

## LESS-DIMENSIONS AND THE ORIGIN OF DARK MATTER

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We extend the concept of matter parity  $P_M = (-1)^{3(B-L)}$  to non-supersymmetric theories and argue that  $P_M$  is the unique explanation to the existence of Dark Matter of the Universe. The argument is general but we motivate it using constraints on GUT particle content from lower-dimensional field theories. The non-supersymmetric Dark Matter must be contained in scalar **16** representation(s) of  $SO(10)$ , thus the unique low energy Dark Matter candidates are  $P_M$ -odd complex scalar singlet(s)  $S$  and inert scalar doublet(s)  $H_2$ . We have calculated the thermal relic Dark Matter abundance of the model and shown that its minimal form may be testable at LHC via the SM Higgs boson decays  $H_1 \rightarrow DM DM$ . The PAMELA anomaly can be explained with the decays  $DM \rightarrow \nu W$  induced via seesaw-like operator which is additionally suppressed by Planck scale. Because the SM fermions are odd under matter parity too, Dark Matter sector is just our scalar relative.

### 1 Introduction

While the existence of Dark Matter (DM) of the Universe is now established beyond doubt<sup>1</sup>, its origin, nature and properties remain obscured. In most models beyond the standard model (SM), such as the minimal supersymmetric SM, additional discrete  $Z_2$  symmetry is imposed by hand to ensure the stability of the lightest  $Z_2$ -odd particle. There is no known general principle for the origin of DM which could discriminate between DM models.

In this Letter we propose that there actually might exist such a common physics principle for the theories of DM. It follows from the underlying unified symmetry group for all matter fields in grand unified theories (GUTs) and does not require supersymmetry. One can classify all matter fields in Nature under the discrete remnant of the matter symmetry group which is nothing but the matter parity  $P_M$ . Thus the existence of DM might be a general property of Nature rather than an accidental outcome of some particular model. As a general result, there

is no “dark world” decoupled from us, rather we are part of it as the SM fermions are also odd under the matter parity  $P_M$ .

We argue that, assuming all matter fields to respect  $SO(10)$ <sup>2</sup>, the gauged GUT group  $SU(5)$ <sup>3</sup> is complemented by an additional discrete symmetry  $Z_n$ . For the simplest case,  $n = 2$ , the GUT symmetry is broken to  $SU(5) \times P_M$  and all the fermion and scalar fields of the GUT theory, including the SM particles plus the right-handed neutrinos  $N_i$ , carry well defined discrete quantum numbers uniquely determined by their original representation of  $SO(10)$ . Therefore non-supersymmetric DM candidates can come only from **16** scalar representations of  $SO(10)$ , and the unique low energy DM fields are new  $SU(2)_L \times U(1)_Y$   $P_M$ -odd scalar doublet(s)  $H_2$ <sup>4</sup> and singlet(s)  $S$ <sup>5,6</sup>.

While our argument is general, we motivate it with an example new physics scenario called less-dimensions. In the less-dimensional scenario new physics effects arise from lower-dimensional quantum field theories (QFT) as opposed to the extra dimensions in which new space dimensions are added. The consistency of field theory in three dimensions implies constraints on the number of fermions and gauge bosons of the theory. The 3-dimensional constraints apply also in 4-dimensions<sup>7</sup> if one space dimension is very small and compactified to a circle, and the number of fermion generations is odd. During inflation the compactified dimension can be expanded as much as needed in order to achieve the present flat and homogeneous Universe. However, particle physics “remembers” the initial conditions. It is interesting that the WMAP observations indeed point to a preferred direction in space<sup>8,1</sup>.

We formulate and study the minimal matter parity induced phenomenological DM model which contains one inert doublet  $H_2$  and one complex singlet  $S$ . We show that the observed DM thermal freeze-out abundance can be achieved for wide range of model parameters. We also show that the PAMELA<sup>9</sup> and ATIC<sup>10</sup> anomalies in  $e^+/(e^- + e^+)$  and  $e^- + e^+$  cosmic ray fluxes can be explained by DM decays to the SM leptons via Planck scale suppressed  $P_M$ -violating seesaw-like operator of the form  $m/(\Lambda_N M_P) LLH_1 H_2$ , where  $m/M_P$  is  $P_M$ -violating heavy neutrino mixing. In this model the SM Higgs boson  $H_1$  is the portal<sup>11</sup> to the DM. We show that for well motivated model parameter values the DM abundance predicts the decay  $H_1 \rightarrow DM DM$ , which allows to test the model at LHC<sup>12</sup>.

## 2 Less-dimensions

To simplify the presentation we first discuss our example scenario and then present the general argument. If the topology of our 4-dimensional space-time is actually  $\mathcal{M}^3 \times S^1$ , *i.e.*, the usual Minkowski space-time with one spatial coordinate compactified to a circle, topological anomalies occur in non-Abelian gauge theories with *odd* number of massless fermion generations in a direct analogy with the corresponding theories in three dimensions<sup>7</sup>. Therefore the consistency of QFT in four dimensions must follow from the consistency of 3-dimensional QFT. In three dimensions there occur topological Chern-Simons terms in total non-Abelian gauge action as well as in the gravitational action which must be quantized<sup>13</sup>. At one loop level, corrections to the gravitational Chern-Simons term require<sup>14</sup>

$$\frac{1}{16} N_F - \frac{1}{8} N_G = 0, \quad (1)$$

where  $N_{F,G}$  is the number of fermions and gauge bosons, respectively. Eq. (1) implies constraints on the number of fermions as well as on the gauge group of the theory.

At early Universe at very high energies the  $\mathcal{M}^3 \times S^1$  space-time topology leads to CPT violation and generates a topological mass for the photon. However, today, after inflation, the

$$m_\gamma \sim 10^{-35} \text{ eV} \left( \frac{\alpha}{1/137} \right) \left( \frac{1.5 \cdot 10^{10} \text{ lyr}}{R} \right). \quad (2)$$

where  $R$  is the radius of the compact spatial coordinate. For the observable Universe,  $R_{\text{obs}}$ ,  $m_\gamma$  is a factor  $\mathcal{O}(10^2)$  smaller than the present experimental bound. Since inflation can generate  $R \gg R_{\text{obs}}$ , the topology of the Universe may remain unknown. Nevertheless, the WMAP results may support this scenario<sup>1</sup>.

### 3 Matter parity as the origin of DM

An immediate consequence of Eq. (1) is that chiral fermionic matter must come in multiples of sixteen. This is in a perfect agreement with experimental data as there exist 15 SM fermions plus right-handed  $N$  for the seesaw mechanism<sup>15</sup>, and fermions of every generation naturally form one  $SO(10)$  multiplet **16<sub>i</sub>**,  $i = 1, 2, 3$ . As a result, Eq. (1) implies that there must be 24 gauge bosons. This is the dimension of adjoint representation of  $SU(5)$  and suggests that  $SU(5)$  is the gauge group of GUT. In that case, if all matter fields, fermions and scalars, respect  $SO(10)$ , the group theoretic branching rule,

$$SO(10) \rightarrow SU(5) \times U(1)_X \rightarrow SU(5) \times Z_2, \quad (3)$$

implies that every  $SU(5)$  GUT matter multiplet carries an additional uniquely defined quantum number under the (global or gauged)  $U(1)_X$  symmetry. The  $U(1)_X$  symmetry is further broken to its subgroup  $Z_n$  by order parameter carrying  $n$  charges of  $X$ <sup>16</sup>. The simplest case  $Z_2$ , which also allows for the seesaw mechanism induced by heavy neutrinos  $N_i$ <sup>15</sup>, yields the new parity  $P_X$  with the field transformation  $\Phi \rightarrow \pm \Phi$ . Therefore the actual GUT group is  $SU(5) \times P_X$ .

Of course, the group theory in Eq. (3) is general and does not necessarily require less-dimensions nor global  $U(1)_X$  because the  $Z_2$  can also be gauged<sup>16,17</sup>. We do not speculate on details of GUT model building here and adopt the breaking chain (3).

Under Pati-Salam charges  $B - L$  and  $T_{3R}$  the  $X$ -charge is decomposed as

$$X = 3(B - L) + 4T_{3R}, \quad (4)$$

while the orthogonal combination, the SM hypercharge  $Y$ , is gauged in  $SU(5)$ . Because  $X$  depends on  $4T_{3R}$  which is always an even integer for  $T_{3R} = 1/2, 1, \dots$ , the  $Z_2$   $X$ -parity of a multiplet is determined by  $3(B - L) \bmod 2$ . Therefore one can write

$$P_X = P_M = (-1)^{3(B - L)}, \quad (5)$$

and identify  $P_X$  with the well known matter parity<sup>18</sup>, which is equivalent to  $R$ -parity in supersymmetry. While  $U(1)_X$ ,  $X = 5(B - L) - 2Y$ , has been used to consider and to forbid proton decay operators<sup>19</sup>, so far the parity (5) has been associated only with SUSY phenomenology.

Due to Eq. (3) a definite matter parity  $P_M$  is the general intrinsic property of every matter multiplet. The decomposition of **16** of  $SO(10)$  under (3) is **16** = **1<sup>16</sup>(5)** + **5<sup>16</sup>(-3)** + **10<sup>16</sup>(1)**, where the  $U(1)_X$  quantum numbers of the  $SU(5)$  fields are given in brackets. This implies that under the matter parity all the fields **10<sup>16</sup>**, **5<sup>16</sup>**, **1<sup>16</sup>** are odd. At the same time, all other fields coming from small  $SO(10)$  representations, **10**, **45**, **54**, **120** and **126**, are predicted to be even under  $P_M$ . Thus the SM fermions belonging to **16<sub>i</sub>** are all  $P_M$ -odd while the SM Higgs boson doublet is  $P_M$ -even because it is embedded into **5<sup>10</sup>** and/or **5̄<sup>10</sup>**, and **10** = **5<sup>10</sup>(-2)** + **5̄<sup>10</sup>(2)**. Although  $B - L$  is broken in nature by heavy neutrino Majorana masses,  $(-1)^{3(B - L)}$  is respected by interactions of *all* matter fields.

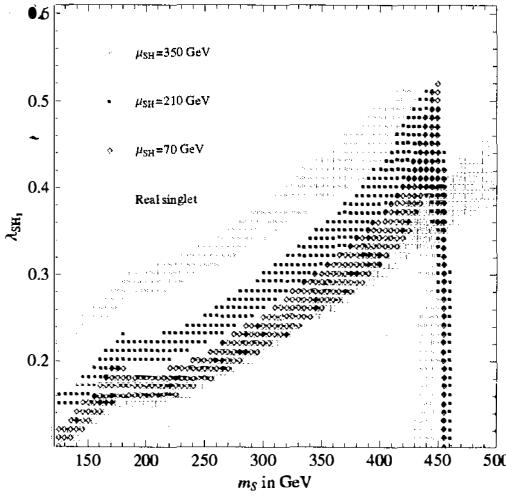


Figure 1: Allowed  $3\sigma$  regions for predominantly singlet DM in  $(m_S, \lambda_{SH_1})$  plane for  $b_S = 5$  GeV,  $m_{H_0} = 450$  GeV.

As there is no DM candidate in the SM, we have to extend the particle content of the model by adding new  $SO(10)$  multiplets. The choice is *unique*, only **16** contains  $P_M$ -odd particles. Adding new fermion **16** is equivalent to adding a new generation, and this does not give DM. Thus we have only one possibility, the scalar(s) **16** of  $SO(10)$ . Because DM must be electrically neutral, **16** contains only two DM candidates. Under  $SU(2)_L \times U(1)_Y$  those are the complex singlet  $S = 1^{16}$  and the inert doublet  $H_2 = \bar{5}^{16}$ .

#### 4 DM predictions of the minimal model

GUT symmetry groups are known to be very useful for classification of particle quantum numbers, and this is sufficient for predicting the DM candidates. Unfortunately GUTs fail, at least in their minimal form, to predict correctly coupling constants between matter fields. Therefore we cannot trust GUT model building for predicting details of DM phenomenology. Instead we study *phenomenological low-energy* Lagrangian for the SM Higgs  $H_1$  and the  $P_M$ -odd scalars  $S$  and  $H_2$ ,

$$\begin{aligned}
 V = & -\mu_1^2 H_1^\dagger H_1 + \lambda_1 (H_1^\dagger H_1)^2 + \mu_S^2 S^\dagger S + \lambda_S (S^\dagger S)^2 \\
 & + \lambda_{SH_1} (S^\dagger S) (H_1^\dagger H_1) + \mu_2^2 H_2^\dagger H_2 + \lambda_2 (H_2^\dagger H_2)^2 \\
 & + \lambda_3 (H_1^\dagger H_1) (H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2) (H_2^\dagger H_1) \\
 & + \frac{\lambda_5}{2} \left[ (H_1^\dagger H_2)^2 + (H_2^\dagger H_1)^2 \right] + \frac{b_S^2}{2} \left[ S^2 + (S^\dagger)^2 \right] \\
 & + \lambda_{SH_2} (S^\dagger S) (H_2^\dagger H_2) + \frac{\mu_{SH}}{2} \left[ S^\dagger H_1^\dagger H_2 + S H_2^\dagger H_1 \right], \tag{6}
 \end{aligned}$$

which respects  $H_1 \rightarrow H_1$  and  $S \rightarrow -S, H_2 \rightarrow -H_2$ . The doublet terms alone form the inert doublet model<sup>4</sup>. To ensure  $\langle S \rangle = 0$ , we allow only the soft mass terms  $b_S, \mu_{SH}$  and the  $\lambda_5$  term to break the internal  $U(1)$  of the odd scalars<sup>6</sup>. Thus the singlet terms in (6) alone form the model A2 of<sup>6</sup>. The two models mix via  $\lambda_{SH}, \mu_{SH}$  terms. Notice that mass-degenerate scalars are strongly constrained as DM candidates by direct searches for DM. The  $\lambda_5, b_S^2$  and  $\mu_{SH}$  terms in Eq. (6) are crucial for lifting the mass degeneracies.

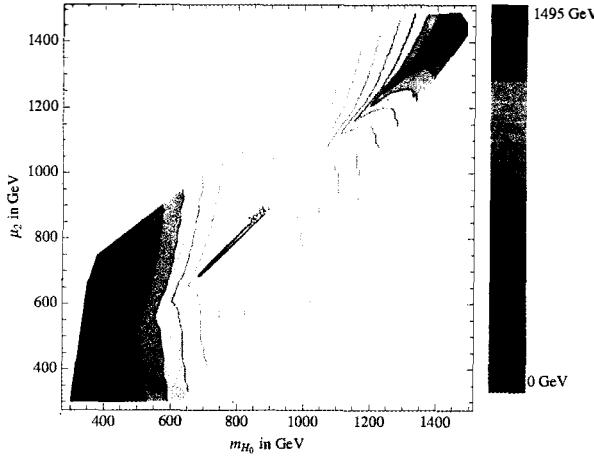


Figure 2: Allowed  $(m_{H_0}, \mu_2)$  parameter space for  $\mu_{SH} = 0$  and different values of  $m_S$  represented by color code.

In the following we assume that DM is a thermal relic and calculate its abundance using MicrOMEGAs package<sup>20</sup>. The DM interactions (6) were calculated using FeynRules package<sup>21</sup>. To present numerical examples we fix the doublet parameters following Ref.<sup>22</sup> as  $m_{A_0} = m_{H_0} = 10$  GeV,  $m_{H^+} - m_{H_0} = 50$  GeV and treat  $m_{H_0}$  and  $\mu_2$  as free parameters. For predominantly singlet DM we present in Fig. 1 the allowed  $3\sigma$  regions in the  $m_S^2 = \mu_S^2 + \lambda_{SH_1} v^2/2 - b_S^2$  and  $\lambda_{SH_1}$  plane for  $b_S = 5$  GeV,  $m_{H_0} = 450$  GeV and the values of  $\mu_{SH}$  as indicated in the figure. For comparison we also plot the corresponding prediction of the real scalar model (light green band). For those parameters the observed DM abundance can be obtained for  $m_S < m_{H_0}$ . Due to the mixing parameter  $\mu_{SH}$ , large region in the  $(m_S, \lambda_{SH_1})$  plane becomes viable.

To study DM dependence on doublet parameters we present in Fig. 2 the  $(m_{H_0}, \mu_2)$  parameter space for which the observed DM abundance can be obtained. Values of the singlet mass are presented by the colour code and we take  $\mu_{SH} = 0$ ,  $b_S = 5$  GeV. Without singlet  $S$ , in the inert doublet model<sup>22</sup>, the allowed parameter space is the narrow region on the diagonal of Fig. 2 starting at  $m_{H_0} \approx 670$  GeV. In our model much larger parameter space becomes available.

## 5 PAMELA, ATIC and FERMI data

PAMELA satellite has observed a steep rise of  $e^+/(e^- + e^+)$  cosmic ray flux with energy and no excess in  $\bar{p}/p$  ratio<sup>9</sup>. ATIC experiment claims a peak in  $e^- + e^+$  cosmic ray flux around 700 GeV<sup>10</sup>, a claim that will be checked by FERMI satellite soon. To explain the cosmic  $e^+$  excess with annihilating DM requires enhancement of the annihilation cross section by a factor  $10^{3-4}$  compared to what is predicted for a thermal relic. Non-observation of photons associated with annihilation<sup>23</sup> and the absence of hadronic annihilation modes<sup>24</sup> constrains this scenario very strongly. However, the PAMELA anomaly can also be explained with decaying thermal relic DM with lifetime  $10^{26}$ s<sup>25</sup>, 3-body decays in our case.

In our scenario the global  $Z_2$  matter parity can be broken by Planck scale effects<sup>16</sup>. If there exists, at Planck scale, a  $SO(10)$  fermion singlet  $N'$ , its mixing with the  $SU(5)$   $P_M$ -odd singlet neutrinos  $N$  via a mass term  $mNN'$  breaks  $P_M$  explicitly but softly. The exchange of  $N$  now induces also a seesaw-like<sup>15</sup> operator

$$\frac{\lambda_N}{M_N} \frac{m}{M_P} LLH_1H_2 \rightarrow 10^{-30} \text{GeV}^{-1} \nu l^- W^+ H_2^0, \quad (7)$$

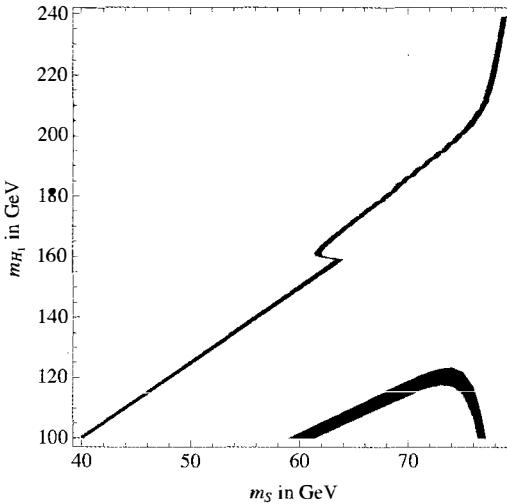


Figure 3: Allowed  $3\sigma$  regions in the singlet DM and SM Higgs boson mass plane for  $\mu_S = 0$  and  $b_S = 5$  GeV.

where we have taken  $\lambda_N \sim 1$ ,  $M_N \sim 10^{14}$  GeV and  $m \sim v \sim 100$  GeV. Such a small effective Yukawa coupling explains the long DM lifetime  $10^{26}$ s.

### 3 LHC phenomenology

In our scenario the DM couples to the SM only via the Higgs boson couplings Eq. (6). Therefore, discovering  $\sim 1$  TeV DM particles at LHC is very challenging. However, if DM is relatively light the SM Higgs decays  $H_1 \rightarrow DM DM$  become kinematically allowed and the SM Higgs branching ratios are strongly affected. Such a scenario has been studied by LHC experiments<sup>12</sup> and can be used to discover light scalars.

In our model such a scenario is realized for  $\mu_S = 0$ , small  $b_S \ll v$  and heavy doublet. In this case the DM is predominantly split singlet and, in addition, the DM abundance relates the DM mass  $m_S^2 \approx \lambda_{SH_1} v^2/2 + b_S^2$  to the SM Higgs boson mass  $m_{H_1}$ , as seen in Fig. 3. For  $m_{H_1} = 120$  GeV,  $b_S = 5$  GeV we predict  $m_S = 48$  GeV with the Higgs branching ratios  $BR(H_1 \rightarrow b\bar{b} + c\bar{c} + \tau\bar{\tau}) = 14.2\%$ ,  $BR(H_1 \rightarrow DM DM) = 42.4\%$  and  $BR(H_1 \rightarrow S_2 S_2) = 42.4\%$ . The second heaviest singlet  $S_2$  with the mass  $m_{S_2}^2 \approx \lambda_{SH_1} v^2/2 + b_S^2$  decays via the SM Higgs exchange to  $S_2 \rightarrow DM \mu\mu$  or  $S_2 \rightarrow DM c\bar{c}$  with almost equal branching ratios. Thus the SM Higgs boson decay modes are very strongly modified. This makes the  $H_1$  discovery more difficult at LHC but, on the other hand, allows the scenario to be tested via the Higgs portal<sup>11</sup>.

### Conclusions

We have extended the concept of  $Z_2$  matter parity,  $P_M = (-1)^{3(B-L)}$ , to non-supersymmetric GUTs and argued that  $P_M$  is the unique origin of DM of the Universe. Assuming that  $SO(10)$  is the matter symmetry group, the matter parity of all  $SU(5)$  GUT matter multiplets is determined by their  $U(1)_X$  charge under Eq. (3). We have motivated this scenario with the constraint Eq. (1) from lower-dimensional effective field theories but our argument is general. Consequently, the non-supersymmetric DM must be contained in the scalar representation **16** of  $SO(10)$ . This implies that the theory of DM becomes completely predictive and the only possible low energy DM candidates are the  $P_M$ -odd scalar singlet(s)  $S$  and doublet(s)  $H_2$ . We have calculated the

DM abundances in the minimal DM model and shown that it has a chance to be tested at LHC via Higgs portal. Planck-suppressed  $P_M$  breaking effects may occur in the heavy neutrino sector leading to decays  $DM \rightarrow \nu l W$  which can explain the PAMELA and FERMI anomalies.

Our main conclusion is that there is nothing unusual in the DM which is just scalar relative of the SM fermionic matter. Although  $B - L$  is broken in Nature by heavy neutrino Majorana masses,  $(-1)^{3(B-L)}$  is respected by interactions of all matter fields implying stable scalar DM.

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