY_{11}) = 0$. It suffices to prove that $\sigma^{\oplus 8} \to B\mathbb{Z}_2$ admits a string structure, where σ is the tautological line bundle. To see this, recall that $\sigma^{\oplus 4}$ (like the sum of 4 copies of any vector bundle) is spin, so $\sigma^{\oplus 8} \cong \sigma^{\oplus 4} \oplus \sigma^{\oplus 4}$ factors $\sigma^{\oplus 8}$ as the direct sum of two spin vector bundles. Then use the Whitney sum formula for $\frac{1}{2}p_1$ of a direct sum of spin vector bundles [46, Lemma 1.6] to conclude that in $H^4(B\mathbb{Z}_2; \mathbb{Z})$,
$$\frac{1}{2}p_1(\sigma^{\oplus 8}) = 2 \cdot \frac{1}{2}p_1(\sigma^{\oplus 4}). \tag{4.25}$$
Maschke's theorem implies that for $k \geq 1$, multiplication by 2 kills all elements in $H^k(B\mathbb{Z}_2; \mathbb{Z})$, so $\frac{1}{2}p_1(\sigma^{\oplus 8}) = 0$.

- The bundles L and L^* over S^4 have inverse values of p_1, hence also of $\frac{1}{2}p_1$ (since $H^4(S^4; \mathbb{Z})$ is torsion-free, the latter follows from the former). Therefore when we descend from $S^4 \times S^4 \times S^3$ to Y_{11}, the class $\frac{1}{2}p_1(V^L) + \frac{1}{2}p_1(V^R)$ is 0.

Finally, we need to verify $\int_{Y_{11}} w_4^L w_4^R x^3 = 1$. Since $H^{11}(Y_{11}; \mathbb{Z}_2) \cong \mathbb{Z}_2$, it suffices to show that the pullback of $w_4^L w_4^R x^3 \in H^{11}(BG_{16,16}; \mathbb{Z}_2)$ along the classifying map $f_P \colon Y \to BG_{16,16}$ for $P \to Y_{11}$ is nonzero. To do this, first factor f_P into the following diagram of three fibrations:

$$\begin{tikzcd}
S^4 \times S^4 \arrow[r, equal] \arrow[dr] & S^4 \times S^4 \arrow[r, "j"] \arrow[dr] & B\mathrm{Spin}(16) \times B\mathrm{Spin}(16) \arrow[dr] & \\
& Y_{11} \arrow[r] \arrow[d] \arrow[dr, phantom, very near start, "\lrcorner"] \arrow[rr, dashed, bend right, "f_P"'] & \mathcal{X} \arrow[r] \arrow[d] & BG_{16,16} \arrow[d] \\
& \mathbb{RP}^3 \arrow[r] & B\mathbb{Z}_2 \arrow[r, equal] & B\mathbb{Z}_2.
\end{tikzcd} \tag{4.26}$$

Here $\mathcal{X}$ is the $S^4 \times S^4$-bundle over $B\mathbb{Z}_2$ defined analogously to Y_{11} but using $S^\infty = E\mathbb{Z}_2$ instead of S^3. The map $j \colon S^4 \times S^4 \to B\mathrm{Spin}(16) \times B\mathrm{Spin}(16)$ is the map classifying L and L^*.

The diagram (4.26) induces maps between the Serre spectral sequences of the three fibrations; using it, one can compute the pullback of $w_4^L w_4^R x^3$ to Y_{11} and see that it is nonzero, as promised. $\square$

Corollary 4.27. *In the Adams spectral sequence in figure 10, d survives to the E_∞-page; in particular, $d_2(d) = 0$.*

Proof. We reuse the strategy from lemma 3.59: since d is in filtration 0, it corresponds to some characteristic class $c \in H^{11}(BG_{16,16}; \mathbb{Z}_2)$, and d survives to the E_∞-page if and only if there is some closed 11-dimensional $\mathbb{G}_{16,16}$-manifold M such that $\int_M c = 1$. By inspection of figure 9, $c = w_4^L w_4^R x^3$, so by proposition 4.23 we can take $M = Y_{11}$. $\square$

The last step in this calculation is to address extensions. The argument is nearly identical to [46, Lemma 2.59 and Proposition 2.60], though one now has the extra classes $h_0^k a_1$ for $k \geq 0$, $h_1 a_1$, and $h_1^2 a_1$ in degrees 8, 9, and 10 respectively which were not present in the $E_8 \times E_8$ spectral sequence. Fortunately, this new ambiguity is fully resolved by applying the "$2\eta = 0$ trick" to classes of the form $h_1 x$ in standard ways, for example as in [159, Corollary F.16(2)], [160, (5.47)], [51, Lemmas 14.29 and 14.33], and [46, Lemma 2.59], and one learns that there are no hidden extensions in degrees 10 and below. Unfortunately, just like in the $E_8 \times E_8$ case [46, Theorem 2.62], we have not ruled out the possibility of a hidden extension in $\Omega_{11}^{\mathbb{G}_{16,16}}$. $\square$

The generators described in [46, section 2.2.1, section 2.2.2] for $\Omega_*^{\xi^{\mathrm{het}}}$ pull back to generate most of the corresponding $\mathbb{G}_{16,16}$ bordism groups: the difference between a $\mathbb{G}_{16,16}$-structure and a ξ^{het}-structure is that in the latter, Spin(16) is replaced by E_8, so to support our claim that the generators there pull back to generators of $\mathbb{G}_{16,16}$-bordism, we must argue that the $(E_8 \times E_8) \rtimes \mathbb{Z}_2$-bundles are induced from $G_{16,16}$-bundles. As usual we may replace BE_8 with $K(\mathbb{Z}, 4)$, so this amounts to checking that for the generating manifolds in [46, section 2.2.1, section 2.2.2], the degree-4 classes entering the Green-Schwarz mechanism can be written as $\frac{1}{2}p_1(V)$ for some rank-16 spin vector bundle V. By adding trivial summands, we may use lower-rank spin vector bundles.

By inspection of the list of generators in [46, section 2.2.1, section 2.2.2], it suffices to show this for $\mathbb{HP}^2$: the rest of the list of generators there either have degree-4 classes equal to 0, have their ξ^{het}-structure induced from a spin structure (so that we may use the tangent bundle to define the $\mathbb{G}_{16,16}$-structure as in the proof of proposition 4.10), or are products of manifolds otherwise accounted for. For $\mathbb{HP}^2$, the degree-4 classes come from the tautological quaternionic line bundle, hence define a $\mathbb{G}_{16,16}$-structure.

Thus the list of generators in [46, section 2.2.1, section 2.2.2] accounts for most of the generators of the $\mathbb{G}_{16,16}$-bordism groups we have computed. A few manifolds are as yet unaccounted for.

1. There is an 8-dimensional $\mathbb{G}_{16,16}$-manifold Y_8 generating a $\mathbb{Z}$ and whose image in the Adams E_∞-page is a_1. The $\mathbb{Z}_2$ summands lifting $h_1 a_1$ and $h_1^2 a_1$ are also unaccounted for, and can be generated by $Y_8 \times S^1_{nb}$, resp. $Y_8 \times S^1_{nb} \times S^1_{nb}$.

2. Depending on the fate of $d_2(c)$, there may be a $\mathbb{Z}_2$ summand in $\Omega_9^{\mathbb{G}_{16,16}}$ whose generator lifts the class $c \in E_\infty^{9,0}$. In [46, section 2.2.1] no generator was provided and we also do not know what manifold this would be.

3. A generator lifting the class $d \in E_\infty^{0,11}$ was left as an open question in [46, section 2.2.1(11)]. Thanks to proposition 4.23, we can choose Y_{11}.

Proposition 4.28. *Let $V \to S^8$ be the rank-16 spin vector bundle whose classifying map is either generator of $[S^8, B\mathrm{Spin}(16)] = \pi_8(B\mathrm{Spin}(16)) \cong \mathbb{Z}$ (by Bott periodicity), and let $P \to S^8$ be the $G_{16,16}$-bundle induced by $V^L := V$, $V^R = 0$, and the trivial $\mathbb{Z}_2$-bundle. Then (S^8, P) admits a $\mathbb{G}_{16,16}$-structure, and for any such structure, its $\mathbb{G}_{16,16}$-bordism class is linearly independent from the classes of $\mathbb{HP}^2$, B, $\mathbb{RP}^7 \times S^1_{nb}$, and X_8 described in [46, section 2.2.1(8), section 2.2.2], and is not a multiple of any other class.*

Thus (S^8, P) is the generator lifting $a_1 \in E_\infty^{0,8}$.

Proof. It suffices to find a bordism invariant $\psi\colon \Omega_8^{\mathbb{G}_{16,16}} \to \mathbb{Z}^m$ for some m such that the value of ψ on (S^8, P) is linearly independent from the values on the other generators, and also not a multiple of any other element of $\mathbb{Z}^m$. For X_8 and $\mathbb{RP}^7 \times S^1_{nb}$, this will be vacuously true, because the $\mathbb{G}_{16,16}$-bordism classes of these manifolds are torsion, so we focus on the two $\mathbb{HP}^2$s and the Bott manifold described in [46, section 2.2.1(8)].

Let $m = 2$ and ψ be given by the two $\mathbb{Z}$-valued invariants $\int p_2(M)$ and $\int (p_2(V^L)+p_2(V^R))$. The latter is a priori an invariant of manifolds with a $\mathrm{Spin}(16) \times \mathrm{Spin}(16)$-bundle, but it survives the Serre spectral sequence to define an invariant of $G_{16,16}$-bundles and therefore of $\mathbb{G}_{16,16}$-manifolds.

For the $\mathbb{G}_{16,16}$-structure on the Bott manifold and both $\mathbb{G}_{16,16}$-structures on $\mathbb{HP}^2$ specified in [46, section 2.2.1(8)], $\int p_2(M) \neq 0$. However,

$$\int_{S^8} p_2(S^8) = 0 \tag{4.29a}$$

$$\int_{S^8} (p_2(V^L) + p_2(V^R)) = \int_{S^8} p_2(V) = \pm 1, \tag{4.29b}$$

the former because TS^8 is stably trivial and the latter somewhat tautologically from the definition of V. Therefore $\psi(S^8, P)$ is linearly independent from ψ evaluated on the other bordism classes we have considered. Finally, we know that the bordism class of (S^8, P) is not a multiple of some other class because (4.29b) is ± 1, and if the class of (S^1, P) were a multiple, the values of all $\mathbb{Z}$-valued bordism invariants on it would be divisible by some natural number greater than 1. $\square$

Thus we have found generators for all classes except for c and $h_1 c$, which may or may not be trivial, depending on the value of an Adams differential.

4.2 Cancelling the anomaly

Now that we have the generators of $\Omega_{11}^{\mathbb{G}_{16,16}}$ in hand, we proceed to calculate the partition function of the anomaly theory on these generators and show that it is trivial. We are able to do this without knowing the isomorphism type of $\Omega_{11}^{\mathbb{G}_{16,16}}$, similarly to Freed-Hopkins' approach in [41].

Theorem 4.30. *Let α denote the anomaly field theory for the $\mathrm{Spin}(16)^2$ heterotic string on $\mathbb{G}_{16,16}$-manifolds. Then α is isomorphic to the trivial theory.*

Proof. Recall that $\alpha \cong \alpha_f \otimes \alpha_{X_8}$, where α_f is the anomaly of the fermionic fields and α_{X_8} is the anomaly coming from the term $-\int B_2 \wedge X_8$ (2.17) that the Green-Schwarz mechanism adds to the action, as we discussed in section 2.

We will calculate α on a generating set for $\Omega_{11}^{\mathbb{G}_{16,16}}$. Based on [51] and the above discussion, the two generators are

$$B \times \mathbb{RP}^3 \quad \text{and} \quad Y_{11} \tag{4.31}$$

with $\mathbb{G}_{16,16}$-structures described in the previous subsection, corresponding physically to turning on appropriate gauge bundles. Here B is a *Bott manifold*, i.e. a closed spin 8-manifold satisfying $\widehat{A}(B) = 1$, and indeed any choice of B that admits a string structure may be used in this computation. We will use the Bott manifold constructed by Freed-Hopkins in [41, section 5.3]; those authors show $\frac{1}{2}p_1(B) = 0$, so B is string, and that $p_2 = -1440b$, where b is a generator of $H^8(B;\mathbb{Z}) \cong \mathbb{Z}$. Any other Bott manifold is cobordant to this one, so we will not botter studying them all.

The first generator we evaluate α on is $B \times \mathbb{RP}^3$. As discussed in [46, section 2.2.1], this generator has $G_{16,16}$-bundle induced from the principal $\mathbb{Z}/2$-bundle $S^3 \times B \to \mathbb{RP}^3 \times B$ and any inclusion $\mathbb{Z}_2 \hookrightarrow G_{16,16}$ complementary to the normal $\mathrm{Spin}(16)^2$ subgroup. From a physics point of view, this means the Spin(16) gauge bundles are trivial: the $\mathbb{Z}_2$ symmetry switches two copies of the trivial bundle. This implies $X_8 = 0$, so α_{X_8} is trivial. For α_f, we must calculate the η-invariants of the spinor bundles associated to the gauge bundles. We will first dimensionally reduce our theory on B, to obtain a 2d effective theory, and study the corresponding anomaly, which is the dimensional reduction of α_f, on $\mathbb{RP}^3$. As the defining property of a Bott manifold is that the Dirac index is 1, and the gauge bundle is switched off in our example, the 2d spectrum is identical to the ten-dimensional one, so showing that the anomaly on $\mathbb{RP}^3$ is trivial will imply $\alpha_f(B \times \mathbb{RP}^3) = 1$.

We need to know the gauge bundle on $\mathbb{RP}^3$,[25] but because the gauge bundle is trivial on B, we can describe the $G_{16,16}$-bundle on $\mathbb{RP}^3$ as induced from the $\mathbb{Z}_2$-bundle $S^3 \to \mathbb{RP}^3$. Thus we should see how the $G_{16,16}$-representations describing the fermions branch when we restrict to $\mathbb{Z}_2$.

- The 10d fermions in the $(\mathbf{128}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{128})$ give a total of 128 2d fermions transforming as singlets of the swap, and another 128 transforming in the sign representation.
- For the 10d fermions in the $(\mathbf{16}, \mathbf{16})$, the swap is implemented via a matrix with sixteen blocks each having eigenvalues ± 1, again giving 128 fermions in each of the trivial and sign representations of the swap $\mathbb{Z}_2$. Since the 10d fermions have opposite chirality to those in the previous point, the resulting 2d fermions also come in opposite chirality.

With these matter assignments, we obtain a total of 128 $\mathbb{Z}_2$ charged fermions of each chirality, which collectively are anomaly-free (and therefore, gravitational anomalies cancel). Therefore, there is no anomaly under the swap on any background, such as $\mathbb{RP}^3$: the η-invariants all cancel out. Thus $\alpha_f(B \times \mathbb{RP}^3)$ vanishes and the overall anomaly $\alpha_f \otimes \alpha_{X_8}$ vanishes on $B \times \mathbb{RP}^3$.

For Y_{11}, which is an $(S^4 \times S^4)$-bundle over $\mathbb{RP}^3$, we perform a twisted compactification on $S^4 \times S^4$ and study the anomaly of the resulting 2d theory on $\mathbb{RP}^3$. Because Y_{11} is not

[25] In general keeping track of tangential structures on dimensional reductions can be complicated (see, e.g., [161, section 9], but because B has a string structure and the tangential structure of the theory is a twisted string structure, we do not need to worry about this detail.

a product, we must take a little more care with this procedure, but it is not so difficult to show that the assignment from a string 3-manifold N with principal $\mathbb{Z}/2$-bundle $P \to N$ to the manifold

$$\kappa(N) := (S^4 \times S^4) \times_{\mathbb{Z}_2} N, \tag{4.32}$$

where the two copies of S^4 are given the same $\mathbb{Z}_2$-action and Spin(16)-bundles as we used in the construction of Y_{11}, produces a $\mathbb{G}_{16,16}$-manifold for all N and is compatible with bordism, allowing κ to define a functor of bordism categories and therefore a twisted compactification as promised.

The covering S^4 has $\mathrm{Spin}(16)^2$ bundles characterized by a second Chern class

$$c_2^{\mathrm{SO}(16),i} = (-1)^i(b_1 + b_2), \tag{4.33}$$

where b_1, b_2 are the volume forms of both S^4 factors. Now, rather than explicitly computing the dimensional reductions of α_f and α_{X_8} on $\mathbb{RP}^3$, we take advantage of the fact that α is a deformation invariant, so we may deform our 2d theory into something where the value of the anomaly on $\mathbb{RP}^3$ is more obviously trivial.[26] Specifically, we can take a limit in moduli space where the instantons become singular and pointlike, turning into a non-supersymmetric version of the heterotic NS5-brane; as explained in section 3.4.2, the resulting theory becomes symmetric between the two Spin(16) factors, implying that, just like in the supersymmetric heterotic string theories, small instantons of both gauge factors are identified. After deforming in this way the gauge bundle on both Spin(16) factors, we are left with a single pointlike NS5 and a single anti-NS5 in each sphere, which annihilate, leading to a trivial and therefore anomaly-free configuration for the compactified theory, and implying that $\alpha(Y_{11}) = 1$. □

As a bonus, we can answer a question of [46], giving a bordism-theoretic argument for the analogous anomaly cancellation question for the $E_8 \times E_8$ heterotic string. This anomaly cancellation result was first established by Tachikawa-Yamashita in [42] by a different argument.

Corollary 4.34. *The anomaly field theory α for the $E_8 \times E_8$ heterotic string theory taking into account the $\mathbb{Z}_2$ swap symmetry is trivial.*

Proof. The argument for Y_{11} also works in the supersymmetric $E_8 \times E_8$ theory, since the instantons may also be embedded in E_8. In the supersymmetric case, the pointlike limit of the instanton is the ordinary, supersymmetric heterotic NS5-brane, as illustrated in [150], so the E_8^2 anomaly vanishes on Y_{11}.

For $B \times \mathbb{RP}^3$, dimensional reduction leads to 248 singlet and 248 fermions (from the E_8 adjoints) charged under the sign representation. Since the relevant anomaly is controlled by

$$\Omega_3^{\mathrm{Spin}}(B\mathbb{Z}_2) = \mathbb{Z}_8, \tag{4.35}$$

[26] The $\mathrm{SO}(16) \times \mathrm{SO}(16)$ string is non-supersymmetric, and therefore the deformations we have just outlined in the previous paragraph may be obstructed dynamically; for instance, there may be a potential obstructing the small instanton limit. However, since we only wish to compute the anomaly, we may ignore such effects; the only ingredient we really need is the fact, proven in section 3.4.2, that in the small instanton limit the anomaly becomes symmetric between both Spin(16) factors.

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and 248 is a multiple of 8, we conclude there is no swap anomaly either. Finally, we already know that gravitational anomalies cancel in $B \times \mathbb{RP}^3$, since if we forget about the swap this is just an ordinary string background. $\square$

In summary, we have shown that anomalies vanish under both generators of the swap bordism group, both for $\mathrm{Spin}(16)^2$ and $E_8 \times E_8$. The supersymmetric case is covered by the worldsheet analysis in [42], which takes into account twists including the swap we just discussed. Thus, we recover a special case of the general anomaly cancellation result there. On the other hand, our approach covers the non-supersymmetric $\mathrm{SO}(16)^2$ case (for the case of geometric target spaces only).

5 Conclusions

Our world is non-supersymmetric, and that fact alone means that non-supersymmetric corners of the string landscape warrant much more attention than they have received so far, both as a source of interesting backgrounds that might connect more directly to our universe, as well as a new trove of data to check and refine Swampland constraints. In this paper we have moved a bit in this direction by computing the bordism groups and anomalies associated to twisted string structures in the three known non-supersymmetric, tachyon free string models in ten dimensions. The results we obtained are summarized in table 1 for the Sugimoto and $\mathrm{Spin}(16)^2$ groups; for the more complicated Sagnotti 0'B model, we were just able to show that there is a potential $\mathbb{Z}_2$ anomaly.

From the results of the table, it is clear that both $\mathrm{Spin}(16)^2$ and Sugimoto models are free of global anomalies. One might have expected this from the fact that they have a consistent worldsheet description. However, there can be non-perturbative consistency conditions that are not automatically satisfied by the existence of a consistent worldsheet at one-loop, see for instance [162], where a K-theory tadpole is not detected by the closed string sector. It would be very interesting to determine in full generality whether existence of a consistent worldsheet is sufficient to guarantee consistence of the target spacetime. Although we have not settled the question of consistency in the Sagnotti string, we expect that it is also free of anomalies; for instance, upon circle compactification, it can be related to a "hybrid" type I' setup involving an $O8^+$ plane and an $\overline{O8^-}$, both of which are individually consistent [139].

Perhaps the more interesting result of our work is table 1 itself, listing the bordism groups of the $\mathrm{Spin}(16)^2$ and Sugimoto theories. An obvious follow-up to this paper is to use the Cobordism Conjecture [48] together with the groups in table 1 to predict new, non-supersymmetric objects in the non-supersymmetric string theories, similarly to what has been done in type II in [51]. While it is natural to expect these new branes to be non-supersymmetric, it may be worthwhile to pursue this direction in more detail.

One subtlety that must be kept in mind, when considering our results, is that we did not necessarily use the correct global form of the gauge group in our calculations. With the exception of the $\mathrm{SO}(16)^2 \rtimes \mathbb{Z}_2$, we focused on simply connected versions of all the groups, which immensely simplified the calculations. Since any bundle before taking a quotient is still an allowed bundle after taking the quotient, our results show that a very large class of

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k	$\Omega_k^{\text{String}-\text{Spin}(16)^2}$	$\Omega_k^{\mathbb{G}_{16,16}}$	$\Omega_k^{\text{String}-\text{Sp}(16)}$	$\Omega_k^{\text{String-SU}(32)\langle c_3\rangle}$
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
2	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
3	0	$\mathbb{Z}_8$	0	0
4	$\mathbb{Z}^2$	$\mathbb{Z}\oplus\mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}$
5	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
6	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0 or $\mathbb{Z}_2$
7	0	$\mathbb{Z}_{16}$	$\mathbb{Z}_4$	$\mathbb{Z}_2$ or $\mathbb{Z}_4\oplus\mathbb{Z}_2$
8	$\mathbb{Z}^6$	$\mathbb{Z}^3\oplus\mathbb{Z}_2^i$	$\mathbb{Z}^3\oplus\mathbb{Z}_2$	$\mathbb{Z}^3\oplus\mathbb{Z}_2$ or $\mathbb{Z}^3\oplus\mathbb{Z}_2^2$
9	$\mathbb{Z}_2^5$	$\mathbb{Z}_2^j$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$
10	$\mathbb{Z}_2^7$	$\mathbb{Z}_2^k$	$\mathbb{Z}_2^3$	$\mathbb{Z}\oplus\mathbb{Z}_2^2$ or $\mathbb{Z}\oplus\mathbb{Z}_2^3$
11	0	A	0	0 or $\mathbb{Z}_2$

Table 1. Twisted string bordism groups computed in this paper for the $\text{Spin}(16)^2$ theory with and without including the swap (second and third columns), for the Sugimoto string (fourth column), and for the Sagnotti string (fifth column). In the second column, i, j, k are unknown integers, and A is an abelian group of order 64 (see section 4 for details). In the fifth column, there are ambiguities due to undetermined differentials in the Adams spectral sequence; see section 3.3.2 for details. In some cases, the bordism group vanishes in degree 11, which automatically implies the corresponding theory has no anomalies; we also show the anomaly can be trivialized for the $\mathbb{Z}_2$ outer automorphism of the $\text{Spin}(16)^2$ string, even though the bordism group is nonzero. The results in this table can be further used to classify bordism classes and predict new solitonic objects in these non-supersymmetric string theories following [48, 51].

allowed bundles in the Sugimoto and $\text{SO}(16)^2$ theories are anomaly free,[27] but particularly in the Sugimoto case there may be more bundles to check if the gauge group is actually $\text{Sp}(16)/\mathbb{Z}_2$. In the type I theory, we know the group is $\text{Spin}(32)/\mathbb{Z}_2$ and not just $\text{SO}(32)$ due to the presence of K-theory solitons transforming in a spinorial representation [163]. In the Sugimoto theory, the relevant K-theory is symplectic, and there do not seem to be any such solitonic particles [4], suggesting that the group might actually be $\text{Sp}(16)/\mathbb{Z}_2$. It would be interesting to elucidate this point and figure out whether there really are any global anomalies beyond those studied here.

Another result of our paper is a series of arguments and checks that in any heterotic string theory, the Bianchi identity must hold at the level of integer coefficients. Furthermore, satisfying the Bianchi identity even at the level of integer coefficients is not enough to guarantee consistency of the string background; there is also a consistency condition (tadpole) that is detected by $H^3(M;\mathbb{R})$. The general consistency condition is of course that the anomaly of probe strings vanishes; more generally, it is natural to expect that all consistency conditions (tadpoles) of any quantum gravity background come from consistency of probe branes in said background.

Another limitation of our study is that, by following a (super)gravity approach, we must restrict to studying anomalies on *smooth* backgrounds. String theories, both with and without

[27] The equivalence discussed at the beginning of subsection 3.4.2 shows that anomalies vanish for $\text{SO}(16)^2$ even when the correct global form is taken into account.

spacetime supersymmetry, make sense on much larger classes of backgrounds that do not admit a geometric description, such as orbifolds, and which are only analyzed from a worldsheet perspective. These cases are not covered by our analysis. Using modular invariance one can show that the Green-Schwarz mechanism always cancels local anomalies in any consistent worldsheet background [71, 164], with or without spacetime supersymmetry. The question of whether global anomalies also cancel in these non-geometric backgrounds was addressed in [42, 45], where *all* global anomalies are shown to cancel for all gauge groups and dimensions in the ordinary supersymmetric heterotic string theories. This remarkable result rests on the validity of the Segal-Stolz-Teichner conjecture [165], which connects deformation classes of worldsheet theories (or, more generally, two-dimensional $(0,1)$ supersymmetric QFTs) to the spectrum of (connective) *topological modular forms* (TMFs) [117, 166] (see also [167]). The physical interpretation of this more refined generalized cohomology theory is related to "going up and down RG flows" [168], and it includes the familiar string bordism deformations of the target space manifold of a sigma model as well as more exotic, "non-geometric" deformations.

To construct an ordinary, spacetime-supersymmetric heterotic model, all that one needs is a $(0,1)$ SQFT. Such a QFT always has a notion of a right-moving worldsheet fermion number F_R^w, which is gauged by the usual GSO projection to construct a modular-invariant partition function. The original Segal-Stolz-Teichner conjecture applies precisely to $(0,1)$ SQFT's. If we wanted to make such an argument for a spacetime non-supersymmetric string theory (tachyonic or not), we face the obstacle that the GSO projection is different, and it involves *additional* worldsheet symmetries. For instance, the $\mathrm{SO}(16)^2$ theory has a "diagonal" modular invariant partition function, which requires a notion of a left-moving worldsheet fermion number in addition to the $(0,1)$ SQFT structure. Thus, valid $\mathrm{SO}(16)^2$ worldsheet theories are equipped with an additional left-moving $\mathbb{Z}_2$ symmetry, or equivalently, they are equipped with both a spin structure and a $\mathbb{Z}_2$ symmetry. To repeat the argument of [42, 45], one must work with $\mathbb{Z}_2$-equivariant TMF; it would be very interesting to do so.

When we started this project we were actually quite surprised that we could not find a comment on global anomalies of non-supersymmetric tachyon-free strings[28] anywhere in the literature. After all, these constructions are all 25+ years old, and they have a quite distinguished role in the string landscape. In a sense, they look more like our universe than the more familiar, supersymmetric theories! Maybe the reason for this neglect is simply lack of workforce; the last 25 years have brought so much progress on so many areas that the community just had to focus on the most novel or promising ones, and simply left many important questions unanswered. The physics of non-supersymmetric string theories was a victim to this rapid progress. Despite this, recent research in this direction has yielded e.g. metastable vacua [147, 169], novel end-of-the-world defects [170–176], and checks of Swampland constraints [12, 20, 177]. The results that we have presented in this paper are yet another step in this direction. We believe (and hope to have convinced at least some readers) that non-supersymmetric string theories constitute a very interesting arena where there seems to be an abundance of low-hanging fruit that is likely to yield novel lessons both in the Landscape and the Swampland.

[28] Other than [36]; maybe we just missed it.

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4
Probes of Quantum Gravity

This chapter contains the articles:

- *Black holes as probes of moduli space geometry*,
  M. Delgado, Miguel Montero, Cumrun Vafa
  JHEP 04 (2023) 045 arXiv:2212.08676 - Inspire
- *Emergence of Species Scale Black Hole Horizons*,
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Black holes as probes of moduli space geometry

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Abstract: We argue that supersymmetric BPS states can act as efficient finite energy probes of the moduli space geometry thanks to the attractor mechanism. We focus on 4d $\mathcal{N} = 2$ compactifications and capture aspects of the effective field theory near the attractor values in terms of physical quantities far away in moduli space. Furthermore, we illustrate how the standard distance in moduli space can be related asymptotically to the black hole mass. We also compute a measure of the resolution with which BPS black holes of a given mass can distinguish far away points in the moduli space. The black hole probes may lead to a deeper understanding of the Swampland constraints on the geometry of the moduli space.

Keywords: Black Holes in String Theory, String and Brane Phenomenology

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Contents

1 Introduction

One of the basic features of String Theory compactifications is the ubiquity of moduli fields — massless or light scalars that parametrize the internal compactification space and that control the masses and couplings of the theory. The geometry of moduli fields is relevant to the physical content of the theory and is captured by aspects of the Swampland Program including the Distance Conjecture [1]. Given their relevance, it seems an important question how would we probe the global geometry of moduli fields in terms of physical data at one point in moduli space.

More concretely, suppose one has a theory with an exact or approximate moduli space, and we live in a vacuum where the moduli ϕ take some value ϕ_0. One can study directly the physics of the vacuum at $\phi = \phi_0$ by means of scattering experiments, and even learn about the local geometry of the moduli space by studying these couplings. However it is, in general, very difficult to design an experiment to probe the physics at some value $\phi = \phi_1$ which is very far away from ϕ_0. It is precisely in faraway regions where interesting physics (such as decaying towers of states, emergence of perturbative string or decompactification limits, etc.) is supposed to take place. In most string theory literature, one is satisfied with studying the family of vacua parametrized by ϕ, as well as the ϕ dependence of various observables such as masses and couplings. Yet this approach is somewhat unphysical: changing the vev of ϕ everywhere at once costs infinite energy, and once one starts considering configurations where ϕ only changes in a region of finite size, other challenges can appear. One difficulty in designing setups that will probe large variations $\Delta\phi$ is that, when these are transplanckian, they will have a significant gravitational backreaction of their own, and whatever setup we consider is in danger of collapsing into a black hole. This was studied in [2], as well as in the more recent series of papers [3, 4], where it was pointed out that probing arbitrarily large $\Delta\phi$ is in principle possible in the effective field theory, but requires resources (masses, size of the laboratory...) exponentially large in $\Delta\phi$. Furthermore, the constructions described in [2] are not solving Einstein's equations, and so are regarded at best as interesting initial conditions, but they do not provide any concrete way to probe the faraway regions in moduli space.

The basic goal of this short note is to explain how the attractor mechanism allows one to overcome this challenge and do a form of "black hole spectroscopy", where properties of black holes at any one given point in moduli space can be used to probe the vacuum in faraway regions of the moduli space. Indeed, in a 4d $\mathcal{N} = 2$ theory, thanks to the attractor mechanism (first constructed in [5] and further studied in [6–10]), the properties of the vector multiplet moduli space far away from any given vacuum can be studied reliably and in a robust way — simply engineer a black hole such that the near-horizon values of all scalars X^I_h differ significantly from those at infinity. The resulting geometry has the same asymptotics as the vacuum, but the near-horizon geometry constitutes a very long $AdS_2 \times S^2$ geometry, where the scalars are stabilized at the attractor value. The two regions are joined by an intermediate throat in which the fields X^I run. The size of the S^2 (or equivalently, the curvature of the AdS_2) are furthermore controlled by the total value of the black hole charge, which may be safely rescaled to arbitrarily large values without affecting the attractor solution. What this means is that one can, while keeping the attractor value fixed, engineer an $AdS_2 \times S^2$ region where the size of the S^2 is arbitrarily large, and in which the physics looks locally like the vacuum solution on $\mathbb{R}^4$ with the attractor values of the moduli, thus achieving a concrete "laboratory" in which the asymptotic vacuum can be probed. To make this picture concrete, we will show explicitly how the triple intersection numbers of the infinite distance limit, as well as the first subleading correction, can be encoded in term of mass and degeneracy of charged BPS states that can be physically measured in the asymptotic region.

If one has access to arbitrary mass/charge states, it becomes possible to study points in moduli space which are arbitrarily far away and with any desired precision. It is more interesting to study how the distance in moduli space and the resolution of the probing depend on the resources — how well can we do if we have a maximum allowed mass, charge for the states. We study this question in a simple two-parameter family of black holes, finding agreement with the results in [2] that an exponential field range in moduli space are intimately related to the masses which trigger the flow. However, unlike in [2], we have a concrete setup where transplanckian field ranges are attained in the context of a supersymmetric solution, in a time-independent way, thanks to the attractor mechanism.

The rest of the note includes a review of 4d $\mathcal{N} = 2$ black holes and the attractor mechanism in section 2, followed by the main application of black hole spectroscopy in section 3, where we obtain the triple intersection numbers of an asymptotic limit in terms of degeneracy of states. Section 4 quantifies just how far can we go in probing the moduli space geometry for a given mass, and section 5 explains how we quantify the resolution of points on moduli space using physical data at far away points. Section 6 contains a few concluding remarks.

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2 Review of 4d $\mathcal{N}=2$ black hole solutions

We will start by reviewing some elements of Calabi-Yau three-fold X compactifications of Type IIB string theory and their associated black hole solutions, which will be the core of this paper. The reader interested in further details is encouraged to check [11, 12]; here we will only describe the essentials of what we need. At low energies, the effective field theory describing a Calabi-Yau compactification is a four-dimensional $\mathcal{N}=2$ supergravity, coupled to $n_V = h^{1,2}$ abelian vector multiplets and $n_H = h^{1,1}+1$ hypermultiplets, where $h^{i,j}$ are the Hodge numbers of the Calabi-Yau three-fold X. The vector multiplet moduli space is a special Kähler manifold, and its scalars parametrize the complex structure of X. The dynamics of the hypermultiplets decouples completely from that of the vectors in the black holes we will consider, due to the 4d $\mathcal{N}=2$, so we will mostly ignore them in the following.

As explained e.g. in [13], the intersection pairing in the middle cohomology of the Calabi-Yau defines a symplectic (antisymmetric) inner product. Constructing the complex structure moduli space comes down to choosing a symplectic basis $\{A^I, B_I\}$ of 3-cycles in $H_3(X,\mathbb{Z})$ (and the corresponding basis of three forms $\{\alpha_I, \beta^J\}$ of $H^3(X,\mathbb{Z})$), which we take to be orthonormal in the following sense:

$$\begin{gathered}\langle \alpha_I, \beta^J\rangle = -\langle \beta^J, \alpha_I\rangle = \int_X \alpha_I \wedge \beta^J = \delta_I^J \\ \int_{A^I} \alpha_J = -\int_{B_J} \beta^I = \delta_J^I \, ; \quad \int_{A^I} \beta^J = \int_{B_J} \alpha_I = 0 \, ,\end{gathered} \tag{2.1}$$

where $\{I,J\} \in \{0,\dots,h^{2,1}\}$. Every Calabi-Yau manifold has a holomorphic (3,0)-form (see e.g. [11]) that can be decomposed as follows in terms of its A- and B-periods $\{X^I, F_J\}$:

$$X^I = \int_{A^I} \Omega_3 \qquad F_J = \int_{B_J} \Omega_3 \quad \longleftrightarrow \quad \Omega_3 = X^I \alpha_I - F_J \beta^J \, . \tag{2.2}$$

Performing a change $\Omega_3 \to e^f \Omega_3$ has no impact on the complex structure of X. In terms of the scalars X^I, this amounts to an overall re-scaling $X^I \to e^f X^I$ from which it is clear that only $h^{2,1}$ of these $h^{2,1}+1$ scalars are independent. The Kähler potential is given by (see. eg [14])

$$\mathcal{K} = -ln \left(i \int_X \Omega_3 \wedge \bar{\Omega}_3 \right) = -ln \, i \left(\bar{X}^I F_I - X^I \bar{F}_I \right) \, . \tag{2.3}$$

One can now see that a rescaling of Ω_3 corresponds to a Kähler transformation in 4d $\mathcal{N}=2$ language:

$$\Omega_3 \to e^f \Omega_3 \qquad \mathcal{K} \to \mathcal{K} - f - \bar{f} \, . \tag{2.4}$$

The fact that the complex structure of X is unchanged by a rescaling of Ω_3 translates to the 4d $\mathcal{N}=2$ Lagrangian being invariant under Kähler transformations. One can therefore define a Kähler metric on the complex structure moduli space that is invariant under rescalings of Ω_3 by using $\mathcal{K}$ as a Kähler potential. The metric obtained in this way on complex structure moduli space $g_{I\bar{J}} \sim \partial_I \partial_{\bar{J}} \mathcal{K}$ coincides with what can be read off of the kinetic term of the 4d Lagrangian for the complex structure moduli. One can choose a symplectic basis such that a single holomorphic function, the so-called prepotential

$F = F(X)$, encodes all the data of the topological theory. The B-periods can be reexpressed in terms of the prepotential as:

$$F_J(X) = \frac{\partial F(X)}{\partial X^J} . \tag{2.5}$$

One can construct black hole solutions in the 4d $\mathcal{N} = 2$ effective theory by wrapping D-branes on the various cycles of X (see eg. [9, 10, 15, 16]). These are generalizations of Reissner-Nordström black holes, charged under the $n_V = h^{1,2}$ abelian vector multiplets. These black holes have the remarkable property that they are *attractors* for the vector multiplet moduli. This means that these moduli, in general, run along the radial direction until they reach the black hole horizon where their value is entirely determined by the supersymmetric equations of motion, in what is known as the attractor mechanism [5]. The attractor equations that describe this flow relate the charges of the black hole to the values of the moduli at the horizon. Throughout this work, we will use the attractor mechanism as a tool to map black hole thermodynamic properties to the complex structure moduli space.

Let us now review the attractor mechanism of extremal 4d $\mathcal{N} = 2$ black holes in more detail. In type IIB language, one constructs such black holes by wrapping D3 branes on a general 3-cycle $\mathcal{C}$ in X. Indeed, a black hole is identified by the decomposition of $\mathcal{C}$ onto the basis $\{A^I, B_I\}$ or equivalently by its corresponding electric and magnetic charges $\{p^I, q_J\}$. Take Γ to be the 3-form that is Poincaré dual of $\mathcal{C}$, then the corresponding splitting of magnetic and electric charges $\{p^I, q_J\}$ is given by:

$$p^I = \int_{A^I} \Gamma \qquad q_J = \int_{B_J} \Gamma \, . \tag{2.6}$$

Consider a BPS solution charged under the 3-form Γ. Then, the central charge of the black hole is given by:

$$Z = e^{\mathcal{K}/2} \int_X \Omega_3 \wedge \Gamma = e^{\mathcal{K}/2} (p^I F_I - q_I X^I) \, . \tag{2.7}$$

The attractor mechanism acts as a potential for the moduli and drives them to minimizing the central charge at the horizon of the black hole (note that the horizon values of the moduli will differ significantly, in general, from their values at spatial infinity) [8]. This minimization procedure leads to the attractor equations at the horizon, which relate the holomorphic periods to the charges of the black hole and can be written as follows:

$$\begin{aligned} p^I &= \mathrm{Re}\left[C_h X_h^I\right] = C_h X_h^I + \bar{C}_h \bar{X}_h^I \, , \\ q_I &= \mathrm{Re}\left[C_h F_{h\,I}\right] = C_h F_{h\,I} + \bar{C}_h \bar{F}_{h\,I} \, , \end{aligned} \tag{2.8}$$

where the "h" subscripts emphasize that these quantities are evaluated at the horizon and where we have introduced $C \equiv -2i\bar{Z}e^{\mathcal{K}/2}$. The vector multiplet moduli, which can be expressed in terms of the X^I, have arbitrary values infinitely far from the black hole, they vary along the radial direction and are fixed by the attractor equations at the horizon. Solving these equations for the periods at the horizon allows one to obtain the entropy of the black hole (equivalently, its area), which is expressed in terms of the central charge as:

$$S = \pi |Z_h|^2 \, . \tag{2.9}$$

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One can also compute the ADM mass of the black hole, which turns out to be

$$M_{ADM}^2 = |Z_\infty|^2, \tag{2.10}$$

where we have introduced the subscript to emphasize that the ADM mass is obtained by evaluating the central charge Z, viewed as a function of the charges p^I, q_I and the scalar values X^I given in (2.7), with the scalars X^I taken to have their asymptotic values, i.e. evaluated at infinite distance from the black hole. In the particular case where the asymptotic and near-horizon values of the scalars coincide, (2.10) and (2.8) agree: the attractor value of the mass is just given by the near-horizon dynamics. When they do not, the difference is due entirely to the running scalars outside of the horizon contributing to the mass. This follows from the attractor equations, which imply [8]

$$|Z_\infty|^2 - |Z_h|^2 = \int_{-\infty}^{0} d\tau\, e^{-U/2} \sqrt{g_{I\bar{J}} \frac{dt^I}{d\tau} \frac{dt^{\bar{J}}}{d\tau}}. \tag{2.11}$$

In this expression, $t^I \equiv X^I/X^0$ are the physical moduli, e^{2U} is the time-component of the black hole metric, and τ is a certain parametrization of the radial coordinate in which the horizon sits at $\tau = -\infty$ and spatial infinity is at $\tau = 0$. Thus, we see that the difference in mass above the attractor value is just the backreaction of the running moduli.

Finally, we also note that the attractor equations and in particular the charges are invariant under Kähler transformations, which act on the periods and C as follows:

$$\mathcal{K} \to \mathcal{K} - f - \bar{f}, \quad C \to e^{-f} C, \quad X^I \to e^f X^I, \quad F \to e^{2f} F. \tag{2.12}$$

One can obtain the $h^{2,1}$ physical, invariant, moduli t^I by choosing special coordinates such as $X^I = t^I X^0$. Throughout the next sections we will be solving the attractor equations (2.8) by choosing a constant C_h. Solving the attractor equations with different values of C_h will generate a set of black hole solutions with the same attractor point in moduli space but different charges and masses.

In the next section we will exploit the attractor equations in an attempt to map topological data of the Calabi-Yau moduli space to thermodynamic properties of black holes.

3 Probing the prepotential with large black holes

Armed with the attractor mechanism described in the previous section, we will explain how it can be used to achieve a simple form of black hole moduli space spectroscopy, where we relate the properties of faraway points in moduli space to statistical, thermodynamic properties of large charge BPS states in a given vacuum. As described above, the attractor mechanism produces near-horizon $AdS_2 \times S^2$ geometries where the value of the moduli are controlled by the attractor mechanism and can in general be very different from the asymptotic values of the moduli. For concreteness and simplicity, we will be interested in black holes that take the vector multiplet moduli to near-infinite distance limits in their moduli space. In these regions, the prepotential is constrained to take the well-known form:

$$F(X) = -D_{IJK} \frac{X^I X^J X^K}{X^0}\,, \tag{3.1}$$

where $C_{IJK} = 6D_{IJK}$ are integers which, in the Calabi-Yau context, receive the interpretation of the triple intersection numbers of the mirror Calabi-Yau. But it is expected that this structure follows from general quantum gravity principles, even when a Calabi-Yau description is not present (see [17]).

With this, we see from (2.5) that the attractor equations will relate the charges of the black hole directly to the X^I at the horizon and the parameters D_{IJK} (we take C_h to be a constant at the horizon). Naturally, these charges will be very large since the moduli are reaching near-infinite values. Turning things around, solving these equations for the moduli at the horizon would allow us to express the entropy of the black hole (2.9) in terms of the charges $\{p^I, q_J\}$ and D_{IJK}. This would show that if one could measure the entropy and charge of one of these large black holes experimentally, it would be possible to deduce the values of the D_{IJK}. One would therefore recover topological data of the underlying Calabi-Yau from measuring black hole observables at a very different point in moduli space. Furthermore, quantities like electric and magnetic charges, or the degeneracy of charged BPS states (i.e., entropy of the black holes), are actual observables, which one could measure experimentally.

We will just illustrate this method in the simplest example, and assume that we have a single vector multiplet $n_V = 1$. Then, there are just four periods, and from the prepotential (3.1) we have

$$F_1 = -3D_{111}\frac{(X^1)^2}{X^0} \quad \text{and} \quad F_0 = D_{111}\frac{(X^1)^3}{(X^0)^2}. \tag{3.2}$$

One can set $C_h = 1$ at the horizon by a Kähler transformation, and then the attractor equations are given by:

$$\begin{aligned} p^0 &= \text{Re}\left[X^0\right] & \qquad q_0 &= D_{111}\ \text{Re}\left[\frac{(X^1)^3}{(X^0)^2}\right] \\ p^1 &= \text{Re}\left[X^1\right] & \qquad q_1 &= -3D_{111}\ \text{Re}\left[\frac{(X^1)^2}{X^0}\right]. \end{aligned} \tag{3.3}$$

Solving these equations yields the central charge at the horizon in terms of the X^I fields,

$$|Z|^2 = \frac{D_{111}|X^1\bar{X}^0 - X^0\bar{X}^1|^3}{4|X^0|^4}\,. \tag{3.4}$$

Equivalently, one can solve (3.3) for the periods and express the entropy in terms of the charges and D_{111}. For simplicity, we will assume that one of the charges vanishes ($p_0 = 0$), in which case we obtain the entropy as:

$$S = \pi|p^1|\frac{\sqrt{|q_1^2 - 12D_{111}p^1q_0|}}{\sqrt{3}}\,. \tag{3.5}$$

The argument of the square root is always positive if we pick charges such that the attractor equations have a solution. We emphasize that an expression such as (3.5) is anyway only expected to hold for very large charges, and in a one-parameter family of solutions such that the attractor values of the scalar are approaching the infinite distance limit in which (3.1) is approximately valid. One example of such a family can be parametrized as follows: in terms of the physical modulus $t = X^1/X^0$, take the charges that scale, in the $y \sim |t| \to \infty$ limit, as

$$\mathcal{Q}_\infty = \begin{cases} p^0 = 0 \\ p^1 = N \\ q_0 = -y^2 N \\ q_1 = 0 \end{cases} . \tag{3.6}$$

Here, N is an overall rescaling of the charges, that does not affect the attractor value, but which will be important in a number of applications in what follows. Importantly, we have chosen a family of black holes whose charges solve (3.3) but do not depend on D_{111} explicitly. We are trying to encode D_{111} in terms of observables such as charges and the degeneracy of BPS states, and therefore, choosing charges depending on D_{111} would amount to assuming the answer. For the family (3.6), the entropy formula simplifies to

$$S = 2\pi\sqrt{D_{111}}\, yN^2 . \tag{3.7}$$

By counting the number of BPS states with such charges in a 4d $\mathcal{N} = 2$ world, one could use this formula to obtain an experimental evaluation of the triple-intersection number D_{111}, and provides a direct link between moduli space properties and properties of the prepotential. This is an interesting result since one would not expect to be able to probe far away moduli data from measurements in the middle of moduli space, without any knowledge of the underlying compactification. Naturally, the formula (3.7) is to be taken as proof of concept that such a relation can be made. The exact expression will change if we consider a large black hole with charges that scale differently, or if one considers a different Calabi-Yau for the compactification.

A natural next step is to determine whether this framework can also be used to detect subleading corrections in the prepotential. To this effect, we will repeat the above analysis, now including the first correction to the prepotential as one moves slightly into the bulk of moduli space:

$$F(X) = -D_{IJK}\frac{X^I X^J X^K}{X^0} + d_I X^I X^0. \tag{3.8}$$

In the Calabi-Yau case, we can relate d_I to topological properties of the mirror Calabi-Yau via the formula

$$\int_{CY} c_2 \wedge \alpha_I = 24\, d_I, \tag{3.9}$$

where c_2 and α_I are the second Chern class and the corresponding two-form of the mirror Calabi-Yau. Again, one can write the attractor equations in the simplified case where there is only one modulus, where they become

$$p^0 = \operatorname{Re}\left[X^0\right] \qquad q_0 = D_{111}\operatorname{Re}\left[\frac{(X^1)^3}{(X^0)^2}\right] + d_1 \operatorname{Re}\left[X^1\right] \tag{3.10}$$

$$p^1 = \operatorname{Re}\left[X^1\right] \qquad q_1 = -3D_{111}\operatorname{Re}\left[\frac{(X^1)^2}{X^0}\right] + d_1 \operatorname{Re}\left[X^0\right] . \tag{3.11}$$

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One can solve these equations for the periods at the horizon and express the entropy in terms of the charges. For a black hole with a single vanishing charge $p^0 = 0$, one obtains:

$$S = \frac{\pi}{\sqrt{3}} |p^1| \sqrt{12 d_1 D_{111} (p^1)^2 - 12 D_{111} p^1 q_0 + (q^1)^2} \,. \tag{3.12}$$

It is easy to see that this reduces exactly to (3.5) when d_1 is set to zero. One can evaluate the entropy of the large black hole with charges that scale as (3.6) in the $y \sim |t| \to \infty$ limit and obtain:

$$S = 2\pi N^2 \sqrt{D_{111}(y^2 + d_1)} \,. \tag{3.13}$$

Expanding this near $y \to \infty$, one obtains

$$S = \pi N^2 \left[2\sqrt{D_{111}} y + d_1 \sqrt{D_{111}} \frac{1}{y} + \mathcal{O}(y^{-3}) \right]. \tag{3.14}$$

Having previously measured D_{111} using (3.7) with extremely large black holes, one could measure deviations of this expression for slightly smaller black holes and obtain d_1 from (3.13).

An important point in all of the above is to note that we have been using the leading behavior of the black hole entropy. This is valid for large N and one expects to receive corrections suppressed by powers of $1/N^2$ [18]. So for example if we want the D_{111} term to be measurable, we need to ensure that

$$\sqrt{D_{111}} y \gtrsim \mathcal{O}(1/N^2) \,, \tag{3.15}$$

which is automatically satisfied in our case since $y \gg 1$ and D_{111} is an integer. For the subleading term to be measurable we need to make sure

$$d_1 \sqrt{D_{111}}/y \gtrsim \mathcal{O}(1/N^2) \,, \tag{3.16}$$

which would be achievable if we pick $N \gtrsim \sqrt{y}$. In addition to polynomial corrections in prepotential, there are also exponential corrections:

$$\frac{F}{X_0^2} = -D_{IJK} t^I t^J t^K + d_I t^I + K_{\alpha_I} e^{-\alpha_I t^I} + \dots \,, \tag{3.17}$$

where the coefficients K_{α_I} are quantized in the primitive directions [19]. In the one modulus example we have studied (where there is a single coefficient, $\alpha_1 = 1$), to get to the precision to be able to measure K_{α_I}, we need to measure the degeneracy of large charge states, where

$$\sqrt{D_{111}}\, N^2 \gtrsim \mathcal{O}(y^3\, e^{2y}) \,, \tag{3.18}$$

which is consistent with the intuition that measurement of exponentially small corrections require exponentially large charged BPS states. As we will discuss in the next section y itself is exponential in distance in moduli space, so this is a double exponential in terms of distance.

The above procedure could be refined indefinitely, recovering more and more information about the Calabi-Yau by measuring the sizes of smaller and smaller black holes. At

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each order in y, one can solve the attractor equations with the corrected prepotential and obtain the entropy in terms of the charges, the previously determined parameters and the new undetermined ones. Measuring the size of an appropriate selection of black holes will allow one to obtain the value of the undetermined parameters. The examples above describe the case with a single vector multiplet; in general, when $n_V > 1$, one will need a multi-parameter family of black holes at each step. For instance, in the first step, one would need a large black hole with charges analogous to (3.6) for each direction in moduli space in order to recover all of the triple intersection numbers D_{IJK}. Of course, this procedure becomes increasingly more complicated as we go further away from the controlled corners of the moduli space, though see [20], where a similar iterative procedure was used to find generic solutions to the attractor equations at all orders, also incorporating instanton contributions. At low orders, this method is equivalent to our own. Nevertheless, the fact that such a procedure can be carried out in principle suggests that there is no obstruction in recovering the geometry of moduli space at any point, using physical measurements of charged BPS states at other points. However, the BPS degeneracy of states is a function of the attractor point alone, and it is not helpful in relating the asymptotic and attractor values of the moduli in a meaningful way. In the next section we will address this question by considering the energetics of the BPS states and relating it to asymptotic distances in moduli space.

4 Asymptotic black hole properties in moduli space

As we saw in the previous section, one can directly relate the prepotential, and thus, the usual moduli space metric, to degeneracy of BPS states. This suggests that it might be possible to obtain the full geometry of moduli space from other physical measurements. We now show that the attractor flow can correctly capture the notion of asymptotic distance near the boundaries of moduli space; namely, we will show that asymptotically in moduli space, the entropy of large BPS states and also their masses can be directly related to the distance in moduli space to their attractor points.

To do this, we once more consider black holes whose attractor point is near the boundary of moduli space, as in the previous section. Using again the family of black holes with charges (3.6), the corresponding periods at the horizon are given by:

$$\begin{aligned} \mathrm{Re}\left[X^0\right] &= 0\,, \quad \mathrm{Im}\left[X^0\right] = -N D_{111}^{1/2} y^{-1}\,, \\ \mathrm{Re}\left[X^1\right] &= N\,, \quad \mathrm{Im}\left[X^1\right] = 0\,. \end{aligned} \tag{4.1}$$

From these expressions, it is straightforward to obtain the Kähler potential (2.3) in this limit:

$$\mathcal{K} = -\log\left(8N^2\sqrt{D_{111}}y\right)\,. \tag{4.2}$$

The metric in moduli space is

$$ds^2 = 2\partial_t\partial_{\bar{t}}K|dt|^2 = \frac{3}{2}\frac{|dt|^2}{\Im(t)^2} = \frac{3}{2}\frac{dy^2}{y^2}\,, \tag{4.3}$$

so that the distance in moduli space is given by:

$$d \sim \sqrt{\frac{3}{2}} \log y\,. \tag{4.4}$$

From the formula for the entropy (3.7) obtained for this set of charges, one immediately obtains

$$S \sim N^2 e^{\sqrt{\frac{2}{3}} d}\,. \tag{4.5}$$

for this family of black hole solutions. A similar exponential relation holds asymptotically for the ADM mass: using the general expression (2.10), one gets that

$$M_{BPS} = e^{\mathcal{K}_\infty/2} |p_1 F^1_\infty - q_0 X^0_\infty| = N e^{\mathcal{K}_\infty/2} |F^1_\infty + y^2 X^0_\infty|, \tag{4.6}$$

where the subscript $_\infty$ in any quantity denotes its asymptotic values. For large y, the last term is leading, giving a dependence on the ADM mass that agrees parametrically with the entropy, and so

$$M_{BPS} \sim N e^{\sqrt{\frac{8}{3}} d}, \tag{4.7}$$

as well.

These expressions relate the mass of a large BPS state to the attractor point lying at a far away distance in moduli space. In practice, it means that if one wants to probe the moduli space at a large distance d, one needs to create a massive BPS state with energy which is exponentially large in distance. This is reminiscent of the work by Nicolis [2] where it was shown that, in a purely Newtonian setting, it was possible to construct setups with scalar sources that lead to arbitrarily large field ranges, with a size that grows exponentially on the field range. What we are finding is not only in agreement with this, but more broadly, with the findings of [3, 4], which studied transplanckian field displacements in a variety of setups (including 4d dilatonic black holes). As proposed in [4], there really seems to be a universal feature of quantum gravity that arbitrarily large field ranges can be probed at an exponential cost in physical resources. Other instances where one can see this include the extended objects of [21, 22] which probe infinite distances in moduli space; their tension goes exponentially with the distance. It would be very interesting to find the physical mechanism underlying this phenomenon.

A scaling similar to (4.5) was recovered in [23] in relation to the Black Hole Entropy Distance conjecture proposed in [24], a generalization of the ADC to black hole spacetimes. Both [23, 24] proposed identifying the logarithm of the black hole entropy (the horizon area) with a notion of distance, encoded in the change of the metric when the flux changes one unit. The connection to (4.7) is precisely that, when the quantized black hole charges change and the black hole area readjust, so do the vevs of the vector multiplet moduli, and the notion of distance using these or directly the metric as in [23, 24] agree. In any case, equation (4.5) shows that the black holes provide a thermodynamic interpretation for the distance in moduli space, if only asymptotically.

Appendix A discussed the precise realization of the general discussion in this paper in the context of a specific model, namely the mirror quintic M. The results agree with (4.5), as they should.

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5 Resolution of the probing

As we have seen, BPS states can serve as effective probes of far away regions of moduli space via their attractor geometry. However, when solving the attractor equations (3.3), one needs to take into account Dirac quantization, which demands that the charges p^I, q_I are quantized. In the Calabi-Yau picture, the quantization simply maps to the fact that the D3 branes that form the black holes we study must wrap an integer homology class.

Dirac quantization implies that the picture of the moduli space provided by the black holes is not continuous; rather, it is naturally a mesh of points in the moduli space. These issues of quantization are however often ignored in the study of 4d $\mathcal{N}=2$ black holes, simply because of the Kähler transformation (2.12). This transformation tells us that a homogeneous rescaling of the charges does not affect attractor values; as a result, one may simply scale the charges up, to very large values, achieving an arbitrarily dense mesh. While this is true, if one is constrained to finite resources (finite black hole mass, charge, or equivalently, entropy), the mesh allowed by Dirac quantization will be finite, leading to a finite "resolution" in the probing of moduli space. We will determine this resolution momentarily.

In more detail, consider the attractor equations near the boundary of moduli space (3.3). The general solution with non-vanishing charges $p^1>0, q^0<0$ is, asymptotically,

$$X^1 = p_1, \quad X_0 = \sqrt{-\frac{p_1^3}{q_0}D_{111}}, \quad t = \frac{X^1}{X^0} = \sqrt{-\frac{q_0}{p_1 D_{111}}}. \tag{5.1}$$

We see that the attractor value of the physical modulus is insensitive to an overall rescaling of the charges. As we make a small change in the charges, the value of t in (5.5) changes, and we probe a nearby point of moduli space. The smallest such change that can take place, compatible with Dirac quantization, is changing p_1 by one unit while keeping q_0 constant. Under such a change, we obtain that the infinitesimal change in moduli space distance, δd, is given by

$$\delta d = \sqrt{\frac{3}{2}}\frac{\delta t}{t} = \sqrt{\frac{3}{8}}\frac{1}{p_1}, \tag{5.2}$$

where we have used the asymptotic form of the Kähler potential, $\mathcal{K} \sim -3\log t$. Using the asymptotic relation between the moduli space distance and the change in t,

$$d \sim \sqrt{\frac{3}{2}}\log t \quad \Rightarrow \quad t \sim e^{\sqrt{\frac{2}{3}}d}, \tag{5.3}$$

combined with (5.1), one can rewrite (5.2) as

$$\delta d = \sqrt{\frac{3}{8}}\frac{t^2 D_{111}}{|q_0|} \sim \sqrt{\frac{3}{8}}\frac{D_{111}e^{\sqrt{\frac{8}{3}}d}}{|q_0|}, \tag{5.4}$$

in terms of the moduli space distance traversed by the black hole. Now, close to the infinite distance limit, $|t|\to\infty$, (5.1) tells us that p_1 is much smaller than q_0, and so the ADM mass

$$M_{BPS} = e^{\mathcal{K}_\infty/2}\left|p_1 F^1_\infty - q_0 X^0_\infty\right|, \tag{5.5}$$

can be approximated by the second term,

$$M_{BPS} \approx e^{\mathcal{K}_\infty/2} X^0_\infty |q_0|. \tag{5.6}$$

This last equation allows us to replace $|q_0|$ by the black hole mass M in (5.4), yielding an expression

$$\delta d = \sqrt{\frac{3}{8}} D_{111} e^{\mathcal{K}_\infty/2} X^0_\infty \frac{e^{\sqrt{\frac{8}{3}}d}}{M}, \tag{5.7}$$

and finally, defining the resolution of the moduli space probing as the inverse spacing (in analogy with optics), we get

$$r \equiv \frac{1}{\delta d} \sim \frac{M_{BPS}}{e^{\sqrt{\frac{8}{3}}d}} = N, \tag{5.8}$$

where in the last equality we have used (4.7). This equation gives us a notion of how the resolution in moduli space scales with the size of a large black hole whose attractor point is at a distance d in moduli space. Taking d to be a constant, we see that the resolution increases with the amount of energy (black hole mass) at one's disposal. This makes sense, as higher masses and bigger black holes naturally mean higher charges and so the "mesh" of points in moduli space becomes smaller. Furthermore, keeping the mass of the black hole fixed, we see that the resolution will decrease exponentially as we try to explore farther points in moduli space. This fits naturally with previous discussions in [25] relating the Distance Conjecture to the Bekenstein bound and finiteness of quantum gravity amplitudes. The number of states that quantum gravity admits in a box should be finite, and bounded by the area of the box. This means that it should not be possible to construct distinguishable states which probe infinite swaths of moduli space with arbitrarily large resolution in a box of given size. The resolution of this puzzle is, precisely, that the resolution drops quickly and makes far away points indistinguishable without increasing the size of the box.

6 Conclusions

Although the physics of moduli spaces is arguably one of the most important aspects of the Swampland program and string compactifications, the question of how these moduli spaces could be probed in practice, if one was found, has received comparatively little attention. In this short note we have shown how, in 4d $\mathcal{N} = 2$ theories, asymptotic properties of moduli spaces are encoded in black hole solutions in a possibly very far away vacuum in moduli space, finding that quantitative features of the prepotential can be deduced from measurements of BPS degeneracies with appropriate charges. On top of this, the 4d $\mathcal{N} = 2$ BPS states are able to reach arbitrarily large regions in moduli space, at a finite energy cost.

We also studied the quality of the moduli space picture provided by the BPS black holes — how far in moduli space can we go, and with which resolution, given finite resources. While this question is in general a complicated optimization problem, we studied a couple of one-parameter families of black holes, finding that both the distance goes logarithmically with the mass of the BPS black hole and that the resolution of the probing (which is finite due to charge quantization) depend linearly on the size of the black hole. This is

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in general agreement with the findings in [3, 4], and shows that studying a transplanckian field range is possible in gravity, but takes an exponential amount of resources. It would be interesting to explore potential connections between this finding and more information-theoretic approaches to the Distance Conjecture recently put forth in [26, 27].

It is tantalizing that the distance conjecture is also an exponential relation between the mass of the tower and distance in moduli space. However, that involves the mass going exponentially down in distance, unlike the BPS mass that we need to probe such distances, which increases exponentially with distance. It would be interesting to see if there is a relation between these two facts.

One outstanding question is how to generalize our analysis to setups where supersymmetry is not protecting the answer, such as non-supersymmetric string theories or even questions involving hypermultiplets in 4d $\mathcal{N}=2$ theories, for which the attractor mechanism does not provide protection. Although it is likely that qualitative different ingredients are needed, we suspect that the basic point — that black holes are appropriate probes of the moduli space — is likely to apply, too.

Finally, the perspective we have taken in this manuscript is reminiscent of the moduli space holography picture of [28, 29]. In that reference, it was proposed that the bulk of moduli space could be reconstructed from asymptotic data; in our setup, we have done the reverse, studying asymptotic regions from a bulk point in moduli space. And much as in the setup of [21, 22], we have a one-to-one mapping between moduli space and physical space, sourced by the gradients of the fields. As these gradients can in principle be studied via ordinary holography, our perspective may help bridge the gap between these two disparate notions of holography.

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A Asymptotic mass formulae for the mirror quintic

In this section we will particularize the general discussion in this paper to a specific model, namely the mirror quintic M. This is a Calabi-Yau threefold with hodge numbers $h^{1,1} = 101$ and $h^{2,1} = 1$. One can define it by considering the following quotient in $\mathbb{P}^4$:

$$M = \left(\Sigma_i z_i^5 - 5\psi \Pi_i z_i\right) / \mathbb{Z}_5^3 \,. \tag{A.1}$$

The four periods of the mirror quintic were famously studied in [19]. In particular, they can be combined into a period vector with respect to an integer symplectic basis (A^i, B_j) of $H^3(M,\mathbb{Z})$, which in the large complex structure limit, $\psi \to \infty$, take the form [19]:

$$\Pi = \begin{pmatrix} F_0 \\ F_1 \\ X^0 \\ X^1 \end{pmatrix} \xrightarrow{\psi\to\infty} \Pi_{LCSL} = \left(\frac{2\pi i}{5}\right)^3 \begin{pmatrix} \frac{5}{6}t^3 + \frac{25}{12}t \\ -\frac{5}{2}t^2 - \frac{1}{2}t \\ 1 \\ t \end{pmatrix}, \quad \text{with} \;\; t = -\frac{5}{2\pi i}\log(5\psi)\,. \tag{A.2}$$

The Kähler potential at the large complex structure point is thus given by

$$e^{-\mathcal{K}}|_{LCSL} = \frac{32\pi^3 \log^3(5|\psi|)}{75}\,. \tag{A.3}$$

Now, we will solve the attractor equations (2.8) in a slightly different way than in the main text; rather than the choice of charges in (3.6), we will use the choice of charges that exactly leads to the attractor values in (A.2), in a gauge where $C_h = N(\frac{2\pi i}{5})^{-3}$ and writing $t = x + iy$. One immediately obtains, in the limit $y \to \infty$:

$$\mathcal{Q}_\infty = \begin{cases} p^0 = N \\ p^1 = Nx \\ q_0 = -\frac{5}{2}Nxy^2 \\ q_1 = \frac{5}{2}Ny^2 \end{cases}. \tag{A.4}$$

From (A.3), one obtains the distance in moduli space as $d = \sqrt{\frac{3}{2}}\log y$. Finally, we obtain the entropy from (2.9) which, at leading order in y, is:

$$S \sim N^2 y^3 \sim N^2 e^{\sqrt{6}d}\,. \tag{A.5}$$

This agrees with (4.5), with a different exponent since we picked a different set of charges.

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Emergence of species scale black hole horizons

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Abstract: The scale at which quantum gravity becomes manifest, the species scale Λ_s, has recently been argued to take values parametrically lower than the Planck scale. We use black holes of vanishing horizon area (small black holes) in effective field theories coupled to quantum gravity to shed light on how the three different physical manifestations of the species scale Λ_s relate to each other. (i) Near the small black hole core, a scalar field runs to infinite distance in moduli space, a regime in which the Swampland Distance Conjecture predicts a tower of exponentially light states, which lower Λ_s. (ii) We integrate out modes in the tower and generate via Emergence a set of higher derivative corrections, showing that Λ_s is the scale at which such terms become relevant. (iii) Finally, higher derivative terms modify the black hole solution and grant it a non-zero, species scale sized stretched horizon of radius Λ_s^{-1}, showcasing the species scale as the size of the smallest possible black hole describable in the effective theory.

We present explicit 4d examples of small black holes in 4d $\mathcal{N} = 2$ supergravity, and the 10d example of type IIA D0-branes. The emergence of the species scale horizon for D0-branes requires a non-trivial interplay of different 8-derivative terms in type IIA and M-theory, providing a highly non-trivial check of our unified description of the different phenomena associated to the species scale.

Keywords: String and Brane Phenomenology, Black Holes in String Theory, D-Branes

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Contents

1 Introduction

Recent activity in the Swampland Program (see [1–4] for reviews) is providing strong support that the cutoff of an effective field theory (EFT) consistent with quantum gravity is not the Planck scale, but potentially a much lower one, the species scale Λ_s [5–9]. This implies that objects which can be reliably described in the EFT have a lower bound in their size of order Λ_s^{-1}. This is a very profound statement since it leads to quantum gravity effects at length scales which can potentially be parametrically larger than the Planck scale. This happens for instance near infinite distance points in moduli spaces, where the (Swampland) Distance Conjecture (SDC) [10] predicts the appearance of an infinite tower of light particles. Additionally, bounds on how fast these towers and the species scale become light have been proposed and tested in [11–14].

A natural setup in which to explore and sharpen these ideas is in black hole physics. In particular, charged extremal black holes with scalar dependent gauge kinetic functions have an attractor mechanism [15–18] that fixes the vevs of such scalars at the horizon. By appropriately choosing the charges of the black hole, one can tune the point of moduli space that is probed by the horizon. A particular choice of charges leads to small black holes, which classically have zero horizon area (i.e. a singularity), and for which scalars run off to infinite distance as one approaches the black hole core (see [19–21] for other uses of small black holes to explore swampland constraints and [20–29] for other spacetime-dependent configurations probing infinite distances in field space). One would then expect that quantum gravity effects smooth out the singularity by generating a stretched horizon of small but finite size. According to our discussion above, this size should be given by the species scale Λ_s^{-1} (hence parametrically larger than the Planck length). Indeed, it was argued in [19] that in order for small black hole solutions not to violate entropy bounds, they should acquire a finite size, of the order of the UV cut-off.

Black holes have been used at multiple occasions in the literature to probe the species scale [30, 31]. And conversely, the species scale itself is often defined as being the size of the smallest possible black hole (SPBH) that one can reliably describe within an EFT description [5, 32]. One of our objectives is to study how quantum gravitational effects at the species scale come to modify small black hole solutions and ultimately promote them to these non-singular, finite-sized SPBHs, in perfect agreement with the works of [19].

The appearance of stretched horizons for small black holes has long been explored in the context of string theory, e.g. for supersymmetric (and non-supersymmetric) charged black holes in 4d $\mathcal{N} = 4, 2$ supersymmetric compactifications, by the inclusion of higher derivative corrections, resulting in string scale stretched horizons.[1] Actually this corresponds in those examples to precisely the species scale in the infinite distance limit of the corresponding small black hole, in agreement with our discussion above (see [30, 31] for related works).

In the above discussion, the appearance of the species scale associated to the SDC tower seems accidental. We will however argue that there is a direct link, by showing that integrating out the SDC tower produces, in the spirit of Emergence [35–38] (see [39–43] for recent discussions), the higher curvature terms which ultimately lead to the stretched horizon, which is thus naturally set by the species scale. Hence, we provide an explicit microscopic mechanism for the emergence of species scale horizons for this class of small black holes.[2] Our result moreover matches the approach in [47], where the species scale has been argued to be the scale at which higher curvature terms become relevant.

Hence, our work allows us to bring together three definitions of the species scale: the original definition accounting for the number of species in the SDC tower, the scale defining the size of SPBHs, and the scale of higher curvature corrections. The central idea is that small black hole solutions get resolved as SPBHs at the species scale, by including the effect of higher derivative corrections emerging from integrating out the SDC tower. Given that our main tool is small black holes, our results hold in asymptotic regimes in moduli space;

[1]For a complementary viewpoint, see [33, 34].

[2]It would be interesting to explore connections with other proposals to produce stretched horizons parametrically larger thank the Planck scale for small black holes, such as the fuzzball proposal (see [44–46] for reviews).

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it would be interesting to extend our understanding to test recent proposal involving the species scale in the interior of moduli space [30, 47, 48].

We expect that the emergence of species scale stretched horizons should hold beyond 4d systems. In fact, the main point of this work is to explore this question for D0-brane solutions of 10d type IIA. These behave as 10d charged small black holes, of classically zero size, and at whose core the dilaton runs off to infinitely strong coupling. Swampland considerations imply that a species scale horizon should arise. However, despite the fact that systems of N D0-branes have been under scrutiny for decades from diverse perspectives, including supersymmetric quantum mechanics [49, 50], M(atrix) theory [51], holography [52], their combination [53], and MonteCarlo simulations (see [54] for a recent update), the only discussion about D0-brane horizons seems to have appeared in [55].

We will provide several arguments for the existence of this species scale horizon for D0-brane solutions; our arguments include microstate counting, finite temperature considerations and the analysis of higher curvature terms using (a 10d version of) the entropy function. On the other hand, we compute the familiar higher curvature R^4 terms arising from emergence by integrating out the SDC tower of BPS particles (which are D0-branes themselves) and show that it does not suffice to generate the horizon. Hence, the appearance of the species scale horizon implied by Swampland considerations demands the inclusion of further higher derivative corrections. Indeed, we show that one indeed gets a finite size species scale horizon once we include 8-derivative terms involving curvatures and RR 2-form field strength, a computation which we carry out using a lift to 11d. This also nicely dovetails with the fact that the species scale is the 11d Planck scale.

We regard this as impressive evidence of the non-trivial power of the species scale proposal, which provides a rationale for the presence of these extra terms (which are otherwise largely ignored in the literature), and hence probes deep UV properties of M-theory.

One general aspect in our analysis is that the limited knowledge of higher derivative corrections implies that the computations can be carried out including only the leading terms. However, the scales probed, and in particular the species scale, are such that in principle the whole set of higher derivative terms contribute in comparable amounts (hence, the use of *leading* is misleading). The key point however is that the main effect of the correction is to turn the singular behaviour of the two-derivative approximation into a smoothed out behaviour controlled by the species scale, and it suffices to truncate the infinite set of higher derivative terms to a tractable finite subset which captures the essence of this change, namely the presence of the species scale horizon, with the expectation that the remaining terms modify only order 1 numerical factors, but not the parametric dependence producing the species scale. In fact, in specific setups, such as 4d small black holes, this has been substantiated using scaling and other arguments [56, 57]. We expect this lesson to apply to other setups as well, including the 10d type IIA D0-branes. For this case, we do not have a clear argument why a truncation of the higher derivative terms suffices to capture the existence and parametric dependence of the stretched horizon. It would be interesting to argue for this, for instance by looking for general scaling properties of the higher derivative terms in the type IIA effective action with the dilaton.

The paper is organized as follows. In section 2 we consider small black holes in 4d $\mathcal{N}=2$. Section 2.1 reviews the attractor mechanism, including higher curvature corrections, and in

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section 2.2 we study a class of 4d small black holes, the appearance of a species scale horizon from the higher curvature corrections, and the emergence of the latter from integrating out an SDC tower of KK gravitons of M-theory on $\mathbf{S}^1$ (times the CY_3). In section 3 we turn to the case of 10d type IIA D0-branes, regarded as small black holes. In section 3.1 we carry out a microscopic computation of the entropy of the system of N D0-branes and of the appearance of the species scale scaling as $N^{1/2}$. Section 3.2 presents a complementary discussion of the computation of the entropy from the perspective of the finite temperature deformation of the D0-brane supergravity solution.

In section 4 we study the appearance of the horizon from higher derivative terms in the action. In section 4.1 we perform an entropy function analysis of the spacetime solution, and show that general R^4 corrections can lead to the appearance of a species scale stretched horizon matching the microscopic results. In section 4.2 we consider the emergence of certain supersymmetric R^4 terms from integrating out the SDC tower in the limit explored by the solution (which is a D0-brane tower, or equivalently M-theory KK gravitons). In section 4.3 we apply the entropy function to these terms and show they still do not lead to the appearance of the horizon. Finally, in section 4.4 we show that including terms involving the RR 2-form field strength one recovers the species scale horizon for the system. We offer some final thoughts and future prospects in section 5.

2 Warmup example: 4d small black holes

In this section, we illustrate our picture of the diverse appearances of the species scale using small black holes in 4d $\mathcal{N} = 2$ EFTs obtained from compactifications of type IIA theory on a Calabi-Yau threefold. As is well known, there are large classes of BPS black hole solutions displaying the attractor mechanism [15]: vector multiple moduli vary along the radial direction, and, in the $\mathrm{AdS}_2 \times \mathbf{S}^2$ near-horizon geometry, attain values which are fixed in terms of the black hole charges and are independent of the asymptotic value of the moduli (while hypermultiplets remain fixed all along the flow). Small black holes correspond to solutions where the attractor mechanism drives the scalars to infinite distance in field space, leading to a horizon of formally zero size and to a singularity. As has been studied extensively in the literature (see [57, 58] for reviews), higher curvature corrections generically lead to a stretched horizon and a finite entropy, matching microscopic computations in many classes of examples. In section 2.1, we summarize these developments and describe the case of a specific class of small black hole solutions in full detail.

Our new angle is discussed in section 2.2, where we argue that the Swampland Distance Conjecture infinite tower of states becoming light in the infinite distance limit corresponding to small black holes is precisely responsible, via emergence, for the appearance of higher derivative terms producing the stretched horizon. We thus link the appearance of the species scale as derived from the SDC tower, as controlling higher derivative corrections, and as setting the size of the smallest possible black hole in the effective field theory. It is amusing to see that swampland ideas play such a central role in the almost three decade-long success story of microstate explanation of black hole entropy.[3]

[3] For other approaches to the Distance Conjecture and Black Holes, see [19–21, 28, 30, 59, 60]. For the interplay of higher curvature corrections and swampland constrains, mainly the Weak Gravity Conjecture [61], see e.g. [62–64].

2.1 4d $\mathcal{N}=2$ supergravity, attractors and higher derivative corrections

2.1.1 $\mathcal{N}=2$ supergravity black holes

Type II string theory on a Calabi-Yau threefold gives 4d $\mathcal{N}=2$ supergravity theories. We will focus on the physics in vector multiplet moduli space, as hypermultiplets are inert. Including the graviphoton, there are n_V+1 gauge bosons, labelled by $I=0,\dots,n_V$. The structure of the bosonic Lagrangian is

$$\mathcal{L} = R - 2g_{i\bar{j}}\,\partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} + \mathrm{Im}\mathcal{N}_{IJ}\,F^I_{\mu\nu}\,F^{J\,\mu\nu} + \mathrm{Re}\mathcal{N}_{IJ}\,F^I_{\mu\nu}\,\frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{-g}}F^J_{\rho\sigma}\,, \tag{2.1}$$

where the different quantities are defined using special geometry (see [65] for reference). The scalars are parametrized by a set of projective coordinates X^I, and the Kähler potential is determined by the prepotential $F(X)$, a holomorphic function of degree 2, as

$$\mathcal{K} = -\log\, i\left(\bar{X}^I F_I - X^I \bar{F}_I\right)\,, \quad \text{with } F_I = \partial_I F\,. \tag{2.2}$$

The gauge kinetic functions $\mathcal{N}_{IJ}$ are also determined by special geometry, but we will skip its detailed structure.

The microscopic description in type IIA theory is as follows. There are $h_{1,1}$ vector multiplets whose gauge bosons arise from the 10d RR 3-form integrated over 2-cycles ω_i, $i=1,\dots,h_{1,1}$. The Kähler moduli, complexified with the integrals of the NSNS 2-form over the 2-cycles, give rise to the vector moduli; morally, the affine coordinates Z^i. The corresponding $F_i \equiv \partial_i F$ are asociated to the dual 4-cycles.

The structure of the prepotential in the large volume regime is

$$F_0(X) = -\frac{1}{6}C_{ijk}\frac{X^i X^j X^k}{X^0}\,, \tag{2.3}$$

where C_{ijk} are the integer triple intersection numbers of the Calabi-Yau. Away from the large volume limit, the prepotential receives corrections from worldsheet instantons. Also, as discussed later, certain higher derivative corrections can also be encoded in a corrected prepotential.

The theory contains BPS black holes constructed out of D-branes wrapped on holomorphic cycles in the CY. There is a vector of electric and magnetic charges $\Gamma=(q_I,p^I)$. The central charge is

$$\mathcal{Z} = e^{\mathcal{K}/2}\left(q_I X^I - F_I p^I\right)\,. \tag{2.4}$$

Regular black holes have a near-horizon $\mathrm{AdS}_2\times\mathbf{S}^2$ geometry. The values of the moduli at the horizon and the entropy are fixed by the attractor equations

$$\partial_i \mathcal{Z}(X^I,\bar{X}^{\bar{I}},q_I,p^I)|_h = 0\,, \qquad S = \pi|\mathcal{Z}(q_I,p^I)|_h\,, \tag{2.5}$$

where the subindex h indicates the value at the horizon. The solution can be expressed as

$$\begin{aligned} p^I &= \mathrm{Re}[C_h X_h^I] = C_h X_h^I + \bar{C}_h \bar{X}_h^I\,,\\ q_I &= \mathrm{Re}[C_h F_{h\,I}] = C_h F_{h\,I} + \bar{C}_h \bar{F}_{h\,I}\,, \end{aligned} \tag{2.6}$$

where we have introduced $C = -2i\bar{\mathcal{Z}}e^{\mathcal{K}/2}$.

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In the type IIA setup, the charges q_0 and q_i correspond[4] to D0-branes, and D2-branes on 2-cycles, while the charges p^0 and p^i correspond to D6-branes on the CY and D4-branes on 4-cycles.

A celebrated class (given its simple lift to M-theory [66]) is obtained by considering sets of D4-branes wrapped p_i times on the i^{th} 4-cycle and a number q_0 of D0-branes, for $|q_0| \gg p^i \gg 1$. Its attractor behaviour can be easily analyzed in the large volume limit (2.3), see [30] for a recent application. The attractor values for moduli, and the entropy, are (we introduce $q \equiv -q_0$)

$$X^0 = -\frac{1}{2}\sqrt{\frac{\frac{1}{6}C_{ijk}p^ip^jp^k}{q}}\,, \qquad X^i = -\frac{i}{2}p^i\,, \qquad S = 2\pi\sqrt{\frac{1}{6}q\,C_{ijk}p^ip^jp^k}\,. \tag{2.7}$$

The CY volume modulus at the horizon is thus

$$\mathcal{V}_h = \sqrt{\frac{q^3}{\frac{1}{6}C_{ijk}p^ip^jp^k}}\,. \tag{2.8}$$

Note that we need $q \gg p^i$ for a reliable large volume expansion. In section 2.2 we exploit this class of models to build small black holes and discuss its corrections.

2.1.2 Higher derivative corrections and quantum attractors

The discussion of higher derivative corrections to the attractor mechanism has been studied extensively in the literature. In particular, there is a class of $\mathcal{N} = 2$ R^2F^{2g-2} F-term corrections which are computed by the topological string, and whose effects on black holes can be included systematically [67–69], see also [58, 70] for reviews. Following these references, one uses the Weyl superfield $W_{\mu\nu} = F^+_{\mu\nu} - R^+_{\mu\nu\lambda\rho}\theta\sigma^{\lambda\rho}\theta + \ldots$, whose lowers component is the (self-dual piece of the) graviphoton, and defines $\Upsilon = W^2$. The F-term corrections can be encoded in a degree 2 generalized prepotential

$$F(X^I,\Upsilon) = \sum_{g=0}^{\infty} F_g(X^I)\Upsilon^g\,. \tag{2.9}$$

The lowest term F_0 corresponds to the usual prepotential, and higher F_g's are the genus g topological string amplitude. In the presence of these corrections, the attractor equations and the entropy are modified so we have

$$\begin{aligned} p^I &= i(X^I_h - \bar{X}^I_h)\,, \qquad & q_I &= i(F_I(X,\Upsilon)_h - \overline{F}_I(\overline{X},\overline{\Upsilon}_h))\,,\\ \Upsilon|_h &= -64\,, & S &= \pi(\,|\mathcal{Z}_h|^2 + 4\mathrm{Im}(\Upsilon\partial_\Upsilon F)_h\,)\,. \end{aligned} \tag{2.10}$$

The structure of corrections near the large volume limit is

$$F(X,\Upsilon) = -\frac{1}{6}C_{ijk}\frac{X^iX^jX^k}{X^0} + d_i\frac{X^i\Upsilon}{X^0}\,, \tag{2.11}$$

[4]One should take into account induced D-brane charges, e.g. a single D4-brane wrapped on a K3 has -1 units of induced D0-brane charge due to Chern-Simons couplings to background curvature.

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where the C_{ijk} are the triple-intersection numbers of the Calabi-Yau and the d_i are given in terms of the second Chern classes of the Calabi-Yau by

$$d_i = -\frac{1}{24}\frac{1}{64}\int_{\mathbf{X}_6} c_2(T\mathbf{X}_6) \wedge \omega_i \tag{2.12}$$

with ω_i a basis of $H^{1,1}(\mathbf{X}_6)$. While these corrections are present for general black holes, for regular ones they are suppressed with respect to the two-derivative result. However, they are crucial for small black holes for which the classical piece vanishes, so that the extra pieces produce a non-zero entropy, namely a stretched horizon. In the following we consider a particular class of small black holes, and show that the leading correction F_1, which contain 4-derivative R^2 corrections, produce a stretched horizon. In section 2.2 we will show that these corrections precisely follow from the SDC tower and that the horizon size is controlled by the species scale.

We would like to point out that the results can also be derived in the formalism of minimization of the entropy function in an $\mathrm{AdS}_2 \times \mathbf{S}^2$ ansatz near horizon geometry [71]. The added terms in the Lagrangian encoded in the corrected prepotential contribute to a modified entropy function. For small black holes, there is a run away but once the corrections are included, the entropy function can actually be minimized, signaling the presence of a horizon. This approach will be exploited in the 10d setup in section 4.1.

2.2 Small black holes, species scale horizons and the SDC tower

2.2.1 An illustrative class of stretched small black holes

We now consider a specific class of models, based on a 2-modulus version of the large volume expansion (2.3). We take the prepotential

$$F = -\frac{X^1(X^2)^2}{X^0}\,, \tag{2.13}$$

where we have chosen $C_{122} = 6$ for simplicity. This is a simple template that can model an internal space $\mathbf{X}_6$ given by K3$\times\mathbf{T}^2$ (for which extensive studies of small black holes and stretched horizons have been carried out, see [57] for a review), or K3-fibered CY threefolds. The affine coordinates $Z^1 = X^1/X^0$ and $Z^2 = X^2/X^0$ correspond to the sizes of the $\mathbf{T}^2$ (or the base $\mathbf{P}_1$ of the K3 fibration) and of the K3 (fiber), respectively.

We can easily build small black holes with a choice of non-zero charges $p \equiv p^1$, $q \equiv -q_0$, and setting $q_i, p^0, p^2 = 0$. The central charge, in terms of $t_i = \mathrm{Im}Z^i$, is

$$\mathcal{Z} = \frac{q + p(t_2)^2}{2\sqrt{2}\,(t_1)^{1/2}t_2}\,. \tag{2.14}$$

The attractor equations imply that t_2 is stabilized at the horizon at

$$t_2 = \sqrt{\frac{q}{p}}\,, \tag{2.15}$$

which is still in the large volume regime if $q \gg p$, as we assume in the following. The attractor mechanism also leads to $t_1 \to \infty$, so this scalar runs off to infinite distance in moduli space. Consequently the overall volume in string units at the horizon also diverges:

$$e^{-\mathcal{K}_h}|X_h^0|^{-2} = 8\mathcal{V}_h = 8t_1t_2^2 \to \infty\,, \tag{2.16}$$

where the $|X_h^0|^{-2}$ factor accounts for the choice of projective coordinates on the special Kähler manifold such that $\mathcal{V}$ is independent of the Kähler gauge. Using the formulas (2.5), the area of the horizon and therefore the entropy vanish:

$$S = \lim_{\substack{t_1 \to \infty \\ t_2 = \sqrt{\frac{q}{p}}}} \pi |\mathcal{Z}| \to 0 \,. \tag{2.17}$$

We now show that the small black hole acquires a stretched horizon upon the inclusion of the correction of the kind (2.11). In particular, we focus on the correction d_1, so the prepotential is

$$F = -\frac{X^1 (X^2)^2}{X^0} + d_1 \frac{X^1 \Upsilon}{X^0} \,, \tag{2.18}$$

where the last two terms correspond to F_1, the genus 1 topological string amplitude. This corresponds to the class of solutions considered in [30] with $p^i = 0$, $i \neq 1$ (see also [57] for a related class).

The moduli at the horizon and entropy are given by

$$Z^1 = i \frac{\sqrt{p\, q}}{2\sqrt{d_1 \Upsilon_h}} \,, \qquad Z^2 = 0 \,, \qquad S \sim 4\pi \sqrt{d_1 \Upsilon_h \, p\, q} \,. \tag{2.19}$$

We see that the horizon no longer explores an infinite distance in moduli space and that the entropy is non-vanishing. Hence the solution develops a stretched horizon that cloaks up the singularity. The overall volume at the stretched horizon is modified by the curvature corrections (see [58] and references therein) and is given by:

$$e^{-\mathcal{K}_h} |X_h^0|^{-2} = 8 \mathcal{V}_h \sim \frac{q (t_1)_h}{p} \sim \frac{q^{3/2}}{\sqrt{p\, d_1 \Upsilon_h}} \,, \tag{2.20}$$

which is finite, and still in the large volume regime if $q \gg p$.

Recall that the above result could have been obtained using the entropy function formalism, but we skip its explicit discussion. Rather we turn to the discussion of the appearance of the higher derivative correction from the SDC tower.

2.2.2 The species scale horizon

In a d-dimensional EFT weakly coupled to Einstein gravity, the species scale is given by:

$$\Lambda_s \sim \frac{M_d}{N_s^{\frac{1}{d-2}}} \,, \tag{2.21}$$

where N_s is the number of species below the species scale and M_d is the d-dimensional Planck mass. In EFTs obtained from string theory (or compactifications thereof), the number of light species changes as one moves along the moduli space. Thus, the species scale is dependent on the region of moduli-space under consideration. In particular, in infinite distance limits in moduli space where infinite towers of states are becoming exponentially light [10], the species scale falls as the number of light species increases drastically. In order to determine the species scale, we must therefore study what infinite distance limit the (formerly) small black hole was probing.

Recall that the small black holes described in the previous section probes the infinite distance limit $t_1 \to \infty$, with t_2 fixed. Note that in this case, the overall volume modulus (2.16) $\mathcal{V} \sim t_1(t_2)^2$ also goes to infinity at the horizon, where $\mathcal{V}$ is the volume of the CY $\mathbf{X}_6$ in string units. At fixed 4d Planck scale $M_p^2 = M_s^2\mathcal{V}/g_s^2$, the large $\mathcal{V}$ limit corresponds to a large g_s limit in string variables (a similar conclusion follows from the fact that the 4d dilaton lies in a hypermultiplet, so it is constant along the attractor flow, hence $\mathcal{V}/g_s^2$ remains fixed). Hence, the limit of our interest correspond to a large g_s regime. There is therefore a tower of D0-brane particles becoming light; this also follows from the fact that the central charge for k D0-branes is given by $Z_{\text{D0}} = k(t_1)^{-1/2}(t_2)^{-1}$, which goes to zero in the limit. Actually, there may be other towers of particles becoming light, but they correspond to branes wrapped on some internal cycles, so they are suppressed in the regime $q \gg p$. The D0-brane particles thus encode the leading correction. Incidentally, we also note that the infinite distance limit explored by the singular two-derivative solution is a standard decompactification limit to M-theory compactified on $\mathbf{X}_6$, rather than an emergent string limit [72, 73]. Indeed, one can show that the volume of the Calabi-Yau in M-theory units is related to the 4d dilaton and is therefore constant, signalling that there is no decompactification to 11d M-theory.

The computation of the species scale in this infinite distance limit can be done in different ways. We choose to focus on the approach of [48], where it was argued that higher derivative corrections to the EFT could be used as a proxy for determining the species scale. In this framework, the species scale acts as the energy scale that suppresses higher derivative corrections to the EFT. This recipe was applied to the case of Calabi-Yau compactifications of type II string theories to find that the species scales is given by:

$$\Lambda_s = \frac{M_4}{\sqrt{F_1}}\,, \tag{2.22}$$

where once more, F_1 is the genus 1 topological free energy. Taking (2.19) and $F_1 \sim Z^1 d_1 \Upsilon$, we see that at the horizon of the stretched black hole, we have:

$$S \sim \left(\frac{\Lambda_s}{M_4}\right)^{-2}\,. \tag{2.23}$$

The radius of the stretched horizon is therefore given exactly by the species scale.

Evaluating the species scale can also be done by counting the number of D0 states that are light in the EFT, in the spirit of [39–43], using (2.21). In the next section we will bridge these two interpretations of the species scale by showing how the higher derivative corrections to the EFT Lagrangian originate from integrating out the tower of D0 states, in the spirit of Emergence.

2.2.3 The SDC tower and higher derivative emergence

The SDC implies that the description of infinite distance limits in moduli space require the inclusion of microscopic physics, in particular an infinite tower of states whose masses decrease exponentially with the field distance [10]. On the other hand, in the previous section we have argued that the singular behaviour of the solution is cloaked by a stretched horizon if suitable higher derivative corrections are included. In this section we show that the two UV ingredients are related, and in fact the distance conjecture *implies* the appearance of

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higher derivative corrections of the necessary kind to generate the stretched horizon. Hence, the two explanations are two sides of the same phenomenon and link the different physical interpretations of the species scale.

The key idea is that the R^2 terms can be seen as generated by an infinite tower of 4d BPS D0-brane states, namely KK gravitons running on an $\mathbf{S}^1$ compactification of the 5d theory given by M-theory on $\mathbf{X}_6$. In fact, they correspond to the leading term in the Gopakumar-Vafa expansion [74, 75] in the large volume limit (where the contribution from D2-branes is subleading).

Indeed, M-theory on $\mathbf{X}_6$ contains 5d BPS states which correspond to 11d gravitons on the groundstate of the $\mathbf{X}_6$ compactification. These are given by the K3 cohomology classes, giving an overall multiplicity of 24, times a linear dependence on the Kähler modulus of Z^1. These 5d BPS states can run with KK momentum in the $\mathbf{S}^1$ compactification to 4d, leading to the 4d tower of D0-brane states.[5]

Hence the problem is just the computation of 1-loop diagram, see [76] for computations in this spirit (see also [77–82] for related computations in other setups). For later convenience, we present the general n graviton scattering amplitude in d dimensions in the worldline formalism:

$$\mathcal{A}_{d,n} = \frac{1}{n!\,\pi^{d/2}} \sum_k \int d^d\mathbf{p} \int_0^\infty \frac{d\tau}{\tau}\, \tau^n\, e^{-\tau\left(\mathbf{p}^2+\frac{k^2}{R^2}\right)} . \tag{2.24}$$

Here τ is the worldline parameter, k is the KK momentum, and R is the $\mathbf{S}^1$ radius.

We want to obtain the R^2 corrections to the effective action, which are encoded in two graviton scattering. For pedagogical reasons, it is easier to start from the 6d perspective of M-theory on K3$\times\mathbf{S}^1$ and postpone compactifying on the $\mathbf{T}^2$. Thus, we particularize to $d = 6$, $n = 2$. The above expression corresponds to the contribution of a single K3 ground state, and diverges in two ways: because of the integral and because of the infinite sum. Since the sum over momentum modes is infinite, we can perform a Poisson resummation, so that both are nicely combined. We get

$$\frac{1}{\pi^{3/2}}\mathcal{A}_{6,2} = \pi^{3/2}\tilde{K}\int_0^\infty d\hat{\tau}\,\hat{\tau}^{1/2}\sum_l e^{-\pi\hat{\tau}R^2l^2} = C\tilde{K} + \frac{\zeta(3)}{\pi R^3}\tilde{K}\,, \tag{2.25}$$

where $\hat{\tau} = \tau^{-1}$. This Poisson summation trades the KK momentum k for the winding number l of the worldline along the $\mathbf{S}^1$. The only divergence is now in the $l = 0$ piece, which has been isolated as the first terms in the second equality; C is an unknown coefficient regularizing the divergence. Including the K3 ground state multiplicity of 24 and the $\mathbf{T}^2$ modulus dependence upon compactification to 4d, we have

$$\frac{1}{\pi^{3/2}}\mathcal{A}_{4,2} = 24Z^1\left[C\tilde{K} + \frac{\zeta(3)}{\pi R^3}\tilde{K}\right], \tag{2.26}$$

where $\tilde{K}$ is a 4d kinematical matrix. This is of the form introduced in the prepotential (2.18), hence it is precisely of the form required to produce a stretched horizon. We note that the same 4d result (2.26) could have been obtained via a 1-loop computation in the 10d theory

[5] The D0-brane tower is actually analogous to that we will consider in the 10d context in section 4.2, as they are simply related by compactification.

compactified directly on K3$\times\mathbf{T}^2$, with the Z^1 prefactor arising from the momentum integral over the $\mathbf{T}^2$. Hence, the 4d coupling of interest arises from higher derivative emergence.

This nicely illustrates the links among the appearance of the species scale from the SDC tower, the higher-curvature corrections and the stretching of a small black hole into the SPBH.

We finish with a comment in hindsight. It is a well-known fact that the R^2 corrections encoded in the corrected prepotential can be seen to arise from the compactification of suitable 10d R^4 terms [83]. For concreteness, focusing on the particularly simple case of compactification on $\mathbf{X}_6$ =K3$\times\mathbf{T}^2$, we start from the familiar 10d supersymmetric R^4 terms, and put two curvature insertions in K3 to get a 6d R^2 term, with a prefactor of $\chi(K3)=24$; upon further integration over $\mathbf{T}^2$ (which provides no background curvature) we obtain the 4d R^2 term with the Z^1 prefactor. It is easy to generalize this argument to general CY$_3$ compactifications and obtain the corrections[6] (2.11) with d_i essentially determined by the second Chern classes of $\mathbf{X}_6$

$$d_i \sim c_{2\,i} = \int_{\mathbf{X}_6} R\wedge R\wedge \omega_i\,, \tag{2.27}$$

where ω_i are the Poincaré dual 2-forms of the basis 2-cycles. In fact, the above is the simplest way to derive the 4d R^2 corrections [83] (see [30] for a recent application). The dimensional reduction just described underlies the fact that the 4d R^2 terms can be obtained from integrating out a tower of 5d KK gravitons is related to a similar derivation of 10d R^4 terms from 11d M-theory gravitons, as we describe in section 4.2.

3 D0-branes and the species scale horizon

We now turn to the discussion of the system of N D0-branes in 10d type IIA theory. We will argue for the existence of a stretched horizon controlled by the species scale, and explore its appearance from higher derivative couplings and their interplay with emergence upon integrating out the states in the corresponding SDC tower.

We start by describing D0-brane systems as small black holes in the theory. The relevant part of the string-frame type IIA action is

$$S=\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left[e^{-2\phi}\left(R+4\partial_\mu\phi\partial^\mu\phi\right)-\frac{1}{2}|F_2|^2\right]\,, \tag{3.1}$$

where $2\kappa^2=(2\pi)^7\alpha'^4$. The D0-brane solution is

$$\begin{aligned} ds^2 &= f(r)^{-1/2}(-dt^2)+f(r)^{1/2}(dx_\perp^2)\,,\\ e^{(\phi-\phi_\infty)} &= f(r)^{3/4}\,,\\ f(r) &= 1+\frac{\rho^7 g_s^\infty N}{r^7},\quad \text{with } \rho^7=(4\pi)^{\frac{5}{2}}\,\alpha'^{\frac{7}{2}}\,\Gamma\!\left(\frac{7}{2}\right). \end{aligned} \tag{3.2}$$

[6]Actually, there is one further constant correction to the prepotential, proportional to the Euler characteristic $\chi(\mathbf{X}_6)$, arising from integrating R^3 over the space $\mathbf{X}_6$, and reabsorbing the resulting correction to the 4d Einstein term via a change of frame [83]. This will not be relevant for our discussion.

We will be more interested in the Einstein frame action

$$S_{\rm IIA} = \frac{1}{2}\int d^{10}x\sqrt{-g}\,\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{\frac{3}{2}\phi}|F_2|^2\right)\,, \tag{3.3}$$

where, from now on, we use 10d Planck units. The D0-brane solution reads

$$\begin{aligned} ds^2 &= f(r)^{-7/8}(-dt^2) + f(r)^{1/8}(dx_\perp^2)\,,\\ e^{(\phi-\phi_\infty)} &= f(r)^{3/4}\,,\\ f(r) &= 1 + \frac{\rho^7 g_s^\infty N}{r^7}, \quad \text{with } \rho^7 = (4\pi)^{\frac{5}{2}}\,\alpha'^{\frac{7}{2}}\,\Gamma\left(\frac{7}{2}\right). \end{aligned} \tag{3.4}$$

Although not usually described from this viewpoint, this solution provides an interesting analogy with small black holes in lower dimensional theories. It has an $\mathbf{S}^8$ horizon of zero size at its core, where a scalar (the dilaton) goes off to infinite distance in moduli space, resulting in a singularity. The latter is not worrisome, because it simply reflects the presence of a source, whose microscopic description is beyond the two-derivative supergravity approximation.

On the other hand, swampland arguments would imply that, as we are driven to the core of the solution, namely near infinite distance limit in the dilaton moduli space, the effective field theory should include additional degrees of freedom, corresponding to an SDC tower of states. Inclusion of these in the effective field theory leads to a lowering of the cutoff scale and hence a minimal size for the system, which would naturally be associated to a species scale horizon. In the following sections we provide several arguments that such a horizon exists and is given by the species scale.

3.1 Microscopic derivation of the entropy and of the species scale

The most direct way to argue that the D0-brane system develops a finite size horizon is to show that it has a large number of microstates, to which one can associate an entropy. This provides a lower bound on the effective size of the system, by holographic bounds.[7] In this section we perform the microscopic computation of the entropy of a system of D0-branes with total charge N (which is familiar from similar combinatorics problems in string theory), and also explain the microscopic derivation of the species scale (which is a novel result).

Recall that consistency of the description of 10d type IIA as M-theory on $\mathbf{S}^1$ requires that, for each value of k, there is exactly one threshold bound state in the k D0-brane system (corresponding to the 11d graviton with k units of KK momentum) [84]. Hence, for a total D0-brane charge N, the number of microstates is given by the number of partitions $p(N)$. The asymptotic behaviour for large N is given by the celebrated Hardy-Ramanujan formula

$$p(N) \sim \frac{1}{4\sqrt{3}\,N}\exp\left(\pi\sqrt{\frac{2}{3}}\sqrt{N}\right)\,. \tag{3.5}$$

The entropy of this thermodynamic ensemble for large charge is thus

$$S \sim \sqrt{N}\,, \tag{3.6}$$

[7]Note that in general the Bekenstein-Hawking area law gets corrected when the action includes higher curvature terms; hence, this relation will be made more precise in section 4.1.

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up to $\log N$ corrections. It is tantalizing to speculate the latter may be related to similar log corrections to the species scale discussed in [85].

The above expression implies that the corresponding horizon size in a gravitational description of the system is parametrically larger than the Planck scale, as we will explain in later sections. In fact, the expectation that it corresponds to the species scale can already be supported from the viewpoint of microscopic combinatorics, as we show next.

One may have expected that, in order for a charge N black hole to be describable in an effective field theory, the latter should include at least N species (i.e. different D0-brane bound states). However, this is not correct and highly overestimates the number of species. In fact, rephrasing the definition in [48], the number of species N_s should be the minimum necessary to explain the entropy of black hole entropy. In other words,[8] if we consider a theory with the number of species given by k and want to build a charge N black hole, the number of ways to do so is given by $p_k(N)$, the number of partitions of N with each part not larger than k. This is equivalent to the number of partitions of N into at most k parts.[9] A classic result in [88] gives the asymptotic behaviour

$$\lim_{N\to\infty} \frac{p_k(N)}{p(N)} = \exp\left(-\frac{2}{C}e^{-\frac{1}{2}Cx}\right) \tag{3.7}$$

for

$$k = C^{-1}N^{\frac{1}{2}}\log N + xN^{\frac{1}{2}}\,, \quad \text{with } C = \pi\sqrt{\frac{2}{3}}\,. \tag{3.8}$$

Summing up, on the one hand the entropy of a black hole made out of N D0 branes in a theory with k species is given by $p_k(N)$ in (3.7). On the other, we know the entropy of such a black hole is given by $p(N) \sim \sqrt{N}$ by microstate counting. We now show that this scaling can be also recovered if and only if the number of species in the theory is at least of order $\sqrt{N}$. Indeed, equation (3.7) gives an order one number as $N \to \infty$ (so that both $p(N)$ and $p_k(N)$ are of the same order) unless $x \to -\infty$. Imposing that x should be bounded from below, (3.8) automatically implies that $k \gtrsim \sqrt{N}$ asymptotically, where we are ignoring a multiplicative $\log N$ correction. Hence, the number of species $N_s = \sqrt{N}$ suffices to explain the black hole entropy scaling. Equivalently, most microscopic configurations can be built using blocks of bound states of at most $N_s = \sqrt{N}$ D0-branes.

In fact, we incidentally note that the distribution is dominated by states containing at least one bound state of *exactly* $N_s = \sqrt{N}$ D0-branes. Following [89], we introduce the number $p(N,k)$ as the number of partitions of *exactly* k parts (equivalently, partitions with at least one part *exactly equal* to k). One can define a probability measure for the likelihood of a random microstate containing at least one bound state of k D0-branes:

$$f_{N,k} = \frac{p(N,k)}{p(N)}\,, \quad \sum_{k=1}^{N} f_{N,k} = 1\,. \tag{3.9}$$

[8]Similar counting problems arise in many other setups, for instance characterizing 1/2 BPS states with potentially large R-charges in AdS/CFT, see e.g. [86, 87].

[9]The equivalence is clear by using the relation of partitions with Young diagrams. A partition of $N = l_1 + \ldots + l_k$ with $l_i \geq l_{i+1}$ with *at most* k parts can be depicted as a Young diagram with N boxes arranged in k rows of lengths l_i. By conjugating the diagram, i.e. exchanging the roles of rows and columns, it corresponds to a partition of N with arbitrary number of parts, but each part being *at most* k.

Defining the quantity

$$X(k) = \frac{k}{\sqrt{N}} - \frac{1}{C}\log n\,, \tag{3.10}$$

the probability distribution takes the asymptotic form

$$f_{N,k} \sim e^{-\frac{C}{2}e^{-\frac{2}{C}X(k)}} - e^{-\frac{C}{2}e^{-\frac{2}{C}X(k-1)}}\,. \tag{3.11}$$

The probability was shown in [90] to be maximized at a value

$$k_0 \sim \frac{\sqrt{6}}{\pi}\sqrt{N}L + \frac{6}{\pi^2}(3(L+1)/2 - L^2/4) - \frac{1}{2}\,, \quad \text{with } \; L = \log(\sqrt{6N}/\pi)\,. \tag{3.12}$$

This shows that the species scale is exactly given by $N_s = \sqrt{N}$ (and for instance, not smaller), as we had anticipated.

In the previous description, we simply used the microscopic D0-brane combinatorics, and the abstract definition of species scale. In an actual spacetime description of the system, it will be manifest that the D0-brane stack is a small black hole exploring the infinite distance limit of strong coupling, i.e. decompactification to 11d Mtheory. This will allow for the interpretation of the species scales as the 11d Planck scale. We thus turn to the description of the stretched horizon and the species scale from the spacetime perspective.

3.2 Hot D0-branes

In the previous section we have argued that the D0-branes solution develops an species scale-sized stretched horizon. Naively, a direct approach to confirm the existence of a stretched horizon would be to solve the equations of motion including all the higher derivative corrections. However, given the lack of knowledge about the precise form of all such corrections, this is actually not feasible. In this section, we propose an alternative way to use spacetime equations of motion to explore the finite entropy of the system, in a universal way insensitive to the details of such corrections. We can explore the resolution of the singularity by studying the system at a finite temperature, and determine its properties as a function of N in the limit of vanishing temperature. We will find that the entropy goes as $N^{\frac{1}{2}}$, in perfect agreement with the microstate counting of the previous section. This provides further evidence that the would-be stretched horizon of a stack of D0 branes should have an area that scales as $N^{\frac{1}{2}}$.

We take the near-extremal (finite temperature) solution for D0-branes (see e.g. [52]), with a horizon r_0. This comes down to introducing factors of

$$f_0 = \left(1 - \frac{r_0^7}{r^7}\right) \tag{3.13}$$

in the metric (3.4), where r_0 is tied to the energy density ϵ of the branes above extremality:

$$r_0^7 \sim \alpha'^4 g_s^2\,\epsilon\,. \tag{3.14}$$

In the limit where $\epsilon \to 0$, we recover the extremal (zero temperature) stack of D0 branes.

In the spirit of the gauge/gravity correspondence for general D-branes in [52], we will carry out the computation of the entropy in the near-horizon limit

$$r \to 0\,, \qquad \alpha' \to 0\,, \qquad U = r/\alpha' = \text{fixed}\,, \qquad g_{QM}^2 = (2\pi)^{-2} g_s \alpha'^{-3/2} = \text{fixed}\,. \tag{3.15}$$

Here g_{QM} is the coupling of the D0-brane worldvolume quantum mechanics. It is useful to define the dimensionless coupling at some energy scale U

$$g_{\text{eff}}^2 \sim g_{QM}^2 N U^{-3} \sim \lambda U^{-3}\,, \tag{3.16}$$

where $\lambda = g_{QM}^2 N$ is the (dimensionful) 't Hooft coupling.

The solution in the limit becomes

$$\begin{aligned} \frac{ds^2}{\alpha'} &= -\frac{U^{7/2}}{\sqrt{d_0 \lambda}} f_0\, dt^2 + \frac{\sqrt{d_0\lambda}}{U^{7/2}} \left(\frac{dU^2}{f_0} + U^2 d\Omega_8^2 \right)\,, \\ e^{\phi} &= \frac{(2\pi)^2}{d_0} \frac{1}{N} \left(\frac{\lambda d_0}{U^3} \right)^{7/4} \sim \frac{g_{\text{eff}}^{7/2}}{N}\,, \end{aligned} \tag{3.17}$$

where

$$d_0 = 240\pi^5\,, \qquad U_0^7 = a_0\, g_{QM}^4\, \epsilon, \qquad a_0 = \frac{4480\,\pi^7}{3}\,. \tag{3.18}$$

The horizon U_0 can be expressed in terms of the temperature as:

$$T^{-1} = \frac{4}{7}\pi \sqrt{\lambda d_0} U_0^{-5/2}\,. \tag{3.19}$$

We now move on to the Einstein frame to compute the entropy. The Einstein frame spatial metric reads [91]:

$$ds_E^2 = C U^{-7/8} \left(\frac{dU^2}{f_0} + U^2 d\Omega_8^2 \right) \quad \text{with} \quad C = \frac{l_s^2 N^{1/2} d_0^{1/8}}{2\pi \lambda^{3/8}}\,. \tag{3.20}$$

It is convenient to introduce a new radial variable R, with dimension of length, defined as $U^{9/8} = (R^2/C)$. The metric becomes:

$$ds_E^2 = \frac{256}{81} \frac{dR^2}{1 - \left(\frac{R_H}{R}\right)^{112/9}} + R^2 d\Omega_8^2\,, \tag{3.21}$$

where R_H has the parametric dependence

$$R_H \sim \left(\frac{T}{\lambda^{1/3}} \right)^{9/40} N^{1/4} l_s\,. \tag{3.22}$$

So the entropy of the finite temperature black hole in terms of the dimensionless temperature $T/\lambda^{1/3}$ is given by

$$S \sim N^2 \times (T/\lambda^{1/3})^{9/5}\,. \tag{3.23}$$

Restoring the powers of N hidden in T and λ, we get the scaling behaviour

$$S \sim (U_0^{5/2}/g_{QM}^{5/3})^{9/5}\, N^{1/2}\,. \tag{3.24}$$

In the limit $U_0 \to 0$, one recovers the familiar zero entropy result. However, the point that the above computation shows is that entropy of the system, when cloaked with a fixed horizon size, scales as $N^{1/2}$, in agreement with our discussion in section 3.1.

4 Species scale horizon from emergent higher derivative terms

It is useful to have a more direct intuition, at the level of the geometry, about the appearance of the species scale horizon for D0-branes uncovered in the previous section. In analogy with the 4d small black holes in section 2.2, we expect this horizon to show up once higher derivative corrections are included in the effective theory, which become relevant precisely at the species scale. Moreover, such new terms are expected to arise from integrating out the towers of light particles associated to the infinite distance limit of the formerly small black hole, in the spirit of Emergence.

In this section we work out in detail this picture for the D0-brane system. We first show that generic 10d R^4 corrections lead to a finite size horizon. We then consider the specific R^4 couplings arising from integrating out a tower of D0-brane states (i.e. 11d KK gravitons), which constitute the SDC tower associated to the infinite distance limit probed by the small black hole solution (decompactification to 11d M-theory). Interestingly we find that these familiar R^4 terms do not suffice to generate the finite size horizon for the system (let alone a species scale one). Turning things around, this suggests that the appearance of the species scale horizon demanded by the swampland considerations requires a crucial role of new physics beyond the usually considered R^4 terms. In fact, we include a specific set of supersymmetric 8-derivative terms involving the RR field strength 2-form, tractable via a lift to 11d Mtheory, and show that they do produce the species scale horizon, thus confirming the swampland expectations.

4.1 Entropy function analysis of general R^4 terms

It is well established that the existence and properties of stretched horizons for 4d small black holes in theories with higher-curvature corrections can be studied using the entropy function formalism [71]. The idea is to use the ansatz for a putative near horizon AdS_2 solution to evaluate an entropy function, and to extremize it with respect to its parameters. In this section we apply a similar logic to the solution of D0-branes in 10d type IIA, and argue that the introduction of higher curvature corrections leads to a stretched horizon for the system.

Before entering the discussion, let us make a general comment. In this section we only include general terms involving only the curvature, but no other fields. This is for tractability, since curvature terms lead to closed expressions which can be analyzed in a model-independent way, and suffice to illustrate the fairly generic appearance of stretched horizons. The discussion of other corrections can be carried out for instance if a specific set of such correction is fixed, as we do in section 4.4 for a set of 8-derivative corrections including curvature and the RR 2-form field strength.

4.1.1 The D0-brane solution at two-derivative level

We start by recovering the key features of the two-derivative solution of the D0-brane system from the entropy function formalism [71], as warmup for coming sections. As explained above, we introduce an $\mathrm{AdS}_2 \times \mathbf{S}^8$ ansatz:

$$\begin{aligned} ds^2 &= v_1 \left(-r^2 dt^2 + \frac{1}{r^2} dr^2\right) + v_2 d\Omega_8^2 \,, \\ F_{rt} &= e \,, \quad \tilde{F}_{\theta\phi_1\cdots\phi_7} = 0 \,, \\ e^{\phi} &= g_s \,, \end{aligned} \tag{4.1}$$

where v_1 and v_2 control the AdS and sphere length scales. For the 2-form field strength this introduces electric but not magnetic charge at the horizon. Finally, the last equation sets the value of the string coupling at the horizon.

The entropy function is given (modulo some irrelevant overall factor) by evaluating the action at the near horizon ansatz (4.1) and applying a Legendre transform with respect to e, namely

$$\mathcal{E}(N, g_s, v, \beta, e) = eN - \int_{S^8} d\Omega_8 \sqrt{-g}\ \mathcal{L}|_h \ . \tag{4.2}$$

To evaluate this quantity for the ansatz (4.1) we compute

$$\sqrt{-g} = v_1 v_2^4 \sqrt{g_{\Omega_8}} = \frac{v^5}{\beta} \sqrt{g_{\Omega_8}} \,, \tag{4.3}$$

$$R = -\frac{2}{v_1} + \frac{56}{v_2} = \frac{56 - 2\beta}{v} \,, \tag{4.4}$$

$$|F_2|^2 = \frac{1}{2!} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{v_1^2} e^2 = -\frac{\beta^2}{v^2} e^2 \,. \tag{4.5}$$

With hindsight, we have introduced new variables $v = v_2$ and $\beta = v_2/v_1$, to describe the overall length scale of $\mathrm{AdS}_2 \times \mathbf{S}^8$ and the relative scale separation. The result is

$$\mathcal{E}(N, g_s, v, \beta, e) = eN - \frac{8\pi^4 v^3 \left(\beta^2 e^2 g_s^{3/2} - 4(\beta - 28) v\right)}{105\beta} \,. \tag{4.6}$$

The goal is to extremize this function with respect to g_s, v, β and e, expressing the result in terms of the D0-brane charge N. We start with the extremization with respect to e, which gives

$$\frac{\partial \mathcal{E}}{\partial e} = 0 \quad \rightarrow \quad e = \frac{105N}{16\pi^4 \beta v^3 g_s^{3/2}} \,. \tag{4.7}$$

Substituting this result, the entropy function reads

$$\mathcal{E}(N, g_s, v, \beta) = \frac{105N^2}{32\pi^4 \beta v^3 g_s^{3/2}} + \frac{32\pi^4 (\beta - 28) v^4}{105\beta}. \tag{4.8}$$

We now extremize with respect to v and β, and get

$$\frac{\partial \mathcal{E}}{\partial v} = \frac{\partial \mathcal{E}}{\partial \beta} = 0 \quad \rightarrow \quad v^7 = \frac{1575N^2}{4096\pi^8 g_s^{3/2}} \,, \ \beta = 49 \,. \tag{4.9}$$

Incidentally, the approximately order 1 value of β (taking volume factors into account) is natural from the absence of scale separation in AdS vacua [92]. The resulting entropy function now reads

$$\mathcal{E}(N, g_s) \sim \frac{N^{8/7}}{g_s^{6/7}}\,. \tag{4.10}$$

We see that there is no well-defined extremum for the parameter g_s, it lies at $g_s \to \infty$. This simply reflects the fact that the D0-brane solution drives the dilaton to infinity in moduli space at its core, as observed above from the 10d solution. Indeed, we can recover other features of the 10d solution from the entropy function, by studying the scaling of different quantities with g_s. For instance, from (4.9) and (4.4), we have that in the $g_s \to \infty$ limit

$$v_2 = v \sim \frac{1}{g_s^{3/14}} \to 0\,, \qquad R \sim g_s^{3/14} \to \infty\,. \tag{4.11}$$

Namely, we recover that the horizon volume goes to zero, and the scalar curvature blows up with exactly the same scalings as would be obtained directly from the D0 solution at its core. Note that the g_s we consider in this section is to be compared to the dilaton near the core of the D0 stack in the solution (3.4). Indeed, g_s is the value of the string coupling at the horizon of the would-be black hole, whilst g_s^∞ in (3.4) is the string coupling infinitely far away from the stack of branes.

We therefore recover that the process of extremization of the Wald entropy function of the near-horizon geometry of a black hole electrically charged under the RR 2-form in 10d describes the near-core behaviour of a stack of D0 branes. The fact that the Wald entropy is only extremized for a value of the string coupling at the core that is blowing up, with a horizon that tends to zero size is exactly the behaviour expected from small black holes, as in section 2. We conclude from this that the Wald entropy formalism constitutes a reliable description of D0-brane stacks as small black holes. In the next section, we will introduce higher derivative corrections to the 10d supergravity which will affect the Wald entropy in such a way that the same exact minimization procedure will lead to a finite sized black hole instead of a small one.

4.1.2 The D0-brane solution and higher derivative terms

In analogy with the 4d small black hole case, we may expect that the D0-brane solutions can develop a stretched horizon upon the inclusion of suitable higher derivative corrections. In 10d type IIA the first corrections allowed by supersymmetry arise at 8-derivative level. There is a vast literature devoted to the computation of these corrections at tree and one-loop level (see e.g. [76, 93–98]). However, for the present purposes it will be more efficient to assume the presence of general R^4 corrections (the example of supersymmetric R^4 corrections will be discussed explicitly in section 4.3). Also, as explained earlier, we focus on terms involving only curvatures, which allows for a tractable model-independent analysis. The inclusion of terms involving other fields is discussed in section 4.4 for a specific set of supersymmetric 8-derivative terms including the RR 2-form field strength.

We now describe the entropy function computation in the presence of general R^4 corrections. The corrections can be written as a linear combination of all possible contractions

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between eight inverse metrics and four Riemann tensors with all the indices down. Inspecting the ansatz in (4.1), we see that each inverse metric gives either a $1/v_1$ or a $1/v_2$ contribution, while each Riemann gives either a v_1 or a v_2. In terms of v and β, this means that any of these contributions will take the form $\beta^a\, v^{-4}$, with a ranging from -4 to 8. For instance, the $a=-4$ one corresponds to the case in which all the Riemanns give v_1 and all the metrics give $1/v_2$, while the $a=8$ one corresponds to all the Riemanns giving v_2 and all the metrics giving $1/v_1$.

Following this reasoning, we can write the R^4 term evaluated at the horizon as

$$\mathcal{L}_{R^4}|_h = -g_s^{1/2}\,\frac{p_{12}(\beta)}{\beta^4}\,\frac{1}{v^4}\,, \tag{4.12}$$

where p_{12} denotes a degree 12 polynomial, and the minus sign is there for future convenience. Note that we already make explicit the dependence on the string coupling, which is fixed to that of the R^4 terms arising at first loop in perturbative Type IIA string theory. Motivated by having $g_s\to\infty$ at the core of the D0 solution at the two-derivative level, we are neglecting the tree level piece. One could worry that even higher loop contributions would dominate as $g_s\to\infty$, but they actually vanish. This has been considered previously in the literature from the Type IIA perspective [76, 99, 100] and, as it will become clearer in section 4.4, it is required for a consistent uplift to M-theory in this limit.

Including the extra term in (4.12), the entropy function (4.2) reads

$$\mathcal{E}(N,g_s,v,\beta,e) = eN - \frac{8}{105}\pi^4\beta e^2 g_s^{3/2}v^3 + \frac{16\pi^4 v\left(g_s^{1/2}p_{12}(\beta)+2(\beta-28)\beta^4v^3\right)}{105\beta^5}\,. \tag{4.13}$$

The extremization with respect to e is insensitive to the new R^4 term, so the result is again (4.7). Plugging this result, the entropy function reads

$$\mathcal{E}(N,g_s,v,\beta) = \frac{105N^2}{32\pi^4\beta g_s^{3/2}v^3} + \frac{16\pi^4\left(g_s^{1/2}p_{12}(\beta)v+2(\beta-28)\beta^4v^4\right)}{105\beta^5}\,. \tag{4.14}$$

In order to display the effect of the higher derivative term in the dilaton, let us now consider the extremization with respect to g_s. We obtain

$$\frac{\partial\mathcal{E}}{\partial g_s}=0 \quad\to\quad g_s = \frac{105}{16\pi^4}\sqrt{\frac{3}{2}}\frac{\beta^2 N}{\sqrt{p_{12}(\beta)}v^2}\,. \tag{4.15}$$

We see that the new contributions generically stabilize the dilaton. We note that the solution only exist if $p_{12}(\beta_0)>0$, where β_0 is the yet to be determined value for β extremizing the entropy function. The latter is determined by the extremization with respect to v, namely

$$\frac{\partial\mathcal{E}}{\partial v} = \frac{128\pi^4(\beta-28)v^3}{105\beta} = 0 \quad\to\quad \beta=28\,. \tag{4.16}$$

Note that we again obtain a value compatible with the absence of scale separation, suggesting that this property is fairly robust against inclusion of higher derivative corrections.

Finally, the extremization with respect to β gives

$$\left.\frac{\partial\mathcal{E}}{\partial\beta}\right|_{\beta=28}=0 \quad\to\quad v^4 = \frac{4\,p_{12}(28)-21\,p_{12}'(28)}{175616\cdot 6^{1/4}\,\pi^2\,p_{12}(28)^{1/4}}\sqrt{\frac{5N}{7}}\,, \tag{4.17}$$

where $p'_{12}(\beta)$ is the derivative of the polynomial with respect to β. Hence, the existence of the solution requires

$$p_{12}(28) > 0\,, \quad 4\,p_{12}(28) > 21\,p'_{12}(28)\,. \tag{4.18}$$

We will see the explicit evaluation of such polynomials for a particular R^4 correction in section 4.3.

In conclusion, under fairly general circumstances, the presence of higher derivative corrections suggests the existence of a stretched horizon for 10d D0-brane solutions. From the above expressions, the scaling of its properties with N is

$$S \sim \sqrt{N}\,, \qquad v^4 \sim \sqrt{N}\,, \qquad \beta = 28\,, \qquad g_s \sim N^{3/4}\,, \qquad e \sim N^{-1/2}\,. \tag{4.19}$$

The entropy of the system scales as $\sqrt{N}$, in agreement with the microstate counting in section 3.1. Notice that this, as well as all the scalings above, rely heavily in the string coupling dependence made explicit in (4.12) and corresponding to the one-loop contribution in perturbative Type IIA string theory. The string coupling at the horizon blows up as $N \to \infty$, which is consistent with neglecting the tree level contribution in this regime. In the next section we will shed some further light on these scalings, since they lead to the appearance of the species scale.

4.1.3 The species scale and cosmic censorship

The species scale. Here we shortly note that the above horizon scale indeed corresponds to the species scale of the SDC tower associated to the infinite distance limit probed by the solution. This infinite distance limit corresponds to a decompactification to 11d M-theory (consistently with the above mentioned fact that the horizon value of g_s gets strong in the large N regime). The extra dimension is reconstructed by the SDC tower of D0 branes, whose species scale corresponds to the 11d Planck scale. Restoring the 10d Planck scale, we have

$$\Lambda_s \sim M_{11} \sim g_s^{-1/12} M_{10} \sim N^{-1/16} M_{10}\,, \tag{4.20}$$

where we have used the scaling of g_s with N in equation (4.19). It is indeed easy to show that this coincides with the inverse radius of the stretched horizon. Using the scaling of v with N in (4.19), we have

$$r_h^{-1} \sim v^{-1/2} M_{10} \sim N^{-1/16} M_{10}\,. \tag{4.21}$$

Let us note that this agreement is easily understood by the fact that the stretched horizon arises from the competition between the classical and the R^4 terms in the effective action. The scale at which these two can compete indeed corresponds to one of the notions of the species scale [47]. The fact that it matches the species scale associated to the tower of D0s can be directly checked in the effective action and, as we explain in section 4.2, can be understood from the emergence of higher derivative terms coming from integrating out the tower of D0s. Here we see that these two notions of species scale also coincide with the one of the smallest BH describable within the EFT, in this case the stretched horizon of the D0 black hole.

The D0-brane bound state size scale. We have emphasized the appearance of a finite size for the system from the spacetime viewpoint, via the appearance of a stretched horizon. However, as any quantum system, the D0-brane system has a characteristic size, in particular that associated to the size of the D0-brane bound states. This has been studied in particular in the context of M(atrix) theory. In this context, the outcome of [51, 101–103], is an estimate of the size of a bound state of N D0-branes scaling as

$$R \gtrsim N^{1/3} M_{11}^{-1} \,. \tag{4.22}$$

This grows with N in a way seemingly stronger[10] than the stretched horizon size $r_h \sim M_{11}^{-1}$ discussed above. However, an important observation is that (4.22) is derived in the limit of $g_s \to 0$ with M_{11} fixed, with g_s the coupling in flat space. Hence, for a proper comparison, in the evaluation of the horizon size $r_h \sim N^{1/16} M_{10}^{-1} \sim g_s^{-1/12} N^{1/16} M_{11}^{-1}$, one should not use the horizon value $g_s \sim N^{3/4}$ in matching M_{10}, but rather consider g_s as a free asymptotic parameter. Thus, in the $g_s \to 0$ and M_{11} fixed limit, the horizon size is parametrically larger than the size of the bound state of gravitons, which are effectively cloaked behind the horizon.

A cosmic censorship interpretation. The above considerations motivate a cosmic censorship interpretation of our result, which provides a different angle to other uses of cosmic censorship in the analysis of swampland constraints (see e.g. [104, 105]). The classical 2-derivative solution displays a singularity. In the full theory, the pathological implications of such singularities must be avoided, either by some desingularization [106], addition of extra degrees of freedom as in orbifolds, or the appearance of a horizon cloaking it. We have shown that in our system the latter occurs via the use of higher derivative interactions, despite the fact that it does not happen in the low-energy two-derivative approximation. Interestingly, this happens in the realm of the effective field theory, on the verge of its validity. This is again tied up to the notion of the species scale as that at which the tower of higher derivative interactions becomes relevant.

4.2 The D0-brane SDC tower and higher derivative emergence

The infinite distance limit in moduli space explored by the core of the solution is the strong coupling limit of type IIA theory, which corresponds to the decompactification limit of 11d M-theory on $\mathbf{S}^1$. The tower of states predicted by the swampland distance conjecture corresponds to D0-brane states,[11] namely the KK momentum states of 11d graviton multiplets. The computation is indeed simpler in this 11d picture, and is a higher-dimensional version of that encountered in section 2.2.3. In fact is a well-established story that this set of states can lead, at the 1-loop level, to the appearance of higher derivative corrections to the 10d effective action, which can be interpreted as tree- and one-loop level (in g_s) terms of type IIA theory [76], as we now discuss.

[10] Note that the stronger growth also holds even if we restrict to the dominant bound state of $k \sim N^{1/2}$ D0-branes, as in section 3.1, which would yield $R \sim N^{1/6}$.

[11] We expect readers not to confuse the D0-brane states in the SDC tower with the D0-branes sourcing the solution in the previous sections.

The terms we are going to compute are the $t_8 t_8 R^4$ terms,[12] in principle of the kind included in the analysis in section 4.1.2 (see also section 4.3 for more details on their structure). They are protected by supersymmetry, so only the 1-loop diagram of BPS 11d graviton KK modes can contribute. These can be easily computed from a worldline formalism. Using the general expression (2.24), integrating over continuous momenta, and particularizing for $d = 10$, $n = 4$, we get

$$\mathcal{A}_{10,4} = \frac{\sqrt{\pi}}{2R} \tilde{K} \int_0^\infty \frac{d\tau}{\tau^2} \sum_k e^{-\pi\tau R^{-2} k^2} \,, \tag{4.23}$$

where the prefactor $\tilde{K}$ contains all the kinematics, and is the linearized approximation to the $t_8 t_8 R^4$ term in the effective action. As in the 4d case, the above expression diverges in two ways: because of the integral and because of the infinite sum. We can perform a Poisson resummation, so that both are nicely combined and we get

$$\frac{1}{\pi^{3/2}} \mathcal{A}_{10,4} = \tilde{K} \int_0^\infty d\hat{\tau}\, \hat{\tau}^{1/2} \sum_l e^{-\pi\hat{\tau} R^2 l^2} = C\tilde{K} + \frac{\zeta(3)}{\pi R^3} \tilde{K} \,, \tag{4.24}$$

where $\hat{\tau} = \tau^{-1}$. This Poisson summation trades the KK momentum k for the winding number l of the worldline along the $\mathbf{S}^1$. The only divergence is now in the $l = 0$ piece, which has been isolated as the first terms in the second equality; C is an unknown coefficient regularizing the divergence.[13]

As anticipated, the quantum corrections coming from the SDC tower lead to a higher derivative R^4 correction, in principle of the kind invoked to lead to a stretched horizon. There remains to perform a detailed evaluation of these terms and their effect in the entropy function argument, which we postpone to section 4.3.

An important remark is that the R^4 correction obtained in this way is ambiguous. The 4-graviton scattering is an on-shell quantity that is invariant under local field redefinitions $g_{ij} \to g_{ij} + \delta g_{ij}$, which is usually used to get rid of terms involving Ricci tensors (see e.g. [55]). However, since the $\mathrm{AdS}_2 \times \mathbf{S}^8$ ansatz has a non-vanishing Ricci tensor, doing this is not innocuous for our purposes. We will thus stick to the full $t_8 t_8 R^4$ term, including Ricci terms, as it is the natural kinematic function that appears in this computation à la Emergence. That this tensor structure gives the full off-shell R^4 correction to the effective action was also suggested in [95]. As evidence for this, it was found that the terms containing no more than one Ricci tensor, which were reliably computed from the vanishing of the worldsheet beta function, precisely match the ones appearing in the $t_8 t_8 R^4$ structure.

Higher derivative emergence. When translated to the Type IIA frame, the first and second terms in (4.24) precisely reproduce the $t_8 t_8 R^4$ tensor structure of the 1-loop and

[12] As we will explain in section 4.3, the actual 10d type IIA R^4 terms contain a further piece with an $\epsilon_{10}\epsilon_{10}$ structure. This however does not contribute to 4-graviton scattering amplitudes, or conversely is not captured by our 4-graviton 1-loop amplitude (it would require a 5-point computation). This is analogous to the similar statements for the odd-odd structure in the R^4 computation from scattering amplitudes in string theory, see e.g. [107].

[13] In [76], it was fixed by T-duality upon further $\mathbf{S}^1$ compactification. In our present context, there is no way of fixing it without this kind of extra UV information. It would be interesting if performing the sum over KK modes up to the species scale one could reproduce its precise value, along the lines of [39–43].

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tree-level string perturbation theory contributions, respectively. Therefore, this establishes that these can be interpreted as coming from integrating out the full tower of D0-particles, as well as the fast-movers in the massless 10d sector. Let us remark that this result is very much in the spirit of the Emergence Proposal, albeit for higher derivative terms rather than just the kinetic terms.

Note also that the above computation integrates out the full tower of states, i.e. not just up to the species scale as in [39–43]. In fact, interpreting the latter recipe as a regularization procedure to be applied when integrating out infinite number of states, one could hope to recover the right value for C; we leave this as an interesting open question. On the other hand, notice that integrating the full tower is in the spirit of the recent proposals in [42, 43]. It would be interesting to explore these connections further.

4.3 Supersymmetric 10d R^4 terms are not enough

In this section, we particularize the analysis in section 4.1.2 to the specific R^4 terms appearing in the Type IIA effective action (for a concrete choice fixing all the already mentioned ambiguities concerning these terms). By plugging the $\mathrm{AdS}_2 \times \mathrm{S}^8$ ansatz, we will compute the polynomial appearing in equation (4.12) and check the conditions in equation (4.18).

We will show that, perhaps surprisingly, these supersymmetric R^4 terms do not satisfy the constraints in section 4.1.2 required to generate a finite size horizon. Note that this does not invalidate the analysis of the entropy function, but rather shows that the appearance of the horizon from pure higher curvature terms demands the use of other possibly non-supersymmetric terms. Alternatively, one can maintain the problem in the realm of supersymmetric corrections, and include higher derivative couplings involving the RR 2-form field strength; these go beyond the analysis in section 4.1.2, and will be discussed in section 4.4.

Hence, we focus on the supersymmetric R^4 corrections in 10d type IIA theory. Following the notation in [98], at the eight derivatives level, the type IIA action gets supplemented by additional terms quartic in the Riemann tensor at both tree level and first loop in the string coupling g_s:

$$S_{\mathrm{IIA}} = S_{\mathrm{IIA}}^{class} + \frac{1}{3 \cdot 2^{11}} (S^{\mathrm{tree}} + S^{loop}) \,, \tag{4.25}$$

where S^{tree} and S^{loop} both arise at level α'^3. In fact, it has been conjectured that these are the only two contributions to this type of term in string perturbation theory [76, 99, 100]. They can be written as follows in the Einstein frame and in 10d Planck units:

$$S^{\mathrm{tree}} = \frac{\zeta(3)}{2} \int d^{10}x \sqrt{-g}\, e^{-\frac{3}{2}\phi} \left(t_8 t_8 R^4 + \frac{1}{8} \epsilon_{10}\epsilon_{10} R^4 \right) \,, \tag{4.26}$$

$$S^{loop} = \frac{\pi^2}{6} \int d^{10}x \sqrt{-g}\, e^{\frac{1}{2}\phi} \left(t_8 t_8 R^4 - \frac{1}{8} \epsilon_{10}\epsilon_{10} R^4 \right) \,, \tag{4.27}$$

where the form of the $t_8 t_8 R^4$ and $\epsilon_{10}\epsilon_{10} R^4$ structures can be found in appendix A. Let us only consider the 1-loop term, which is the leading one as $g_s \to \infty$.

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Evaluating $t_8t_8R^4$ and $\frac{1}{8}\epsilon_{10}\epsilon_{10}R^4$ on the $AdS_2 \times S^8$ near-horizon geometry (4.1), one obtains:

$$t_8t_8R^4 = \frac{192\,(3\beta^4 + 56\beta^2 + 2520)}{v^4}\,, \tag{4.28}$$

$$\frac{1}{8}\epsilon_{10}\epsilon_{10}R^4 = \frac{161280\,(4\beta - 1)}{v^4}\,. \tag{4.29}$$

Therefore, the 1-loop term can be put in the form (4.12) if

$$p_{12}(\beta) \sim -\beta^4 \left(3\beta^4 + 56\beta^2 - 3360\beta + 3360\right)\,, \tag{4.30}$$

where we are ignoring an irrelevant positive-defined prefactor. Now we can check the conditions in (4.18) for the existence of an extremum of the entropy function. The second one is satisfied, while the first one is not. This implies that the familiar R^4 terms used so far do not suffice to generate the stretched horizon for the D0 solution.

This does not invalidate the main claim that the system should develop a species scale horizon. As explained earlier, we expect other higher derivative couplings to contribute non-trivially, and the horizon may very well develop when they are ultimately included. Indeed, we will show in the next section that a simple modification of the above computation promoted to M-theory allows to include couplings involving the RR 2-form F_2, and lead to a non-trivial species scale stretched horizon for the D0-brane system.

4.4 Including F_2: species scale D0-brane horizon from M-theory couplings

In the previous section we showed that 10d supersymmetric R^4 terms do not generate a finite size horizon. Hence, the requirement that there should arise species scale horizon, as demanded by swampland considerations, implies that one should look further. One possibility is to consider other possibly non-supersymmetric pure higher curvature terms. A more attractive avenue is to stay in the realm of supersymmetric terms but include higher derivative couplings involving the RR 2-form field strength F_2. These go beyond the analysis in section 4.1.2, but, once a specific set of couplings is fixed, are amenable to a direct study again via the entropy function.

In this section we include such terms, completing the analysis at the supersymmetric eight derivatives level of the effective action. The most efficient way of doing this is by encoding these terms in the 11d M-theory effective action, since both the 10d metric and RR 1-form become part of the 11d metric. This is, by encoding the extremal near-horizon ansatz in the 11d metric and plugging it into the M-theory effective action, supplemented by R^4 terms, we can effectively recover the 10d Lagrangian evaluated at the horizon including both R^4 and terms involving F_2. Let us remark that we will keep the 10d point of view at all times to focus in the presence of a stretched horizon for the D0 solution. This is, here we think about the 11d effective action as just an efficient way of encoding the eight derivatives terms involving F_2 and evaluating them at the extremal near-horizon ansatz.

Our analysis is close to that in [55], with the difference that we emphasize the 10d perspective in the computation. Even though we find analogous results, an important difference in the analysis is that we keep the full $t_8t_8R^4$ structure, while only purely Riemannian terms were included in [55]. As already discussed in section 4.2, the Wald entropy analysis is sensitive to the terms including the Ricci tensor which, via Emergence, are naturally encoded in the $t_8 t_8 R^4$ structure. This is consistent with the results obtained in [95], where the terms in $t_8 t_8 R^4$ including one Ricci tensor were unambiguously shown to appear in the Type IIA effective action.

Following again the notation in [98], and setting the three-form to zero for our purposes, the M-theory effective action can be written at the eight derivatives level as

$$S_M = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-\hat{g}} \left(\hat{R} + \frac{\left(4\pi\kappa_{11}^2\right)^{2/3}}{(2\pi)^4 \cdot 3^3 \cdot 2^{13}} \left(\hat{t}_8 \hat{t}_8 \hat{R}^4 - \frac{1}{24} \epsilon_{11}\epsilon_{11} \hat{R}^4 \right) \right) . \qquad (4.31)$$

To simplify the computations, we will take $\kappa_{11} = 1$.[14] The $\hat{t}_8$ and ϵ_{11} tensors are defined in complete analogy with the Type IIA case. In fact, after writing these terms as contractions of four Riemann tensors, the 11d R^4 term look exactly the same as the 10d one. More details about these structures can be found in appendix A.

As it is well-known, the circle compactification ansatz from 11 to 10 dimensions is:

$$d\tilde{s}^2 = e^{-\frac{1}{6}\phi} ds^2 + e^{\frac{4}{3}\phi} \left(dz - C_\mu dx^\mu\right)^2 . \qquad (4.32)$$

Plugging this into the effective action recovers the 10d Einstein frame Type IIA effective action up to the eight-derivative level. Instead of doing this and then taking the extremal $AdS_2 \times S^8$ near-horizon ansatz, we perform the computation at the level of M-theory. That is, we first encode the $AdS_2 \times S^8$ ansatz into the 11d metric, and then we plug it into the 11d effective action. This allows us to read off the 10d Lagrangian evaluated at the horizon. For this last step, we need to take into account the relation between the 11d and 10d determinant of the metric appearing in the action. This gives the following relation:

$$\mathcal{L}_{IIA}|_h = g_s^{-1/6} \mathcal{L}_M|_{11d\, ansatz} . \qquad (4.33)$$

To encode the extremal $AdS_2 \times S^8$ near-horizon geometry in the 11d metric ansatz, we just plug equation (4.1) into (4.32) to get

$$d\tilde{s}^2 = -g_s^{-1/6} v_1 \left(r^2 dt^2 + \frac{1}{r^2} dr^2 \right) + g_s^{-1/6} v_2 \, d\Omega_8^2 + g_s^{4/3} \left(dz \; - e\, r\, dt\right)^2 . \qquad (4.34)$$

Evaluated on this ansatz, the 11d Ricci scalar reads

$$\hat{R} = g_s^{1/6} \frac{\beta^2 e^2 g_s^{3/2} - 4v(\beta - 28)}{2v^2} , \qquad (4.35)$$

where we introduced new variables $v = v_2$ and $\beta = v_2/v_1$ as we did in section 4.1.1. When plugged into (4.33), this recovers the two-derivative piece of the Type IIA Lagrangian evaluated at the horizon (cf. equation (4.6)).

[14] Even though this might seem to imply that the results will be in 11d Planck units, when mapping to the type IIA Einstein frame, the 11d Planck scale gets traded for the 10d one. Thus, the final results are actually in 10d Planck units.

Similarly, we evaluate the M-theory R^4 terms in this ansatz, getting

$$\begin{aligned}\left(\hat{t}_8\hat{t}_8 - \frac{1}{24}\epsilon_{11}\epsilon_{11}\right)\hat{R}^4 = \frac{3g_s^{2/3}}{4v^8}\Big(&287\beta^8 e^8 g_s^6 - 1392\beta^7 e^6 g_s^{9/2} v \\ &+ 32\left(83\beta^2+308\right)\beta^4 e^4 g_s^3 v^2 \\ &- 768\left(3\beta^3+28\beta-280\right)\beta^2 e^2 g_s^{3/2} v^3 \\ &+ 256\left(3\beta^4+56\beta^2-3360\beta+3360\right)v^4\Big)\,.\end{aligned} \tag{4.36}$$

As a check, we can recover the Type IIA R^4 result from the previous section by setting $e = 0$ and plugging (4.36) into (4.33) (cf. equation (4.28) with the $g_s^{1/2}$ factor appearing in (4.12)).

Putting these two results together in the 11d Lagrangian, using equation (4.33) and plugging the result into the definition of the entropy function in (4.2), we finally obtain:

$$\begin{aligned}\mathcal{E}(N,g_s,v,\beta,e) = e\,N - &\frac{8\pi^4 v^3\left(\beta^2 e^2 g_s^{3/2} - 4(\beta-28)v\right)}{105\beta} \\ &-\left(\frac{\pi}{2}\right)^{2/3}\frac{g_s^{1/2}}{2580480\,v^3\beta}\Big(287\beta^8 e^8 g_s^6 - 1392\beta^7 e^6 g_s^{9/2} v + 32\left(83\beta^2+308\right)\beta^4 e^4 g_s^3 v^2 \\ &-768\left(3\beta^3+28\beta-280\right)\beta^2 e^2 g_s^{3/2} v^3 + 256\left(3\beta^4+56\beta^2-3360\beta+3360\right)v^4\Big)\,.\end{aligned} \tag{4.37}$$

Here we have already performed the integral in equation (4.2), taking into account the determinant of the metric evaluated in the ansatz as given in (4.3).

The entropy function turns out to be rather involved, and we cannot extremize it analytically. Doing so numerically would require fixing N to a set of different values, extremizing the entropy function for each of these and then fitting the dependence on N of the various quantities $(g_s, v, \dots)$. Instead of doing so, let us perform a change of variables inspired by the scalings found in section 4.1.2. We define

$$g_s = \tilde{g}_s\,N^{3/4}\,,\qquad v = \tilde{v}\,N^{1/16}\,,\qquad \beta = \tilde{\beta}\,,\qquad e = \tilde{e}\,N^{-1/2}\,. \tag{4.38}$$

Introducing this into the entropy function, we get

$$\begin{aligned}\mathcal{E}(N,\tilde{g}_s,\tilde{v},\tilde{\beta},\tilde{e}) = \sqrt{N}\Bigg[\tilde{e} - &\frac{8\pi^4\tilde{v}^3\left(\tilde{\beta}^2\tilde{e}^2\tilde{g}_s^{3/2} - 4(\tilde{\beta}-28)\tilde{v}\right)}{105\tilde{\beta}} \\ &-\left(\frac{\pi}{2}\right)^{2/3}\frac{\tilde{g}_s^{1/2}}{2580480\,\tilde{v}^3\beta}\Big(287\tilde{\beta}^8\tilde{e}^8\tilde{g}_s^6 - 1392\tilde{\beta}^7\tilde{e}^6\tilde{g}_s^{9/2}\tilde{v} + 32\left(83\tilde{\beta}^2+308\right)\tilde{\beta}^4\tilde{e}^4\tilde{g}_s^3\tilde{v}^2 \\ &-768\left(3\tilde{\beta}^3+28\tilde{\beta}-280\right)\tilde{\beta}^2\tilde{e}^2\tilde{g}_s^{3/2}\tilde{v}^3 + 256\left(3\tilde{\beta}^4+56\tilde{\beta}^2-3360\tilde{\beta}+3360\right)\tilde{v}^4\Big)\Bigg]\,.\end{aligned} \tag{4.39}$$

This change of variables does the important job of factoring out all the N-dependence in the entropy function, which is very non-trivial since all the terms had to conspire to give the same overall $\sqrt{N}$ factor. Thanks to this, we can now focus on the terms in parenthesis and look for its N-independent extrema numerically. If an extremum exist, then we see that the scalings we found in section 4.1.2 are automatically guaranteed. Furthermore, given that the overall factor in front of the entropy function is $\sqrt{N}$, the scaling of the entropy from the microscopic counting in section 3.1 is also automatically guaranteed.

Finally, we look for extrema of the terms in parenthesis in the previous equation or, equivalently, of $\mathcal{E}(1, \tilde{g}_s, \tilde{v}, \tilde{\beta}, \tilde{e})$. This cannot be done analytically, so we perform a numerical search. The result is the following extremum:

$$\tilde{g}_s \approx 2.90365\,, \qquad \tilde{v} \approx 0.159653\,, \qquad \tilde{\beta} \approx 14.0556\,, \qquad \tilde{e} \approx 0.0479134\,. \tag{4.40}$$

For these values, one can check that the derivatives of the entropy function are 14 orders of magnitude smaller than the value of function itself. This indicates that, to a very good approximation, this is an extremum of the entropy function. To make this extremum more apparent, we can fix one of the variables to its value at the extremum and display the three-dimensional gradient flow. This is shown in figure 1, where we can see the presence of a non-trivial extremum. As it can be checked also by direct computation, we also see that the extremum is in fact a saddle point. This fact is not relevant for the Wald entropy formalism, that only requires the presence of an extremum. It would be interesting to understand the thermodynamical stability of this stretched horizon, somewhat along the lines of [55]. However, this analysis presumably requires a systematic inclusion of even higher derivative terms to achieve control of order one factors.

In conclusion, we found that the Type IIA action including *all* terms up to the eight derivative level captures the presence of the species scale stretched horizon for the D0 solution. In addition, the entropy associated to the horizon nicely fits the microscopic counting and the near-horizon $\mathrm{AdS}_2 \times \mathrm{S}^8$ is not scale separated. This gives support to the idea that, even though a proper geometric treatment of a stretched horizon requires the inclusion of all the infinite amount of higher derivative operators, the classical level and the (complete) first non-trivial higher derivative level seem to suffice for capturing its presence and its scaling properties with the species scale.

As in the case of 10d R^4, the new set of higher derivative terms can be understood as arising from integrating out the SDC tower of D0-branes, in the spirit of Emergence. Indeed, since all these couplings are related by 11d Poincaré invariance, the computation of the 4-point amplitude is essentially unchanged. From the 10d perspective, the difference is that instead of computing 4-graviton scattering one should consider processes involving the RR 1-form field. Their worldline vertex operators are directly related [108], hence the structure of the 1-loop integral is essentially unchanged, and the changes only involve the kinematic tensor structure. This computation is directly not available in the literature, but it can be regarded as a 10d decomposition of the computation of the 11d R^4 correction from the 4-point 1-loop superparticle amplitude in [109]. This suffices to show the emergence from the SDC tower of the required higher derivative terms beyond the purely gravitational sector.

5 Conclusions

In this work we have clarified the links among the different avatars of the species scale in the swampland program: (i) as cutoff of the effective theory specially in the presence of the

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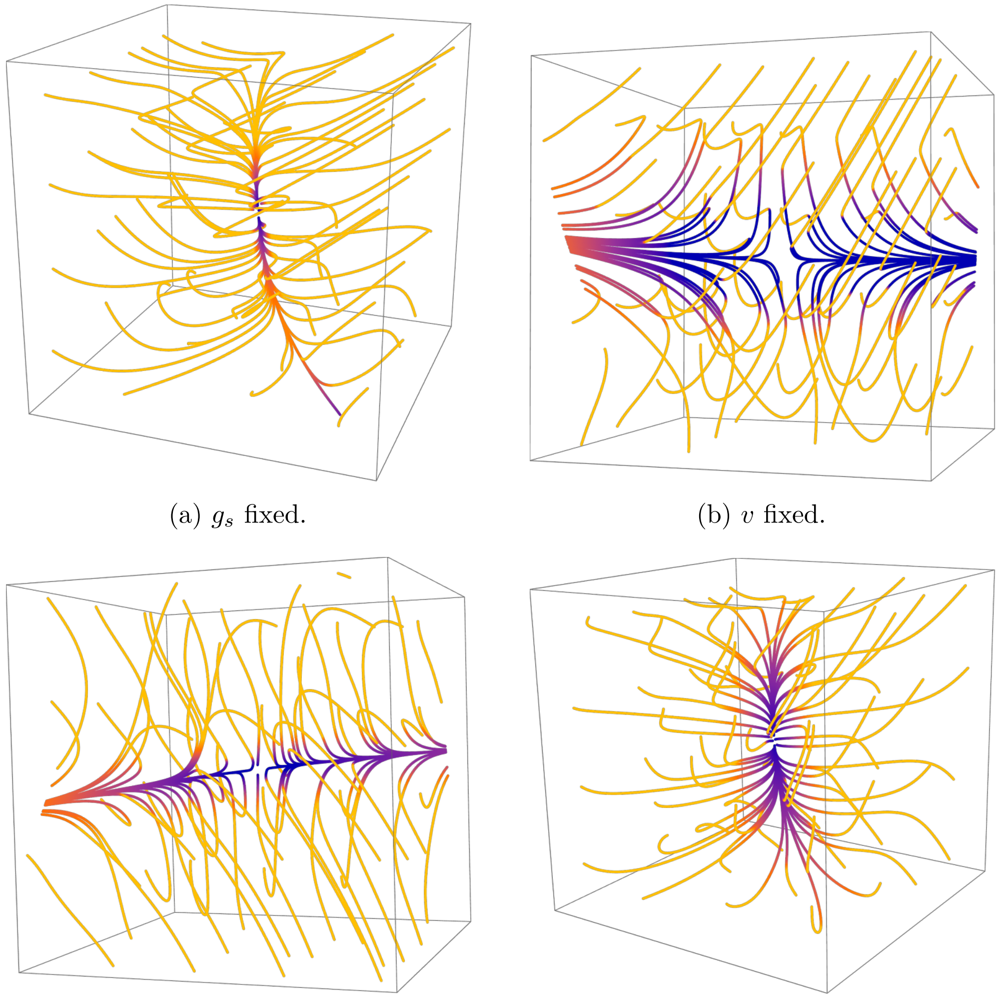
(a) g_s fixed.
(b) v fixed.
(c) β fixed.
(d) e fixed.

Figure 1. Gradient flows of the entropy function with one of the variables fixed. Lighter/yellow colors denotes bigger values for the gradient, while darker/blue colors denote smaller ones.

infinite SDC tower of light particles near infinite distance points in moduli space, (ii) the scale at which terms in the infinite series of higher derivative corrections in the gravitational sector start competing with the two-derivative level, (iii) as the size of the smallest black hole which admits a description within the EFT.

Our analysis uses small black holes, whose cores probe infinite distance limits in field space, at which we can characterize the SDC tower of states leading to the species scale in avatar (i). Upon integrating out the tower of states, the theory generates a series of higher-dimensional operators, naturally leading to avatar (ii) of the species scale. Finally, including these corrections in the black hole solution triggers the appearance of a stretched horizon of a size given by the species scale in avatar (iii).[15]

[15]It is worth noticing that we consider BPS small black holes, and that the quantum gravity effects we are taking into account come in the form of supersymmetry protected higher derivative corrections. It would be interesting to see how our results might extend to cases with less supersymmetry.

We have provided quantitative examples of this phenomenon, both in a class of 4d small black holes, and in the fascinating case of 10d type IIA D0-branes. The latter case turned out to be highly non-trivial, as the appearance of the species scale horizon required understanding of 10d 8-derivative terms involving not only the curvature but also the RR 2-form field strength. The nice packaging of those terms into an 11d R^4 correction is a tantalizing hint that the species scale can serve as a powerful probe of the UV physics of M-theory.

The 10d D0-brane example displays an interesting feature: the objects making up the small black hole are precisely of the same kind as those in the SDC tower for the corresponding infinite distance limit. This feature arises in the microscopic explanation of the entropy for other black holes in string theory; but the remarkable fact about the D0-brane case is that the higher derivative terms coming from integrating out the tower of D0-branes are crucial to generate a geometric horizon for the D0-brane black hole itself!

This motivates us to entertain the following picture. According to the most radical interpretation of the Emergence Proposal [35–38], not only these higher derivative terms but the very dynamics of gravity would emerge from integrating out the towers of states. In this spirit, the emergence of species scale stretched horizons studied in this work would potentially apply to any black hole horizon. This suggests a possible general picture for the emergence of horizons in quantum gravity: in the UV, the natural quantum system to consider would be that formed by the states in the tower, with no dynamical gravity. When going down to the IR, the species in the tower are integrated out. It is reasonable to expect that this effective description cannot describe the previous physical system in full detail, but it should be able to codify its coarse-grained properties, i.e., its macrostate. How is this made possible? In the IR, gravity emerges and the system would be replaced by a black hole that precisely reproduces this macrostate. In this sense, the emergence of a horizon would be somehow required for the effective description to be able to keep the macroscopic information about the formerly considered physical system.

It is also tantalizing to speculate that the entropy of the black hole is nothing but a reflection of the entanglement entropy of the species after their degrees of freedom are traced out. This goes in the spirit of ER=EPR [110], the N-portrait picture [111], or of the literature on thermal M(atrix) theory black holes, see e.g. [112–118] (also [119, 120] for more recent developments). On a related note, it would be interesting to understand if and how this system made out of species could radiate and reproduce the evaporation of the black hole, perhaps along the lines of [31].

Back to more earthly matters, our work suggest several interesting questions for further research, for instance:

- We have focused on the interpretations of the species scale in asymptotic regions of moduli space, which are those naturally explored by small black holes. It would be interesting to extend our concept of unification of the three notions of species scale and gain a similar understanding in the interior of moduli space, connecting with the proposal in [30, 47, 48, 121], perhaps using large black hole probes as in [28].
- Even in asymptotic regimes, there is a strong ongoing activity in understanding the precise treatment of the species scale in the process of integrating out the SDC tower of states, with different proposals [39–43]. We hope the appearance of such computations in our setup can serve to clarify the proper physical procedures.

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- The non-trivial constraints on higher derivative corrections for the appearance of stretched horizons are reminiscent of similar conditions derived from positivity constraints in the swampland program or in the S-matrix bootstrap, see e.g. [62, 122–124]. It would be interesting to explore this possible connections further.
- The $\mathrm{AdS}_2 \times \mathbf{S}^8$ ansatz in the entropy function analysis suggests, via holography, the possible appearance of IR superconformal behaviour in the worldvolume theory on the D0-brane, once the equivalent to the higher derivative corrections are taken into account. It would be interesting to find further quantitative evidence for this proposal from the field theory side.
- It would be interesting to see whether some of our results extend to lower codimension cases. Higher-dimensional objects that probe infinite distance in field space and are singular in the EFT (such as the End-of-the-World branes of [20, 24, 26, 125–128][16]) could, in the same way as the small black holes, be shown to develop a species scale sized horizon using a modified version of the entropy function formalism.
- In some instances, higher derivative corrections have been shown to create new singularities on the horizon of large black holes [139]. This is to be put in contrast with our work, where curvature corrections instead smooth out singularities. It would be interesting to better understand the mechanisms that make it such that curvature corrections can seemingly both create and smooth out singularities in the EFT.
- There are interesting recent development regarding the possibility of log corrections to the species scale [41, 48, 85]. In this respect, it is tantalizing that our microscopic considerations in section 3.1 lead to such corrections. It would be interesting to exploit our techniques to shed some light on this question.

We hope to come back to these questions in the near future.

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A Tensor structure of R^4 corrections

In this appendix we recall the form of the $t_8 t_8 R^4$ and $\epsilon_{10}\epsilon_{10} R^4$ terms arising in 10d type IIA theory (and closely related to those in 11d M-theory). We follow closely the notation in [36].

Let us start with the $t_8 t_8 R^4$ structure, which denotes

$$t_8 t_8 R^4 = t_8^{A_1 \cdots A_8} t_8^{B_1 \cdots B_8} \, R_{A_1 A_2 B_1 B_2} \cdots R_{A_7 A_8 B_7 B_8} \,, \tag{A.1}$$

[16]For the related topic of solutions in theories with dynamical tadpoles, see [129–132] for early work and [133–138] for related recent developments.

where the t_8 tensor can be written in terms of the metric as[17]

$$\begin{aligned} t_8^{A_1\cdots A_8} = \frac{1}{5}\Big[& -2\left(g^{A_1A_3}g^{A_2A_4}g^{A_5A_7}g^{A_6A_8}+g^{A_1A_5}g^{A_2A_6}g^{A_3A_7}g^{A_4A_8}+g^{A_1A_7}g^{A_2A_8}g^{A_3A_5}g^{A_4A_6}\right) \\ & +8\left(g^{A_2A_3}g^{A_4A_5}g^{A_6A_7}g^{A_8A_1}+g^{A_2A_5}g^{A_6A_3}g^{A_4A_7}g^{A_8A_1}+g^{A_2A_5}g^{A_6A_7}g^{A_8A_3}g^{A_4A_1}\right) \\ & -(A_1\leftrightarrow A_2)-(A_3\leftrightarrow A_4)-(A_5\leftrightarrow A_6)-(A_7\leftrightarrow A_8)\Big]\,. \end{aligned} \tag{A.2}$$

After contracting all the metrics in t_8 with the Riemman tensors in (A.1), the $t_8t_8R^4$ structure can be written as:

$$\begin{aligned} t_8t_8R^4 = 12\, & \left(R^{A_1A_2A_3A_4}R_{A_1A_2A_3A_4}\right)^2 \\ & +192\, R_{A_1}{}^{A_5}{}_{A_3}{}^{A_6}R^{A_1A_2A_3A_4}R_{A_2}{}^{A_7}{}_{A_4}{}^{A_8}R_{A_5A_7A_6A_8} \\ & -192\, R_{A_1A_2A_3}{}^{A_5}R^{A_1A_2A_3A_4}R_{A_4}{}^{A_6A_7A_8}R_{A_5A_6A_7A_8} \\ & +24\, R_{A_1A_2}{}^{A_5A_6}R^{A_1A_2A_3A_4}R_{A_3A_4}{}^{A_7A_8}R_{A_5A_6A_7A_8} \\ & +384\, R_{A_1}{}^{A_5}{}_{A_3}{}^{A_6}R^{A_1A_2A_3A_4}R_{A_2}{}^{A_7}{}_{A_6}{}^{A_8}R_{A_4A_8A_5A_7} \\ & -96\, R_{A_1A_2}{}^{A_5A_6}R^{A_1A_2A_3A_4}R_{A_3A_5}{}^{A_7A_8}R_{A_4A_6A_7A_8}\,. \end{aligned} \tag{A.3}$$

Similarly, $\epsilon_{10}\epsilon_{10}R^4$ structure denotes

$$\begin{aligned} \epsilon_{10}\epsilon_{10}R^4 &= \epsilon_{10}^{C_1C_2A_1\cdots A_8}\,\epsilon_{10\;C_1C_2B_1\cdots B_8}\,R^{B_1B_2}{}_{A_1A_2}\cdots R^{B_7B_8}{}_{A_7A_8} \\ &= -2\cdot 8!\,\delta^{A_1}{}_{[B_1}\cdots\delta^{A_8}{}_{B_8]}R^{B_1B_2}{}_{A_1A_2}\cdots R^{B_7B_8}{}_{A_7A_8}\,, \end{aligned} \tag{A.4}$$

where, in the last line, we have used the usual identity for contracting Levi-Civita tensors (cf. equation (A.1) in [36]). Contracting all the Kronecker delta functions with the Riemann tensors in (A.4), the $\epsilon_{10}\epsilon_{10}R^4$ structure can be written as:

$$\begin{aligned} & \frac{1}{8}\epsilon_{10}\epsilon_{10}R^4 = \\ & -12\left(R^{A_1A_2A_3A_4}R_{A_1A_2A_3A_4}\right)^2 \\ & -192\, R_{A_1}{}^{A_5}{}_{A_3}{}^{A_6}R^{A_1A_2A_3A_4}R_{A_2}{}^{A_7}{}_{A_4}{}^{A_8}R_{A_5A_7A_6A_8} \\ & +192\, R_{A_1A_2A_3}{}^{A_5}R^{A_1A_2A_3A_4}R_{A_4}{}^{A_6A_7A_8}R_{A_5A_6A_7A_8} \\ & -24\, R_{A_1A_2}{}^{A_5A_6}R^{A_1A_2A_3A_4}R_{A_3A_4}{}^{A_7A_8}R_{A_5A_6A_7A_8} \\ & +384\, R_{A_1A_2}{}^{A_5A_6}R^{A_1A_2A_3A_4}R_{A_3}{}^{A_7}{}_{A_5}{}^{A_8}R_{A_4A_8A_6A_7} \\ & +384\, R_{A_1}{}^{A_5}{}_{A_3}{}^{A_6}R^{A_1A_2A_3A_4}R_{A_2}{}^{A_7}{}_{A_5}{}^{A_8}R_{A_4A_7A_6A_8}-768\, R^{A_1A_2}R_{A_1}{}^{A_3}{}_{A_2}{}^{A_4}R_{A_3}{}^{A_5A_6A_7}R_{A_4A_5A_6A_7} \\ & +384\, R^{A_1A_2}R_{A_1}{}^{A_3A_4A_5}R_{A_2A_3}{}^{A_6A_7}R_{A_4A_5A_6A_7}+96\, R_{A_1A_2}R^{A_1A_2}\, R_{A_3A_4A_5A_6}R^{A_3A_4A_5A_6} \\ & -1536\, R^{A_1A_2}R_{A_1}{}^{A_3A_4A_5}R_{A_2}{}^{A_6}{}_{A_4}{}^{A_7}R_{A_3A_7A_5A_6}+768\, R^{A_1A_2}R^{A_3A_4}R_{A_1}{}^{A_5}{}_{A_2}{}^{A_6}R_{A_3A_5A_4A_6} \\ & -768\, R_{A_1}{}^{A_3}R^{A_1A_2}R_{A_2}{}^{A_4A_5A_6}R_{A_3A_4A_5A_6}-32\, R\, R_{A_1A_2}{}^{A_5A_6}R^{A_1A_2A_3A_4}R_{A_3A_4A_5A_6} \\ & +128\, R\, R_{A_1}{}^{A_5}{}_{A_3}{}^{A_6}R^{A_1A_2A_3A_4}R_{A_2A_6A_4A_5}-768\, R^{A_1A_2}R^{A_3A_4}R_{A_1}{}^{A_5}{}_{A_3}{}^{A_6}R_{A_2A_5A_4A_6} \\ & -384\, R^{A_1A_2}R^{A_3A_4}R_{A_1A_3}{}^{A_5A_6}R_{A_2A_4A_5A_6}+1536\, R_{A_1}{}^{A_3}R^{A_1A_2}R^{A_4A_5}R_{A_2A_4A_3A_5} \\ & +384\, R\, R^{A_1A_2}R_{A_1}{}^{A_3A_4A_5}R_{A_2A_3A_4A_5}-24\, R^2\, R_{A_1A_2A_3A_4}R^{A_1A_2A_3A_4} \\ & -384\, R\, R^{A_1A_2}R^{A_3A_4}R_{A_1A_3A_2A_4}-192\left(R_{A_1A_2}R^{A_1A_2}\right)^2+384\, R_{A_1}{}^{A_3}R^{A_1A_2}R_{A_2}{}^{A_4}R_{A_3A_4} \\ & -256\, R\, R_{A_1}{}^{A_3}R^{A_1A_2}R_{A_2A_3}+96\, R^2\, R_{A_1A_2}R^{A_1A_2}-4\, R^4\,. \end{aligned} \tag{A.5}$$

[17] Notice the change of prefactor with respect to equation (B.3) of [36], so that, after expressing this structure as contractions of four Riemann tensors, the result matches that in [95].

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Evaluating on Mathematica each of the contractions in (A.3) and (A.5) on the $\mathrm{AdS}_2 \times \mathbf{S}^8$ ansatz in (4.1), adding them up, and substituting $v = v_2$ and $\beta = v_2/v_1$, we get to the result reported in equation (4.28).

As we used in section 4.4, both the $\hat{t}_8\hat{t}_8\hat{R}^4$ and the $\frac{1}{24}\epsilon_{11}\epsilon_{11}\hat{R}^4$ structures appearing in the M-theory effective action in equation (4.31) look exactly the same as the terms we just discussed. This is, once written in terms of contractions of 11d Riemman tensors, they precisely reduce to the ones in (A.3) and (A.5). Evaluating these contractions on the 11d metric ansatz in (4.34) with Mathematica, we recover the result in (4.36).

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5 Global summary of results and discussion

In this thesis, we investigated the implications of implementing dynamical change of the topology of spacetime, as should be allowed by a background independent theory of QG. In a first part, we studied dynamical realizations of the processes predicted by the cobordism conjecture, in the context of the Swampland Program. This allowed us to characterize the singular extended objects that play the role of implementing these processes. Then, we studied how bordism groups can detect global gauge/diffeomorphism anomalies in theories of quantum gravity. Finally we explored the different ways in which (singular and smooth) extended objects in string theory can, or not, be used as probes of UV physics. Let us briefly summarize the key points and the main results of each of the seven articles.

In Chapter 2, we studied various aspects of dynamical realizations of cobordisms to nothing. These are solutions to the equations of motion that run along a spacetime dimension and where the compact space pinches of at finite distance in spacetime.

In the first article, we considered string theory vacua with tadpoles for dynamical fields in the scalar potential and uncovered universal features of the resulting spacetime-dependent solutions. We argued that the solutions can extend only a finite spacetime distance Δ away in the spacetime dimensions over which the fields vary, scaling as $\Delta^n \sim \mathcal{T}$ with the strength of the tadpole $\mathcal{T}$. We showed that naive singularities arising at this distance scale are physically replaced by ends of spacetime, related to the cobordism defects of the swampland cobordism conjecture and involving stringy ingredients like orientifold planes and branes, or exotic variants thereof. We illustrated these phenomena in large classes of examples, including $\mathrm{AdS}_5 \times T^{1,1}$ with 3-form fluxes, 10d massive IIA, M-theory on K3, the 10d non-supersymmetric $USp(32)$ strings, and type IIB compactifications with 3-form fluxes and/or magnetized D-branes. We also described a 6d string model whose tadpole triggers spontaneous compactification to a semirealistic 3-family MSSM-like particle physics model.

In the second article, we continued studying spacetime-dependent solutions to string theory models with tadpoles for dynamical fields. We confirmed in many new explicit string theoretic examples, that such solutions have necessarily finite extent in spacetime, and are capped off by boundaries at a finite distance, in a dynamical realization of the Cobordism Conjecture. We showed that as the configuration approaches these cobordism walls of nothing, the scalar fields run off to infinite distance in moduli space, allowing to explore the implications of the Swampland Distance Conjecture. We uncovered new interesting scaling relations linking the moduli space distance and the SDC tower scale to spacetime geometric quantities, such as the distance to the wall and the scalar curvature.

We show that walls at which scalars remain at finite distance in moduli space correspond to domain walls separating different (but bordant) theories/vacua; this still applies even if the scalars reach finite distance singularities in moduli space, such as conifold points. We illustrated our ideas with explicit examples in massive IIA theory, M-theory on CY threefolds, and 10d non-supersymmetric strings. In 4d $\mathcal{N} = 1$ theories, our framework reproduces a recent proposal to explore the SDC using 4d string-like solutions.

In the third article, we explain why one observes these universal patterns in the dynamical realization of cobordism, as spacetime dependent solutions of Einstein gravity coupled to scalars containing such end-of-the-world 'branes'. The latter appear in effective theory as a singularity at finite spacetime distance at which scalars go off to infinite field space distance. We provided a local description near the end-of-the-world branes, in which the solutions simplify dramatically and are characterized in terms of a critical exponent, which controls the asymptotic profiles of fields and the universal scaling relations among the spacetime distance to the singularity, the field space distance, and the spacetime curvature. The analysis did not rely on supersymmetry. We studied many explicit examples of such Local Dynamical Cobordisms in string theory, including 10d massive IIA, the 10d non-supersymmetric $USp(32)$ theory, Bubbles of Nothing, 4d $\mathcal{N} = 1$ cosmic string solutions, the Klebanov-Strassler throat, Dp-brane solutions, brane configurations related to the D1/D5 systems, and small black holes. Our framework encompasses diverse recent setups in which scalars diverge at the core of defects, by regarding them as suitable end-of-the-world branes. We also explored the interplay of Local Dynamical Cobordisms with the Distance Conjecture and other swampland constraints.

Finally, in the fourth article, we took a step in the direction of studying time-dependent dynamical cobordisms. We described timelike linear dilaton backgrounds of supercritical string theories as time-dependent Dynamical Cobordisms in string theory, with their spacelike singularity as a boundary defining the beginning of time. We proposed and provided compelling evidence that its microscopic interpretation corresponds to a region of (a strong coupling version of) closed tachyon condensation. We argued that this beginning of time is closely related to (and shares the same scaling behaviour as) the bubbles of nothing obtained in a weakly coupled background with lightlike tachyon condensation. As an intermediate result, we also provided the description of the latter as lightlike Dynamical Cobordism.

In Chapter 3, we studied Dai-Freed anomalies in non-supersymmetric string theories. These anomalies arise in theories with dynamical gravity and where spacetime topology change should be allowed.

More specifically, in the article, we studied the three tachyon-free non-supersymmetric string theories in ten dimensions. These theories provide a handle on quantum gravity away from the supersymmetric lamppost. Despite having been around for decades, they have not been shown to be fully consistent; although local anomalies cancel due to versions of the Green-Schwarz mechanism, there could be global anomalies, not cancelled by the Green-Schwarz mechanism, that could become fatal pathologies. We computed the twisted string bordism groups that control these anomalies via the Adams spectral sequence, showing that they vanish completely in two out of three cases (Sugimoto and $SO(16)^2$) and showing a partial vanishing also in the third (Sagnotti 0'B model). We also computed lower-dimensional bordism groups of the non-supersymmetric string theories, which are of interest to the classification of branes in these theories via the Cobordism Conjecture.

Using an anomaly inflow argument, we were able to propose a worldvolume content of the $SO(16)^2$ NS5-brane. As a byproduct of our techniques and analysis, we also reprove that the outer $\mathbb{Z}_2$ automorphism swapping the two E_8 factors in the supersymmetric heterotic string is also non-anomalous.

Lastly, in Chapter 6, we considered different types of black holes that could be used as probes of UV physics.

In the first article, we argued that supersymmetric BPS black holes can act as efficient finite energy probes of the moduli space geometry thanks to the attractor mechanism. We focused on 4d $\mathcal{N}=2$ compactifications and captured aspects of the effective field theory near the attractor values in terms of physical quantities far away in moduli space. Furthermore, we illustrated how the standard distance in moduli space can be related asymptotically to the black hole mass. We also computed a measure of the resolution with which BPS black holes of a given mass can distinguish far away points in the moduli space. The black hole probes may lead to a deeper understanding of the Swampland constraints on the geometry of the moduli space.

In the second paper, we instead considered singular black holes, that take us to the limits of the domain of validity of the whole notion of an EFT of QG, the species scale Λ_s. We used black holes of vanishing horizon area (small black holes) in effective field theories coupled to quantum gravity to shed light on how the three different physical manifestations of the species scale Λ_s relate to each other. (i) Near the small black hole core, a scalar field runs to infinite distance in moduli space, a regime in which the Swampland Distance Conjecture predicts a tower of exponentially light states, which lower Λ_s. (ii) We integrate out modes in the tower and generate via Emergence a set of higher derivative corrections, showing that Λ_s is the scale at which such terms become relevant. (iii) Finally, higher derivative terms modify the black hole solution and grant it a non-zero, species scale sized stretched horizon of radius Λ_s^{-1}, showcasing the species scale as the size of the smallest possible black hole describable in the effective theory. We present explicit 4d examples of small black holes in 4d $\mathcal{N}=2$ supergravity, and the 10d example of type IIA D0-branes. The emergence of the species scale horizon for D0-branes requires a non-trivial interplay of different 8-derivative terms in type IIA and M-theory, providing a highly non-trivial check of our unified description of the different phenomena associated to the species scale.

6

Resumen global de los resultados y discusión

En esta tesis, investigamos las implicaciones de implementar el cambio dinámico de la topología del espacio tiempo, como debería permitirlo una teoría independiente del fondo de gravedad cuántica (QG). En una primera parte, estudiamos realizaciones dinámicas de los procesos predichos por la conjetura de cobordismo, en el contexto del Programa Swampland. Esto nos permitió caracterizar los objetos extendidos singulares que desempeñan el papel de implementar estos procesos. Luego, estudiamos cómo los grupos de bordismo pueden detectar anomalías globales de calibre/difeomorfismo en teorías de gravedad cuántica. Finalmente, exploramos las diferentes formas en que los objetos extendidos (singulares y suaves) en la teoría de cuerdas pueden, o no, ser utilizados como sondas de la física ultravioleta (UV). Resumamos brevemente los puntos clave y los principales resultados de cada uno de los siete artículos.

En el Capítulo 2, estudiamos varios aspectos de las realizaciones dinámicas de cobordismos hacia la nada. Estas son soluciones a las ecuaciones de movimiento que recorren una dimensión del espacio tiempo y donde el espacio compacto se estrangula a una distancia finita en el espacio tiempo.

En el primer artículo, consideramos vacíos de la teoría de cuerdas con "tadpoles" para campos dinámicos en el potencial escalar y descubrimos características universales de las soluciones resultantes dependientes del espacio tiempo. Argumentamos que las soluciones solo pueden extenderse una distancia finita en el espacio tiempo Δ en las dimensiones del espacio tiempo sobre las cuales los campos varían, escalando como $\Delta^n \sim \mathcal{T}$ con la fuerza del "tadpole" $\mathcal{T}$. Mostramos que las singularidades ingenuas que surgen a esta escala de distancia son físicamente reemplazadas por extremos del espacio tiempo, relacionados con los defectos de cobordismo de la conjetura de cobordismo y que involucran ingredientes de la teoría de cuerdas como planos orientables y branas, o variantes exóticas de estos. Ilustramos estos fenómenos en grandes clases de ejemplos, incluyendo $\mathrm{AdS}_5 \times T^{1,1}$ con flujos de 3-formas, IIA masivo en 10d, M-teoría en K3, las cuerdas $USp(32)$ no supersimétricas en 10d, y compactificaciones de tipo IIB con flujos de 3-formas y/o branas magnetizadas. También describimos un modelo de cuerda en 6d cuyo "tadpole" desencadena una compactificación espontánea a un modelo de física de partículas MSSM-like semi-realista de 3 familias.

En el segundo artículo, continuamos estudiando soluciones dependientes del espacio tiempo para modelos de teoría de cuerdas con "tadpoles" para campos dinámicos. Confirmamos en muchos nuevos ejemplos explícitos de teoría de cuerdas, que tales soluciones tienen necesariamente una extensión finita en el espacio tiempo, y están tapadas por límites a una distancia finita, en una realización dinámica de la Conjetura de Cobordismo. Mostramos que a medida que la configuración se acerca a estas paredes de cobordismo hacia la nada, los campos escalares se alejan a una distancia infinita en el espacio de móduli, permitiendo explorar las implicaciones de la Conjetura de Distancia. Descubrimos nuevas relaciones de escala interesantes que vinculan la distancia en el espacio de móduli y la escala de la torre SDC a cantidades geométricas del espacio tiempo, como la distancia a la pared y la curvatura escalar. Mostramos que las paredes en las que los escalares permanecen a una distancia finita en el espacio de móduli corresponden a paredes de dominio que separan diferentes teorías/vacíos (pero bordantes); esto sigue aplicándose incluso si los escalares alcanzan singularidades a una distancia finita en el espacio de móduli, como puntos de conifolds. Ilustramos nuestras ideas con ejemplos explícitos en la teoría IIA masiva, M-teoría en tresfolds CY, y cuerdas no supersimétricas en 10d. En teorías 4d $\mathcal{N}=1$, nuestro marco reproduce una propuesta reciente para explorar la SDC usando soluciones tipo cuerda en 4d.

En el tercer artículo, explicamos por qué se observan estos patrones universales en la realización dinámica del cobordismo, como soluciones dependientes del espacio tiempo de la gravedad de Einstein acoplada a escalares que contienen tales 'branas' al final del mundo. Estas últimas aparecen en la teoría efectiva como una singularidad a una distancia finita en el espacio tiempo en la que los escalares se alejan a una distancia infinita en el espacio de campo. Proporcionamos una descripción local cerca de las branas al final del mundo, en la que las soluciones se simplifican drásticamente y se caracterizan en términos de un exponente crítico, que controla los perfiles asintóticos de los campos y las relaciones de escala universales entre la distancia en el espacio tiempo a la singularidad, la distancia en el espacio de campo y la curvatura del espacio tiempo. El análisis no se basó en la supersimetría. Estudiamos muchos ejemplos explícitos de tales Cobordismos Dinámicos Locales en la teoría de cuerdas, incluyendo IIA masivo en 10d, la teoría $USp(32)$ no supersimétrica en 10d, Burbujas de la Nada, soluciones de cuerdas cósmicas 4d $\mathcal{N}=1$, la "throat" de Klebanov-Strassler, soluciones de Dp-branas, configuraciones de branas relacionadas con los sistemas D1/D5, y pequeños agujeros negros. Nuestro marco abarca diversas configuraciones recientes en las que los escalares divergen en el núcleo de los defectos, al considerarlos como branas adecuadas al final del mundo. También exploramos la interacción de los Cobordismos Dinámicos Locales con la Conjetura de Distancia y otras restricciones del Swampland.

Finalmente, en el cuarto artículo, dimos un paso en la dirección de estudiar cobordismos dinámicos dependientes del tiempo. Describimos fondos de dilatón lineal timelike de teorías de cuerdas supercríticas como Cobordismos Dinámicos dependientes del tiempo en la teoría de cuerdas, con su singularidad espacial como un límite que define el comienzo del tiempo. Propusimos y proporcionamos evidencia convincente de que su interpretación microscópica corresponde a una región de (una versión de acoplamiento fuerte de) condensación cerrada de taquiones. Argumentamos que este comienzo del tiempo está estrechamente relacionado con (y comparte el mismo comportamiento de escala que) las burbujas de la nada obtenidas en un fondo débilmente acoplado con condensación de taquiones null. Como resultado intermedio, también proporcionamos la descripción de esta última como Cobordismo Dinámico lightlike.

En el Capítulo 3, estudiamos las anomalías Dai-Freed en teorías de cuerdas no supersimétricas. Estas anomalías surgen en teorías con gravedad dinámica y donde se debería permitir el cambio en la topología del espacio tiempo.

Más específicamente, en el artículo, estudiamos las tres teorías de cuerdas no supersimétricas sin taquiones en diez dimensiones. Estas teorías proporcionan un manejo de la gravedad cuántica lejos del poste de luz supersimétrico. A pesar de haber estado presentes durante décadas, no se ha demostrado que sean completamente consistentes; aunque las anomalías locales se cancelan debido a versiones del mecanismo de Green-Schwarz, podría haber anomalías globales, no canceladas por el mecanismo de Green-Schwarz, que podrían convertirse en patologías fatales. Calculamos los grupos de bordismo de cuerdas torcidas que controlan estas anomalías a través de la secuencia espectral de Adams, mostrando que desaparecen por completo en dos de los tres casos (Sugimoto y $SO(16)^2$) y mostrando un desvanecimiento parcial también en el tercero (modelo 0'B de Sagnotti). También calculamos grupos de bordismo de dimensiones inferiores de las teorías de cuerdas no supersimétricas, que son de interés para la clasificación de branas en estas teorías mediante la Conjetura de Cobordismo. Utilizando un argumento de flujo de anomalías, pudimos proponer un contenido de volumen mundial de la NS5-brana de $SO(16)^2$. Como subproducto de nuestras técnicas y análisis, también demostramos que el automorfismo externo $\mathbb{Z}_2$ que intercambia los dos factores E_8 en la cuerda heterótica supersimétrica también es no anómalo.

Por último, en el Capítulo 6, consideramos diferentes tipos de agujeros negros que podrían utilizarse como sondas de la física UV.

En el primer artículo, argumentamos que los agujeros negros BPS supersimétricos pueden actuar como eficientes sondas de energía finita de la geometría del espacio de móduli gracias al mecanismo atrayente. Nos centramos en compactificaciones $\mathcal{N}=2$ de 4d y capturamos aspectos de la teoría efectiva cerca de los valores atrayentes en términos de cantidades físicas lejanas en el espacio de móduli. Además, ilustramos cómo la distancia estándar en el espacio de móduli puede relacionarse asintóticamente con la masa del agujero negro. También calculamos una medida de la resolución con la que los agujeros negros BPS de una masa dada pueden distinguir puntos lejanos en el espacio de móduli. Las sondas de agujeros negros pueden llevar a una comprensión más profunda de las restricciones del Swampland en la geometría del espacio de móduli.

En el segundo artículo, en cambio, consideramos agujeros negros singulares, que nos llevan a los límites del dominio de validez de toda la noción de una EFT de QG, la escala de especies Λ_s. Utilizamos agujeros negros con área de horizonte que tiende a cero (agujeros negros pequeños) en teorías efectivas acopladas a la gravedad cuántica para arrojar luz sobre cómo se relacionan entre sí las tres diferentes manifestaciones físicas de la escala de especies Λ_s. (i) Cerca del núcleo del agujero negro pequeño, un campo escalar se aleja a una distancia infinita en el espacio de móduli, un régimen en el que la Conjetura de Distancia predice una torre de estados exponencialmente ligeros, que disminuyen Λ_s. (ii) Integramos los modos en la torre y generamos mediante la Emergencia un conjunto de correcciones de derivadas superiores, mostrando que Λ_s es la escala en la que dichos términos se vuelven relevantes. (iii) Finalmente, los términos de derivadas superiores modifican la solución del agujero negro y le otorgan un horizonte estirado de tamaño de escala de especies de radio Λ_s^{-1}, mostrando la escala de especies como el tamaño del agujero negro posible más pequeño que se puede describir en la teoría efectiva. Presentamos ejemplos explícitos de agujeros negros pequeños en supergravedad $\mathcal{N} = 2$ de 4d y el ejemplo de 10d de las D0-branas de tipo IIA. La emergencia del horizonte de escala de especies para las D0-branas requiere una interacción no trivial de diferentes términos de 8 derivadas en tipo IIA y M-teoría, proporcionando una verificación altamente no trivial de nuestra descripción unificada de los diferentes fenómenos asociados a la escala de especies.