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Magnetic Moments of Quarks, Leptons and Hadrons: A Serious Difficulty for Composite Models

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ABSTRACT

The coupling of three elementary spin 1/2 particles to make a composite state of spin 1/2 arises in (a) models of baryons made of three quarks, (b) recently proposed models for quarks and leptons made of three more fundamental building blocks. Reproducing the observed magnetic moments of physical particles provides stringent constraints on these models. Obtaining the observed Dirac moments of the electron and muon is particularly difficult because the mass scale of the moment must be precisely the mass of the composite system rather than some function of basic building block masses, and because the total spin and magnetic moment is not obtained by simple addition of constituent properties but involves Clebsches. The Clebsch problem also arises in obtaining u and d quark moments proportional to their charges. Difficulties in the quark description of recent values of baryon magnetic moments are also discussed.

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I. MAGNETIC MOMENTS OF QUARKS AND LEPTONS FROM COMPOSITE MODELS

The possibility that leptons and quarks might not be elementary but could have a composite structure has been considered for a long time[1,2] and has been recently revived.[3] The purpose of this note is to emphasize and discuss the implications of the observed magnetic moments of leptons and the implied magnetic moments of quarks from experimental hadron moments. These values, which are equal to the Dirac moment, are very simply obtained for Dirac particles, but are not easily obtained from composite models.[2] In particular the precise measurements of g - 2 for leptons which agree so well with QED predictions must be also taken as evidence for a remarkable precision in the value $g = 2$, which is obtained before the higher order corrections are included. This value of 2 comes from the Dirac equation, but does not come naturally in any composite model. Thus the magnetic moment provides strong constraints on any proposed composite model for quarks and leptons. There are two separate aspects of the problem, the mass scale problem and the Clebsch problem.

1. The Mass Scale Problem. Magnetic moments have a mass scale, the magneton, which is simply the mass of the particle in the Dirac equation. However, there is no simple relation between the mass of a composite system and the masses of the constituents. If the constituents have different electric charges and are coupled therefore with different coupling constants to the electromagnetic field, it would seem to be a very peculiar accident for the magnetic moment to come out to be exactly that of a Dirac particle with a mass equal to the mass of the bound state.

The mass parameter which enters into the magnetic moment of a Dirac particle is the same as the one which enters into the kinetic energy and in the behavior of the particle under Lorentz transformations. For a composite system in which each constituent has its own four-momentum and its own coupling to the electromagnetic field, it would be a peculiar accident for these couplings to combine into a minimal coupling depending only on the total four-momentum of the system and completely independent of the masses or reduced masses of the constituents.

2. The Clebsch problem. A composite model of a spin ½ particle with constituents having spin must necessarily have some spins parallel and some spins antiparallel to the spin of the bound state. The magnetic moment of the bound state is a complicated linear function of the magnetic moments of the constituents, with coefficients determined by the angular momentum Clebsches coupling the constituent spins to give a total spin of ½. The electric charge of the bound state is given simply by the algebraic sum of the constituents. Thus the ratio of the magnetic moment of a composite system to its charge Q is given by

$$\frac{\mu}{Q} = \frac{\sum_i C_i \mu_i}{\sum_i q_i} \tag{1}$$

where μ_i and q_i are the magnetic moment and electric charge of constituent i, and C_i is a function of Clebsch-Gordan coefficients in the coupling of the spins of the constituents to the total spin. Since the denominator of (1) is a simple sum, while the numerator is a linear combination involving coefficients generally unequal and often with opposite signs, the resultant ratio would be expected to be very different from the ratio μ_i/q_i for any constituent.

This difficulty is most easily seen in the case of a neutral particle constructed from two particles with charge Q and one with charge -2Q. Since the spins of the three constituents are not parallel, a very peculiar coupling of the spins would be needed to cancel the magnetic moment exactly and obtain the Dirac value of zero moment. For the particular case where the two particles with charge Q are identical fermions of spin ½, they would be expected from Fermi statistics to be coupled either to spin zero or to spin one, depending upon the symmetry of the other degrees of freedom, but not to a coherent linear combination of zero and one. For either the spin one or spin zero case the magnetic moment would not vanish.

An instructive example which shows both difficulties is the deuterium atom in the state of total angular momentum $J = ½$. The mass of the atom is determined primarily by the mass of the deuteron, which plays essentially no role in determining the scale of the magnetic moment. The scale of the magnetic moment is determined almost completely by the magnetic moment of the electron. However, because the deuteron has spin one and the electron has spin ½, the electron spin and magnetic moment are oriented antiparallel to the spin of the atom, and the sign of the magnetic moment of the atom is opposite to that which would be obtained from an electron. Its exact value is neither zero, which would be the Dirac moment of the atom, nor the magnetic moment of the electron. It is the magnetic moment of the electron multiplied by Clebsch factors arising from the coupling of the electron and deuteron spins. The result for the magnetic moment of the atom μ_A in terms of the magnetic moment of the deuteron μ_d and electron μ_e is

$$\mu_A = (2/3)\mu_d - (1/3)\mu_e \quad . \tag{2}$$

The origin of the Clebsch factors (2/3) and -(1/3) is easily seen. Although the deuteron and electron spins are called "antiparallel" the exact wave function is more complicated. For an atom with spin up, the naive "antiparallel" coupling with the deuteron spin up and the electron spin down describes only 2/3 of the wave function; the other 1/3 has the deuteron spin "sideways" and the electron up. The factors 2/3 and 1/3 are just the squares of the Clebsch-Gordan coefficients arising in the coupling of two spins of 1 and 1/2 to a total spin of 1/2. Thus the deuteron contribution to the magnetic moment (2) is weighted by the factor 2/3 because there is a full contribution from the first term and no contribution from the second term which has the deuteron spin pointing "sideways." The electron contribution is opposite in the two terms, pointing down 2/3 of the time and up 1/3 of the time. The net result is -1/3. Note that this result (2) is general and applies to the magnetic moment of any system of a spin 1 and a spin 1/2 constituent coupled to a total spin of 1/2.

In this example the following general features are evident: 1) The mass of the bound state bears no relation to the scale of the magnetic moment. 2) The constituents have equal and opposite charge, but not equal magnetic moments; therefore the charges cancel but the magnetic moments do not. 3) The constituents have antiparallel spins; therefore their magnetic moments add up in a completely different way from their charges. Even if the magnetic moments of the electron and deuteron were equal, they would not cancel one another in the total magnetic moment, like the charges cancel. These three features can be expected in any composite model for quarks and leptons and should be checked very carefully in testing the credibility of any model.

Many composite models propose coupling three spin ½ constituents to make spin ½ quarks or leptons. We therefore consider this case in detail and write down the appropriate Clebsch factors for the magnetic moment. We denote the three fundamental objects by a, b and c. There are two independent coupling schemes for three angular momenta of ½ to a total angular momentum of ½. One basis of two states are those in which particles a and b are coupled to spin zero and spin one. For these two cases the magnetic moments are given by

$$\mu_o \equiv \mu\,[(ab)_{S=0};\, c\,]_{S=½} = \mu_c \tag{3a}$$

$$\mu_1 \equiv \mu\,[(ab)_{S=1};\, c\,]_{S=½} = (2/3)(\mu_a + \mu_b) - (1/3)\mu_c \quad . \tag{3b}$$

For the case (3a), the particles a and b coupled to spin zero make no contribution to the magnetic moment or total spin of the system and all comes from c. In the case (3b), particles a and b have their spins parallel to the total spin, and particle c is antiparallel. This is exactly the same coupling scheme as the deuterium atom (2) and the same factors of (2/3) and -(1/3) appear.

Note that for three different constituents, Eqs. (3) can be used for any choice of a, b and c, and therefore gives the magnetic moments for six different states which are pairs of orthogonal states in three different bases. Magnetic moments for states in which no pair is coupled to a definite total angular momentum are slightly more complicated because they involve off-diagonal matrix elements between the two basic states (3a) and (3b). However, if particles a and b are identical, the magnetic moment operator has no off-diagonal matrix elements between the two states and the magnetic moment of any linear combination of the two states (3) is given by combining the two values (3) with the weighting factors given by the wave function.

If the individual constituents a, b and c all have Dirac moments, the results (3) are in general very different from the Dirac moment. However, there are two special cases where the Clebsch problem simplifies and a Dirac moment can be obtained if the mass scale problem is solved.

One special case is when the magnetic moments and charges of all three constituents are equal

$$q_a = q_b = q_c \tag{4a}$$

$$\mu_a = \mu_b = \mu_c \tag{4b}$$

Then for both coupling schemes (3), the total magnetic moment is given by

$$\mu = \mu_a = \mu_b = \mu_c = (1/3)(\mu_a + \mu_b + \mu_c) \tag{5a}$$

while the total electric charge is

$$Q = 3q_a = 3q_b = 3q_c = q_a + q_b + q_c \tag{5b}$$

Thus

$$(\mu/Q) = (1/3)(\mu_a/q_a) \tag{5c}$$

If the constituents have Dirac moments corresponding to a mass m, the composite system then has a Dirac moment corresponding to mass 3m.

A similar result is obtained for the case of a wave function which is a 50-50 mixture of the two spin couplings (3a) and (3b). The magnetic moment is then given by the mean of the two,

$$\mu = (1/2)(\mu_o + \mu_1) = (1/3)(\mu_a + \mu_b + \mu_c) \quad . \tag{6a}$$

If the ratio of the charges to the magnetic moments of all constituents is the same,

$$(\mu_a/q_a) = (\mu_b/q_b) = (\mu_c/q_c) \quad , \tag{6b}$$

then Eq. (5c) holds for the ratio of the total charge to the total magnetic moment also for this case.

For these cases where Eq. (5c) holds, the magnetic moment of the bound state will be the Dirac moment, as if the state were an elementary particle, if the "effective mass" m of the constituent is exactly one third of the total mass, as is often assumed for quarks in nucleons. However, one must still explain why the magnetic moment of the constituent is given by the Dirac moment with a scale of one third of the bound state mass.

The case of equally charged constituents (4) has been considered for the leptons, and avoids the Clebsch problem. However, it is unsuitable for quarks unless new objects with charges of -(1/9) and +(2/9) of the electron charge are introduced. The case of the 50-50 mixture (6) might be used for quarks, but some reason must then be found for the peculiar spin couplings. Note that if constituents a and b are identical and satisfy some kind of statistics, the wave function should be either symmetric or antisymmetric under the permutation of the particles, depending upon the symmetry of the other degrees of freedom including hidden degrees of freedom like color. One would not expect a coherent linear combination of states having opposite permutation symmetry. Since the wave functions (3a) and 3(b) are odd and even respectively under spin permutations of particles a and b, and one would expect ground state configurations to be symmetric under spatial permutations, the mixing of (3a) and 3(b) appears to be forbidden for particles obeying normal statistics. However, we have seen examples of constituents apparently obeying peculiar statistics before, and should not discard this possibility too quickly.

II. BARYON MAGNETIC MOMENTS

The same Clebscherei applies to magnetic moments of baryons made of three constituent quarks. Eq. (3a) applies to the Λ, with constituent c being a strange quark; Eq. (3b) applies to the remaining seven octet baryons, with (a, b, c) being:

(u, u, d) and (d, d, u) for the proton and neutron,

(u, u, s), (u, d, s) and (d, d, s) for the Σ^+, Σ^0 and Σ^-

and

(s, s, u) and (s, s, d) for the Ξ^0 and Ξ^-.

Substituting these into Eq. (3b) gives the results

$$\mu_p = (4/3)\mu_u - (1/3)\mu_d = -(8/3)\mu_d - (1/3)\mu_d = -3\mu_d \tag{7a}$$

$$\mu_n = (4/3)\mu_d - (1/3)\mu_u = (4/3)\mu_d + (2/3)\mu_d = 2\mu_d = -(2/3)\mu_p \tag{7b}$$

$$\mu_\Lambda = \mu_s \tag{7c}$$

$$\mu_{\Sigma^+} = (4/3)\mu_u - (1/3)\mu_s = \mu_p + (1/3)(\mu_d - \mu_s) = \mu_p - (1/9)(\mu_p + 3\mu_\Lambda) \tag{7d}$$

$$\mu_{\Sigma^o} = (2/3)(\mu_u + \mu_d) - (1/3)\mu_s = -(2/3)\mu_d - (1/3)\mu_s = (2/9)\mu_p - (1/3)\mu_\Lambda \quad (7e)$$

$$\mu_{\Sigma^-} = (4/3)\mu_d - (1/3)\mu_s = -(4/9)\mu_p - (1/3)\mu_\Lambda \quad (7f)$$

$$\mu_{\Xi^o} = (4/3)\mu_s - (1/3)\mu_u = (4/3)\mu_s + (2/3)\mu_d = (4/3)\mu_\Lambda - (2/9)\mu_p \quad (7g)$$

$$\mu_{\Xi^-} = (4/3)\mu_s - (1/3)\mu_d = (4/3)\mu_\Lambda + (1/9)\mu_p \quad . \quad (7h)$$

The mass scale problem arises in this case as well and has no obvious solution since free quark masses are not known and may not be relevant. However, the assumption that the mass scale is the same for the u and d quarks and that the quark magnetic moments are proportional to their charges gives $\mu_u = -2\mu_d$, which has been substituted into Eqs. (7) above. This gives the well-known successful result (7b) for the ratio of the neutron and proton moments.

The magnetic moments of all the Σ and Ξ baryons are seen to be given in terms of the proton and Λ magnetic moments. However, the magnetic moment of the Λ cannot be related to that of the nucleon without some assumption about mass scale. The general result obtained from Eqs. (7) is

$$\mu_\Lambda = -(1/3)\mu_p(\mu_s/\mu_d) = -(1/3)\mu_p(m_d/m_s) \quad , \quad (8)$$

where m_d and m_s are effective constituent quark masses which set the mass scales for the corresponding magnetic moments, and which are not known a priori.

Two independent determinations[4,5] of the mass scale factor (m_d/m_s) have been shown to give surprising agreement with experiment when substituted into the relation (8)

$$m_d/m_s = (M_{\Sigma^*} - M_\Sigma)/(M_\Delta - M_p) \tag{9a}$$

$$m_s - m_d = M_\Lambda - M_p \quad . \tag{9b}$$

Equation (9a) follows from the assumption that the spin splittings on the right-hand side are hyperfine splittings inversely proportional to quark masses because they come from the "color magnetic" force from one gluon exchange. Equation (9b) follows from the assumption that the only flavor dependence in hadron masses comes from the quark mass difference (9b) and the hyperfine splittings (9a). That these hadron mass differences should give such precise values for the mass scale which determines the hadron magnetic moments is very surprising. The possible theoretical implications of this have been discussed.[5]

Recently there have been new measurements of the Ξ^o and Σ^+ magnetic moments which do not fit this simple picture.[6] Both moments are smaller in magnitude by about 15% from the predicted values. One can immediately draw some qualitative conclusions.

1. The discrepancy between predictions and experiment cannot be fixed by adjusting the magnetic moment of the strange quark, because the moments of the Λ, Σ^+ and Ξ^0 depend upon this in very different ways. The value of μ_Λ is exactly equal to μ_s. The value of μ_{Σ^+} depends mainly on μ_u and is very insensitive to μ_s. The value of μ_{Ξ^o} depends roughly equally on μ_d and μ_s. Thus, even if one of the three experimental values for strange magnetic moments is incorrect; the other two cannot be fit by adjusting μ_s.

2. The discrepancies of μ_{Σ^+} and μ_{Ξ^o} both being reduced in magnitude by about the same amount is suggestive. Both these hyperons have only u and s

quarks, which have charges of opposite sign, coupled with spins antiparallel, so that their magnetic moments add. This spin coupling thus gives the maximum possible magnetic moment. Any admixture of a different configuration with a different spin coupling would be expected to reduce the magnetic moment. Thus the deviation from experiment might be explained by configuration mixing, provided that a mixing mechanism is found which does not spoil the good results for the nucleon and Λ.

3. The predictions (7) all assume the same mass scale factor for all baryons. If the mass scale factor depends upon the baryon mass, then heavier hyperons will have larger mass scales and smaller magnetic moments. This would then reduce the Σ^+ and Ξ^o moments relative to the nucleon and Λ.

The obvious modifications of the model giving Eq. (7) are thus seen to push the Σ^+ and Ξ^o moments in the right direction. We must therefore be very careful before jumping to conclusions about any model. With only two pieces of data to fit, and models which go in the right direction anyway, it is too easy to get a fit which is not really significant.

One way to try to stay honest while looking for modifications in the model is to examine models already proposed for other reasons before these new data were available. One finds two candidates, one with configuration mixing[7] and one with a new mass scale.[8]

Configuration mixing of a d-wave into the baryon octet, analogous to the d-wave in the deuteron has been proposed. This mixing has the desirable feature of affecting only the Σ and Ξ, without affecting the nucleon and the Λ. The reason is that unlike the deuteron which has spin one and can have an $L = 2$ admixture without recoupling spins, the baryons with spin 1/2 can only admix $L = 2$ and keep

$J = 1/2$ by recoupling the quark spins to $S = 3/2$. But if the maximum space symmetry is kept, as seems reasonable for attractive forces, then $S = 3/2$ means a decuplet in SU(3). Octet-decuplet mixing is forbidden by SU(3), but expected to take place like the usual octet-singlet mixing as a result of the SU(3) symmetry breaking due to the $m_s - m_u$ mass difference. However, such octet-decuplet mixing can affect only the Σ and the Ξ, which have states of the same isospin in both octet and decuplet. The nucleon and Λ have no counterparts in the decuplet with the same isospin, and therefore cannot be mixed without violating isospin conservation.

Although this mechanism gives a qualitative effect in the right direction for the right states, it is difficult to see how effects as large as 15% can be obtained. Precise quantitative values can be obtained only by specific model calculations, which have not yet been carried out. However, back-of-the-envelope estimates suggest that this effect is still too small.

The other alternative is to suggest that the magnetic moment of a quark of a given flavor has a mass scale which depends upon the mass of the hadron. The nucleon magnetic moments give the magnetic moments of u and d quarks in the nucleon. The Λ magnetic moment gives the moment of the strange quark in the Λ. If we now assume that the mass scale of the magneton in each case is proportional to the mass of the hadron, Eqs. (7d-h) are modified by replacing μ_p on the right-hand side by $\mu_p M_p/M_Y$ and μ_Λ by $\mu_\Lambda M_\Lambda/M_Y$, where M_Y is the mass of the appropriate hyperon, Σ in Eqs. (7d-f) and Ξ in Eqs. (7g-h). With this modification, the predicted value for μ_{Σ^+} is reduced from 2.68 to 2.15, and for μ_{Ξ^o} from -1.43 to -1.13. These new predictions are in agreement within one standard deviation of the new experimental results.

Measurements of other hyperon magnetic moments would give a better insight into the possible mechanism for violation of the simple predictions. In particular, one can examine the following linear combinations of magnetic moments which depend only on either the strange or nonstrange quark contributions

$$\mu_{\Sigma^+} - \mu_{\Sigma^-} = -3\mu_d = (4/3)\mu_p \quad (g^d_\Sigma/g^d_N) \tag{8a}$$

$$\mu_{\Sigma^+} + 2\mu_{\Sigma^-} = -\mu_s = -\mu_\Lambda \quad (g^s_\Sigma/g^s_\Lambda) \tag{8b}$$

$$\mu_{\Xi^o} - \mu_{\Xi^-} = \mu_d = -(1/3)\mu_p \quad (g^d_\Xi/g^d_N) \tag{8c}$$

$$\mu_{\Xi^o} + 2\mu_{\Xi^-} = 4\mu_s = 4\mu_\Lambda \quad (g^s_\Xi/g^s_\Lambda) \tag{8d}$$

$$\mu_{\Omega^-} = 3\mu_s = 3\mu_\Lambda \quad (g^s_\Omega/g^s_\Lambda) \quad . \tag{8e}$$

where $g^f_{H'}/g^f_H$ is the ratio of the magnetic moment of a quark of flavor f in hadron H' to the magnetic moment of the same quark in hadron H. There are a sufficient number of predictions here so that systematics in any disagreements can appear. For example, if the mass scale effect is responsible for the decreased moments of the Σ^+ and Ξ^o, there are a sufficient number of cross checks in the predictions (8) which should be reduced by the same mass factors to definitely prove or disprove this model.

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