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Mass Properties Measurement for Drag-free Test Masses

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Abstract. Space-borne gravitational wave observatories like the Laser Interferometer Space Antenna (LISA) and those beyond, which may utilize a Modular Gravitational Reference Sensor (MGRS), greatly benefit from precise knowledge of the mass center location and moment of inertia tensor of the test mass prior to launch. The motion of the mass center of a drag-free test mass, which follows a pure geodesic, must be inferred from measurements of the surface. Therefore, knowledge of the mass center is critical for calibration of the cross-coupling between rotational and translational degrees of freedom. Together with the moment of inertia tensor, the mass center can also provide an estimate of the material density inhomogeneity to quadratic order, and the gravitational potential to second order, which improves modeling of self gravitation forces. These benefits, which are independent of the test mass shape, motivate the development of three new techniques for improving mass center and moment of inertia measurements beyond the current state of the art. A static pendulum is proposed to determine the mass center of a cubic test mass to ~ 1 μm by measuring the equilibrium position with the cube in up to 24 different orientations relative to the pendulum platform. Measuring the natural frequency of a dynamic torsion pendulum can determine both the mass center and moment of inertia tensor of arbitrarily shaped objects to ~ 5 μm and 1 part in $\sim 10^4$ respectively. The velocity modulation technique for measuring the mass center of a sphere has raised the bar in precision to ~ 150 nm, a factor of 20 improvement over the work presented at the LISA 6th symposium. This new technique involves rolling the sphere down a set of parallel rails to spectrally shift the mass center offset information to the rolling rate frequency, in order to avoid the $1/f$ noise that typically prevents other techniques from achieving precision below 1 μm.

1. Introduction & Motivation

A drag-free spacecraft [1] shields an internal free-floating test mass (TM) from external disturbances, while minimizing disturbances caused by the spacecraft itself. A gravitational reference sensor contains and shields the test mass, and senses its position, enabling a feedback control system to command thrusters to keep the spacecraft fixed with respect to the test mass. In principle, the test mass is then completely freed from non-gravitational disturbances, so that its mass center (MC) follows a perfect geodesic.

One exciting application of drag-free technology is space-borne gravitational wave detectors such as the joint NASA and ESA Laser Interferometer Space Antenna (LISA) [2–4]. LISA will require unprecedented ~ 10 pm level accuracy in the measurement of the distances between the mass centers of drag-free test masses housed in three spacecraft separated by 5 Gm. In addition to the challenging metrology, the disturbances applied to the test masses must be reduced four orders of magnitude below what has been demonstrated by Gravity Probe B [5], and over four orders of magnitude in frequency (0.1 mHz to 1 Hz). LISA will detect and observe the emission of gravitational radiation from massive black holes in the distant Universe and compact binaries within our own galaxy, and consists of three drag-free spacecraft in a heliocentric orbit. Beyond LISA is the proposed Big Bang Observer (BBO) [6; 7], which will detect primordial gravitational waves generated during the birth of the Universe. BBO hopes to achieve acceleration noise levels two additional orders of magnitude below that of LISA, but at higher frequencies (0.1 - 10 Hz).

LISA (and LISA Pathfinder [8]) will use two $\sim$ 40 mm cubic test masses per spacecraft, one aligned with each of the two remote spacecraft. The choice of cubes is based on the flight heritage of precision accelerometers manufactured by ONERA. Cubes also provide a flat surface for reflecting laser light. Alignment of the cubes is maintained by electrostatic actuation so that changes in their orientation do not mimic translation of the mass center.

The Modular Gravitational Reference Sensor (MGRS) is an alternate design for future gravitational missions, including gravitational wave detection [7; 9; 10]. The MGRS uses a single spinning sphere as the gravitational reference, no electrostatic suspension, and a relatively large gap ($\sim$ 20 mm) between the spacecraft and TM. All of these features make the MGRS especially sensitive below 1 mHz, where acceleration noise dominates the sensitivity limit [11].

Several sources of acceleration noise are related, in some way, to the density distribution of the test mass material. These include the translation of the TM mass center coupling with the orientation control, gravitational attraction with the spacecraft, and magnetic interactions [12]. We will discuss now how precise determination of the mass center (MC) and moment of inertia tensor of the test mass improves our ability to estimate the magnitudes of these effects

1.1. Cross-coupling of Test Mass Degrees of Freedom

The science signal for LISA, and almost any drag-free mission, is the position of the mass center of the test mass. In the case of LISA it is the variations in the distance between the mass centers of test masses housed in three drag-free spacecraft. Measurement of the mass center position must be inferred from measurements of the TM surface. Therefore, precise knowledge of the mass center of the test mass is crucial for achieving the desired science signal.

There are two general situations that can potentially confuse rotation of the test mass with translation of the mass center. The first is rotation of the TM about its mass center, without any real mass center motion. For a faceted test mass, if the direction of the distance measurement to the TM surface does not pass through the mass center, then rotation about the mass center will produce a false measurement of mass center position variations. The second case occurs when the TM orientation control rotates the TM about an axis not coincident with the mass center. This will inadvertently produce a force acting at the mass center, which corrupts the science measurement. Both of these systematic effects can be mitigated by incorporating accurate mass center information in the orientation control software and in the data reduction.

1.2. Density Inhomogeneity

Measurements of the mass, mass center and moment of inertia tensor can provide some information regarding the density distribution of the TM. To be more quantitative, let us choose a particular model for the true density distribution, $\rho(x, y, z)$,

$$\rho(x,y,z) \approx c_0 + c_1\,x + c_2\,y + c_3\,z + c_4\,x^2 + c_5\,y^2 + c_6\,z^2 + c_7\,yz + c_8\,xz + c_9\,xy + \dots \quad (1)$$

This model is simply a polynomial expansion of the density distribution to quadratic order, including cross product terms.

Assume that the three mass properties are measured, which represent ten numbers: the mass, three components of the mass center location and six independent components of the moment of inertia tensor. These ten numbers directly specify the parameters c_i in the model for $\rho(x, y, z)$ which minimizes the error in density (true density distribution minus the quadratic model) squared and integrated over the TM volume. In other words the mass properties provide the best fit polynomial model, to quadratic order, to the true density in a least-squares sense.

Knowledge of the density inhomogeneities is vital for materials like gold-platinum, where spinodal decomposition during the material cooling process, can cause the gold and platinum to separate. A uniform mixture of the diamagnetic gold and paramagnetic platinum is required for a near-zero magnetic susceptibility. A low susceptibility reduces the TM to spacecraft stiffness and TM disturbances due to fluctuations in the magnetic field in the LISA frequency band.

1.3. Gravitational Self-attraction

The gravitational self-attraction force between the satellite and the TM is a dominant contribution to the disturbance budget. Although the attraction force can not be easily measured directly, the mass attraction formula through a second order expansion consists of the measurable quantities of mass, mass center, and moment of inertia about the mass center. Thus, the gravitational self-attraction force on the drag free reference due to the satellite can be indirectly measured. By computing the attraction force from the measured mass properties, variations in density as well as geometric variations will be included in the gravitational self-attraction calculation. As a result, the exact density distribution within the test mass need not necessarily be known or measured in order to determine the actual gravitational attraction properties.

2. Measuring Mass Center & Moment of Inertia

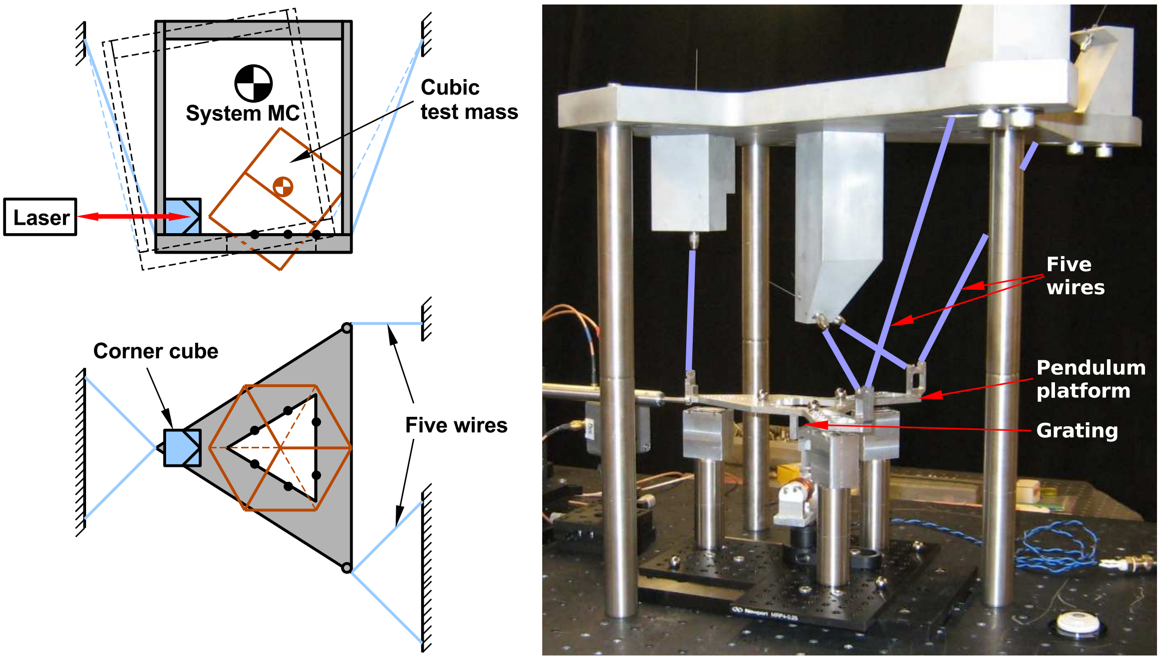
The LISA laboratory at Stanford University is exploring three methods of measuring the mass center and moment of inertia tensor of objects on the same scale as the LISA test masses.

2.1. Mass Center Measurement for a Cube with a Static Pendulum

The first technique is a static pendulum specifically designed to quickly and accurately determine the mass center of a cube. A schematic of the apparatus, which is currently in the development stages at Stanford, is shown in Figure 1 (a). Five wires are used to suspend a triangular platform. The position and orientation of the five wires are chosen so that they constrain all but one degree of freedom. The free motion of the pendulum, shown in Figure 1 (a) by the dashed lines, is primarily lateral translation, with a small amount of tilt. The pendulum platform has a triangular hole to accommodate a cubic test mass. Six gauge spheres are fixed to the edges of the triangular hole so that the cube is kinematically constrained to the platform.

The measurement is the equilibrium position of the pendulum, which depends on the mass center locations of both the pendulum platform and cubic TM. The equilibrium position can be measured using a laser interferometer, shown in Figure 1 (a), or by an optical shadow sensor. To separate the mass center of the platform from that of the cube, and to determine the mass center of the cube in all three dimensions, many measurements are taken, each time with the cube in a different orientation. Each time the orientation of the cube is changed, the cube's mass center is changed relative the mass center of the platform. There a total of 24 possible orientations of the cube on the pendulum platform. This provides more than enough measurements to determine the mass centers of both the pendulum platform and the cubic TM in all three directions, and provide an estimate of the measurement error using the post-fit residuals.

The static pendulum can potentially measure the MC location of the cube to better than 1 μm. This assessment is based on the sensitivity of the optical detection, which can be less than 10 nm for either an interferometric or a shadow sensor, and the geometry and masses of the the pendulum and cubic TM. In addition, the measurement procedure is easy since it only involves changing the orientation of the cube and measuring the resulting equilibrium position. The data



(a) (b)

Figure 1. (a) Elevation view (top) and plan view (bottom) of the static pendulum for measuring the mass center of a cubic test mass. (b) Schematic of the dynamic five-wire pendulum for measuring the moment of inertia tensor and mass center of arbitrary objects.

reduction entails the proper coordinate transformations for each of the cube orientations, and the proper weighting of the mass of the cube with respect to that of the platform.

2.2. Mass Center & Moment of Inertia Measurement with a Dynamic Pendulum
An apparatus which is designed to generate a pure rotation about an axis provides the ability to measure the mass center location and the moment of inertia. For example, in order to measure the moment of inertia, I, the object is rotated about an axis and the pendulum natural frequency, $\omega/2\pi$, is used to determine the radius of gyration, R_g, about that rotational axis:

$$R_g^2 = \frac{I}{m} = \frac{I_o + m_o r^2}{m} = \frac{k}{\omega^2} \tag{2}$$

Here, k is the torsion coefficient or stiffness constant of the pendulum and m is the total mass.

By measuring the natural frequency of rotation, the mass center offset from the geometric center of an object is also obtained. In Eq. (2), we replace the moment of inertia about the geometric center, I, by the moment of inertia about the objects mass center, I_o, plus the parallel axis theorem components of the object mass, m_o, times the square of the distance from the rotation center to the mass center, r^2. Therefore, placing the object on the pendulum far from the rotation axis amplifies the contribution due to the mass center offset from the geometric center, due to its quadratic dependence on r. By changing the orientation of the object with a fixed geometric location relative to the pendulum rotation center, the mass center offset is determined by measuring the change in the natural frequency, $\omega/2\pi$.

The difficulty in the measurements lies within the ability to generate pure rotation about one degree of freedom. Care must be taken to minimize the extra degrees of freedom in the system. In a five-wire torsion pendulum shown in Figure 1 (b), five wires are arranged to minimize translational degrees of freedom in order to reduce errors due to tilt and translation [13; 14]. By reducing the errors associated with generating a pure rotation, the five-wire pendulum has demonstrated a consistent frequency measurement to better than 10^{-5} Hz for a 2.5 Hz pendulum frequency. As show in Ref. [13], the precision on the frequency measurement results in a repeatable moment of inertia measurement on the order of a few parts in 10^4. For the mass center measurement, the five wire pendulum has demonstrated the ability to determine the mass center offset to within 5 μm.

2.3. The Velocity Modulation Technique for Mass Center Measurement

A new technique for measuring the mass center of a sphere to $\sim$ 100 nm, called velocity modulation, was presented in 2006 [15; 16]. Previous methods of determining the mass center, involving measurement of the sphere's natural pendulous period, are typically limited to MC offsets $\sim>$ 1 μm due to low frequency ($1/f$) instrumentation noise. In the velocity modulation technique, the sphere is rolled down a set of parallel rails to spectrally shift the MC offset information to the rolling rate frequency. The sphere's trajectory is measured optically by recording the times that the sphere crosses five optical gates.

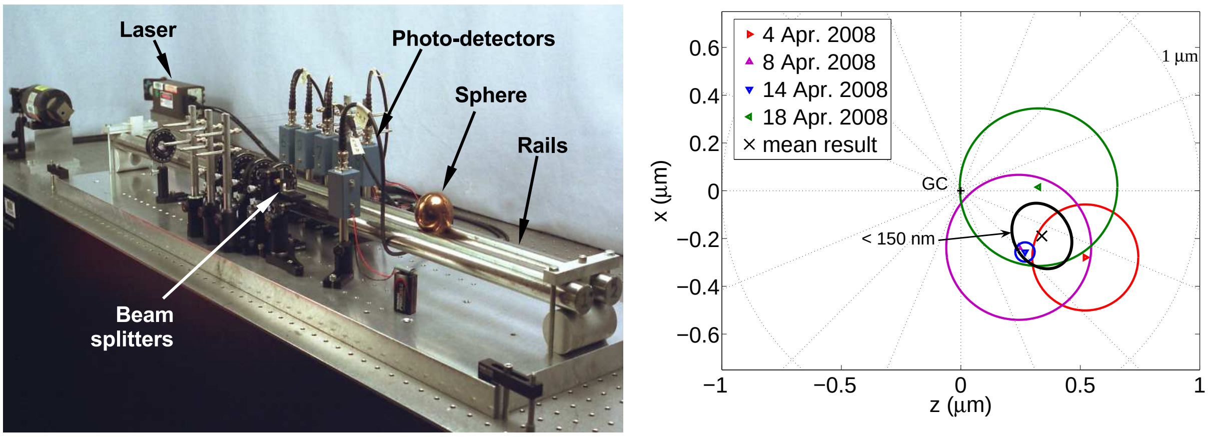
The experimental apparatus is shown in Figure 2 (a). The two $\sim$ 1 m long steel rails are supported above a base plate mounted on an optical bench. The light from a single laser is broken into five beams, by five polarizing beam splitters, so that the beams cross the rails at right angles at the exact height of the sphere's geometric center. Measurement of the sphere's location on the rails is made by *pairs* adjacent photo-detectors mounted opposite to the beam splitters. The reflected laser beams and photo-detectors are space one diameter of the sphere apart so that exactly half of the incident light is occulted when the sphere is precisely between two detectors. The voltage produced by the pair of photo-detectors is differenced so that when the voltage difference is zero, the sphere is exactly between the two optical gates. Fluctuations in laser intensity are consequently eliminated in the differential signal because laser intensity variations are common to all detectors. The sphere is rolled three times, each time with a different initial phase, in order to separate the geometric irregularities in the rails, which are fixed in the laboratory, from the MC offset, which depends on the initial conditions. A Monte-Carlo parameter search is used to fit all relevant parameters in a simplified mechanics model to the measured data, including the optical gate locations, initial velocities and MC location.

The velocity modulation technique can also determine the MC of an arbitrarily shaped test mass (including a cube) to $\sim$ 1 μm by placing the TM inside a spherical fixture. To separate the MC of the TM from that of the spherical fixture, several measurements are made, each time changing the orientation of the TM with respect to the fixture. In this case, the measurement error is limited by the repeatability of the placement of the test mass inside the spherical fixture.

The precision of the velocity modulation technique is demonstrated by the repeatability of independent measurements. Figure 2 (b) shows four independent measurements of the mass center of a stainless steel gauge sphere. All four measurements are consistent within their uncertainties estimated by the RMS of the post-fit residuals. The mean result and one standard deviation error ellipse with maximum radius less than 150 nm are also shown in Figure 2 (b).

3. Summary

Future drag-free missions like LISA, with strict requirements for both metrology and disturbance reduction greatly benefit from precise determination of the mass center and moment of inertia tensor of its test masses prior to launch. This allows verification of several design requirements, including degree of freedom cross-coupling, gravitational disturbances and density inhomogeneity. Three techniques developed at Stanford for measuring these mass properties represent the state of the art, with accuracies listed in Table 1.



(a) (b)

Figure 2. (a) Velocity modulation apparatus for mass center measurement. (b) Precision of mass center measurement demonstrated through the repeatability of four independent measurements. The mean result and error ellipse are shown in black.

Mass property	Technique	Accuracy
Mass center (any shape)	Static/dynamic pendulum or velocity modulation	$\sim 1\ \mu$m
Mass center (sphere)	Velocity modulation	$\sim$ 150 nm
Moment of inertia (any shape)	Dynamic pendulum	$\sim 10^{-4}$

Table 1. Mass center and moment of inertia measurement capability.

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References

[1] DeBra D B 1997 *Classical and Quantum Gravity* **14** 1549–1555

[2] Danzmann K and Rüdiger A 2003 *Classical and Quantum Gravity* **20** S1–S9

[3] LISA Study Team 1998 LISA, Laser Interferometer Space Antenna for the detection and observation of gravitational waves LISA Pre Phase A Report, 233 Max-Plank-Institut für Quantenoptik Garching, Germany

[4] Jennrich O (ed) 2004 *Proceedings of the 5th International LISA Symposium and the 38th ESLAB Symposium* (*Classical and Quantum Gravity* vol 22) (Institute of Physics Publishing)

[5] Conklin J W 2008 *Proceedings of the Sixth International Conference on Gravitation and Cosmology* Journal of Physics: Conference Series

[6] Phinney E S *et al.* 2004 The Big Bang Observer: Direct detection of gravitational waves from the birth of the Universe to the present NASA OSS Vision Missions Program, Proposal VM03-0021-0021

[7] Sun K X, Allen G, Buchman S, DeBra D B and Byer R L 2004 *Proceedings of the 5th International LISA Symposium and the 38th ESLAB Symposium* (*Classical and Quantum Gravity* vol 22) pp S287–S296

[8] Anza S, Armano M, Balaguer E, Benedetti M, Boatella C, Bosetti P, Bortoluzzi D, Brandt N, Braxmaier C, Caldwell M, Carbone L, Cavalleri A, Ciccolella A, Cristofolini I, Cruise M, Da Lio M, Danzmann K, Desiderio D, Dolesi R, Dunbar N, Fichter W, Garcia C, Garcia-Berro E, Garcia Marin A F, Gerndt R, Gianolio A, Giardini D, Gruenagel R, Hammesfahr A, Heinzel G, Hough J, Hoyland D, Hueller M, Jennrich O, Johann U, Kemble S, Killow C, Kolbe D, Landgraf M, Lobo A, Lorizzo V, Mance D, Middleton K, Nappo F, Nofrarias M, Racca G, Ramos J, Robertson D, Sallusti M, Sandford M, Sanjuan J, Sarra P, Selig A, Shaul D, Smart D, Smit M, Stagnaro L, Sumner T, Tirabassi C, Tobin S, Vitale S, Wand V, Ward H, Weber W J and Zweifel P 2005 *Classical and Quantum Gravity* **22** 125–+

[9] Sun K X, Allen G, Buchman S, Byer R L, Conklin J W, DeBra D B, Gill D, Goh A, Higuchi S, Lu P, Robertson N and Swank A 2006 *2007 AAS/AAPT Joint Meeting, American Astronomical Society Meeting 209, #74.07* (*Bulletin of the American Astronomical Society* vol 38) p 991

[10] Sun K X, Allen G, Buchman S, Byer R L, Conklin J W, DeBra D B, Gill D, Goh A, Higuchi S, Lu P, Robertson N A and Swank A J 2006 *Laser Interferometer Space Antenna: 6th International LISA Symposium* (*American Institute of Physics Conference Series* vol 873) pp 515–521

[11] Gerardi D, Allen G, Conklin J W, Sun K X, DeBra D B, Buchman S, Gath P, Byer R and Johann U Achieving Disturbance Reduction for Future Drag-Free Missions to be submitted

[12] Schumaker B L 2003 *Classical and Quantum Gravity* **20** S239–S253

[13] Swank A J, Hardham C, Sun K X and Debra D 2006 (ASPE)

[14] Swank A J, Sun K X and DeBra D 2006 *Laser Interferometer Space Antenna: 6th International LISA Symposium* (*American Institute of Physics Conference Series* vol 873) pp 588–592

[15] Conklin J W, Sun K X and DeBra D B 2006 *Laser Interferometer Space Antenna: 6th International LISA Symposium* (*American Institute of Physics Conference Series* vol 873) pp 566–570

[16] Conklin J W, Sun K X and DeBra D B 2006 *Proceedings of the 21st Annual Meeting of the American Society for Precision Engineering, Monterey, CA* American Society of Precision Engineers (Raleigh, NC)