Isoscalar giant monopole resonance in ^{40,48}Ca

N. N. Arsenyev[1] **and A. P. Severyukhin**[1,2]

[1]Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia

[2]Dubna State University, 141982 Dubna, Moscow Region, Russia

E-mail: arsenev@theor.jinr.ru

Abstract. Starting from the Skyrme energy-density functional, the microscopic model is developed for the monopole excitations. We take into account the coupling between one-, two-, and three-phonon terms in the wave functions of excited 0^+ states. As an example, the properties of the isoscalar giant monopole resonance (ISGMR) for the doubly magic nuclei ^{40,48}Ca are discussed. The inclusion of complex configurations leads to a fragmentation of the ISGMR strength to lower energy 0^+ states and also higher energy tail.

Among the giant resonances discovered a long time ago in the very intensive study of this field of nuclear research the isoscalar giant monopole resonance (ISGMR) has attracted special attention [1]. The resonance energy is sensitive to the nature of nuclear interactions and related to the incompressibility of nuclear matter [2]. The latter quantity is of fundamental importance for the understanding of the saturation properties of nuclear matter and because of that also is of astrophysical interest. Thus, the experimental source of information regarding incompressibility is the ISGMR excited by the inelastic scattering of various projectiles at small angles (see, e.g., [1]). A complete list of references on that subject is given in [3]. On the theoretical side, the random phase approximation (RPA) with the Skyrme-type energy-density functional (EDF) is the most widely used theoretical model for such collective excitation in nuclei, see e.g. [2, 3]. These RPA calculations enable one to describe the properties of the ground state and excited 0^+ states using the same EDF. The wave functions of nuclear collective 0^+ states are associated with the coherent effect of one-particle–one-hole ($1p1h$) excitations in terms of the mean-field and particle-hole interaction [4]. A comparison of such calculations with the experimental data demonstrates that the RPA approach cannot correctly reproduce the ISGMR strength distributions. It is necessary to take into account a coupling with more complex configurations. The decay evolution along the hierarchy of more complex configurations ($2p2h$, $3p3h$, etc.) till compound states determines the fine structure of the ISGMR and its damping properties [3, 5, 6]. Recently, the Skyrme-RPA has been generalized to take into account the phonon-phonon coupling (PPC) in the case of the ISGMR strength distribution [7, 8]. It was found that the PPC inclusion is crucial for the description of the gross properties of the ISGMR in medium-heavy mass spherical nuclei. It is worth mentioning that the PPC follows the basic ideas of the quasiparticle-phonon model (QPM) [9], but the single-particle (SP) spectrum and the residual interaction are derived from the same Skyrme EDF (see details in Refs. [10, 11]).

To describe the damping properties of the ISGMR and recent experimental data in ^{40,48}Ca isotopes [12, 13], we analyze the effects of the coupling between one-, two-, and three-phonon configurations in the present work.

Our model consists in the Hartree-Fock (HF) calculation of the ground state with the effective Skyrme interaction [4], assuming spherical symmetry for the nuclei considered here. To build the RPA equations on the basis of HF states using consistently the residual interaction (derived from the Skyrme force in the particle-hole channel) is a standard procedure [14]. The dimensions of the RPA matrix grow rapidly with the size of the nucleus. Using the finite-rank separable approximation (FRSA) [15] for the residual interactions, the eigenvalues of the RPA equations can be obtained as the roots of the secular equation. It enables us to perform RPA calculations in very large configuration spaces. In particular, the cutoff in the discretized SP continuum can be chosen at 100 MeV. Excited states of even-even nuclei are treated in the terms of the phonon excitations built upon the ground state that is considered as the RPA phonon vacuum $|0\rangle$. The wave functions of the one-RPA phonon excited states given by $Q^{+}_{\lambda\mu i}|0\rangle$ as a superposition of the $1p1h$ configurations.

In the next step, the wave functions of the excited states are written as a sum of configurations of different complexity by the number of phonons. If we limit this sum to one-, two-, and three configurations only, the wave function has the form [16, 17]

$$\Psi_\nu(JM) = \Bigg(\sum_i R_i(J\nu)Q^{+}_{JMi} + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\left[Q^{+}_{\lambda_1 i_1}Q^{+}_{\lambda_2 i_2}\right]_{JM} + \sum_{\lambda_1 i_1 \lambda_2 i_2 \lambda_3 i_3 J'} T^{\lambda_1 i_1\, \lambda_2 i_2}_{J'\, \lambda_3 i_3}(J\nu)\left[\left[Q^{+}_{\lambda_1 i_1}Q^{+}_{\lambda_2 i_2}\right]_{J'}Q^{+}_{\lambda_3 i_3}\right]_{JM}\Bigg)|0\rangle, \quad (1)$$

where $Q^{+}_{\lambda\mu i}$ is the phonon creation operator for multipolarity $\lambda\mu$. Using the variational principle and taking into account the normalization of the wave function (1), we get the secular equation for the energies of excited states and the system of equations for finding the wave function coefficients $R_i(J\nu)$, $P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)$, and $T^{\lambda_1 i_1\, \lambda_2 i_2}_{J'\, \lambda_3 i_3}(J\nu)$. The resulting system of equations obtained with the chosen Skyrme EDF has a similar form as the system of equations within the QPM by using the Woods-Saxon potential [9, 18, 19]. The rank of the determinant is determined by the number of the complex configurations included in the wave function (1).

In the present work, the calculations are performed by using the SLy5 EDF [20]. To construct the wave functions (1) of the 0^+ excitations up to 30 MeV we use the natural parity phonon with $\lambda^\pi = 0^+ - 5^-$. In Ref. [17], we found the main mechanisms of the ISGMR formation. The approach takes into account correctly and consistently the coupling between one-, two- and three-phonon terms in the wave functions of excited 0^+ states.

As an illustration of the method described above, let us demonstrate the fragmentation of the ISGMR strength function due to the coupling of the one-phonon component over complex configurations of the ISGMR. The strength function describing the fragmentation of the 0^+ states are defined as

$$S(\omega) = \sum_\nu \left|\langle 0^+_\nu|\hat{M}_{\lambda=0}|0^+_{g.s.}\rangle\right|^2 \frac{1}{2\pi}\frac{\Delta}{(\omega - E_\nu)^2 + \Delta^2/4}, \quad (2)$$

where the monopole isoscalar transition operator $\hat{M}_{\lambda=0}$ is given as $\sum_{i=1}^{A} r_i^2$, E_ν denotes the excitation energies and $\left|\langle 0^+_\nu|\hat{M}_{\lambda=0}|0^+_{g.s.}\rangle\right|^2$ are the transition probabilities. The smearing parameter Δ in Eq. (2) is equal to 1 MeV in all the calculations. In order to discuss the various integral characteristics, we introduce the energy-weighted moments $m_k = \sum_\nu (E_\nu)^k \left|\langle 0^+_\nu|\hat{M}_{\lambda=0}|0^+_{g.s.}\rangle\right|^2$. These values are useful in estimating the resonance centroid, $E_c = m_1/m_0$, and also in checking numerical calculations.

The ISGMR strength distributions of ^{40}Ca are displayed in Fig. 1(a). Both theoretical and experimental results show the fragmentation and splitting of the ISGMR strength. One can see that three regions can be distinguished in the strength distribution: the low-energy part

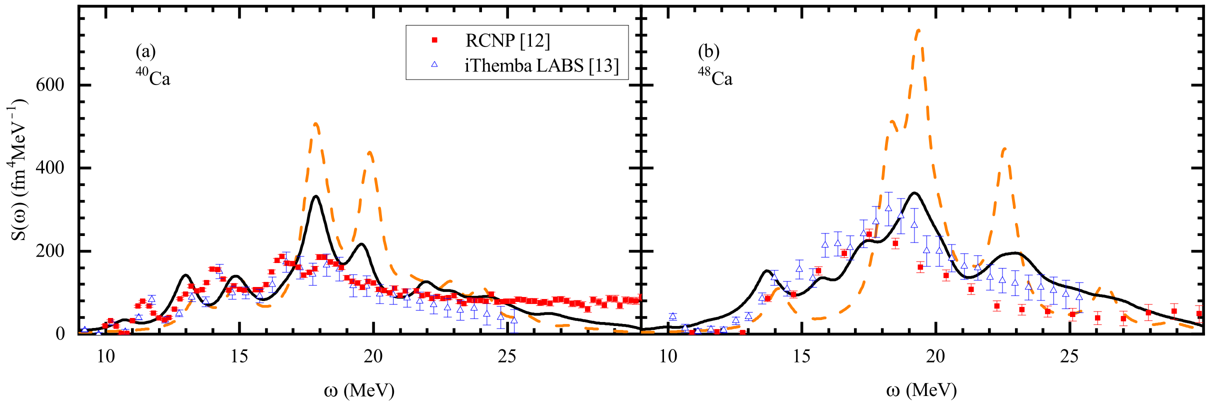

Figure 1. The phonon-phonon coupling effect on the ISGMR strength distributions in ^{40}Ca (left panel) and ^{48}Ca (right panel). The dashed and solid curves correspond to the RPA and PPC calculations, respectively. The smoothing parameter 1 MeV is used for the strength distribution described by the Lorentzian function. Experimental data (open triangles and filled squares) are taken from [12, 13].

up to 10 MeV, the resonance region (main peaks) in the range from 10 to 25.5 MeV and the high-energy part of the ISGMR located above 25.5 MeV. The RPA results demonstrate that the lowest state arises at the energy of 13 MeV. At the same time, the summed strength in the resonance region exhausts 97% of the total monopole strength, i.e., 97% of the non-energy-weighted sum rules (NEWSR) which we obtained below 30 MeV. In the case of the one-phonon approximation, there are two main peaks at 17.8 MeV and 19.9 MeV, which exhausts about 58% of the total monopole strength. Figure 1(a) demonstrates the PPC impact on the ISGMR strength distribution. The coupling of the $1p1h$ configurations to more complex $2p2h$ and $3p3h$ configurations leads to a noticeable redistribution of the ISGMR strength distribution in comparison with the RPA results. In the resonance region, we found the decrease in the fraction of NEWSR to be 89%. The quasiparticle-phonon interaction induces fragmentation in the high-energy peaks (about 4% of the NEWSR fraction). The 4% contribution of NEWSR is also moved downwards. Moreover, the PPC effect is a 600-keV downward shift in the ISGMR centroid energy ($E_c = 19.0$ MeV in the one-phonon approximation). The experimental centroid is 17.78 ± 0.17 MeV [13].

Moving from ^{40}Ca to ^{48}Ca, the RPA calculations predict the lowest 0^+ state above 14 MeV. As can be seen from Fig. 1(b), the RPA strength distribution is practically concentrated on three states at 18.3, 19.4, and 22.6 MeV, which exhausts or about 79% of the NEWSR. In general, the ISGMR strength in the energy range 10.0–25.5 MeV exhausts a bit less than 93% of the NEWSR. The PPC inclusion leads to the fragmentation of the RPA strength distribution in the resonance region of ^{48}Ca and thus to the formation of the width of the ISGMR [17]. Moreover, about 1% and 3% of the total monopole strength is shifted to the regions of low-energy (up to 10 MeV) and high-energy (above 25.5 MeV) excitations, respectively. As a consequence, about 89% of the total monopole strength remains in the ISGMR region of 10.0–25.5 MeV. The PPC effect induces a 700-keV downward shift of the ISGMR centroid energy compared to the RPA ($E_c = 19.8$ MeV). The calculated integral characteristic of the ISGMR is in satisfactory agreement with the experimental data on ^{48}Ca ($E_c = 18.40 \pm 0.13$ MeV) [13].

The ISGMR strength distributions in ^{40,48}Ca were measured using the high energy-resolution capabilities at the Research Center for Nuclear Physics (RCNP) [12] and the iThemba Laboratory for Accelerator Based Sciences (iThemba LABS) [13]. As can be seen from Fig. 1, the general shapes of the ISGMR obtained in the PPC are somewhat close to those observed in the RCNP and iThemba LABS experiments. The PPC description is definitely better than that obtained in the one-phonon approximation. It is worth mentioning that the effect of the complex configurations on basic peculiarities of the ISGMR strength distributions of ^{40,48}Ca were already qualitatively discussed in [6].

In conclusion, we have been applied the self-consistent microscopic approach by using the Skyrme interaction for calculations of the ISGMR strength distributions in ^{40,48}Ca isotopes. The model Hamiltonian has diagonalized on the basis of the wave functions of excited 0^+ states which include one-, two- and three-phonon configurations. The calculations performed in this paper have shown that the phonon-phonon coupling leads to a considerable redistribution of strength within the energy interval 10–25.5 MeV. About 4% of the ISGMR strength in ^{40}Ca and 1% in ^{48}Ca is shifted to the energy region below 10 MeV. Inclusion of complex configurations is responsible for the shift of 3-4% of the ISGMR strength from the main peak towards higher excitation energies. The calculations show that the quasiparticle-phonon interaction leads to the formation of the low-lying 0^+ states ($E_x < 10.0$ MeV). The main properties of the ISGMR in ^{40,48}Ca isotopes are described within the presented approach and found to be in reasonable agreement with available experimental data, including the latest ones.

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