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The Horizon Problem

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Abstract. The standard model of cosmology describes relatively in a satisfactory way, the major stages of the evolution of the observable universe, over time. Despite its success, it presents some problems that constitute a puzzle nowadays. Among them, we cite the horizon problem, which is the purpose of this paper to study and present the different solutions that are available in the literature.
The inflationary model, originally introduced by A. Guth in 1981, was designed to solve the horizon, flatness and entropy problems. Since then, we are faced now with more than 200 inflationary models. However, space missions, for to test the validity of some cosmological models (Planck 2013, 2015...) show that the universe follows the simplest proposed inflationary theories.
In this work, we will study some models proposed as a solution to the horizon problem.

1. Introduction

The standard model of cosmology (ΛCDM), also known as the concordance model, is the mathematical support of the big bang cosmological model. It is relatively successful in describing the evolution of the observable universe, but it has shortcomings such as: horizon problem, flatness, entropy of the universe, origin of the homogeneity of the universe at large scales. Inflationary model was proposed by A. Guth (1981) [1] to cure these shortcomings, especially the horizon and flatness problems.

But still the horizon problem a field of interest for cosmologists. This paper is organized as follows: in section two we review briefly the standard model of cosmology. In section three we present the different types of horizon. We will show how the horizon problem arises in section four. Next we will show different solutions of the horizon problem. Then, in section six we estimate the particle and event horizons for CMB from the surface of last scattering. Finally we conclude.

2. The standard model of cosmology

The relativistic cosmology is based essentially in these three ingredients [2]:

- The cosmological principle which states the homogeneity and isotropy of the universe. This leads to adopt the Friedman-Robertson-Walker metric:

$$ds^2 = c^2dt^2 - a(t)\left[\frac{dr^2}{1-kr^2} + r^2d^2\theta + r^2\sin^2\theta d^2\varphi\right] \quad (1)$$

where $a(t)$ is the scale factor, measuring how the universe is expanding, t is the proper time measured by an observer at rest in co-moving coordinates, that is $(r,\theta,\varphi) = \text{constant}$, in other words, coordinates that remain fixed with the expansion of the universe and k is the spatial curvature, taking on values $k = 0,\ +1,\ -1$ (after rescaling) which correspond to a flat universe, a closed universe and an open universe respectively.

- Weyl postulate according to which the universe should considered as a perfect fluid,
- The general relativity is assumed to be the correct theory of gravity on cosmological scales.

We suppose also that the perfect fluid, with energy density $\rho(t)$ and pressure $p(t)$ that is filling the universe is made of different constituents i, each having an energy density $\rho_i(t)$ and pressure $p_i(t)$. These constituents are: matter (ordinary and dark), radiation and dark energy.

The equations describing the evolution of the universe are the Friedman equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2} \tag{2}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{3}$$

where $H = \dot{a}/a$ is the Hubble parameter and Λ is the cosmological constant.

By combining between these two equations, we find

$$\dot{\rho} = -3H(\rho + p) \tag{4}$$

which translates the energy conservation of the universe. Note that a dot placed in a variable means derivative of this variable with respect to time.

To describe the evolution of the universe entirely from the big bang to now, we need to set an equation of state relating the pressure to the energy density of the universe:

$$p = \omega\rho \tag{5}$$

with ω a proportionality constant, such that

$$\omega = \begin{cases} 0 & \text{for relativistic matter} \\ 1/3 & \textit{for radiation} \\ -1 & \text{for cosmological } \textit{constant} \end{cases} \tag{6}$$

It is also possible to consider other constituents such as quintessence, a certain hypothetical form of dark energy with $-1 < \omega < -1/3$, proposed to explain the acceleration of the universe [2].

According to recent cosmological data, the universe is flat [3], so we put $k = 0$ in the Friedman equation (2). The scale factor evolves differently from a dominated era to another. The table 1 below shows its evolution for different dominated era.

Table 1. Scale factor for different dominated era.

	Density ρ_i	Scale factor $a(t)$
Radiation domination	a^{-4}	$t^{1/2}$
Matter domination	a^{-3}	$t^{2/3}$
Dark energy domination	$\Lambda c^4/8\pi G$	e^{Ht}

Astronomers define the red shift z of an object as the ratio between the detected wavelength to the emitted wavelength:

$$1+z=\frac{\nu_{\det}}{\nu_{em}}=a\left(t_{\det}\right)/a\left(t_{em}\right) \tag{7}$$

For one constituent i of the universe, we can show that the energy density ρ_i reads in terms of the red shift as:

$$\rho_i=\rho_{i0}\left(1+z\right)^{3\left(1+\omega_i\right)} \tag{8}$$

We define the critical density:

$$\rho_c=\frac{3c^2}{8\pi G}H^2 \tag{9}$$

and the dimensionless density parameter:

$$\Omega=\frac{\rho}{\rho_c} \tag{10}$$

For the component i, it reads:

$$\Omega_{\mathrm{i}}=\frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{c}}} \tag{11}$$

Substituting (8), (9) and (10) in (11) we get:

$$\Omega_i=\Omega_{i0}H_0^2\frac{\left(1+z\right)^{3\left(1+\omega_i\right)}}{H^2\left(z\right)} \tag{12}$$

Adding all the Ω_i leads to the following Hubble parameter:

$$H\left(z\right)=H_0\left(\sum_{i=1}^{\infty}\frac{\Omega_{i0}}{\Omega}\left(1+z\right)^{3\left(1+\omega_i\right)}\right)^{1/2} \tag{13}$$

3. Types of Horizons

The proper distance is defined as the distance between two simultaneous events at proper time t measured by a fundamental or an inertial observer [2].

$$d_p=a\left(t\right)\int_0^R dr\ \left(1-kr^2\right)^{-1/2}=ca\left(t\right)\int_{t_e}^{t_0}dt'\ \left[a\left(t'\right)\right]^{-1} \tag{14}$$

R is the radial co-moving coordinate of the measured point. We have to mention, here, that it is the physical distance defined as $R\left(t\right)=a\left(t\right)$ r . t_e is the time of emission of the photon, whereas t_0 is the time of detection, at present.

3.1. Event horizon

It is the hyper-surface boundary between the future events that will be observable by A, and those that will never be seen by A. So event horizon can be expressed as [2]:

$$H_e=\lim_{t\to\infty}\left(-d_p\right) \tag{15}$$

3.1.1. Event horizon at the present time.

At infinitely time, the red shift is zero. So the event horizon now can be expressed as:

$$H_e=\int_{-1}^{0}dx\ \left(\Omega_{m0}\left(1+x\right)^3+\Omega_{\Lambda 0}\right)^{-1/2} \tag{16}$$

Note that we have neglected the parameter of density of radiation , as its value is small compared to that of matter and dark energy.

3.1.2. Event horizon at arbitrary time.

At arbitrary time, we can show that the expression of the event horizon is:

$$H_e = \frac{1}{H_0(z)} \int_{-1}^{z} dx \left[\left(\Omega_m (1+x)^3 (1+z)^3 + \Omega_\Lambda \right) \right]^{-1/2} \tag{17}$$

where we corrected the upper limit of integration in the formula given in reference [4].

3.2. Particle horizon

If we assume that the universe has finite age and because of the limited value of the light velocity, any observer A can define two regions in 3D ($t=t_0$), for some fixed value of time. The one contains co-moving points already seen by A, the other is its complement. Particle horizon takes into account only the past events with respect to A. The horizon particle is defined [4]

$$H_p = \lim_{t_e \to 0} d_p(t_e) = c\int_0^{t_0} dt' \left[a(t') \right]^{-1} \tag{18}$$

So, in this sense, particle horizon at present is the proper distance a photon of light emitted initially at $t = 0$ travelled until now t_0.

3.2.1. Particle horizon at the present time.

By using the fact that light is infinitely red shifted initially at $t = 0$, following reference [4], we obtain the expression of the particle horizon at the current proper time:

$$H_p(z) = \frac{1}{H_0} \int_0^{\infty} dx \left[\left(\Omega_{m0} (1+x)^3 + \Omega_{\Lambda 0} \right) \right]^{-1/2} \tag{19}$$

where the subscript 0 means the values of the Hubble and density parameters as measured at current time.

3.2.2. Particle horizon at arbitrary time.

At arbitrary time, we can show that the expression of the particle horizon is:

$$H_p(z) = \frac{1}{H_0} \int_z^{\infty} dx \left[\left(\Omega_{m0} (1+x)^3 (1+z)^3 + \Omega_{\Lambda 0} \right) \right]^{-1/2} \tag{20}$$

Note that we corrected a mistake that appeared in reference [4] regarding the lower limit of integration.

3.3. The co-moving horizon and the Hubble radius

It is important to differentiate between the co-moving horizon and the Hubble radius. We define a Hubble sphere as a spherical region in the universe delimiting two regions, in such a way, objects outside this sphere are receding at a speed greater than the speed of light c. Thus the Hubble radius is defined as

$$R_H = cH^{-1} \tag{21}$$

The Hubble radius has no physical meaning except that it is used as a useful length scale. It is often confused, by mistake, with the size of the universe, though this later is larger.

We define the co-moving Hubble radius [5]:

$$r_{co-Hubble} = (aH)^{-1} \tag{22}$$

4. The Horizon problem

The universe displays a pronounced degree of large-scale homogeneity. CMB is the observational evidence. Measurements show that it has a thermal blackbody spectrum with a temperature highly homogeneous.

$$T = (2.7 \pm 10^{-5})K \tag{23}$$

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The photons we see today are those emitted from the surface of last scattering (380 000 years). At that time the particle horizon was small enough that if we could see it today at the spherical surface of last scattering, its diameter will subtend an angle of 1°. So, how did different opposite regions in the universe get into causal contact?

For some cosmologists, in Friedman-Robertson-Walker metric the horizon problem doesn't occur because, by assumption, we postulated a homogeneous and isotropic universe [2].

5. Solutions to the Horizon problem

To solve the problem of horizon, several ideas emerged in the literature. The well known is the solution involving inflationary scenario in the early universe.

5.1. Inflationary models

It was A. Guth who, first, used the concept of inflation [1]. According to him, from 10^{-45} s to 10^{-30} s, the universe underwent an accelerated expansion during which the scale factor increased by 10^{50}. The solution to the horizon problem is found by bringing the initially disconnected regions into a region where they are causally in contact.

5.1.1. Mechanism of inflation.

The universe was in causal contact during the decoupling and the Hubble radius was about 1**°.** To solve the problem, the horizon should be greater than the size of the universe.

The necessary condition is

$$\frac{d\left(aH\right)^{-1}}{dt} < 0 \tag{24}$$

where $\left(aH\right)^{-1}$ is the co-moving Hubble radius. Relation (19) has an easy interpretation: During inflation, the Hubble radius as measured in co-moving coordinates decreases.

5.1.2. Condition for inflation.

We should mention, here, that inflation is imposed in the standard model of cosmology only in the early universe from 10^{-45} s to 10^{-30} s. Elsewhere the standard model is not changed. So we should find a condition for inflation to occur. We define the parameter $\varepsilon = -\dot{H}/H^2$.

The condition for inflation is:

$$\varepsilon = -\dot{H}/H^2 = -dln\ H/dN < 1 \tag{25}$$

We define the e-folding

$$a(t) \sim \exp\left[\int \mathrm{H}(\mathrm{t})\mathrm{dt}\right] = \mathrm{e}^{-N(t)} \tag{26}$$

The e-folding measures how much inflation needed to solve the horizon problem (and flatness problem). In reference [1], it is found that the e-folding should be $N{\sim}60$ to solve the horizon problem. During inflation, the $\sim 60H^{-1}$ Hubble time elapsed only $\sim 60H^{-1} \sim 10^{-34}s$ During this period, the Hubble volume increased from $10^{-28}m$ to $10^{-2}m$. The duration of inflation should be enough so that the parameter ε remains small for large Hubble time. So we need to define another parameter which takes account of this fact:

$$\eta \equiv \frac{\dot{\varepsilon}}{\varepsilon H} = \frac{dLn\varepsilon}{dN} \rightarrow |\eta| \ll 1 \tag{27}$$

Despite the success of the inflationary model of A. Guth to solve the horizon problem, soon it showed some discrepancies (Problem of formation of bubbles in the early universe, eternal inflation ...).

So other inflationary models were proposed by postulating new fields. We have now in the literature more than 200 models that differ from each other.

For this reason, one can argue that inflationary concept is not a final solution to the horizon problem.

5.2. Other solutions

5.2.1. Solution of $R_h = ct$ universe without inflation

In series of paper [6], [7], [8],F. Melia showed that

- The horizon problem emerges only for ΛCDM model,
- No need of inflation scenario,
- Invoking Birkhoff theorem, he argued that the time t involved in the expression of the proper distance should differ from one observer to another. So, we can write:

$$R(t) = a(t)r(t) \rightarrow \dot{R} = \dot{a}r + a\dot{r} \tag{28}$$

Melia arrived to the result

$$R_h = cH^{-1} = ct \tag{29}$$

R_h being the Hubble radius.

The solution of F. Melia solves the horizon problem naturally. However, his ideas are criticized in the literature [9].

5.2.2. Solution by P. Magain.

Another idea emerged in the literature by P. Magain [10]. He argued that cosmic time doesn't flow uniformly. In reference [10], the author supposes that an observer in an expanding universe will feel a cosmic time τ flowing at a variable rate so that the he will always measure a null apparent curvature. His idea leads to solve the horizon problem without inflation, as regions in the universe not causally connected will evolve independently for each other and each one will has its own proper cosmic time. Also the acceleration of the expansion of the universe is explained without introducing dark energy or dark matter.

6. Cosmological horizons for CMB

Isotropy is exhibited by the Cosmic Microwave Background (CMB) radiation and it can be shown that isotropy guaranties homogeneity [2]. This radiation was studied by COBE satellite. The CMB that we observe today comes from the surface of last scattering with a red shift $z_{ls} = 1089$.

Hubble constant:

$$H_0 = 70.1 kms^{-1} Mpc^{-1} \tag{30}$$

Density parameters:

$$\Omega_{m_0} = 0.278,\ \Omega_{r_0} = 5\times10^{-5},\ \Omega_{\Lambda_0} = 0.722. \tag{31}$$

The proper distance to the mean value of the surface of last scattering is:

$$d_p = \frac{8.016\ 10^5}{70.1}\int_0^{1089} dy\left[\Omega_{m0}(1+y)^3 + \Omega_{\Lambda 0}\right]^{-1/2} = 14042.35\text{Mpc} \tag{32}$$

The result found, by using maple 2016, is in agreement with that in reference [4].

The present value of the event horizon is, by using relation (17), is:

$$H_e = \frac{2.998\ 10^5}{70.1}\int_{1089}^{-1} dx\left[\Omega_{m0}(1+x)^3(1+1089)^3 + \Omega_{\Lambda 0}\right]^{-1/2} = 17.769 Mpc \tag{33}$$

Notice that equation (12) can be used to calculate density parameters at any time t of red shift z .

The present value of the event horizon is, by using relation (20):

$$H_p\left(z\right)=\frac{2.998\ 10^5}{70.1}\int_{1089}^{\infty}dx\ \left[\left(\Omega_{m0}\left(1+x\right)^3\left(1+1089\right)^3+\Omega_{\Lambda 0}\right)\right]^{-1/2}=0.0004Mpc$$

These values are different from that found in reference [4], because of an error estimating the upper and down limits of integration, there. The value found there is $14577Mpc$ for particle horizon and $4825Mpc.$ for event horizon.

Both the values of event and particle horizons to the surface of last scattering are smaller than the proper distance.

7. Conclusion

In this paper, we have studied the horizon problem and presented three different solutions : solution using inflationary model (A. Guth), a solution without inflation (F. Melia) in the framework of R_h=ct model, and a solution with proper time flowing non uniformly. We estimated the particle and event horizons of CMB Radiation at surface of last scattering and found that their values are smaller than the proper distance from SLS to now.

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