A BOOTSTRAP CALCULATION FOR THE MASS OF THE $\pi\pi$ $(T=0,\ l=0)$ ANTIBOUND STATE

S. Ciulli, Gr. Ghika

Institute for Atomic Physics — Bucharest, Rumania

(Presented by S. CIULLI)

In the last four years many papers [1, 2] have been published regarding the existence or nonexistence of a particle related to the enhancement of the $\pi\pi \to \pi\pi$ $(l = 0,\ T = 0)$ cross section at zero kinetic energy (near the elastic threshold), observed in the Abashian — Booth — Crowe experiment (ABC particle).

Three months ago, Dr. Atkinson [8] showed that there is a strong indication that an antibound state will occur in the $(l = 0,\ T = 0)$ partial wave of the $\pi\pi$ scattering, just under the elastic threshold $(s = 4)$ *. The effect of such an antibound state for the $\pi\pi$ cross section could be similiar to that produced by a. stable $\pi\pi$ bound state. Nevertheless such a «particle» would not be detectable experimentaly, the antibound state being a pole on the second Riemann sheet of the amplitude.

A complete bootstrap calculation would yield all the resonance masses and coupling constants (scattering lengths). For such a bootstrap calculation one must take properly into account the left hand cut produced by the crossed reaction *, but in the present work we shall use experimental values for $T = 0$ scattering length restricting ourselves to the Chew pole approximation for the left hand cut. By the N/D method

$$D(s) = D(4) - \frac{s-4}{\pi}\int_4^{\infty}\frac{N(s')\sqrt{\frac{s'-4}{s'}}}{(s'-4)(s'-s)}\,ds' \qquad (1)$$

we get in the pole approximation $[N(s) = \frac{1}{s+s_0}\ ;\ s_0 > 0]$

$$A_{l=0}^{T=0}(s) = \frac{1}{D(4)(s+s_0)\frac{2}{\pi}\sqrt{\frac{4-s}{s}}\,\text{arctg}\sqrt{\frac{s}{4-s}} - \frac{1}{\pi}\frac{4-s}{\sqrt{s_0(s+4)}}\ln\frac{\sqrt{s_0+4}+\sqrt{s_0}}{\sqrt{s_0+4}-\sqrt{s_0}}},\quad 0<s<4 \qquad (2)$$

In order to estimate the position of the antibound state, Atkinson approximated the behaviour of the partial wave amplitude between $s = 0$ and $s = 4$ by a constant. By analytical continuation [4] a pole is found on the real axis of the second Riemann sheet, close to the elastic threshold.

This possible interpretation of the ABC-«particle» is very attractive. A less crude estimation becomes however necessary in order to compute more accurately the position of this second Riemann sheet pole. It is the purpose of the present paper to perform a bootstrap calculation of the mass of this antibound state.

where $D(4)$ and s_0 are two independent parameters. $D(4)$ is determined by the value of the amplitude in point number 1 (see fig.):

$$D(4) = \frac{1}{a^0(s_0+4)} \times (a^0\text{-scattering length for } T=0). \qquad (3)$$

If an antibound state exists, its position m^2 is completely determined by s_0. Indeed, the form of the amplitude on its second Riemann sheet for $0 < s < 4$ is [4]:

$$^{II}A_0^0(s) = \frac{A_0^0(s)}{1-2\sqrt{\frac{4-s}{s}}A_0^0(s)} \qquad (4)$$

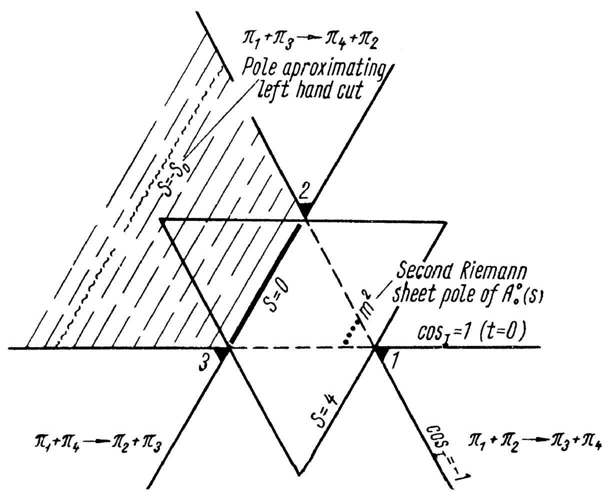
* In the following we shall use natural units with the mass of the meson equal to one. As usual, $s = (p_1 + p_2)^2 = 4(\nu + 1)$, $t = (p_1 + p_3)^2 = -2\nu(1 - \cos\theta)$, $u = (p_1 + p_4)^2 = -2\nu(1 + \cos\theta)$; $\nu = |\mathbf{p}|^2$.

* This problem will be discussed in the last part of this paper. A more detailed version of the present work will appear in Revue Roumaine de Physique.

having a pole at $s = m^2$ where m (antibound state mass) is given by [3]:

$$\frac{1}{2}\sqrt{\frac{m^2}{4-m^2}} = A_0^0(m^2), \quad 0 < m^2 < 4. \qquad (4')$$

Now, to obtain a «feed-back» equation for m, we equate the value of $A_{l=0}^{T=0}(s)$ from equation (2) at $s = 0$, to that obtained by integration of $A_{l=0}^{T=0}(s, t)$ on the line conecting points



No. 2 and 3 (the solid line in fig. 1). This enables us to take advantage of the fact that on the solid line $A_{l=0}^{T=0}(s, t)$ can be written as a dispersion integral over the imaginary part of the forward (backward) amplitude of the crossed processes. Although the pole approximation is a somewhat rough approach, having requested for $A_0^0(s)$ to have correct values at $s = 0$ and $s = 4$ it is expected that (2) would be a good approximation in the interval $0 \leqslant m^2 \leqslant 4$. We feel this procedure «more stable» then that which uses [2] the symmetry relations of the derivatives of A in a single point ($s = 4/3$, $t = 4/3$). The estimation of the error in the position of the antibound pole (see later) confirms this statement.

Hence at $s = 0$ we have

$$\left\{\frac{s_0}{(s_0+4)\,a^0} + \frac{2}{\pi} - \frac{4}{\pi\sqrt{s_0\,(s_0+4)}} \times \right.$$
$$\left. \times \ln\frac{\sqrt{s_0+4}+\sqrt{s_0}}{\sqrt{s_0+4}-\sqrt{s_0}}\right\}^{-1} = \frac{1}{2}\int_{-1}^{+1} d\cos\theta_I \times$$
$$\times\left[\frac{1}{3}a^0 + \frac{u-4}{3\pi}\int_4^\infty \frac{\operatorname{Im} A_0^0(a^0;\, u')\,du}{(u'-4)(u'-u)} + \right.$$
$$+\frac{t}{3\pi}\int_4^\infty \frac{\operatorname{Im} A_0^0(a^0;\, t')\,dt}{t'(t'-t)} +$$
$$+\frac{3(u-4)}{\pi}\int_4^\infty \frac{\operatorname{Im} A_1^1(u')}{(u'-4)(u'-u)}\,du' +$$
$$+\frac{3t}{\pi}\int_4^\infty \frac{\operatorname{Im} A_1^1(t')\,dt'}{(t'-t)\,t'} + \frac{5}{3}a^2 +$$
$$+\frac{5(u-4)}{3\pi}\int_4^\infty \frac{\operatorname{Im} A_0^2(a^2;\, u')\,du'}{(u'-4)(u'-u)} +$$
$$\left. +\frac{5}{3}\frac{t}{\pi}\int_4^\infty \frac{\operatorname{Im} A_0^2(a^2;\, t')\,dt'}{t'(t'-t)}\right], \qquad (5)$$

where $u = 2(1 + \cos\theta_I)$. The sign of $T = 1$ coefficient implies that the forward scattering of the third channel was defined along $s = 0$. Therefore $\operatorname{Im} A'(u', s = 0) = \sum_l (2l+1) \operatorname{Im} A_l^I(u')$. Only S and P waves were retained, higher waves tending rapidly to zero near the elastic threshold. In fact, even the P wave ($T = 1$) contribution to the right hand side of (5) is rather small *.

The $T = 1$ wave was calculated from the known exierimental parameters of the ϱ meson. One could write for the ϱ meson amplitude a Breit — Wigner expression satisfying unitarity

$$-\frac{2\sqrt{\nu+1}\operatorname{Im}\sqrt{\nu_0}}{\nu-|\nu_0|+2i\sqrt{\nu}\operatorname{Im}\sqrt{\nu_0}}. \qquad (6)$$

This expression was then corrected to take properly into account the actual threshold behaviour [3]:

$$A_1^1(\nu=0)=0, \quad A_1^{1\prime}(\nu=0) \sim 0.05. \qquad (7)$$

In this purpose (6) was written in a N/D form and a CDD pole was added to D just at $\nu = 0$, ($A_1^1 = \frac{N}{D+\lambda/\nu}$) in order to fit (7). Hence, the ϱ meson contribution becomes

$$A_0^1(\nu) = \frac{-\sqrt{\nu+1}}{1.65\nu-7.476-\frac{20}{\nu}+i\sqrt{\nu}} \qquad (8)$$

corresponding to a $T = 1$ cross section

$$\tau_\rho = \frac{48\pi\nu^2}{(1.65\nu^2-7.476\nu-20)^2+\nu^3}$$

reaching its maximum at $s = 29{,}26$ and having $\gamma = \frac{1}{2}\sqrt{s_\rho\Gamma} = 2{,}325$ ($\Gamma = 120$ MeV).

* For the $T = 0$, 1, and 2 contribution we found respectively 0.489, 0.015, 0.802 for $a^0 = 2{,}5$ — and 0.257, 0.015, 0.401 for $a^0 = 1$.

The $T = 2$ term would give the most important contribution because of its large isotopic spin factor. For A^2 we take a $1/D$ approximation. Now, owing to the lack of experimental data on the $T=2$ scattering length a^2, in order to determine it, we shall use the well known relation $A^0(s, t) = \frac{5}{2} A^2(s, t)$ in the point $s = 2$, $t = 0$, (which is not to far from the maximum symmetry point $s = \frac{4}{3}$, $t = \frac{4}{3}$ where the above relation actually holds).

$$-a^0 + \frac{2}{\pi} \int_4^\infty \frac{\operatorname{Im} A_0^0(a^0; s')\, ds'}{(s'-4)(s'-2)} + \frac{5}{2} a^2 - \frac{5}{\pi} \int_4^\infty \frac{\operatorname{Im} A_0^2(a^2; s')\, ds'}{(s'-4)(s'-2)} + \frac{1}{\pi} \int_4^\infty \frac{\operatorname{Im} A_0^0(a^0: s')\, ds'}{s'(s'-2)} + \frac{9}{2\pi} \int_4^\infty \frac{3 \operatorname{Im} A_1^1(s')\, ds'}{s'(s'-2)} - \frac{5}{2\pi} \int_4^\infty \frac{\operatorname{Im} A_0^2(a^2; s')\, ds'}{s'(s'-2)} = 0. \qquad (9)$$

Equations (5) and (9) are to be solved together. The experimental value of the $T = 0$ scattering length being not known exactly *, the equations were solved for a set of values of a^0 ranging from 1 to 2.5. The corresponding results are listed in Table togetger with the mass m of the $T = 0$ antibound state (ABC «particle») given by (4′).

To estimate the errors, we compared the above listed a^2 values to those obtained by using the $A^0 = \frac{5}{2} A^2$ condition in $s = 4$, $t = 0$, which is two times farther from the maximum symmetry point than $s = 2$, $t = 0$. The obtained discrepancies exceed considerably those produced by neglecting the P-wave term, and represent therefore an indication for the order of magnitude of the errors connected with the hitherto neglected higher waves. The errors of s_0 are great, but those affecting the value of the ABC mass, are insignificant (ranging from 0.9 to 3.7 MeV), owing to its extreme neighbourhood to the elastic threshold.

Of course, although the errors of the antibound state mass are probably very low, expression (2) does not represent a fair approximation for the scattering amplitude at all possible values of s. We see two different possibilities to improve the results, taking properly into account the left hand cut.

Firstly one can use for the amplitude (and its cosine derivative) a dispersion relation at $t = 0$, where the crossing transformation can be performed simply. Some trouble would be produced by unitarity (which relates real and imaginary parts of the forward amplitude derivatives) because the cosine power series around $t = 0$ would not in general converge over the whole physical region $0 > t > 4 - s_0$. This point can be solved by expanding the amplitude into power of the function $w(s, t)$ which maps the cosine singularities on the boundary of the unit circle [5, 6].

The second possible way is to use Legendre expansions only in the s-channel, and w series in the crossed channels. This later ones will permitt analytic continuation of the amplitude in the dashed region of fig. 1, in order to calculate the left-hand cut of the partial waves. If one isolates the marginal square

a^0	1	1.25	1.50	1.75	2	2.25	2.50
a^2	0.26	0.32	0.38	0.43	0.47	0.52	0.56
s_0 (s_0^1)	57 (24)	37 (11)	29 (9.5)	24 (8.5)	21 (7.5)	19 (7)	17 (6.5)
m_{ABC} (in MeV)	230±2.8	243±3.7	251±2.5	257±1.9	261±1.5	264±1.2	266±0.9

(The errors of m_{ABC} and the values (s_0^1) were obtained using condition $a^2 = {}^2/_5\, a^0$ instead of the above listed values of a^2. The elastic threshold is 279.16 MeV.)

* A probable value for a^0 is 1.50.

root singularity of the spectral function, the conformal mapping series are still convergent even in the spectral function region [7] and the analytical continuation is therefore quite possible. This second approach takes advantage of the N/D method, which is especially suitable for studying the behaviour of the partial waves on the second Riemann sheet, where resonant and antibound states may occur.

We wish to express our gratitude to Dr. Atkinson for informing us in advance about his work.

The authors are indebted to their colleague Dr. M. Micu for an illuminating discussion. It is a pleasure to acknowledge our thanks to Prof. H. Hulubei and Prof. S. Tzitzeica for their interest in this work.

REFERENCES

1. Abashian A., Booth N. E., Crowe K. M. Phys. Rev. Lett., **5**, 258 (1960); Phys. Rev. Lett., **7**, 35 (1961); Phys. Rev., **132**, 2314 (1963).
2. Desai B. R. Phys. Rev. Lett., **6**, 497 (1961).
3. Atkinson D. Phys. Lett., **9**, 69 (1964).
4. Oehme R. Phys. Rev., **121**, 1840 (1961).
5. Fischer J. Ciulli S. JETF, **41**, 256 (1961); Soviet Physics, **14**, 185 (1962).
6. Atkinson D. Phys. Lett., **7**, 65 (1963).
7. Ciulli I., Ciulli S., Fisher J. Nuovo cimento, **23**, 1129 (1962).